



**SCHOOL OF ECONOMIC AND REGIONAL STUDIES
DEPARTMENT OF ECONOMICS**

***OUTPUT VERSUS EMISSIONS TAX WHEN THE
PRODUCT'S ENVIRONMENTAL QUALITY IS
ENDOGENOUS AND CONSUMERS HAVE GREEN
PREFERENCES***

Author: Martis Apostolos-Ioannis

Supervisor: Prof. Constantatos Christos

Co-supervisor: Prof. Filippiadis Eleftherios

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“It always seems impossible until it’s done”

-Nelson Mandela

ΠΕΡΙΛΗΨΗ

Στην συγκεκριμένη εργασία εξετάστηκε η επίδραση δύο φόρων: ο ένας εφαρμόστηκε επί της παραγόμενης ποσότητας, ενώ ο άλλος επί των εκπομπών ρύπων, όταν οι καταναλωτές ενδιαφέρονται για το περιβάλλον. Χρησιμοποιήθηκε ένα θεωρητικό μοντέλο σε μονοπωλιακό πλαίσιο. Κατασκευάσαμε ένα παίγνιο 3 σταδίων κατά το οποίο ο μονοπωλητής στο πρώτο στάδιο επιλέγει το άριστο επίπεδο απορρύπανσης, ακολουθούμενος από τον κοινωνικό σχεδιαστή ο οποίος στο δεύτερο στάδιο επιλέγει άριστα επίπεδα των δύο φόρων. Στο τρίτο στάδιο, ο μονοπωλητής επιλέγει τα άριστα επίπεδα παραγόμενης ποσότητας. Τα παρακάτω αποτελέσματα προέκυψαν: υπό την εφαρμογή ενός φόρου επί της παραγωγής, η κατάσταση του περιβάλλοντος είναι καλύτερη σχετικά με αυτή που προκύπτει όταν εφαρμόζεται ένας φόρος επί των εκπομπών ρύπων. Επιπλέον, αυξήσεις της περιβαλλοντικής συνείδησης των καταναλωτών μειώνουν τις καθαρές εκπομπές ρύπων που δημιουργούνται από την επιχείρηση όταν χρησιμοποιείται ένας φόρος επί της παραγωγής, ενώ αυτές αυξάνονται υπό την εφαρμογή ενός φόρου επί των εκπομπών ρύπων. Ακόμη, η κατανάλωση είναι μεγαλύτερη σε περίπτωση εφαρμογής φόρου επί της παραγωγής. Ωστόσο, παρότι ο προαναφερθείς φόρος δημιουργεί καλύτερη κατάσταση περιβάλλοντος και μεγαλύτερη κατανάλωση, η κοινωνική ευημερία υπό φόρου εκπομπών ρύπων βρίσκεται υψηλότερα σε σχέση με τον φόρο επί της παραγωγής.

ABSTRACT

In this work, we examined the effects of two types of tax; one applied on per unit of output, while the other on emissions generated, when the consumers care about the environment. We used a theoretical model under a monopolistic framework. We construct a three-stage game where the monopolist chooses the optimal abatement level, followed by the regulator who chooses the optimal tax rates for both taxes. In the third stage, the monopolist chooses the optimal quantity produced. The following results arise: under the output tax regime, the environmental condition is better compared to that of the emissions tax case. Moreover, increases in environmental consciousness of the consumers decreases net emissions generated by the firm when the tax is applied on output, whereas it increases the net emissions generated under the emissions tax regime. Additionally, the consumption is greater under the output tax. However, even though the aforementioned tax yields better environmental conditions and increased consumption, the social welfare under the emissions tax lies much higher than the one under output tax regime.

1 INTRODUCTION

The environmental crisis that has emerged over the last years, has raised substantially consumers' worries and awareness about the environmental impact of the products they use. Empirical evidence shows that almost 94% of Europeans think that protecting the environment is an important issue for them personally, while almost half of Europeans (53%) say that protecting the environment is a very important issue. The remaining 41% believe that protecting the environment is a fairly major issue (Eurobarometer, 2020). The existence of green consumers changes the traditional consumption patterns by adding the environmental awareness parameter as a factor that determines final consumption. In this work, the consumption patterns due to voluntary environmental action of the consumers is reflected in their utility function. Namely, there is a negative relation between the consumption and the net emissions generated by the product. Given that the consumers take into consideration the environmental impact of the product, governments should take that parameter into account when they design the environmental policies.

In this work, we examine how a tax aiming to correct the environmental externality is affected by consumers' willingness to contribute in ameliorating the environmental condition. We consider a polluting firm that has the option to reduce its emissions by either increasing its product's environmental quality in anticipation of the upcoming tax and/or by simply reducing the output produced after the tax is imposed. We construct a four-stage game (if we assume that the regulator's choice of tax is a separate stage) where at the first stage the regulator announces the type of tax but not the tax-rate to be applied. At the second stage the monopolist decides how much to spend on abatement technology; the cost of abatement is treated as a fixed cost. At the third stage the regulator decides the tax rate and at the final stage the monopolist decides the quantity of the product it sells. This specification allows us to represent situations where the monopolist knowing that an environmental tax is imminent, adjusts its abatement policy at a level higher than even that of the first-best in order to obtain a more favorable tax rate. Within this context, we examine the relative efficiency of output *versus* emissions taxes. Several interesting results arise. First, when the monopolist has the option to adjust its abatement level before the tax rate is announced, the finally imposed optimal output-tax rate is negative independently of the importance of environmental

damage in social welfare. In other words, the regulator's optimal second-best policy is to boost consumption. This happens because in anticipating the tax, the monopolist overinvests in abatement. Additionally, when the consumers become more environmentally aware, the net emissions under the emissions tax tend to rise, even though the optimal abatement and quantity levels drop. In this case, at each increment of environmental consciousness, the regulator chooses to increase the emissions-tax rate, in order to punish the monopolist for the bad product quality. Under the output tax policy, things are a little different. The monopolist does a lot of abatement, which makes the regulator to provide an output subsidy. The high abatement level is observed by the consumers who increase the consumption of the clean product, making the monopolist to produce more. As a result, net emissions drop. While this might look a happy outcome-higher consumption with fewer emissions, even compared to the first-best- it is not so, since it is obtained at the expense of excessive abatement cost. Viewed from a general equilibrium perspective, the sector in question withdraws too many resources from the rest of the economy. Thus, the social welfare drops as well, due to overprovision of quality, or else the excessive use of abatement.

Second, when the monopolist knows that an emissions tax is to be imposed also increases its commitment in abatement, but not to the same extent as in the case of an output tax. Compared to first-best, both abatement and quantity are inferior; overall welfare is much closer to its first-best level than with an output tax.

Third, while welfare enhancing when combined with an output tax, consumer awareness is counterproductive when an emissions tax is in use. All interesting welfare indices-environmental protection, consumption and overall welfare-are lowered as individual environmental consciousness increases. This is a result also found in Constantatos *et al.* (hereafter CPS) and is due to the fact that while environmental consciousness nicely complements the workings of an output tax, it works antagonistically with the emissions tax. By reducing consumer willingness-to-pay for polluting products, environmental consciousness plays a similar role to an emissions tax in that it affects abatement directly, thus leaving more room to the regulator to correct the resulting quantity distortion. While this can be easily done with a subsidy, reductions in the emissions tax rate can never be sufficient for two reasons: a) they cancel the effects of an increase in consciousness, and b) the emissions tax rate can never become negative, since an "emissions subsidy" is not an acceptable solution.

The rest of the thesis is structured as follows. Section 2 presents the literature review, section 3 shows the model which we use to extract our results, section 4 demonstrates the second-best equilibrium values, section 5 shows the results of the comparison between the emissions and the output tax. The last section contains our main conclusions.

2 LITERATURE REVIEW

The environmental pollution is a major problem which it is contained in a general concept in economics called *externalities*. An *externality* is “the effect that an action of any decision maker has on the well-being of other consumers or producers beyond the effects transmitted by changes in prices” (Besanko & Braeutigam, 2014). In other words, an externality is the consequences of one’s actions to others in a society, either directly or indirectly observed. There are two types of externalities, positive and negative. Negative externalities describe a case where an action of one agent affects others in a society in a negative way (for example environmental pollution). In that case, if the agent who causes the externality is not obliged to pay for the damage that she caused, the marginal private cost will be less than the marginal social cost, due to the fact that the cost of the externality is not imputed to the agent that caused it, due to non-excludability. As a result, the quantity produced will be greater than the optimal one. On the other hand, positive externalities describe the case where an agent’s actions benefit others in a society as well. In that case, the marginal social benefit is greater than the marginal private benefit (for example education). If only the latter is taken into account, the quantity produced will be less than the Pareto optimal. The existence of the externality makes the market equilibrium to diverge from the Pareto efficient outcome. The latter is determined by the equalization between marginal benefits and marginal costs of an action. In a negative externality, such as an environmental hazard, the divergence of the optimum is due to the existence of the marginal social cost of the action, which cannot be directly observed. At its very core, environmental pollution appertains to a more specific type of externality which is called “*The tragedy of the commons*” (Hardin, 1968).

Without resolving this issue, the quantity produced is greater than the optimal one, leading to dead weight loss. To avoid this inefficient allocation, economists and policymakers have introduced many instruments. We can categorize those in *non-economic* and *economic* instruments. Some examples of the former are the *command-and-control* policy and the *quality standards*, while for the latter there is the *tax* policy and *Cap-and-Trade*. Since this work focuses on the tax policy, we have to mention the pioneering work of the economist Arthur Pigou, who introduced a *tax* (the now-called *Pigouvian tax*) in order to correct the market distortion generated by the negative externality (Pigou, 1932). An extensive and always expanding literature has been devoted to analyzing the role of the *Pigouvian* tax in various situations.

Another source of distortion is the structure of the market. While –at least theoretically– perfect competition leads to the Pareto optimal outcome, the presence of market power distorts the equality between marginal benefits and costs. However, one would expect the presence of a second distortion to make things worse, Buchanan (1969) shows that in the presence of a negative externality a monopolistic structure may be better than competition. This is a direct application of the Theorem of 2nd best¹. Therefore, the different market structure and the different order of moves with which the players interact, made the authors to create various models in order to approach the real-world cases with greater accuracy. Consequently, the firms and consumers’ environmental awareness was introduced in the models in order to depict the interaction of the latter with a tax policy.

The literature has shown that the first-best *Pigouvian* rule no longer holds in environments with market structure distortions: it needs to be modified in the second-best world by adding an adjustment term. Cremer & Gahvari (2001) compare the emissions tax with an output tax in order to identify if the former is used only on the basis of correcting the distortion generated by the externality. As they find out, taxing a commodity is not always needed, since it can be concluded in an emissions tax. Moreover, the output tax exists only for optimal tax objectives reasons. Since the authors discuss about emission and commodity taxes, they conclude that when the

¹ According to OECD: “The theory of the second best suggests that when two or more markets are not perfectly competitive, then efforts to correct only one of the distortions may in fact drive the economy further away from Pareto efficiency”. (OECD, 2002)

consumer's preferences for emissions and other goods are weakly separable a proportional emissions tax eliminates the distortion caused by the externality. Therefore, no modified *Pigouvian* rule is required. In case where the marginal rate of substitution (MRS) is identical between consumers, an income tax is good enough to serve for optimal tax problems, while the emissions tax abolishes the externality distortion. There is no need for an output tax to be set. (Cremer & Gahvari, 2001).

However, most of the papers, even though they focus on the comparison between commodity tax and emissions tax, they also concentrate on a duopolistic market with vertical differentiation of the product on the basis of the cleanup levels of the firm. By constructing a two-stage game, Conrad (2005) tries to identify the market effects of product differentiation, when the consumers develop *green consciousness*. The term *green consumer* refers to the fact that the consumers loses satisfaction, which is reflected as a reduction in utility, when she observes that a product is harmful for the environment. Therefore, there is a positive correlation between the environmental aspects of the product and the consumer's conscience, which is reflected in her utility function. On the basis of that, Conrad (2005) finds that the equilibrium values are affected both by the consumer's environmental awareness and the cost of producing the product.

Following the vertical differentiation market framework on the basis of environmental quality, Lombardini-Riipinen (2005) studies the impact of emission and output taxes, when the consumers have a higher willingness-to-pay for environmentally friendly variants. An interesting result is that by applying combinations of uniform commodity and emissions tax, the first-best quality level can be achieved. Moreover, the society is better off with a uniform ad valorem tax, if the quality of the product stays at low levels. Bansal (2008) attempts to examine the effects on welfare of two policies, an ad valorem and an emission taxes, when consumers have green preferences. The criterion under which the optimal policy will be pursued is the severity of the environmental externality generated though the product. It turns out that, as the environmental damage increases, it is socially optimal to provide a subsidy. In our work, we obtain almost the same result. In contrast to Bansal (2008) however, the analysis is based on the environmental consciousness. The environmental damage is exogenously defined. Therefore, our work differs from Bansal (2008) in terms of the basis which we make the analysis. At low levels of the negative externality, the emission tax is dominated by the commodity tax.

Bansal & Gangopadhyay (2003) attempt to find the optimal policy that ameliorates the environmental condition, when consumers care about the environment. They examine the effects of a uniform and a discriminatory ad valorem tax. When a discriminatory tax policy is in effect, the “cleaner” firm receives a subsidy, while the bad-performance firm receives a tax. As for the uniform tax policy, in terms of environmental condition and social welfare, a uniform negative tax rate (i.e. a subsidy) dominates a positive one.

Another interesting prospect is the existence of socially responsible manufacturers interacting with environmental aware consumers. According to Garcia-Gallego & Georgantzis (2009), the existence of a firm’s socially responsible behavior can be considered as a vertical differentiation strategy. In their work, they examine a case where the consumers have a higher willingness-to-pay (WTP) for products that derive from socially responsible firms. Increases in WTP causes changes in the consumers’ heterogeneity. They examine three types of market structures: (1) Monopoly with full coverage, (2) duopoly with complete coverage and (3) duopoly with incomplete coverage of the market. Considering unaffected market structure, the former yields increases in social responsible firm’s profits and social welfare. The only change if the market structure changes, is that the social welfare decreases. Following the model above, Doni & Ricchiuti (2013) once again attempt to examine the equilibrium changes when the consumers’ awareness and/ or firms’ responsible behavior changes but only in a vertically differentiated duopoly market with a clean and a dirty good. The consumers’ awareness is expressed according to their WTP, which the authors assume that it differs across consumers. Increases in a firm’s corporate social responsibility (CSR) makes the firm to increase its abatement level, while the other firm reduces it. However, the magnitude of these effects depends on the level of CSR. High levels of the latter enhance the magnitude of the “dirty” firm, making the aggregate cleanup levels to drop. However, high levels of CSR in accordance with high levels of consumers’ environmental awareness yield overprovision of the abatement level and a reduction in social welfare.

