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ΟΙΚΟΝΟΜΙΚΩΝ ΚΑΙ ΚΟΙΝΩΝΙΚΩΝ ΕΠΙΣΤΗΜΩΝ

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Thesis

**IMMIGRATION AND UNEMPLOYMENT:  
A SEARCH AND MATCHING APPROACH**

Επιβλέπων Καθηγητής: Παλυβός Θεόδωρος

Εξεταστής Καθηγητής: Κατρανίδης Στέλιος

Επιμέλεια Εργασίας: Πουρνάζης Κωνσταντίνος Α.Μ: 03/05

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## Contents

1. Introduction.....	1
2. Migration in macroeconomic (or not) models.....	4
3. Theories of Unemployment.....	9
3. 1. Implicit-Contract Theory.....	9
3. 2. Efficiency-wage models.....	11
3. 3. Search and Matching Theory.....	13
4. The Basic Model.....	18
5. Numerical Solution.....	27
6. Effects of Government Policies on Unemployment and Welfare.....	29
6.1. Increase in border enforcement.....	30
6.2. Increase in internal enforcement.....	31
6.3. Government funding towards firms.....	32
6.4. Effects of a change in the unemployment benefit.....	33
6.5. Different bargaining power between natives and immigrants.....	34
6.6. Effects of a change in the entry fee.....	35
7. Conclusion.....	36
Appendices.....	38
References.....	40

## **1. Introduction**

Immigration is emerging as a pivotal issue—like race, taxes, and crime—that defines political conflict over the basic values of our society. Illegal migrants are present in almost every country's workforce. A large part of the existing literature, like the paper of Harris and Todaro (1970), highlight a negative effect of immigration in the condition of natives, especially in their welfare and their unemployment rate. Thus, it is clear that in order to improve its welfare, the host-country's government must apply a variety of policies in order to prevent illegal migrants from entering the country and increase the host-country's welfare while reducing the unemployment rate. In order to achieve a sufficient analysis of the impact illegal migrants have into the native's unemployment rate, we must combine immigration policy in an unemployment theory context.<sup>1</sup>

But looking thoroughly almost in any economy and almost at any time we will find that there is unemployment. This unemployment pool consists of people who remain unemployed, but who desire to work in a job similar to that held by another identical worker at any time, receiving exactly the same wages with them.

From macroeconomic perspective, we should examine whether existing unemployment is a feature of non-clearing markets. This market failure might arise either due to friction in the matching process or due to the non-Walrasian features of the economy. Alternatively, we should study the labor market and its cyclical behavior. It seems that the real wage is only moderately procyclical. So, on the one hand, the labor market is a Walrasian one only if the shifts in labor supply are crucial to employment fluctuations.

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<sup>1</sup> Although, the idea that immigrants have negative influence in the host-country's welfare has been accepted by many economists, due to a variety of social, political and economic factors, many countries find it unnecessary to spend more money to search for, deport or punish illegal migrants. Thus, their presence in the host-country is ignored by enforcement authorities (both of internal and border jurisdiction) and they are employed openly in selected industries.

On the other hand, the labor market is not a Walrasian one when the most important role in defining its cyclical behavior is played by its non-Walrasian characteristics.

Considering that we have unemployment in a Walrasian market, to reach equilibrium unemployed workers must have lower demands in their wages. Following Romer (2001), firms' reaction to such an offer can be summarized in the following four approaches:

- i. *The firm can accept this offer and employ the unemployed worker.*
- ii. *The firm wants to cut wages, but it is bounded by a contract.<sup>2</sup>*
- iii. *The firm can refuse to reduce wages.<sup>3</sup>*
- iv. *The firm doesn't believe that unemployed workers are identical to the employed ones.<sup>4</sup>*

Here, in our paper we work in a search and matching context, where firms and workers search for each other aiming to form a pair. When something like this is happening, a surplus arises, which in turn must be shared among both parties. The sharing follows that bargaining process and through its Nash solution.

Our novelty in the literature lies into the fact that we introduce illegal migration into a similar search and matching approach. In this context, firms face a dilemma about which of the two different groups of workers should be employed. Their wage differences play an important role in the firms' decision. From the workers' point of view, their bargaining power lies into their "outside options", i.e. their valuation of time, unemployment benefits etc. Our analysis illustrates the influence the migrant group has into the unemployment rate and the vacancies of the host country.

In this paper, we will follow the analysis of Ortega (2000) who uses a dynamic two-country economy in order to show that immigration has a

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<sup>2</sup> For a further analysis into the "implicit contract theories" see Baily (1974), Gordon (1974) and Azariadis (1975).

<sup>3</sup> For further analysis in the literature of "efficiency-wage models" see Yellen (1984) and Katz (1986). An important paper in the literature is also that of Shapiro and Stiglitz (1984) and its extension by Kimball (1994).

<sup>4</sup> For representative reading, see Diamond (1982a, 1982b), Pissarides (1985) and Hosios (1990).

positive effect on natives' wages and unemployment rate. Here, we extend his analysis by examining specific policy changes and how do these affect our models' variables as well as the host-country's welfare condition.

This paper is structured as follows. Section 1 is used as an introduction of what exactly is going to follow. Section 2 gives an extensive literature review concerning theories of unemployment, which is the exact cases of labor market which deviate from the Walrasian analysis, like implicit contract theory, efficiency wage theory and search and matching theories. Section 3 gives a descriptive review concerning matters of migration in a variety of models, macroeconomic or not. Section 4 provides an analysis of our model and its most important equations. Section 5 provides the solution of the model's simplified version in order to derive some analytical results. Section 5 gives all the aspects of the model using calibration, while Section 6 illustrates some policy measures applied by the host-government and its implications. Section 7 provides some concluding remarks and finishes the analysis. In the end of the paper there is also an Appendix and the reference list.

## **2. Migration in macroeconomic (or not) models**

The impact of immigration on the host economy is discussed in many countries. Borjas (1994) stated that this discussion focuses in three key questions

- i. How do immigrants perform in the host country's economy?
- ii. What impact do immigrants have on the employment opportunities of natives?
- iii. Which immigration policy benefits the most the host country?

Highly productive migrants can have positive effects on the economic growth of the host while less productive migrants increase the costs concerning income maintenance programs. Depending on the worker's impact on the host economy each country chooses the best policy measures in order to support or discourage migration

The beginning, concerning migration in the labor market, was done with the literature on the rural-urban migration in less developed countries. Three of the most representative papers concerning rural-urban migration are those of Todaro (1969, 1976) and Harris and Todaro (1970). Their central idea is that individuals migrate from rural to urban areas because of their need to improve their well-being. In this case, migration results from rational economic decisions based on the difference between the benefits and costs to be taken.

Although these models offer some important conclusions, there are also many problems arising from these concerning the social outcomes they derive. Firstly, since the creation of one job spot will cause rural-urban migration by more than one migrant, there will be an increase in urban unemployment showing that the social costs of migration outweigh the individual financial benefits. Secondly, migration out of rural areas will reduce rural output as a result of the young migrants moving to urban areas.

In the same idea, Zarembka (1970) supported the Todarian analysis, as do Bartlett (1983) introducing the more appropriate specification of the induced-migration hypothesis suggested by Todaro into the dual market. Comparing

the Todaro model with that of Harris and Todaro, Blomqvist (1978) showed the conflict of the model's conclusions concerning the relation between urban job creation and of urban unemployment. Due to Todaro paradox, the job creation increases urban unemployment while a subsidy for the employment of the manufacturing sector improves the welfare of the economy although it appears to co-exist with rural-urban unemployment.

Supporting the Harris and Todaro, there was an attempt to apply this into a growth model. Such an attempt was carried out by Mas-Colell and Razin (1973) and Gupta (1984) giving a dynamical interpretation of migration, unemployment and development of an economy. A similar approach is followed by Bencivenga and Smith (1997) where rural-urban migration and urban unemployment are important aspects of the development process. In this model there exists underemployment resulting from the adverse selection in labor markets.

Djajic (1985) extended the Harris and Todaro model applying it in a developed economy with the help of a minimum wage. He also focused into the difference of worker's skills. Although there are many similarities with their analysis, Djajic rejected the idea that factors other than labor are sector-specific and stated that capital is also mobile not only among sectors but also internationally. Since Djajic uses two types of labor in his analysis (high- and low-skilled) assuming that skills consume time in order to be obtained, he had the ability to analyze the model either in the short-run assuming immobile labor or in the long-run with perfectly mobile labor.

Djajic (1989) also examines the impact of labor mobility from the side of migrants who try to maximize their utility. He compared the consumption and leisure levels between migrants, natives in the host country and non-migrants in the source country. He also showed that the distinction between permanent and temporary migration.

Analyzing migration from a welfare perspective, we need to see whether migrants have a positive or negative influence in the host country's economy. Borjas (1995) provided a simple economic framework to describe how natives

benefit from migration. This is done by indicating that natives do benefit from immigration because of production complementarities between migrants and other factors of production, and these benefits are larger when immigrants have sufficient difference from the natives. He stated that if the U.S moves to immigration policy that supports the entrance of high-skilled migrants, then the gains from migration will increase so as the welfare in the host country.

Ethier (1986) distinguishes three important aspects of migration.

- i. Whether migration is a permanent or a temporary phenomenon
- ii. Whether migration involves skilled or unskilled labor
- iii. Whether migration is legal or not.

He especially focuses on the illegal migration of unskilled labor. In order to prevent migration Ethier studies the effects of border and internal enforcement.

