A GARCH MODEL FOR MONERO

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Abstract

The purpose of this research was to find the best fit model between GARCH, GJR , TGARCH , EGARCH models for the log-closing prices Monero(XMR) from 09/11/2017 – 31/08/2023. We estimated all the models mention above, in the Normal, students t, GED, skewed t and skewed GED distributions and using the AIC information criterion we tried to select the best model. Then we repeated the same process after we split the sample into two sub-samples and the date of the split (12/03/2020) was provided by the Bai-Perron Structural Break test.

1.Introduction

In econometrics one of the most used model of the Ordinary Least Squares. The model thought cannot actually deal with the matters that we can observe in the real-world data. One of the main defaults is that it assumes that the expected value of all error terms, when squared, is the same at any given point. This assumption is called homoskedasticity but what happens when this condition is not met? How can an econometrician overcome this problem? While conventional time series and econometric models operate under an assumption of constant variance, the ARCH (Autoregressive Conditional Heteroskedastic) process introduced in Engle (1982) allows the conditional variance to change over time as a function bf past errors leaving the unconditional variance constant. In the real-world data more times than often the variances of the error terms are not equal, when this condition is met the we have to deal with the problem of heteroscedasticity. An answer to the question above was provided by Eangle (1982) with the ARCH (Autoregressive Conditional Heteroscedasticity) and later generalized by Bolleslev (1986) who introduced the GARCH model (Generalised Autoregressive conditional heteroscedasticity). The main point of these models is that the regression coefficients for an ordinary least squares regression are still unbiased but the variance gets to be modeled. In this way both the deficiencies of the least squares are corrected and we can also have a prediction of each error term. In the modern era of internet things have gone beyond the classical stocks and bonds. Nowadays the financial products are several and sometimes very complicated both in understanding and in modelling them.

The introduction of Bitcoin in 2009 by Nakamoto revolutionized the financial exchanges. Bitcoin is an open source software which allows people to conduct economic transaction without an intermediate institution. Prior to Bitcoin there were others digital cash technologies although proven to be insufficient or detectable or even susceptible to software attacks. The first bitcoin transaction was concluded in 12 January 2009 after that the rest is history. Unlike fiat currency, Bitcoin is created, distributed, traded, and stored using a decentralized ledger system known as a blockchain. Bitcoin and its ledger are secured by proof-of-work (PoW) consensus, which also secures the system and verifies transactions. At the moment bitcoin is the largest cryptocurrency in circulation with over 600 billion in market cap and a price of over 30.000 $ per bitcoin with all time high $67,566.83 per unit.

In this paper we focus on another cryptocurrency called Monero(XMR).

Monero is also a cryptocurrency which uses blockchain technology with privacy-enhancing properties in order to achieve anonymity and fungibility. The transaction is a Peer – to- Peer process and Observers cannot decipher addresses trading Monero, transaction amounts, address balances, or transaction histories. Like the Bitcoin Monero is an open source software based on the CryptoNote technology. The Monero protocol includes various methods to obfuscate transaction details, though users can optionally share view keys for third-party auditing. Transactions are validated through a miner network running RandomX, a prof of work (PoW) algorithm. The algorithm issues new coins to miners and was designed to be resistant to Application-specific integrated circuit. (ASIC) mining. An Interesting fact is that Monero currently has the Third largest community of developers after Bitcoin and Ethereum.

Although the cryptocurrencies are a form of digital asset they experience the same or better say, some of the same properties of the traditional financial assets. That menas that in the ways of financial analysis someone could say that they have similarities such as volatility clustering or bubbles or breaks or phenomena of extreme leverage etc. so an analyst can use the same tools to analyze them. Under the term of risk management and volatility analysis and forecasting there has been a very big discussion and literature of researchers trying to model and forecast these parameters using a variety of tools such as MS-GARCH models (ardia et all 2019)

AR-CGARCH (katsiampa 2017), other asummetrical GARCH models such as Egarch, regime switching type, APARCH FIGRCH, IGARCH, TARCH and many more (Dyhberg 2016, Caporale 2019, Panagiotidis 2018,2022, and many others). Results that can be drawn from the existing literature that we encompassed is that there is not one specific model that is the best to analyze the volatility, rather it depends on the kind of the dataset, the size, the frequency, the economic shocks and the kind of specification one wants to include in his research.

In this paper our main focus will be to find a model that fits good to our dataset. We will estimate a GARCH , a GJR GARCH, a TARCH and EGRCH model for our dataset and try to determine which is better based on the provided information criteria. The analysis will be done for every distribution that is available on the econometric program we will use, GRETL. Then we will check for any possible structural breaks if there are any we are going to split the sample and repeat the same analysis for each given sub-sample. After that we will try to determine which of the 2 types analysis gives better results and try to provide the News Impact Curve for the best selected models.

From this point forward section 2 is the literature review, section 3 presents the data and methodology of our research, the results are presented and discussed in section 4 and section 5 concludes.

2.Literature review

(Caporale & Zekokh, 2019) fitted over 1000 GARCH models for four cruptocurrencies (BTC, ETH, RIPPLE,) in order to estimate the one step ahead Value at Risk and the Expected Shortfall using the method of rolling stats. The results of their work is that the classical Garch models may lead to not good predictions in reguadrs of the VAR and ES and tha of course is not a good sign for the risk management of the volatility, instead they suggest models that allow the assumetries and the switching of regimes.

(Ardia, Bluteau, Boudt, & Catania, Forecasting risk with Markov-switching GARCH models: A large-scale performance study, 2018) in their empirical analysis compare the standard one regime GARCH model to the MS-GARCH models to see which one responds better in aspects of risk management. They used the daily, weekly, and ten-day equity log-returns in order to prove that the MS-GARCH models fit better and forecast better the Value at Risk while taking into account the left fat tales. This research was made with stock data and suggest the use of MS-GARCH models but state that the traditional ones have a good forecasting ability if the parameter <<uncertainty>> is included.

(Ardia, Bluteau, & Ruede, Regime changes in Bitcoin Garch volatility Dynamics, 2019) used an MS_GARCH modes (MARKOV-SWITCHING) to test if the with this regime change the volatility an the Value at risk can be better forecasted than the tradition GARCH models. The research was done using 2 regimes and the daily log-returns of Bitcoin and proved that the models that use this technique are better at forecastin and modellin volatility and VaR. They found that the only in the normal distribution a 3 regime model overcomes the other that are mentioned before.

In (Katsiampa , 2017) study we find that she used many GARCH-type models to measure the conditional heteroscedasticity of Bitcoin. The estimated results suggested that the best fit model for the data was an AR-CGARCH model and states that it is very important to include both a short term and a long-term component for the conditional variance.

(Dyhberg, 2016) classified the Bitcoin as an asset somewhere between gold and American dollar he researcheded the vollatitliy of bitcoin using a garch model and an asummetrical garch model (egarch) and found that there are summilarities in to gold an the American dollar an said that bitcoin can be used for hedging for risk averse investors.

(Trucios & Taylor, 2021)used a thre stage analysis for the Value at Risk and Expected shortfall estimation. tin the first stage they find the best model to estimetate the Value at Risk and evalueate their out of sample performance secondary they extend the use of these models to estimate the Expected shortfall and finally they evaluate the use of forecast combination strategies as a way to deal with misspecification. The data that they used is the daily closing prices of the four major cryptocurrencies Bitcoin, Ethereum, ,Litecoin and Ripple.

(Mensi, Al-Yahyaee, & Kang, 2019) used the daily spot prices of Ethereum (ETH) and Bitcoin (BTC) for over 7 years to determine wether the structural breaks impact on the long memory of the volatility of these cryptos. They used 4 different Garch models, classic Garch , Fractionally Integrated Garch (FIGARCH) , the Fractionally Integrated Assymetric Power ARCH (FIAPARCH) and the Hyperbolic GARCH (HYGARCH) and found that the dual long memory property of Bitcoin and Ethereum contrast the market efficiency and random walk hypothesis,meaning that after the structural breaks there is long memory in both the mean and the variance.

(Panagiotidis, Stengos, & Papapanagiotou, On the volatility of cryptocurrencies, 2022) take 292 cryptocurrencies and run 27 GARCH-type models in each of them in an attempt to evaluate in a large-scale the performance of the traditional MS-GARCH modles in concerns of the volatility modelling. They use a 3-regime analysis and test for goodness of fit forecasting capabilities and Value at Risk. They perform an in sample and out of sample analysis and come to the conclusion that the biggest percentage of the cryptocurrencies (52%) the time varriyng models out perform the traditional ones and are a better modelling selection for the volatility.

(Panagiotidis, Stengos, & Papapanagiotou, A Bayesian Approach for the Determinants of Bitcoin Returns, 2023) try to determine the factors that drive the price of the Bitcoin. More specific 32 factors that affect Bitcoin are taken into account and using a Bayesian LASSO with stochastic volatility approach examine the potential determinats of Bitcoin returns. This model takes into account the time-varying volatility of Bitcoin and also shrinks the coefficients of weakly related variables faster than other methods(ML,OLS). The result of this research is that the main determinamts of bitcoin returns is the attractiveness and the difficulty of mining and that Bitcoin is related to stock market returns and the exchange rates.

In another research (Cicbaric, 2020) tests which type of ARCH/GARCH models better fit the cryptos Ethereum, Neo, Ripple, Litecoin, Dash, Zcash, Dogecoin. the results claims that the EGARCH model is the best fit for Ethereum, Zcash, Neo, PARCH is the best for Ripple while fore the others the results are inconclusive and depend on the selected distribution and information criteria.

Our research will be focused on the Monero (XMR) cryptocurrency. Monero was first introduced by Ricardo Spagni in 2014 and is based on the idea of the Cryptonote tecnology which is a concept first described in 2013. Monero promotes privacy, decentralization and scalability. For our analysis we used the daily closing prices of Monero (XMR) from 09/11/2017 up to 31/08/2023. We tried to perform a GARCH analysis on the sample of 2127 observations and determine using Information criteria which Garch fits better for the modelling of volatility and one-step ahead forecast.

3.DATA AND METHOGOLOGY:

In this point we are going to review the theory of the models used for our analysis.