Most of the papers assume that once the quality of the product has been chosen by the firms, the consumers know it precisely. In Kehoe (1996) this is not the case. The author examines the price strategy of a monopolistic firm, when the consumers do not know exactly the firm’s product quality. The existence of homogenous consumers on the basis of tastes with low levels of uncertainty about the quality causes a negative correlation

between the firm's price and the quality uncertainty, while the correlation becomes positive if the consumers taste differ significantly and/or the uncertainty levels are high.

3 THE MODEL

3.1 PRODUCTION AND CONSUMPTION

Following Constantatos, Pargianas & Sartzetakis (2021), assume a monopolist who produces a private good X . For simplicity, the marginal cost of production is considered to be equal to zero. The production of one unit of the good, generates δ units of harmful pollutant. The monopolist has the option to remove some of the pollutant, by adopting an abatement technology at some positive cost. We consider the cost function to be constant with respect to production, yet increasing with respect to total abatement:

$$C = kv^2, \quad k \geq 1 \quad (1)$$

where v represents the monopolist's choice of abatement. Higher investments in abatement technology yield lower levels of the net emissions that the firm generates through the production process. Keeping the above in mind, we can introduce the net emissions function as:

$$e = \delta Q - v, \quad \delta > 0 \quad (2)$$

where Q is the total amount of units that the firm produces. In order to avoid the absurd case of negative emissions we restrict v in the $[0, \delta Q]$ interval. Moreover, to avoid further complexity in the mathematical analysis, we normalize $\delta = 1$. Therefore, v can take any value from the $[0, Q]$ interval. The total environmental damage generated is:

$$D(e) = ze^2 \quad (3)$$

where z indicates the transformation of units of net emissions into environmental damage.

Having already constructed the supply side of the model, as well as the pollution that generates, we move on to the demand side. Assume that there are $n \geq 1$ number of consumers with identical preferences represented by the following utility function:

$$U(q) = \alpha q - \frac{1}{2}q^2 + M, \quad \alpha > 0, \quad (4)$$

with $q \geq 0$ being the individual consumption of the product and M being the amount of numéraire-good consumed. The utility function introduced above shows that the

representative consumer cares only about her consumption. In this work we allow consumers to care to some extent of the environmental consequences of their consumption. While they cannot coordinate their actions, we assume that they can observe the total pollution generated by X and *act consciously*, in the sense that they may voluntarily reduce their utility consumption. Thus, the individual decision of the representative consumer is given by:

$$\tilde{U}(q; \phi) = \begin{cases} (\alpha - \phi e)q - \frac{1}{2}q^2 + M, & e > 0, \\ U(q), & e \leq 0, \end{cases} \quad (5)$$

where ϕ denotes magnitude of how much environmental conscious a representative consumer is².

The first part of the function expresses the partial internalization of the environmental externality. When the environmental consciousness is equal to zero ($\phi = 0$), then the consumers do not care at all about the environmental consequences generated by the product. Therefore, the upper and the lower branch of the function coincide. However, $\phi > 0$ implies that consumers have developed some consciousness about the environmental issues, partly internalizing some of the externality created by the firm. The above leads the socially responsible consumers to develop a different consumption pattern than that led by the strict utility-maximization problem using (4). In order to avoid over-internalization of the externality, we assume that $\phi \leq z$.

By maximizing (5) with respect to q , for every $e \geq 0$, we obtain the representative-consumer's inverse demand function:

$$p(q; \phi) = \begin{cases} (\alpha - \phi e) - q, & e > 0, \\ \alpha - q, & e \leq 0. \end{cases} \quad (6)$$

Thus, the representative consumer behaves according to (5) but the satisfaction she gains derives from (4).

In order to extract the aggregate demand function, we use the regular form of the demand function dictated by the upper branch of (6) multiplied by n .

$$q = (\alpha - \phi e) - p \Leftrightarrow Q = (\alpha - \phi e)n - np.$$

² Consumers may of course have different attitudes (different ϕ 's), but the potential effects of ϕ 's distribution lie outside the scope of this work.

Afterwards, we substitute (22) to the function above to get:

$$Q = [\alpha - \phi(Q - v)]n - np.$$

By performing some manipulations, we get the inverse aggregate demand function:

$$p(Q; v, \phi, n) = (\alpha + \phi v) - \frac{1 + \phi n}{n} Q \quad (7)$$

It is reasonable to assume that current technology is too expensive for firms to generate zero emissions. Therefore, we restrict hereafter our attention to cases represented by the upper branch of (6), or equivalently by (7).

3.2 SOCIAL WELFARE FUNCTION

The social welfare function borrowed from Constantatos *et al.* (2021) is:

$$W = \sum_{i=1}^n u_i - (D + C).$$

The regulator uses a welfare function that contains the total utility from consumption minus the environmental damage and the cost of abatement. We assume that the regulator takes into account the gross value of consumption, ignoring any consumption-value reductions due to ethical considerations. This implies that the u_i function above is represented by the expression in (4) rather than that in (5). Substituting (1), (3) and (4) in the above yields:

$$W = \sum_{i=1}^n (\alpha q_i - \frac{1}{2} q_i^2) - z(Q - v)^2 - kv^2.$$

Concerning the use of (4) instead of (5) in the social welfare function, note that in this work we consider that consumers act out of social responsibility and not because the good procures them less utility. Therefore, we follow the social responsibility (SR) approach, rather than the hedonic one³. In the former approach, the socially responsible consumers, at any price, consume the quantity dictated by the upper part of (6), but still value their consumption according to the lower part. In other words, while they

³ For more information, see Constantatos, Pargianas & Sartzetakis (2021)

consciously decide to reduce their consumption, the value of the units consumed remains unaffected by ethical considerations and the damage suffered is only that due to environmental degradation⁴, already contained in the 2nd term above. Thus, social welfare is not directly affected by ϕ .

In order to simplify our calculations, we apply some normalizations. First, it turns out that α doesn't significantly affect the results and therefore, we set $\alpha = 1$. Second, the use of the representative-consumer model allows to set $n = 1$. Therefore, the inverse demand function becomes $p = (\alpha + \phi v) - (1 + \phi)q$. Moreover, we set $k = 1$ ⁵. Thus, the welfare function becomes:

$$W = \alpha q - \frac{1}{2}q^2 - z(q - v)^2 - v^2 \Leftrightarrow$$

$$W(q, v; z) = (1 + 2vz)q - \left(\frac{1}{2} + z\right)q^2 - (1 + z)v^2. \quad (8)$$

A key assumption of our model is that the regulator values the environment damage at least as much as the consumption, i.e., $z \geq \alpha = 1$. Having all the pieces in order, we can now proceed to the game between the monopolist and the regulator.

4 SUBGAME PERFECT NASH EQUILIBRIUM

We consider a four-stage perfect-information game where: at the first stage the regulator announces the type of tax to be applied (either on emissions or on the output), but not the tax rate. At the second stage the monopolist decides her optimal investment in abatement. Having observed the latter, at the third stage the regulator decides the optimal tax rate in each case. At the last stage, the monopolist decides the optimal product quantity.

The monopolist's profit function is:

$$\Pi^O = (p - t)q - v^2, \quad (9)$$

⁴ This contrasts the *hedonic* approach, where consumers act under warm-glow effects. In such a case, consuming a polluting good causes negative warm glow (guilt) that reduces the utility of the all that good's units finally consumed. Thus, the hedonic approach would imply a double environmental damage: the real damage measured by D and the psychological damage due to guilt, contained in (5).

⁵ This is not a mere normalization but greatly simplifies the analysis; its impact will be discussed in the robustness section.

in case of an output tax and

$$\Pi^E = pq - t(q - v) - v^2, \quad (10)$$

in case of an emissions tax.

We use backwards induction method. Substituting (7) to (9) and (10) and manipulating the profit functions, we obtain the maximization problem that the monopolist faces⁶:

$$\max_{q \geq 0} \{\Pi^O = ((1 + \phi v - t) - (1 + \phi)q)q - v^2\},$$

or,

$$\max_{q \geq 0} \{\Pi^E = ((1 + \phi v) - (1 + \phi)q)q - t(q - v) - v^2\},$$

where t is the tax rate. It is shown that the optimal quantity decision under each tax regime is⁷:

$$q_3 = \frac{1 - t + v\phi}{2 + 2\phi}. \quad (11)$$

Moving on to the second stage, the problem that the regulator faces is:

$$\max_{t \in \mathbb{R}} \{W = (1 + 2vz)q - \left(\frac{1}{2} + z\right)q^2 - (1 + z)v^2, \quad s.t. q = q_3\}$$

By substituting (11) and setting the derivative of the above with respect to t equal to zero, we can obtain the optimal tax-rate as function of the abatement level already chosen⁸:

$$t_2 = \frac{-1 + (-2 + v)\phi - 2z(-1 + v(2 + \phi))}{1 + 2z}. \quad (12)$$

Now that the optimal responses have been set, we are going to separate the main game.

4.1 EQUILIBRIUM WITH OUTPUT TAX

In this case, the regulator has already announced that the imposed tax will be on output. Therefore, in the first stage the monopolist maximizes (9) with respect to v , in order to

⁶ We can maximize either (9) or (10), due to the fact that v is treated as exogenous.

⁷ The subscript "3" denotes that the optimal quantity derives from the third stage of the game.

⁸ Following the same idea, the subscript here denotes that the optimal tax-rate function is extracted from the second stage of the game.

find the optimal abatement level. Mathematically, the problem that the monopolist faces is:

$$\max_{v \geq 0} \{\Pi^O = (p(q) - t)q - v^2, \quad s.t. \quad q = q_3, t = t_2\}$$

Lemma 1. For all $\phi \in \left[0, \frac{1+4z}{4z^2}\right)$:

- i. The optimal abatement level that the monopolist will choose is:

$$v^O = -\frac{2z(1 + \phi)}{-1 + 4z(-1 + z\phi)}. \quad (13)$$

- ii. The derivative $\frac{\partial v^O}{\partial \phi} > 0 \forall \phi \geq 0, z > 1$. The second-order condition holds.

Proof. See the appendix □

The aforementioned lemma shows that when the environmental consciousness of the socially responsible consumers increases, it is in favor of the monopolist to increase her abatement level as well. However, in order to obtain the value of v that maximizes the profits function, we have to restrict the environmental consciousness, as shown in Lemma 1. Introducing (13) to (12) yields the optimal output-tax rate that the regulator will impose.

Proposition 1. The optimal output-tax rate is:

$$t^O = \frac{1 + 2\phi(1 + z + 2z^2 - z\phi)}{-1 + 4z(-1 + z\phi)}. \quad (14)$$

- i. The optimal output-tax rate decreases as the environmental consciousness increases.
- ii. t^O is always negative and therefore the regulator chooses to provide a subsidy to the monopolist.

Proof. See the appendix. □

Proposition 1 indicates a negative relation between environmental consciousness and the optimal output tax rate. More interesting, the regulator always chooses to reward the monopolist for her abatement level with a subsidy. To further investigate the cause of this behavior we take the derivative of $t^O = t_2(\phi, v(\phi))$:

$$\frac{dt^o(\phi, v(\phi))}{d\phi} = \frac{\partial t_2}{\partial \phi} \bigg|_{v=v^o} + \frac{\partial t_2}{\partial v} \frac{dv^o}{d\phi}. \quad (15)$$

On the right-hand side of (15), the first term illustrates the direct effect, while the second term shows the strategic effect. Beginning with the direct effect, it can be shown that it is negative (proof is shown in the appendix). The strategic effect is consisted of two components, the derivative of t_2 with respect to the abatement level function and the derivative of the optimal abatement level with respect to the environmental awareness. Intuitively, the first component is negative, since a raise in the abatement that the firm performs induces the regulator to reduce the tax. From (15) the specific form of that component is:

$$\frac{\partial t_2}{\partial v} = \frac{\phi - 2z(2 + \phi)}{1 + 2z},$$

which is negative $\forall \phi \geq 0, z > 1$ (proof in the appendix). As for the second component of the strategic effect Lemma 1 shows that is positive. Thus, since both the direct and the strategic effect are negative, t^o has a negative slope. Note that $t^o(\phi = 0) = \frac{1}{-1-4z} < 0$, meaning that t^o is always negative.

Turning to the equilibrium quantity under output tax, we obtain:

Proposition 2. *The equilibrium quantity is*

$$q^o = \frac{1 + 2z}{1 + 4z - 4z^2\phi}, \quad (16)$$

and it is increasing in ϕ , $\frac{dq^o}{d\phi} > 0$.

Proof. By substituting (13) and (14) to (11), we obtain (16) which is increasing in ϕ . □

Intuitively, a rise in ϕ has three effects. First, consumers reduce *ceteris paribus* their consumption. Second, final consumption is *ceteris paribus* affected by the change in the tax rate. Third, the monopolist adjusts her abatement level and this causes a further change in consumption. Since the tax rate is affected both directly and indirectly through changes in abatement, the expression in (16) can be considered as reduced form of

$$q^o(\phi) = q_3(\phi, v(\phi), t(\phi, v(\phi))).$$

Hence, the total derivative of final quantity with respect to ϕ can be decomposed as:

$$\frac{dq^o}{d\phi} = \underbrace{\left. \frac{\partial q_3}{\partial \phi} \right|_{t=t^o, v=v^o}}_{\text{Direct Effect}} + \underbrace{\frac{\partial q_3}{\partial v} \frac{dv^o}{d\phi}}_{\text{Indirect Effect}} + \underbrace{\frac{\partial q_3}{\partial t} \left(\frac{\partial t^o}{\partial \phi} + \frac{\partial t_2}{\partial v} \frac{dv}{d\phi} \right)}_{\text{Strategic Effect}} \quad (17)$$

The monopolist in the first stage raises the abatement level as the environmental consciousness increases. In a world of perfect information, both the regulator and the consumers would observe that move. This yields a decrease in the output-tax rate as shown in Proposition 2. The monopolist then has an incentive to produce higher quantities of her product. Moreover, with regard to the demand side, the consumers see that the product becomes more environmentally friendly and as they internalize more of the externality (increases in ϕ), they choose to consume more of that product. Therefore, the quantity function isn't affected only by the environmental consciousness directly, but through abatement and tax levels as well.

