OPTIMIZING NETWORK DESIGN: DETERMINISTIC VS. PROBABILISTIC TRAFFIC SIMULATION (ASSIGNMENT)

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Abstract

The Optimization of Transport Network Design belongs to the area of non-linear optimization and mathematical programming within the field of Operations Research. The elegant inclusion of Traffic Simulation Methods within the Network Design Problem improves modeling realism, in terms of actual human travel behavior, for purposes of regional development analysis. Actual evaluation of existing and widely recognized traffic simulation methods, in real large-sparse transport networks, is presented in this paper.

Keywords
Optimal Network Design, Traffic Simulation Methods, OR.

1. Introduction

The Transport Network Design problem is mathematically quite complex but substantial for regional economic development planning. Thus, the *transport network traffic flow constraints* involved, for both mathematical optimization and traffic simulation modeling are based on the Maxwell – Kirchhoff theory of current distribution in electrical circuits. In this article, we follow the principle that designing networks is immaterial, unless traffic flow considerations are taken into explicit account (Athanasenas, 1991, 1997).

Here, then, we emphasize on the following subjects, as follows. Section I is a short summary on Network Design. Section II presents a short reference on the very basics of Deterministic and Probabilistic Traffic Simulation (Assignment) methods and their shortcomings. Last, Section III, describes some very basic results on optimizing network design by commenting on *actual traffic assignment applications.*

2. Section I: Modeling Network Design.

The *optimum* network design problem is based on improving road network links and traffic flows simultaneously, so as to minimize the total cost of transportation, and *not to minimize simply travel time.* The intrinsic objective is to assign traffic, and thus design the regional urban highway and combined rural road network by eliminating and/or adding links, in a way that we get minimum travel cost and minimum link maintenance and investment cost(s). Hence, the fundamental aim is to apply the *steady-state optimal transport network design formulation*, in order to develop a regional[1] cost-effective prioritization scheme in order to guide economic investment decisions about transport network improvements. These kinds of measures direct investments and research towards establishing the regional economic basis for national development (Athanasenas, 1991, 1997).

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1. Regional is one aspect, but not the only one! Citywide specific networks or national are also possible schemes.

Consequently, we must start our analysis from the *classic* methodology on network improvements, such as introduction of new network links and decrease of free-flow travel time on the *existing links* by following Dantzig *et al.,* 1979. Next, we *continue here* by following Dantzig *et al., (1979)*, and Steenbrink (1974a, and b), by optimizing users' travel and investments cost(s) (i.e.: the total transport costs) using traffic simulation methods instead of Linear Programming, given an exogenous origin-destination traffic demand (trip matrix of users). We apply this approach by choosing link dimensions D_{ij} on link *ij,* and traffic flows f_{ij}, with d_{ij} the investment decision on link ij. Let then $C_{ij}\ (d_{ij})$ be the convex function describing the investment cost associated with the investment decision d_{ij}.

Thus, the conventional total transport cost (total user travel cost $F_{ij}\ (x_{ij},\ d_{ij})$, plus the network improvement costs $C_{ij}\ (d_{ij})$ for link *ij*), is expressed as:

$$F_{ij}(x_{ij},\ d_{ij}) + C_{ij}(d_{ij}) \tag{1}$$

as an additive-separable objective function; where, is the conversion factor, in corresponding terms.

Given the total traffic flow on each link ij, estimated by the trip matrix (origin-destination O – D matrix), the network design problem can, then, be stated as follows: determine the investment decisions *dij* and the total traffic flows on origin-destination nodes a, b, with a, b S (defined below), for the aggregate system (i.e.: for each traveler, all nodes, and vehicle types), so that the system's total transport cost is minimized as:

$$Min \sum_{d_{ij},x_{ij}^{ab}} \{F_{ij}(x_{ij},d_{ij}) + \lambda C_{ij}(d_{ij})\} \tag{2}$$

$$\forall i, j \in L,\ \forall a,b \in S$$

with respect to $d_{ij},\ x_{ij}^{ab}$.