ARCH:

$$\sigma_t^2 = \omega + \sum_{\iota=1}^{q} \alpha_\iota \varepsilon_{t-i}^2$$

GARCH:

$$\sigma_t^2 = \omega + \sum_{\iota=1}^{q} \alpha_\iota \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \ \sigma_{t-j}^2$$

condition:

$\alpha_\iota + \beta_j < 1$ (stationary condition)

IGARCH:

$$\sigma_t^2 = \omega + \varepsilon_{t-1}^2 + \sum_{i=2}^{q} a_i \left(\varepsilon_{t-i}^2 - \varepsilon_{t-1}^2 \right) + \sum_{j=1}^{p} \beta_j \left(\sigma_{t-j}^2 - \varepsilon_{t-1}^2 \right)$$

GJR-GARCH:

$$\sigma_t^2 = \omega + \sum_{\iota=1}^{q} \alpha_\iota + \gamma_i I e_{t-1}^2 + \sum_{j=1}^{p} \beta_j \ \sigma_{t-j}^2$$

Condition:

$$e_{t-i} > 0$$

EGARCH:

$$\log(\sigma_t^2) = \omega + \sum_{\iota=1}^{q} [\alpha_\iota e_{t-i} + \gamma_i (|e_{t-i}|) - E(|e_{t-i}|)] + \sum_{j=1}^{p} \beta_j \log(\sigma_{t-j}^2)$$

TGARCH:

$$\sigma_t^2 = \omega + \sum_{\iota=1}^{q} \alpha_\iota \left[(1-\gamma_i)e_{t-i}^{+} - (1+\gamma_i)e_{t-i}^{-}\right] + \sum_{j=1}^{p} \beta_j \ \sigma_{t-j}$$

Equations provided by (Hansen & Lunde, 2005)

Analysis:

We used the daily closing prices of Monero (XMR) from 09/11/2017-31/08/2023 which we downloaded from YahooFinance. As a first step we take a look at the time series plot and the descriptive statistics (close), next we are going to create the logarithmic first differences of our times series (Id_close) in order to deal with trend. This is the variable that we are going to use for the rest of our analysis. After we check again the descriptive stats and time series and correlogram we are going to test if our series is appropriate for GARCH. In order to do so we are going to test first the stationarity of our series with an ADF test (Augmented Dickey-Fuller). If our series is stationary (I(0)) then we will test for volatility clustering. This can be examined from the autocorrelation-partial-autocorelation which we get from the correlogram. Next we are going to run an OLS one our series and determine if there are ARCH effects on the series.

If all the previous conditions are met then we can move on to a GARCH model.

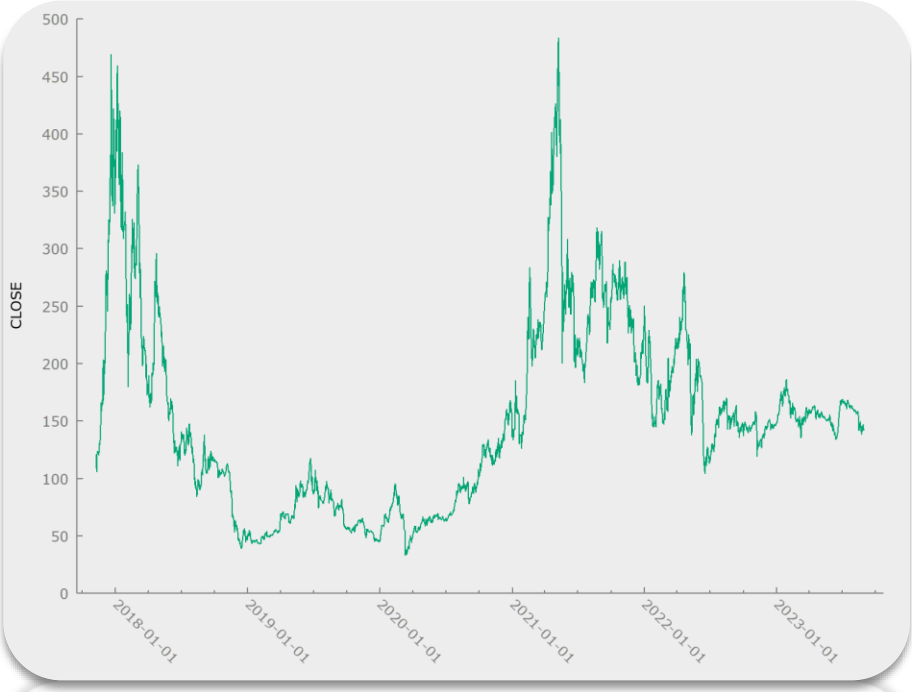
As we will see later on the results all the conditions are met, the series is stationary we have volatility clustering and ARCH effects so we move one to a GARCH. The model we are going to be estimating is going to be a GARCH(1,1). This decision was made firstly by looking at the correlogram and was later confirmed after we run higher orders of GARCH models and came to the find that the coefficients for the ARCH and GARCH terms were not statistically significant at 5% level of confidence. For the evaluation of the GARCH(1,1) we are going to conclude in our model some dummy variables in order to compensate for the most extreme spikes (outliers) in our sample. So the dummy variables I1,I2,I3,I4 will be inserted and each of them is going to take an value of 1 for the two most extreme positive observations(I3,I4) and zero for all the other observations , in the same way for the most negative observations (I1,I2).

The results of the tests portray that when using a single regime sample the GARCH(1,1) is produces a better fit model than the other asymmetrical, GJR, TARCH, EGARCH, models in every distribution that we tested. We test our data and perform our analysis in all the possible distribution because we want to take a better grasp of the probability of extreme outliers occurring and if so which one can take a better measure of them. As the results came back we noticed that most of the distributions can incorporate the extreme outliers except the student's T distribution and the skewed version of it, in which significantly larger coefficients were produced.

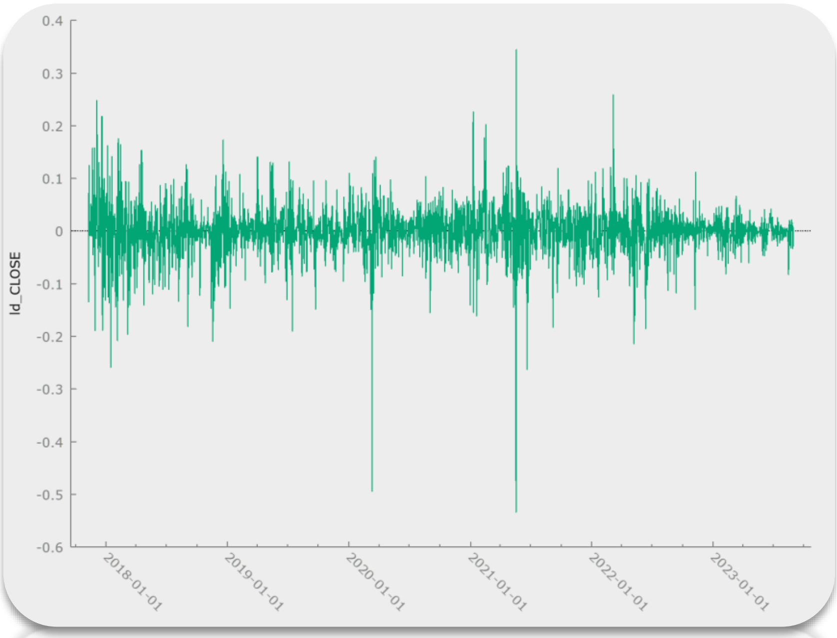
In the second phase of our analysis we are going to use the same dataset but this time with a different approach. We perform a bai-perron test for structural breaks. The test reveals three dates of possible structural breaks. The date chosen was the 12/03/2020 and is the date of the biggest volatility clustering and extreme outlier of the three dates. Next we are going to split the sample into two sub-samples the first one starts at 09/11/2017 and finishes at 12/03/2020 and the second starts at 13/12/2020 and ends at 31/08/2023. Now we are going to perform the same analysis as we did when we treated the sample as a whole.

At this point we present all the descriptive stats, diagrams and test we performed in order to verify that the series was appropriate for GARCH analysis.