Following Ethier, both Djajic (1987) and Bond and Chen (1987) study the influence of immigration policy upon the welfare of the host-country economy. Djajic focused on the interaction of migrant labor, unemployment rate and enforcement spending either in the short-run or in the long-run, while Bond and Chen explore the optimal level of enforcement by the host-country economy. Their results show that enforcement costs have less efficient impact than that of a tax on wages. Assuming mobile capital, migrants are better off after an increase in enforcement in the home country since capital can move away from the home country.

Similarly, Ozler and Waldman (1987) study the immigration policies either through border enforcement or internal enforcement. They treated border enforcement as a one-time cost paid in order to prevent a migrant to enter in the home-country while the internal enforcement as a cost paid for many periods in order to "clear" the economy from migrants. They especially focus in their adjustment to permanent increases in both enforcement types. They result that internal enforcement has an advantage over border enforcement.

Another paper about immigration policy and welfare implications is that of Levine (1999). He worked in a Harris and Todaro framework where labor

markets never clear. He examined two different cases, where there are flexible wages and full employment, and fixed wages in the host country. Through free migration or just immigration he examined the welfare of the host-country based on the migrants' incentives to migrate.

A more trade theoretic approach concerning migration is that of Chesney and Hazari (1998) and Chesney, Hazari and Sgro (1999). Chesney and Hazari established a relationship between illegal migrants and tourism, where under certain conditions expanded tourism increases the welfare of domestic agents. Similarly, Chesney, Hazari and Sgro analyzing a trade theoretic model examined two different models concerning either skilled or unskilled workers, being unemployed. They showed that under certain conditions, migration raises both skilled and unskilled employment and welfare.

Another category where migration and unemployment has been studied is that of an efficiency wage framework. Carter (1999) used an efficiency-wage model in a dual labor market in order to see the influence an increase in migration has on domestic economy. He followed the Shapiro and Stiglitz analysis extended by Kimball in its dynamic form and resulted that at the beginning of migration, natives gain since then there is an increase in primary sector jobs. As the number of migrants increases more than natives are hurt due to the migrant's expansion in the primary sector jobs. He proposed a policy of deporting migrants working only in primary sector jobs.

In a similar context, Müller (2003a, 2003b) used the efficiency-wage approach in order to show the impact illegal migration has in a small country in a dual labor market with unemployment when there is discrimination against foreign workers. He displayed three regimes concerning the number of illegal migrants where in the first regime the number of immigrants is small and they only work at the secondary sector while natives work on both sectors, in the second regime there is a sectoral segregation between immigrants and natives, and in the third regime, natives work only in the primary sector while immigrants work in both sectors. Müller showed that the intermediate regime gives the best gains to the natives. Although, similar

to that of Carter's, these models studied the welfare analysis in a dynamic framework while Carter's model focuses only on steady-state and the transition phase.

An alternative approach, different from Carter's, is also the model analyzed by Kondoh (2004). In his model, when migrants are complementary to the native workers, the optimal policy should be to increase the spending for border or internal enforcement in order to have a higher employment level, larger wages in both sectors and greater unemployment allowances. But, when legal migrants are substitutable for natives, increasing their number will reduce the unemployment rate of natives. On the other hand, when illegal migrants are complementary to natives, and they are free to immigrate, then the optimal policy should be to restrict legal migrants who are substitutable to them.

Finally, we should mention the literature concerning migration in a standard growth modeling. An important attempt was that of Hazari and Sgro (2003), analyzing how illegal migration influences domestic consumption and domestic welfare. When migrants and natives are perfect substitutes, illegal migration lowers the per-capita consumption of natives. On the other hand, when migrants and natives are imperfect substitutes, the influence on the per-capita consumption is ambiguous, although in the Cobb-Douglas case migration raises per-capita domestic consumption.

Extending this model, Moy and Yip (2006) resulted in an ambiguous welfare result after an increase in the stock of migrants, not in a negative one as that of Hazari and Sgro. This is happening due to the opposing effects of the positive exploitation effect and the negative intertemporal effect. Again in the Cobb-Douglas case, migration raises per-capita domestic consumption. Finally, Palivos (2006) studied all the above not from a social planner's point of view but in a competitive equilibrium framework. Studying migration in a competitive equilibrium framework, Palivos arrived at a conclusion that migration unambiguously raises per-capita domestic consumption and thus domestic welfare.

### **3. Theories of Unemployment**

#### *3.1. Implicit-Contract Theory*

In this chapter, we consider long-term relationships between firms and workers. Firms hire workers looking for permanent employment. Long-term relationships imply that the wage does not need to fluctuate so as to clear the labor market each period. From the workers' point of view, they wish to stay as long as possible but only at periods where their wages are larger than their outside opportunities.

Azariadis (1975) points out that risk-neutral firms can act either as employers or as insurers of homogenous, risk-averse laborers. The use of such an employment contract as an insurance contract has been one of the most important issues in implicit contract theory. Implicit-contract theory lies in the belief that fluctuations in wages and employment cannot be explained by a competitive labor market in which the labor market is always in equilibrium. Instead, the observation in the labor market is that over the economic cycle we see stable wages even if employment changes gradually.

The most representative papers in the "implicit-contract" approach are written by Baily (1974), Gordon (1974) and Azariadis (1975). The basic idea of implicit-contract theory is that with regard to employers, they are less risk-averse than workers. This is happening because the owners of capital who represent the employers can distribute their capital holdings among many different firms, and by this diversification they obtain insurance against the risks faced by individual firms.

On the other hand, it is more difficult for workers to diversify their human capital since they work for only one employer at the time. That's why they cannot insure themselves against wage fluctuations. However, a risk-averse employee is willing to pay for income certainty because an action like this increases his utility. But, since workers have imperfect information of the insurance market, they cannot successfully insure themselves. In that way, firms enjoy more insurance than workers. The most crucial feature of implicit-

contract models is how risk is shared between workers and firms. The best move towards a joint improvement of their condition is to replace a wage which tends to change from time to time with a fixed wage contract.

So as it seems, the implicit contract theory gives adequate explanations about wage stickiness, one of the stylized facts of the labor market. In the optimal contract, the wage is rigid and does not vary when the marginal revenue product of labor does. We know that the marginal revenue product is thought to be high in good times and low in bad times. Following this, the employment contract must have an insurance element against bad times by collecting premiums from them in good times.

An implicit contract describes all the labour services a firm receives at any moment and all the corresponding payments to the workers for their services. In implicit-contract theory with the word "implicit" we try to show that an agreement has been made that is clear and understood from both parties. But, contrary to an explicit contract where everything is specified and written, implicit contracts have informal meaning and it is rather confusing whether it provides insurance or not. Unless, the most important feature of a contract is not its implicit form but the way the risk is distributed among both parties.

The problem with implicit contracts compared to explicit ones lies into the fact that there is no mediator to enforce such an agreement on behalf of firms and workers. In such an agreement there is always a common incentive for one of the parties to deviate from the implicit contract. A firm could always replace workers with marginal product lower than their fixed wage with cheaper workers, while workers could always quit and search for a better-paid job when their marginal product surpasses their fixed wage.

Exploring these incentives, Arnot, Hosios and Stiglitz (1988) stated that when firms cannot observe their workers' searching, their insurance against firm-specific shocks has an adverse impact on their incentives to find better-paid jobs. The deviations from implicit contracts could be avoided through self-enforcement using labor market institutions such mobility costs (Baily (1974) and reputation Bull (1987)).

### 3. 2. Efficiency-wage models

As we noted previously, the most crucial characteristic of the efficiency-wage approach is that there is a cost and a benefit to a firm when it pays a higher wage. For a review in the literature of “efficiency-wage models” Yellen (1984) and Katz (1986) are the more representative papers.

At any time, firms are trying to advance their workers efforts in return for wages. In this process, we have conflict of interest among both parties since firms are profit maximizers while workers are utility maximizers. This means that workers try to avoid difficulties during their working time such as a hard task by providing less effort known to the literature as “shirking”. From their own perspective, firms try to locate shirkers who damage production and replace them with others who won’t shirk.

To avoid the above malfunction, firms must monitor their workers’ effort. But such an attempt creates great costs to the firm. Consequently, with imperfect monitoring and full employment, workers will always have an incentive to shirk. Of course, firms could instead offer to workers some compensation plans so that they will keep on the hard working.

Another way to avoid shirking is to give workers a wage above the one clearing the market. This method has been the heart of the efficiency-wage theories. A higher wage is highly probable to generate a rise in the workers’ productivity. This is happening due to a feeling of gratitude by workers who raise their efforts so as to pay-back for their improved wages. On the contrary, there is the notion that it is not the higher wage that makes workers avoid shirking but the fear that they will be caught shirking and they will lose that wage after being fired. Thus, the cost of earning less in another job elsewhere leads them in hard working. The wage that firms must pay to avoid shirking from workers depends upon the alternatives their workers have.

Shapiro and Stiglitz (1984) in their seminal work, pointed out that in the case where all firms move to an identical strategy, that is raising wages, then all workers have an incentive to shirk. If a worker is caught shirking he gets fired. But that is not a problem since the same workers can be rehired

instantly by a firm at the same wage something which means that we don't have unemployment. But, since all firms raise their wage offers there will be excess labor supply, and thus the existence of unemployment. Subsequently, at the presence of unemployment, even if all firms pay the same wages the workers will choose not to shirk because if they will be caught to shirk and get fired they won't find another job immediately.

Extending models such as that of Salop (1979), Shapiro and Stiglitz analyzed a model where firms cannot monitor their workers' effort and follow an efficiency-wage strategy so as to avoid shirking. Under the cost of being fired, the workers have an incentive not to shirk, while firms examine which is the optimal wage and employment tactic they should follow so as to avoid it. The Shapiro-Stiglitz model arrives through its analysis to the following results. Firstly, we absolutely have unemployment when we are in equilibrium. Secondly, this unemployment is always involuntary.