The first derivative on the right-hand side of (17) represents the direct effect, while the second derivative is the indirect effect caused by the change in the product's environmental attributes. The last two derivatives represent the effect of a change in ϕ on equilibrium consumption via the change in the tax rate. Thus, while the direct effect of an increase in consciousness points to a reduction in consumption, the indirect effect points to the opposite direction. The direct effect tells us that changes in environmental awareness tend to reduce the equilibrium⁹, while the whole indirect is strictly positive. The final sign depends on the magnitude of the two effects, which it turns out to be positive. Proposition 2 indicates that the equilibrium quantity and the environmental consciousness are positively correlated. Thus, the total effect is positive.

By substituting (13) and (16) to (2), optimal level of net emissions arises:

Proposition 3. *The optimal net emissions are:*

$$e^o = \frac{1 - 2z\phi}{1 + 4z - 4z^2\phi}. \quad (18)$$

⁹ While intuitive obvious, the proof of this result is a bit complex and for this reason relegated to the appendix.

The net emissions under an output-tax regime decrease as the socially responsible consumers become more environmentally aware, i.e., $\frac{\partial e^O}{\partial \phi} < 0$.

Proof. See the appendix □

Proposition 3 is important, since it announces that when an output tax is imposed the net emissions drop as the consumers become more conscious. Intuitively, the reduction of net emissions derives from Lemma 1 and Proposition 2, where they show that both q^O and v^O increase in ϕ , however the latter increases at a higher rate than the former. Therefore, one can say that the more the consumers internalize the externality generated, the more the monopolist is pushed to adopt a more efficient abatement technology in order to accelerate the abatement process.

In order to rule out corner solutions due to negative net emissions in equilibrium (see section 3.1), we further restrict the admissible range of the social consciousness parameter ϕ :

Lemma 2. *Net emissions, e^O , remain non-negative when $\phi \in \left[0, \frac{1}{2z}\right)$, $z > 1$.*

Proof. From Lemma 1, we can be sure that the denominator of e^O is positive for $z > 1$, while the numerator remains positive $\forall \phi \in \left[0, \frac{1}{2z}\right)$. Since the upper bound proposed in Lemma 1 is higher than that proposed in Lemma 2, optimal net emissions remain positive. □

Finally, the optimal social welfare can be generated by introducing (13) and (16) to (8):

$$sw^O = \frac{1 + 2z(3 + 2z - 4z(1 + z)\phi(2 + \phi))}{2(1 + 4z - 4z^2\phi)^2}. \quad (19)$$

Proposition 4. *The social welfare in the output-tax equilibrium decreases as environmental consciousness increases, i.e., $\frac{\partial sw^O}{\partial \phi} < 0$ and may*

even become negative $\forall \phi > \phi_W$, where $\phi_W \equiv -1 + \frac{\sqrt{(1+2z)^3}}{2\sqrt{z^2(1+z)}}$.

Proof. See the appendix □

The proposition above shows mainly that in the output-tax equilibrium, social welfare decreases as social consciousness increases! This is seemingly a paradox since the output tax generates both higher consumption and lower net emissions compared to even their first-best level. The paradox disappears if one does not take into account the

cost of abatement (see section 5.5). The decreasing social welfare happens because, knowing that her subsidy for higher levels of ϕ increases more than proportionally with its level of abatement, the monopolist responds to a rise in ϕ by overinvesting in abatement in order to manipulate the subsidy rate. By “overinvesting” here we mean abatement levels at which the marginal benefit is less than the marginal cost. Consequently, the optimal social welfare can be negative, meaning that any increase of environmental consciousness makes the society worse off. The lower bound of ϕ in the proposition above doesn’t consist a restriction of ϕ , rather than just a simple observation that when the regulator chooses an output tax, relatively too much of environmental consciousness generates unhappiness.

4.2 EQUILIBRIUM WITH EMISSIONS TAX

In this case, the regulator has already announced that the imposed tax will be on emissions. Therefore, the monopolist maximizes (10) with respect to v in order to find the optimal abatement level. Thus, the monopolist’s problem can be represented mathematically as:

$$\max_{v \geq 0} \{\Pi^E = p(q)q - t(q - v) - v^2, \quad s.t. \ q = q_3, t = t_2\}.$$

Lemma 3. *The optimal abatement level that the monopolist will choose is:*

$$v^E(\phi; z) = \frac{-1 + 4z(1 + z) - 2\phi}{2 + 16z(1 + z) - 2\phi}. \quad (20)$$

The derivative of (20) remains negative $\forall \phi \geq 0$, i.e.,

$$\frac{\partial v^E}{\partial \phi} < 0.$$

Proof. See the appendix. □

Lemma 3 says that the abatement level decreases when ϕ increases. This is a counter-intuitive result in the sense that, when the consumers care more about the environment, it is in favor of the monopolist to reduce her abatement level, by investing less to the respective technology.

By introducing (20) to (12), we get the optimal emissions-tax rate:

$$t^E(\phi; z) = \frac{-2 + 4z(-1 + 2z) - 3\phi - 4z(4 + z)\phi + 2\phi^2}{2(1 + 8z(1 + z) - \phi)} \quad (21)$$

By simply observing the optimal emissions-tax rate, it is not obvious whether it is positive. Since we do not allow for emission subsidies, the following proposition restricts the interval of ϕ in order to rule out corner solutions with $t^E(\phi; z) \equiv 0$.

Lemma 4. *The optimal emissions-tax rate is non-negative $\forall \phi \in [0, \bar{\phi}_{tE}]$ when $1 < z \leq \frac{1}{8}(7 + \sqrt{33})$ or $\forall \phi \in [0, \bar{\phi}_{eO})$ when $z > \frac{1}{8}(7 + \sqrt{33})$. For $1 < z \leq \frac{1}{8}(7 + \sqrt{33})$, $\bar{\phi}_{tE} < \bar{\phi}_{eO}$ ¹⁰, while the opposite is true for $z > \frac{1}{8}(7 + \sqrt{33})$.*

Proof. See the appendix □

Lemma 4 helps us identify in which intervals of ϕ , we avoid emission subsidy.

Proposition 5. *Moreover, the first derivative of the emissions-tax rate is negative:*

$$\frac{dt^E}{d\phi} < 0.$$

Proof. See the appendix □

Proposition 5 shows that as ϕ increases the regulator relies more heavily in consumer awareness and less on the emissions tax for limiting the environmental damage. As a result, he reduces *ceteris paribus* the tax rate in order to stimulate consumption. The monopolist, anticipates this change in regulator's behavior and reduces her abatement level. The impact that a change in ϕ has on the regulator's decision is analyzed by separating the two effects of an increase in ϕ : the direct effect and the indirect effect through change in abatement.

$$\frac{dt^E(\phi, v(\phi))}{d\phi} = \underbrace{\frac{\partial t_2}{\partial \phi} \bigg|_{v=v^E}}_{\text{Direct Effect}} + \underbrace{\frac{\partial t_2}{\partial v} \frac{dv^E}{d\phi}}_{\text{Strategic Effect}}. \quad (22)$$

As shown in Proposition 5, in equilibrium the direct effect dominates when the representative consumer becomes more environmentally conscious, the best-response tax tends to decrease. This may be happening because an increased ϕ reduces the consumption, which we will demonstrate later on, and therefore, since the regulator predicts that the quantity produced will be less, she will reduce the tax. When it comes

¹⁰ $\bar{\phi}_{tE} = \frac{1}{4}(3 + 16z + 4z^2) - \frac{1}{4}\sqrt{25 + 128z + 216z^2 + 128z^3 + 16z^4}$, $\bar{\phi}_{eO} = \frac{1}{2z}$. The subscripts denote where these upper bound came from, i.e., from t^E and from e^O respectively.

to the strategic effect, things are more simple. As shown on Lemma 3, the optimal abatement level when the regulator uses an emissions tax, reduces with respect to ϕ . Furthermore, as it is essential, a decrease in the abatement level increases the tax that the monopolist has to pay. In order to verify that this is the case, we present the derivative of t_2 with respect to v : $\frac{\partial t_2}{\partial v} = \frac{\phi - 2z(2+\phi)}{1+2z}$. The denominator remains positive $\forall z > 1$. Therefore, we must focus on the analysis of the numerator. Let us suppose that, the numerator is nonnegative (≥ 0). By performing some manipulation, we conclude that $\phi \leq \frac{4z}{1-2z}$. The aforementioned upper bound of ϕ is negative $\forall z > 1$, meaning that it must be rejected. Thus, the numerator can never be positive $\forall \phi \geq 0, z > 1$. As it emerges from the analysis above, the whole strategic effect positive. The final sign depends on the magnitude of each effect. The direct effect of ϕ to the tax has a stronger impact than that of the indirect effect, making the total effect negative (proof in the appendix)

The optimal quantity when an emissions tax is applied by the regulator, is the one that follows:

Proposition 6. *The equilibrium quantity under emissions tax is:*

$$q^E = \frac{1 + 5z + 2z^2 - \phi}{1 + 8z + 8z^2 - \phi}. \quad (23)$$

The optimal quantity when an emissions-tax rate is imposed has a negative slope $\forall \phi \geq 0, z > 1$.

Proof. By introducing (20) and (21) to (11), we extract the optimal quantity under the emissions tax □

Proposition 6 simply shows that as the consumers become more environmentally conscious, a reduction in the abatement will also reduce the quantity consumed. While the statement above is an intuitive justification of the consumers' consumption patterns, this is just the effect of the environmental consciousness through the abatement level. There is a direct effect and an effect through tax as well. Generally, the quantity is a function of the environmental consciousness, the tax rate and the abatement level:

$$q^E = q_3(\phi, v(\phi), t(\phi, v(\phi))).$$

Thus, in order to further examine the behavior of q^E , we decompose the total effect into the direct, indirect and strategic effect of ϕ to quantity. The derivative is:

$$\frac{dq^E}{d\phi} = \underbrace{\left. \frac{\partial q_3}{\partial \phi} \right|_{t=t^E, v=v^E}}_{\text{Direct Effect}} + \underbrace{\frac{\partial q_3}{\partial v} \frac{dv^E}{d\phi}}_{\text{Indirect Effect}} + \underbrace{\frac{\partial q_3}{\partial t} \left(\frac{\partial t^E}{\partial \phi} + \frac{\partial t_2}{\partial v} \frac{dv^E}{d\phi} \right)}_{\text{Strategic Effect}}. \quad (24)$$

We begin with the first partial derivative on the right-hand side of (24). The direct effect displays a negative relation between environmental consciousness and quantity, since the consumers show aversion to pollution (proof in the appendix). Moving on to the second derivative, which consists the indirect effect. The first component of the indirect effect operates as the one in (17), i.e., when the abatement level increases, the quantity produced and consumed increases, which is consistent with what intuition dictates. From Lemma 3, we conclude that the second component of the indirect effect is negative, making the whole indirect effect negative. Last but certainly not least, we have the strategic effect which has to individual components: one that is the effect of ϕ through the tax at equilibrium and the other is the effect that the environmental awareness has through the abatement level to the best-response tax function and the effect that the latter has to the quantity. Beginning with the first component of the strategic effect, from (17) we have that the first derivative is negative. From the extensive analysis that followed (22), we got that the effect that ϕ has on t^E is negative, making the first component of the strategic effect positive. When it comes to the second component of the strategic effect, we know from the analysis of (22), that the abatement level and tax are negatively correlated (the signs of the other two derivatives $\frac{dv^E}{d\phi}$ and $\frac{\partial q_3}{\partial t}$ are already known from Lemma 1 and the analysis above). Thus, the second component of the strategic effect is negative.

From the analysis of (22), the whole strategic effect is positive, meaning that we must perform a comparison between the direct and indirect effects combined and the strategic effect. This analysis leads us to the conclusion that the direct plus the indirect effect dominates the strategic effect, making the total effect negative. Therefore, in total, when the consumers have a better understanding about the existence of the externality, they choose to consume less at equilibrium. Thus, it is in favor of the monopolist to produce less.

Turning to the optimal net emissions, we get

Proposition 7. *The optimal net emissions under an emissions tax are:*

$$e^E = \frac{3 + 6z}{2 + 16z(1 + z) - 2\phi} \quad (25)$$

The net emissions under an emissions tax regime, e^E , is an increasing function of the environmental consciousness, ϕ .

Proof. By introducing (20) and (23) to (2), we get optimal net emissions. As for the rate of change of e^E : the quantity's rate of change is greater than that of the abatement level. Remember that, the net emissions function is $e = q - v$, therefore the total effect of ϕ to net emissions can be expressed as the subtraction of the total effect in quantity minus the total effect in abatement,

$$\begin{aligned} \frac{de^E}{d\phi} &= \frac{dq^E}{d\phi} - \frac{dv^E}{d\phi} \Leftrightarrow \frac{de^E}{d\phi} = -\frac{3z(1+2z)}{(-1-8z(1+z)+\phi)^2} - \left(-\frac{3(1+2z)^2}{2(-1-8z(1+z)+\phi)^2} \right) = \\ &= \frac{3+6z}{2(-1-8z(1+z)+\phi)^2} > 0. \end{aligned}$$

□

Proposition 7 is an important one, since it shows that even though an emissions tax is imposed in order to push the firm to be more environmentally friendly, increases in ϕ tend to decrease all of our variables of interest (v, t, q). The fact that the tax decreases in ϕ doesn't give the incentive to the monopolist to change her optimal choice and increase the abatement level. The latter leads to a reduction in the quantity produced and consumed. Even though all the variables are reduced, the optimal net emissions end up increasing. This is a counter-intuitive result, since the emissions tax combined with the environmental awareness should reduce the total net emissions generated. Thus, the quantity has a greater influence on the emissions than the abatement level. Therefore, in order to generate less emissions two actions must take place: Either the consumers become more sensitive about consumption and push the monopolist to produce less (quantity curve becomes more elastic) or the monopolist increase the optimal abatement level in a rate of change that is greater than that of quantity.

The net emissions have to remain positive, therefore:

Lemma 5. *In order to have positive net emissions, $e^E > 0$, ϕ must be in the $[0, 1 + 8z + 8z^2)$ interval.*

Proof. The numerator is always positive for $z > 1$. However, the same doesn't hold for the denominator. The denominator stays positive only if $\phi \in [0, 1 + 8z + 8z^2)$. Using Lemma 4, which contains the strictest restrictions, we can assume that e^E is always positive. □

Finally, the last piece of the analysis is the optimal social welfare when an emissions tax is imposed. By substituting (20) and (23) to (8), we get:

Proposition 8. *The optimal social welfare function under an emissions tax is:*

$$sw^E = \frac{1 + z[31 + 2z(49 + 58z + 20z^2)] - 8\phi - 16z(1 + z)\phi - 2\phi^2}{4(-1 - 8z(1 + z) + \phi)^2}. \quad (26)$$

The optimal social welfare function when an emissions tax is imposed reduces with respect to ϕ .