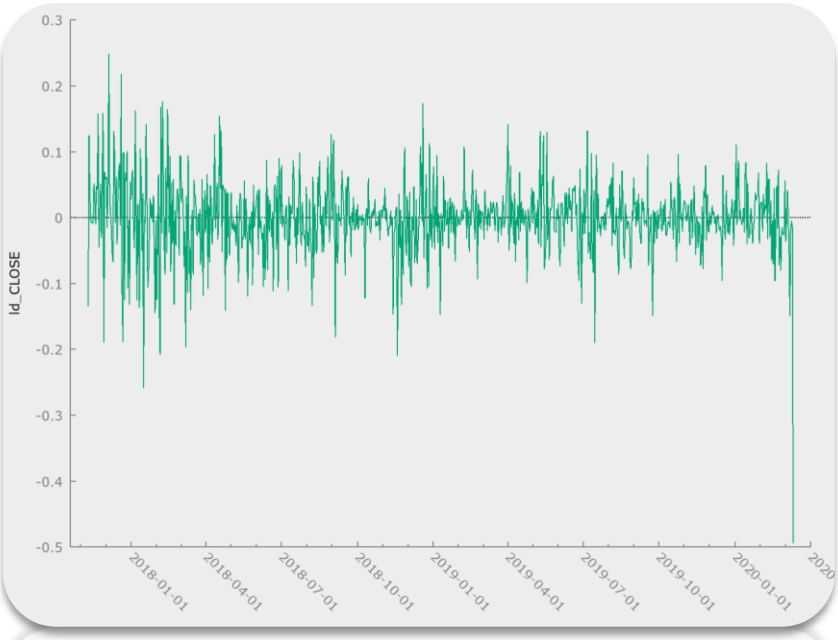
1.time series plot of close



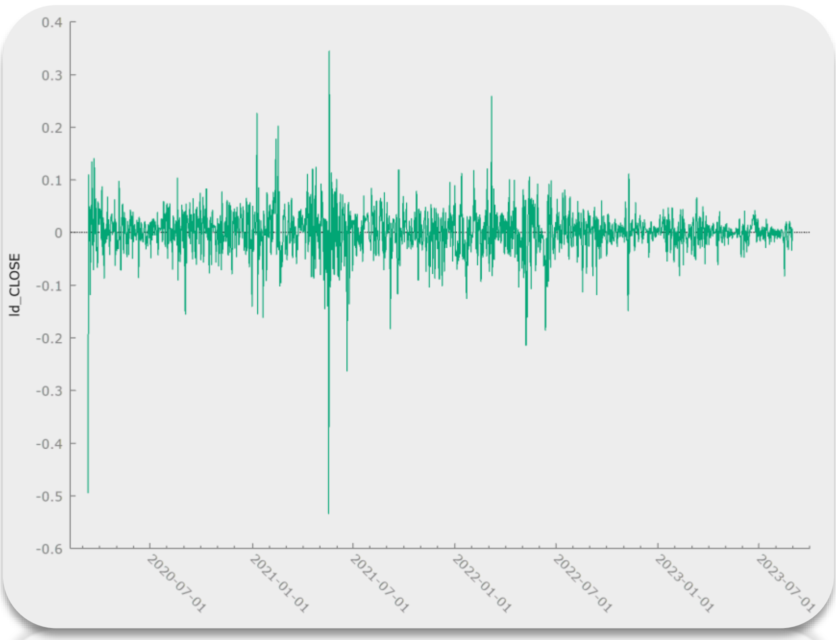
2.time series plot of ld_close



3.time series plot of sub-sample 1

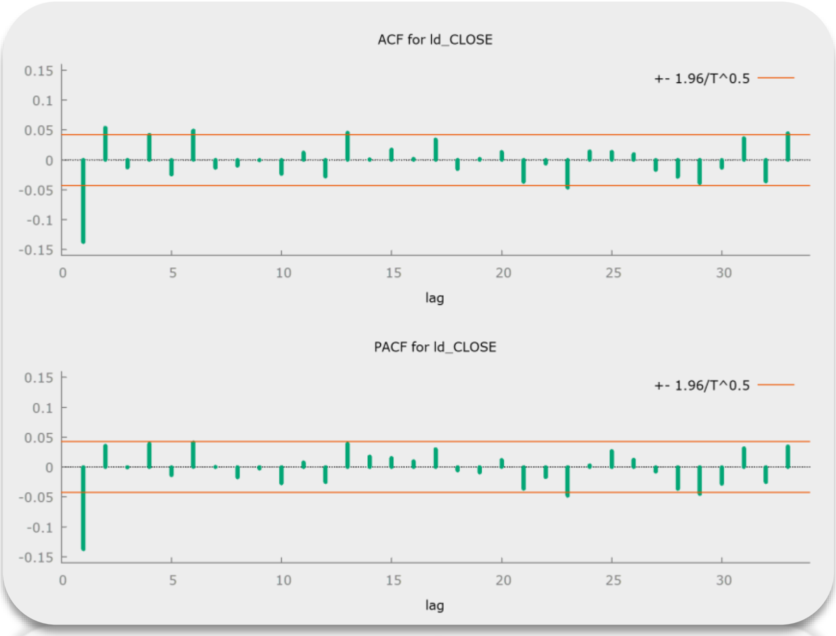


4.time series plot of sub-sample 2

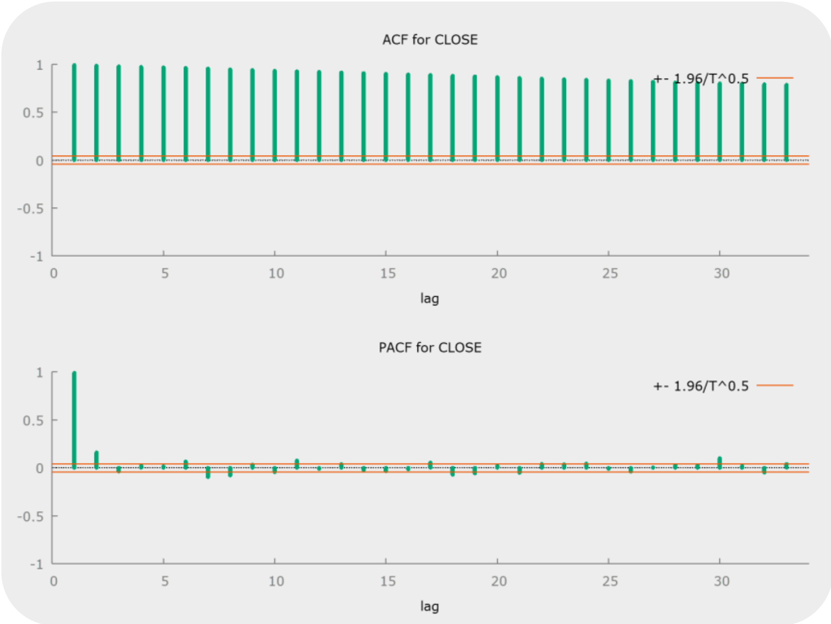


Corellogram for close and ld_ close

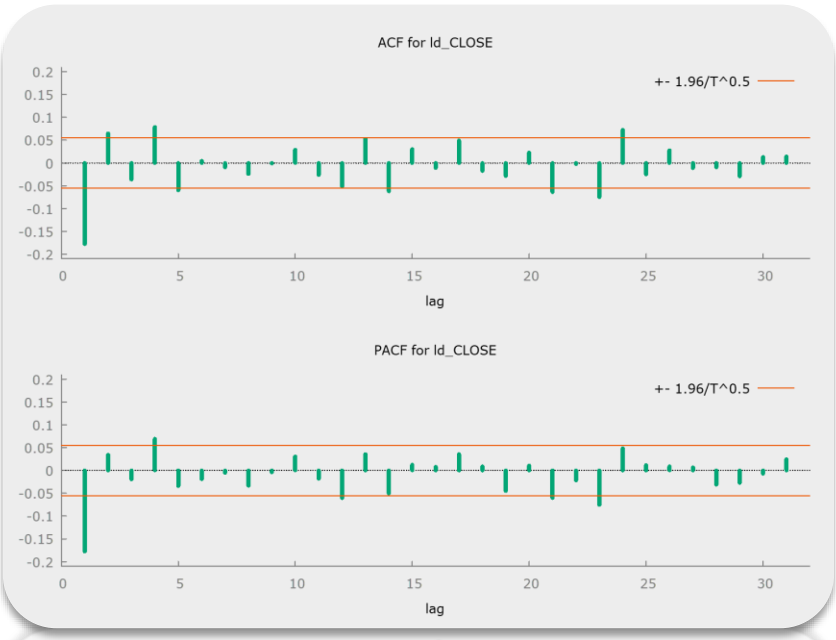
5. correlogram of ld_close



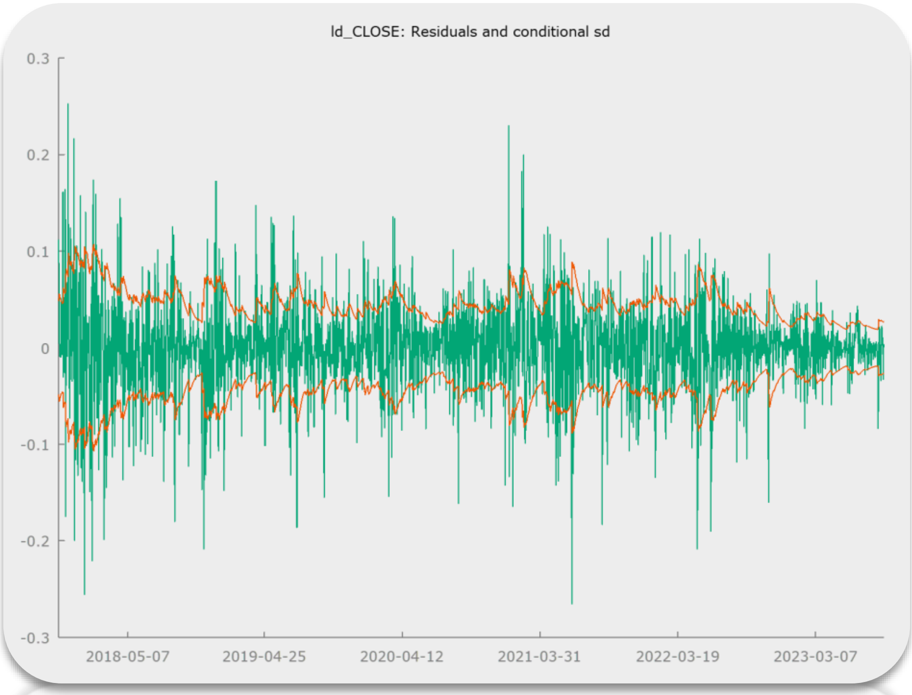
6.correlogram of close



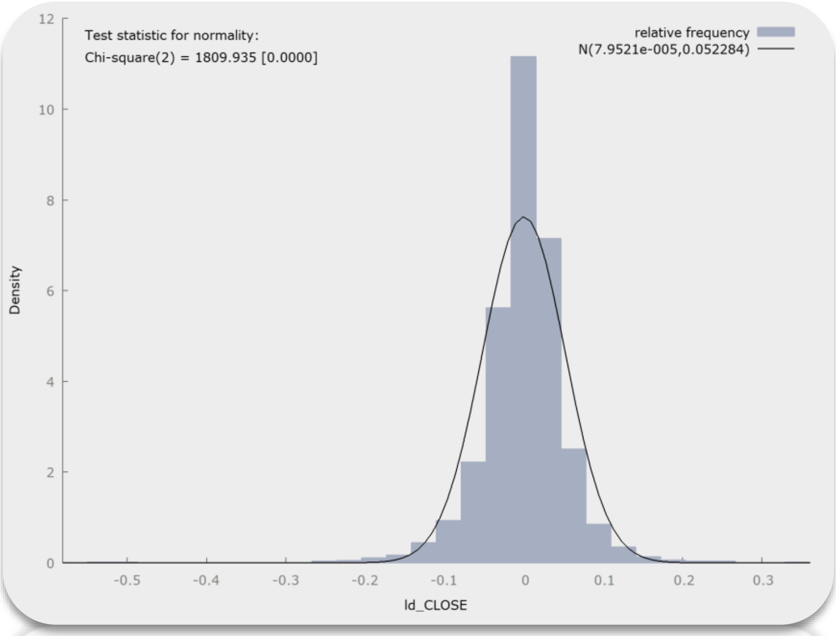
7.correlogram of sub-sample 1



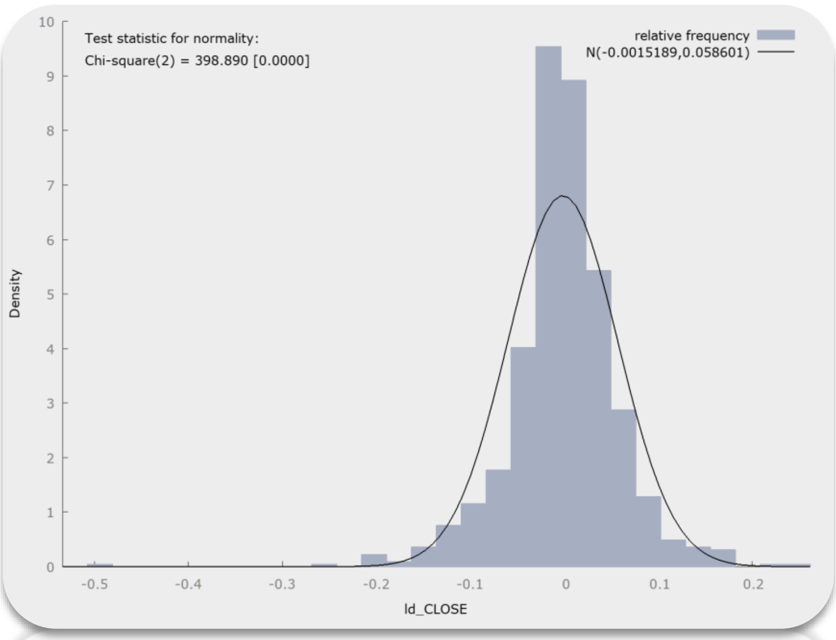
8.correlogram of sub-sample



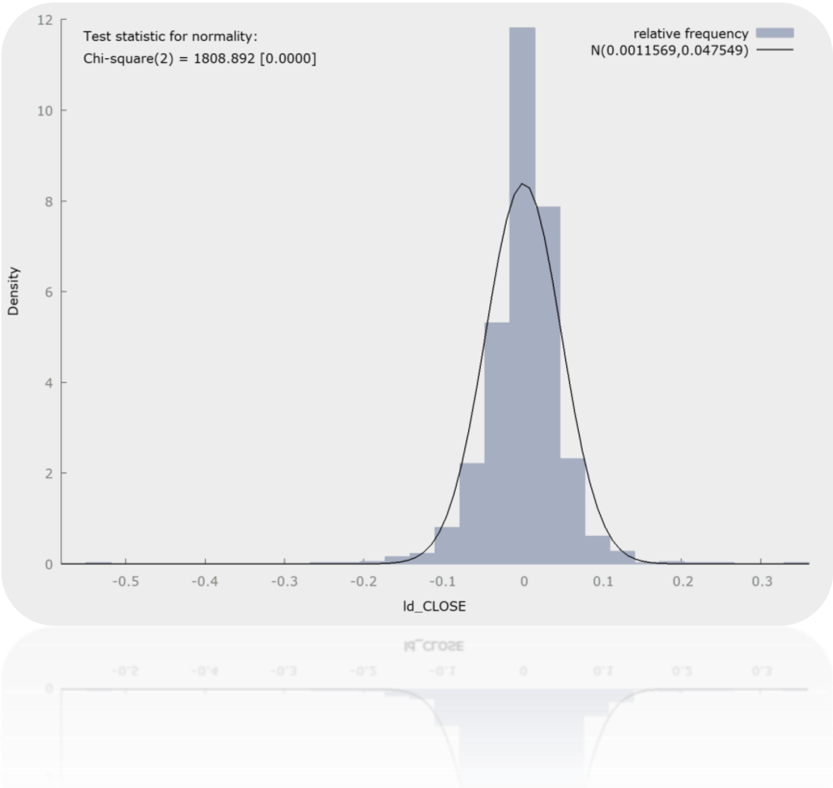
9.ld_close frequency Distribution



10.sub-sample 1 frequency Distribution



11.sub-sample 2 frequency Distribution



4.Results

Descriptive stats:

At this point we present the descriptive statistics for the series of closing price of Monero (close), the log differences of this price which we created afterwards (ld_close), the log differences of the same price after we spit the sample into to two sub-samples, sub-sample 1 represents the log differences of the price from 11/09/2017-12/03/2020 and sub-sample 2 the log differences of the price from 13/03/2020-31/08/2023

Table 1. Descriptive stats

	close	ld_close	sub-sample 1	sub-sample 2
Mean	150.10	7.9521e-005	-0.0015189	0.0011569

Median	143.84	0.0019990	-0.0012807	0.0031854
Minimum	33,01	-0.53418	-0.49421	-0.53418
Maximum	483.58	0.34493	0.24824	0.34493
Standard deviation	82,334	0.052284	0.058601	0.047549
C.V.	0.54852	657.49	38,581	41,099
Skewness	0.99726	-0.95662	-0.82962	-1.0486
Ex. kurtosis	0.96808	11,859	7,367	17,221
5% percentile	49,938	-0.083870	-0.10271	-0.072587
95% percentile	305.21	0.074749	0.091937	0.065933
Interquartile range	111.86	0.048139	0.053347	0.043616

In this point we present the results of the analysis described above. In the first stage we present the GARCH tables of every distribution as and the News Impact Curve of the selected models of each distribution.

Full range analusis

NORMAL DISTRIBUTION

Table 2. Normal-Distribution

coefficients	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	2.30402e-05	2.05742e-05	4.45975e-05	-0.290929
	(0.1370)	(0.1501)	(0.0534)*	(0.0015)***
Alpha	0.0755498	0.0718638	0.0865865	0.201464
	(0.0014)***	(0.0031)***	(0.0004)***	(5.76e-05)***
Beta	0.917773	0.922665	0.916597	0.976046
	(2.26e-281)***	(1.10e-289)***	(2.25e-277)***	(0.0000)***
Gamma	-	-0.126844	-0.0626801	-0.0217485
		(0.0708)*	(0.5482)	(0.4235)
AIC	-7180.33008	-7202.86435	-7207.17038	-7031.09766

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

$$\sigma_t^2 = 0.0755498\varepsilon_{t-1}^2 + 0.917773\sigma_{t-1}^2$$

We chose the GARCH (1,1) model because it’s the only model in this table that meets the theoretical condition. In the GARCH (1,1) we can see that a+b<1 and are both statistically significant in the 5% confidence band. The GJR and TGARCH (EGARCH) models fail to produce a statistical significant (in 5% confidence band) and positive (negative) gamma term as the theory suggests so the are excluded.

t-DISTRIBUTION

Table 3. t-Distriburion

coefficients	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	9.10613e-06	8.73541e-06	2.21939e-05	-0.250676
	(0.2719)	(0.2321)	(0.0648)*	(0.0013)***
Alpha	0.103130	0.106324	0.115833	0.230576
	(2.21e-05)***	(3.68e-06)***	(4.12e-08)***	(3.66e-08)***
Beta	0.900081	0.908492	0.910284	0.985081
	(0.0000)***	(0.0000)***	(0.0000)***	(0.0000)***
Gamma	-	-0.185062	-0.162359	0.0362042
		(0.0002)***	(0.0264)**	(0.2307)
AIC	-7446.86505	-7458.14009	-7468.72350	-7421.80518

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