Although the Shapiro-Stiglitz model is being used by many authors, it is limited by its non-dynamic form. In 1994 Kimball extended this model introducing in that kind of efficiency-wage model. He also modified their analysis by allowing changes in total hours that a worker has to work per week as well as in employment. Using endogenous changes in weekly hours of work he managed to estimate the lag length and the short- and medium-run macroeconomic labor supply elasticities. Kimball resulted that after introducing dynamics into the Shapiro-Stiglitz model; he could then focus on that equilibrium in which employment changes gradually.

Of course, this equilibrium is not the only one as shown by Bulow and Summers (1986) although their result depend on guesses on the nature of dynamic equilibria. Their efficiency-wage model explores some of the microeconomic implications of this kind of models, showing how these can clear out the existence of involuntary unemployment in an economy where there are specific types of work which are always available to unemployed workers. Another issue analyzed by them is that of discrimination. Unlike other theories of discrimination, their model shows the reasons of its

persistence although there is tendency to eliminate it. Their conclusion is that disadvantaged groups get “equal pay for equal work” but do unequal work.

An alternative approach concerning efficiency-wage models is that of Summers (1988). This paper suggested that the best way to understand unemployment and its fluctuations is through relative wages. This is happening because workers’ productivity is closely related to their relative wages which in their turn have similarities with the efficiency-wage theories.

Finally, we should mention the application of efficiency-wage theories into a standard growth model. Specifically, Hoon (1993) attempted to do the above by making the assumption that the elasticity of substitution between capital and labor is less than unity. He showed that an increase in the accumulation of capital decreases the natural rate of unemployment in order to reach a lower level in the new steady-state. The decline in the unemployment rate is also a result of a fall in the public debt or a fall in the Harrod-neutral technical progress which increases the accumulation of capital.

### 3. 3. Search and Matching Theory

A third approach in explaining the theories of unemployment are that of “search and matching models”. In these categories of models, trading frictions play a crucial role. In order to find a “good” and “well-paid” job a worker and also a firm must both spend time and money. This is happening because there is not a centralized economy where workers and firms could meet in order to trade for a job at a single price, exactly like in the case of classical equilibrium theory. Although there are several important areas for research, the most agreed areas in equilibrium search theory focus into the way workers and firms finally meet and into the way of wage determination.<sup>5</sup>

The literature in search theory begins with the problem of a single worker looking for a job. We can see that it is time-consuming to find a suitable job and that this job is always randomly selected. This results into different wages

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<sup>5</sup> Search theory has been also applied into monetary economics, industrial organization, finance etc.

among workers. Search models can be given either in discrete or continuous time. The first important models concerning search theory are those dealing with “information” as that of Stigler (1961, 1962). Information models try to analyse the microeconomic behaviour of individuals and the macroeconomic behaviour of economic variables such as unemployment fluctuations.

It is clear that search theory is opposed to the classical analysis since it cancels some its basic assumptions such as the existence of information costs and of search activity. As we said above, the main characteristic of these models is the absence of a Walrasian auctioneer in the labor market where all transactions take place between firms and workers. We assume that labor is a homogeneous quantity and that there are risk neutral agents looking for a job, accepting or rejecting the wage offered by firms each period. Due to different worker skills, firms move to different wage offers even to the same job applicants. A worker pays each period a fixed cost for his search activity and receives a distribution of wage offers which are independently and identically distributed (i.i.d). Since the worker doesn't know the wage offers but only their distribution, he samples from that known distribution hoping that he will eventually choose the best available wage offer.

Workers try to maximize their net income while firms try to maximize their profits. A worker has only two choices, to accept the wage offer and receive an “acceptance” wage or to refuse and receive a “reservation” wage.

The most important contribution to that kind of search models is that of McCall (1970) and Lippman and McCall (1976). They showed that if there is an expected wage distribution there would also be a maximum expected wage offer taking a finite value, and a stopping rule which yields the maximum wage offer among all stopping rules. Following a “take-it-or-leave-it” process, a worker passes up a wage offer and continues sampling if this given offer is smaller than the reservation wage. Even when the worker is allowed to sample with recall, the optimal strategy will still be to continue searching until his wage offer surpasses his reservation wage.

Mortensen (1970) adopted the optimal stopping process as well as the notion that firms set an optimal wage dependent on the response of workers. He showed that the larger the wage set by the firm the larger the amount of workers sampling this wage offer, and the higher the amount of unemployed accepting this wage.

Kohn and Shavell (1974) presented a general formulation of the search problem assuming that search is a sequential sampling from a population. The optimal decision rule of an expected utility maximizer takes the form a switch point level of utility. When the wage offer is higher than this switch point then the search ends, otherwise if the wage offer is lower than the switch point then search continues.

Modifying the standard search model, Burdett (1979) assumed that there is an unemployment insurance subsidy. Using an infinite-horizon model where workers' objective is to maximize their expected lifetime utility depending on income and leisure. He showed that as workers reach the end of their unemployment insurance subsidy, they reduce their acceptance wages, consume less leisure and increase their searching. According to Burdett, a worker can search while he is unemployed, while he is still at work, or not search at all. He stated, the higher is the wage enjoyed by a worker the less likely is to quit, because it is very difficult to find a better one, showing that the acceptance wage is a decreasing function of the time consumed in search.

On the other hand, there is also an alternative approach concerning search models. This approach involves random matching between firms and workers and wage determination through bargaining. So, the most important question for this approach is in what way do firms and workers meet and how is the wage eventually determined. The most representative papers for this kind of analysis are those of Diamond (1982a, 1982b), Pissarides (1984, 1985, 1987, 2000) and Hosios (1990).

The central idea of this approach is again that trade between workers and a firm in the labor market is happening in a decentralized economy. This means that search needs time and money in order to create a match. Firms

spend money in order to create a job while workers pay a search cost in their attempt to find a suitable job. Thus, labor market ceases to be a Walrasian one.

In this kind of models, it is used the so-called “matching function” for their analysis. The matching function has the same meaning as the aggregate neoclassical production function and illustrates the number of successful matches formed every period as a function of the number of workers looking for a job and the number of vacant jobs looking for worker to get filled.

In a non-Walrasian labor market, trade is a non-trivial economic activity because of the heterogeneous agents, frictions and asymmetric information. This leads both workers and firms in spending money in order to find a match. Because of the existence of uncertainty in these models agents must decide whether to form a match or wait for a better alternative, or spend money seeking for further information aiming in a better job offer.

Although there is a lack of microeconomic foundation in the matching function, it is still thought as the most effective tool in this kind of analysis. Following the above characteristics and giving it an aggregate form, the matching function illustrates the outcome which result after all this spending done by both parties as a function of the inputs during the trading process. But, trade and production are not connected with each other. Agents could either search for a pair and thus trade or already create a match and move to production. In simple words, there is full specialization in either of these activities. Of course, it is clear that these are not firms that specialize but jobs, and that it is not necessary for a worker to be unemployed in order to search for a job, while there is also on-the-job search.

As long as vacant jobs and unemployed workers meet each other they can form a match or continue search. The existence of unemployment in the steady-state is a result of a job destruction, which is the separation effect in matched pair due to firm-specific shocks, which provide a flow into unemployment at any period of time.

Aware of the matching function and the separation probability, firms and workers continue searching in order to create a matched pair. Their actions

are independent resulting in rational expectations equilibrium, where the flow into unemployment is the same as the flow into employment. Upon this equality, a unique unemployment rate exists defining our economy. Due to constant returns to scale, this rate is constant along the balanced growth path.

It is assumed that the matching success between firms and workers is a random procedure following the Poisson distribution, drawing from the vacancy and unemployment pool. The easiness or difficulty of matching depends on the “tightness” of the labor market, a constructed variable giving the rate of number of vacancies to the number of unemployed workers.

Now, it is clear to say that the two key activities in their analysis is that of job creation, where a firm and a worker form a pair agreeing at a negotiated wage, and of job destruction, where the pair separates and the worker returns to the unemployment pool. Another useful tool in this kind of analysis is the so-called “Beveridge curve” which can be represented either in tightness-unemployment space or in vacancy-unemployment space.

Finally, in these models the wage determination is happening through the Nash solution to the bargaining problem, as that of Binmore, Rubinstein and Wolinsky (1986). In equilibrium, the returns from a matched pair are greater than the sum of expected returns of both searching firms and workers.

Thus, there is a surplus in the form of a monopoly-rent which needs to be shared among agents in order to compensate them for spending money in the search process. Because all jobs are equally productive and leisure has the same value for all workers, the wage resulting from the bargaining process will be the same everywhere.

For an alternative wage setting we must see the papers of Burdett and Mortensen (1998) and Moen (1997). From an empirical point of view representative papers are those of Pissarides (1986), Blanchard and Diamond (1989), Broersma and Van Ours (1998) and Petrongolo and Pissarides (2001). Finally, alternative models concerning search theory are those of Laing, Palivos and Wang (1995, 2003) that focus on the human capital side through learning connecting those models with growth.

#### 4. The Basic Model

We assume that in our economy the total labor force is given by

$$N = E + U \tag{1}$$

or by

$$N = L + M \tag{2}$$

where  $N$  : total labor force

$L$  : total local labor force

$M$  : total migrant labor force

$E$  : total employment

$U$  : total unemployment

$$\text{But, since } U = U_L + U_M \quad \text{and} \quad E = E_L + E_M \tag{3}$$

$$\Rightarrow N = E_L + E_M + U_L + U_M \tag{4}$$

We assume that at any point of time there are  $U$  unemployed workers and  $V$  vacancies trying to find each other so as to form a matched-pair.

