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Empirical Investigation of the nexus between the price of oil and the US real exchange rate

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Abstract

The purpose of this thesis is to empirically investigate the relationship between the US real exchange rate and the price of oil. In order to do so, the analysis starts by conducting unit root tests on the examined series by applying both conventional and break point unit root tests. Then, the casual relationship between the variables in examined by applying the Toda Yamamoto procedure. Results revealed that the real exchange rate Granger causes the price of oil and not vice versa. Next, the long-run relationship between the variables in investigated by firstly conducting conventional cointegration tests. In particular, the Engle-Granger and Johansen cointegration tests revealed that no long-run relationship exists between the selected variables. Since the breakpoint unit root tests revealed the presence of structural breaks in the series, the Gregory-Hansen cointegration test was applied in order to take into account the possible breaks in the series. Findings suggest, that the null hypothesis of no cointegration cannot be rejected. As a final step, the asymmetric cointegration between the variables was tested by applying nonlinear ARDL models for various periods in order to investigate the presence of asymmetric responses in the US real exchange rate. After estimating the models, findings revealed the presence of asymmetry and that positive changes in the US real effective exchange rate have a negative impact on oil prices, and negative changes in the former exert a negative positive impact on the latter.

1. Introduction

The role of crude oil, as an energy commodity, has been very important in shaping the global economic growth, particularly after the 1960s. Furthermore, the US dollar plays a major role in determining the oil prices, since it is used as the main invoicing currency in international crude oil trading (Zhang, 2013, p. 341). Therefore, it is safe to assume that fluctuations in the US dollar display a strong influence on the crude oil price.

Historically, oil prices have started to follow a rising trend at the same time that the value of the US dollar have fallen and vice versa, especially since 2002. These facts shade light upon the relationship between oil prices and the US exchange rate, indicating that a long-run relationship between these variables exists. More precisely, a fall in the value of the US dollar should be associated with higher oil prices and vice versa. The idea is that a depreciation of the US dollar relative to other major currencies will render oil prices cheaper and therefore will put upward pressure on the demand for oil.

The purpose of this thesis is to empirically investigate the relationship between the US real exchange rate and the price of oil. In order to do so, this thesis employs firstly the conventional cointegration methods developed by Engle-Granger and Johansen and secondly the Gregory-Hansen that allows for structural breaks in the cointegrating equation and the nonlinear ARDL model in order to examine whether there are asymmetries in the long-run relationship between the examined variables. Before proceeding to cointegration analysis, the order of integration between the examined variables is tested by applying the conventional unit root tests, namely Augmented-Dickey Fuller, Phillips-Perron and the Kwiatkowski–Phillips–Schmidt–Shin unit root tests. In addition, as a robustness check, the breakpoint unit root test developed by Vogelsang and Perron (1998) in order to ensure that the order of integration resulted from the conventional unit root tests is true under the presence of structural breaks. Next, the casual relationships between the variables is tested by applying the Toda Yamamoto procedure to Granger non-causality. Also, the effect of a shock in each variable on the variable of interest is tested by computing the impulse response functions from the estimate VAR model.

The rest of the thesis is structured as follows: Section 2 critically reviews the literature on the nexus between oil prices and the US dollar. Section 3 describes the data and the adopted empirical methodology in order to tackle the research questions addressed above. Section 4 presents the empirical findings. Finally, Section 5 concludes.

2. Literature Review

Oil price movements are affected by various factors such as the demand and supply of oil which are considered as fundamental market factors, speculation in the crude oil markets and less common factors that have the potential to disrupt its flow, such as geopolitical crisis or adverse weather conditions. In particular, the latter events may lead to increased uncertainty about the future oil demand or supply, which in turn may lead to significant oil price volatility. This thesis, however, will focus on the relationship between the exchange rate of the US and the oil price.

There is a vast literature, both theoretical and empirical, that examines the impact of oil price moments on the real effective exchange rate (see for example the study of Coudert et al., 2008). In particular, the authors conduct Granger causality tests between the U.S. real effective exchange rate and WTI concluding that causality runs from the latter to the former.

Amano and Norden (1998) examined the links between the US real effective exchange rate and oil prices by conducting cointregration analysis. In detail, after utilizing a single-equation Error-Correction Model, the authors concluded that the above variables are cointegrated and more importantly that the arrow of causality runs only from oil prices to the real effective exchange rate.

Benassy-Quere et al. (2007) provided a significant and positive long-run relationship between oil price and the US dollar exchange rate by conducting cointegration analysis. After estimating a Vector Error-Correction Model during the period 1974-2004, the authors concluded that oil prices affect the US exchange rate and not vice versa. In addition, the estimated model implied that a 10% increase in the oil price leads approximately to 4% appreciation of the real effective exchange rate in the long-run. Moreover, the estimated speed of adjustment was negative and statistically significant, but the adjustment process of the real exchange rate to its long-run target was rather slow and equal to around 6.5 years.

Krichene (2005) studied the relationship between the US exchange rate and interest rate with oil prices. More precisely, the author put particular focus on the effects of monetary policy shocks on the US nominal effective exchange rate using monthly, quarterly and annually data. Results indicated that at least one cointegrating relationship exists between the examined variables; however, the cointegrating coefficients changed signs and significance when the author conducted the analysis on different frequencies, lags and sample periods. In general, the study of Krichene (2005) concluded that interest rates affect negatively and statistically significant the oil prices for most of the chosen sample periods, and the effect of a nominal effective exchange rate shock affects negatively the oil prices, during periods of significant interest and exchange rate changes.

Chen and Chen (2007) also concluded that oil prices and exchange rates are positively and statistically significant linked in the long-run. Contrary to the previous authors, Cheng (2008) utilized a Dynamic Ordinary Least Squares estimator and concluded that changes in the oil prices have a negative and statistically significant impact on exchange rates both in the short-and-long run.

In addition to the previous studies, a vast literature regarding the asymmetric relationship between oil prices and exchange rates arose. To begin with, Enders and Dibooglu (2001) suggested higher monetary interventions may lead to asymmetric adjustments in exchange rates. Furthermore, asymmetries that stem from various economic or political shocks, institution factors (for example the decisions of OPEC) regarding the pricing and production schemes, may have different impact on the exchange rates (see also the study of Ewing et al., 2006).

Research in asymmetric cointegration between oil prices and exchange rates provided significant evidence in favor of the existence of a long-run asymmetric relationship between these variables. In particular, Coleman et al. (2010) used a Smooth Transition Regression Model and found significant evidence in favor of the existence of a non-linear long-run relationship between oil prices and exchange rates. Similarly, Ahmad and Hernandez (2013) concluded that these variables are non-linearly cointegrated by conducting Threshold Autoregressive and Momentum Threshold Autoregressive Models in a large sample of major oil importing and exporting countries.

Kumar (2019) investigated the casual relationship and asymmetric impact of oil price on exchange rate and stock prices in the Indian economy using the Hiemstra and Jones (1994) nonlinear Granger causality test and nonlinear ARDL tests. Results suggested that a bidirectional nonlinear causality between oil prices and exchange rates exists and that previous month positive and negative oil price shocks in oil prices exert a positive and statistically significant impact on the exchange rate of India. However, the positive shock appears to have a stronger impact in terms of magnitude on the exchange rate than the negative shock in oil prices.

Finally, Rafailidis and Katrakilidis (2016) investigated the long-run relationship between the US real effective exchange rate and oil prices during the period 01/1986 to 08/2014 by employing the so called hidden cointegration technique of Granger and Yoon (2002) and Schorderet (2004) that account for structural breaks and asymmetric responses in the variables. The authors concluded that results reveal an asymmetric long-run relationship between the examined variables.

3. Data and Empirical Methodology

3.1 Data

The variables of interest used in this study are the West Texas Intermediate (WTI) spot price and the real broad effective exchange rate (REER) for the US. Both variables are monthly covering the period between 01/1994 to 05/2022. Both variables are expressed in logarithmic form and are obtained from the Federal Reserve Bank of St. Louis (FRED).

The REER is the weighted average of the US dollar relative to an index or basket of other major currencies. According to the FRED, the REER is calculated as a weighted average of bilateral exchange rates adjusted for by relative consumer prices[1]. The weights are determined by comparing the relative trade balances, in terms of one country's currency, with each other country within the index (Rafailidis and Katrakilidis, 2016, p. 136).

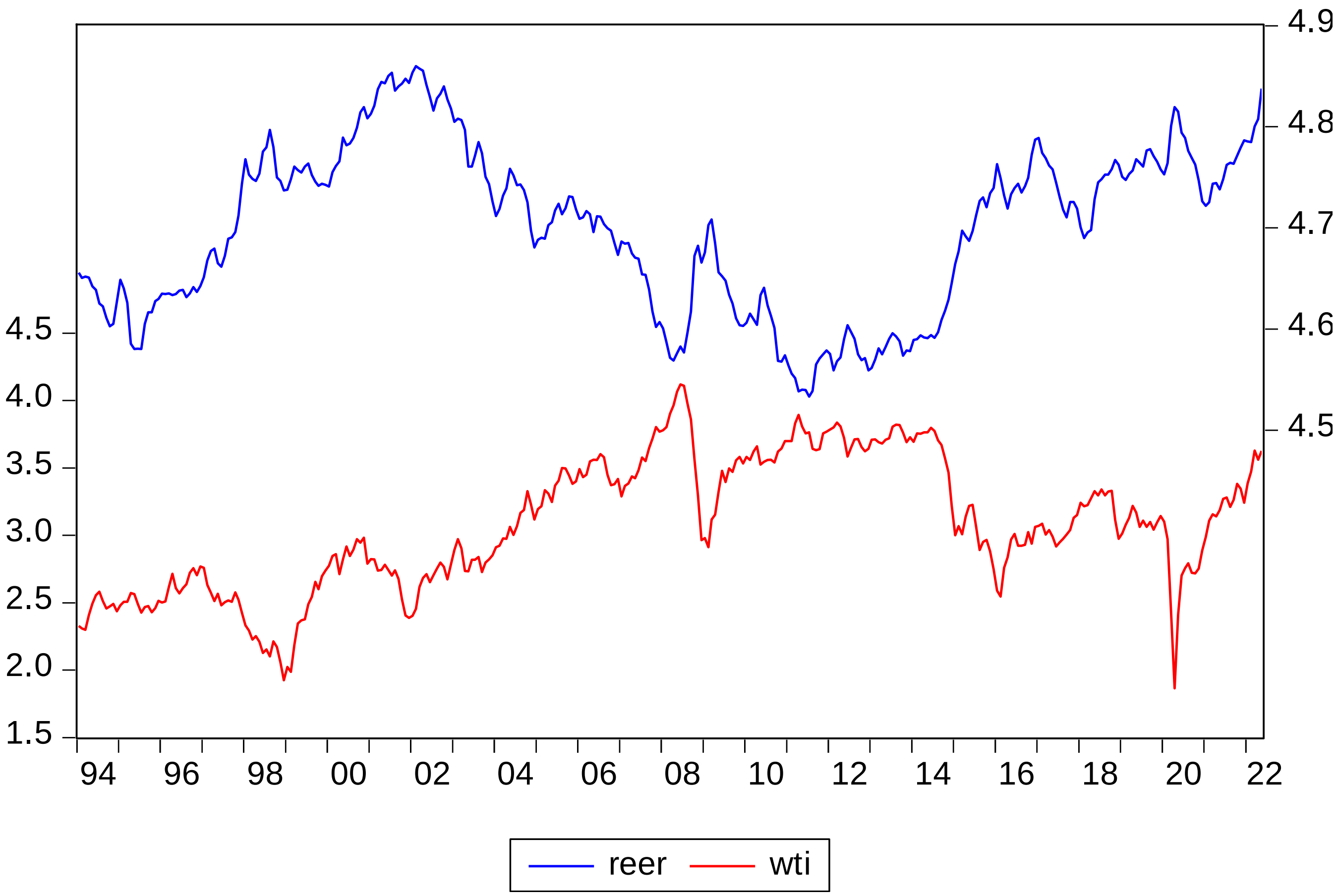
The historical movements of both variables are graphically illustrated in **Figure 1** below. At first, it seems that both series display a reverse relationship over the examined period. However, by