Proof. See the appendix □

The optimal social welfare, proposition above says, is a reducing function of ϕ . However, unlike sw^O , it becomes can become negative at a much higher level of environmental consciousness, which exceeds the upper bound of ϕ introduced in Proposition 5 or at a negative level. Therefore, we cannot consider the negative part in our analysis. This statement can be easily proven by observing the numerator of (26). The numerator is a trinomial in ϕ , with a positive discriminant:

$$\Delta = 72(1 + 7z + 18z^2 + 20z^3 + 8z^4),$$

which is positive. Therefore, there are two solutions to the equation where the numerator is equal to zero. The Vieta's formulas show that the addition of the roots of the equation is: $S = \phi_1 + \phi_2 = -\frac{\beta}{\alpha}$, whereas the multiplication of the roots is equal to $P = \phi_1 \cdot \phi_2 = \frac{\gamma}{\alpha}$, where ϕ_1, ϕ_2 are the roots of the equation, α is the coefficient of ϕ^2 , β is the coefficient of ϕ and γ is the constant term. We begin with P which is negative, because the constant term and the α coefficient has different signs, which leads us to the conclusion that we have one negative and one positive root. By solving the equation and performing some manipulations, the positive root that arises is:

$$\phi_2 = -2 - 4z - 4z^2 + \frac{3}{\sqrt{2}}\sqrt{1 + 7z + 8z^2 + 20z^3 + 8z^4},$$

which can be proven that doesn't belong to the intervals introduced in Proposition 5.

5 COMPARISONS

In this section, we compare the equilibrium values of q, v, t, e, sw , between the two tax regimes. Before proceeding, we need to determine the first-best to be used as a

benchmark. Direct maximization of (8) with respect to v , q , will give us the first-best abatement and quantity level:

$$v^* = \frac{z}{1 + 3z}, \quad (27)$$

$$q^* = \frac{1 + z}{1 + 3z}, \quad (28)$$

where the superscript “*” denotes the first-best outcome. Armed with the above we can obtain the first-best net emissions:

$$e^* = \frac{z}{1 + 3z}. \quad (29)$$

By substituting (27) and (28) to (8), the first-best social welfare arises:

$$sw^* = \frac{1 + z}{2(1 + 3z)}. \quad (30)$$

We can see that all of the first-best outcomes are independent of the level of environmental consciousness. This is due to the fact that we are using the SR approach according to which any eventual reduction in the purchased quantity attributed to consumers’ willingness to internalize part of the environmental externality, rather than to a change in their preferences in favor of environmentally friendly goods. Thus, despite quantity reductions, consumers still value the finally consumed quantity according to (4) instead of (5). This in return affects the value from consumption than the social planner must take into account in computing optimal tax rates. In order to enhance comparability, in all of the plots that follow the propositions, we set $z = 2$.

5.1 EQUILIBRIUM ABATEMENT LEVEL

In this section, we discuss the difference between the equilibrium abatement levels under both tax regimes and compare them to their first-best level. As shown by Lemma 1 and Lemma 3, v^O has a positive relation with the environmental consciousness, whereas the relation between v^E and ϕ points to the opposite direction. However, from the aforementioned lemmata we cannot know their relative position, i.e., which tax regime generates higher abatement level.

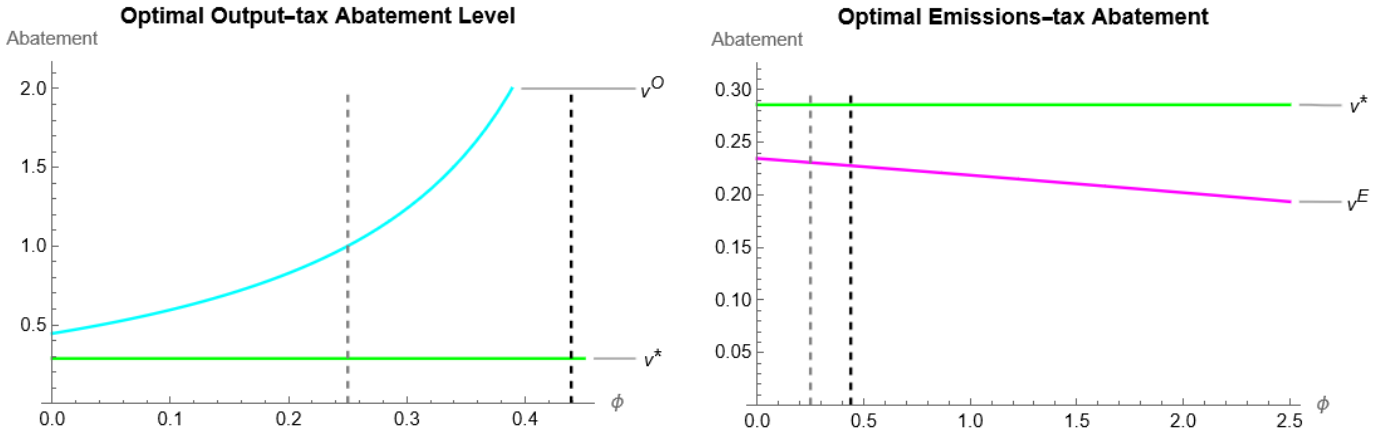
Proposition 9. *For the intervals of ϕ proposed in Proposition 5, the optimal abatement level in case of an output tax is greater than that when*

an emissions tax is in effect, i.e., $v^O > v^E \forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\})$, $z > 1$.

Proof. See the appendix □

Proposition 9 shows that when the regulator decides to apply an output subsidy, the monopolist not only abates at a positive rate, but she abates more than if the regulator chooses an emissions tax. An intuitive explanation of the proposition is that, an increase in abatement increases both the subsidy rate and the quantity. Meanwhile, in case of an emissions tax more abatement reduces the tax rate but at the same time it reduces also the base (emissions) to which the rate is applied.

Figure 1 illustrates the statement above. The green line represents the first-best outcome, while with cyan and magenta we represent v^O and v^E respectively. In addition, the black-dashed line shows $\bar{\phi}_{tE}$, while the gray-dashed one displays $\bar{\phi}_{eO}$, introduced in Proposition 5. The first two figures display each optimal abatement level individually compared to v^* , whereas below those figures we can see v^O and v^E , top to bottom respectively.



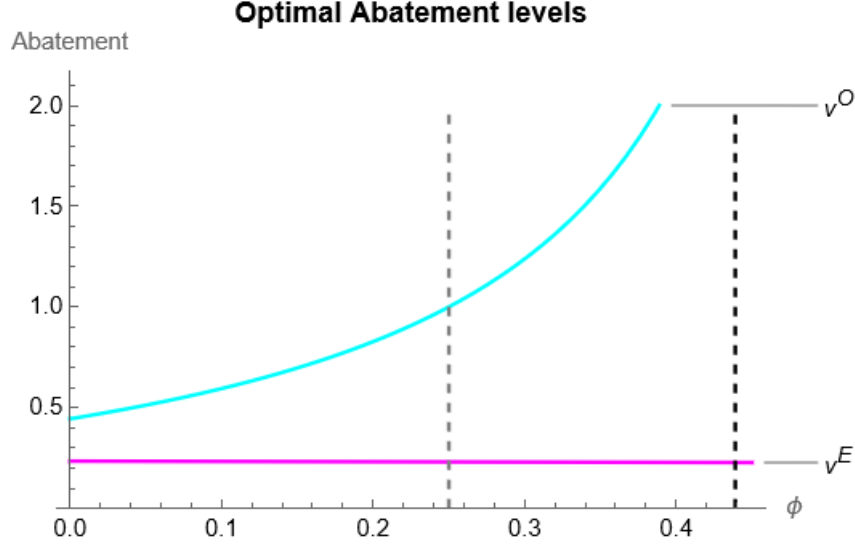


Figure 1. Optimal abatement levels when $z=2$.

In Figure 1, as in all subsequent figures we set z to be equal to 2, implying that it is gray-dashed line that limits the range of ϕ . Therefore, in this case the gray-dashed line is the one that is in effect. One can observe that the equilibrium abatement level in case of an output tax is greater not only than v^E , but than v^* as well, meaning that the subsidy is a great incentive for the monopolist. However, even though v^E reduces with respect to ϕ , it is very close to the first-best outcome, meaning that the monopolist's performance in case of an emissions tax is not as bad as one might think.

5.2 EQUILIBRIUM TAX RATES

Now that we have proved that $v^O > v^E$, we can compare the magnitude of the equilibrium tax-rates. Intuitively, since the optimal abatement level in case of an output tax increases with such high rate and it lies above the emissions-tax and the first-best abatement level, the output-tax rate must be less than the emissions-tax rate.

Proposition 10. *For all $\phi \geq 0$, the equilibrium output-tax rate is greater than the equilibrium emissions-tax rate, $t^E > t^O$.*

Proof. From the second part of Proposition 1 and from Proposition 5, we get that t^O is always negative and t^E is positive at the intervals proposed in the later. Therefore, since the two equilibrium tax functions have opposite signs $t^E > t^O$. □

Essentially, Proposition 10 is a combination of Proposition 1 and Proposition 5 and indicates that indeed the optimal commodity tax is less than the optimal emissions tax. Figure 2 illustrates graphically that difference. Once again, the cyan and the magenta lines represent t^O and t^E respectively. The behavior of the former was introduced in Proposition 1. As the environmental consciousness increases, optimal abatement level increases as well, leading to a reduced negative tax, or else, an increased positive subsidy. Accordingly, we can see from Proposition 5 that the emissions tax is a downward sloping function of the environmental consciousness.

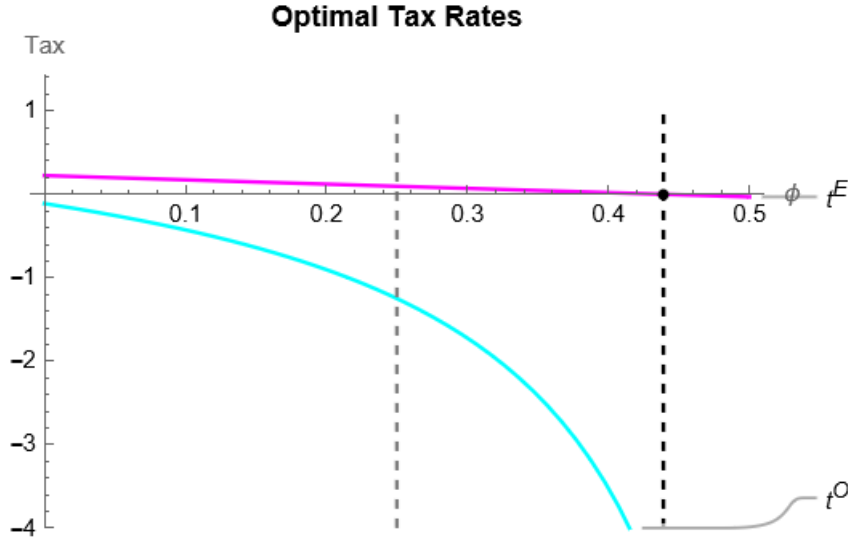


Figure 2. Optimal tax rates when $z=2$.

However, a per-unit-of-emissions tax can never be negative. In order to avoid an emissions subsidy, Proposition 5 poses limits the admissible range of ϕ -values. In the plot that follows, the second interval of the aforementioned proposition is in effect, which is depicted by the gray-dashed line, i.e., $\phi \in [0, \bar{\phi}_{e0})$. In that interval, the emissions-tax rate is always greater than the output-tax rate. In fact, even if the black line is in effect, t^E remains greater than t^O .

5.3 EQUILIBRIUM QUANTITIES

Having already shown the relationship between the abatement levels and the tax rate under the two policies, we will demonstrate difference that exists between q^O and q^E . Intuitively, we expect that since under the commodity tax, the abatement level is higher accompanied by a lowered tax, the quantity produced will follow the same direction.

Namely, we expect that $q^O > q^E$, as a result of the above. Armed with the above, we can deduce the following proposition:

Proposition 11. *For all $\phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}]$, $z > 1$, the equilibrium quantity in case of an output subsidy is greater than the equilibrium quantity under an emissions tax.*

Proof. See the appendix □

The two panels of Figure 3 present the equilibrium quantity as a function of ϕ under each tax regimes and compare them both to the first best. They also show that while q^O increases at an exponential-like rate, q^E decreases in an approximately linear fashion, which is not obvious by observing the bigger plot. Those statements were already proven in Proposition 2 and Proposition 6. The large plot below show the difference between q^O and q^E .