The matching process takes place using the following matching function

$$m = M(U, V) \tag{5}$$

Since a vacancy is filled by only one worker, it follows that

$$\mu U = \eta V \tag{6}$$

where  $\mu$  is the probability that an unemployed worker moves to employment and  $\eta$  is the probability that a firm finds a worker to fill its vacancy. Job vacancies and unemployed workers are matched randomly from their unemployment and vacancy groups.

We should also point that the flow of workers out of the unemployment pool must be equal to the growth rate of the population,  $n$ , which we assume to be constant and the same for both natives and migrants.

Thus, it must hold that

Next, we introduce the variable

$$\theta \equiv \frac{V}{U} \tag{7}$$

as the vacancies to unemployment ratio, which also measures the “tightness” of the labor market. Now, with the help of (7), we can rewrite  $\mu$  and  $\eta$  like

$$\mu = \frac{m(U, V)}{U} = m(1, \theta) \quad (8)$$

and

$$\eta = \frac{m(U, V)}{V} = \frac{m(U, V) U}{U V} = \frac{\mu}{\theta} \quad (9)$$

We assume that unemployment changes according to the following equation

$$\dot{U} = \dot{N} + s_L E + s_M E - \mu U \quad (10)$$

where  $s_L$  is the separation probability for the native workers and  $s_M$  the separation probability for migrants, i.e. the probability that a matched-pair is separated by a firm-specific shock, and  $\dot{N}$  gives change into the labor force.

Since,  $u = \frac{U}{N}$  is the unemployment rate we see that it can also be written as

$$U = uN \Rightarrow \dot{U} = \dot{u}N + u\dot{N} \Rightarrow \dot{u} = \frac{\dot{U}}{N} - u \frac{\dot{N}}{N} \quad (11)$$

So, combining (10) and (11) we get that

$$\dot{u} = \frac{\dot{N}}{N} + s_L \frac{E}{N} + s_M \frac{E}{N} - \mu \frac{U}{N} - u \frac{\dot{N}}{N} \quad (12)$$

Using (1), we can see that  $\frac{E}{N} = 1 - u$  and thus from (11) and (12) we get that

$$\dot{u} = n + (s_L + s_M) - [n + s_L + s_M + \mu]u \quad (13)$$

Since our analysis takes place in the steady state (13) is getting

$$u = \frac{n + s_L + s_M}{n + s_L + s_M + \mu} \quad (14)$$

We denote the value of a vacant job as  $\Pi_V$ , and the value of a filled job by  $\Pi_F$ . Similarly, the value of an unemployed worker is  $J_E$  and the value of an employed worker is  $J_U$ . Although,  $\Pi_V$  is the same for all kind of workers,  $\Pi_F$  and  $J_E$  and  $J_U$  will differ depending on whether a worker is a migrant or not taking the notation of  $M$  and  $L$  next to the value functions.

Thus,  $\Pi_V$ : the value equation of firm that fails to match

$\Pi_F(M)$ : the value equation of a firm that matches with a migrant

$\Pi_F(L)$ : the value equation of a firm that matches with a native

- and,  $J_U(M)$ : the value equation of an unemployed migrant  
 $J_U(L)$ : the value equation of an unemployed local worker  
 $J_E(M)$ : the value equation of an employed migrant  
 $J_E(L)$ : the value equation of an employed local worker

When a firm searches for a worker it meets an unemployed migrant with probability  $\frac{U_M}{U_L + U_M}$  and a local unemployed worker with probability  $\frac{U_L}{U_L + U_M} = \left(1 - \frac{U_M}{U_L + U_M}\right)$ . We also assume that there is a perfect capital market with discount rate  $r$ .

Thus, we could write a firm's value equation when its job is vacant as

$$r\Pi_V = -\gamma y + \dot{\Pi}_V + \frac{\mu}{\theta} \left[ \frac{U_M}{U_M + U_L} \Pi_F(M) + \left(1 - \frac{U_M}{U_M + U_L}\right) \Pi_F(L) - \Pi_V \right] \quad (15)$$

$$r\Pi_F(M) = y - w_M + \dot{\Pi}_F(M) + s_M (\Pi_V - \Pi_F(M)) \quad (16)$$

$$r\Pi_F(L) = y - w_L + \dot{\Pi}_F(L) + s_L (\Pi_V - \Pi_F(L)) \quad (17)$$

$$rJ_E(M) = w_M + \dot{J}_E(M) + s_M (J_U(M) - J_E(M)) \quad (18)$$

$$rJ_E(L) = w_L + \dot{J}_E(L) + s_L (J_U(L) - J_E(L)) \quad (19)$$

$$rJ_U(M) = 0 + \dot{J}_U(M) + \mu (J_E(M) - J_U(M)) \quad (20)$$

$$rJ_U(L) = z + \dot{J}_U(L) + \mu (J_E(L) - J_U(L)) \quad (21)$$

where  $y$  is the output resulting from a matching pair,  $-\gamma y$  are the forgone earnings a firm has to suffer when its job remains unemployed,  $w_M$  and  $w_L$  are the wages of migrants and local workers, respectively, while 0 and  $z$  are the unemployment benefits when workers remain unemployed. The dot

above the value functions illustrates their change in time and can either be negative or positive.

We assume that  $\Pi_v = v_0$ , an assumption that illustrates a free-entry condition into the labor market. As soon as firms pay that entry fee they can start their search. If the expected value of entry exceeds the entry fee ( $\Pi_v > v_0$ ), there is a large flow of new entrants in the labor market lowering  $\Pi_v$  by making it more difficult for firms to fill their vacancies. On the other hand, if the expected value of entry is less than the entry fee ( $\Pi_v < v_0$ ), then entry stops since it is not profitable to enter.

Finally, we assume that the match between a firm and a worker will lead to a positive surplus. This surplus is divided between these two parties according to a Nash bargaining rule. Thus, wages can be determined through the Nash bargain and are different depending on whether a worker is native or migrant.

The equation that illustrates the Nash bargaining rule must equalize the gains of a worker when he achieves employment ( $J_E - J_U$ ) with the gains of a firm when it has its job filled ( $\Pi_F - \Pi_v$ ). Thus, it must hold that

$$\max (J_E - J_U)^\beta (\Pi_F - \Pi_v)^{1-\beta} \quad (22)$$

$$\Rightarrow (J_E - J_U) = \frac{\beta}{1-\beta} [\Pi_F - \Pi_v] \quad (23)$$

(22) and (23) show the general case where we don't have to bother whether a worker is native or not. In the symmetric Nash bargaining rule  $\beta = \frac{1}{2}$  and gives equal shares to both parties. In our analysis, we could deviate from the symmetry case assuming that there is a different  $\beta$  depending on the worker's ethnicity.

If a worker is a native (23) is becoming

$$(J_E(L) - J_U(L)) = \frac{\beta_L}{1-\beta_L} [\Pi_F(L) - \Pi_v] \quad (24)$$

while if he is an illegal migrant (23) is changing into

$$(J_E(M) - J_U(M)) = \frac{\beta_M}{1 - \beta_M} [\Pi_F(M) - \Pi_V] \quad (25)$$

Since our model's analysis takes place in steady-state equilibrium we have

$$\dot{\Pi}_V = \dot{\Pi}_F(M) = \dot{\Pi}_F(L) = \dot{J}_E(M) = \dot{J}_E(L) = \dot{J}_U(M) = \dot{J}_U(L) = 0 \quad (26)$$

In steady state, it also holds that

$$\frac{U_M}{U_M + U_L} = \frac{M}{N} = (1 - \alpha) \text{ and that } 1 - \frac{U_M}{U_M + U_L} = 1 - \frac{M}{N} = \frac{L}{N} = \alpha \quad (27)$$

Substituting (26) and (27) into (15)-(21) along with  $\Pi_V = v_0$  we have

$$rv_0 = -\gamma y + \frac{\mu}{\theta} [(1 - \alpha)\Pi_F(M) + \alpha\Pi_F(L) - v_0] \quad (28)$$

$$r\Pi_F(M) = y - w_M + s_M (v_0 - \Pi_F(M)) \quad (29)$$

$$r\Pi_F(L) = y - w_L + s_L (v_0 - \Pi_F(L)) \quad (30)$$

$$rJ_E(M) = w_M + s_M (J_U(M) - J_E(M)) \quad (31)$$

$$rJ_E(L) = w_L + s_L (J_U(L) - J_E(L)) \quad (32)$$

$$rJ_U(M) = \mu (J_E(M) - J_U(M)) \quad (33)$$

$$rJ_U(L) = z + \mu (J_E(L) - J_U(L)) \quad (34)$$

From (31) and (33) we have that

$$r(J_E(M) - J_U(M)) = w_M - (s_M + \mu)(J_E(M) - J_U(M)) \Rightarrow (r + s_M + \mu)(J_E(M) - J_U(M)) = w_M$$

$$\Rightarrow (J_E(M) - J_U(M)) = \frac{w_M}{(r + s_M + \mu)} \quad (35)$$

Using (29) we get that 
$$\Pi_F(M) = \frac{y - w_M + s_M v_0}{r + s_M} \quad (36)$$

Now, substituting (35), (36) and the fact that  $\Pi_V = v_0$  into (25) we get

$$w_M = \frac{\beta_M (r + s_M + \mu)(y - r v_0)}{[(1 - \beta_M)(r + s_M) + \beta_M (r + s_M + \mu)]} \quad (37)^6$$

Following the exact same process we can obtain the wage the native workers get after the Nash bargaining.