Equation (2) is then minimized with respect to the classic conservation of flow (COF) constraints for each node j and each O – D pair $a,b \in S$, as:

$$\sum_i x_{ij}^{ab} \nwarrow \sum_k x_{jk}^{ab} \; {}^{ab}_{jk} \;=\; \begin{cases} 0 & \forall j \neq a,b \\ -x_{\upsilon}^{ab} & \text{if } \; j=a \\ x_{\upsilon}^{ab} & \text{if } \; j=b \end{cases} \tag{3}$$

$$\forall i,j \in L,\ \forall a,b \in S,\ x \in N$$

and, subject to the total flow in each link being equal to the sum of the flows from the sources as:

$$x_{ij} = \sum_{a=1}^{a=0} x_{ij}^{ab} \qquad (i, j \in L) \tag{4}$$

and, upper-lower bounds on the maximum investment possible,

$$L_{ij} \leq d_{ij} \leq P_{ij} \quad (i, j \in L), \tag{5}$$

and, non-negativity constraints on traffic flows as,

$$x_{ij} \geq 0 \quad (i, j \in L), \tag{6}$$

where: L = set of links in the network,
N = set of nodes in the network,
S = number of origin nodes,
x_{ij}^{ab} = flow on link ij from origin (a) to destination (b),
O_{ij} = number of trips made from node i to node j (link i, j), i.e.

$$O_{ij} = \begin{cases} 0 & \forall j \neq a,b \\ -x_{\upsilon}^{ab}, & \text{if } (j=a),\ \forall \upsilon \in N \\ x_{\upsilon}^{ab}, & \text{if } (j=b),\ \forall a,b \in S, \end{cases}$$

and, d_{ij} = investment decision for link ij;
L_{ij} = minimum value for investment decision for link ij;
P_{ij} = maximum value for investment decision for link ij;
O_p = set of links originating at node p N;
D_p = set of links terminating at node p N;

The system can be solved by employing any continuous convex functions
$F_{ij}(.,.)$, $C_{ij}(d_{ij})$.

Now, for a given value of d_{ij}, the objective function (2) minimizes the total transport costs, subject to the conservation of flow constraints (COF) based on the foundations of the Maxwell-Kirchhoff theory of current distribution in electrical networks.

The above formulation has been investigated by Steenbrink (1974) in the Dutch Integral Transportation Network Study. Steenbrink's decomposition method placed particular emphasis on a normative modeling setting, equivalent to the traffic assignment problem, while optimizing all social transport costs to society. The approach begins by defining as $D_j(x_j)$ the optimal investment decision for link j, and flow x_j, and $F_j(x_j)$ the minimum travel and investment costs, while estimating them $(D_j(.), \& \ F_j(.))$ as:

$$F_j(x_j) = \min_{x_j}\left[f_j(x_j, d_j) + \lambda C_j(x_j) \right] \tag{7}$$

subject to:

$$L_j \leq d_j \leq P_j, \tag{8}$$

where L_j, P_j, d_j, $C_j(.)$, are defined before.

Cases (7) and (8) here, represent a sub-problem to be solved *for each link j,* while their solutions compose the master problem, which is to compute the total flow in each link j.

Dantzig et al. (1979) investigated further three cases of which the first was the application of a BPR[2] curve with change in link capacity, for the $F_j(.,.)$ function. Specifically, Dantzig et al. (1979) applied the so-called BPR curve to

2. See below for details concerning the BPR curve.

model travel time on a link as a function of flow on the link as:

$$T_j(f_j) = t_j f_j \left[1 + r\left(f_j / c_j\right)^k\right],$$

where:

$T_j(f_j)$= total travel time for all users on link j,
f_j= flow on link j,
t_j = free-flow travel time parameter for link j,
c_j = capacity parameter for link j,
r= constant,
k= constant.

Steenbrink (1974) used a similar function in his work of road investments in the Netherlands. The optimum design problem is based on improving network parts and traffic-flows simultaneously, so as to minimize the cost of transportation. The user-optimum network design problem is based on the same objective, but the travel cost is always evaluated at the user-optimum flows x_{ij}^*, corresponding to the proposed improvements of the network. That is, in the user-optimum case the computational difficulties arise since network improvements represent the decision variables, while network flows are determined whenever improvements are chosen by a traffic assignment method, which finds the user-optimal flows x_{ij}^* *(.).* (Dantzig *et al.,* 1979; Steenbrink, 1974b).

The linearity of the objective function on network improvements remains the acceptable solution that guarantees decreasing marginal costs and convexity of the optimization problem.

3. Section II: Traffic Simulation (Assignment) Methods

II.1: Equilibrium Traffic Simulation: A Note.

Our principle in optimizing network design is the need for accurate modeling of network users' travel behavior. The latter requires a specialized description of each user's route choice in accordance with the other network users. Essentially, traffic simulation (assignment) models can be classified in two distinct categories: (a) deterministic equilibrium, where their accepted postulate about the equilibrium notion is that of Wardrop's First Principle (Wardrop, 1952); and (b) probabilistic oriented multi-path traffic assignments where transport network users are assumed to select non-minimum cost routes, such as Dial's (1971) model.