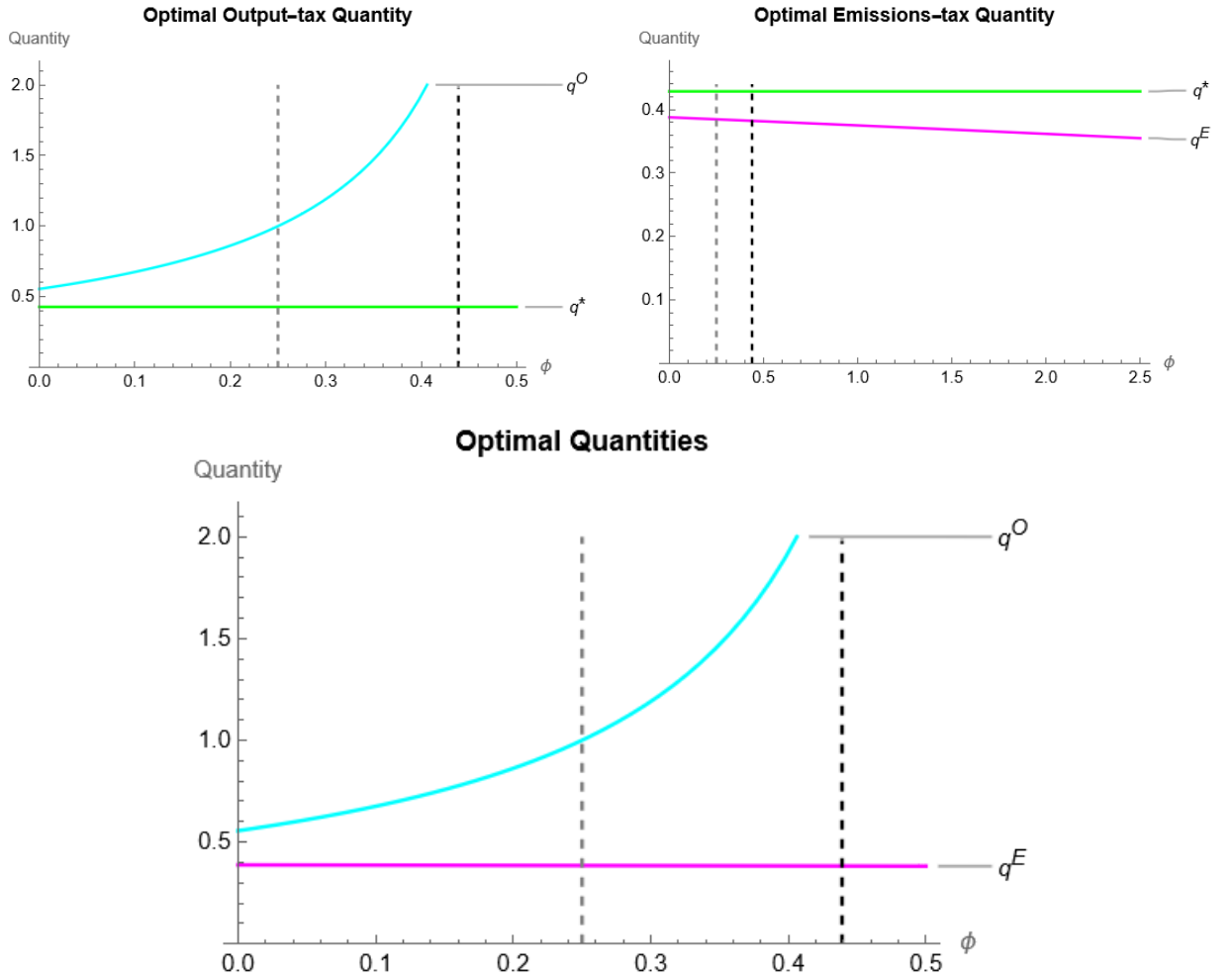


Figure 3. Optimal quantities when $z=2$.

Further, we can see that the optimal quantity under an output tax regime is greater than the first-best outcome as well, while in case of an emissions tax the quantity is below but pretty close to the latter. At some point the excessive use of abatement and therefore the large cost which follows the former, exceeds the utility gained from the increased consumption. Once again, the gray line is the one that dictates the upper bound of ϕ .

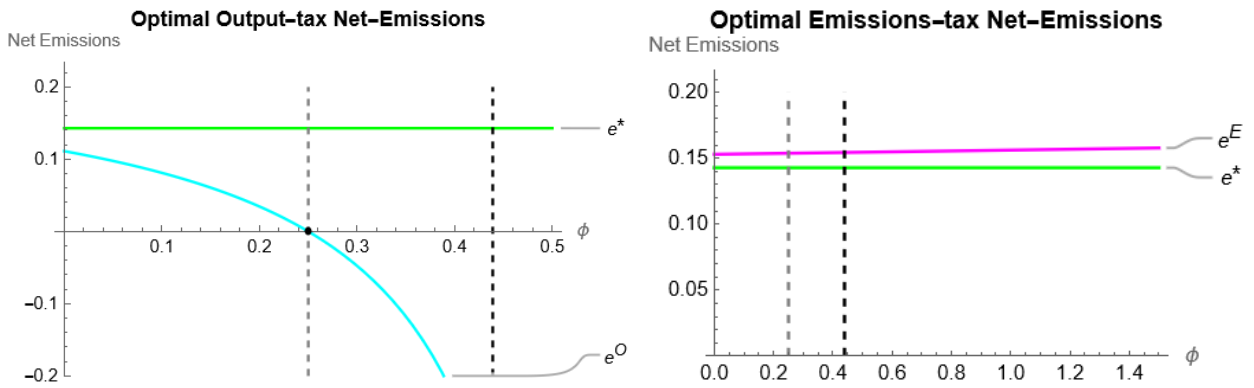
5.4 EQUILIBRIUM NET EMISSIONS

In this section, we are going to compare the net emissions generated under both tax regimes, when the consumers become more environmentally conscious. Let us concentrate on the comparison between the two optimal net emissions and the first-best net emissions.

Proposition 12. *An output subsidy regime combined with the environmental consciousness of the consumers, generates lower emissions than both the first-best and the net emissions under an emissions tax regime, i.e., $e^O < e^E \forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}], z > 1$.*

Proof. See the appendix □

We can see that in the appropriate interval of ϕ , the output tax always creates better environmental conditions, unlike e^E , which is always higher than the first-best. Figure 4 is consisted of 3 plots. As in some of the previous subsections, the first two plots show each function individually in order to have a clearer vision of the slopes. In the bigger plot, the difference between e^O and e^E is depicted.



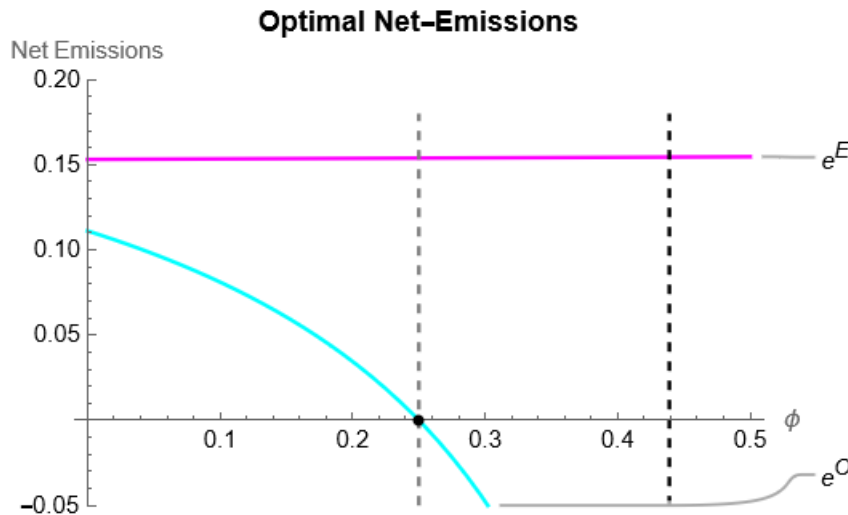


Figure 4. Optimal net emissions under both tax regimes when $z=2$.

Let us begin the analysis from the e^O, e^E difference. Initially, both optimal net emissions are in high levels. As the environmental consciousness increases, the monopolist increases each abatement level as well as the quantities produces (in the output tax case). However, as shown in section 4.1, the abatement increases at a higher rate than the quantity produced, thus decreasing the net emissions generated. This is not the case in the emissions-tax net emissions, where not only the emissions do not reduce as the environmental consciousness rises, but on the contrary, they seem to increase at a linear-like rate! This counter-intuitive result implies that as far as the environment is concerned, when the government imposes an emissions tax, the consumers better refrain from conscious action in terms of consumption adjustments! On the other hand, under the output subsidy regime, the more the consumers care about the environment, the better the environmental condition.

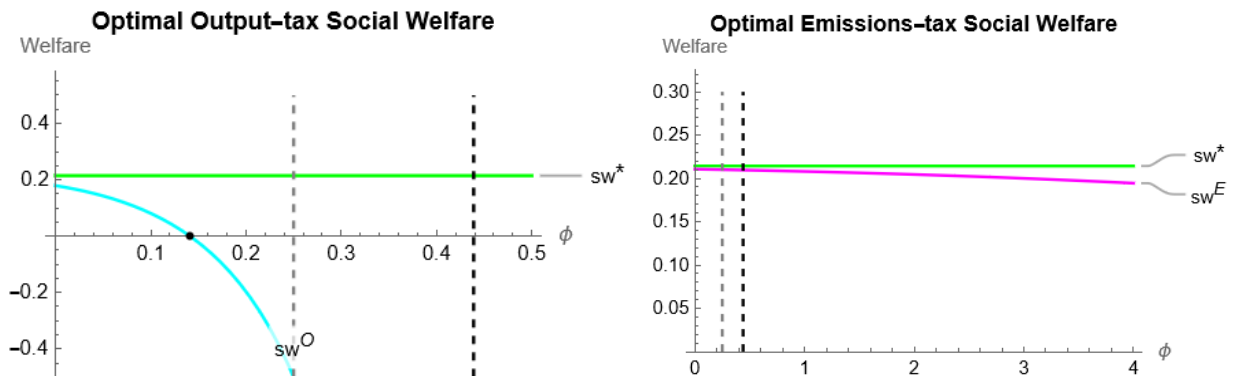
5.5 SOCIAL WELFARE: The Regulator's choice of tax type

In the first stage of the game (if we consider the regulator's choice of the appropriate tax as a separate stage) the regulator must make a discrete choice between output and emissions tax. This decision is based on which tax regime yields higher social welfare. Therefore, it is necessary to compare the equilibrium social welfare functions, in order to predict which tax should the regulator choose. Thus, in this part we are going to compute the equilibrium social welfare under output and emissions taxes and compare them to that of the first best. The regulator is going to choose the policy that yields the highest (and closest to the first-best) social welfare.

Proposition 13. *The optimal social welfare under an output subsidy lies below the optimal social welfare under an emissions tax regime $sw^O < sw^E \forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}], z > 1$.*

Proof. See the appendix □

The proposition above dictates that even though the environmental conditions are better and consumption is greater in case of an output tax, the optimal social welfare lies below the social welfare under an emissions tax. Figure 5, depicts graphically Proposition 13 alongside with Proposition 4 and Proposition 8. We can see that both social welfare functions have a downward slope. While sw^E reduces with respect to ϕ , it does not become negative, unlike sw^O , which becomes negative for $\phi > \phi_W^{11}$. The fact that both the abatement and the quantity under the output subsidy are way greater than the first-best, may cause the negative optimal social welfare. Thus, any further development of environmental consciousness leads to unhappiness, meaning that in the case of an output subsidy, the environmental consciousness has to stay at low levels in order to obtain positive social welfare.



¹¹ See Proposition 4

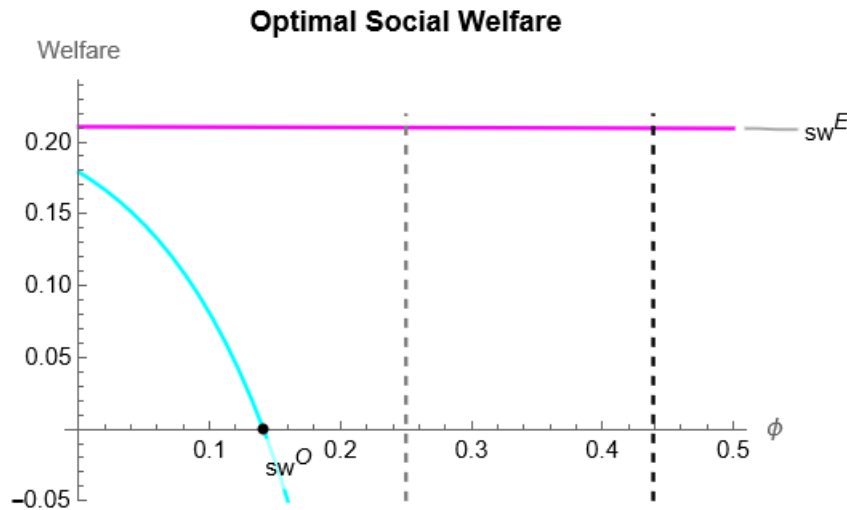


Figure 5. Optimal Social Welfare when $z=2$.

On the contrary, net emissions under an emissions tax generate much higher social welfare. As showed in Proposition 8 environmental consciousness and optimal social welfare are negatively correlated, even though this statement is not clear by observing the third plot. However, what is pretty clear is that sw^E lies higher than sw^O and much closer to the first-best. Even though the consumption is decreased and the environmental condition is worse, the cost function, which enters the social welfare function with a negative sign, stays at low levels due to low abatement performance. Therefore, it is possible that the utility gained combined with the total environmental damage to be higher than the cost function, thus generating higher social welfare. Thus, the analysis above can lead us to the conclusion that in terms of social welfare, the regulator would choose an emissions tax over a commodity subsidy, even if the former generates higher levels of net emissions.

It is surprising that, despite the fact that in equilibrium the output tax results in higher output and lower net emissions, its use yields lower social welfare than the use of an emissions tax. The explanation to this paradox lies in the fact that, since by construction the first-best level cannot be surpassed, the more efficient tax type is the one that yields variable levels that diverge as little as possible from their first-best levels. Such tax in our case is the emissions tax. Under the output tax, quantity is *too high* and net emissions *too low* if the abatement cost is taken into account. Thus, while the output tax produces higher welfare from the production of the specific good X, it drains too many resources from the production of other goods. The above remark is illustrated on Figure 6, where along with the full social welfare functions (19), (26) and (30), we also

present social welfare free of abatement cost, i.e., $sw_{NC} = U - D^{12}$. Dropping abatement cost from total welfare and simplifying yields the following expressions for the first-best, output and emissions tax respectively:

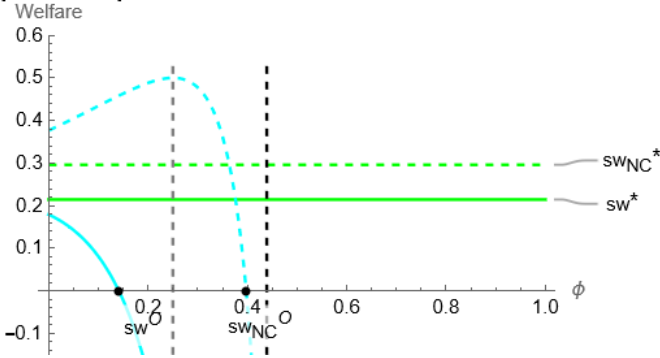
$$sw_{NC}^* = \frac{1 + z(4 + 5z)}{2(1 + 3z)^2},$$

$$sw_{NC}^O = \frac{1 + 2z(3 + 6z - 4z^2\phi(2 + \phi))}{2(1 + 4z - 4z^2\phi)^2},$$

$$sw_{NC}^E = \frac{1}{2} - \frac{9z(1 + 2z)^3}{4(-1 - 8z(1 + z) + \phi)^2}, z > 1.$$

The aforementioned proposition show that in case of a commodity tax, the society is better off as environmental consciousness increases. However, by excluding the abatement cost function, optimal emissions-tax social welfare doesn't perform as well. In the plots that follow, the solid lines represent the usual with-cost optimal social welfare functions, while the dashed lines, except of the black and gray ones, represent the optimal no-cost social welfare. The left one depicts the output subsidy case, whereas the right one illustrates the emissions tax case.

