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ΠΑΝΕΠΙΣΤΗΜΙΟ ΜΑΚΕΔΟΝΙΑΣ

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Replication of Rothman 2021

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Abstract

In this study we revisit a significant work conducted by Rothman in 1998. With a view to finding the most suitable model for the interpretation of the asymmetric unemployment rates, various nonlinear ARMA models were introduced estimated and examined for their ability to account for this asymmetric behaviour plus forecast its future values. We aim on replicating the estimations, with data of the U.S. unemployment rates corresponding to the same as well as an extended time period, for the purpose of finding out how the models vary when we add additional and more modern observations. Data of the Greek unemployment rates is also used, demonstrating how these models look for this particular case and if there is a difference with the first one. Graphs showing the course of the coefficients values and t-statistics over time are also included. The models used are: Autoregressive (AR), Self-Exciting autoregressive (SETAR), Exponential smooth transition Autoregressive (ESTAR), Exponential Autoregressive (EAR), Generalized Autoregressive (GAR) and the Bilinear. We begin with the replication of the models using the same dataset, of the US quarterly unemployment rates of the period 1949:1 - 1979:4. The results indicate small deviations mainly on the thresholds of the two Threshold autoregressive models (TAR). Afterwards we expand the dataset to 2008:2 and repeat the estimations for the same models, this time finding only two models to be significant. Out of sample forecasts are then performed and interesting results are found for the HP-Filtered data. Lastly, we repeat the same analysis using Greek unemployment rates for the time period of 2001:1 - 2021:4 and conclude with similar results for the two countries, with the ESTAR model proving to be the best expansion of the Autoregressive model for both cases, even though it still needs a lot of improvement.

Keywords: Unemployment, Nonlinearity, Forecast, Nonlinear ARMA models, Model selection.

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1. Introduction

Unemployment has many terminologies and ways of measurement, though according to OECD it is the amount or percentage of people of working age who are eligible but not currently working and are actively searching for one. It has always been one of the main, if not the main focus of every government and direct evidence of this constitutes the fact that it is comprehended in every politician's speech. It can be seen as an index of how well an economy uses her main resource, the labour and as a result of the general economic activity. It is of great importance as it affects everyone individually but also the economy as a whole. That being the case, the measuring and interpreting of the unemployment rates has become the center of attention for many researchers. Special attention is given to its behaviour during the business cycles.

But what is so special about the unemployment rates, that makes it so difficult to be forecasted? The answer lies in its asymmetric and as a result nonlinear behaviour which makes the linear models unable to predict efficiently its future values. According to Neftçi (1984, p. 309) «*if a nonlinear prediction problem is treated (by mistake) as a linear one, then the estimate of the "unpredictable components" of a time series would contain too much information*».

The paper is organized as follows: Section 2 reports the advances of the topic offered by this paper; Section 3 details the peculiarities in the behaviour of the unemployment rates found in the empirical literature; Section 4 presents the datasets used and the modification that was needed; Section 5 offers an adequate analysis of the models used; Section 6 provides the empirical results both for the estimated models and the forecasts; Section 7 concludes; Section 8 contains the Appendices.

2. Contributions

The whole analysis of this paper is based on the work of Rothman (1998). Its contribution to the existing literature is double.

Firstly, we report how the models presented in the original word correspond to the more modern period by introducing the estimations for more modern as well as increased amount of observations. Furthermore we perform out of sample forecasts which we then examine to find how accurately each model can predict the future values of unemployment.

Secondly, using data from the Greek economy, we now show how the models correspond to the Greek case but also compare the results to that of the U.S. economy. Additionally we present how the models in both cases evolve over time.

3. Literature Review

Keynes (1936) was the first to mention asymmetries and cyclical movements on economic variables. Referring to the business cycles, which he called “trade cycles”, he reported that although the switch from an upwards tendency to a downwards happens swiftly and explosively, no such a movement happens for the switch from a downwards to an upwards tendency. The analysis was proceeded by the work of Burns and Mitchell (1946) which added the perception that neither the duration nor the absolute value of the slope is equal for the expansions and depressions during the business cycle. Neftçi (1984) later made an in depth analysis and confirmed the existence of asymmetries in the behaviour of various economic time series and especially on employment. Using the statistical theory of finite-state Markov processes, he was able to confirm that the duration of expansions, when unemployment drops, is longer than the recessions, when unemployment rises. In the bibliography there are reports of 3 types of asymmetry in the behaviour of the time series, each with different properties.