From (32) and (34) we have that

$$r(J_E(L) - J_U(L)) = (w_L - z) - (s_L + \mu)(J_E(L) - J_U(L)) \Rightarrow (r + s_L + \mu)(J_E(L) - J_U(L)) = w_L - z$$

$$\Rightarrow (J_E(L) - J_U(L)) = \frac{w_L - z}{(r + s_L + \mu)} \quad (38)$$

Using (30) we get that 
$$\Pi_F(L) = \frac{y - w_L + s_L v_0}{r + s_L} \quad (39)$$

Now, substituting (38), (39) and  $\Pi_V = v_0$  and  $z = b w_L$  into (25) we get

$$w_L = \frac{\beta_L (r + s_L + \mu)(y - r v_0)}{[(1 - \beta_L)(r + s_L)(1 - b) + \beta_L (r + s_L + \mu)]} \quad (40)^7$$

If  $\beta_M = \beta_L$ , then since  $0 \leq b \leq 1$ , we can see from (37) and (40) that the wage native workers get after the bargaining process exceeds the one migrants get.

Thus, 
$$w_L \geq w_M \quad (41)^8$$

Substituting (37) and (40) back to (36) and (39) we get the following expressions of  $\Pi_F$  as a function of  $\mu$ .

$$\Pi_F(M) = \frac{(1 - \beta_M)(y + s_M v_0) + \beta_M (r + s_M + \mu)v_0}{(1 - \beta_M)(r + s_M) + \beta_M (r + s_M + \mu)} \quad (42)$$

Or 
$$\Pi_F(M) = \frac{(1 - \beta_M)(y - r v_0)}{(1 - \beta_M)(r + s_M) + \beta_M (r + s_M + \mu)} + v_0 \quad (43)$$

<sup>6</sup> For the derivation of the immigrant workers' wage see Appendix A at the end of the paper.

<sup>7</sup> For the derivation of the native workers' wage see Appendix B at the end of the paper.

<sup>8</sup> The "exploitation effect" pointed out by Hazari and Sgro (2003), Moy and Yip (2006) and Palivos (2006) holds.

And, 
$$\Pi_F(L) = \frac{(1-\beta_L)(1-b)(y+s_L v_0) + \beta_L(r+s_L+\mu)v_0}{(1-\beta_L)(1-b)(r+s_L) + \beta_L(r+s_L+\mu)} \quad (44)$$

Or 
$$\Pi_F(L) = \frac{(1-\beta_L)(1-b)(y-rv_0)}{(1-\beta_L)(1-b)(r+s_L) + \beta_L(r+s_L+\mu)} + v_0 \quad (45)$$

Substituting (43) and (45) into (28) we can get an expression of  $\mu$ .<sup>9</sup>

$$\begin{aligned} & \frac{(rv_0 + \gamma y)\theta}{\mu}(r+s_M)(r+s_L)(1-b+\beta_L b) + \frac{(rv_0 + \gamma y)\theta}{\mu}(r+s_L)(1-b+\beta_L b)\beta_M \mu \\ & + \frac{(rv_0 + \gamma y)\theta}{\mu}(r+s_M)\beta_L \mu + \frac{(rv_0 + \gamma y)\theta}{\mu}\beta_L \beta_M \mu^2 = \\ & (1-\alpha)(1-\beta_M)(y-rv_0)(r+s_L)(1-b+\beta_L b) + (1-\alpha)(1-\beta_M)(y-rv_0)\beta_L \mu \\ & + \alpha(1-\beta_L)(1-b)(y-rv_0)(r+s_M) + \alpha(1-\beta_L)(1-b)(y-rv_0)\beta_M \mu \end{aligned} \quad (46)$$

Assuming a Cobb-Douglass matching function we have

$$m = U^\delta V^{1-\delta} \quad (47)$$

Thus, (8) and (9) are becoming

$$\mu = \frac{m(U, V)}{U} = m(1, \theta) = \frac{U^\delta V^{1-\delta}}{U} = U^{-(1-\delta)} V^{1-\delta} = \frac{V^{1-\delta}}{U^{1-\delta}} = \theta^{1/2} \quad (48)$$

$$\eta = \frac{m(U, V)}{V} = m\left(\frac{1}{\theta}, 1\right) = \frac{U^\delta V^{1-\delta}}{V} = U^\delta V^{-\delta} = \frac{U^\delta}{V^\delta} = \theta^{-1/2} \quad (49)$$

Substituting (48) and (49) into (46) we have

$$\begin{aligned} & \Rightarrow \theta \left[ (rv_0 + \gamma y)(r+s_L)(1-(1-\beta_L)b)\beta_M + (rv_0 + \gamma y)(r+s_M)\beta_L \right] \\ & + \theta^{1/2} \left[ (rv_0 + \gamma y)(r+s_L)(r+s_M)(1-(1-\beta_L)b) + (rv_0 + \gamma y)\beta_M \beta_L \right] \\ & + \left[ -(1-\alpha)(1-\beta_M)(y-rv_0)\beta_L - \alpha(1-\beta_L)(1-b)(y-rv_0)\beta_M \right] \\ & + \left[ -(1-\alpha)(1-\beta_M)(y-rv_0) \right] (r+s_L)(1-(1-\beta_L)b) - \alpha(1-\beta_L)(1-b)(y-rv_0)(r+s_L) = 0 \end{aligned} \quad (50)$$

Equation (50) is of type  $A\theta + B\theta^{1/2} + C = 0$ , where

<sup>9</sup> In order to get to (46) see Appendix C at the end of the paper.

$$A = (rv_0 + \gamma y)(r + s_L)(1 - (1 - \beta_L)b)\beta_M + (rv_0 + \gamma y)(r + s_M)\beta_L,$$

$$B = (rv_0 + \gamma y)(r + s_L)(r + s_M)(1 - (1 - \beta_L)b) + (rv_0 + \gamma y)\beta_M\beta_L \quad \text{and}$$

$$-(1 - \alpha)(1 - \beta_M)(y - rv_0)\beta_L - \alpha(1 - \beta_L)(1 - b)(y - rv_0)\beta_M$$

$$C = -(1 - \alpha)(1 - \beta_M)(y - rv_0)(r + s_L)(1 - (1 - \beta_L)b) - \alpha(1 - \beta_L)(1 - b)(y - rv_0)(r + s_L)$$

We know that from (50) we arrive in two different roots of  $\theta$  where

$$\theta^{\frac{1}{2}} = \frac{-B \pm \sqrt{D}}{2A} \quad \text{where } D = B^2 - 4AC, \text{ holding only the one that is positive.}$$

Thus, from (51) we get

$$\theta = \left( \frac{-B \pm \sqrt{D}}{2A} \right)^2 \quad (51)$$

Unfortunately, the roots given when solving (50) have a largely extensive form and cannot be analyzed. To simplify our results we are making the following assumptions.

- i. The bargaining share of both local and migrant workers is identical. Thus,  $\beta$  is the same for both parties and it holds that  $\beta_M = \beta_L = \frac{1}{2}$ . It is clear now that we apply the symmetric Nash bargaining solution.
- ii. There is no need to pay a cost of posting a vacancy. Thus,  $\Pi_v = v_0 = 0$ .
- iii. There is a difference between separation probabilities when it comes to see whether a worker is native or not. To clear this out we assume that when a firm matches a native worker and forms a pair we have to deal with a permanent, not a temporary match. This is not the case when it comes to migrants. When the match is taking place between a firm and a migrant worker and form a pair, this pair is not permanent and thus there is always a positive probability that this migrant worker will be separated from his job due to job-specific shock. This means that must hold  $s_L = 0 \neq s_M > 0$ .
- iv. Finally, we will also assume for simplicity that the unemployment benefit,  $b$ , is also zero in order to help us obtain a more simple form of  $\theta$ .

Thus,  $\theta$  will take the following form

$$\theta = \left[ \frac{\left( \frac{-\gamma yr(r+s_M) - 0.25\gamma y + 0.25y(1-\alpha) + 0.25y\alpha \pm \sqrt{(\gamma yr(r+s_M) + 0.25\gamma y - 0.25y(1-\alpha) - 0.25y\alpha)^2 + 4(\gamma yr + 0.5\gamma ys_M)(0.5\gamma y(1-\alpha) + 0.5y\alpha)}}{2(\gamma yr + 0.5\gamma ys_M)} \right)^2}{2(\gamma yr + 0.5\gamma ys_M)} \right] \quad (52)$$

Let's see now how the number of migrants,  $(1-\alpha)$ , and their separation probability affects,  $s_M$ , affects the market tightness,  $\theta$ , and what kind of policy measure the host country's government should take against migrants in order to improve the total welfare.

$$\frac{\partial \theta}{\partial (1-\alpha)} = \frac{(0.5y + \frac{1}{\sqrt{-0.5y + 2\gamma y^2 r^2 + \gamma y^2 r s_M}})}{[2(\gamma yr + 0.5\gamma ys_M)]^2} > 0 \quad (53)$$

$$\frac{\partial \theta}{\partial s_M} = \frac{2-\gamma yr + \frac{1}{2} \{2\gamma yr + 2\gamma(0.5\gamma(1-\alpha) + 0.5y\alpha)\}^{\frac{1}{2}} \times [2(\gamma yr + 0.5\gamma ys_M)]^{\frac{1}{2}}}{[2(\gamma yr + 0.5\gamma ys_M)]^4} - \frac{\left[ \frac{-\gamma yr(r+s_M) - 0.25\gamma y + 0.25y(1-\alpha) + 0.25y\alpha + \sqrt{(\gamma yr(r+s_M) + 0.25\gamma y - 0.25y(1-\alpha) - 0.25y\alpha)^2 + 4(\gamma yr + 0.5\gamma ys_M)(0.5\gamma y(1-\alpha) + 0.5y\alpha)}}{2(\gamma yr + 0.5\gamma ys_M)} \right]^2 \times 4\gamma y}{[2(\gamma yr + 0.5\gamma ys_M)]^4} < 0 \quad (54)$$

It is clear from (53) and (54) that the number of illegal immigrants has a positive effect upon the market tightness while the immigrant workers' separation effect on the market tightness is negative. Thus, the domestic government should apply policies that increase the number of immigrants or decrease their separation probability, or a combination of these policies.