The equilibrium traffic simulation (assignment), in terms of an algorithmic estimation structure, embodies a set of iterations of *"All-or-Nothing"- "AON"* traffic assignments, with an internal travel time adjustment, reflecting the relationship between the assigned traffic volumes versus the road practical traffic capacity. Equilibrium traffic assignments are multi-path, representing a sequence of linear combinations of "All-or-Nothing" traffic assignments.

The equilibrium process, in its general form (Eash, W. R.; Janson, N. B.; and Boyce, E. D.; 1979), can be described as follows:

STEP1: Let VOL1 (1, LINKS) be a set of initial link volumes (VOL), for a set of one way links (LINKS), in the transport network.

STEP2: Calculate the link impedances (time, distance, or cost), as congestion delays, and denote them as IMPEDANCE (VOL1(1, LINKS)).

STEP3: Perform an "All-or-Nothing" ("corridor") assignment, to obtain a new set of assigned link volumes, as VOL2 (2, LINKS).

STEP4: Combine *STEP2* with *STEP3* volumes, by a Fractional constant (k), as:
VOL3 (2, LINKS) = (1.0 – k)*VOL1(1, LINKS)+
+k*VOL2(2, LINKS)
with 0.0 < k < 1.0

STEP5: Use VOL3 (2, LINKS) to estimate the new link Impedances, IMPEDANCE (VOL3 (2, LINK)).

STEP6: Continue the iteration process, until a ratio of travel hours during iterations 1-to-I-1 over travel hours iteration I is achieved.

The time adjustment (travel time) is achieved through the use of the *standard BPR*[3] road link capacity formula, as:

$$T_i = T_{i-1} \left[1.0 + 0.15 \left(\frac{\text{ASSIGNED TRAFFIC VOLUME}}{\text{PRACTICAL LINK CAPACITY}}\right)\right]$$

for the i^{th} iteration[4].

II.2: Probabilistic Traffic Simulation: A Note.
Although efficient trip makers travel in a manner that a shorter path should have the higher probability of use, some travelers use paths which are not cost-optimal due to *inaccurate*[5] *information or habitual preference.* The most common probabilistic traffic assignment for such modeling is Dial's (1971) version, which attempts to circumvent the path enumeration problem and its computational cost. Even though probabilistic assignments lead to questionable model stability in highly congested networks, such a problem is generally avoided in large-sparse rural transport networks and highways, as it is the case with the empirical application of this study in Polk County Minnesota, in the early 90's (Athanasenas, 1991-92).

The Dial type of probabilistic traffic assignment is a logit-modeling scheme, which never examines a path by explicit enumeration, by assigning expected volume of trips to each alternative link based on diversion probabilities. Dial's proposed model is an untested hypothesis (Dial 1971). Probabilistic modeling in large-sparse rural transport networks is empirically untested thus far, and the first application of this method in comparison to the deterministic equilibrium traffic assignment approach has given some very substantial results (Athanasenas, 1997), very briefly presented in Section III.

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3. BPR is the letter initials for "Bureau of Public Roads".
4. In summary, the equilibrium notion developed above could be envisaged as an aggregate time-depended outcome of individual decisions, such that no single driver can further reduce cost of travel by choosing another alternative route in the network. This definition implies two fundamental premises while simulating traffic movement, and thus optimizing network design. See Athanasenas, 1991, pg: 78-79, for details.
5. Inaccurate or asymmetric information, as it stands in pure economics.

Consequently, a managerial network design strategy will require a transport network baseline scheme, with the main purpose to describe the base-case of current "as-it-is" traffic flow pattern, and link maintenance and investment cost(s). Hence, the baseline network solution is not the least-cost routes (as generalized minimum paths). *The baseline-case need be the existing traffic flow scheme, and as such need be modeled under stochastic configurations.*

The general strategy of Dial's probabilistic simulation is to avoid explicit path enumeration for computational efficiency and applicability. The *origin-destination* trips can be assigned, given all the link diversion probabilities for all network nodes. Consequently, the algorithmic procedure is proposed as follows (Dial, 1971; Athanasenas, 1991, for detailed applications in actual modeling):

Algorithm 1: Multi-path Probabilistic Assignment

STEP0: Given origin node 0, and destination node d, let y trips to be assigned, when it is required to know:

- i: p(i) = the shortest path distance from 0 to i node.
- ii: q(i) = the shortest path distance from i to d.
- iii: I_i = the set of all links with initial node i.
- iv: Fi = the set of all links with final node i.

