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**“ HENRYK GROSSMANN AND THE THEORY OF CRISIS:
AN OVERVIEW”**

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1. INTRODUCTION.

Henryk Grossmann was born in 1881, in Cracow and he was the son of a Jewish mine owner in Galicia. He studied law and economics in Cracow and Vienna where he developed a great interest in economic theory. As a student, he became a member of the left-wing student organization Ruch in Cracow while in 1905 he was one of the founders of the Jewish Social Democratic Party of Galicia. In 1918, he became a member of the Polish Communist Party and worked in Warsaw for the Central Statistical Office and the Free Polish University. In 1924, Grossmann was arrested because of his connection with the Communist Party and in 1925 he moved in Germany where he worked with the Institute for Social Research in Frankfurt. He was forced to leave Germany in 1933 when Hitler took possession of power in Germany. He was deported at first in Paris (1933-1935), in London (1935-1937). In 1937, he moved in New York with the Institute. After the end of World War II, he returned to Europe (1949) and he was declared Professor of Political Economy at the University of Leipzig. Henryk Grossmann died in 1950.

Henryk Grossmann became widely known by his principal work "Das Akkumulations-und Zusammenbruchsgesetz des Kapitalistischen Systems" which has been published in 1929. He also wrote "Sismonde de Sismondi et ses theories economiques"(1924), "Die Anderung des ursprunglichen Aufbauplans des Marxschen Kapital und ihre Ursachen"(1929), "The Evolutionist Revolt against Classical Economics" (1943) and many other writings. His principal work "The Law of Accumulation and Breakdown of the Capitalist System", as translated in English, has been criticized heavily by many Marxian economists. The centre of the

controversy was whether Grossmann's arguments about the breakdown of the capitalist system are consistent with Marx's theory of crisis.

Grossmann adopted Marx's thesis that the collapse of the capitalist process of production is unavoidable because of the tendency of the rate of profit to fall. Grossmann's main purpose was to demonstrate that the accumulation process of capital cannot be continued indefinitely because of this decline in the rate of profit. To maintain his task, he expanded Otto Bauer's reproduction schemes to 36 periods and he concluded that in year 36 the accumulation process has to be interrupted because the capitalist mode of production can no longer yield the appropriate surplus value which secures the balanced reproduction of the system. This implies that capitalist consumption- the part of surplus value which capitalists keep to satisfy their own needs- falls down to zero and capitalists have no more incentives to continue the accumulation process. This is a version of the well-known theory of breakdown which became the centre of debate in the '30s. In his book, Sweezy exerted a vigorous criticism on Grossmann's thoughts and he accused him for incapacity to elucidate the basic elements of his theory which led him to conclude that the collapse of the capitalist system is unavoidable, a conjecture that cannot be admissible. It seems that Sweezy misconceived Grossmann's theory of breakdown as he considered it as a theory of automatic collapse of the capitalist system. It is true however that, the utilization of the term "collapse" by Grossmann is problematic and it is the cause of the existing agitation. Grossmann intended to develop a theory of crisis related to a theory of cycles and not to advance a theory of automatic collapse of the capitalist system. So, the above terminology awkwardly used by him.

Grossmann's theory of breakdown is not the only notable piece of his essay. Grossmann gets near to reality by analyzing the influences of Countertendencies in the accumulation process. The capitalist mode of production is characterized by the existence of interacting forces many of which are contradictory. When these forces begin to operate, accumulation is not any longer a continuous process but it takes the form of periodic cycles. So the breakdown tendency of the capitalist system is suspended and it is altered into a temporary crises. Grossmann claims that these driving forces derive from the inner mechanism of capital, they influence the mechanism of capital's movement in the world market and they create conditions of intense competition in capitalist world economy.

Grossmann's work is significant and deserves our respect, not only because of the way he confronted and elaborated Marx's theory but also because of , he was one of the first who attempted to give a mathematical formula of his theory of breakdown. In this essay, we will try to analyze and evaluate Grossmann's theory of crisis and we will discuss subsequent debates about his work.

2. THE GROUND OF THE DEBATE.

First of all, we have to mention that there are two types of economic crisis: Those which arise from the tendency of the rate of profit to fall and those which are mentioned as “crisis of realization” and emanate from the incapacity of capitalists to realize completely the value of the commodities they produce. The crisis of realization are distinguished in:

- a. Crisis due to the anarchic character of capitalist mode of production which implies the unequal development of different branches of economy and
- b. Crisis that spring from the underconsumption of masses.

At the beginning the discussion focused in crisis of realization and there was an intense controversy between those who sustained that the unequal production is the real cause for the overflowing of economic crises and those who supported that crisis occurs when masses lose their power of consumption. The first theory, which is well known as “ theory of disproportionalities”, became popular by Tugan Baranowsky, a Russian economist who was contending that the disproportionality theory was the only possible explanation of economic crisis. He rejected the theory of underconsumption with the justification that as long as production is distributed equally among the different branches of economy, overproduction or inadequacy of demand cannot be sustained. He also refuted Marx’s statement that crisis is the emanation of the tendency of the rate of profit to fall by using the argument that the rising organic composition of capital implies an increase in the rate of profit and not a reduce of it as Marx believed. In his analysis, he used Marx’s reproduction schemes in order to prove the necessity of existence of coordinated development of different branches of production and to verify that capitalism contains unlimited

possibilities

of

development..

2.1 DISPROPORTIONALITY THEORY

If we want to penetrate in Tugan-Baranowsky's syllogistic, it is useful to examine the way he exposes his thoughts by using Marx's reproduction schemes. Firstly, he supposes that there are two sectors in the capitalist economy. Sector I consists of the industries which produce means of production and Sector II includes the industries that produce means of consumption. In the case of simple reproduction, at the end of productive process, capitalists of sector I possess a quantity of means of production and capitalists of sector II occupy a quantity of means of consumption.

In sector I $A_1 = c_1 + v_1 + s_1$

In sector II $A_2 = c_2 + v_2 + s_2$

where A_i , $i=1,2$ denote the value of the commodity produced in the two sectors.

c_i , $i=1,2$ is the constant capital that has been used in the productive process

v_i , $i=1,2$ is the variable capital and

s_i , $i=1,2$ is the surplus value that has been produced at the end of the productive process. In the second phase of capitalist process, capitalists of sector I will use only one part of the means of production they have produced. In conditions of Simple reproduction, there is no accumulation of capital. So capitalists of sector I will use the surplus value that has been produced in order to buy consumption goods from sector II. Similarly, capitalists of sector II have already secured the necessary means for their preservation from their own industry, but in

order to enter in the second phase of productive process they have to buy the necessary mechanical equipment and raw materials from sector I.

In every year, the constant capital which has been wasted during the productive process has to be replaced, so

$$C_1 + V_1 + S_1 = C_1 + C_2 \quad (1)$$

Also, the supply of the consumption goods has to absorb the capitalists' and workers' income, that means

$$C_2 + V_2 + S_2 = V_1 + S_1 + V_2 + S_2 \quad (2)$$

From equations (1) and (2), the equilibrium condition in the simple reproduction is:

$$C_2 = V_1 + S_1 \quad (3)$$

which insures the existence of balance between the two sectors in the economy.

In the extensive reproduction, a part of surplus value that has been produced in the previous year, has to be used as additional constant capital in the following phase of the productive process. Another part of the produced surplus value has to be used for covering the additional labor power necessary for the operation of the additional constant capital. Equilibrium condition require that demand of department I for consumption goods must be equal with demand of department II for means of production. Capitalists and workers of department I will consume commodities of department II but in the initial consumption of the workers has to be included the consumption of the additional labor power which has been occupied by the new capital. Capitalists of department II are obliged to dispose a part of surplus value to buy raw materials and equipment in order to replace the spoiled constant capital.