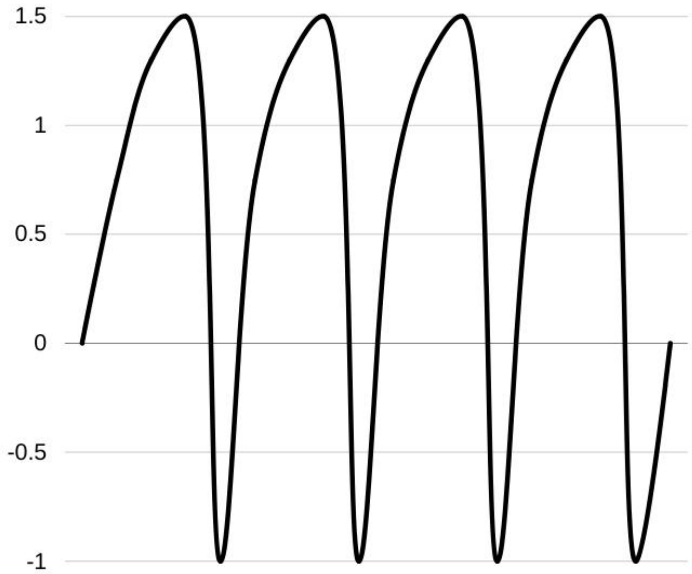
3.1. Types of asymmetry

There are 3 types of asymmetries in the behaviour of the unemployment. We are not going to go into many details, as an extensive analysis can be found in Rothman and Ramsey (1996). We make only a brief summary of each one.

3.1.1. Steepness

Reported originally by Burns and Mitchell (1946), “Steepness” refers to the asymmetry in the duration of the expansions and depressions during the business cycles. In particular periods of expansions, when employment rises last longer than those of depressions, when employment falls. Figure 3.1 depicts the phenomenon, with the positive slope being lower than the violent negative slope.

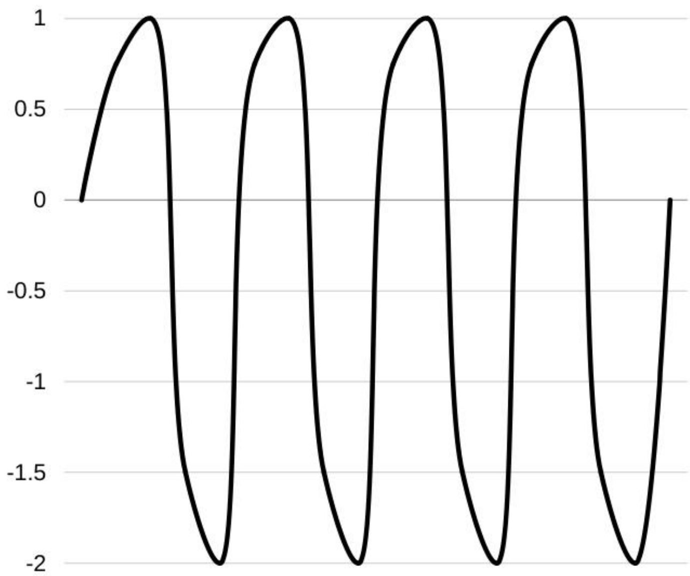
Figure 3.1 Time series plot with differences in steepness



3.1.2. Deepness

“Deepness” refers to the phenomenon in which the distance between the peaks and the mean is significantly different from the distance between the troughs and mean. Regarding the employment, we would expect the troughs to be deeper than the peaks. This can be seen in Figure 3.2 in which we have deeper troughs, with an absolute value of 2, than peaks, with a value of 1.

Figure 3.2 Time series plot with differences in deepness

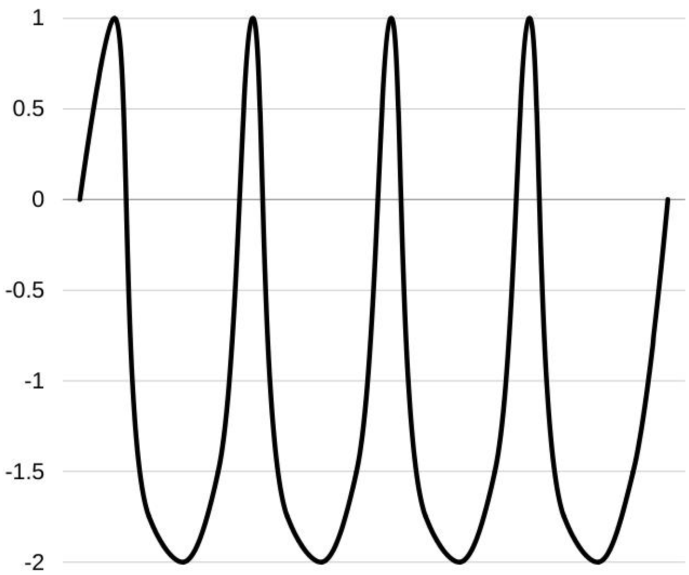


3.1.3. Sharpness

The last type was identified by McQueen and Thorley (1993) and given the definition of “Sharpness”. Specifically this type of asymmetry occurs when the time series has significant differences in the curvature of the peaks and troughs. When examining the employment, we would expect to find out more smooth curves during peaks and more sharp turns during troughs, a scenario well illustrated in Figure 3.3.