Let link e = (i,j) with length (i,j). Then, link likelihood can be designed as:

$$a(e) = \begin{cases} e^{\vartheta[p(j)-p(i)-t(i,j)]} \quad \textit{if} \quad p(i)<p(j), \text{ and } q(j)<q(i), \\ 0, \text{ otherwise.} \end{cases} \tag{1}$$

STEP1. (Forward). Examine all nodes i in ascending order with respect to p(i), and for each link e in I_i calculate its weight, as:

$$w(e) = \begin{cases} a(e) & \text{if } i=0 \text{ (origin)} \\ a(e) \sum_{e' \in F_i} w(e') & \text{otherwise,} \end{cases} \tag{2}$$

when d is reached, go to:

STEP2. (Backward). Start at d, examine all nodes j in descending order with respect to p(j). Assign trip volume x(e) to each link e in F_j as:

$$x(e) = \begin{cases} y\, w(e) / \sum_{e' \in F_j} w(e'') & \text{if } j = d \text{ destination,} \\ w(e) \sum_{e' \in I_i} x(e') / \sum_{e' \in F_j} w(e') & \text{otherwise.} \end{cases} \tag{3}$$

when the origin node 0 is reached, stop. The probabilistic assignment is complete.

PROOF: Assume that the probability of using path P is directly proportional to the product of the likelihood of the links in the path P, i.e.:

$$\text{Prob}(P) = k \prod_{e \in P} a(e) \tag{4}$$

Thus, Prob(P) $\neq 0$, iff P is efficient. The Functional Specification (i) is completed[6]. Substituting (1) into (4), we get:

$$\text{Prob}(P) = k \exp \vartheta [p(d) - \sum_{(i,j) \in P} t(i,j)] \tag{5}$$

ϑ is a positive constant (parameter), that permits the model's user to affect diversion probabilities, and thus satisfy Specification (iv). Hence, since p(d) is the shortest path, the above model satisfies the Functional Specifications (ii) and (iii).[7]

6. See the following note for details concerning the Functional Specifications.
7. A probabilistic multi-path traffic assignment model should conform to a set of Functional Specifications (Dial, 1971, pp: 89-92): (i) Reasonable Paths-Efficient Paths (All reasonable travel paths should be given a non zero probability of use); (ii) Equal Path- Equal Volume (Reasonable paths of equal length should have an equal probability of use); (iii) Longer Path-Less Volume (Among several alternative reasonable paths, the shorter paths have higher probability of use, assuming that the number of efficient travelers is the maximal in the set of all travelers); (iv) Diversion Probabilities (that is probabilities of a given path of length L being used, can be controlled by the modeler); and, (v) Algorithmic Efficiency and Computational Feasibility (which guarantee applicability, require non-enumeration of travel paths).

In order to prove that Specification (v) is also satisfied, the algorithm must divert trips from each node according to condition link probabilities, as:

$$\text{Prob}[(i,j)|j] = \text{prob}\ [(i,j)]/\text{prob}\ (j) \qquad (6)$$

Now, let i: P_i= (all links topologically preceding link (i,j)),
ii: P_j= (all links topologically following link (i,j)).
Let R_i the family of P_i, R_j the family of P_j, then, equation (6) becomes:[8]

$$\text{Prob}[(i,j)|j] = \frac{a(i,j)\sum_{p\in R_i}\prod_{e\in P} a(e)}{\sum_i [a(i,j)\sum_{p\in R_i}\prod_{e\in P} a(e)]} \qquad (7)$$

Hence, $\text{Prob}\ [e|j] = w(e) / \sum_{e'\in F_j} w(e'),$

where $y(j) = \sum_{e\in I_j} x(e)$ (8),

All trips from origin 0 to destination d, are passing through node j, QED.[9]

4. Section III: Basic Results on Network Design Optimization.

With the co-operation of the Minnesota DOT[10] and Polk County DOT Officials, a specialized Research Project Staff at The University of Minnesota, Department of Applied Eco-

8. See Dial, 1971, pg: 98, for details.
9. Thus, even though travelers' behavior is modeled with preference to the minimum travel cost route choice, probabilistic simulation (assignment) allows for non-minimum cost path route selection. This type of traffic assignments presumes that : (I) Not all travelers are strictly travel cost minimizers; (II) Full information on travel cost, and minimum travel paths is rejected; (III) Alternative traveling preferences are assumed existing. See Athanasenas, 1991, for additional details and explanations in actual modeling applications.
10. DOT stands for Department of Transportation, in US.

nomics and myself, an area of 600 miles^2 and approximately a dozen townships was divided into a *large-sparse* transport network of 186 zones, 1676 nodes and1778 links to be studied extensively, for alternative road investment, abandonment, maintenance and restructuring strategies[11]. That is a pioneering study on managerial strategies focusing on (a) mathematical optimization of actual road network design, (b) evaluation, and proposed improvement of traffic simulation methods, (c) evaluation of actual County proposed road investment plans, within the overall County Five Years Investment Planning, (d) evaluation and proposed improvement of existing specialized traffic simulation computer packages, and clearly, (e) regional development analysis, has provided some substantial results.