$$\sigma_t^2 = 0.103130\ \varepsilon_{t-1}^2 + 0.900081\sigma_{t-1}^2$$

We chose the GARCH (1,1) model because it's the only model in this table that meets the theoretical condition. In the GARCH (1,1) we can see that a+b=1 and are both statistically significant in the 5% confidence band. The GJR and the TGARCH models although they have a statistical a,b,g coefficients the gamma term is negative which would mean that the higher the risk the lower the return which violates the theory that suggests that gamma should be positive, so these models are excluded. The EGARCH model is also excluded because the gamma term is not statistically significant.

GED-DISTRIBUTION

Table 4. GED-distribution

	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	1.36015e-05	1.29838e-05	2.96240e-05	-0.254048
	(0.1595)	(0.1405)	(0.0424)**	(0.003)***
Alpha	0.0836388	0.0852155	0.0990137	0.203176
	(9.76e-05)***	(4.14e-05)***	(3.45e-06)***	(3.69e-07)***
Beta	0.916080	0.915925	0.915005	0.982683
	(0.0000)***	(0.0000)***	(0.0000)***	(0.0000)***
Gamma	-	-0.169506	-0.127951	0.0162484
		(0.0014)***	(0.1045)	(0.2812)
AIC	-7421.26981	-7429.27447	-7436.58102	-7367.11240

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

$$\sigma_t^2 = 0.0836388\ \varepsilon_{t-1}^2 + 0.916080\sigma_{t-1}^2$$

We chose the GARCH (1,1) model because it's the only model in this table that meets the theoretical condition. In the GARCH (1,1) we can see that a+b<1 and are both statistically significant in the 5% confidence band. The GJR and (EGARCH) models fail to produce a statistical significant (in 5% confidence band) and positive (negative) gamma term as the theory suggests so they are excluded. The TGARCH model has statistical significant gamma term but it is not positive which is inconsistent with the theory so it is also excluded.

Skewed t-DISTRIBUTION

Table 5. Skewed t-Distribution

	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	8.33632e-06	7.66496e-06	2.14763e-05	-0.254875
	(0.3129)	(0.2852)	(0.0759)*	(3.84e-05)***
Alpha	0.103804	0.106492	0.118527	0.235327
	(4.47e-05)***	(5.86e-06)***	(8.46e-08)***	(5.56e-09)***
Beta	0.910047	0.907739	0.908138	0.985209
	(0.0000)***	(0.0000)***	(0.0000)***	(0.0000)***
Gamma	-	-0.182926	-0.158329	0.0360031
		(0.0001)*	(0.0273)**	(0.0239)**
AIC	-7452.48539	-7463.83279	-7474.93136	-7428.64623

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

$$\sigma_t^2 = 0.103804\varepsilon_{t-1}^2 + 0.910047\sigma_{t-1}^2$$

The GJR , the TGARCH and (EGARCH) models although they have a statistical a,b,g coefficients the gamma term is negative (positive) which would mean that the higher the risk the lower the return which violates the theory that suggests that gamma should be positive (negative), so these models are excluded. The GARCH(1,1) model has a+b>1 which suggests that the model has extreme outliers. The a,b coefficients are statistically significant and this is the model chosen.

Skewed GED-DISTRIBUTION

Table 6. Skewed GED-Distribution

	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	1.24035e-05	1.16756e-05	2.81481e-05	-0.251134
	(0.0472)**	(0.0381)**	(8.30e-010)***	(1.34e-010)***
Alpha	0.0826707	0.0838383	0.0996497	0.202881
	(1.03e-041)***	(1.37e-035)***	(1.48e-042)***	(1.32e-019)***
Beta	0.917498	0.916597	0.914629	0.983325
	(0.0000)***	(0.0000)***	(0.0000)***	(0.0000)***
Gamma	-	-0.168260	-0.131237	0.0168428
		(3.97e-010)***	(0.0444)**	(0.2423)
AIC	-7427.72285	-7435.86554	-7444.16861	-7376.33359

Note. P-values are presented in the parenthesis

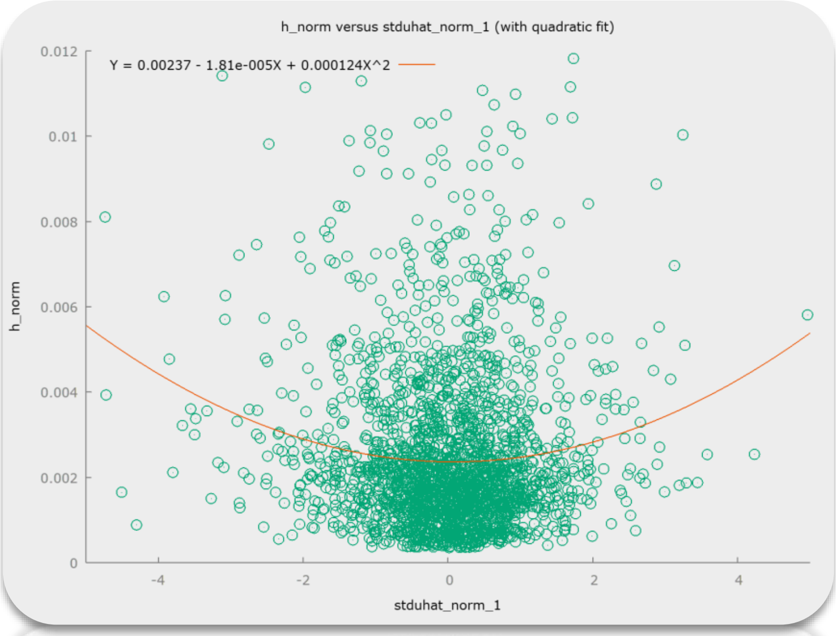
The variance equation of the selected GARCH(1,1) model is :

$$\sigma_t^2 = 1.24035e - 05 + 0.0826707\varepsilon_{t-1}^2 + 0.917498\sigma_{t-1}^2$$