Neftçi (1984) mentioned «if asymmetric behavior is indeed systematic then one needs to develop theoretical models that can generate such behavior endogenously» as well as «if there is some evidence that nonlinear ARMA models produce predictions with lower mean squared errors due to asymmetry in economic time series, then this would imply some caution in the way asymptotic theory is being used.». This was the basis for the work of Rothman (1998)

Figure 3.3 Time series plot with differences in sharpness



4. Data

The U.S. dataset consists of 295 observations of U.S. quarterly unemployment rates, ranging from the first quarter of 1949 to the third quarter of 2021 (1949:1 - 2021:3). The data source is FRED - Federal Reserve Bank of ST. Louis. The Greek dataset consists of 84 observations, ranging from the first quarter of 2001 to the last quarter of 2021, with the source of the data being ELSTAT - Hellenic Statistical Authority. The stationarity of the data is a requirement for its usage, thence we continue by checking whether this hypothesis is true.

4.1. *Stationarity test*

At first glance, it is clear from Figure 4.1 that the U.S. unemployment rate is not stationary as it contains a trend. However, in order for this to be confirmed we will use the two tests of stationarity that were used in the original work (Dickey-Fuller, Kwiatkowski–Phillips–Schmidt–Shin (KPSS)) as well as two additional ones (Phillips-Perron, Elliott-Rothenberg-Stock (ADF-GLS)).

Table 4.1 Tests of stationarity in quarterly unemployment rates, 1949:1 -2021:3

Test	Test Statistic	Level of significance	Critical value
Dickey-Fuller	-3.80	1% level	-3.45
		5% level	-2.87
		10% level	-2.57
KPSS	0.33	1% level	0.73
		5% level	0.46
		10% level	0.34
Phillips-Perron	-3.94	1% level	-3.45
		5% level	-2.87
		10% level	-2.57
ADF-GLS	2.22	1% level	1.94
		5% level	3.21
		10% level	4.40

With the Dickey-Fuller test statistic being -3.8 we cannot reject the null hypothesis of non stationarity due to a unit root for significance levels of 10%, 5% and 1%, similarly the Phillips-Perron and ADF-GLS test show the same result, since with critical values of -3.94 and 2.22 respectively we cannot reject the same hypothesis as the latter test for any significance level. The KPSS test, which utilizes the opposite null hypothesis, of stationary data, confirms our findings as it has a test statistic of 0.33. Therefore we identify our time series as non stationary, for every level of statistical significance (1%, 5%, 10%).

4.2. *Detrendation*

For the detrendation of our data we will use the log linear method, as it was suggested by Rothman (1998). To begin with, we create the logarithm of the unemployment rates and then we use them as a dependent variable on an OLS regression with the independent variables being the constant and trend. Following this we save the residuals as our log linear detrended unemployment rates. Once more, we use the same tests, in order to confirm the stationarity of our variables.

Table 4.2 Tests of stationarity in quarterly log-linear detrended US unemployment rates, 1949:1 - 2021:3