## 5. Numerical Solution

A more suitable approach for better results is to give values to all our variables and then find  $\theta$ , which is the most crucial variable in this kind of models. Along with our previous simplifying assumptions (that is  $\beta_M = \beta_L = 0.5$ ,  $\Pi_V = v_0 = 0$  and  $s_L = 0$ ), we will make some additional assumptions concerning variables such as  $\gamma$ ,  $\alpha$ ,  $b$ ,  $r$ ,  $s_M$ ,  $y$  and  $n$ . In the benchmark case illustrated in Table 1, the exact prices given to our variables are  $\gamma = 0.7$ ,  $\alpha = 0.8$ ,  $b = 0.6$ ,  $r = 0.05$ ,  $s_M = 0.03$ ,  $y = 5$  and  $n = 0.01$ .<sup>10</sup>

Substituting these values into equation (51) we get a numerical expression of  $\theta$ . From backward substitution we can then calculate all the other variables concerning our model such as  $\mu$  and  $\eta$ ,  $\Pi_F(L)$  and  $\Pi_F(M)$ ,  $w_L$  and  $w_M$ ,  $J_E(L)$  and  $J_U(L)$ , as well as  $J_E(M)$  and  $J_U(M)$ . Finally, we will be able to calculate the unemployment rate by using (14), and combining it with (48), we get

$$\Rightarrow \quad u = \frac{n + s_L + s_M}{n + s_L + s_M + \theta^2} \quad (14')$$

After the calculation of  $\theta$ , we must find the prices of  $\mu$  and  $\eta$  substituting it into (48) and (49). Using these, we can then go and substitute into (40) and (37) in order to find  $w_L$  and  $w_M$ . Then we can check out whether there is a difference between them (the so-called "exploitation effect") and how is this connected with the other variables. Having found the wages paid to the workers (migrants or not), we can continue with the calculation of the firms' and the workers' value functions,  $\Pi_F(L)$ ,  $\Pi_F(M)$ ,  $J_E(L)$ ,  $J_U(L)$ ,  $J_E(M)$  and  $J_U(M)$ . Our remaining variable to be calculated is that of the unemployment rate from (14'), which seems to have a negative relationship with  $\theta$ .

In Table 1, we can also examine the welfare implications of both firms and workers. The welfare measure concerning firms is given by the value function

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<sup>10</sup> The variable prices are taken so as to closely related with those of Broersma and Van Ours (1999), Ortega (2000), Bertola and Garibaldi (2001), Garibaldi and Wasmer (2001) and Azariadis and Pissarides (2006).

of both natives and migrants multiplied with their analogous population rates. That is,

$$W_f = \bar{\Pi} = (1 - \alpha)\Pi_F(M) + \alpha\Pi_F(L) \quad (55)$$

On the other hand, the welfare measure concerning workers is given by the value function of the workers when they are employed and when they are unemployed, multiplied by their probabilities when they are at these states.

That is,  $W_w = \frac{E_L}{E} J_E(L) + (1 - \frac{E_L}{E}) J_U(L)$  and this in the steady-state is written as

$$W_w = \bar{J} = (1 - u)J_E(L) + uJ_U(L) \quad (56)$$

where  $\frac{E_L}{E}$  is the employment rate of the native workers and  $(1 - \frac{E_L}{E})$  is their unemployment rate.

We must focus into the change of total welfare (workers' welfare plus that of firms') after a change in one of our variables in our benchmark case caused by the application of a specific policy by the host-country's government.

Thus, we should examine the following welfare equation

$$\Delta W_T = \Delta W_f + \Delta W_w \quad (57)$$

where  $\Delta W_T$  is the change in the total welfare,  $\Delta W_f$  is the change in the welfare of firms and  $\Delta W_w$  is the change in the welfare of workers, after a policy, all resulting from the weighted welfare equations (55) and (56).

variables	The Benchmark case
$\theta$	0.04320488
$u$	0.16138284
$\Pi_F(L)$	7.19792565
$\Pi_F(M)$	13.5922076
$W_F$	12.3133512
$J_E(L)$	368.368328
$J_U(L)$	339.7969
$J_E(M)$	70.0971432
$J_U(M)$	56.5049356
$W_w$	363.75739

*Table 1*

## **6. Effects of Government Policies on Unemployment and Welfare**

The following chapter illustrates how government's policy can influence our model's important variables. We focus especially on policies concerning the phenomenon of migration and its connection to the unemployment rate as well as with the welfare implications of those changes both in native workers' and firms' welfare. Such policies can be either to loose or tighten the border enforcement by applying specific measures aiming to the prevention of additional entrance from illegal migrants and thus affecting the number of migrants  $(1-\alpha)$ . Another kind of policy can be to prevent a possible rise in the stock of migrants through spending for stricter internal enforcement, affecting the separation probability of the migrants,  $s_M$ .<sup>11</sup>

But, we will not stand only in these cases. We will examine the effect of a government fund towards firms to assist them in their search procedure and help them overcome a possible loss from staying with a vacant job, a policy which clearly affects  $\gamma$ , or similarly a government policy concerning a change in the unemployment benefit workers receive when they join the unemployment pool.

Additionally, we will check out how changes in the entry fee,  $v_0$ , affects all of our model's variables, and what happens in the case where the bargaining power is not symmetric when it comes to the migrants.

Finally, we will see how these changes in our model's variables due to a government policy, can affect the welfare of the host country's workers and firms and whether such a policy benefits or harms the total welfare of the host-country.

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<sup>11</sup> For an analysis of policy enforcement and welfare implication and how illegal immigration affects the host-country's welfare, see the papers of Ethier (1986), Djajic (1987), Bond and Chen (1987), Ozler and Waldman (1987), Borjas (1995) and Levine (1999). For a more trade theoretic approach see Chesney and Hazari (1998) as well as Chesney-Hazari and Sgro (1999).

6.1. Increase in border enforcement

We assume that domestic government enforces a policy against illegal migrants following an increase in its spending for border enforcement. Thus, the more the government spends trying to prevent the additional entrance of immigrants in their country, the stricter will be the enforcement and the lower will be the stock of illegal migrants in the host-country.

Effects of a change in border enforcement			
variables	$\alpha=0.7$	$\alpha=0.8$	$\alpha=0.9$
$\theta$	0.07990024	0.04320488	0.02477608
$u$	0.12396708	0.16138284	0.20262996
$\Pi_F(L)$	5.67108325	7.19792565	8.79491312
$\Pi_F(M)$	11.2951898	13.5922076	15.7527856
$W_F$	9.60795787	12.3133512	15.0569984
$J_E(L)$	475.237585	368.368328	296.291685
$J_U(L)$	446.666156	339.7969	267.720256
$J_E(M)$	75.1505823	70.0971432	65.3438716
$J_U(M)$	63.8553925	56.5049356	49.5910859
$W_W$	471.695668	363.75739	290.502258

Table 2

Such a policy has a direct effect in the population of immigrants as well as in the population of natives, since stricter border enforcement results in the reduction of  $(1-\alpha)$  and in the increase of  $\alpha$ . We assume that  $(1-\alpha)$  is reduced from 0.2 to 0.1, which means that  $\alpha$  is increased from 0.8 to 0.9. Due to the reduction of immigrants' number entering the country, we can see that there is also a decrease in the market tightness,  $\theta$ .

Thus, there is a positive (negative) relationship between  $\theta$  and  $(1-\alpha)$  ( $\alpha$ ). An increase in border enforcement yields higher value functions for firms and lower value functions for workers. Additionally, since  $u$  and  $\theta$  have a negative relationship, a decrease in  $(1-\alpha)$  will increase the unemployment. Finally, we see that stricter border enforcement harms the welfare of both types of workers and that increases the welfare of firms of the host country.

Examining Table 2 and (54), we can see that  $\Delta W_T$  is negative, which means that increasing border enforcement is not the proper policy measure in order to improve the host-country's welfare.

6.2. Increase in internal enforcement

We assume that domestic government wishes to reduce the number of illegal migrants through hardening the internal enforcement. Through this policy can detect and locate immigrants working in a firm at this period of time and then deport them.

Such a policy has a great impact on the migrants' separation probability,  $s_M$ , by increasing it. So, assuming an increase in government spending in internal enforcement would increase  $s_M$  from 0.03 to 0.05.

Effects of a change in internal enforcement			
variables	$S_M=0.02$	$S_M=0.03$	$S_M=0.04$
$\theta$	0.0445684	0.04320488	0.04192674
$u$	0.12442335	0.16138284	0.196263
$\Pi_F(L)$	7.11459438	7.19792565	7.27907466
$\Pi_F(M)$	14.2404581	13.5922076	12.9951071
$W_F$	12.8152853	12.3133512	11.8519006
$J_E(L)$	373.017573	368.368328	363.943141
$J_U(L)$	344.446145	339.7969	335.371712
$J_E(M)$	74.3671755	70.0971432	66.2127215
$J_U(M)$	60.1267175	56.5049356	53.2176143
$W_W$	369.462621	363.75739	358.335627

Table 3

Analysing Table 3 we can see that increasing  $s_M$  results in a decrease of  $\theta$ . A higher separation probability for migrants leads to a higher (lower) value function for a firm when it matches native (immigrant) workers and to lower value functions for both types of workers. Additionally, an increase in  $s_M$  increases  $u$  through the fall of  $\theta$ . Finally, we can see that increasing internal enforcement results in a lower welfare for workers as well as for firms. Thus, such a policy worsens the welfare state of the host economy. This means that it is not recommended to strengthen its internal enforcement measures.