[1] https://fred.stlouisfed.org/series/RBUSBIS

carefully examining **Figure 1**, it is apparent that during specific time periods both series appear to be moving together over time. For example, since early 2000s the spot price of real WTI follows a rising trend, whereas *reer* is downward sloping up until the mid 2008s. After the financial crisis in 2008-2009 and until late 2014s both variables are stable display similar movements with minor fluctuations. Finally, reer and wti move in different directions until mid 2021s. but during the last year of the examined period it appears that both variables start to move together following an upward trend.

By visually inspecting Figure 1, there is no doubt that both series display nonlinearities and therefore this issue will be carefully examined in the empirical section of this thesis.

Figure 1. The evolution over time of the US real effective exchange rate and real WTI



Notes: *reer* denotes the US real effective exchange rate and *wti* denotes the real spot price of West Texas Intermediate. All variables are in logarithmic form. Both the calculations and the graph were obtained using the EViews software.

3.2 Empirical Methodology

3.2.1 Unit Roots

In order to perform causality and cointegration tests, it is important to ensure that order of integration of the selected variables. Therefore, the first step of the analysis involves conducting unit root tests. In particular the unit root tests that will be used in this thesis are namely, the Augmented Dickey Fuller (ADF), the Phillips-Perron (PP) and the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) unit root tests.

The ADF test (Dickey and Fuller, 1979) is based on the following regression

$$\Delta y_t = \mu + \lambda t + \psi y_{t-1} + \sum_{i=1}^{p} a_i \Delta \mathrm{y}_{t-i} + u_t \qquad (1)$$

Where y_t is the dependent variable[2], μ is the constant term or drift, t is the trend, u_t the error term and p is the optimal lag length that eliminates the autocorrelation in the above regression and is chosen based on the Akaike or Bayesian Information criterion (AIC and BIC respectively). The null and the alternative hypothesis for the existence of unit root in the real exchange rate are tested as follows

$$H_0\colon \psi = 0\ (non - stationay)$$

$$H_1\colon \psi < 0\ (stationary)$$

In order to check the robustness of the ADF unit root test, this study also employs the PP and KPSS unit root tests on the real exchange rate. The PP unit root test (Phillips and Perron, 1988) corrects for both autocorrelation and heteroskedasticity in the error term and is based on the following formula

$$\Delta y_t = a + bt + \pi y_{t-i} + \varepsilon_i \qquad (2)$$

where a is the constant term, t is the time trend, ε_ι is the error term and y_t is the dependent variable. The null and alternative hypothesis are formed as follows:

$$H_0\colon \pi = 0\ (non - stationay)$$

$$H_1\colon \pi < 0\ (stationary)$$

[2] The choice of the dependent variable, i.e. real effective exchange rate or WTI, will be based on the results of the causality tests.

In contrast to ADF and PP which test whether a time series is I(1), i.e. nonstationary, the KPSS tests whether the examined time series are under the null hypothesis I(0), i.e. stationary. The KPSS statistic (Kwiatokowski et al., 1992) is based on the following model

$$y_t = \xi t + r_t + \varepsilon_t$$

$$r_t = r_{t-1} + u_t, u_t \sim N(0, \sigma_u^2) \qquad (3)$$

where y_t is the real interest rate, t is a deterministic trend, r_t is a random walk process, ε_t and u_t are the error terms of the first and second equation respectively. The null hypothesis that y_t is stationary is tested against the alternative as follows

$$H_0: \sigma_u^2 = 0$$

$$H_1: \sigma_u^2 > 0$$

The next econometric technique used is the Engle-Granger (1987) cointegration method between the real US exchange rate and the price of oil. In general, cointegration analysis tells us that any two nonstationary series, which are found to be integrated of the same order are cointegrated if a linear combination of the two exists which is itself stationary.

3.2.2 Unit Roots with Structural Breaks

In order to examine the presence of a structural break in the examined series and to ensure that they have the “true” order of integration even if a structural break occur, this thesis adopts the breakpoint unit root test proposed by Vogelsang and Perron (1998) . The authors proposed two different forms of structural breaks, namely the Additive Outlier and the Innovational Outlier models (hereinafter AO and IO models respectively). More precisely, the IO model performs better in capturing structural breaks in a time series, when the break is smooth over time. In contrast, the AO model is more reliable, when a sudden break occurs in the mean of the examined variable.

The IO model can be decomposed into two sub-categories, one that captures gradual changes in the intercept and a second that captures changes in both intercept and the trend (see also Perron and Vogelsang, 1992 and Perron, 1997). These IO models ($IO_1$and IO_2 respectively) are formed as follows

$$IO_1 = x_t = \mu + \theta DU_t + \beta t + \delta D(T_b)_t + \alpha x_{t-1} + \sum_{i=1}^{K} c_i \Delta x_{t-i} + e_t \quad (4)$$

$$IO_2 = x_t = \mu + \theta DU_t + \beta t + \gamma DT_t + \delta D(T_b)_t + \alpha x_{t-1} + \sum_{i=1}^{K} c_i \Delta x_{t-i} + e_t \quad (5)$$

Where T_b denotes an unknown breakpoint that is determined endogenously, DU_t is the intercept dummy taking value equal to 1 if $t > T_b$ and zero otherwise, DT_t is the slope dummy, $DT_t = T_t$ if $t > T_b$ and zero otherwise, and $D(T_b)_t$ is the crash dummy, $D(T_b)_t = 1$ if $t = T_b + 1$ and zero otherwise. The null hypothesis states that the variable under consideration is nonstationary. In other words, the null hypothesis is tested for $a = 1$, minimizing the value of the t-statistic.

Regarding the AO model, it involves a two-step estimation procedure (see Perron, 1994). More precisely, the first step involves the detrending of y_t by regressing y_t on a linear trend

$$y_t = \mu + \beta t + \gamma DT_t^* + \widetilde{y_t} \quad (6)$$

where $\widetilde{y_t}$ is the detrended series[3]. After estimating the residuals on the first step, the AO model continues to the second step where the following regression is estimated using the residuals from the first step

$$\widetilde{y_t} = \alpha \tilde{y}_{t-1} + \sum_{i=1}^{K} \Delta y_{t-i} + e_t \quad (7)$$

3.2.3 Toda Yamamoto Granger non-causality test

The next step of the econometric analysis in this thesis involves the examination of the casual relationship between the US real exchange rate and oil prices. Since the usual bivariate Granger non-causality tests are sensitive to the number of chosen variables and lags (Toda and Phillips, 1994) and due to the fact that many variables have stochastic trends and are cointegrated, the

[3] The time that the structural break occurs, that is T_b, is endogenously determined and estimated by minimizing the value of t-statistic when $\gamma = 0$ (see Harris and Solis, 2003). Each equation in the IO and AO models are estimated sequentially for all possible break points in order to choose the optimal T_b.

usual F-tests do not provide valid results, because the test statistics do not have a standard distribution (see also Enders, 2014). Therefore, to test for linear causality between the variables of interest, this thesis implements the Toda Yamamoto (1995) procedure to Granger non-causality. In particular, this procedure involves the estimation of an augmented Vector Autoregressive (VAR) model which can be estimated regardless the order of integration of the variables. The estimation of the augmented VAR model guarantees the asymptotic distribution of the Wald statistic. The Toda Yamamoto (TY) Granger non-causality test involves the estimation of the following regressions

$$Y_t = \omega + \sum_{i=1}^{m} \beta_i Y_{t-i} + \sum_{i=m+1}^{m+d_{\max}} \beta_i Y_{t-i} + \sum_{i=1}^{m} \lambda_i X_{t-i} + \sum_{i=m+1}^{m+d_{\max}} \lambda_i X_{t-i} + \varepsilon_{1t} \tag{8}$$

$$X_t = \psi + \sum_{i=1}^{m} \varphi_i X_{t-i} + \sum_{i=m+1}^{m+d_{\max}} \varphi_i X_{t-i} + \sum_{i=1}^{m} \delta_i Y_{t-i} + \sum_{i=m+1}^{m+d_{\max}} \delta_i Y_{t-i} + \varepsilon_{2t} \tag{9}$$

where X_t and Y_t are the endogenous variables of the model, ω and ψ are the constant terms, d_{max} is the maximum order of integration of each variable based on the unit root tests, m is the optimal lag length based on the AIC or SIC, and $\varepsilon_{1t}, \varepsilon_{2t}$ are the error terms in each equation. Finally, the null hypothesis of granger non-causality can be expressed as$H_0: \lambda_i = 0, \delta_i = 0$, implying that, if the lagged values of X_i are jointly significant, then X can Granger-cause Y, whereas, if the lagged values of Y_i are jointly significant, then Y can Granger-cause X (Granger, 1969).

3.2.4 Conventional Cointegration Tests

The next econometric technique used to investigate the existence of a long-run relationship between the US exchange rate and oil price is the Engle-Granger (1987) cointegration method. Generally, cointegration analysis implies that any two nonstationary series, which are found to be integrated of the same order are cointegrated, if a linear combination of the two exists which is itself stationary.

The Engle-Granger approach to cointegration is a two-step procedure. To begin with, the long-run relationship is estimated as follows

$$lny_t = a_0 + a_1 lnx_t + u_t \tag{10}$$

where y_t, x_t are the dependent and independent variables, a_0 is the constant term, a_1 is the slope coefficient and u_t is the error term. After estimating equation (10) by applying OLS, the residuals are obtained and tested for unit roots. Therefore, cointegration holds, if $\hat{u}_t$ is I(0), i.e. stationaty.