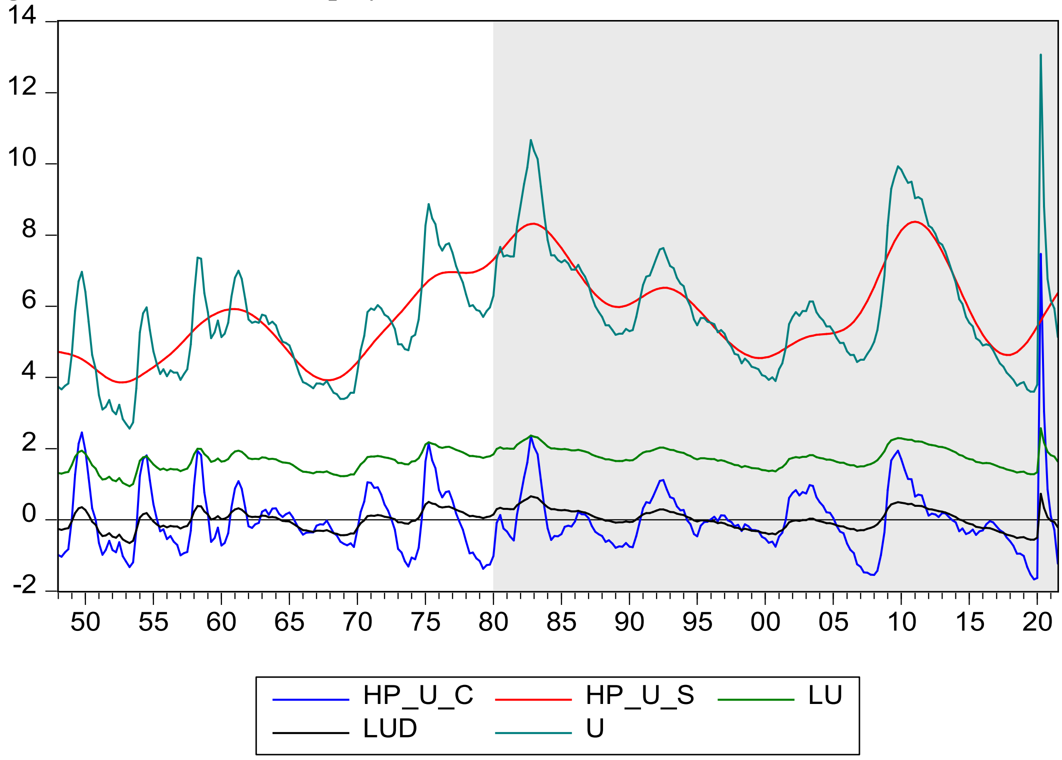
Test	Test Statistic	Level of significance	Critical value
Dickey-Fuller	-3.96	1% level	-2.57
		5% level	-1.94
		10% level	-1.61
KPSS	0.15	1% level	0.73
		5% level	0.46
		10% level	0.34
Phillips-Perron	-3.79	1% level	-2.57
		5% level	-1.94
		10% level	-1.61
ADF-GLS	1.40	1% level	1.94
		5% level	3.21
		10% level	4.40

Although we stayed loyal to the methodology of the original work and used the linear detrendation, the logarithmic unemployment rates were also tested for stationarity. The results are given in the Appendix 1.

With the Dickey-Fuller test statistic now being -3.97 we reject the null hypothesis of unit root and in the same manner we reject the same null hypothesis for the Phillips-Perron and ADF-GLS tests with test statistics of -3.79 and 1.40 respectively. With LM value of 0.15 we cannot reject the null hypothesis of stationary data of the KPSS test. Consequently we have achieved stationarity on our variable for every level of statistical significance (1%, 5%, 10%).

Undoubtedly, the log-linear detrended unemployment rate is the correct form of our non stationary data, as all of the tests above and Figure 4.1 indicate. We follow the same detrendation method for the Greek unemployment rates, presenting the results and the graph in Appendix 1.

Figure 4.1 Time series plot for U.S. unemployment rates, log unemployment rates, log-linear detrended unemployment rates, 1949:1 - 2021:3



Note: U - Standard unemployment rates, LU - Logarithm of the unemployment rates, LUD - Log linear detrended unemployment rates, HP_U_C - HP filtered unemployment rates - cycle series, HP_U_S - HP filtered unemployment rates - smoothed series

5. Methodology

The methodology used for the estimation of most of the models is the nonlinear least squares, with the only exceptions being the TAR models, which have a unique estimation method. The subsection below, provides further information on our models.

5.1. Models

We continue with an introductory analysis of the models that are being utilized for the interpenetration of the unemployment rates. The books that had a significant role for the following analysis are Tong (1990), Priestley (1988), Enders (2014). Subsection 5.1.1. refers to the pure autoregressive model, subsections 5.1.2. and 5.1.3. to the two TAR models, subsections 5.1.4. and 5.1.5. to the GAR and EAR models respectively and lastly subsection 5.1.6. refers to the special case of the Bilinear model.

5.1.1. Autoregressive (AR)

The standard Autoregressive (AR) model, estimates the dependent variable's behaviour based linearly on its lags, a constant and an error term.

$$(1 - \alpha_1 L - \alpha_2 L^2 - \ldots - \alpha_n L^n) y_t = \alpha_0 + \varepsilon_t$$

Where L is the lag operator

$$y_t = \alpha_0 + \sum_{i=1}^{n} a_i y_{t-i} + \varepsilon_t$$

5.1.2. Self-Exciting Threshold Autoregressive (SETAR)

The Self-Exciting Threshold Autoregressive (SETAR) is a regime switching model, which belongs to the family of the Threshold autoregressive TAR models, developed by Howell Tong. The behaviour of the dependent variable y_t is affected by different autoregressive processes that depend on the values of the threshold variable $y_{t-\theta}$, where θ is the lag of the dependent variable that acts as the threshold. Through this regime switching, the model achieves a higher degree of flexibility in its parameters, as their values differ based on the current regime. Each of the autoregressive processes may be linear, however the regime switching ability makes the model display nonlinear behaviour.

$$y_t = \begin{cases} \alpha_0 + \sum_{i=1}^{n} a_i y_{t-i} + \varepsilon_{1t} & if\ y_{t-\theta} \geq \gamma \\ \beta_0 + \sum_{i=1}^{n} \beta_i y_{t-i} + \varepsilon_{2t} & if\ y_{t-\theta} < \gamma \end{cases}$$

Where θ is the delay, γ is the threshold

The regime switching is mostly triggered by shocks on the stochastic terms ε_{1t},ε_{2t} which are the cause for the $y_{t-\theta}$ to fall below/grow above the threshold γ. Thus the bigger (lower) the variance of the stochastic terms, the more (less) times the regime will switch.