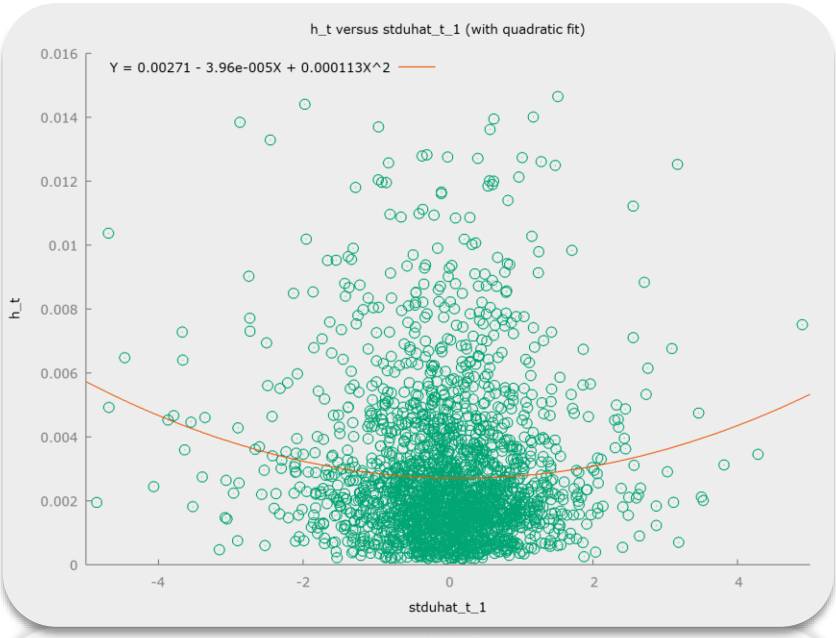
In this distribution we choose the GARCH(1,1) model, a+b<1 and both statistically significant. GJR and TGARCH once again have the same problem with the non positive gamma coefficient and the EGARCH model doesn't have a statistically significant gamma term so they are excluded.

- The results of these GARCH tables for the 5 distributions tested are that the best fit according to this dataset and research is the GARCH(1,1) model. As we can see above the main problem with the othe models are that the TARCH and GJR model although they produce some statistically significant leverage coefficients in some distributions those coefficients are not positive as the theory a priori demands. The negative “gamma” coefficients would suggest that the higher the risk the lower the return which is not compatible we the theory. As for the EGARCH model no stastistically significant and negative leverage terms (gamma) were conducted from these tests. so the results conclude that the best – fit model is the GARCH (1,1) model because all the alpha and betas are positive and the sum a+b<1 in most distribution an equal to 1 in the t-ditribution which makes this an Integrated Garch model and is probably caused by extreme volatility clustering and outliers.

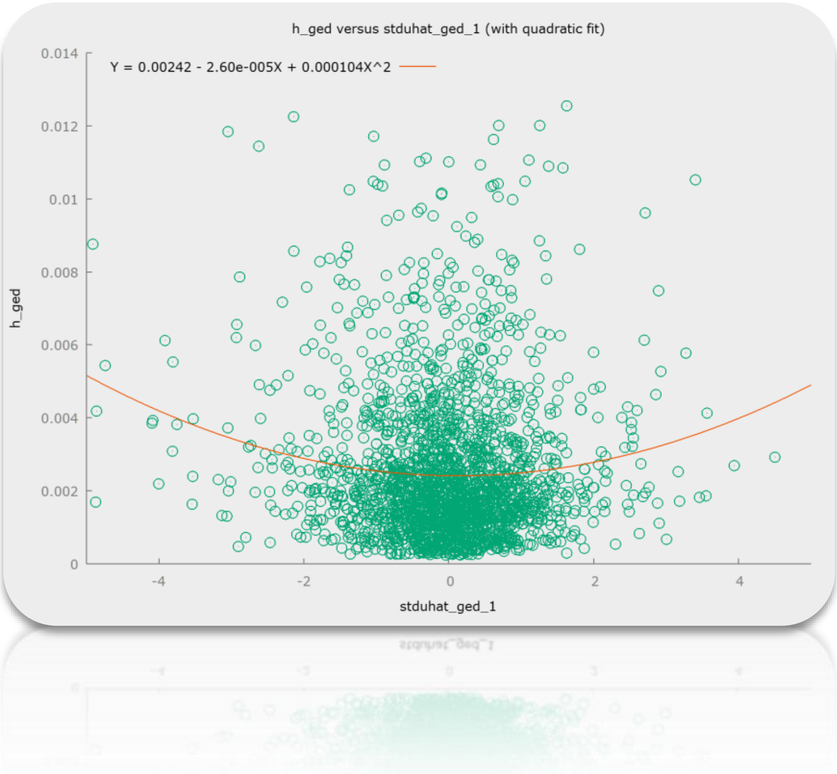
12.News Impact Curve normal distribution GARCH(1,1)



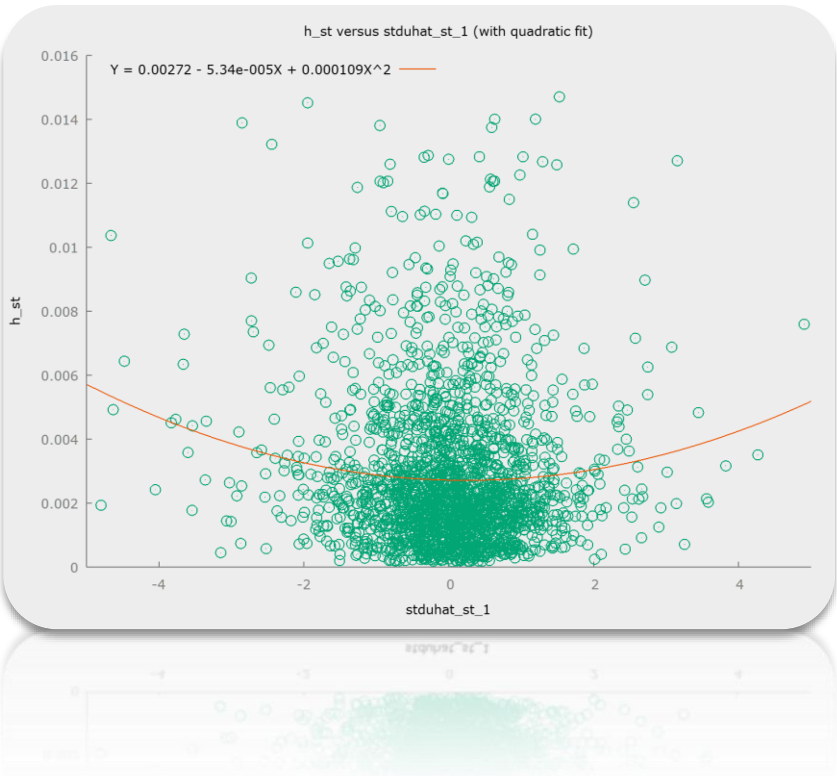
13.News Impact Curve t-distribution GARCH(1,1)



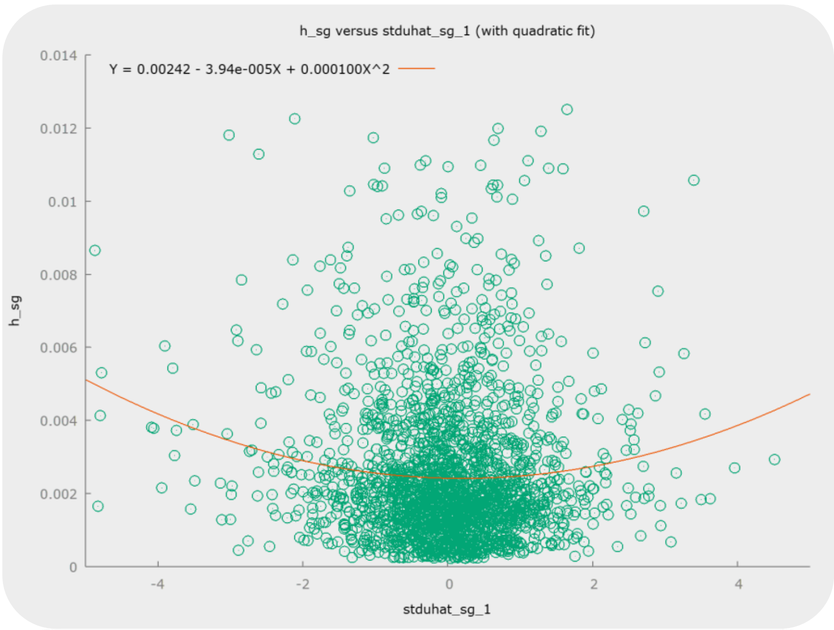
14.News Impact Curve GED-distribution GARCH(1,1)



15.News Impact Curve skewed t-distribution GARCH(1,1)



16.News Impact Curve skewed ged-distribution GARCH(1,1)



Structural break analysis:

We performed a bai-perron test for structural breaks and the results suggested that there are 3 possible breaks in our series. We are going to split our sample at the date of the biggest outlier. So the sub-sample 1 is from 11/09/2017-12/03/2020 and the second from 13/03/2020-31/08/2023. The date of the break is 12/03/202. In each sub-sample we are going to deploy the use of two dummy variables in order to compensate for any extreme outliers. So in sub sample I1 is the dummy which takes 1 at the minimum lowest observation (12/03/2020) and I2 takes 1 for the maximum observation (15/12/2017). The results from the GARCH tables for each distribution are demonstrated below.

Sub-sample 1:

Date: 11/09/2017-12/03/2020

NORMAL DISTRIBUTION

Table 7. Normal Distribution

	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	7.57722e-05	8.39958e-05	0.000118650	-0.329257
	(0.4588)	(0.1900)	(0.1507)	(0.0289)**
Alpha	0.0617474	0.0658477	0.0785107	0.168387
	(0.0510)*	(0.0613)*	(0.0297)**	(0.0095)***
Beta	0.912438	0.905463	0.902356	0.963711
	(1.10e-092)***	(6.58e-070)***	(5.26e-072)***	(0.0000)***
Gamma	-	-0.112929	-0.126628	-0.0294060
		(0.3019)	(0.4500)	(0.5005)
AIC	-2603.73128	-2603.64843	-2600.46662	-2521.76954

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

$$\sigma_t^2 = 0{,}0617474\varepsilon_{t-1}^2 + 0.912438\sigma_{t-1}^2$$

The selected model here is GARCH(1,1) with a+b<1 and are statistically significant in the 10% confidence band (for less the alpha is statistically not significant). The GJR, TGARCH, EGARCH do not have a statistically significant leverage term (gamma) so they are excluded.