The capital has to be distributed equally among the two sectors of production and demand for means of production must equal the supply of means of

consumption in order capitalists to avoid the existence of excesses or shortages between the two sectors. The model of extensive reproduction takes the form:

$$A_1 = C_1 + V_1 + S_{k1} + S_{dk1} + S_{v1} + S_{c1} \quad (1)$$

$$A_2 = C_2 + V_2 + S_{k2} + S_{dk2} + S_{v2} + S_{c2} \quad (2)$$

where s_k is the part of surplus value necessary to keep up the capitalists consumption in the previous levels.

s_{dk} denotes the rise in capitalists' consumption which results from the increase of labor power and from the increase in capitalists' income.

s_v is the part of surplus value that capitalists use in order to acquire the necessary additional variable capital.

s_c is the part of surplus value needed for the replacement of the ruined constant capital.

The demand for constant capital must equal the total production of constant capital, or

$$C_1 + S_{c1} + C_2 + S_{c2} = C_1 + V_1 + S_{k1} + S_{dk1} + S_{v1} + S_{c1} \quad (3)$$

The demand for consumption goods must equal the total production of consumption goods, or

$$V_1 + S_{k1} + S_{dk1} + S_{v1} + V_2 + S_{k2} + S_{dk2} + S_{v2} = C_2 + V_2 + S_{k2} + S_{dk2} + S_{v2} + S_{c2} \quad (4)$$

Combining equations (3) and (4) we take the equilibrium condition which is:

$$C_2 + V_{c2} = V_1 + S_{k1} + S_{dk1} + S_{v1} \quad (5)$$

By using the above scheme of reproduction Tugan-Baranowski claimed that crisis will occur if surplus value is not divided between the various branches of production in the right proportions and it will be avoided if additional constant capital is distributed correctly among the sectors. Tugan-Baranowski and later the socialist

neo-harmonists Rudolf Hilferding and Otto Bauer turned down the idea that capitalism contains unsurpassable economic limits. Tugan-Baranowski states:

“ there are no grounds for supposing that capitalism will ever meet a natural death; it has to be destroyed through conscious, human will, destroyed by the class exploited by capital, by the proletariat”.

Baranowski's point of view has been espoused by Otto Bauer who, in his basic work “ Kapitalismus und Sozialismus nach dem Weltkrieg”, cites the followings:

“ The conclusive cause of the industrial cycle is that capitalistic society is unable to distribute the renewal, the expansion and the finishing of productive mechanism equally in different years, but it centralizes them in some years while it suspends them completely at amongst periods”.

According to Bauer, the size of a given population imposes objective limits on accumulation of capital. Within these limits, accumulation of capital appears to be unbounded and crisis occur only when accumulation goes beyond these limits. Relatively with the population at the disposition of capital, he esteems that there is either an overaccumulation or an underaccumulation of capital. So those crises that occur under a periodic form are nothing else but temporary ruptures of the equilibrium of capital accumulation. He believes that:

“ The periodic occurrence of phases of prosperity, crisis and depression is only the empirical expression of the fact that the mechanism of the capitalist mode of production automatically eliminates over or under-accumulation and always adjusts the accumulation of capital to the growth of population”.

In a similar way, Hilferding supported disproportionality theory as the only possible explanation of crises by saying that overproduction would never appear if there was a proportional distribution of capital in the branches of industry, so production can be expanded indefinitely without leading to overproduction of commodities. He also argued that breakdown of capitalism is not an economic matter but a political and social phenomenon.

2.2 THEORY OF UNDERCONSUMPTION

Rosa Luxemburg disputed Tugan-Baranowski's reasoning declaring that his view that : " the production of means of production does not depend on consumption", is a vulgarized economic illusion of him. According to Luxemburg's opinion, the realization of surplus value constitutes the main problem of the capitalist accumulation process. Crises of underconsumption appear as crises of realization. Supporters of this theory purport that there is a discrepancy, inherent in the capitalist mode of production. There is a tendency for unlimited development of production and simultaneously, there is a tendency for the consumption of masses to be restricted.

The main purpose and the basic motive of capitalists is the augmentation of their profits and the increase of accumulation. To maintain their task, they try to develop their labor productivity by introducing improved technical methods of production and new equipment. Labor power is reduced as human labor is replaced by machines. The purchasing capacity of the working class diminishes while at the same time, production of commodities keeps on rising. The commodities that are disposed in the market remain unsold as the consumptionist power of masses has been restrained. As a consequence of this, an surplus supply of commodities is displayed in the market which will lead capitalists to restrain the production of consumption goods. In that point, crises will occur. Because of the existing economic situation, the owners of less developed industries will pursue to lessen the active working population and a reserve army of unemployed will occur in the labor market.

Luxemburg based on the above theory, tried to establish that accumulation of capital is impossible within the borders of a closed capitalist system which is constituted exclusively by capitalists and workers. She states that if capitalists want to realize the accumulated surplus value then they are obliged to sell it in consumers who are found out of the capitalist system either because the country they live remains intact by capitalism or because they belong to classes other than capital and labor. The development process leads the underdeveloped nations to adopt capitalist mode of production. Luxemburg concludes that “ **capitalism cannot exist without non-capitalist markets and the system will collapse by its own**”. This was the argument she used in order to reject neo-harmonists’ theory and a great historical contribution began.

Few years later, Grossmann will context Luxemburg’s reasoning by referring that “ **If her line of reasoning were true, the breakdown tendency would have been a constant symptom of capitalism from its very inception, and it would be impossible to explain either periodic crises or the characteristic features of the latest stage of capitalism called ‘imperialism’. Yet Luxemburg herself had the feeling that the breakdown tendency and imperialism only appear at an advanced stage of accumulation and find their sole basis in this stage**”

and he continues as follows:

“ **Her own deduction of the necessary downfall of capitalism is not rooted in the immanent laws of accumulation process, but in the transcendental fact of an absence of non-capitalist markets. Luxemburg shifts the crucial problem of capitalism from the sphere of production to that of circulation. Hence the form in which she conducts her proof of the absolute economic limits to capitalism**

comes close to the idea that the end of capitalism is a distant prospect because the capitalization of the non-capitalist countries is the task of centuries”.

In his book “ Theory of capitalist development”, Paul Sweezy contends Tugan’s point of view that since accumulation is related indissolubly with capitalist development, crises are inevitable as long as capitalists does not establish an effective scheme of production. He also disputed the way Tugan choosed to found his convictions. Sweezy believes that the reproduction schemes do not offer reliable conclusions. In order to prove it, he used Tugan’s reproduction scheme

$$A_1=C_1+V_1+S_{k1}+S_{dk1}+S_{v1}+S_{c1} \quad (6)$$

$$A_2=C_2+V_2+S_{k2}+S_{dk2}+S_{v2}+S_{c2} \quad (7)$$

and he assumed that consumption remains constant or that $s_{dk1}=s_{dk2}=0$ and that accumulated capital is used only as additional constant capital or $s_{v1}=s_{v2}=0$. Under these assumptions, the equilibrium condition (5) becomes:

$$V_1+S_{k1}=C_2+S_{c2} \quad (8)$$

and by using the equilibrium condition of simple reproduction (which is also valid in expanded reproduction) follows that $s_{c2}=0$. In other words, there is no accumulation process in department II, a conclusion that contradicts the initial assumption.