This model serves well the interpretation of the unemployment, as it allows for its persistence to differ based on the current regime, making it competent of mimicking its asymmetric behaviour during the business cycles.

5.1.3. Exponential Smooth Transition Autoregressive (ESTAR)

The Exponential Smooth Transition Autoregressive (ESTAR) model is another threshold autoregressive model that achieves a higher degree of flexibility in its parameters, similar to SETAR, by changing based on the regime, however instead of a sharp shift of regimes, it allows for a slow, smooth one. It is used when the hypothesis of the sharp transition appears to be false. The first row of the equation constitutes the linear part, where the second row constitutes the nonlinear one. The parameter γ represents the smoothness of the transition.

$$y_t = [\alpha_0 + \sum_{i=1}^{n} a_i y_{t-i}\ ] +$$
$$[1 - exp(-\gamma(y_{t-\theta} - c)^2)][\beta_0 + \sum_{j=1}^{n} \beta_j y_{t-j}] + \varepsilon_t$$

Where θ, is the delay, γ the slope, c the threshold

As $y_{t-\theta}$ gets further from c, the term $[1 - exp(-\gamma(y_{t-\theta} - c)^2)]$ approaches 1 and the second part becomes linear as well, so the model consists of a combination of the two autoregressive processes:

$$y_t = [(\alpha_0 + \beta_0) + \sum_{i=1}^{n} (a_i + \beta_i) y_{t-i}\ ]$$

As $y_{t-\theta}$ approaches c, the term $[1 - exp(-\gamma(y_{t-\theta} - c)^2)]$ approaches 0 and the model becomes symmetric and once again linear, a standard Autoregressive AR(n) process with n lags of the dependent variable:

$$y_t = [\alpha_0 + \sum_{i=1}^{n} a_i y_{t-i}\ ]$$

The above is also true when the smoothness parameter γ approaches zero or infinity.
In any other case, the model is nonlinear.

5.1.4. *Exponential Autoregressive (EAR)*

The Exponential Autoregressive (EAR) model is a nonlinear autoregressive process with dynamics comparable to those of the previous ESTAR model. The behaviour of the dependent variable y_t is affected by smooth functions of its past values. In the same way, it can too interpret efficiently the unemployment rates, as it can account well for its asymmetries.

$$y_t = \alpha_0 + \sum_{i=1}^{n} \alpha_i exp[f(y_{t-i})]\, y_{t-i} + \varepsilon_t$$

5.1.5. *Generalized Autoregressive (GAR)*

The Generalized Autoregressive (GAR) model constitutes a more generalized form of the nonlinear autoregressive models. Based on the Taylor series, it attempts to account for a wide variety of functional forms with the introduction of various powers and cross-products of interpreted variable's lags in the autoregressive process.

$$y_t = \alpha_0 + \sum_{i=1}^{n} a_i y_{t-i} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{r} \beta_{ijkl} {y^k}_{t-i}\, {y^l}_{t-j} + \varepsilon_t$$

5.1.6. *Bilinear*

The Bilinear models are an extension of the ARMA models and similar to the GAR models, uses various lags of the error term. It includes cross-products of the autoregressive and moving average processes to the pure ARMA model.

$$y_t = \alpha_0 + \sum_{i=1}^{n} a_i y_{t-i} + \sum_{j=0}^{m} \beta_j \varepsilon_{t-j} + \sum_{i=1}^{k}\sum_{j=1}^{l} \gamma_{ij} y_{t-i} \varepsilon_{t-j}$$

Where $\beta_0 = 1$

Furthermore, the model can be viewed as an ARMA model where the AR coefficients are also affected by random shocks. The expected value of the AR coefficient might be a_i however random shocks will have a big impact on its value. Assuming that the random error term and lag have a positive and significant correlation, big shocks will decrease the AR coefficient, given that γ_{ij} is negative, making them less persistent than small ones. A scenario that describes greatly the asymmetric behaviour of the unemployment rates. This applies the same for both positive and negative shocks.

$$y_t = \alpha_0 + \sum_{i=1}^{k} \sum_{j=1}^{l} (a_i + \gamma_{ij}\varepsilon_{t-j}) y_{t-i} + \sum_{j=1}^{m} \beta_j \varepsilon_{t-j} + \varepsilon_t$$

Now that we have a better understanding of the models as well as the reason for their use, we are able to go on to their estimation and explain any possible deviation.