T-DISTRIBUTION

Table 8. T-distribution

	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	5.44344e-05	5.25513e-05	7.64567e-05	-0.285242
	(0.2476)	(0.2600)	(0.1733)	(0.0342)**
Alpha	0.109574	0.112929	0.131253	0.245632
	(0.0231)**	(0.0175)**	(0.0035)***	(0.0008)***
Beta	0.891850	0.890231	0.886306	0.979148
	(4.93e-082)***	(2.81e-086)***	(7.44e-093)***	(0.000)***
Gamma	-	-0.146499	-0.154464	0.0321613
		(0.0755)*	(0.2056)	(0.2573)
AIC	-2684.27218	-2685.13162	-2689.11159	-2673.78249

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

$$\sigma_t^2 = 0{,}109674\varepsilon_{t-1}^2 + 0.891850\sigma_{t-1}^2$$

In this distribution we choose the GARCH(1,1) with a+b<1 and are statistically significant in the 5% confidence band. The GJR, TGARCH, EGARCH do not have a statistically significant leverage term (gamma) so they are excluded.

GED-DISTRIBURION

Table 9. Ged-Distribution

	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	6.17640e-05	6.54198e-05	9.46807e-05	-0.285242
	(0.1749)	(0.1855)	(0.1318)	(0.0342)**
Alpha	0.0802347	0.0842506	0.100883	0.245632
	(0.0222)**	(0.0212)**	(0.0081)***	(0.0008)***
Beta	0.901617	0.896460	0.894631	0.979148
	(2.96e-096)***	(2.77e-085)***	(8.92e-089)***	(0.0000)***
Gamma	-	-0.0456473	-0.142906	0.0321613
		(0.1253)	(0.2783)	(0.2573)
AIC	-2683.03389	-2683.08472	-2683.80005	-2673.78249

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

$$\sigma_t^2 = 0{,}109674\varepsilon_{t-1}^2 + 0.891850\sigma_{t-1}^2$$

In the GED distribution we choose the GARCH(1,1) with a+b<1 and are statistically significant in the 5% confidence band. The GJR, TGARCH, EGARCH do not have a statistically significant leverage term (gamma) so they are excluded.

SKEWED T-DISTRIBUTION

Table 10. Skewed t-Distribution

	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	5.61171e-05	5.34765e-05	7.66733e-05	-0.285548
	(0.2425)	(0.2553)	(0.1740)	(0.0333)**
Alpha	0.110420	0.112721	0.130917	0.244354
	(0.0237)**	(0.0176)**	(0.0037)***	(0.0008)***
Beta	0.890409	0.889305	0.886168	0.979082
	(2.61e-079)***	(1.21e-084)***	(9.87e-092)***	(0.0000)***
Gamma	-	-0.142373	-0.151373	0.0308951
		(0.0884)*	(0.2207)	(0.2769)
AIC	-2682.85308	-2683.54697	-2687.25818	-2672.03298

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

$$\sigma_t^2 = 0{,}110420\varepsilon_{t-1}^2 + 0.890409\sigma_{t-1}^2$$

We chose the GARCH (1,1) model because it's the only model in this table that meets the theoretical condition. In the GARCH (1,1) we can see that a+b=1 and are both statistically significant in the 5% confidence band the fact that a+b=1 is because of the extreme outliers and the ability of the distribution to encompass them. The GJR, TGARCH, EGARCH do not have a statistically significant leverage term (gamma) so they are excluded.

SKEWED GED-DITRIBUTION

Table 11. Skewed GED-Distribution

	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	6.20529e-05	6.54808e-05	9.50115e-05	-0.302520
	(0.0270)**	(0.0517)*	(0.0657)*	(0.0001)***
Alpha	0.0802102	0.0840613	0.101265	0.197236
	(3.62e-05)***	(0.0003)***	(0.0023)***	(6.27e-07)***
Beta	0.901404	0.896407	0.894333	0.972313
	(0.0000)***	(8.09e-226)***	(2.55e-127)***	(0.0000)***
Gamma	-	-0.134705	-0.144855	0.00669235
		(0.0584)*	(0.2320)	(0.7481)
AIC	-2681.05762	-2681.11453	-2681.80905	-2654.39008

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

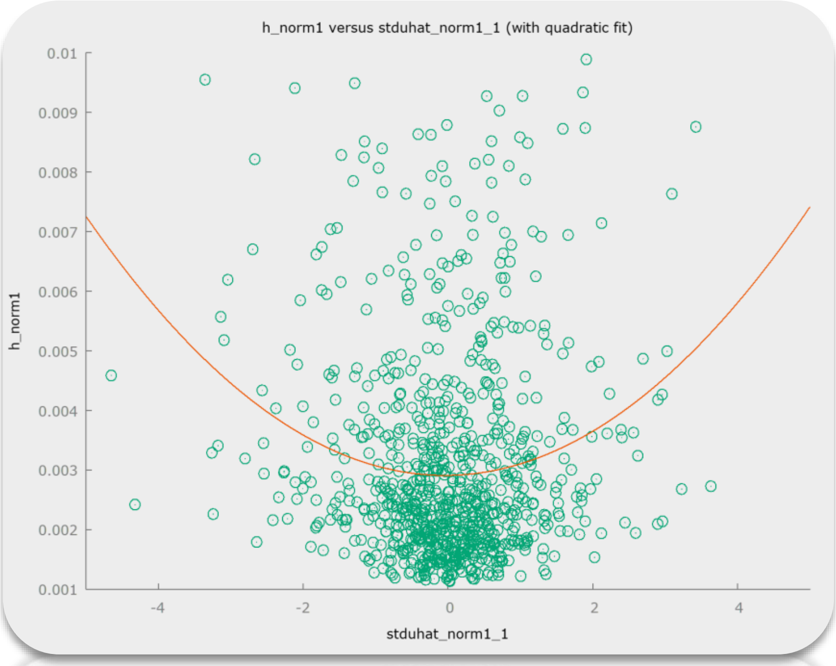
$$\sigma_t^2 = 6.20529\text{e-}05 + 0{,}0802102\varepsilon_{t-1}^2 + 0.901404\sigma_{t-1}^2$$

In this distribution we choose the GARCH(1,1) with a+b<1 and are statistically significant in the 5% confidence band. The GJR, TGARCH, EGARCH do not have a statistically significant leverage term (gamma) so they are excluded.

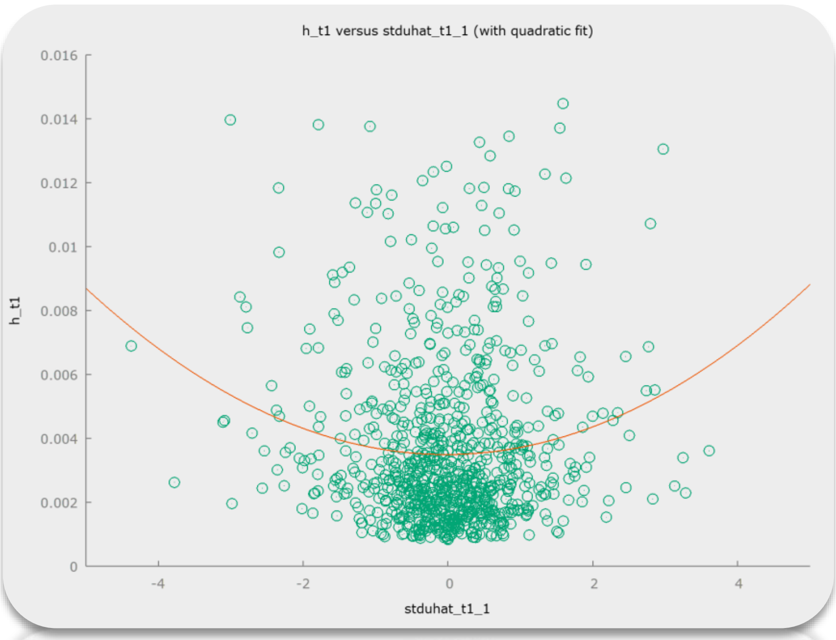
- In the first sub-sample we examined we found the following results. First the GARCH(1,1) is still the best fit model for our data. Second the asymmetrical models still cannot produce a significant leverage coefficient. Third the smaller sample with the dummies seems to compensate better for the extreme outliers since all the GARCH(1,1) coefficients are smaller and last we still observe that the problem with the a+b=1 coefficients is still present only in the T and skewed T distributions.

Next we present all the News Impact Curves of the selected models.

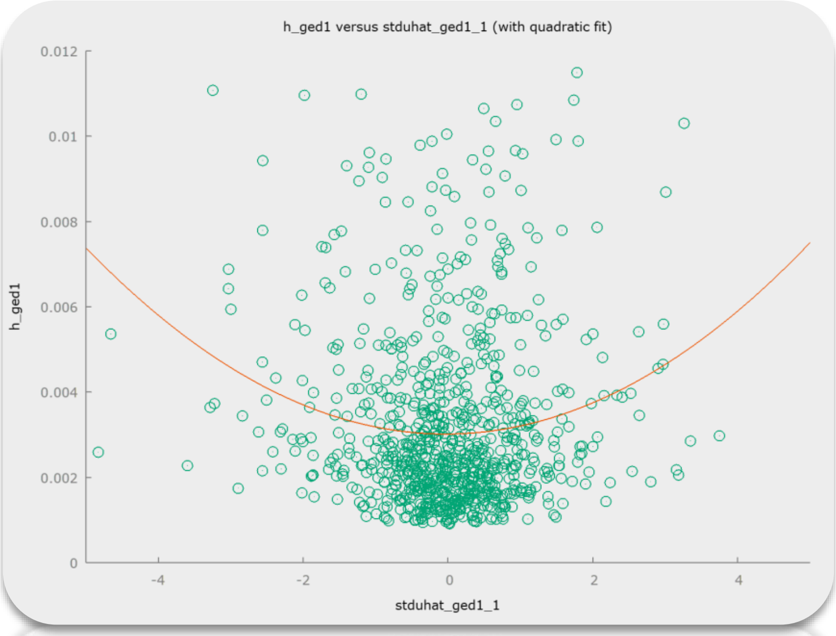
17.News Impact Curve normal distribution GARCH (1,1)