Sweezy argued that Tugan’s arguments are baseless and that the changes in demand for consumption goods are the real causes of crises. He also criticized Luxemburg’s theory and he indicates that she make a basic mistake which overshadows all the others: In her analysis of expanded reproduction, Luxemburg preserves tacitly the assumptions of simple reproduction.

2.3 THE REPRODUCTION SCHEMES

In the previous part, we analyzed the crises of realization and the assertions of Tugan-Baranowski, Otto Bauer and Hilferding in relation to the theory of disproportionality and we mentioned the aspects of Rosa Luxemburg and Paul Sweezy who adopted a theory of underconsumption. We have already mentioned that Bauer's will was to prove that capitalism contains unlimited possibilities of development and he tried to demonstrate it by constructing a reproduction scheme which, according to Grossmann's opinion " **it represents a distinct progress over all earlier attempts of this kind**".

Bauer constructed his scheme based on the following assumptions:

1. There is an ever increasing organic composition of capital which certifies the existence of continual technological advances.
2. Constant capital grows at 10% every year while variable capital grows at 5%.
3. Increases in productivity and the mass of surplus value permit the using of a progressively greater portion of surplus value for the purposes of accumulation.
4. There is a symmetry between department I and II, in the sense that both departments exact, annually, the same percentage of surplus value to accumulation.
5. The rate of profit satisfies the law of its continual fall according to Marx's theory.

Bauer starts his analysis with an initial constant capital $c_0 = 200.000$ and variable capital $v_0 = 100.000$ and distributes the available capital among the two departments as follows: 120.000 of constant capital is disposed in department I and 80.000 of it, in department II. The variable capital is distributed equally between the two departments. Another assumption he makes is that the rate of surplus value

remains constant at 100 per cent every year and that the rate of accumulation is equal in the two sectors. Bauer's task was to find out what will be the development of accumulation after four years of productive process. His results are represented in the following table.

Table 1. Bauer's reproduction scheme.

Year	Department	c	v	k	a _c	a _v	AV	k/s
1	I	120.000	+ 50.000	+ 37.500	+10.000	+2.500	=220.000	
	II	80.000	+ 50.000	+ 37.500	+10.000	+2.500	=180.000	
		200.000	+100.000	+75.000	+20.000	+5.000	=400.000	75%
2	I	134.666	+53,667	+ 39.740	+11.244	+2. 683	= 242.000	
	II	85.000	+51.337	+ 38.010	+10.756	+2.567	=188.000	
		220.000	+105.000	+77.750	+22.000	+5.250	= 430.000	74.05%
3	I	151.048	+57.567	+ 42.070	+12.638	+2.868	= 266.200	
	II	90.952	+ 52.674	+ 38.469	+11.562	+2.643	=196.300	
		242.000	+110.250	+ 80.539	+24.200	+5.511	=462.500	73.04%
4	I	169.124	+61.738	+ 44.465	+14.186	+3.087	=292.600	
	II	96.876	+54.024	+ 38.909	+12.414	+2.701	=204.924	
		266.000	+115.762	+ 83.374	+26.600	+5.788	=497.524	72.02%

where

c=constant capital

v=variable capital

k=capitalist consumption

a_c=accumulated as constant capital

a_v=accumulated as variable capital

AV=value of annual product

By constructing this scheme, Bauer believed that he has given a perfect answer to Luxemburg's crucial question about the necessity of existence of non-capitalist markets for the realization of surplus value. He claimed that if capital expands proportional to the growth of population then surplus value can be realized within capitalism. He also argues that as long as the rate of accumulation rises sufficiently fast to make possible that variable capital expand as promptly as population despite the rising organic composition of capital, capitalist process of production can only be interrupted by the political struggle of the working class. As he supports " **Socialism can only be the product of masses' conscious will**".

According to Bauer, crises appear only when capitalist are unable to preserve the indispensable proportionality between accumulation and population. He argues that the required condition of proportionality c/v can be maintained even in the long run and whichever disturbances of equilibrium occur during the accumulation process can be automatically corrected by the mechanism of capital. Proceeding this way, he concludes that underaccumulation and overaccumulation are only passing phases of the industrial cycle.

Grossmann acknowledges the significance of Bauer's scheme but contrary to Bauer's and Luxemburg's perspective, he alleges that these schemes cannot describe the real process of accumulation in terms of value and use value or, in other words, they can reflect only the value side of the reproduction process. He indicates, parallelly, that Bauer mistakenly believes that his schemes are somehow an illustration of the actual processes in capitalism as he ignores the simplified assumptions he made in order to construct them .

Finally, he notices that if Bauer's reproduction scheme are completely correct and describe precisely the real accumulation process, then the tendency of the rate of profit to fall ceases to be a real threat to capitalism, because it seems that capitalists can achieve the growth of the absolute mass of their profits with the presupposition that they accumulate at a rate which exceeds the fall in the rate of profit. So the tendency of the rate of profit to fall plays no role. This deduction opposes completely to Marx's theory. Few years later, Grossmann expanded Bauer's scheme and he proved the essential role of the tendency of rate of profit to fall and he examined in what way this tendency can affect the development of the accumulation of capital.

2.4 THE LAW OF THE TENDENCY OF THE RATE OF PROFIT TO FALL

The law of the rate of profit to fall is one of the central debate of Marxist economists. Marx considered this law to be of great significance and he believed that it is the most important law from the historical stand-point. According to Marx, the rate of profit tends to fall during the capitalist process of production because the increases of the organic composition of capital are not accompanied by proportionate increases of the rate of surplus value. Many Marxist economists believe that Marx considered that the fall of the rate of profit was the primordial principle for economic crises.

The rate of profit can be expressed as a function of the rate of surplus value and of the organic composition of capital in the following way:

$$r = s' / (c+v) = s' / (occ+1)$$

where $s' = s/v$ is the rate of surplus value which represents the portion of labor in net product and $occ = c/v$ is the organic composition of capital and expresses the proportion of constant capital by worker.

Marx distinguished three types of composition of capital:

- The technical composition of capital which is the ratio of the mass of means of production consumed per period to the mass of workers.
- The value composition of capital which is the ratio of constant capital to variable capital measured in terms of the current values of means of production and wage goods consumed.
- The organic composition of capital which is the ratio of constant capital to variable capital measured in terms of the old values of means of production and wage goods

consumed or, in other words, the organic composition of capital abstracts from the changes in values that derive from increases in productivity.

Marx maintains that the law of the tendency of the rate of profit to fall is a consequence of a rising organic composition of capital and in order to formulate his law, he abstracts from the effects of:

- a. the rising technical composition of capital,
- b. of changes in the rate of surplus value and finally
- c. the effects of prices and wage changes on the rate of profit. This means that if we assume that the rate of surplus value remains constant then an increase in the organic composition of capital leads to a fall in the rate of profit.

Marx analyses five counteracting tendencies which prevent and revoke the general law of the tendency of the rate of profit to fall. Marx's law has the character of a tendency which means that this law cannot predict the actual falls in the rate of profit. These counteracting tendencies are:

1. Increasing intensity of exploitation.

The intensity and the expansion of labor increases the rate of surplus value by compressing the necessary labor in shorter time. As a consequence, a greater part of the working day is used for the production of surplus value and the rate of profit rises.

2. Depression of wages.

When capitalists inflict the retrenchment of wages, the ratio c/v is reduced and the rate of surplus value s' increases and tends to increase the rate of profit.

3. Relative overpopulation.

The rising organic composition of capital throws out workers and an over-population is created. The creation of the reserve army of unemployed promote the installation of new industries which begin to operate with a low organic composition of capital and consequently, with a relative high rate of profit. The rate of profit rises in total. The existence of over-population has one more influence it impacts the value of wages through the competition in the labor market and tends to increase the rate of surplus value which, by its order, signifies the rise in the rate of profit.