If $\hat{u}_t$ is found stationary, then the next step involves the estimation of the Error-Correction Model (ECM) using the following formula:

$$\Delta y_t = \varphi_0 + \sum_{j=1} \varphi_j \Delta y_{t-1} + \sum_{h=0} \theta_h \, \Delta x_{t-h} + a\hat{u}_{t-1} + \varepsilon_t \tag{11}$$

where $a < 0$ is the speed of adjustment towards equilibrium,

The second conventional cointegration method applied in this thesis is the Johansen (1998) approach to cointegration. More precisely, the Johansen cointegration test is based on the following Vector Error-Correction Model (VECM)

$$\Delta \mathrm{Z}_t = \mu + \Gamma_1 \Delta \mathrm{Z}_{t-1} + \cdots + \Gamma_{k-1} \Delta \mathrm{Z}_{t-(k-1)} + \Pi \mathrm{Z}_{t-1} + u_t \tag{12}$$

where Z_t is a n x 1 vector containing the I(1) variables, μ is a n x 1 vector which contains constant terms, u_t is a n x 1 vector of Gaussian errors, $\Gamma_i = (I - A_1 - \cdots - A_k)$ is a n x n matrix of coefficients, $\Pi = -(\mathrm{I} - A_1 - \cdots - A_k)$ is a n x n matrix of coefficients which contains all the information about the long run relationship between the variables. Matrix Π can be decomposed into $\Pi = a\beta'$, where β the matrix of long-run coefficients that contains the cointegrated vectors r and a is the speed of adjustment to equilibrium on each cointegrated vector r.

The Johansen approach to cointegration, is a test for the rank of matrix Π. If $\mathit{\Pi} = 0$, then the variables are not cointegrated, therefore cointegration is a test of whether $rank(\mathit{\Pi}) = 0$. On the other hand, if variables are cointegrated, then $rank(\mathit{\Pi}) \neq 0$ or else $rank(\mathit{\Pi}) = r$. In order to test the rank of matrix Π, this study employs the trace and maximum eigenvalue tests which are computed using the following formulas

$$\lambda_{trace}(r,k) = -T \sum_{i=r+1}^{k} \ln(1-\lambda_i)$$

$$\lambda_{max}(r, r+1) = -T\ln(1-\lambda_{r+1}) \tag{13}$$

where T is the number of observations, k is the number of variables, r is the number of cointegrated vectors and λ are the eigenvalues. The null hypothesis of the trace test is $rank(\Pi) = r$ and the alternative is $r < rank(\Pi) \leq k$. If the null hypothesis is rejected, then $rank(\Pi) = r + 1$ is tested against $r + 1 < rank(\Pi) \leq k$ and so on. The null hypothesis of the maximum eigenvalue test states that $rank(\Pi) = 0$, whereas the alternative $rank(\Pi) = 1$. In detail, if $rank(\Pi) = 0$ holds, then there is no cointegration between the examined variables, since the maximum eigenvalue is zero. However, if the largest eigenvalue is non zero, then $rank(\Pi) \geq 1$, suggesting that there might be more cointegrating vectors. Therefore, the procedure continues by testing whether the second largest eigenvalue is zero and so forth.

3.2.5 Gregory-Hansen Cointegration Test

The Gregory-Hansen approach to cointegration is a residual based cointegration test and is mainly utilized in order to test for structural break in the cointegrating relationship among the examined variables. Gregory et al. (1996) showed that the rejection of the null hypothesis of the ADF test for cointegration is primarily account for the presence of structural breaks in in the cointegration relationship. Since the examined series exhibit some structural breaks, the Gregory-Hansen (hereinafter GH) cointegration test appears to be the appropriate test for cointegration. More precisely, the residual-based cointegration test proposed by Gregory and Hansen (1996a, b) is superior to the Engle-Granger or the Johansen approach, since it permits the occurance of the structural break to endogenously determined by the data. In this study, three break specifications are going to be tested, namely: level break, trend and level break, and a regime break where the break in the constant, slope and trend occurs

$$lnwti = a_1 + a_2 lnreer_t + \varepsilon_t \qquad (14)$$

$$lnwti = a_1 + a_2 D_t + \beta trend + \gamma lnreer_t + \varepsilon_t \qquad (15)$$

$$lnwti = a_1 + a_2 D_t + \beta_1 trend + \beta_2 trend D_t + \gamma_1 lnreer_t + \gamma_2 lnreer_t D_t + \varepsilon_t \qquad (16)$$

where D_t^{τ} is a dummy variable and τ is the date that the structural breaks occurs, $D_t^{\tau} = 0$, if $t < \tau$ and $D_t^{\tau} = 1$, if $t \geq \tau$.

3.2.6 Asymmetric Cointegration

Conventional cointegration tests may fail to detect a long-run relationship between the examined series, usually due to the presence of structural breaks or nonlinearities. One way to tackle this problem is to apply the non-linear Autoregressive Distributed Lag (NARDL) model proposed by Shin et al. (2014). The main advantage of the non-linear ARDL[4] over the conventional cointegration methods (Engle-Granger, Johansen, linear ARDL) is that the former can capture both short-run and long-run asymmetries, which can potentially shade light on the long-run relationship in the examined variables.

Starting the analysis from the long-run equation

$$y_t = a_0 + a_1 x_t + e_t \qquad (17)$$

In order to introduce asymmetry in equation (17), the positive and negative partial sums of the independent variable are computed and incorporated in equation (18) as follows

$$y_t = a_0 + a_1 x_t^{+} a_2 x_t^{-} + e_t \qquad (18)$$

where y_t is the dependent variable, x_t^{+} and x_t^{-} are the positive and negative partial sums of the independent variable which are computed as follows

$$x_t^{+} = \sum_{i=1}^{t} \Delta x_i^{+} = \sum_{i=1}^{t} max(\Delta x_i^{+}, 0) \qquad (19)$$

$$x_t^{-} = \sum_{i=1}^{t} \Delta x_i^{-} = \sum_{i=1}^{t} min(\Delta x_i^{-}, 0) \qquad (20)$$

[4] Other nonlinear cointegration tests involve the estimation of the MTAR model or the hidden cointegration method proposed by Granger and Yoon (2002) that allow for structural breaks in the model (see also Katrakilidis and Rafailidis, 2016, p. 136-138).

Considering the above specification, a_2 represents the long run impact of a negative shock in the independent variable on the dependent variable, while $a_1$captures the effects of a positive shock in the independent variable on the dependent variable.

Following Shin et al. (2014) and Pesaran et al. (2001), we can rewrite equation (18) in an unrestricted error correction form as follows

$$\Delta y_t = \beta_o + \beta_1 y_{t-1} + \beta_2 x_t^+ + \beta_3 x_t^- + \sum_{i=1}^{q} p_{1_i} \Delta y_{t-i} + \sum_{i=0}^{m} p_{2_i} \Delta x_{t-i}^+ + \sum_{i=0}^{n} p_{3_i} \Delta x_{t-i}^- + + v_t \qquad (20)$$

Regarding equation (12), the long-run multiplier of positive and negative changes in the independent variable are $-\frac{\beta_2}{\beta_1} = \alpha_1$ and $-\frac{\beta_3}{\beta_1} = \alpha_2$ respectively. Thus, the long-run asymmetry implies that $a_1 \neq a_2$. Furthermore, the short-run asymmetries are captured by $\sum_{i=0}^{m} p_{2_i} \Delta x_{t-i}^+$ and $\sum_{i=0}^{n} p_{3_i} \Delta x_{t-i}^-$ respectively and in order for the short-run asymmetry to hold, the short-run positive and negative partial sums must not be equal.

Before proceeding to the short and long-run dynamics of NARDL model, it is essential to ensure that none of the examined variables are $I(2)$. Despite the fact that NARDL is useful for variables that have different order of integration, that is $I(0)$ and $I(1)$, the estimator cannot capture variables that are integrated of higher order i.e. $I(2)$. Thus, unit root tests are mandatory in order to avoid having $I(2)$series, because the presence of such variables invalidates the F-statistic, which is essential in testing for cointegration**.** The next step is to estimate equation (18) using OLS and choose the appropriate lag order using the AIC and SIC respectively or the general to specific method. The lag selection method will be based on the residual diagnostic and stability tests in order to ensure that the model is well specified and does not suffer from A/C, H/S and non normality. After estimating equation (18), we are going to test whether there is a long-run relationship between our variables by applying the Bounds test[5]. The null and alternative hypotheses are specified below:

$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 \text{ (no cointegration)}$$

$$H_1: \beta_1 \neq \beta_2 \neq \beta_3 \neq \beta_4 \neq \beta_5 \sim \text{(cointegration)}$$

[5] The bounds test is an F-test developed by Pesaran et al. (2001).

If the computed F-statistic of the bounds test[6] is higher than the higher bound for the 5% significance level, then there is a long-run equilibrium among our variables, while, if the F-statistic is lower than the lower bound, then there is no cointegration. Likewise, if the F-statistic is between the lower and higher bound, then the existence of cointegration is ambiguous. IF the null hypothesis of no cointegration is rejected, then we are going to examine whether there are asymmetries in both long and short run between the rate of profit and unproductive expenditures, as well as to test for asymmetric causality in our model. The final step of the NARDL analysis involves estimating the asymmetric cumulative dynamic multipliers of a one percent shock in x_t^+ and x_t^- as follows:

$$m_h^+ = \sum_{i=0}^{h} \frac{\partial y_{t+i}}{\partial x_{t-1}^+}, \qquad h = 0, 1, 2, \ldots H \tag{21}$$

$$m_h^- = \sum_{i=0}^{h} \frac{\partial y_{t+i}}{\partial x_{t-1}^-}, \qquad h = 0, 1, 2, \ldots H \tag{22}$$

where m stands for the dynamic multiplier, h is the time horizon which in this case represents months and H is the final chosen time horizon.

3.2.7 Impulse Response Functions

Impulse response functions (hereinafter IRFs) are essential in assessing the impact of a shock on the variable of interest. In particular, after estimating the VAR model and examine the casual relationship between oil prices and the US real exchange rate, the next step is to construct shocks on each variable and study its effect on the other variable over time.

However, in order to proceed in estimating the impact of shocks on each variable in the VAR system, it is essential to ensure that the residuals do not suffer from autocorrelation, are normaly distributed and that the VAR model is stable. The latter is important because, it allows us to write the VAR model as a Vector Moving Average (VMA) as follows

$$y_t = \mu + \varepsilon_t + \Psi_1 \varepsilon_{t-1} + \Psi_2 \varepsilon_{t-2} + \cdots + \Psi_s \varepsilon_{t-p} \tag{23}$$

Or at time $t + s$:

[6] Shin et al. (2014, p. 291) recommend treating positive and negative partial sums as one variable and using the critical values for the case of k=1.

$$y_{t+s} = \mu + \varepsilon_{t+s} + \Psi_1\varepsilon_{t+s-1} + \Psi_2\varepsilon_{t+s-2} + \cdots + \Psi_s\varepsilon_t + \Psi_{s+1}\varepsilon_{t-1} + \cdots \quad (24)$$

Thus, similarly to the dynamic multipliers in the nonlinear ARDL model, the elements of matrix Ψ_s(n x n),$\psi_{ij} = \frac{\partial y_{i,t+s}}{\partial \varepsilon_t}$, are the dynamic multipliers (or impulse responses) that capture the response of the i-th variable of interest at time $t + s$, if the shock variable increases by one unit. It is important to note however, that the responses are valid only if $var(\varepsilon_t) = \Sigma$ or else that errors are uncorrelated.