Analysing Table 3 and with the help of equation (54), we can see that after the application of an increase in internal enforcement will decrease the host country's welfare, since both workers' and firms' welfare is getting worse. Thus, increasing internal enforcement is not the proper policy measure to improve the host-country's welfare.

6.3. Government funding towards firms

We assume that the domestic government in an attempt to help firms, spends resources in order to assist them in the search procedure by giving them money in the forms of funding or subsidizing, in the case where they have failed to create a matched pair and they suffer a loss in the form of forgone earnings.<sup>12</sup>

Effects of a change in subsidy to job advertising			
variables	$\gamma=0.8$	$\gamma=0.7$	$\gamma=0.6$
$\theta$	0.0212221	0.04320488	0.11772267
$u$	0.21542663	0.16138284	0.10440937
$\Pi_F(L)$	9.2730802	7.19792565	4.84135633
$\Pi_F(M)$	16.3570781	13.5922076	9.93823636
$W_F$	14.9402785	12.3133512	8.91886035
$J_E(L)$	279.540094	368.368328	561.581963
$J_U(L)$	250.968665	339.7969	533.010535
$J_E(M)$	64.0144281	70.0971432	78.13588
$J_U(M)$	47.65735	56.5049356	68.1976436
$W_W$	273.385047	363.75739	558.598838

Table 4

Since part of the loss is covered by the government,  $\gamma$  is falling. Assuming that it falls from 0.7 to 0.6, we can see from Table 4 that  $\theta$  increases. Additionally, there is a decline in firms' value function and a rise in the value functions of the workers. The unemployment rate will fall since it depends negatively on  $\theta$ , and  $\theta$  increases.

Finally, we can see that such a policy reduces the welfare of native workers and increases the welfare of firms. This is happening because such a policy gives subsidizing firms an incentive not to increase their efforts in their search procedure. Thus the probability of a match decreases leading to the decline of the unemployment rate.

We can from Table 4 and equation (54) that an increase in the government subsidizing to firms will increase the total welfare of the host-country although it decreases the firms' welfare. Thus, such a policy is a useful tool in order to reduce unemployment and to improve the host-country's welfare condition.

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<sup>12</sup> This entry fee could also have the form of a cost of maintaining the vacancy into the market for another period, or it might be an advertising cost.

6.4. Effects of a change in the unemployment benefit

We assume that the government in an attempt to help unemployed workers raises the unemployment benefit they receive. Such an increase will result in the rise of  $b$  from 60% to 70%.

Effects of a change in the unemployment benefit			
<i>variables</i>	b=0.5	b=0.6	b=0.7
$\theta$	0.11291505	0.04320488	0.01655724
$u$	0.10637496	0.16138284	0.23714253
$\Pi_F(L)$	6.08230546	7.19792565	7.74493635
$\Pi_F(M)$	10.0800689	13.5922076	17.3205204
$W_F$	9.28051624	12.3133512	15.4054036
$J_E(L)$	514.704472	368.368328	274.884507
$J_U(L)$	481.371138	339.7969	251.807584
$J_E(M)$	77.8238483	70.0971432	61.8948551
$J_U(M)$	67.7437794	56.5049356	44.5743347
$W_W$	511.15864	363.75739	269.411987

Table 5

We can see from Table 5 that there is a decline in  $\theta$ . Additionally, there is a rise in the firms' value functions and a decline in the workers' value functions. Finally, there is a rise in the unemployment rate since it depends negatively on  $\theta$ , and  $\theta$  is falling.

This kind of policy increases the workers welfare since the government gives them a higher unemployment wage balancing their loss from staying unemployed, an extra incentive not to increase their search efforts. A higher  $b$  gives an incentive to workers to stay at the unemployment pool. On the other hand, increasing the unemployment benefit has also a positive effect to the firms' welfare condition.

Such a policy results in increasing their welfare because the more workers have an incentive to stay at the unemployment pool the larger is the firms' probability that they will find a match and move to production.

Again, using by Table 5 and equation (54), it is clear that increasing the unemployment benefit harms the host-country's welfare as well as increases the unemployment rate. Thus, this kind of policy is not the proper one to be applied by the government.

6.5. Different bargaining power between natives and immigrants

We assume that although native workers participate in a symmetric bargaining with  $\beta_L = 0.5$ , illegal immigrants participate in an asymmetric bargaining where workers receive 40% of the surplus and firms receive 60%.

Effects of a decrease in the immigrants' bargaining share			
variables	$\beta_M=0.5$	$\beta_M=0.4$	$\beta_M=0.3$
$\theta$	0.04320488	0.14304817	0.60855067
$u$	0.16138284	0.09564411	0.04877474
$\Pi_F(L)$	7.19792565	4.4621234	2.35267386
$\Pi_F(M)$	13.5922076	12.9709067	11.1454684
$W_F$	12.3133512	11.26915	9.38690949
$J_E(L)$	368.368328	611.738711	1185.85225
$J_U(L)$	339.7969	583.167282	1157.28083
$J_E(M)$	70.0971432	74.0581867	79.301273
$J_U(M)$	56.5049356	65.4109156	74.5246437
$W_W$	363.75739	609.006022	1184.45869

Table 6

In this case, a fall in  $\beta_M$ , increases the market tightness,  $\theta$ . We can see that in this case there is a decrease in the unemployment rate, since  $\theta$  increases. Such a policy leads to decreases in both firms' value functions either it matches a native worker or an illegal immigrant due to its larger bargaining share. We can see that this leads to a decrease in the firms' welfare.

On the other hand, both natives' and immigrants' value functions increase. This is happening because the market tightness is pretty close to unity and this leads in balancing the number of vacancies to the number of the unemployed worker. It is clear from the third column of Table 6, that when the immigrants' bargaining share decreases even more the unemployment rate falls almost to 4%.

Concerning the total welfare, it is clear that a decrease in the immigrants' bargaining share results into a better welfare condition for the host-country and also into a lower unemployment rate. This means that such a policy can be used successfully by the government in order to improve the welfare of the host-country.

6.6. Effects of a change in the entry fee

We assume that in order to enter a labor market a firm must pay an entry fee equal to  $v_0$ . When  $v_0 = 0$  then there is free entry condition for all firms aiming to enter the market as we assumed in our benchmark case.

In this point we will assume that the government applies a policy which thinks of entry fee as a necessary condition in order to enter the market. We assume that the fee a candidate firm has to pay for allowing her to enter the market and post a job vacancy is  $v_0 = 0.1$ .

Effects of a change in the entry fee			
variables	$v_0=0$	$v_0=0.2$	$v_0=0.4$
$\theta$	0.04320488	0.04200531	0.04084782
$u$	0.16138284	0.16233964	0.16521523
$\Pi_F(L)$	7.19792565	7.45944975	7.72061111
$\Pi_F(M)$	13.5922076	13.8730321	14.1527867
$W_F$	12.3133512	12.5903157	12.8663516
$J_E(L)$	368.368328	363.488664	358.714281
$J_U(L)$	339.7969	334.974378	330.257139
$J_E(M)$	70.0971432	69.7193293	69.3438693
$J_U(M)$	56.5049356	56.0462971	55.5910826
$W_W$	363.75739	358.832358	354.012728

Table 7

We can see that an increase in the entry fee firms have to pay in order to enter a labor market leads to a decrease in the market tightness. Another important effect is that it increases the unemployment rate. Both of firms' value functions increase leading to a better welfare condition for firms.

On the other hand, both native and immigrant workers' value functions worsen after an increase in the entry fee. This leads to a subsequent fall in the workers' welfare condition.

Eventually, looking at Table 7 together with (54) we can see that an increase in the entry fee a firm has to pay in order to participate in the labor market has a negative effect in total welfare. That is increasing entry fee results in a worst welfare condition concerning the host-country. Thus, such a policy should not be preferred in order to improve host-country's welfare.

## **7. Conclusion**

In this paper we have examined the case of illegal immigration into a search and matching context, where firms and workers search for each other aiming to form a pair. The existence of different types of workers gives a different twist in the existing literature concerning this kind of unemployment models. Like Ortega (2000), we also use illegal immigration in order to result in whether their existence in the host-country's economy has positive or negative influence.

In this context, firms have to choose whether they will employ an illegal migrant, giving him a smaller wage than the native worker and enjoying greater profits, and whether they will employ a native worker contributing to the decrease of the native workers' unemployment rate. Workers wage differences play an important role in the firms' decision. From the workers' point of view, their bargaining power lies into their "outside options", i.e. their valuation of time, unemployment benefits etc.

Although the biggest part of the literature like the paper of Harris and Todaro (1970) highlight a negative effect of immigration in the condition of natives, we keep up with the results of Ortega (2000) who uses a dynamic two-country economy in order to show that immigration has a positive effect on natives' wages and unemployment rate. Here, we extend his analysis by examining specific policy changes and how do these affect our models' variables as well as the host-country's welfare condition.