4. Cheapening the elements of constant capital.

The increased utilization of machines increases the productivity of labor and depreciates the value of elements of constant capital. This depreciation can counterbalance the initial rise in the organic composition of capital.

5. Foreign trade.

The price of imported raw materials and necessary means of subsistence is possible to be smaller than their price when produced domestically. Foreign trade cheapens the elements of constant capital and tends to increase the rate of profit by increasing the rate of surplus value and by reducing the value of constant capital.

Marx concludes that crises are the implication of the complex contradictions between the tendency of the rate of profit to fall and the counteracting tendencies. As Marx claims “ **this contradiction causes overproduction, speculation, crisis, superfluous capital close to over-population**” and he continuous by saying that “**the significance of the law of the tendency of the rate of profit to fall is that it strains the native discrepancies of the capitalist mode of production**”.

3. GROSSMANN'S THEORY OF BREAKDOWN

As we mentioned in the previous chapter, Grossmann's analysis is based on Marx's law of the tendency of the rate of profit to fall. His intention was to demonstrate how the centripetal forces of accumulation can provoke the breakdown of capitalist process of production by abstracting from the counteracting tendencies and his will was to find out under what conditions the productive process takes the form of cyclical and periodically recurring movements of expansion and decline.

3.1 METHODOLOGICAL ISSUES.

Grossmann extends Bauer's reproduction schemes to 36 periods of productive process and tried to find out in what way accumulation can affect profits and vice versa and if accumulation of capital can be sustained in the long run. His results are represented in table 2.

As we notice from table 2, the accumulation of capital increases continuously while the rate of profit declines. Capitalist consumption rises up to year 20 where it reaches its highest level and in the following years, it declines relatively and absolutely. It reaches its lowest level in year 34 and in year 35 it is set equal to zero. Also, in year 35, the additional capital is not enough to absorb the annual increase in population and a reserve army of unemployed is formed and becomes larger year by year. This violates the initial assumption of the scheme that there can no be a reserve army of labor. Proceeding this analysis, Grossmann concludes that the capitalist system will breakdown in year 36 where the capitalist consumption disappears completely. As he claims:

“ The capitalist class has nothing left for its own personal consumption because all existing means of subsistence have to be devoted to accumulation. In spite of this there is still a deficit of 11.509 on the accumulated variable capital a_v required to reproduce the system for a further year... From year 35 on any further accumulation of capital under the conditions postulated would be quite meaningless. The capitalists would be wasting effort over the management of a productive system whose fruits are entirely absorbed by the share of workers”.

Grossmann observes that the fall in the rate of profit at the early stages of accumulation is accompanied by an expanding mass of profits and an increased capitalist consumption while at the stage of over accumulation, it is accompanied by both, a fall in the part of surplus value that is used for capitalist consumption and an absolute decrease in the mass of profits. At the initial stages, there is a growing valorisation of capital which permits the continuation of the accumulation process while at the last stages there is an imperfect valorisation of capital which causes the crisis of the system.

As we see from the table, in year 35 the capitalist consumption is set equal to zero. The surplus value that remains is equal to 14.756 and is used as additional variable capital which rises from 525.319 to 540.075. But in year 36, the working population has increased to 551.584 and this means that there are:

$551.584 - 540.075 = 11.059$ unemployed workers. Additionally, in year 36, the organic composition of capital, which is the ratio of constant to variable capital is equal to $5.616.200 / 551.584 = 10.18$.

Table 2. Grossmann's reproduction scheme.

Year	c	v	k	a _c	a _v	AV	k/s (%)
1	200.000	+100.000	+ 75.000	+ 20.000	+ 5.000	= 400.000	75.00
2	220.000	+105.000	+ 77.750	+ 22.000	+ 5.250	= 430.000	74.05
3	242.000	+110.250	+ 80.539	+ 24.000	+ 5.511	= 462.500	73.04
4	266.000	+115.762	+ 83.374	+ 26.600	+ 5.788	= 497.524	72.02
5	292.600	+121.500	+ 86.213	+ 29.260	+ 6.077	= 535.700	70.90
6	321.860	+127.627	+ 89.060	+ 32.186	+ 6.381	= 577.114	69.70
7	354.046	+134.008	+ 91.904	+ 35.404	+ 6.700	= 622.062	68.60
8	389.450	+140.708	+ 94.728	+ 38.945	+ 7.035	= 670.866	67.35
9	428.395	+147.743	+ 97.517	+ 42.839	+ 7.387	= 723.881	66.00
10	471.234	+155.130	+100.251	+ 47.123	+ 7.756	= 781.494	64.63
15	758.925	+197.988	+112.197	+ 75.892	+ 9.899	= 1.154.901	56.67
20	1.222.252	+252.961	+117.832	+ 122.225	+ 12.634	= 1.727.634	46.60
21	1.344.477	+265.325	+117.612	+ 134.447	+ 13.266	= 1.875.127	44.30
25	1.968.446	+322.503	+109.534	+ 196.844	+ 16.125	= 2.613.452	33.90
33	4.219.536	+476.480	+ 30.703	+ 421.953	+ 23.284	= 5.172.496	4.20
34	4.641.489	+500.304	+ 11.141	+ 464.148	+ 25.015	= 5.642.097	0.45
35	5.105.637	+525.319	+ 0	+ 510.563	+ 14.756	= 6.156.275	0.00
36	5.616.200	+551.584					

But the active working population is 540.075 and this implies that only 5.499.015=540.075 * 10.18 units of constant capital will be employed and the remaining 5.616.200 - 5.499.015 =117.185 units of constant capital represent a surplus capital with no investment possibilities. These conditions are similar with those that Marx exposed in Capital, Volume III, “ Excess Capital and Excess Population”. At this point, Grossmann makes a distinction between the formation of the reserve army due to insufficient accumulation of capital and the setting free of the workers by machinery. He explains that changes in the technical composition of capital cause the displacement of workers by machinery and this can happen in any economic system. The changes in the organic composition of capital bring on the

existence of a reserve army and they cause the interruption of the capitalist accumulation process.

Grossmann suggests that capitalists will react to overaccumulation when their consumption begins to decline absolutely, by cutting the wages of the workers or by restraining the expansion of constant capital at that level, where it can absorb the increased working population. In this case the system could yield the necessary amount of surplus value which allow the continuation of the accumulation process, but it cannot repel the creation of a reserve army of unemployed as the readjustment of the system is obtained through the change in the organic composition of capital.

Conclusively, Grossmann demonstrated that even if the system preserves the necessary proportionality between the two sectors and even if accumulation proceeds according to the limits imposed by population, the accumulation process cannot be continued indefinitely. Contrary to Bauer, Grossmann finds out that the capitalist mode of production cannot be sustained in the long run.

3.2 THE MATHEMATICAL MODEL OF GROSSMANN.

Although- at least the English (abridged) translation- Grossmann's mathematical formulation of his theory contains some typographical and mathematical errors, it deserves our respect. Grossmann was one of the first Marxist economists who tried to develop a mathematical model in order to found his arguments. The assumptions of his model are:

1. Constant capital grows at 10% and variable capital at 5%, every year.

2. The rate of surplus value is 100% and this means that the every year, the surplus value is equal to the variable capital.

3. One part of surplus value is accumulated as additional constant capital, another part, as additional variable capital and the remaining surplus value is used to cover the individual capitalists consumption.