In order to avoid errors being correlated a transformation if applied on the Σ matrix. In particular, Σ is diagonalized in order to generate orthogonal shocks with the most common method being the Cholesky decomposition which is computed as follows

Let P be a lower triangular matrix such that $\Sigma = PP'$ and $P^{-1}\Sigma P'^{-1} = I_k$. Orthogonalization of the matrix Σ is done through P^{-1}. The Moving Average (MA) representation of the VAR implies that

$$y_t = \mu + \sum_{i=0}^{\infty} \Psi_i \, PP^{-1}\varepsilon_{t-1} \quad (25)$$

Denote $M_i = \Psi_i P$ and $w_t = P^{-1}\varepsilon_t$, so that equation (25) can be rewritten as

$$y_t = \mu + \sum_{i=0}^{\infty} M_i \, w_{t-1} \quad (26)$$

Where w_t are the orthogonalized shocks. After the Cholesky decomposition, a unit shock is a shock of size one standard deviation. However, one major of the Cholesky decomposition is that matrix P highly depends on the order of the variables. The idea is that matrix P creates a casual relationship between the examined variables, because it represents the instantaneous relationship between variables, which means that the ordering of variables will highly affect the results of the impulse responses.

In order to solve the problem of ordering the variables[7] the General Impulse Response Function (hereinafter GIRFs) proposed by Pesaran and Shin (1998) described above are used.

$$GI(n, \delta, \Omega_{t-1}) = \mathrm{E}\left(\mathrm{x_{t+n}} \middle| \varepsilon_{jt} = \delta_j, \Omega_{t-1}\right) - E(x_{t+n}|\Omega_{t-1}) \quad (27)$$

[7] Economic theory regarding the casual relationships between the examined variables can help to overcome this problem too.

where $\Omega_{t-1}$is a set of historical information at time $t-1$, n is the horizon and $\delta_j$is the j-th element of a shock. The impulse response from a one standard deviation shock to the j-th innovation is given by:

$$GI_j = \frac{1}{\sqrt{\sigma_j^2}} \Phi_i U_j \tag{28}$$

4. Empirical Results

As a first step, it is necessary to test for the integration properties of the examined series and thus the ADF, PP and KPSS unit root tests are applied[8]. The results are reported in **Table 1** below, suggesting that when the ADF and PP unit root tests are applied, both variables are nonstationary in levels, whereas they turn stationary in first differences at the 1% significance level. The KPSS unit root test rejects the null hypothesis that reer is stationary in levels at the 10% significance level and wti at the 1% significance level. In contrast, both the test fails to reject the null hypothesis of stationary, when both variables are expressed in first differences, verifying the results of the ADF and PP unit root tests, Therefore, the examined series are nonstationary in levels and stationary in first differences.

Table 1. Unit root tests results

	ADF		PP		KPSS	
Variables/ tests	Level	1st difference	Level	1st difference	Level	1st difference
$lnreer_t$	-1.3098 (0.6260)	-11.7787*** (0.0000)	-1.3642 (0.6001)	-12.3781*** (0.0000)	0.3796*	0.1477
$lnwti_t$	-2.5249 (0.1104)	-14.3118*** (0.0000)	-2.1916 (0.2098)	-13.8996*** (0.0000)	1.0243***	0.0521

Notes: *, **, *** indicate significance at 10%, 5%, and 1% respectively; p-values are reported in parentheses. The critical values of the KPSS test for the 10%, 5% and 1% significance level are 0.739000, 0.463000 and 0.347000 respectively (see Table 1 in Kwiatkowski et al. 1992). The variables *lnreer* and *lnwti* denote the logarithms of the US real effective exchange rate and real spot price of West Texas Intermediate. The optimal lag length for the ADF test was set between 12 and 16 for each variable and chosen based on the SIC. Results are derived using the EViews software.

[8] The tests were conducted including a constant term in the equation of each test. However, the significance of the results do not change, if a trend and a constant term are included in each test. Furthermore, non of the examined variables were found to be I(2).

The next step involves the application of the AO and IO models proposed by Vogelsang and Perron (1998) in order to ensure the integration properties of the examined series under the presence of a structural break. Results are reported in **Table 2** below, suggesting that *reer* are stationary in levels and nonstationary at their first differences. However, the logarithm of real WTI appears to be stationary in levels at the 10% significance level when gradual changes are introduced in both intercept and the trend[9] (IO2 model). The same result holds, when the Additive Outlier model is introduced, implying that the sudden change that occurred in *wti* in 2014M09 is significant regarding the order of integration of this variable. However, the oil price appears to be stationary, if we take account this structural break, at the 10% significance level, therefore it not a valid result and the analysis will proceed as it is a nonstationary variable. Finally, both variables are stationary at their first differences[10].

Table 2. Breakpoint unit root test results

Variables	Model	*t*-stat	*p*-value	T_b
$lnreer_t$	IO1	-1.9551	0.9846	2016M03
	IO2	-3.116	0.9441	2009M12
	AO1	-2.1901	0.9640	2014M06
	AO2	-3.0958	0.9474	2009M02
$dlnreer_t$	IO1	-12.536	<0.01***	2008M10
	IO2	-12.725	<0.01***	2008M10
	AO	-12.844	< 0.01***	2008M10

[9] Different break specifications provide similar results. However, the trend break dummy appears to be insignificant in all specifications, except from the case when only a trend break is introduced. Therefore, it is "safe" to assume that this variable is nonstationary.

[10] The rejection of the null hypothesis holds also at their second differences. Therefore none of the examined variables are I(2), which is a necessary condition to conduct cointegration analysis in the next sections.

	AO2	-12.863	<0.01***	2008M09
$lnwti_t$	IO1	-3.898	0.194	2003M09
	IO2	-4.898	0.099*	2014M09
	AO	-3.918	0.186	2003M08
	AO2	-4.953	0.08*	2014M08
$dlnwti_t$	IO1	-16.124	<0.01***	2020M4
	IO2	-16.368	<0.01***	2020M4
	AO	-14.748	<0.01***	2008M12
	AO2	-14.739	<0.01***	2008M12

Notes: The lag length selection was based on Schwartz information criterion. The breakpoint selection method was the Dickey Fuller minimize t-statistic. The reported p-values are one-sided p-values and taken from Vogelsang (1993). *, ** and *** denote rejection of the null hypothesis at the 10%, 5% and the 1% significance level respectively. IO1 and IO2 models stand for the Innovational Outlier model that capture gradual changes in the intercept and in both the intercept and the trend respectively. The AO model is the Additive Outlier model which captures sudden changes in the mean of the series. T_b denotes the date when a break occurs in each model. Regarding the IO2 and AO2 models, the breaking specification was based on the statistical significance of the results. In both cases, a break was introduced in both the intercept and the trend.

The next step of the empirical analysis involves the examination of the casual relationships between the US real exchange rate and the spot price of WTI. Before proceeding to this step, the VAR model should be stable and the residuals must not suffer from autocorrelation and non-normality. The optimal lag length criteria results are reported in Table 3 indicating that a VAR(1,2) or a VAR(1,3) are appropriate based on the SC and the AIC respectively. Since, autocorrelation vanishes after the first lag, a VAR(1,2) will be used to conduct the TY Granger non-causality, in order to avoid overfitting. Moreover, a VAR(1,2) is stable, since no roots lies outside the unit circle (see Figure 2) and also the residuals are normally distributed, because the jointed Jarque-Berra test does not reject the null hypothesis of normality at the 5% level significance level[11].

[11] The joint test rejects the null hypothesis at the 10% significance level.

Table 3. Lag length Criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
0	200.0908	NA	0.001028	-1.204200	-1.181123	-1.194994
1	1296.581	2172.984	1.34e-06	-7.845479	-7.776250	-7.817862
2	1331.439	68.65508	1.11e-06	-8.033062	-7.917680*	-7.987032*
3	1336.566	10.03661*	1.11e-06*	-8.039915*	-7.878381	-7.975474
4	1338.819	4.381973	1.12e-06	-8.029293	-7.821606	-7.946440
5	1340.434	3.123399	1.13e-06	-8.014799	-7.760959	-7.913535
6	1342.263	3.513506	1.15e-06	-8.001601	-7.701609	-7.881926
7	1345.138	5.486303	1.16e-06	-7.994757	-7.648612	-7.856670
8	1347.355	4.205706	1.17e-06	-7.983921	-7.591623	-7.827422
9	1350.956	6.785465	1.17e-06	-7.981493	-7.543043	-7.806583
10	1350.988	0.059881	1.20e-06	-7.957372	-7.472769	-7.764050
11	1353.706	5.056086	1.21e-06	-7.949579	-7.418823	-7.737845
12	1354.129	0.783013	1.24e-06	-7.927838	-7.350930	-7.697693

Notes: Results were calculated using the EViews software.

Table 4. LM test for autocorrelation

Lags	LM-Stat	P-value
1	9.949734	0.0413
2	7.494653	0.1119
3	7.787010	0.0997
4	2.905881	0.5737
5	3.103794	0.5406
6	5.781273	0.2161
7	5.217326	0.2657
8	5.760949	0.2177
9	0.712032	0.9498
10	5.445238	0.2446