Through the government policies we can realize which one of them has positive influence in the host-country's unemployment rate and welfare condition. We can see that the existence of immigrants in the host-economy has a positive influence. Thus, both border and internal enforcement should not be tightened. The proper policy measures should aim in letting immigrants enter freely into the host-country and get employed by native employees. Lower border and internal enforcement leads to lower unemployment and larger welfare conditions. Similarly, an increase in subsidy to job advertising will lead to lower an unemployment rate and to a

better host-country's welfare condition. On the other hand, an increase in the unemployment benefit will not benefit either the unemployment rate or the welfare condition of the host-country. Thus, the proper government policy should be to decrease the unemployment benefit in order to give unemployed an incentive to search for a job. This eventually leads to a lower unemployment rate and to better welfare conditions. In the case of a different bargaining power between natives and immigrants, we can see that a lower bargaining power by immigrants leads to a decrease in the unemployment rate and to an increase in the host-country's welfare. Finally, a government should not use a policy of increasing the entry fee because then it increases the unemployment rate and lowers the host-country's welfare. Thus, the government should instead lower the entry fee or use a free entry condition in order to achieve lower unemployment rate and better welfare conditions.

Our policy results show that, the number of illegal immigrants has a positive effect upon the market tightness while the immigrant workers' separation effect on the market tightness is negative. Thus, the domestic government should apply policies that increase the number of immigrants or decrease their separation probability, or a combination of these policies.

To sum up, illegal immigrants play an important role in a country's economy, being employed in "bad-quality" jobs and receiving a lower wage. Their existence in the host-country is critical since a slight increase in their number improves the country's welfare and reduces its unemployment rate.

## Appendices

### Appendix A

In order to find the immigrant workers' wage we must substitute (35), (36) and the fact that  $\Pi_v = v_0$  into (25) we get

$$\begin{aligned} \frac{w_M}{(r+s_M+\mu)} &= \frac{\beta_M}{1-\beta_M} \left[ \frac{y-w_M+s_M v_0}{r+s_M} - v_0 \right] \Rightarrow (1-\beta_M)(r+s_M)w_M = \beta_M(r+s_M+\mu)(y-w_M-rv_0) \\ \Rightarrow & \quad [(1-\beta_M)(r+s_M) + \beta_M(r+s+\mu)]w_M = \beta_M(r+s_M+\mu)(y-rv_0) \\ \Rightarrow & \quad w_M = \frac{\beta_M(r+s_M+\mu)(y-rv_0)}{[(1-\beta_M)(r+s_M) + \beta_M(r+s_M+\mu)]} \end{aligned} \quad (37)$$

### Appendix B

In order to find the native workers' wage we must substitute (38), (39) and  $\Pi_v = v_0$  and  $z = bw_L$  into (25) we get

$$\begin{aligned} \frac{(1-b)w_L}{(r+s_L+\mu)} &= \frac{\beta_L}{1-\beta_L} \left[ \frac{y-w_L+s_L v_0}{r+s_L} - v_0 \right] \Rightarrow (1-\beta_L)(r+s_L)(1-b)w_L = \beta_L(r+s_L+\mu)(y-w_L-rv_0) \\ \Rightarrow & \quad (1-\beta_L)(r+s)(1-b)w_L + \beta_L(r+s_L+\mu)w_L = \beta_L(r+s_L+\mu)(y-rv_0) \\ \Rightarrow & \quad [(1-\beta_L)(r+s_L)(1-b) + \beta_L(r+s_L+\mu)]w_L = \beta_L(r+s_L+\mu)(y-rv_0) \\ \Rightarrow & \quad w_L = \frac{\beta_L(r+s_L+\mu)(y-rv_0)}{[(1-\beta_L)(r+s_L)(1-b) + \beta_L(r+s_L+\mu)]} \end{aligned} \quad (40)$$

### Appendix C

In order to find an expression of  $\mu$  we substitute (43) and (45) into (28)

$$\begin{aligned} rv_0 = -\gamma y + \frac{\mu}{\theta} \left[ \frac{(1-\alpha)(1-\beta_M)(y-rv_0)}{(1-\beta_M)(r+s_M) + \beta_M(r+s_M+\mu)} + (1-\alpha)v_0 \right] + \frac{\mu}{\theta} \left[ \frac{\alpha(1-\beta_L)(1-b)(y-rv_0)}{(1-\beta_L)(1-b)(r+s_L) + \beta_L(r+s_L+\mu)} + \alpha v_0 \right] - \frac{\mu}{\theta} v_0 \\ \Rightarrow rv_0 = -\gamma y + \frac{\mu}{\theta} \left[ \frac{(1-\alpha)(1-\beta_M)(y-rv_0)}{(1-\beta_M)(r+s_M) + \beta_M(r+s_M+\mu)} + \frac{\alpha(1-\beta_L)(1-b)(y-rv_0)}{(1-\beta_L)(1-b)(r+s_L) + \beta_L(r+s_L+\mu)} \right] \\ \Rightarrow \frac{(rv_0 + \gamma y)\theta}{\mu} = \frac{(1-\alpha)(1-\beta_M)(y-rv_0)}{(1-\beta_M)(r+s_M) + \beta_M(r+s_M+\mu)} + \frac{\alpha(1-\beta_L)(1-b)(y-rv_0)}{(1-\beta_L)(r+s_L)(1-b) + \beta_L(r+s_L+\mu)} \\ \Rightarrow \frac{(rv_0 + \gamma y)\theta}{\mu} = \frac{(1-\alpha)(1-\beta_M)(y-rv_0)}{(r+s_M + \beta_M\mu)} + \frac{\alpha(1-\beta_L)(1-b)(y-rv_0)}{(r+s_L)(1-b + \beta_L b) + \beta_L\mu} \end{aligned}$$

$$\begin{aligned}
 &\Rightarrow \frac{(rv_0 + \gamma y)\theta}{\mu}(r + s_M + \beta_M \mu)[(r + s_L)(1 - b + \beta_L b) + \beta_L \mu] = \\
 &(1 - \alpha)(1 - \beta_M)(y - rv_0)[(r + s_L)(1 - b + \beta_L b) + \beta_L \mu] + \alpha(1 - \beta_L)(1 - b)(y - rv_0)(r + s_M + \beta_M \mu) \\
 &\Rightarrow \frac{(rv_0 + \gamma y)\theta}{\mu}(r + s_M + \beta_M \mu)(r + s_L)(1 - b + \beta_L b) + \frac{(rv_0 + \gamma y)\theta}{\mu}(r + s_M + \beta_M \mu)\beta_L \mu = \\
 &(1 - \alpha)(1 - \beta_M)(y - rv_0)(r + s_L)(1 - b + \beta_L b) + (1 - \alpha)(1 - \beta_M)(y - rv_0)\beta_L \mu \\
 &+ \alpha(1 - \beta_L)(1 - b)(y - rv_0)(r + s_M) + \alpha(1 - \beta_L)(1 - b)(y - rv_0)\beta_M \mu \\
 &\Rightarrow \frac{(rv_0 + \gamma y)\theta}{\mu}(r + s_M)(r + s_L)(1 - b + \beta_L b) + \frac{(rv_0 + \gamma y)\theta}{\mu}(r + s_L)(1 - b + \beta_L b)\beta_M \mu \\
 &+ \frac{(rv_0 + \gamma y)\theta}{\mu}(r + s_M)\beta_L \mu + \frac{(rv_0 + \gamma y)\theta}{\mu}\beta_L \beta_M \mu^2 = \\
 &(1 - \alpha)(1 - \beta_M)(y - rv_0)(r + s_L)(1 - b + \beta_L b) + (1 - \alpha)(1 - \beta_M)(y - rv_0)\beta_L \mu \\
 &+ \alpha(1 - \beta_L)(1 - b)(y - rv_0)(r + s_M) + \alpha(1 - \beta_L)(1 - b)(y - rv_0)\beta_M \mu \tag{46}
 \end{aligned}$$

### Appendix D

Substituting (48) and (49) into (46) we have

$$\begin{aligned}
 &(rv_0 + \gamma y)(r + s_M)(r + s_L)(1 - b + \beta_L b)\theta^{1/2} + (rv_0 + \gamma y)(r + s_L)(1 - b + \beta_L b)\beta_M \theta^{1/2}\theta^{1/2} \\
 &+ (rv_0 + \gamma y)(r + s_M)\beta_L \theta^{1/2}\theta^{1/2} + (rv_0 + \gamma y)\beta_L \beta_M \theta^{1/2}(\theta^{1/2})^2 = \\
 &(1 - \alpha)(1 - \beta_M)(y - rv_0)(r + s_L)(1 - b + \beta_L b) + (1 - \alpha)(1 - \beta_M)(y - rv_0)\beta_L \theta^{1/2} \\
 &+ \alpha(1 - \beta_L)(1 - b)(y - rv_0)(r + s_M) + \alpha(1 - \beta_L)(1 - b)(y - rv_0)\beta_M \theta^{1/2} \\
 &\Rightarrow \theta[(rv_0 + \gamma y)(r + s_L)(1 - (1 - \beta_L)b)\beta_M + (rv_0 + \gamma y)(r + s_M)\beta_L] \\
 &+ \theta^{1/2}\left[(rv_0 + \gamma y)(r + s_L)(r + s_M)(1 - (1 - \beta_L)b) + (rv_0 + \gamma y)\beta_M \beta_L\right] \\
 &\left[-(1 - \alpha)(1 - \beta_M)(y - rv_0)\beta_L - \alpha(1 - \beta_L)(1 - b)(y - rv_0)\beta_M\right] \\
 &+ [-(1 - \alpha)(1 - \beta_M)(y - rv_0)](r + s_L)(1 - (1 - \beta_L)b) - \alpha(1 - \beta_L)(1 - b)(y - rv_0)(r + s_L) = 0 \tag{50}
 \end{aligned}$$

Having found equation (50) we can now find an expression of  $\theta$  as the positive root of the secondary equation.

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*Immigration and Unemployment: A Search and Matching Approach*  
Pournazis Konstantinos

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*Immigration and Unemployment: A Search and Matching Approach*  
Pournazis Konstantinos

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