He uses, also, the following symbols:

c =constant capital, c_0 = initial constant capital

c_t = value of constant capital after t years of accumulation

v = variable capital, v_0 = initial variable capital

v_t = value of variable capital after t years of accumulation

s = rate of surplus value, k = consumption share of capitalists

a_c = rate of accumulation of constant capital

a_v = rate of accumulation of variable capital

S = mass of surplus value which is given by the following two types:

$$S = k + c (a_c / 100) + v (a_v / 100)$$

$$S = v (s / 100)$$

Ω_0 =organic composition of capital

$r = 1 + (a_c / 100)$ and $w = 1 + (a_v / 100)$.

The constant capital after t years of accumulation is given by:

$$c_t = c_0 r^t \tag{1}$$

and the variable capital is given by:

$$v_t = v_0 w^t \tag{2}$$

The surplus value after the year $t+1$ will be:

$$S = k + c_t (a_c / 100) + v_t (a_v / 100) \quad (3)$$

and
$$S = v_t (s / 100) \quad (4)$$

By equalizing equations (3) and (4), we take:

$$k + c_t (a_c / 100) + v_t (a_v / 100) = v_t (s / 100) \Leftrightarrow k = v_t (s - a_v) / 100 - c_t (a_c / 100) \quad (5)$$

By substituting equations (1) and (2) to (5) we obtain:

$$k = v_0 w^t (s - a_v) / 100 - c_0 r^t (a_c / 100) \quad (6)$$

$$k \geq 0 \Leftrightarrow v_0 w^t (s - a_v) / 100 \geq c_0 r^t (a_c / 100) \quad (7)$$

and $k = 0 \Leftrightarrow v_0 w^t (s - a_v) / 100 = c_0 r^t (a_c / 100) \Leftrightarrow (s - a_v) v_0 w^t = a_c c_0 r^t \Leftrightarrow$

$$\Leftrightarrow (r/w)^t = [(s - a_v) v_0 / a_c c_0] \Leftrightarrow (r/w)^t = (s - a_v) / a_c \Omega_0 \quad (8)$$

Taking the log-form of equation (8) Grossmann calculates the year of breakdown of the system which is:

$$t = \frac{\log\left(\frac{s - a_v}{a_c \Omega_0}\right)}{\log\left(\frac{100 + a_c}{100 + a_v}\right)} \quad \text{with } s \geq a_v \quad (9)$$

From equation (9) we can see that the number of years t of the breakdown of the system depends on:

1. The initial level of the organic composition of capital Ω_0 . If the initial level of the organic composition of capital is high, crisis will be accelerated.
2. The rate of accumulation of constant capital which has the same influence as the initial level of the organic composition of capital. An increase in the rate of accumulation of constant capital will reduce the number of years to breakdown.
3. The rate of accumulation of variable capital which has the opposite impact from the rate of accumulation of constant capital. A rise in the rate of accumulation of variable capital will defer the breakdown of the system.

4. The rate of surplus value. The higher is the rate of surplus value, the greater the number of years t and breakdown is deferred.

Capitalists can apply four alternative methods if they wish to continue the accumulation process. Either to reduce the rate of accumulation a_c , or to devalue the constant capital, or to reduce the value of the wages or finally, to export capital. These movements will interrupt the tendency to breakdown as counteracting tendencies begin to operate. A temporary crisis will occur rather than the absolute collapse of the system and the reproduction process takes a cyclical form. Grossmann examines the impacts of the above four methods to the number of years t by constructing arithmetical examples.

First case.

Let's assume a higher initial organic composition of capital with $c_0 = 200.000$ and $v_0 = 25.000$. Then if the rate of accumulation of constant capital is 10% every year and the rate of accumulation of variable capital is 5% per year, we have the following results:

Table 3.

Year	c	v	k	a_c	a_v	AV					
1	200.000	+	25.000	+	3.750	+	20.000	+	1250	=	250.000
2	220.000	+	26.250	+	2938	+	22.000	+	1312	=	272.500
3	242.000	+	27.562	+	1.984	+	24.200	+	1.378	=	297.124
4	266.000	+	28.940	+	893	+	26.600	+	1.447	=	323.880
5	292.600	+	30.387	+	0	+	29.260	+	1.519	=	

The capitalist system will break down in year 5 where the consumption of capitalists becomes zero.

Second case.

We assume that the initial value of constant capital is $c_0 = 200.000$ and the initial value of variable capital is $v_0 = 100.000$. The constant capital increases at a rate of 20% per year while the rate of accumulation of variable capital remains the same, 5% per year. results are represented in the following table:

Table 4.

Year	c	v	k	a_c	a_v	AV					
1	200.000	+	100.000	+	55.000	+	40.000	+	5.000	=	400.000
2	240.000	+	105.000	+	51.750	+	48.000	+	5.250	=	450.000
3	288.000	+	110.250	+	47.138	+	57.600	+	5.512	=	508.500
4	345.600	+	115.762	+	40.853	+	69.120	+	5.789	=	577.124
5	414.720	+	121.551	+	32.534	+	82.944	+	6.077	=	657.826

$$6 \quad 497.664 + 127.628 + 21.716 + 99.532 + 6.380 = 752.920$$

$$7 \quad 597.196 + 134.008 + 7.869 + 119.439 + 6.700 = 865.212$$

$$8 \quad 716.635 + 140.708 + 0 + 143.326 + 0 = 1.000.669$$

The breakdown of the system occurs in year 8. We notice that the additional constant capital is greater than the available surplus value. So, there is nothing left neither for additional variable capital neither for capitalist consumption.

Third case.

Grossmann states that if the working population remains constant or grows at the assumed rate of 5% every year, an increase in the accumulation of variable capital will accelerate the breakdown of the system. Annual increases in variable capital comes from:

- a, either an increase in the number of workers or
- b. an increase in wages.

Suppose that wages rise by 20% every year while the working population increases by 5% every year. In the following table, the column of the variable capital reflects the annual rise of 5% in the working population while the column of accumulated variable capital reveals the sum required to pay the growing number of workers accumulative annual wage increase of 20%.

Table 5a.

Year	c	v	k	a _c	a _v	AV
1	200.000	+ 100.000	+ 54.000	+ 20.000	+ 26.000	= 400.000
2	220.000	+ 105.000	+ 51.750	+ 22.000	+ 31.250	= 430.000

$$3 \quad 242.000 + 110.250 + 49.288 + 24.200 + 36.762 = 462.500$$

$$4 \quad 266.200 + 115.762 + 46.592 + 26.620 + 42.550 = 497.724$$

$$9 \quad 428.717 + 147.745 + 28.741 + 42.872 + 76.132 = 724.207$$

$$12 \quad 570.623 + 171.033 + 13.386 + 57.062 + 100.585 = 912.689$$

$$14 \quad 690.453 + 188.564 + 527 + 69,045 + 118.992 = 1.067.581$$

$$15 \quad 759.498 + 197.992 + 0 + 75.949 + 128.891 = 1.162.330$$

The capitalist system will break down in year 35.

Grossmann continues his analysis by assuming that the rate of accumulation of variable capital increases at 8% and in order to prevent the assumptions of Bauer's reproduction schemes, he assumes that the constant capital increases at 20% per year.

Table 5b.