6. Empirical Results

This section is divided in three subsections, each one appertaining to a different timeline or country and a fourth one that contains graphs showing the evolution over time of our models. Subsection 6.1 uses observations of the U.S. unemployment rates for the same years as Rothman. Hence we expect the results to be similar to some extent, with the small deviations being the product of differences in the estimation methods. Subsection 6.2 extends the timeline by including more contemporary observations of the unemployment rates and presents how the models deviate. Subsection 6.3 presents the estimated models for all the available data on unemployment rates for Greece. Subsection 6.4 contains the graphs that give us an overview of how the models change thorough the examined time period.

6.1. Identical Timeline

In this part, we present the estimated models, using the same timeline as the original work of Rothman (1998), starting from the first quarter of 1949 up until the last quarter of 1979, in order to look for any major differences. Looking at tables 6.1 and 6.2 we find the most noticeable difference to be in the selection of the threshold's delay in the respective models in which is being used.

Starting from the top, we notice that the pure autoregressive AR model has remained practically the same, not only since the Akaike information criterion AIC keeps yielding the lowest value for the same lags, but also because the values of the coefficients seem untouched. This result was to be expected since the AR is not a very intricate model and thus the estimates should not vary much.

Following the original methodology, we inspect the residuals of the linear model for any significant deviation from the white noise process. As reported by M.B. Priestley (1988) «If the residuals from the regression model were not strict white noise, there would be further structure in the relationship between x and y which we had not "explained" by the model.».

Figure 6.1 shows that the autocorrelations and partial autocorrelations for all lags are close to zero and additionally all the Q-statistics are insignificant. Thereby, we can safely assume that there is no major structure left unexplained by the model. Nevertheless, we will still examine ways and means that may have a reduced residual standard deviation.

Figure 6.1 Residual Correlogram and Q-statistics for the same timeline, 1949:1 - 1979:1

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.063	0.063	0.5132	0.474
		2	-0.002	-0.006	0.5138	0.773
		3	0.027	0.028	0.6108	0.894
		4	-0.125	-0.129	2.6714	0.614
		5	-0.048	-0.032	2.9760	0.704
		6	0.109	0.115	4.5870	0.598
		7	0.117	0.114	6.4413	0.489
		8	-0.099	-0.133	7.7809	0.455
		9	0.158	0.162	11.227	0.260
		10	0.071	0.075	11.918	0.291
		11	0.022	0.059	11.986	0.365
		12	-0.139	-0.212	14.731	0.256
		13	0.095	0.147	16.022	0.248
		14	-0.152	-0.153	19.365	0.151
		15	-0.080	-0.033	20.293	0.161
		16	0.113	-0.007	22.173	0.138
		17	0.069	0.152	22.883	0.153
		18	0.109	0.065	24.649	0.135
		19	-0.008	-0.057	24.660	0.172
		20	0.006	-0.042	24.666	0.215
		21	-0.197	-0.048	30.620	0.080
		22	-0.008	-0.013	30.630	0.104
		23	-0.019	-0.042	30.687	0.131
		24	-0.094	-0.129	32.090	0.125
		25	-0.114	-0.099	34.160	0.105
		26	0.045	0.019	34.493	0.123
		27	-0.027	-0.052	34.612	0.149
		28	-0.061	-0.051	35.224	0.163
		29	0.033	-0.030	35.401	0.192
		30	-0.184	-0.085	41.071	0.086
		31	-0.131	-0.090	43.985	0.061
		32	-0.102	-0.072	45.785	0.054
		33	-0.044	-0.079	46.128	0.064
		34	-0.070	-0.027	46.999	0.068
		35	0.055	-0.016	47.526	0.077
		36	0.028	-0.019	47.671	0.092

When it comes to the Self-Exciting Autoregressive SETAR model, there are major differences between the two tables. Rothman (1998) mentions that according to the AIC, the model selected is the one which uses the first lag of the dependent variable as the threshold variable. On the contrary, our replication using identical timeline provide us with different results. This time the Akaike information criterion suggests that the most fitting variable for regime switch is the second lag. Moreover, the threshold value we find is negative, close to -0.3 indicating that the regime switches with the fall of unemployment, opposed to the original findings where the regime switches for positive values of unemployment, consequently when it rises. The first regime which corresponds to the swift fall of unemployment is more persistent, therefore the effects of the shocks last longer than the ones of the second regime. The ratio $s^2/s^2{}_L$ remains the same, hence its interpretive ability has not changed.