11	2.291810	0.6823
12	0.254649	0.9926

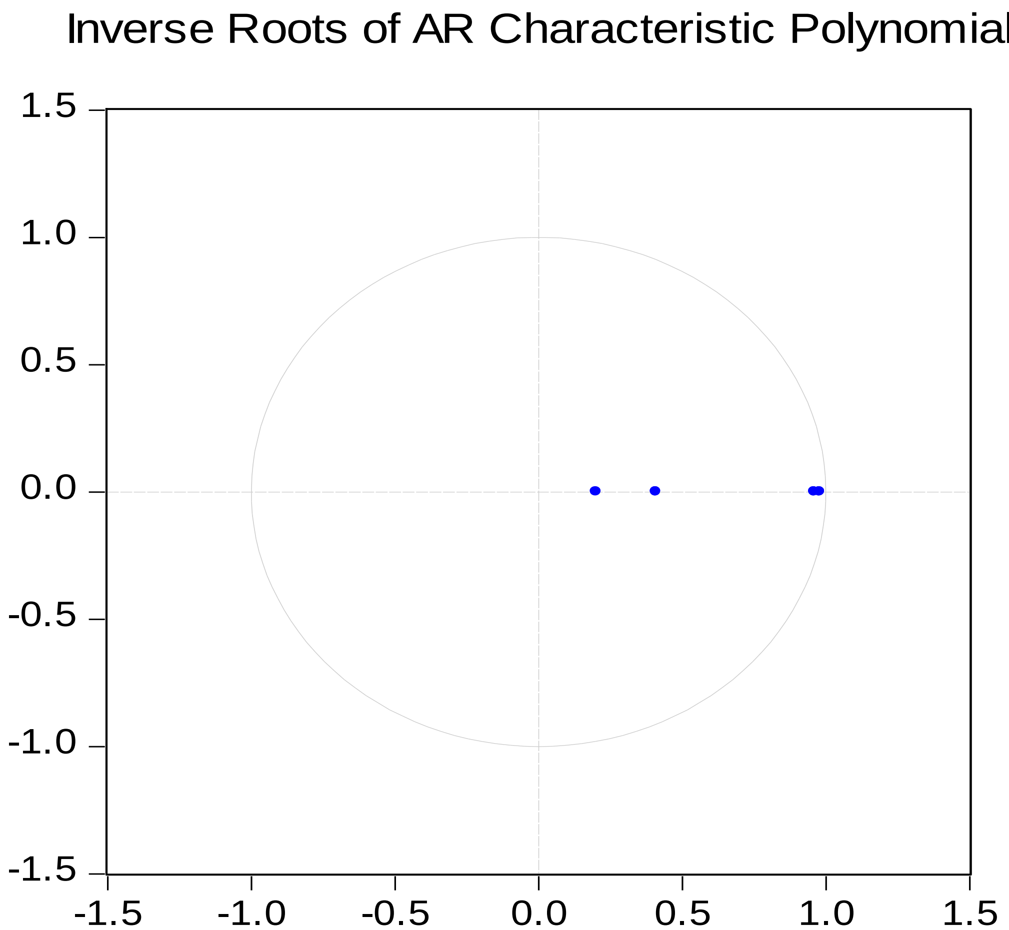
Notes: Results were calculated using the EViews software.

Table 5. Normality and heteroskedasticity tests

Component	Jarque-Bera	df	P-value
1	4.007461	2	0.1348
2	5.164309	2	0.0756*
Joint	9.171769	4	0.0569*

Notes: Results were calculated using the EViews software. * denotes rejection of the null hypothesis at the 10% significance level respectively.

Figure 2. Stability of the VAR model



Notes: Results were calculated using the EViews software

Since the VAR model is well-specified, the econometric analysis proceeds in examining the casual relationships between the variables. The results of the TY Granger non-causality are reported in **Table 6** below suggesting that the spot prices of WTI are statistically significant in Granger-causing the US real effective exchange rate and not vice versa[12]. Therefore, the dependent variable in the cointegration tests will be the real oil price and the independent variable the US real effective exchange rate.

[12] This results holds even after using different number of lags in the VAR model.

Table 6. Toda Yamamoto Granger non-causality results

Dependent variable: $lnreer_t$

Excluded	Chi-sq	df	Prob.
$lnwti_t$	0.878124	2	0.6446
All	0.878124	2	0.6446

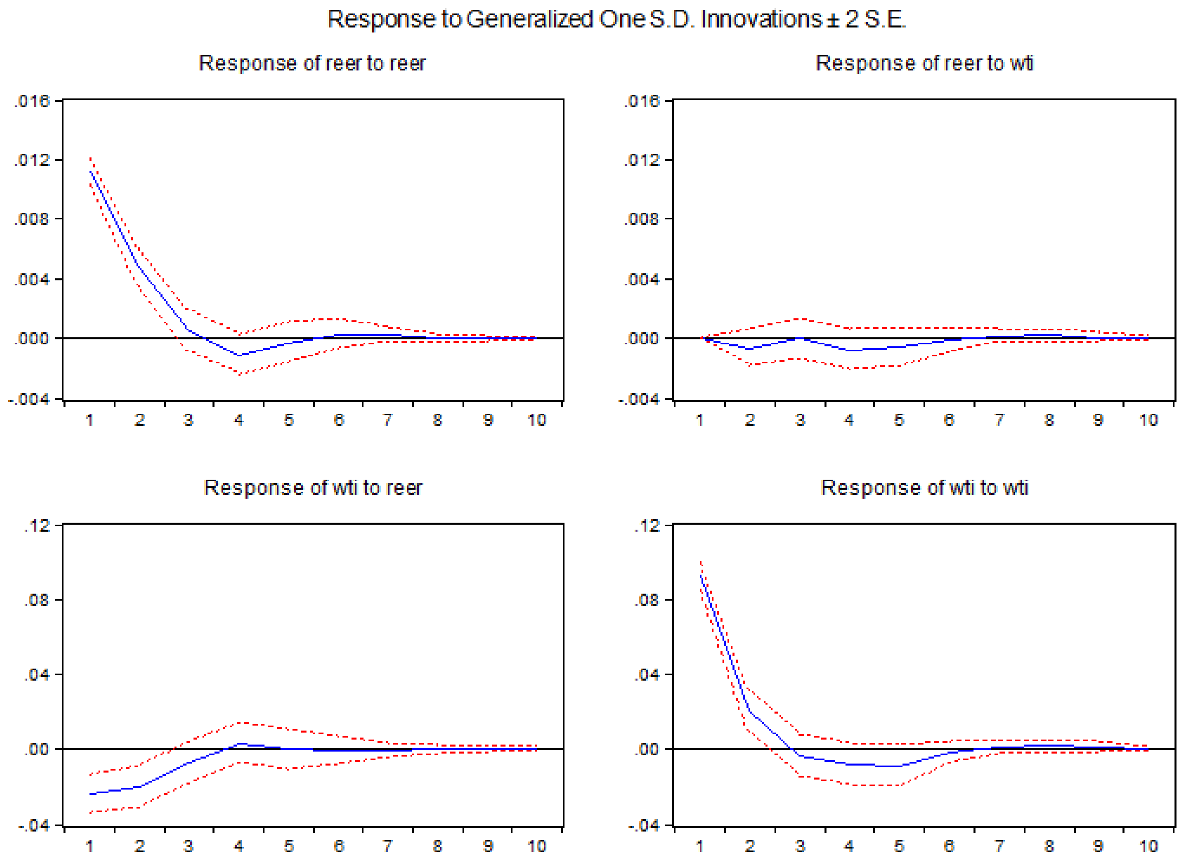
Dependent variable: $lnwti_t$

Excluded	Chi-sq	df	Prob.
$lnreer_t$	6.714884	2	0.0348**
All	6.714884	2	0.0348**

Notes: ** denotes the rejection of the null hypothesis 5% significance level. Results were calculated using the EViews software.

The Generalized Impulse Response Functions (GIRFs) of the estimated VAR model are graphically illustrated in **Figure 3** below. It appears that the price of oil is negatively related to the US real effective exchange rate. In particular, a 1% positive shock in the price of oil results in a depreciation of the US dollar relative to its major trading partners by almost 0.5%. The effect of the shock reaches its highest value 4 months after the initial shock in *wti* and then strarts to decline, becoming however slightly positive but statistically insignificant. Similarly, a 1% positive shock in *reer* exerts a negative initial impact on *wti* during the first 2 months, after the initial shock. Then the effect of the shock starts to gradually fade away, after the 5th month, but the overall effect is statistically insignificant. Therefore, the GIRFs suggest that oil prices are negatively associated with the US real effective exchange rate, verifying the casual relationships found by the Toda Yamamoto test.

Figure 3. GIRFs from a VAR(1,2) model



Notes: Results were obtained from a VAR(1,2) model using the EViews software. Variables are expressed in first differences.

After determining the casual relationships between the examined variables in the dataset, the next step is to conduct cointegration tests in order to determine whether a long-run relationship between *wti* and *reer* exists. First, the Engle-Granger and the Johansen approaches are applied. Results suggest that no cointegration between the examined variables exists at the 5% significance level from both EG and Johansen cointegration methods. However, by introducing a trend and an intercept in the ADF equation, the null hypothesis of no cointegration cannot be rejected at the 10% significance level. Furthermore, the IO and AO breakpoint unit root tests reveal that the estimated residuals are stationary at the 1% significance level when an intercept is introduced in the equation. The stationarity of the error term still holds even if a constant term with a linear trend is introduced in the model; however, the results are statistically significant for the 5% and 10% significance level when the IO and AO models are applied respectively.

Table 7. Cointegration test results

	Engle-Granger					Johansen		
	ADF	ADF Critical values 5%	10%	IO	AO	Number of cointegrated vectors	Trace test	Maximum eigenvalue
none	-2.476	-2.76	-2.45			None	7.232 (0.551)	5.721 (0.649)
Intercept	-2.469	-3.37	-3.07	-5.165*** (<0.01)	-5.104*** (<0.01)	At most 1	1.511 (0.219)	1.511 (0.219)
Trend and intercept	-3.334*	-3.42	-3.13	-5.191** (<0.048)	-5.161* (0.052)			

Notes: *, ** and *** denote rejection of the null hypothesis at the 10%, 5% and the 1% significance level respectively. The critical values used in the ADF test for the Engle-Granger cointegration procedure are described in MacKinnon et al. (1996). The model used in the Johansen cointegration method includes a linear deterministic trend in the data and a constant in the cointegrated equation. Results were derived using the EViews software. The break specification in IO and AO unit root tests include a constant and a time trend, since both were statistically significant at the 5% significance level. The lag length selection was based on Schwartz information criterion. The breakpoint selection method was the Dickey Fuller minimize t-statistic. The break dates were 1999M02 for all models, except from AO model with a constant and a trend in the specification model which produced 1998M01 as the breaking date.

One possible reason that the ADF test did not reject the null hypothesis of a unit root in the estimatd residuals within the EG framework, is the presence of structural breaks in the series. As indicated out by the IO and AO test in the previous sections, both series exhibit structural breaks and more importantly the real price of oil can be stationary at the 10% level if the structural break is taken into account. Therefore, the GH cointegration test seems more robust to the conventional cointegration tests, such as the EG and Johansen cointegration tests Results are reported in **Table 8**, suggesting that the null hypothesis of no cointegration cannot be rejected in most cases. Therefore, when the structural breaks are taken into account, both series appear to have a long-run relationship.

Table 8. Gregory-Hansen cointegration test results

Model	Tests	Test statistic	Break Date	Asymptotic Critical values 1%	5%	10%
Level break	ADF	-5.14	1998m8	-5.13	-4.61	-4.34
	Z_t	-5.29	1999m9	-5.13	-4.61	-4.34
	Z_a	-51.10	1999m9	-50.07	-40.48	-36.19
Level break and trend	ADF	-4.94	1999m10	-5.47	-4.95	-4.68
	Z_t	-4.96	1999m9	-5.47	-4.95	-4.68
	Z_a	-47.96	1999m9	-57.17	-47.04	-41.85
Regime and trend shift	ADF	-5.25	2009m1	-6.02	-5.50	-5.24
	Z_t	-5.33	2009m2	-6.02	-5.50	-5.24
	Z_a	-50.14	2009m2	-69.37	-58.58	-53.31

Notes: t-statistics denotes the ADF minimum test statistic for a unit root across all possible break points and Z_t denotes the PP unit root test. The optimal lag length was chosen based on the AIC. T_b denotes the estimated break date. The critical values are tabulated in Gregory and Hansen (1996a, 1996b).