Year	c	v	k	a _c	a _v	AV
1	200.000	+ 100.000	+ 52.000	+ 40.000	+ 8.000	= 400.000
2	240.000	+ 108.000	+ 51.360	+ 48.000	+ 8.640	= 456.000
3	288.000	+ 116.640	+ 49.709	+ 57.600	+ 9.331	= 521.280
4	345.600	+ 125.971	+ 46.773	+ 69.120	+ 10.078	= 597.542
5	414.720	+ 136.049	+ 42.281	+ 82.944	+ 10.884	= 686.878
6	498.664	+ 146.933	+ 35.646	+ 99.532	+ 11.755	= 792.530
7	597.196	+ 158.687	+ 26.553	+ 119.440	+ 12.694	= 914.570
8	716.636	+ 171.382	+ 14.334	+ 143.327	+ 13.711	= 1.059.390
9	859.963	+ 185.093	+ 0	+ 171.992	+ 14.807	= 1.231.855

By comparing tables 4 and 5b we see that the increase in the rate of accumulation of variable capital deferred the breakdown of the capitalist system by one year.

Fourth case.

We assume that the rates of accumulation of constant and of variable capital remain the same but the rate of surplus value reduces to 5%. The results are the following:

Table 6.

Year	c	v	k	a_c	a_v	AV
1	200.000	+ 100.000	+ 25.000	+ 20.000	+ 5.000	= 350.000
2	220.000	+ 105.000	+ 22.750	+ 22.000	+ 5.250	= 375.000
3	242.000	+ 110.250	+ 20.289	+ 24.200	+ 5.511	= 402.250
4	266.000	+ 115.762	+ 17.612	+ 26.600	+ 5.788	= 431.762
5	292.600	+ 121.500	+ 14.663	+ 29.260	+ 6.077	= 464.100
6	321.860	+ 127.575	+ 11.433	+ 32.186	+ 6.381	= 499.435
7	354.046	+ 134.008	+ 7.896	+ 35.404	+ 6.700	= 538.054
8	389.450	+ 140.708	+ 4.020	+ 38.945	+ 7.035	= 580.158
9	428.395	+ 147.743	+ 0	+ 42.839	+ 7.387	= 626.364

A reduction in the rate of exploitation accelerates the interruption of the accumulation process.

At this point, Grossmann constructs a new model which does not include the initial assumption of proportionality between the rate of accumulation and the rate of growth of population in order to represent clearer the cyclical form that the productive process takes. In this model, the rate of accumulation of constant capital reduced to 5% and capitalists reduce the rate of accumulation of variable capital accordingly. Because of the reduction in the rate of accumulation, the absorption of

all new workers becomes impossible and unemployment inevitably arises. A part of the surplus value will be used by capitalists for individual consumption and the remaining part will be kept in the form of loan capital. This loan capital appears in the money market and is used by capitalists for investment purposes. The rate of interest of loan will fall and the accumulation process will be stimulated. On the other hand, in the labor market, unemployment arises continuously and it will cause the depression of real wages and the increase in the rate of surplus value. The fall in the rate of profit and the rising unemployment will encourage faster accumulation.

Grossmann assumes that the rate of accumulation of constant capital is 5% instead of 10% and the rate of surplus value remains constant at 100%. In the following table Grossmann abstracts the effects of both, the depression of real wages and the fall in the rate of interest.

Table 7.

Year	c	v	s	k	a_c	a_v	Loan Capital	Reserve Army
1	200.000	25.000	25.000	2.500	10.000	56	12.444	0
2	210.000	25.056	25.056	2.505	10.500	58	58	11.993
1.194								
3	220.500	25.113	25.113	2.511	11.025	61	61	11.516
2.389								
4	231.525	25.174	25.174	2.517	11.576	72	72	11.009
3.583								
5	243.101	25.246	25.246	2.524	12.155	57	57	10.510

4.769

6	255.256	25.303	25.303	2.530	12.763	461	10.471
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5.974

7	268.019	24.842	24.842	2.484	13.400	38	8.920
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7.700

8	281.419	24.880	24.880	2.488	14.070	154	8.476
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8.904

9	295.490 + 25.034						
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In the above model, the variable capital increases much more slowly than constant capital as the organic composition of capital must be preserved at that level which is the same with it would have been reached if the constant capital had been accumulated by 10% and variable capital by 5% or

$$220.000 / 262.250 = 210.000 / 25.056 = 8.39$$

At the first year of accumulation, the production process yields a surplus value equal to 25.000 units of which 10.000 are accumulated as constant capital, 56 units as variable capital, 2.500 (the 10% of surplus value) are consumed by capitalists and the remaining 12.444 units of surplus value are used in the form of loan capital for investment purposes. The labor force is assumed to grow by 5% every year. Since the accumulation of variable capital is much slower, the reserve army of unemployed increases continuously. (The unemployment of every year is equal to the 5% of the level of variable capital of the previous year minus the additional variable capital of the previous year.) At the first year of accumulation,

there is no unemployment, at the second year there are 1.194 unemployed workers and at the end of the eighth year of accumulation there are 8.904 unemployed.

The effects of the fall in rate of interest and the decline of real wages on the accumulation of capital are represented in the following table:

Table 8.

Year	c	v	s	k	a_c	a_v	L	R.A	A.W
1	200.000	25.000	25.000	2.500	10.000	56	12.444	0	0
									12.444
2	210.000	25.056	25.056	2.505	14.700	535	7.316	1.194	1.194
									19.760
3	224.700	25.591	25.591	2.559	20.223	1.056	1.753	1.971	1.971
									21.513
4	244.923	26.647	26.647	2.664	26.941	1.565	4.523	2.193	2.193
									16.990
5	271.864	28.212	28.212	2.821	35.342	2.264	12.215	2.175	2.175
									4.823
6	307.206	30.476	30.476	3.047	46.080	2.449	21.100	1.456	1.456
									14.426
7	353.286	32.925	32.925	0			74		

where : R.A = reserve army L= loan capital A.W = accumulated wealth

The rate of accumulation is 5% at the first year and 7%, 9%, 11%, 13% ... at the following years. (The rates are not 6%, 7%, 8% as Howard and King mention.)

At the fourth year of accumulation, the system cannot produce the appropriate surplus value and there is a deficit of 4.523 units which is covered by borrowings. The accumulation ceases at the sixth year and a crisis will occur. Grossmann states that, with the introduction of credit the accumulation process becomes more realistic. By starting his analysis from the sphere of production, Grossmann managed to demonstrate that the inherent laws of accumulation of capital give a cyclical form to the accumulation and this cyclical form penetrate in the sphere of circulation.

In chapter 3 of his book “ The Law of Accumulation and the Breakdown of Capitalist System”, Grossmann examines the influences of the counteracting tendencies to the valorisation of capital. Grossmann developed his theory of breakdown by advancing an abstract analysis in order to find out the tendencies of development of the system in its pure form. By making the following simplified assumptions

1. There is no foreign trade
2. There exist only capitalist and workers
3. There are no landowners
4. The exchange of the commodities becomes without merchants
5. The rate of surplus value remains constant
6. There are only two departments in the production
7. The rate of growth of population is constant
8. The wages do not vary

9. the capital turns over once a year in all branches of production

he proved that the imperfect valorisation of capital due to over accumulation is the real cause of crisis and that the system would breakdown if there were no counteracting influences.

Grossmann suggests that “ **once the system reincorporates by degrees, foreign trade, landowners who live off groundrent, merchants and the middle classes and once the rate of surplus value or the level of wages are allowed to vary**”, the counteracting tendencies begin to operate and the tendency towards to breakdown is transformed into a temporary crisis which take the form of a periodic cycle. Grossmann describes the most important countertendencies and distinguishes them in those which are inherent to the mechanism of capital and those which operate in the world market.