Examining another Threshold Autoregressive model, we notice once again a change in the delay of the threshold used. Reported in the third row for both tables, the ESTAR model presented in the original work shows that the AIC selected delay equals to one, whereas it equals to two when it comes to our replicated results. It should also be noted that our analysis only applies for a confidence level of 90%. For a significance level of 5% we cannot reject the null hypothesis of statistical insignificance of the slope, meaning that it equals to zero and as we have already mentioned in the methodology section that makes the whole nonlinear part to be equal to zero as well. Admittedly only for a significance level of 10% may we have a significant nonlinear part and not just a standard second level autoregressive process. In addition, the second lag of the linear part proves to be statistically insignificant. Also note that the $s^2/s^2{}_L$ is smaller in contrast to the original results, which means that the model was underrated.

Following this, we take a look at the Exponential Autoregressive EAR model, in which once again we reject the statistical significance of the constant. Besides this, the coefficients are very similar with the only minor difference being on the augmented first lag's value. It should also be noted that $s^2/s^2{}_L$ exceeds the value of 1, as a result of the residual standard deviation of EAR model exceeding the one of the autoregressive.

Additionally, the results for the Generalized Autoregressive GAR are extremely identical both for Rothman (1998) and our work. The deviations of the coefficients are so minimal that we can characterize them as insignificant. In terms of the residual standard deviation ratio, we notice a minor augmentation.

Last but not least, we have the unique case of the Bilinear model. Particularly our method of Non-Linear Least Squares fails the task of replication and even though the coefficient's values of the AR process are practically the same, the unique variable that is being added is characterized as statistically insignificant. The main reason for this phenomenon should be the use of the wrong estimation methodology. Despite not analyzing further the problem of estimating Bilinear models, we suggest the works of Subba Rao and Gabr (1984), Tong (1990), Priestley (1981), for a detailed analysis.

It should be noted that the SETAR and ESTAR models were also estimated with the threshold variable set as the first lag of the dependent, as it was originally proposed by Rothman. Since the results were inferior, with many important variables not being statistically significant, we present them in Appendix 2

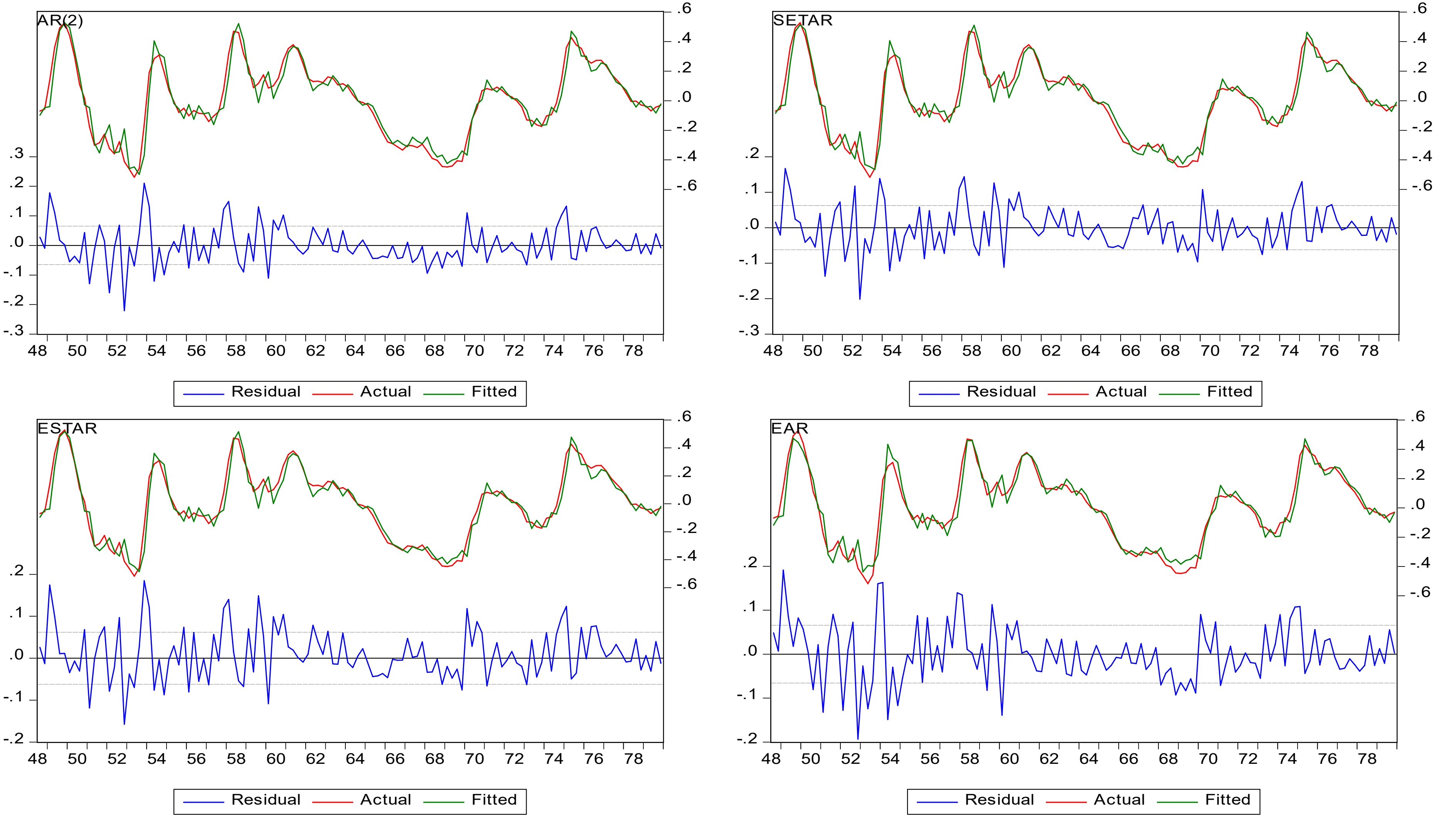
Table 6.1 Estimated models of the original work for quarterly log-linear detrended US unemployment rates, 1949:1-1979:4