Optimal Output-tax Social Welfare with and without cost



Optimal Emissions-tax Social Welfare with and without cost

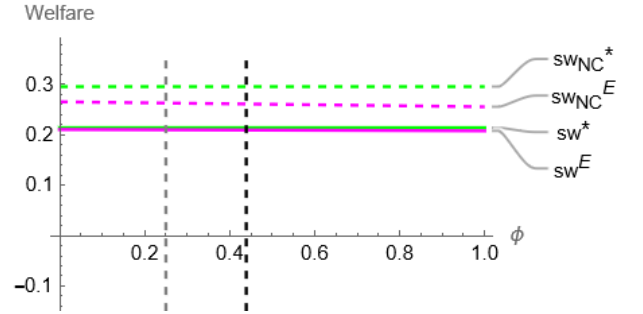


Figure 6. Optimal social welfare function with and without considering the cost when $z = 2$.

Starting with the left plot, we can see that sw^* is greater than sw^O , with the latter being a downward sloping function. When we do not take the cost function into consideration, the optimal social welfare not only is greater than the first-best one, but it is an increasing function of ϕ as well. Within the $[0, \bar{\phi}_{eo})$, sw_{NC}^O is monotonically increasing. After the upper bound, the aforementioned function starts to decrease. Eventually, at

¹² The subscript denotes the “No-Cost” computation.

some point it reaches the first-best outcome. As it happens with sw^O , sw_{NC}^O can become negative. Therefore, if the consumers start to care about the environment (at initial levels of ϕ), they enjoy higher social welfare. Moreover, the monopolist manipulates the tax rate by raising the abatement level in order to obtain higher subsidy. As ϕ rises the abatement level increases at a higher rate than the quantity because of the higher subsidy she receives, pushing the monopolist to overprovide quality in terms of cost. Since the abatement is not directly subsidized, we can calculate the efficient subsidy as the per units of abatement production multiplied with the optimal subsidy: $-t^O \times \frac{q^O}{v^O}$ (the negative sign was put in order to get rid of the negative sign of t^O). The high abatement level accompanied with the high quantity produced and the better environmental conditions lead to better social welfare for $\phi \in [0, \bar{\phi}_{eO})$.

As for the right plot, the optimal with-cost social welfare functions (sw^* , sw^E) is very close, with the latter being a downward sloping function. When we do not take into account the cost function the gap between the optimal social welfare under an emissions tax regime and the first-best outcome is much greater. The gap increment means that when we do not take into account the tax impact in the abatement cost, the performance under an emissions tax looks worse. In layman's terms, the emissions tax has a beneficial impact on cost rationalization leading to a closer to first-best optimal social welfare. Of course, here, there is no direct affection of the tax to the abatement level, rather than the monopolist foresees the tax that she will receive according to her optimal abatement level.

6 CONCLUSIONS

This work presents the effects that the existence of environmentally aware consumers has, when the choice of abatement is an endogenous variable under a monopolistic framework. We set a three-stage game, where the monopolist decides his abatement level first, the regulator chooses the tax rate in the second stage and finally the monopolist decides the quantity produced. We found that the abatement level in case of an output tax is an increasing function while the abatement level under an emissions tax is a decreasing function with respect to environmental consciousness, while the former is always greater than the latter and the first-best outcome. It turns out that the output tax is always negative, i.e., it is a subsidy. Quantity and abatement are both

increasing functions of the level of environmental consciousness. The quantity produced in case of an output tax has similar behavior to the respective abatement level, i.e., it increases when the tax is on output, whereas it decreases when the tax is on emissions. Moreover, the output tax yields both quantity and abatement higher than their first-best corresponding levels, because, loosely speaking, abatement is subsidized: the monopolist knows that by investing in abatement she may not only reduce the output-tax rate, but also turn it into a subsidy that increases with further increases in abatement. On the other hand, consumers know that overall emissions are limited and increase consumption. Not surprisingly, the subsidy leads to levels of abatement and consumption that are above first-best. The welfare provided by the sector increases, but at the expense of welfare from other sectors which is reduced, due to excessive transfer of resources to the sector in question. As expected, despite higher consumption and abatement, the output tax reduces total welfare.

By its nature, the emissions tax is less prone to produce overinvestment in abatement: on the one hand, no emissions subsidy is allowed (*i.e.*, emissions-tax at a negative rate), while on the other hand the emissions tax has an ambiguous effect on consumption since it increases both demand and costs. As it turns out, output, emissions and total welfare are all inferior to, but much closer to their first-best level, compared to their corresponding levels under the optimal output tax. This is an important result contrasting findings in CPS where the output-tax is welfare-superior to the emissions tax, the difference resting of course in the timing of the game. Thus, the answer to the question “which type of tax is more welfare enhancing?” crucially depends on the commitment value of abatement. When abatement can be easily adjusted (as in CPS) the output tax must be preferred. Otherwise, if the monopolist can anticipate the upcoming imposition the government better commit as soon as possible to the fact that the tax will be on the emissions in order to prevent overinvestment in abatement by the monopolist.

While most of the results were derived under specific assumptions, they are quite robust. However, some of our results show sensitivity to changes in values of δ and k parameters, the former representing the units of harmful pollutant per unit of produced good, while the latter is a multiplier of the abatement cost. Complete determination of how the important equilibrium variables are affected by changes in these two variables figures high in our research agenda.

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8 APPENDIX

8.1 Proof of Lemma 1

Lemma 1. *The optimal abatement level that the monopolist will choose is:*

$$v^O = -\frac{2z(1+\phi)}{-1+4z(-1+z\phi)}.$$

The derivative $\frac{\partial v^O}{\partial \phi} = \frac{2z(1+2z)^2}{(1+4z-4z^2\phi)^2} > 0 \forall \phi \geq 0, z > 1$. The second-order condition holds $\forall \phi \in \left[0, \frac{1+4z}{4z^2}\right)$.

Equations (11) and (12) show the best quantity and tax responses that maximize the monopolist's profit functions and the regulator's social welfare function. By substituting them to the monopolist's profits in case of an output tax, we get:

$$\Pi_O(v, \phi; z) = \frac{-((-1+v)(1+v+4vz)) + (1+2vz)^2\phi}{(1+2z)^2}.$$

By maximizing the aforementioned function with respect to v , we get:

$$\frac{\partial}{\partial v} \Pi_O(v, \phi; z) = \frac{2(-v+2z-4vz+2z\phi+4vz^2\phi)}{(1+2z)^2}.$$

By setting the derivative equal to 0, we get:

$$\frac{\partial}{\partial v} \Pi_O(v, \phi; z) = 0 \Rightarrow \frac{2(-v+2z-4vz+2z\phi+4vz^2\phi)}{(1+2z)^2} = 0$$

Since the denominator is positive, the only way that fraction is equal to zero, is by having the numerator equal to 0:

$$2(-v+2z-4vz+2z\phi+4vz^2\phi) = 0 \Leftrightarrow$$

$$-v+2z-4vz+2z\phi+4vz^2\phi = 0 \Leftrightarrow$$

$$2(2z+2z\phi) + 2v(-1-4z+4z^2\phi) = 0.$$

By taking the first part of the left-hand side to the right-hand side and dividing by 2, we get:

$$v(-1-4z+4z^2\phi) = 2z+2z\phi.$$

Dividing by $-1-4z+4z^2\phi$, gives us the optimal output-tax abatement level:

$$v^O = \frac{2z + 2z\phi}{-1 - 4z + 4z^2\phi}.$$

By taking the second derivative of $\Pi(v, \phi; z)$ with respect to v , one can observe that it remains negative and therefore v^O maximizes the profits function $\forall \phi \in \left[0, \frac{1+4z}{4z^2}\right)$:

$$\begin{aligned} \frac{\partial^2}{\partial v^2} \Pi_O(v, \phi; z) &= 2(-1 - 4z + 4z^2\phi) \\ \therefore \frac{\partial^2}{\partial v^2} \Pi_O(v, \phi; z) &< 0 \Leftrightarrow 2(-1 - 4z + 4z^2\phi) < 0 \\ &\Leftrightarrow -1 - 4z + 4z^2\phi < 0 \\ &\Leftrightarrow \phi < \frac{1 + 4z}{4z^2}. \end{aligned}$$

Now that we know that v^O is the optimal abatement level, we investigate its relation with ϕ . The derivative of v^O with respect to ϕ is:

$$\frac{dv^O}{d\phi} = \frac{2z(1 + 2z)^2}{(1 + 4z - 4z^2\phi)^2},$$

which is positive $\forall \phi$.

8.2 Proof that the direct effect of eq. (15) is negative

By taking the first derivative of (12) and substituting v with the optimal abatement level under the output tax, we obtain:

$$\left. \frac{\partial t_2}{\partial \phi} \right|_{v=v^O} = \frac{2 - 2z(-1 + \phi)}{-1 + 4z(-1 + z\phi)}.$$

From Lemma 1 we can extract the sign of the denominator which for sure is negative. Therefore, the final sign of the derivative depends on the numerator. It is easy to observe that the numerator is greater than zero $\forall \phi > \frac{1+z}{z}$. However, is $\frac{1+z}{z}$ less or greater than $\frac{1+4z}{4z^2}$? Let us suppose that the former is less or equal than the latter:

$$\begin{aligned} \frac{1+z}{z} &\leq \frac{1+4z}{4z^2} \Leftrightarrow \\ 4z^2(1+z) &\leq z(1+4z) \Leftrightarrow \\ 4z^2 + 4z^3 &\leq z + 4z^2 \Leftrightarrow \end{aligned}$$

$$4z^3 - z \leq 0,$$

which is not true for $z > 1$. Therefore, by contradiction $\frac{1+z}{z} > \frac{1+4z}{4z^2}$. Thus, since $\phi \in \left[0, \frac{1+4z}{4z^2}\right)$, the numerator is negative, making the whole derivative negative.

8.3 Proof of the sign of eq. (17)

Equation (17):

$$\frac{dq^0}{d\phi} = \underbrace{\left. \frac{\partial q_3}{\partial \phi} \right|_{t=t^0, v=v^0}}_{\text{Direct Effect}} + \underbrace{\frac{\partial q_3}{\partial v} \frac{dv^0}{d\phi}}_{\text{Indirect Effect}} + \underbrace{\frac{\partial q_3}{\partial t} \left(\frac{\partial t^0}{\partial \phi} + \frac{\partial t_2}{\partial v} \frac{dv}{d\phi} \right)}_{\text{Strategic Effect}}$$

By taking the derivative of equation (11), when $t = t^0$ and $v = v^0$:

$$\frac{\partial q_3}{\partial \phi} = \frac{1 + z - z\phi}{(1 + \phi)(-1 - 4z + 4z^2\phi)},$$

it is easy to observe that in order for the derivative to be positive, both the numerator and the denominator have to have the same sign. In the case that both the numerator and the denominator are positive we get that $\phi \in \left(\frac{1+4z}{4z^2}, \frac{1+z}{z}\right)$. If the numerator and the denominator are negative then $\phi \in \left[0, \frac{1+4z}{4z^2}\right) \cap \left(\frac{1+z}{z}, \infty\right)$ which leads to the null set.

Therefore, the direct effect is positive when the environmental consciousness belongs to the first interval. However, Lemma 1 makes it clear that in order to have the abatement level that maximizes the monopolist's profits, $\phi < \frac{1+4z}{4z^2}$. When examine

different signs of numerator and denominator, we get that either $\phi < \frac{1+z}{z}$ and $\phi < \frac{1+4z}{4z^2} \therefore \phi \in \left[0, \frac{1+4z}{4z^2}\right], z > 1$ or that $\phi > \frac{1+z}{z}$ and $\phi > \frac{1+4z}{4z^2} \therefore \phi > \frac{1+z}{z}, z > 1$ which has to be rejected due to Lemma 1. Thus, the direct effect is negative. When it comes to the whole indirect effect (indirect plus strategic effect), we already know the signs of $\frac{dv^0}{d\phi}$ and the whole tax effect $\left(\frac{\partial t^0}{\partial \phi} + \frac{\partial t_2}{\partial v} \frac{dv}{d\phi}\right)$. However, from (11) we know that $\frac{\partial q_3}{\partial v} >$

$0 \forall \phi \geq 0$ and $\frac{\partial q_3}{\partial t} < 0$. By combining all the derivatives above (the signs are shown in (17)) we can see that the direct effect is negative.

8.4 Proof of Proposition 3

Proposition 3. *The optimal net emissions are:*

$$e^0 = \frac{1 - 2z\phi}{1 + 4z - 4z^2\phi}.$$

The net emissions under an output-tax regime decrease as the socially responsible consumers become more environmentally aware, i.e., $\frac{\partial e^0}{\partial \phi} < 0$.

By taking the derivative of (18), we get:

$$\begin{aligned} \frac{\partial e^0}{\partial \phi} &= \frac{-2z(1 + 4z - 4z^2\phi) - (-4z^2)(1 - 2z\phi)}{(1 + 4z - 4z^2\phi)^2} \Leftrightarrow \\ \frac{\partial e^0}{\partial \phi} &= \frac{-2z[1 + 4z - 4z^2\phi - 2z(1 - 2z\phi)]}{(1 + 4z - 4z^2\phi)^2} \Leftrightarrow \\ \frac{\partial e^0}{\partial \phi} &= \frac{-2z(1 + 4z - 4z^2\phi - 2z + 4z^2\phi)}{(1 + 4z - 4z^2\phi)^2}. \end{aligned}$$

By eliminating $-4z^2\phi$ and $4z^2\phi$, we obtain the final form of the derivative:

$$\frac{\partial e^0}{\partial \phi} = -\frac{2z(1 + 2z)}{(1 + 4z - 4z^2\phi)^2},$$

which is negative $\forall \phi \geq 0, z > 1$.