The possibility of asymmetric cointegration between the examined variables in this thesis, is studied by applying a NARDL(2,2) model. Results are reported in **Table 9** suggesting that the lagged value of *wti*, which is essential in computing the asymmetric effects of *reer* on *wti*, is negative statistically significant in all estimated models. By carefully inspecting Table 9, it is apparent that both short-and-long run asymmetry appears to hold, since $\sum_{i=0}^{m} p_{2_i} \Delta x_{t-i}^{+} \neq \sum_{i=0}^{n} p_{3_i} \Delta x_{t-i}^{-}$ and $a_1 \neq a_2$. In other words, the short-run and long-run positive and negative partial sums of the US real effective exchange rate are not equal, and thus there is strong evidence of asymmetric effects. Finally, the inclusion of dummy variables in models (6) and (7) appear to have significant impact on the price of oil.

Table 9. NARDL(2,2) results

	(1) full sample	(2) <2009m1	(3) >=2009m1	(4) <=2019m11	(5) >=1999m9	(6) dummy1	(7) dummy2
$lnwti_{t-1}$	-0.053***	-0.091***	-0.146***	-0.043***	-0.141**	-0.089***	-0.100***
	(0.016)	(0.033)	(0.044)	(0.015)	(0.062)	(0.020)	(0.020)
$lnreer_{t-1}^{+}$	-0.114	-0.122	-0.446*	-0.100	-0.347	-0.302***	-0.067
	(0.076)	(0.089)	(0.237)	(0.067)	(0.264)	(0.101)	(0.076)
$lnreer_{t-1}^{-}$	-0.145*	-0.276**	-0.460	-0.122*	-0.476	-0.322***	-0.191**
	(0.081)	(0.134)	(0.288)	(0.071)	(0.426)	(0.103)	(0.081)
$dlnwti_{t-1}$	0.194***	0.272***	0.211***	0.209***	0.242*	0.207***	0.210***
	(0.054)	(0.079)	(0.079)	(0.056)	(0.127)	(0.053)	(0.053)

$dlnreer_t^+$	-3.793***	-2.068**	-5.680***	-2.898***	-2.410*	-3.764***	-3.500***
	(0.735)	(0.885)	(1.195)	(0.676)	(1.252)	(0.728)	(0.728)
$dlnreer_{t-1}^+$	-1.092	-1.768**	-1.070	-0.390	0.325	-1.005	-1.146
	(0.781)	(0.891)	(1.381)	(0.695)	(1.271)	(0.773)	(0.768)
$dlnreer_t^-$	-0.191	0.262	0.103	-0.295	-0.804	-0.084	0.036
	(0.815)	(0.899)	(1.584)	(0.719)	(1.345)	(0.808)	(0.804)
$dlnreer_{t-1}^-$	0.597	1.326	1.233	0.006	2.849**	0.928	0.839
	(0.819)	(0.919)	(1.543)	(0.719)	(1.305)	(0.819)	(0.808)
dummy1						0.065***	
						(0.023)	
dummy2							-0.091***
							(0.026)
constant	0.174***	0.241***	0.521***	0.136***	0.357**	0.248***	0.272***
	(0.045)	(0.077)	(0.162)	(0.041)	(0.152)	(0.052)	(0.053)
Observations	339	178	161	309	67	339	339
R-squared	0.186	0.184	0.264	0.159	0.211	0.205	0.216

Notes: *, ** and *** denote rejection of the null hypothesis at the 10%, 5% and the 1% significance level respectively. Dummy1 and dummy2 are used in order to capture the structural breaks found using the Gregory-Hansen cointegration test and IO2/AO2 breakpoint unit root tests respectively. In particular, dummy1 takes the value 1 during the period 1998m08-2022m05 and dummy2 takes the value 1 during the periods 2014m08-2022m05.

The estimated long-run coefficients along with the asymmetry and diagnostic tests of each model are presented in **Table 10**. Results reveal that there is an asymmetric both sort-and-long run impact of the US real effective exchange rate on real oil price in almost all estimated models. In particular, the long-run asymmetry effect is statistically significant when the full sample is used in the estimation process (model 1), the period before the financial crisis in 2009 and the pandemic (models 2 and 4), and when a dummy is introduced to capture the structural break occurred in 2014m08[13]. The short-run asymmetry is statistically significant in all estimated models, except from model (5).

Regarding the long-run positive and negative changes in *neer*, findings reveal some interesting results. In particular, the results reveal the long-run impact of the US positive changes in real effective exchange rate on real oil price to be negative and statistically significant in models (1), (3) – (4) and (6). In contrast, negative changes in the US real effective exchange rates exert a

[13] The dummy variables were introduced in order to capture the structural breaks found in the examined series. In particular dumm1 captures the structural break occurred in 1998m08 when the Gregory-Hansen cointegration test is applied. In addition, the break occurred in 2009m1-2009m2 was not used as a dummy variable, but instead two separate models were estimated, one before and one after the financial crisis in 2009. Results did not change significantly. Dummy2 captures the structural break occurred in 2014m08 when the IO2 and AO2 breakpoint unit root tests used In the series.

positive effect on oil prices and the estimate coefficients are statistically significant in all models except from model (5). More importantly, regarding the long-run coefficients, the estimated models suggest that on average, the negative changes in reer display a stronger impact on *wti*, except when the *dummy1* is introduced. Also, it is important to note that the impact of the long-run asymmetry effects become stronger after the financial crisis in 2009 and when the dummy variable that captures the structural break in 1998m08 is introduced. Hau and Rey (2004) point out that the asymmetric response of oil prices to changes in the real exchange rate can arise due to monetary policy actions or due to portfolio re-balancing actions made by investors in order to mitigate their risk exposure.

Overall, the results imply that the price of oil captured by WTI is negatively linked with positive changes in the US real effective exchange rate. In other words, an appreciation of the US dollar leads to lower oil prices, whereas a depreciation in the value of the US dollar results in higher oil prices. The differences observed in the estimated positive and negative long-run coefficients imply asymmetry. These results that are in line with Rafailidis and Katrakilidis (2016), indicate that the US economy highly depends on imports relative to its major trading partners and consequently higher oil prices (due to lower *reer*) may lead to deterioration of the US current account. This in turn, suggests that the value of the US dollar depreciates faster than the value of currency of its major trading partners (Rafailidis and Katrakilidis, 2016, p. 140).

Regarding the asymmetric cointegration between the examined variables, the Bounds F-test suggests that the cointegration between *wti* and *reer* is ambigius during the examined period; however, cointegration between examined variables exists for the 10% significance level after the financial crisis in 2009. The long-run asymmetric relationship between *wti* and *reer* holds for the 5% and 1% significance level if only the structural breaks in 1998m08 and 2014m08 are taken into account respectively.

The diagnostic tests applied to the estimated models, suggest that the residuals do not suffer from autocorrelation. Furthermore, all models, except from models (2) and (7) do not suffer from misspecification and the residuals are normally distributed, except from models (6)-(7) where residuals suffer from non-normality for the 10% significance level. Finally, the estimated residuals in models (1), (3) and (6) suffer from heteroskedasticity for the 5% significance level.

Table 10. Asymmetry statistics and diagnostics

Panel A	(1) full sample	(2) <2009m1	(3) >=2009m1	(4) <=2019m11	(5) >=1999m9	(6) dummy1	(7) dummy2
Long-run effect (+)	-2.137*	-1.334	-3.046*	-2.338*	-2.458	-3.886***	-0.668
	[3.458]	[2.607]	[7.958]	[3.325]	[2.619]	[22.19]	[0.856]
	(0.064)	(0.108)	(0.005)	(0.069)	(0.111)	(0.000)	(0.356)
Long-run effect (-)	2.723**	3.018***	3.148**	2.854**	3.377	3.661***	1.904***
	[5.618]	[12.38]	[4.54]	[5.2]	[1.516]	[27.27]	[8.539]
	(0.018)	(0.001)	(0.035)	(0.023)	(0.223)	(0.000)	(0.004)
Long-run asymmetry	[9.019]***	[34.65]***	[0.045]	[4.4.27]**	[0.419]	[2.234]	[49]***
	(0.003)	0.000	(0.831)	(0.036)	(0.520)	0.136	(0.000)
Short-run asymmetry	[9.145]***	[7.62]***	[5.288]**	[3.071]*	[2.444]	[10.46]***	{10.28]***
	(0.003)	(0.006)	(0.023)	(0.055)	(0.123)	(0.001)	(0.001)
Panel B							
Bounds F-test	4.107	2.707	4.289*	3.016	1.836	6.753**	8.313***
Model Diagnostics							
Portmanteau test	33.9	37.32	23.89	31.43	20.17	30.17	31.9
	(0.7403)	(0.592)	(0.979)	(0.832)	(0.932)	(0.870)	(0.816)
Breush/Pagan heteroskedasticity test	5.567**	3.004*	5.909**	1.470	1.83	4.346**	2.803*
	(0.018)	(0.083)	(0.015)	(0.225)	(0.176)	(0.037)	(0.094)
Ramsey Reset F-Test	2.195*	2.403*	4.002**	2.286*	0.295	2.693*	2.812**
	(0.083)	(0.066)	(0.010)	(0.079)	(0.828)	(0.053)	(0.041)
Jarque-Bera	4.426	4.328	2.842	4.371	0.112	5.824*	5.386*
	(0.103)	(0.115)	(0.241)	(0.112)	(0.946)	(0.055)	(0.068)

Notes: *, ** and *** denote rejection of the null hypothesis at the 10%, 5% and the 1% significance level respectively. Dummy1 and dummy2 are used in order to capture the structural breaks found using the Gregory-Hansen cointegration test and IO2/AO2 breakpoint unit root tests respectively. In particular, dummy1 takes the value 1 during the period 1998m08-2022m05 and dummy2 takes the value 1 during the periods 2014m08-2022m05 Panel A: F-stat in brackets, p-value in parentheses. Panel B: p-values in parentheses. Lower and Upper-bound critical values for the Bounds F-test are obtained from Pesaran et al. (2001), Table CI(ii) and Case II: Lower bound

The dynamic multipliers graphs are reported in Appendix in **Figures A.1-A.7**. By visually, inspecting **Figure A.1**, it is apparent that the positive changes in the US real effective exchange rate display a strongly negative effect on the real oil price during the first 3 months, whereas the effect diminishes in effect, but is still negative in sign, 5 months after the initial change in *reer*. In contrast, a negative change in *reer* have initially (about 3 months) negative effect on *wti*, but the effect turns positive 4 months after the initial change in *reer*. Finally, the combined effect of negative and positive changes in *reer* on *wti* appears to be negative, but with diminishing effect as the horizon increases.