He focused his interest on the analysis of the counteracting tendencies in the world market and he examined their implications to imperialism. He indicates the important role that international exchange, between capitalists and non-capitalists markets, plays in the expansion of productivity. He claims that foreign trade increases the productivity of the economies and rises the profits of capitalists because, in the world market, there is a large number of consumers with unlimited number of needs. The industrialised and economically developed nations are the more favored as they can increase their profits by appropriating the excess surplus value at the underdeveloped countries. The rate of profit of high-industrialised economies increases and the breakdown tendency is postponed. In the world market, the rate of profit can also rise by the sharp competition for monopoly control over raw materials. As he states:

“ The struggle for control over raw materials is thus a struggle for control over processing industries, which is itself finally reducible to the drive for additional surplus value. Because raw materials are only found at specific points on the globe, capitalism is defined by a tendency to gain access to and exert domination over, the sources of supply. This can only take the form of a division of the world. A world economy in raw materials means that more surplus value can be pumped out of the world market. For competitors who face such a monopoly it means that the breakdown of capitalism is intensified. The economic roots of imperialism, of the incessant drive to dominate territories capitalistically and later politically, lie in imperfect valorisation”.

Finally, the international exchange is necessary not only because it increases the rate of profit and reinforces the capitalist system but also because of there are definite limits of capital investments in any country. **“ Not higher profits abroad, but a shortage of investment outlets at home is the basic underlying cause of capital exports”**, as Grossmann maintains.

3.3 A NEW MATHEMATICAL APPROACH WITH DIFFERENCE EQUATIONS

We indicated above the great importance of the mathematical model that Grossmann constructed and we pointed out that it was the first attempt of this kind. But we have to mention that this mathematical formula contains some mistakes and it does not describe precisely the accumulation process of the reproduction schemes. In this chapter, we will try to describe how capitalists consumption evolves in time by using a new approach with difference equations.

Grossmann assumes that the initial value of constant capital is $c_0 = 200.000$ and the initial value of variable capital is $v_0 = 100.000$. If the rate of accumulation of variable capital is a_c then after t years, constant capital reaches the level $c_t = c_0 r^t$. Similarly, the variable capital after t years of accumulation, will be $v_t = v_0 w^t$. The year after $t+1$, the mass of surplus value is given by equations (3) and (4) of the previous chapter. We notice that at the first year of accumulation process the constant capital is $c_1 = c_0 r = 200.000 (1.1) = 220.000$ which is the constant capital of the second year in Bauer's reproduction scheme. By substituting $t = 1$ in equation (4), the mass of surplus value is equal to 105.000 which is the mass of surplus value that the system yields in the second year of accumulation process.

Let's assume that at the beginning of every year, capitalists possess a constant capital c_t and a variable capital v_t and that at the end of the year the system produces a surplus value S_t of which one part will be used as additional constant capital, another part as additional variable capital and the remaining part will be kept by capitalists for individual consumption. Let's also assume that the

initial values of constant and variable capital are $c_0 = 200.000$ and $v_0 = 100.000$ respectively.

The constant capital after t years of accumulation is given by the type:

$$c_t = c_0 r^{t-1}, t \geq 1 \quad (1')$$

and the variable capital will be:

$$v_t = v_0 w^{t-1}, t \geq 1 \quad (2')$$

The surplus value produced is given by the equations:

$$S_t = k_t + (a_c / 100) c_t + (a_v / 100) v_t \quad (3')$$

and
$$S_t = (s / 100) v_t \quad (4')$$

By taking the first differences in equations (3') and (4') and by equalizing their second parts, we obtain a first order difference equation for capitalist consumption:

$$k_{t+1} - k_t = [(s - a_v) / 100] v_0 (w - 1) w^{t-1} - (a_c / 100) c_0 (r - 1) r^{t-1} \quad (5')$$

The general solution of the above first-order difference equation is:

$$k_t = A + [(s - a_v) / 100] v_0 w^{t-1} - (a_c / 100) c_0 r^{t-1} \quad (6')$$

where A is a constant, different than zero. For $t = 1$, we take

$$k_1 = A + v_0 (s - a_v) / 100 - (a_c / 100) c_0, \text{ or}$$

$$A = k_1 + (a_c / 100) c_0 - v_0 (s - a_v) / 100 \quad (7')$$

by substituting (7') to equation (6') the capitalist consumption takes the form:

$$k_t = [k_1 + c_0 a_c / 100 - v_0 (s - a_v) / 100] + v_0 w^{t-1} (s - a_v) / 100 - c_0 r^{t-1} (a_c / 100), t \geq 1$$

The capitalist consumption depends on the initial value of constant and variable capital, it depends on the initial level of capitalist consumption, on the rates of accumulation of constant and variable capital and on the rate of surplus value. We have to note that $k_1 + c_0 a_c / 100 - v_0 (s - a_v) / 100$, cannot be greater than or equal to zero. This means that $k_1 < v_0 (s - a_v) / 100 - c_0 a_c / 100$ or that capitalist

have to keep a part of surplus value in the form of loan capital for investment purposes.

We observe the followings:

1. $\partial k_t / c_0 < 0$.

An increase in the initial value of constant capital reduces the capitalists consumption.

2. $\partial k_t / v_0 > 0$.

An increase in the initial value of variable capital will increase the consumption share of capitalists.

3. $\partial k_t / a_c < 0$.

If the capitalists increase the rate of accumulation of constant capital, they have to reduce their consumption for goods.

4. $\partial k_t / a_v < 0$.

If the rate of accumulation of variable capital rises, the capitalist consumption declines.

5. $\partial k_t / s > 0$.

The capitalist consumption increases when the rate of exploitation rises.

Also, $k_t \leq 0$ when

$$[k_1 + c_0 a_c / 100 - v_0 (s - a_v) / 100] \leq c_0 r^{t-1} (a_c / 100) - v_0 w^{t-1} (s - a_v) / 100.$$

By using L' Hospital rule we find out that: $k_t \rightarrow 0$ as $t \rightarrow \infty$. This means that there are definite limits to the accumulation of capital even if capitalists realize a part of surplus value for investment purposes.

If we assume an initial value of capitalist consumption $k_1 = 70.000$ we have

the followings:

Table 9.

Year	capitalist's consumption k_t
1.	70.000
2.	72.750
3.	75.537
4.	78.354
5.	81.191
6.	84.037
7.	86.878
8.	89.700
9.	92.486
10.	95.218
11.	97.871
16.	108.953
17.	110.474
20.	112.742
34.	5.799
35.	0

4. MODERN VIEWS ABOUT GROSSMANN'S WORK ON THEORY OF CRISIS

In this chapter, we will review the modern views about Grossmann's work on theory of crisis and we will analyse extensively all the different aspects that have been formulated.

In their article "Henryk Grossmann and the Breakdown of capitalism", Howard and King (1988) argue that the most important mistake of Grossmann is that he based his theoretical analysis on Otto Bauer's reproduction scheme and its initial assumptions which, they maintain, "**were too rigid realistically to model an actual capitalist economy**". Their criticism is centralized to the following points:

- Grossmann does not explain clearly why the rates of growth of constant and variable capital should be 10% and 5% respectively. This assumption is too restrictive and it affects the durability of the system.
- 1. The organic composition of capital does not depend on technological changes but it is connected with the profitability of capitalists' investment decisions. By this way, capitalists could avoid the breakdown of the system by receiving convenient investment decisions which would insure the smooth continuation of the accumulation process.
- By holding the rate of surplus value constant, Grossmann ignored the Marxian concept of relative surplus value. If technical progress, which increases simultaneously the organic composition of capital and the labor productivity, is accompanied by constant or falling real wages, then the rate of surplus value and the rate of profit have to increase and the breakdown tendency is effaced.
- Grossmann believes that the effects of technological advances in cheapening the elements of constant and variable capital, are factors that counteracts those

forces which lead the economic system to breakdown and he does not treat them as factors which are inherent in the accumulation process and lower the organic composition of capital and parallelly, raise the rate of surplus value.