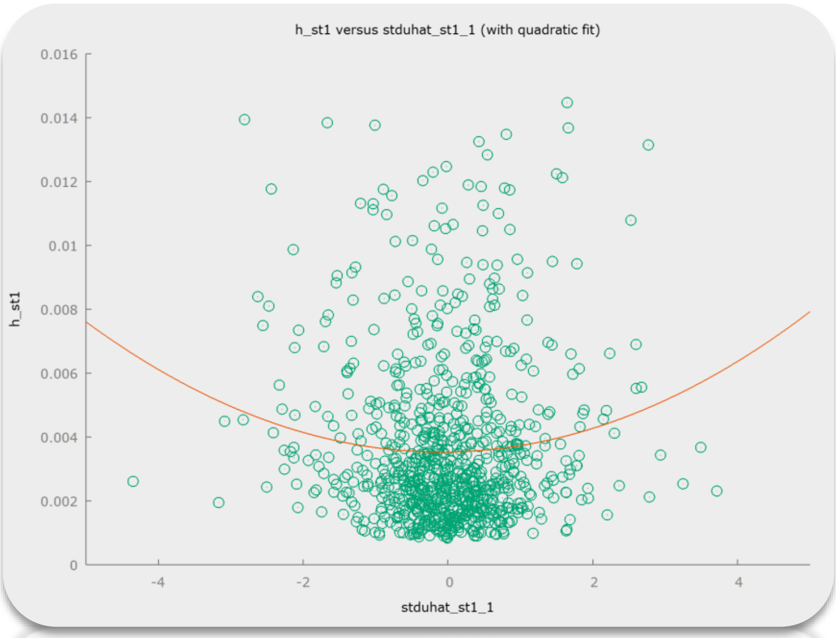
18.News Impact Curve t-distribution GARCH(1,1)



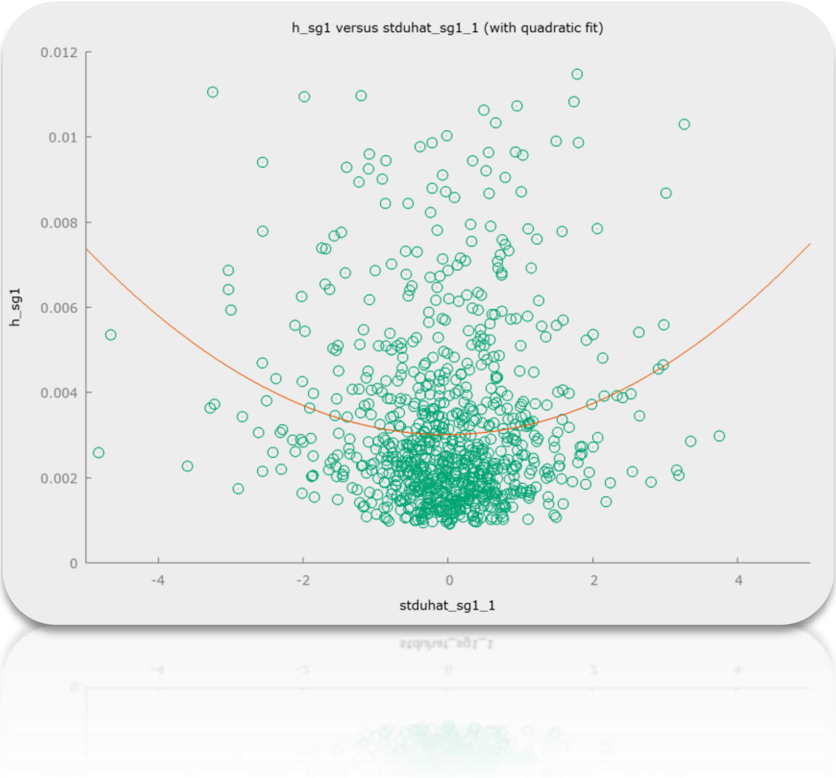
19.News Impact Curve GED-distribution GARCH(1,1)



20.News Impact Curve skewed t-distribution GARCH(1,1)



21.News Impact Curve skewed GED-distribution GARCH(1,1)



In the second sample following the same logic we use the dummies I3=1 at minimum observation (19/05/2021) and I4=1 at maximum (20/05/2021)

Sub-sample 2

Date : 13/03/2020-31/08/2023

NORMAL-DISTRIBUTION

Table 12. Normal Distribution

	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	2.10158e-05	1.95692e-05	3.46783e-05	-0.334142
	(0.1245)	(0.1222)	(0.1057)	(0.0081)***
Alpha	0.105118	0.100263	0.105959	0.238646
	(0.0008)***	(0.0008)***	(0.0007)***	(9.54e-05)***
Beta	0.892348	0.897025	0.902532	0.974879
	(6.48e-214)***	(1.79e-243)***	(1.66e-187)***	(0.0000)***
Gamma	-	-0.138238	-0.0250561	-0.0284478
		(0.1112)	(0.8640)	(0.4791)
AIC	0.892348	-4559.91749	-4570.82600	-4500.98480

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

$$\sigma_t^2 = 0{,}105118\varepsilon_{t-1}^2 + 0.892348\sigma_{t-1}^2$$

First we test the normal distribution and choose the GARCH(1,1) with a+b<1 and are statistically significant in the 5% confidence band. The GJR, TGARCH, EGARCH do not have a statistically significant leverage term (gamma) so they are excluded.

T-DISTRIBUTION

Table 13. t-Distribution

	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	1.15803e-05	1.08848e-05	2.20688e-05	-0.322010
	(0.2246)	(0.1900)	(0.1027)	(0.0007)***
Alpha	0.129640	0.126664	0.123220	0.256155
	(0.0001)***	(2.85e-05)***	(1.23e-05)***	(1.52e-06)***
Beta	0.883026	0.886323	0.899395	0.978625
	(3.02e-243)***	(0.0000)***	(6.00e-299)***	(0.0000)***
Gamma	-	-0.196473	-0.198301	0.0452233
		(0.0047)***	(0.0902)*	(0.0467)**
AIC	-4694.47039	-4700.47204	-4708.87091	-4689.60605

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

$$\sigma_t^2 = 0{,}129640\varepsilon_{t-1}^2 + 0.883026\sigma_{t-1}^2$$

Again, in the T-distribution we have the same problem of a+b=1 and statistically significant in the 5% confidence interval because of the extreme outliers. The GJR model has a statistically significant but negative gamma coefficient so it is excluded for the reasons we have explained before. The TGARCH model does not have a statistically significant gamma coefficient in the 5% confidence interval and therefore is excluded. And last but not least the EGARCH model meets the statistical significant condition but not the negativity of the coefficient so it is also excluded.

GED-DISTRIBUTION

Table 14. GED-Distribition

	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	1.61483e-05	1.53938e-05	2.79764e-05	-0.321427
	(0.1364)	(0.1162)	(0.0824)*	(0.0023)***
Alpha	0.117046	0.114034	0.114149	0.242271
	(0.0001)***	(4.42e-05)***	(4.50e-05)***	(4.42e-06)***
Beta	0.884868	0.888715	0.900609	0.977764
	(6.90e-240)***	(5.80e-303)***	(3.56e-258)***	(0.0000)***
Gamma	-	-0.187236	-0.152194	0.0218453
		(0.0106)**	(0.2250)	(0.3702)
AIC	-4670.80499	-4675.27301	-4682.23600	-4651.65885

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

$$\sigma_t^2 = 0{,}117046\varepsilon_{t-1}^2 + 0.884868\sigma_{t-1}^2$$

In the GED distribution we choose the GARCH(1,1) with a+b<1 and are statistically significant in the 5% confidence band. The GJR has a significant but non negative coefficient and is excluded and the TGARCH, EGARCH do not have a statistically significant leverage term (gamma) so they are excluded.

SKEWED T-DISTRIBUTION

Table 15. Skewed t-Distribution

	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	1.00528e-05	8.65205e-06	2.10470e-05	-0.335112
	(0.2896)	(0.2803)	(0.1400)	(0.0008)***
Alpha	0.135651	0.129691	0.133026	0.274358
	(0.0005)***	(0.0001)***	(3.16e-05)***	(1.42e-06)***
Beta	0.882601	0.886057	0.892976	0.978637
	(4.50e-199)***	(8.50e-280)***	(2.47e-242)***	(0.0000)***
Gamma	-	-0.179834	-0.160194	0.0363171
		(0.0086)***	(0.1585)	(0.1246)
AIC	-4707.47167	-4712.50031	-4723.75970	-4704.51725

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

$$\sigma_t^2 = 0{,}135651\varepsilon_{t-1}^2 + 0.882601\sigma_{t-1}^2$$

In the skewed T-distribution we find the biggest sum of a+b we have encountered this far. Althouth they both are statistically significant at the 5% confidence interval their sum is above 1 (a+b>1). The GJR model has all three coefficients a,b,g statistically significant but gamma is not negative so the model is excluded. The TGARCH model is excluded due to gamma being not significant and so is the EGARCH model.

SKEWED GED-DISTRIBUTION

Table 16. Skewed GED-Distribution

	GARCH (1,1)	GJR(1,1)	TGARCH(1,1)	EGARCH(1,1)
Omega	1.32018e-05	1.21256e-05	2.47539e-05	-0.325822
	(0.1013)	(0.1090)	(0.0031)***	(7.41e-026)***
Alpha	0.121476	0.115422	0.123560	0.256987
	(3.77e-029)***	(2.35e-034)***	(2.53e-010)***	(1.29e-016)***
Beta	0.885464	0.889633	0.895445	0.978814
	(0.0000)***	(0.0000)***	(0.0000)***	(0.0000)***
Gamma	-	-0.158333	-0.0885117	0.00963100
		(8.80e-09)***	(0.3999)	(0.2716)
AIC	-4686.93373	-4689.75542	-4699.54740	-4672.35846

Note. P-values are presented in the parenthesis

The variance equation of the selected GARCH(1,1) model is :

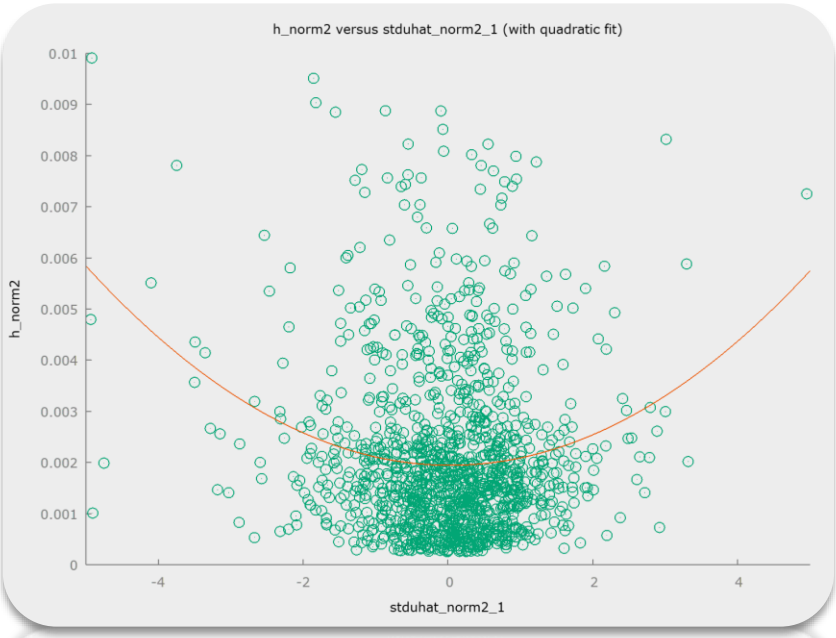
$$\sigma_t^2 = 0{,}121476\varepsilon_{t-1}^2 + 0.885464\sigma_{t-1}^2$$

The GARCH(1,1) here is once again the best choice. The a+b are sliglty above 1 and are statistically significant in the 5% confidence interval.The GJR model has all three coefficients a,b,g statistically significant but gamma is not negative so the model is excluded. The TGARCH model is excluded due to gamma being not significant and so is the EGARCH model.