The same picture applies during the different time periods, but the timing and the magnitude of the change differs. For example, after the financial crisis in early 2009s, the positive changes in *reer* display a stronger effect on *wti* relative to the period before the financial turmoil, and the negative effect diminishes rather slowly compared to the past periods. Similarly, if the pandemic is excluded from the analysis, the positive changes in *reer* exert a strongly negative effect on *wti* during the first two months of the change and the effect appears to be diminishing very slowly since the 4^{th} month of the initial change. In addition, the negative changes in *reer* display strongly positive effect on *wti* at the beginning of the change and the effect does not appear to diminish at all.

Finally, the estimated coefficients from all models appear to be significant, since the estimated CUSUM lies within the 5% error bands.

5. Conclusions

The purpose of this thesis was to empirically investigate the relationship between oil prices and the US exchange rate. In particular, the oil prices were captured by the spot price of West Texas Intermediate and the US real exchange rate by the US real broad effective exchange rate. Both variables were obtained from the Federal Reserve of St. Louis, have monthly frequency and the time span ranges from 1994M01 to 2022M05, yielding a total number of 341 observations.

The first step of the empirical investigation of the relationship between oil prices and the US exchange rate involved the examination of the order of integration of each variable by applying conventional unit root tests. Results revealed that both series are nonstationary at their levels and stationary at their first differences. In addition, the IO and AO breakpoint unit root tests revealed that both series are nonstationary at levels and stationary at their first differences; however, the logarithm of the real price of oil appears to be stationary at the 10% significance level, when a break date is used in both the constant and the trend.

The next step involved the examination of the casual relationship between the selected variables. The Toda Yamamoto approach was applied within a VAR framework revealing that the US real effective exchange rate Granger causes real price of oil and not vice versa. Furthermore, the Generalized Impulse Response Functions revealed a negative relationship between oil prices the US real exchange rate.

The examination of the long-run relationship between the variables of this study was conducted first by applying the conventional cointegration tests developed by Engle and Granger and Johansen. Both procedures indicated that the two variables are not cointegrated. However, the IO and AO unit root tests suggested that both series have structural breaks and therefore the rejection of the null hypothesis of no cointegration may occur due to the existence of structural breaks in the series. For this purpose, the Gregory-Hansen cointegration test was applied in order to take into account the possible structural breaks in the series. Results revealed that the null hypothesis of no cointegration could not be rejected with the consideration of two breakpoints, the first in 1998m08 and the second in 2009m01.

Finally, the asymmetric cointegration analysis was conducted by applying a nonlinear ARDL model in order to investigate whether the series display asymmetries in the long run. Results suggested that a positive change in the US real effective exchange rate has a negative effect on real oil prices, whereas a negative change in the real effective exchange rate results in higher oil prices. In other words, the real oil prices respond asymmetrically in changes in the US real effective exchange rate.

In general, results indicate that an appreciation of the US dollar leads to lower oil prices, whereas a depreciation in the value of the US dollar results in higher oil prices. The differences observed in the estimated positive and negative long-run coefficients imply asymmetry. These results that are in line with Rafailidis and Katrakilidis (2016), indicate that the US economy highly depends on imports relative to its major trading partners and consequently higher oil prices (due to lower *reer*) may lead to deterioration of the US current account. This in turn, suggests that the value of the US dollar depreciates faster than the value of currency of its major trading partners (Rafailidis and Katrakilidis, 2016, p. 140).

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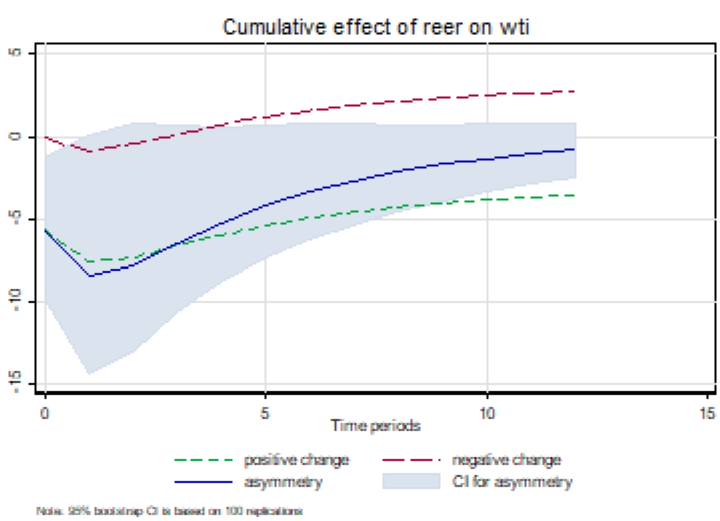
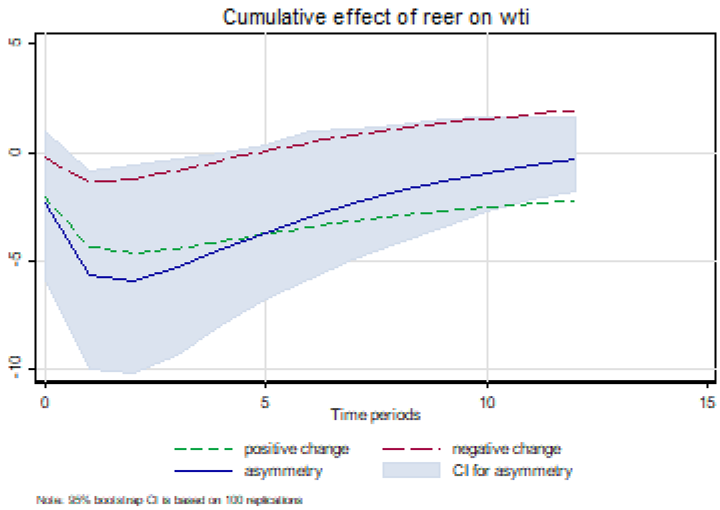
Appendix A. Dynamic Multipliers from NARDL(2,2) models (1)-(7)

Figure A.1 Dynamic Multipliers Full Sample



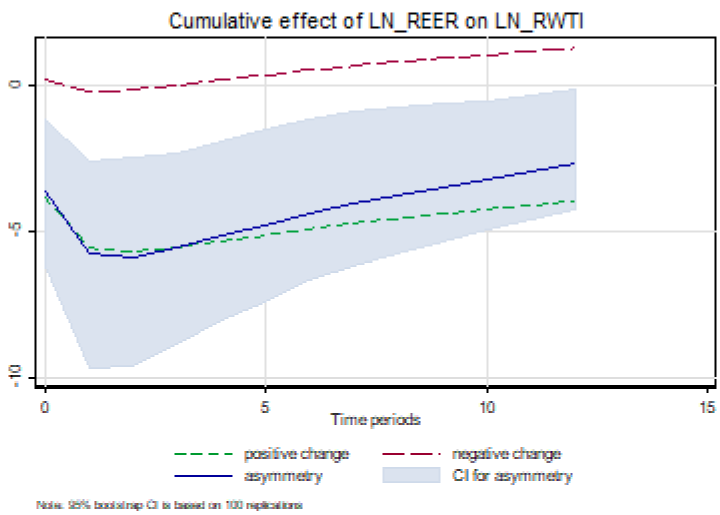
Notes: Dynamic multipliers were computed using the *nardl* module in Stata.

Figure A2. Dynamic Multipliers before 2009m01



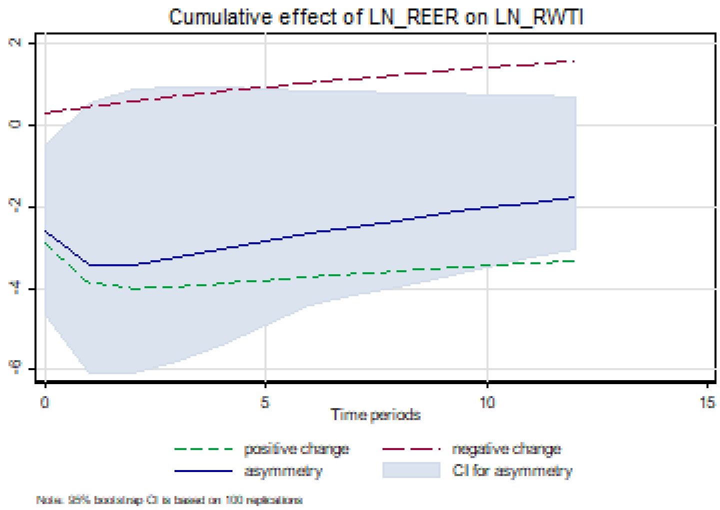
Notes: Dynamic multipliers were computed using the *nardl* module in Stata.

Figure A3. Dynamic Multipliers after 2009m01



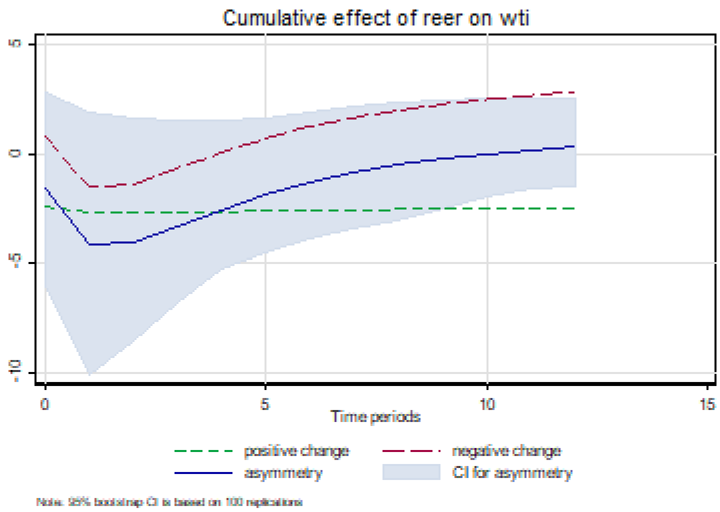
Notes: Dynamic multipliers were computed using the *nardl* module in Stata.

Figure A4. Dynamic Multipliers before the pandemic



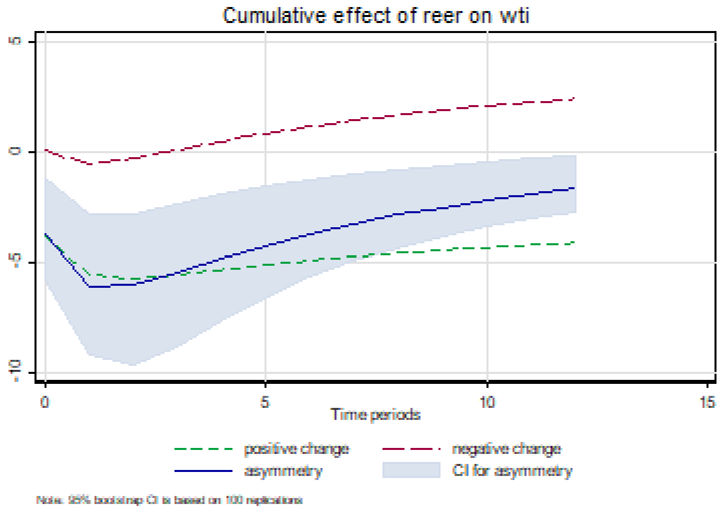
Notes: Dynamic multipliers were computed using the *nardl* module in Stata.