Pending upon the *mathematical subtleties orientation* of this paper, we emphasize here on some very crucial *and mathematically oriented sound* results.

Thus, by employing an extensive "sensitivity analysis" on the diversion parameter ϑ [12] of the Dial type Probabilistic Traffic Simulation (Assignment) Method, we found that by increasing this diversion parameter from 0.2 up to 10.0 inclusive, the specific probabilistic traffic assignment diverts trips from gravel type roads towards paved road types. This indicates *the* "pseudo-deterministic" or "corridor-like" – "All-or-Nothing" (AON) simulation behavior of the model employed for very high diversion parameters.

That is, by increasing sequentially the diversion parameter, the probabilistic traffic simulation performs *as if it were a pseudo-deterministic* traffic simulation, *after the value of 10.0.* These results indicate that by *minimizing the total travel-maintenance-&-investment costs*[13]*, the famous*[14] *Dial's probabilistic traffic simulation technique performs as a "pro-*

11. Specific references are Athanasenas, 1991, 1997.
12. Check, ϑ , as the positive constant (parameter) before, within the multipath probabilistic assignment (simulation).
13. That is, travelers' travel costs, and road & bridge maintenance and alternative specialized costs are all estimated in composite objective functional forms (Athanasenas, 1991, 1997).
14. It is widely used in Business, Consulting, Public Investments, City Planning, etc.

portional" rather than a purely stochastic traffic assignment method, which seems to be the outcome of its "logit" functional structure[15]*.* Thus, by employing a parameter value higher than 10.0, we end up simulating total regional traffic flows under *almost* full deterministic configurations. Whereas, by employing the implemented research *value of 0.2, we perform a traffic simulation under a "wider" non-purely deterministic travel route selection configuration, which conforms to a relatively conservative set of only minimum travel cost route selection. That is, given the "logit" functional structure of Dial's multi-path traffic assignment method, which does not satisfy the purely stochastic travel route selection, the parameter value of 0.2 supports the wider possible non-minimum travel route choice.*

Consequently, the wider trip diversion becomes necessary if we need to model more appropriate "reasonable" route paths. The "reasonable" route paths are those with the non-zero probability of use beyond the minimum travel cost "AON"- *strictly deterministic traffic routes (Dial, 1971, Athanasenas, 1991, 1997) and much better satisfy the regional business and economic development needs.*

Hence, despite some shortcomings of Dial's multi-path probabilistic assignment due to its fundamental dependence on the "independence of irrelevant alternatives axiom"[16], *its immediate advantage over the Wardrop's "AON"-purely deterministic traffic assignment method*[17] *is found to be the selection of alternative reasonable route paths essential to serve regional travel-and-business needs.*

Conclusively, with respect to the modeling and efficient solution of actual problems in transportation planning, with special focus on regional economic development analysis, we can summarize the following:

(a) The estimation of equilibrium traffic flows in urban, and especially rural areas above, is based on the powerful in-

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15. You may consult, Florian and Fox, 1976, Athanasenas, 1991, 1997, for details and applications.
16. See for details, Florian and Fox, 1976, Daganzo and Sheffi, 1977, Athanasenas, 1991, 1997.
17. Check Wardrop, 1952, for early details and theoretical postulates on equilibrium – deterministic traffic system simulation methods.

struments of non-linear optimization and mathematical programming within the field of *Operations Research.*

(b) The "modeling" of future land-use regional plans and future highway urban and combined rural transport networks depends on more elegant methods of evaluation, such as *traffic simulation models*, than "direct" Linear Programming approaches[18].

(c) Dial's *probabilistic logit model* of traffic assignment (simulation) is a strictly convex program in both the link and route flow variables, and thus it has the advantage over the deterministic assignment problem of providing *only unique route flows.* Dial's model has been proved clearly superior to "All-or-Nothing" single pass deterministic traffic assignment in *actual large-sparse transport networks, as it is shown above.*

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18. LP is a different modeling principle. A Linear Programming Formulation cannot capture the very essential effects of restraint capacity, the possible congestion considerations, and the travel time adjustments, which are of very important value in estimating the simulated travel time costs, and the resulting optimal traffic flows. Hence, the model's tractability is constrained to an "optimized" traffic flow under minimum travel cost, while the specific traffic considerations remain completely unexplored.

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