8.5 Proof of Proposition 4

Proposition 4. *The social welfare in the output-tax equilibrium decreases as environmental consciousness increases, i.e., $\frac{\partial sw^0}{\partial \phi} < 0$ and may even become negative $\forall \phi > \phi_W$, where $\phi_W \equiv -1 + \frac{\sqrt{(1+2z)^3}}{z^2(1+z)}$.*

Considering equation (19), and by performing some manipulations, the simple form of the first derivative is:

$$\frac{\partial sw^0}{\partial \phi} = \frac{4z^2(1 + 2z)^2[1 + 2(1 + z)\phi]}{[-1 + 4z(-1 + z\phi)]^3}.$$

Since the numerator consists of positive components, with ϕ, z being positive, it is obvious that the whole numerator ends up being positive. Therefore, we focus on the denominator. Lemma 1 dictates that ϕ is restricted in the $\left[0, \frac{1+4z}{4z^2}\right]$ interval or else $0 \leq \phi \leq \frac{1+4z}{4z^2}$. By taking the right part and multiplying with $4z^2$, we get:

$$\begin{aligned} 4z^2\phi &\leq 1 + 4z \Leftrightarrow \\ -1 - 4z + 4z^2\phi &\leq 0 \Leftrightarrow \\ -1 + 4z(-1 + z\phi) &\leq 0. \end{aligned}$$

Setting the exponent of the above inequality equal to 3, we obtain that:

$$[-1 + 4z(-1 + z\phi)]^3 \leq 0,$$

making the whole derivative negative.

Manipulating the numerator of (19), we extract a trinomial with respect to ϕ . When we set the trinomial above equal to zero we get:

$$1 + 6z + 4z^2 + (-16z^2 - 16z^3)\phi + (-8z^2 - 8z^3)\phi^2 = 0.$$

The discriminant of the equation above is:

$$\Delta = 32(z^2 + 7z^3 + 18z^4 + 20z^5 + 8z^6),$$

or in its simplified form:

$$\Delta = 32z^2(1 + z)(1 + 2z)^3.$$

Since $\Delta > 0 \forall z > 1$, the trinomial has two solutions. According to Vieta's formulas, the product of the solutions of the equation, let them be ϕ_1, ϕ_2 , is equal to the quotient of the gamma coefficient which is equal to $\gamma = 1 + 6z + 4z^2$ and the alpha coefficient, which is equal to $a = (-8z^2 - 8z^3)$, i.e.,

$$P = \phi_1\phi_2 = \frac{\gamma}{\alpha} = \frac{1 + 6z + 4z^2}{(-8z^2 - 8z^3)} < 0,$$

meaning that there is one positive and one negative solution. The positive one is equal

$$\text{to } \phi_1 = -1 + \frac{\sqrt{\frac{(1+2z)^3}{z^2(1+z)}}}{2\sqrt{2}}.$$

8.6 Proof of Lemma 3

Lemma 3. *The optimal abatement level that the monopolist will choose is:*

$$v^E(\phi; z) = \frac{-1 + 4z(1 + z) - 2\phi}{2 + 16z(1 + z) - 2\phi}.$$

The derivative of (20) remains negative $\forall \phi \geq 0$, i.e.,

$$\frac{\partial v^E}{\partial \phi} < 0.$$

The profits function in this case is different than in the case of the output tax. By substituting equations (11) and (12) to $\Pi^E = p(q)q - t(q - v) - v^2$, we obtain:

$$\Pi_E(v, \phi; z) = \frac{1 + \phi - v(1 + v - 4z(1 + z) + 8vz(1 + z) + 2\phi - v\phi)}{(1 + 2z)^2}$$

Taking the first derivative of the aforementioned function with respect to v and performing some manipulations, we get:

$$\frac{\partial}{\partial v} \Pi_E(v, \phi; z) = \frac{-1 + 4z(1 + z) - 2v(1 + 8z(1 + z) - \phi) - 2\phi}{(1 + 2z)^2}.$$

By setting the derivative equal to zero we get the equilibrium abatement level under the emissions-tax regime:

$$\begin{aligned} \frac{\partial}{\partial v} \Pi_E(v, \phi; z) = 0 &\Leftrightarrow \\ \frac{-1 + 4z(1 + z) - 2v(1 + 8z(1 + z) - \phi) - 2\phi}{(1 + 2z)^2} &= 0. \end{aligned}$$

Since the denominator is positive and cannot make our derivative equal to zero (unless $z \rightarrow \infty$), we concentrate on the numerator. Adding both sides with $2v(1 + 8z(1 + z) - \phi)$ and dividing by $2(1 + 8z(1 + z) - \phi)$ yields the optimal emissions-tax abatement level v^E :

$$v^E(\phi; z) = \frac{-1 + 4z(1 + z) - 2\phi}{2 + 16z(1 + z) - 2\phi}.$$

The first derivative of the aforementioned function is defined as:

$$\frac{dv^E}{d\phi} = \frac{(-1 + 4z + 4z^2 - 2\phi)'(2 + 16z + 16z^2 - 2\phi) - (-1 + 4z + 4z^2 - 2\phi)(2 + 16z + 16z^2 - 2\phi)'}{(2 + 16z + 16z^2 - 2\phi)^2} \Leftrightarrow$$

$$\frac{dv^E}{d\phi} = \frac{-2(2 + 16z + 16z^2 - 2\phi) - (-1 + 4z + 4z^2 - 2\phi)(-2)}{(2 + 16z + 16z^2 - 2\phi)^2} \Leftrightarrow$$

$$\frac{dv^E}{d\phi} = \frac{-2(3 + 12z + 12z^2)}{(2 + 16z + 16z^2 - 2\phi)^2} = -\frac{3(1 + 4z + 4z^2)}{2(1 + 8z + 8z^2 - \phi)^2},$$

which is negative. The second derivative is:

$$\frac{\partial^2}{\partial v^2} \Pi_E(v, \phi; z) = \frac{2(-1 - 8z(1 + z) + \phi)}{(1 + 2z)^2}.$$

In order for the above to be positive, ϕ must be in the $[1 + 8z + 8z^2]$ interval, which derives from the numerator of the derivative. This constraint is a loose one since it is greater than the one dictated in Lemma 1. Therefore, the optimal abatement level when an emissions tax is imposed maximizes the profits.

8.7 Proof of Lemma 4

Lemma 4. *The optimal emissions-tax rate is non-negative $\forall \phi \in [0, \bar{\phi}_{tE}]$ when $1 < z \leq \frac{1}{8}(7 + \sqrt{33})$ or $\forall \phi \in [0, \bar{\phi}_{eO})$ when $z > \frac{1}{8}(7 + \sqrt{33})$. For $1 < z \leq \frac{1}{8}(7 + \sqrt{33})$, $\bar{\phi}_{tE} < \bar{\phi}_{eO}$.*

Equation (21) becomes zero when the numerator is equal to zero. The latter is a trinomial with respect to ϕ : $-2 - 4z + 8z^2 + (-3 - 16z + 4z^2)\phi + 2\phi^2 = 0$. The discriminant of the equation above is:

$$\Delta = 25 + 128z + 216z^2 + 128z^3 + 16z^4,$$

which of course is positive, meaning that the equation has two solutions. From Vieta's formulas we can be sure that the solutions are both positive. The solutions are:

$$\phi_{tE}^1 = \frac{1}{4}(3 + 16z + 4z^2) - \frac{1}{4}\sqrt{25 + 128z + 216z^2 + 128z^3 + 16z^4},$$

$$\phi_{tE}^2 = \frac{1}{4}(3 + 16z + 4z^2) + \frac{1}{4}\sqrt{25 + 128z + 216z^2 + 128z^3 + 16z^4}.$$

with, as it is obvious, $\phi_{tE}^2 > \phi_{tE}^1 \forall z > 1$. Now, we have to identify which of these values of ϕ are within the interval proposed in Lemma 2. Taking the first derivative of ϕ_{tE}^2 with respect to z , we can see that it is positive making ϕ_{tE}^2 an increasing function of z :

$$\frac{d\phi_{tE}^2}{dz} = 2 \left(2 + z + \frac{(1+2z)[8+z(11+2z)]}{\sqrt{(1+2z)^2[25+4z(7+z)]}} \right) > 0.$$

We can extract its minimum value by taking the limit of ϕ_{tE}^2 as z approaches 1, i.e.,

$$\lim_{z \rightarrow 1^+} \phi_{tE}^2 \approx 11.4124,$$

which is much greater than $\frac{1}{2}$ which is the maximum value of the upper bound of ϕ proposed in Lemma 2. Thus, ϕ_{tE}^2 is rejected.

As for ϕ_{tE}^1 , we want to identify under which circumstances $\phi_{tE}^1 < \frac{1}{2z}$. After performing some manipulations, the following inequality arises:

$$64z^4 - 48z^3 - 80z^2 - 12z + 4 < 0.$$

or else:

$$4(1+2z)^2(1-7z+4z^2) < 0.$$

By setting the aforementioned polynomial equal to zero, only the trinomial in the last parentheses can be equal to zero at $z_1 = \frac{1}{8}(7 - \sqrt{33})$ and $z_2 = \frac{1}{8}(7 + \sqrt{33})$. Between those two solutions the trinomial is negative. However, $z_1 < 1$ and therefore it has to be rejected. Thus, for $z \in [1, z_2]$, $\phi_{tE}^1 < \frac{1}{2z}$. For $z > z_2$, $\phi_{tE}^1 > \frac{1}{2z}$ and according to Lemma 2, has to be rejected. Therefore, $\forall z \in \left[0, \frac{1}{8}(7 + \sqrt{33})\right]$, $\phi \in [0, \bar{\phi}_{tE}]$ and $\forall z > z_2$, $\phi \in \left[0, \frac{1}{2z}\right)$.

8.8 Proof of Proposition 5

Proposition 5. *Moreover, the first derivative of the emissions-tax rate is negative:*

$$\frac{dt^E}{d\phi} < 0.$$

Using the decomposition analysis described in equation (22), we can extract the total effect of ϕ in the emissions-tax rate. The direct effect is $\frac{\partial t_2}{\partial \phi} = \frac{-5-4z(4+z)+2\phi}{2+16z(1+z)-2\phi}$, which is negative while the strategic effect is $\frac{\partial t_2}{\partial v} \frac{dv^E}{d\phi} = \frac{3(1+2z)(4z-\phi+2z\phi)}{2(1+8z+8z^2-\phi)^2}$, which is positive (as stated in extend in the analysis above). The total effect is:

$$\frac{dt^E(\phi, v(\phi))}{d\phi} = \frac{-5 - 4z(4 + z) + 2\phi}{2 + 16z(1 + z) - 2\phi} + \frac{3(1 + 2z)(4z - \phi + 2z\phi)}{2(1 + 8z + 8z^2 - \phi)^2}$$

If we unite the two fractions, we get:

$$\frac{dt^E(\phi, v(\phi))}{d\phi} = \frac{(-5 - 16z - 4z^2 + 2\phi)(1 + 8z + 8z^2 - \phi) + 3(1 + 2z)(4z - \phi + 2z\phi)}{2(1 + 8z + 8z^2 - \phi)^2}.$$

It is easy to understand that the denominator is always positive. Therefore, the analysis must be focused on the numerator. By performing some algebraic calculations, we get that the numerator, which we are going to name it $N_{\partial t}$, is:

$$N_{\partial t} = (4 + 32z + 32z^2)\phi - 2\phi^2 - 44z - 148z^2 - 160z^3 - 32z^4 - 5.$$

In order to identify its sign in the we are going to take the first derivative with respect to ϕ :

$$\frac{dN_{\partial t}}{d\phi} = 4 + 32z + 32z^2 - 4\phi,$$

which is a downward sloping function. From Proposition 5, we can extract that both of these upper bounds of ϕ are less than a half, using their respective z -intervals. Since the aforementioned derivative has a negative slope and at $\phi = 0$ it is positive, if its minimum value, i.e., as ϕ approaches $\frac{1}{2}$, is positive, then at that interval, $N_{\partial t}$ will be an increasing function $\forall \phi \in [0, \frac{1}{2})$. At $\phi = 0$, $N_{\partial t} < 0, z > 1$, meaning that we have to examine its maximum value:

$$\lim_{\phi \rightarrow \frac{1}{2}} N_{\partial t} = -\frac{7}{2} - 4z(7 + z(33 + 8z(5 + z))),$$

which is negative for $z > 1$. Thus, $\frac{dt^E(\phi, v(\phi))}{d\phi} < 0 \forall \phi \in [0, \frac{1}{2}), z > 1$.

8.9 Proof that the direct effect of eq. (24) is negative.

In its full form it is:

$$\left. \frac{\partial q_3}{\partial \phi} \right|_{t=t^E, v=v^E} = \frac{-5 - 4z(4 + z) + 2\phi}{4(1 + 8z(1 + z) - \phi)(1 + \phi)}.$$

Let us suppose that the derivative above is positive. The fraction above can be positive as long as both the numerator and the denominator has the same sign, whether this is either positive or negative. Let us suppose that are both positive. The interval of ϕ that arises in order for the above to be true is: $\phi \in \left(\frac{1}{2}(5 + 16z + 4z^2), 1 + 8z + 8z^2\right)$, $z > 1$. When both the numerator and the denominator are negative, then $\phi \in \left[0, \frac{1}{2}(5 + 16z + 4z^2)\right) \cap (1 + 8z + 8z^2, \infty)$, and vice versa, which leads to a null set. It is obvious that the lower bound, $\frac{1}{2}(5 + 16z + 4z^2)$, is greater than the upper bound of ϕ which was introduced in Lemma 1. By contradiction, the direct effect is negative. In order to identify that this is the case, we follow the same process when the direct effect is negative. Here, if the numerator is negative and the denominator is positive we get that $\phi < \frac{1}{2}(5 + 16z + 4z^2)$ (which contains both intervals prosed in Lemma 4), while when the numerator is positive and the denominator is negative we get that $\phi > 1 + 8z + 8z^2$.

8.10 Proof of Proposition 8

Proposition 8. *The optimal social welfare function under an emissions tax is:*

$$sw^E = \frac{1 + z[31 + 2z(49 + 58z + 20z^2)] - 8\phi - 16z(1 + z)\phi - 2\phi^2}{4(-1 - 8z(1 + z) + \phi)^2}. \quad (31)$$

The optimal social welfare function when an emissions tax is imposed reduces with respect to ϕ .

Taking the first derivative of (26) with respect to ϕ , we obtain:

$$\frac{\partial sw^E}{\partial \phi} = -\frac{3(1 + 2z)^2(1 + z(-1 + 2z) + 2\phi)}{2(1 + 8z(1 + z) - \phi)^3}.$$

Using Lemma 4, we can be sure that the denominator is positive. Therefore, we have to identify the sign of the numerator. Let $N_{\partial sw} \equiv -3(1 + 2z)^2(1 + z(-1 + 2z) + 2\phi)$ By expanding $N_{\partial sw}$, we get:

$$N_{\partial sw} = -3(1 + 2z)^2 - 3z(-1 + 2z)(1 + 2z)^2 - 6(1 + 2z)^2\phi,$$

which is a decreasing function of ϕ , as the coefficient of ϕ shows, $-6(1 + 2z)^2$. For $\phi = 0$, we get the maximum:

$$N_{\partial sw} = -3(1 + 2z)^2 - 3z(-1 + 2z)(1 + 2z)^2 < 0 \quad \forall z > 1.$$

Thus, since the numerator is negative, the whole derivative is negative, meaning that sw^E is a decreasing function of ϕ .