Figure A5. Dynamic Multipliers after 1999m09



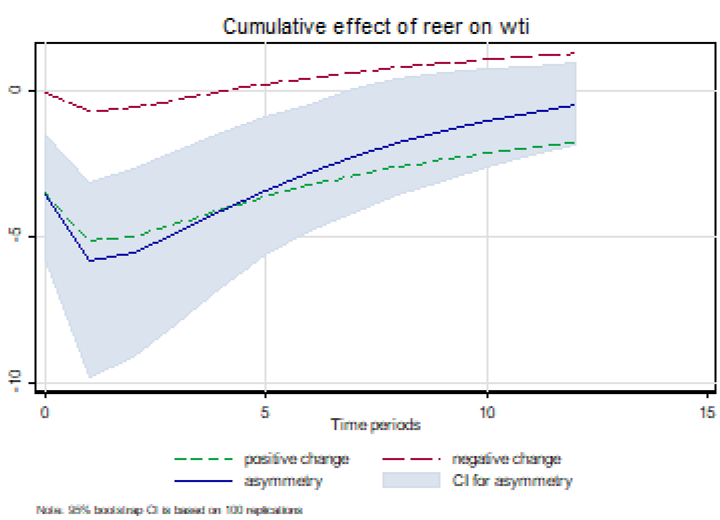
Notes: Dynamic multipliers were computed using the *nardl* module in Stata.

Figure A6. Dynamic Multipliers with dummy1



Notes: Dynamic multipliers were computed using the *nardl* module in Stata. dummy1 takes the value 1 during the period 1998m08-2022m05

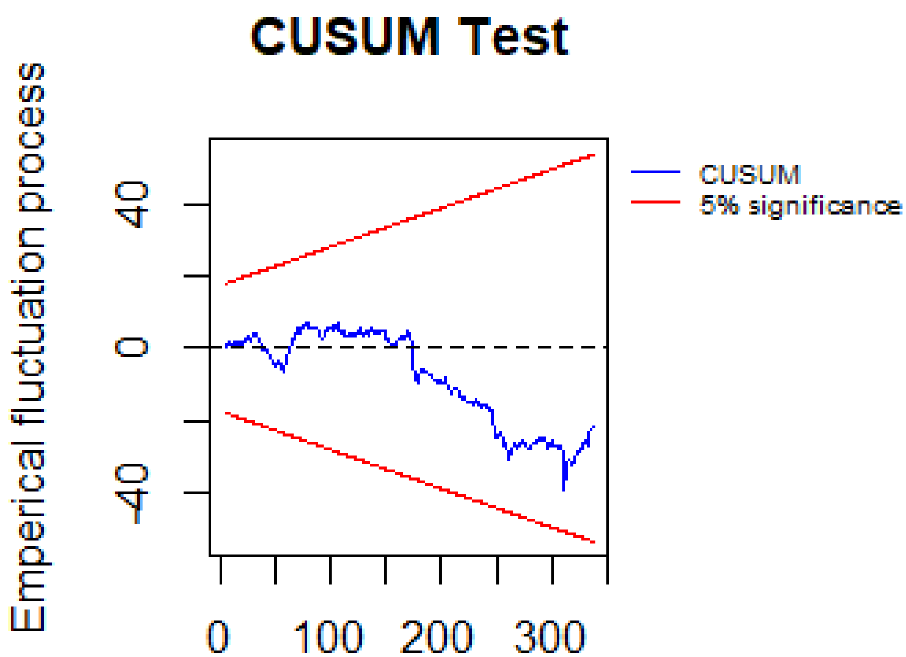
Figure A7. Dynamic Multipliers with dummy2



Notes: Dynamic multipliers were computed using the *nardl* module in Stata. dummy2 takes the value 1 during the periods 2014m08-2022m05

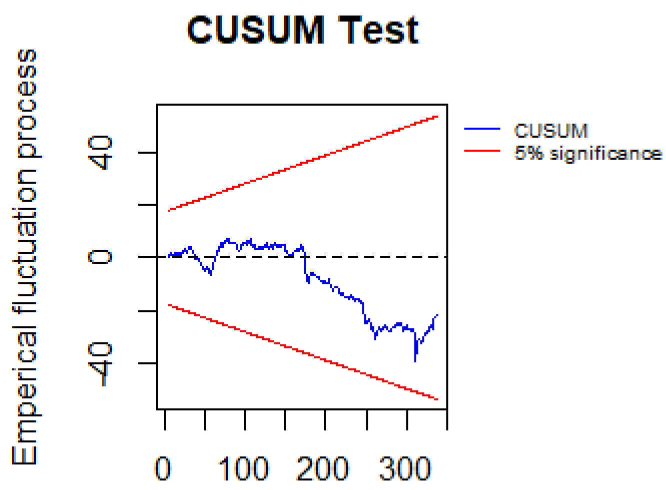
Appendix B. CUSUM stability tests from NARDL(2,2) models (1)-(7)

Figure B1. CUSUM test full sample



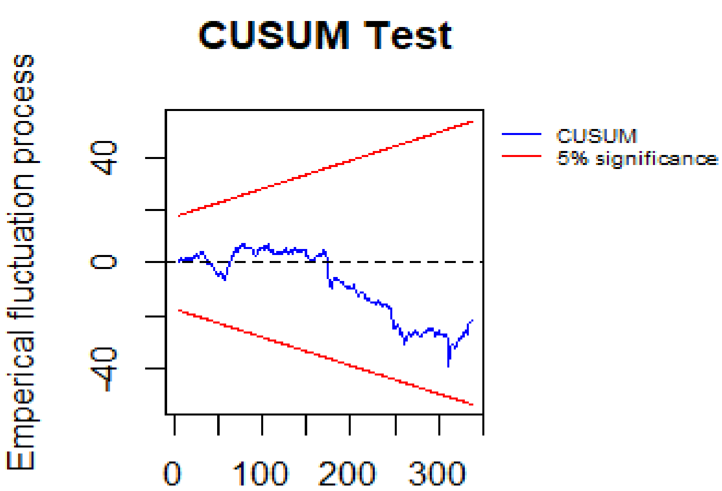
Notes: Results were obtained using the nardl package in R software

Figure B2. CUSUM test before 2009m01



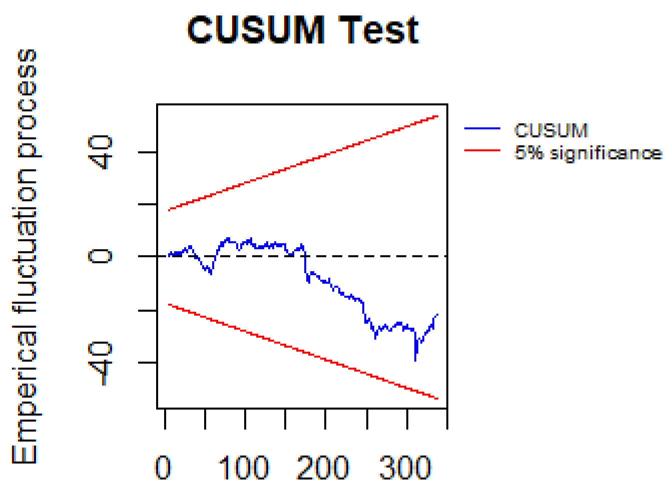
Notes: Results were obtained using the nardl package in R software

Figure B3. CUSUM test after 2009m01



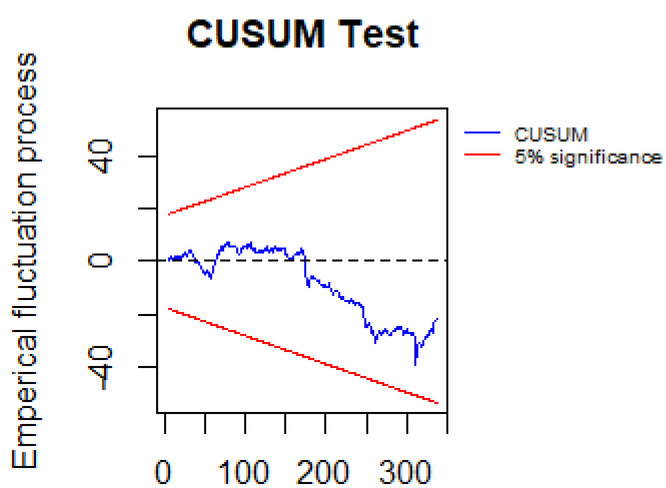
Notes: Results were obtained using the nardl package in R software

Figure B4. CUSUM test before the pandemic



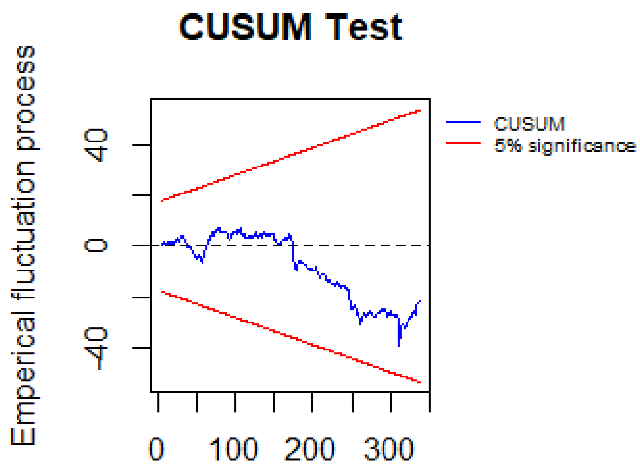
Notes: Results were obtained using the nardl package in R software

Figure B5. CUSUM test after 1999m09



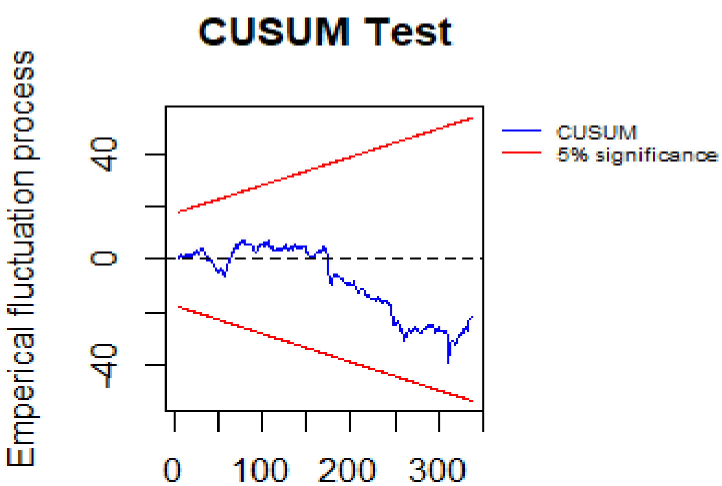
Notes: Results were obtained using the nardl package in R software

Figure B6. CUSUM test with dummy1



Notes: Results were obtained using the nardl package in R software

Figure B7. CUSUM test with dummy2



Notes: Results were obtained using the nardl package in R software