- Grossmann ignores the transformation problem of values into prices of production by assuming that all commodities are sold to their labor values.
- He also ascribed to Marx a theory of breakdown of capitalism rather than a theory of recurrent crises and he misinterpreted Marx's concept of absolute overproduction of capital. Grossmann advanced a theory of relative overaccumulation in which the mass of surplus value increases continuously while in Marx's concept of absolute overproduction, the accumulation process cannot yield the appropriate surplus value when the reserve army of unemployed falls down to zero.
- The part of surplus value which is kept by capitalists as loan capital is not compensated with any investment. There will be difficulties in realizing all the surplus value that is produced, So the commodities will remain unsold and the profits will be reduced.

Howard and King (1988) conclude that “ **Grossmann failed completely to establish the necessity of capitalist breakdown and much of the hostility to him was politically motivated**”. On contrary, Kuhn (1995), in his article “ **Capitalism's Collapse: Henryk Grossmann Marxism's**”, maintains that Grossmann analysis and his approach to Marxism deserves our appreciation and respect as he managed to make “ **a powerful case for revolutionary socialism, in the tradition of Marx and Lenin, against voluntarist, reformist and conservative politics and economics**”. Kuhn is opposed to Howard's and King's criticism. Firstly, he rejects

their argument that capitalists could avoid the rise in the organic composition of capital and the breakdown of the system by slowing down the rate of accumulation. He claims that this argument ignores the competitive advantage of being the first capitalist in a sector who receives and exploits the productivity gains from being the first capitalist to introduce new technology. There is a transfer of surplus value from capitalists who use old technical methods and mechanical equipment in the production process, to capitalists who introduce new technology. Secondly, in relation to their argumentation for the effects of the technical progress, Kuhn suggests that the balance between the tendency of the rate of profit to fall and its counteracting tendencies was for Grossmann an empirical question in the short term and it should be examined at any particular point of time. He also mentions Harman's and Shaikh's statements that: **“ There is a logical limit to which increasing the rate of surplus value can contribute to the rate of profit: when all new value created in a period is appropriated by capitalists and workers do not receive back anything to reproduce their labor power. There is no such limit on the size of constant capital which can grow indefinitely”**.

Thirdly, Kuhn believes that Grossmann deliberately ignored capitalism's realization problems as he was seeking to identify those contradictions which are inherent in the capitalist mode of production. Finally, he points out that Grossmann's treatment of imperialism is of prominent significance and deserves our attention. Grossmann realized that imperialism gives to capitalists the opportunity to increase their profits in periods of stagnation and he developed the theory of “ unequal exchange” in which the international trade involves the transfer of surplus value from less to more developed countries. This theory became, long after, very famous

among third worldists. Kuhn concludes: “ **Grossmann was very far from believing that capitalism would inevitably collapse of its own record... Before, in and after the law of accumulation, the purpose of Grossmann’s argument about capitalism’s collapse was to clarify the necessity for revolutionary practice and the context in which it could take place**”. He also mentions that: “ **By progressively dropping the simplifying assumptions of his initial model, Grossmann brought the analysis to concrete reality. He introduced the credit system and showed how it leads to the characteristic lower interest rates in the early stages of a recovery and, subsequently, as the pace of accumulation grows, to rising rates of interest. This explanation, grounded in developments in the sphere of production, provides a better guide to understanding the excesses of the decade of the 1980s, with its stock exchange bubbles, as well as the current phase of economic recovery, than either the incomprehension of mainstream economists or social democratic depreciation of the irresponsibility of capitalists in the absence of closer state regulation**”.

5. CONCLUSIONS

Henryk Grossmann advanced a theory of breakdown of the capitalist system based on Marx's argument that the tendency to breakdown is inherent in the capitalist productive process because of the tendency of the rate of profit to fall. He demonstrated that capitalist process of production has unsurpassable, absolute limits to accumulation, by using Otto Bauer's reproduction scheme. In Grossmann's model there is a specific pattern of growth and at the initial stage of his analysis, he abstracted from temporary fluctuations in prices, deviations of prices from values due to monopoly and he preserved the assumption of proportional development of productivity in the two sectors of the economy. He run Bauer's reproduction scheme for 36 years and he argued that the capitalism collapses completely in year 36, as the system can no longer produce the appropriate surplus value which insures the continuation of the accumulation process. After year 36, capitalists have no more incentives to continue the accumulation process as the share of surplus value which, they use to satisfy their own needs, diminishes. By this way, Grossmann demonstrated that the mass of surplus value acts as a trigger to economic crises. Beyond a certain point in the accumulation process, the surplus value produced is not sufficiently enough to purchase the required additional constant and variable capital and the continuation of the accumulation process requires the continual reduction of the capitalist consumption. After year 20, capitalists consumption reduces relatively and absolutely and capitalists start seeking other ways out for investment. The breakdown of the system occurs in year 36 when capitalist's private consumption vanishes.

To establish his theoretical arguments, Grossmann develops a mathematical formula and he calculates the time of the breakdown of the capitalist system and he analyses the factors that affect the accumulation of capital. He finds out that the collapse of the capitalist system is accelerated by an increase in the initial level of the organic composition of capital and a rise in the rate of accumulation of constant capital. A higher rate of surplus value defers the breakdown tendency while, according to his opinion, a rise in the rate of accumulation of variable capital is obscure.

In the final chapter of his book, Grossmann examines the counteracting tendencies which lie inherently in the capitalist accumulation process and those which operate in the world market. He argues that once these counteracting tendencies begin to operate the breakdown tendency is interrupted and takes the form of a recurring crises. The counteracting tendencies which operate in the world market exert the market competition for control over raw materials and monopoly. His analysis of imperialism deserves our attention. He claims that imperialism provides capitalists with necessary means for improving their profits when they face an economic stagnation in their country. The international exchange favors more the high, economically, developed and industrialized countries as capitalists can find outlets in non-capitalist and underdeveloped countries for profitable investments and appropriation of the excess surplus value. There is a transfer of surplus value from less developed to highly developed countries. He states that:

“it is therefore clear that the struggle for spheres of investment is also the greatest danger to world peace”.

APPENDIX.

The difference equation we described in chapter 3.3 is of the kind:

$$y_{t+1} + a_1 y_t = c m^t \quad (1)$$

where a_1 and c are constant. In order to find the general solution of equation (1), we solve the homogeneous difference equation:

$$y_{t+1} + a_1 y_t = 0 \quad (2)$$

We try a solution of the form $y_t = A b^t$ (with $A b^t \neq 0$) and the general solution of equation (2) is:

$$y_t = A (- a_1)^t \quad (3)$$

To find the partial solution of equation (1), we choose a solution of the form:

$y_t = B m^t$ and by substituting it to equation (1) we take: $B = c / (m + a_1)$. The general solution of (1) is:

$$y_t = A (- a_1)^t + c / (m + a_1) m^t \quad (4)$$

where A is a constant, different than zero. If we wish to calculate it, we have to substitute to equation (4) the initial condition $t = 0$. Then we have:

$$y_0 = A + c / (m + a_1) \text{ or } A = y_0 - c / (m + a_1).$$

Equation (4) becomes:

$$y_t = [y_0 - c / (m + a_1)] (- a_1)^t + c / (m + a_1) m^t \quad (5)$$

which is the general solution of (1) when the initial condition is given.

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