8.11 Proof of Proposition 9

Proposition 9. *For the intervals of ϕ proposed in Proposition 5, the optimal abatement level in case of an output tax is greater than that when an emissions tax is in effect, i.e., $v^O > v^E \forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\})$, $z > 1$.*

Let $v_{dif}(\phi; z) \equiv v^O(\phi; z) - v^E(\phi; z)$. By manipulating the expressions, we extract the final form of the aforementioned function, which is:

$$v_{dif}(\phi; z) = \frac{-1 - 4z - 12z^2 - 16z^3 - 2\phi - 8z\phi - 28z^2\phi - 48z^3\phi - 16z^4\phi + 4z\phi^2 + 8z^2\phi^2}{2(1 + 8z + 8z^2 - \phi)(-1 - 4z + 4z^2\phi)}.$$

According to Lemma 4, the first part of the denominator, $1 + 8z + 8z^2 - \phi$, is positive. Nevertheless, the second part is negative. Moving on to the numerator, let $N_v \equiv -1 - 4z - 12z^2 - 16z^3 - 2\phi - 8z\phi - 28z^2\phi - 48z^3\phi - 16z^4\phi + 4z\phi^2 + 8z^2\phi^2$. The first derivative of N_v with respect to ϕ is:

$$\frac{\partial N_v}{\partial \phi} = -2 - 8z - 28z^2 - 48z^3 - 16z^4 + 2(4z + 8z^2)\phi.$$

One can observe that the coefficient of ϕ is positive, making the aforementioned derivative an increasing function of ϕ . At its minimum, i.e., when $\phi = 0$, we have:

$$\frac{\partial N_v}{\partial \phi}(\phi = 0) = -2 - 8z - 28z^2 - 48z^3 - 16z^4,$$

which is negative. The derivative becomes zero at $\phi_v = \frac{1+2z+10z^2+4z^3}{4z}$, which is greater than the allowed intervals proposed in Lemma 4. Therefore $\forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}]$, $\frac{\partial N_v}{\partial \phi} < 0$, making N_v a decreasing function of ϕ . At its maximum point, i.e., for $\phi = 0$,

the numerator is negative. Since both the numerator and the denominator are negative, the whole fraction becomes positive, which leads us to the conclusion that:

$$v_{dif}(\phi; z) > 0 \Rightarrow v^O(\phi; z) > v^E(\phi; z) \forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}].$$

8.12 Proof of Proposition 11

Proposition 11. *The equilibrium quantity in case of an output subsidy is greater than the equilibrium quantity under an emissions tax $\forall \phi \in [0, \bar{\phi}_{tE}), 1 < z \leq \frac{1}{8}(7 + \sqrt{33})$ or $\forall \phi \in [0, \bar{\phi}_{eO}), z > \frac{1}{8}(7 + \sqrt{33})$.*

The analysis here is the same as the proof of Proposition 5.

Let $q_{dif}(\phi; z) \equiv q^O(\phi; z) - q^E(\phi; z) = \frac{1+5z+2z^2-\phi}{1+8z+8z^2-\phi} + \frac{1+2z}{1+4z-4z^2\phi}$. By simplifying the expression, we get:

$$q_{dif}(\phi; z) = \frac{-z - 2z^2 - 8z^3 - 2z\phi - 4z^2\phi - 20z^3\phi - 8z^4\phi + 4z^2\phi^2}{(1 + 8z + 8z^2 - \phi)(-1 - 4z + 4z^2\phi)}.$$

Beginning our analysis with the denominator, one can observe that according to Lemma 4, the first part of the denominator is positive, whereas the second one is negative, making the whole denominator negative. Ergo, we focus on the numerator.

Let $N_q(\phi; z) \equiv -z - 2z^2 - 8z^3 - 2z\phi - 4z^2\phi - 20z^3\phi - 8z^4\phi + 4z^2\phi^2$. By taking the first derivative of the function above, we obtain:

$$\frac{\partial N_q(\phi; z)}{\partial \phi} = -2z - 4z^2 - 20z^3 - 8z^4 + 8z^2\phi.$$

Once again, the coefficient of ϕ is positive, making the derivative an increasing function of ϕ . At its minimum,

$$\frac{\partial N_q(\phi; z)}{\partial \phi}(\phi = 0) = -2z - 4z^2 - 20z^3 - 8z^4,$$

which is negative. The derivative becomes zero at $\phi_q = \frac{1+2z+10z^2+4z^3}{4z}$, which is greater than the $\min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}$. Therefore, $\forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}]$, the derivative above is negative, meaning that the numerator is a downward sloping function. At its maximum:

$$N_q(\phi; z) \equiv -z - 2z^2 - 8z^3 < 0,$$

and since it is decreasing, the numerator remains negative $\forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}]$.

Since, both the numerator and the denominator are negative, $q_{dif}(\phi; z) > 0 \Rightarrow$

$$q^O(\phi; z) > q^E(\phi; z) \forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}], z > 1.$$

8.13 Proof of Proposition 12

Proposition 12. *An output subsidy regime combined with the environmental consciousness of the consumers, generates lower emissions than both the first-best and the net emissions under an emissions tax regime, i.e., $e^O < e^E \forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}], z > 1$.*

Let $e_{dif}(\phi; z) \equiv e^O(\phi; z) - e^E(\phi; z) = \frac{1-2z\phi}{1+4z-4z^2\phi} - \frac{3+6z}{2+16z(1+z)-2\phi}$. By manipulating the expression above, we extract:

$$e_{dif}(\phi; z) = \frac{1 + 2z + 8z^2 + 2\phi + 4z\phi + 20z^2\phi + 8z^3\phi - 4z\phi^2}{2(1 + 8z + 8z^2 - \phi)(-1 - 4z + 4z^2\phi)}.$$

Beginning with the denominator, Lemma 4 shows that the denominator is negative. Therefore, once again, we focus on the numerator.

Let $N_e = 1 + 2z + 8z^2 + (2 + 4z + 20z^2 + 8z^3)\phi - 4z\phi^2$, which is the numerator of e_{dif} . The derivative of the former with respect to ϕ is:

$$\frac{\partial N_e}{\partial \phi} = 2 + 4z + 20z^2 + 8z^3 - 8z\phi$$

The derivative above decreases with respect to ϕ . At its maximum:

$$\frac{\partial N_e}{\partial \phi}(\phi = 0) = 2 + 4z + 20z^2 + 8z^3.$$

The derivative is equal to zero:

$$\frac{\partial N_e}{\partial \phi} = 0 \Leftrightarrow$$

$$2 + 4z + 20z^2 + 8z^3 - 8z\phi = 0 \Leftrightarrow$$

$$\phi = \frac{1 + 2z + 10z^2 + 4z^3}{4z},$$

which is already shown that it is greater than $\min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}$. Therefore, $\frac{\partial N_e}{\partial \phi} > 0$, meaning that N_e increases in ϕ . The minimum value of that function is:

$$N_e(\phi = 0) = 1 + 2z + 8z^2 + (2 + 4z + 20z^2 + 8z^3)\phi,$$

which is positive, meaning that $\forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}]$, $N_e > 0$. Since the numerator and the denominator has opposite signs, the whole $e_{dif} < 0 \Rightarrow$

$$e^O < e^E \forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}], z > 1.$$

8.14 Proof of Proposition 13

Proposition 13. *The optimal social welfare under an output subsidy lies below the optimal social welfare under an emissions tax regime, i.e., $sw^O < sw^E \forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}], z > 1$.*

Let sw_{dif} be the difference that exists between sw^O and sw^E , which are described by equations (19) and (26). The mathematical form of the above is:

$$\begin{aligned} sw_{dif}(\phi; z) &\equiv sw^O(\phi; z) - sw^E(\phi; z) \\ \therefore sw_{dif}(\phi; z) &= \frac{1 + z[31 + 2z(49 + 58z + 20z^2)] - 8\phi - 16z(1 + z)\phi - 2\phi^2}{4[-1 - 8z(1 + z) + \phi]^2} \\ &\quad + \frac{1 + 2z[3 + 2z - 4z(1 + z)\phi(2 + \phi)]}{2(1 + 4z - 4z^2\phi)^2}. \end{aligned}$$

Due to its complexity, we are only going to provide the numerator of the function above, knowing that the denominator is always positive due to the fact that it is squared. Let N_{sw} be the numerator of the simplified version of the function above, which is a 4th degree polynomial:

$$\begin{aligned} N_{sw}(\phi; z) &= 1 + 5z - 2z^2 - 52z^3 - 232z^4 - 384z^5 - 128z^6 + \\ &\quad (4 + 24z + 8z^2 - 200z^3 - 1168z^4 - 2592z^5 - 2112z^6 - 768z^7)\phi + \\ &\quad (4 + 28z + 24z^2 - 80z^3 - 1168z^4 - 3824z^5 - 4640z^6 - 2880z^7 - 640z^8)\phi^2 + \\ &\quad (-16z^2 + 192z^3 + 640z^4 + 512z^5 + 256z^6)\phi^3 + \\ &\quad (-16z^2 - 16z^3 + 32z^4)\phi^4. \end{aligned}$$

The fourth degree derivative of N_{sw} is:

$$\frac{\partial^4 N_{sw}}{\partial \phi^4} = 24(-16z^2 - 16z^3 + 32z^4) = 384z^2(1 - z + 2z^2)$$

The derivative becomes zero at $z_1 = -\frac{1}{2}$, $z_2 = 0$ and $z_3 = 1$. One of our main assumptions is that the $z > 1$ and therefore both of the solutions are rejected, meaning that $\frac{\partial^4 N_{sw}}{\partial \phi^4} > 0$ and the third degree derivative is an increasing function of ϕ . Going backwards to the third degree derivative, we have:

$$\frac{\partial^3 N_{sw}}{\partial \phi^3} = 6(-16z^2 + 192z^3 + 640z^4 + 512z^5 + 256z^6) + 24(-16z^2 - 16z^3 + 32z^4)\phi.$$

Its minimum is:

$$\frac{\partial^3 N_{sw}}{\partial \phi^3} = 6(-16z^2 + 192z^3 + 640z^4 + 512z^5 + 256z^6),$$

which is positive. Since the minimum is a positive number and the third degree derivative is increasing in ϕ , $\frac{\partial^3 N_{sw}}{\partial \phi^3} > 0$, meaning that the second degree derivative increases in ϕ . The latter is:

$$\begin{aligned} \frac{\partial^2 N_{sw}}{\partial \phi^2} = & 2(4 + 28z + 24z^2 - 80z^3 - 1168z^4 - 3824z^5 - 4640z^6 - 2880z^7 \\ & - 640z^8) + 6(-16z^2 + 192z^3 + 640z^4 + 512z^5 + 256z^6)\phi \\ & + 12(-16z^2 - 16z^3 + 32z^4)\phi^2. \end{aligned}$$

The minimum of the aforementioned derivative is:

$$\begin{aligned} \frac{\partial^2 N_{sw}}{\partial \phi^2}(\phi = 0) = & 2(4 + 28z + 24z^2 - 80z^3 - 1168z^4 - 3824z^5 - 4640z^6 - 2880z^7 \\ & - 640z^8), \end{aligned}$$

which is negative. The second derivative is a trinomial with a positive discriminant, meaning that it has two real solutions¹³. The fact that the α and γ coefficients have opposite signs, show that the roots of the equation $\frac{\partial^2 N_{sw}}{\partial \phi^2} = 0$, have opposite signs as well. The positive root of the equation is:

¹³ The discriminant is: $\Delta = 3072(2z^2 + 16z^3 + 25z^4 - 128z^5 - 456z^6 + 272z^7 + 3944z^8 + 8896z^9 + 9792z^{10} + 5632z^{11} + 1408z^{12})$

$$\phi_1^{sw} = \frac{-3z^2(1+2z)(-1+2z(7+6z+4z^2)) + \sqrt{3}\sqrt{z^2(1+2z)^4(2+z^2(-23+8z(-1+z(16+11z(2+z))))}}{12(-1+z)z^2(1+2z)}.$$

Due to its complexity, we use the *Reduce* command of Mathematica software in order to identify whether ϕ_1^{sw} is greater than $\min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}$. It turns out that it is greater and therefore, the second derivative remains positive $\forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}]$. Thus, the first derivative decreases in ϕ :

$$\begin{aligned} \frac{\partial N_{sw}}{\partial \phi} = & 4 + 24z + 8z^2 - 200z^3 - 1168z^4 - 2592z^5 - 2112z^6 - 768z^7 + 2(4 \\ & + 28z + 24z^2 - 80z^3 - 1168z^4 - 3824z^5 - 4640z^6 - 2880z^7 \\ & - 640z^8)\phi + 3(-16z^2 + 192z^3 + 640z^4 + 512z^5 + 256z^6)\phi^2 \\ & + 4(-16z^2 - 16z^3 + 32z^4)\phi^3. \end{aligned}$$

The maximum point of the aforementioned function is:

$$\frac{\partial N_{sw}}{\partial \phi}(\phi = 0) = 4 + 24z + 8z^2 - 200z^3 - 1168z^4 - 2592z^5 - 2112z^6 - 768z^7,$$

which negative. Thus, $\forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}]$ the whole derivative is negative. As a result N_{sw} decreases in ϕ . The latter is:

$$\begin{aligned} N_{sw} = & 1 + 5z - 2z^2 - 52z^3 - 232z^4 - 384z^5 - 128z^6 + (4 + 24z + 8z^2 - \\ & 200z^3 - 1168z^4 - 2592z^5 - 2112z^6 - 768z^7)\phi + (4 + 28z + 24z^2 - 80z^3 - \\ & 1168z^4 - 3824z^5 - 4640z^6 - 2880z^7 - 640z^8)\phi^2 + (-16z^2 + 192z^3 + \\ & 640z^4 + 512z^5 + 256z^6)\phi^3 + (-16z^2 - 16z^3 + 32z^4)\phi^4. \end{aligned}$$

The maximum point is:

$$N_{sw}(\phi = 0) = 1 + 5z - 2z^2 - 52z^3 - 232z^4 - 384z^5 - 128z^6,$$

which is negative, and since it is the maximum, the whole function is negative. Thus, $\forall \phi \in [0, \min\{\bar{\phi}_{tE}, \bar{\phi}_{eO}\}], sw_{dif} < 0 \Rightarrow sw^O < sw^E$.