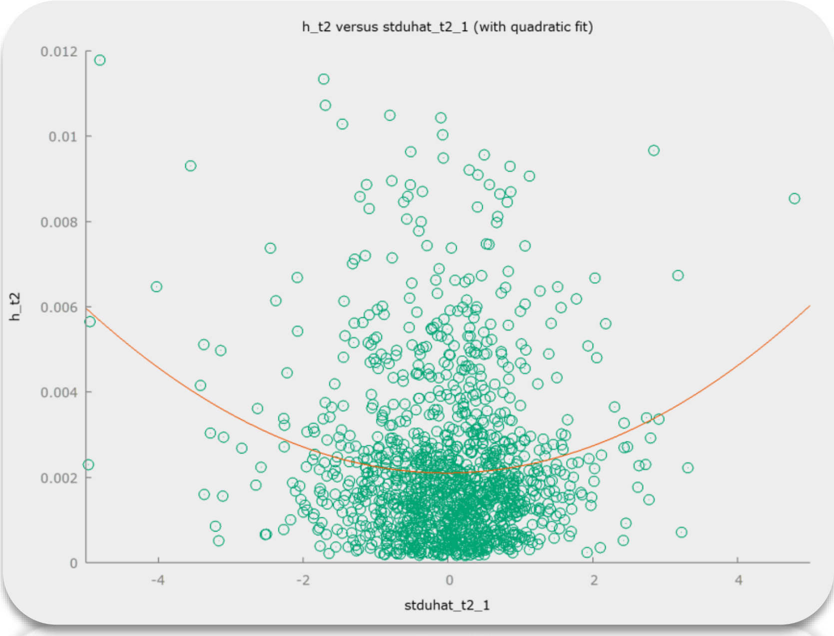
- In the second sub-sample the GARCH(1,1) prevails one more time. The coefficient components of the GARCH(1,1) are slightly amplified in comparison to the first sub-sample especially in the t-distribution. The asymmetrical models fail to produce a good fit for this data set also.

Next we present all the News Impact Curves of the selected models.

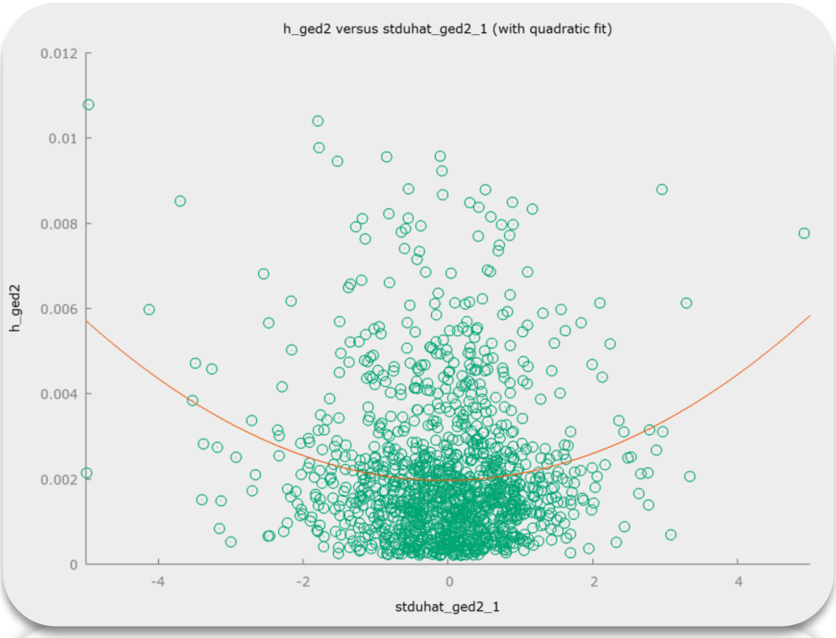
22.News Impact Curve normal distribution GARCH(1,1)



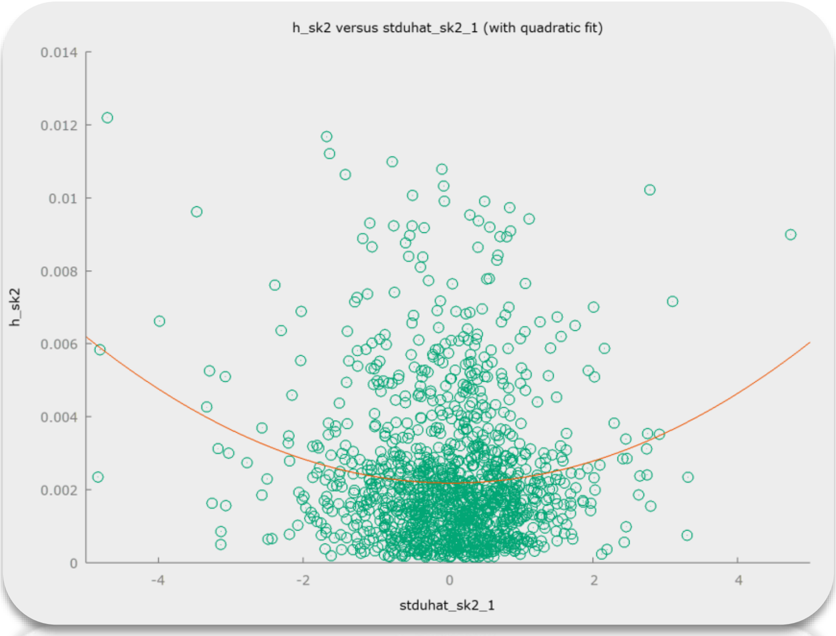
23.News Impact Curve t-distribution GARCH(1,1)



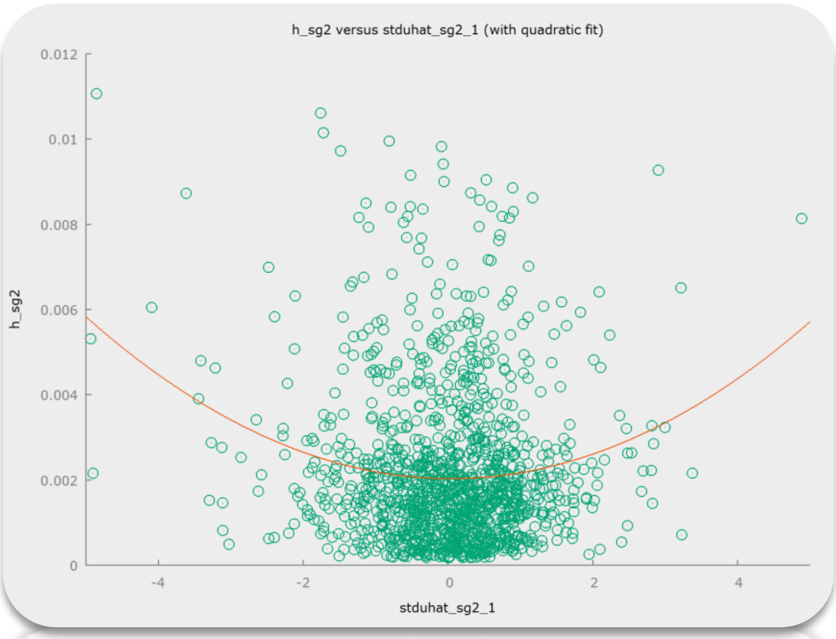
24.News Impact Curve GED-distribution GARCH(1,1)



25.News Impact Curve skewed t-distribution GARCH(1,1)



26.News Impact Curve skewed GED-distribution GARCH(1,1)



As we could see we also added the News Impact Curve for every GARCH model that was selected optimal for every distribution throughout the course of this research. The News Impact Curve is a helpful tool to visualize the response of the variance to the surprise in returns.

- In this stage we can argue that from the empirical results we presented above we had some very good GARCH(1,1) models that met all the theoretical conditions and some less good models especially in the T, skewed T-distributions were we encountered the a+b=1 or a+b>1 problem which was due to extreme outliers and the ability of the distribution to encompass them. This result would mean the selected model is not a mean reverting process, in other words it is not stationary, and probably are measures to adjust this not stationarity problem would lead to a better model specification.

<u>5.Conclusions</u>

We performed 60 GARCH-type models for every distribution in order to find the best fit model for the dataset we analyzed. We performed a two style analysis, in the first stage we tested the entire range of our sample if its fit for GARCH analysis and then estimated the GARCH , GJR, TARCH and EGARCH models for the normal, student's T , GED , skewed T and skewed GED distributions. Based on the information criterion AIC we tried to decide on which model is a best fit for our sample. In order to thought to have a better model specification we deployed four dummy variables (two for the most positive outliers and two for the most negative) I1, I2, I3, I4 . The results suggested that the asymmetrical models GJR, TARCH, EGARCH are not a good fit because the coefficients produced by the test didn't met the conditions of either statistical importance or positivity (negativity) for the leverage coefficient in the GJR , TARCH (EGARCH) models. The selected model for the full range analysis was the GARCH(1,1) because it met these criteria and was only in the student's T distribution that a+b=1 which was due to extreme outliers and the distributions ability to encompass those.

The second stage of our analysis began with a bai perron test which indicated 3 possible date of structural breaks. The one chosen (12/03/2020) was the one of the three which had the most extreme outliers in the time series plot. After we split the sample into to two sub-samples we performed the same analysis as in the full range sample in order to verify if this split would make any difference in the results. We again used 4 dummy variables, two for each sub-sample (one for the lowest and one for the highest observation in each sample) I1, I2, I3, I4. Once again the GARCH(1,1) was selected and in fact it had slighter better smaller coefficients for each term so the split gave a better fit to the data set while the other models had the same outcome not being able to meet the criteria that we established above. In each distribution we also provided the News Impact Curve of the model that was selected which is a very useful

tool in order for someone to understand the impact that information(news) have on the expected returns.
Overall all we would argue that the GARCH(1,1) model is a useful and powerful tool when it comes to modelling the variance. Although the market is constantly changing and new financial products emerge very often such us the cryptocurrencies we can still rely on the traditional models in order to analyze them.

ΒΙΒΛΙΟΓΡΑΦΙΑ

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