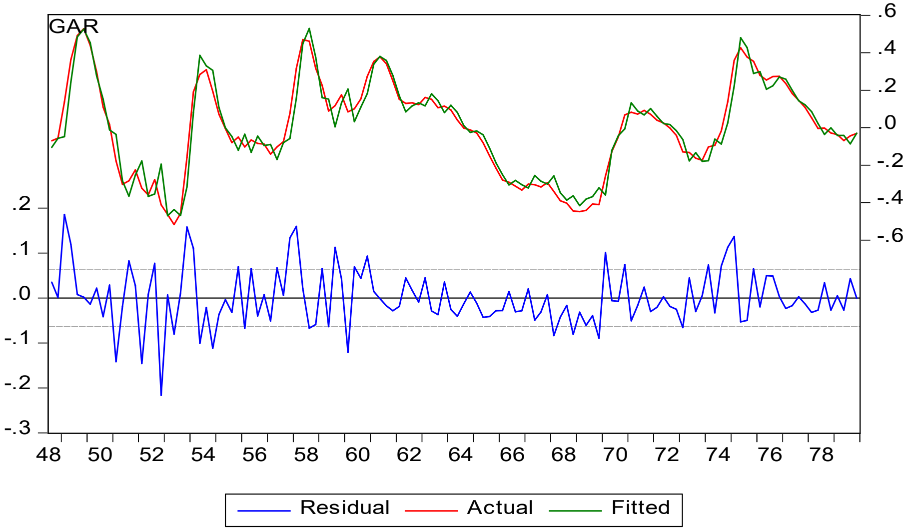
Rothman's Results (1949:1-1979:4)		
Model	Estimated Model	$s^2/s^2{}_L$
AR(2)	$U_t = 1.563 \cdot U_{t-1} - 0.670 \cdot U_{t-2} + \hat{\varepsilon}_t$ (22.46) (-10.06)	
SETAR	$U_t = 0.0529 + 1.349 \cdot U_{t-1} - 0.665 \cdot U_{t-2} + \hat{\varepsilon}_{1t}$ if $U_{t-1} \geq 0.062$ (3.46) (16.03) (-9.37)	0.942
	$U_t = 1.646 \cdot U_{t-1} - 0.733 \cdot U_{t-2} + \hat{\varepsilon}_{2t}$ if $U_{t-1} < 0.062$ (14.27) (-6.37)	
ESTAR	$U_t = 0.325 \cdot U_{t-1} - 1.771 \cdot U_{t-2} + (1.219 \cdot U_{t-1} + 1.124 \cdot U_{t-2}) \times [1 - exp(10.230 \cdot (-200) \cdot U^2{}_{t-1})] + \hat{\varepsilon}_t$ (2.64) (-3.97) (2.34) (2.51) (83.10)	0.953
EAR	$U_t = 0.937 + 0.729 \cdot exp(-U^2{}_{t-1}) - 0.680 \cdot U_{t-2} + \hat{\varepsilon}_t$ (2.89) (1.97) (-10.30)	0.977
GAR	$U_t = 1.500 \cdot U_{t-1} - 0.553 \cdot U_{t-2} - 0.745 \cdot U^3{}_{t-2} + \hat{\varepsilon}_t$ (23.60) (-6.72) (-2.33)	0.965
BILINEAR	$U_t = 1.591 \cdot U_{t-1} - 0.690 \cdot U_{t-2} - 0.585 \cdot U_{t-1} \cdot \hat{\varepsilon}_{t-3} + \hat{\varepsilon}_t$ (24.11) (-10.55) (-2.08)	0.936

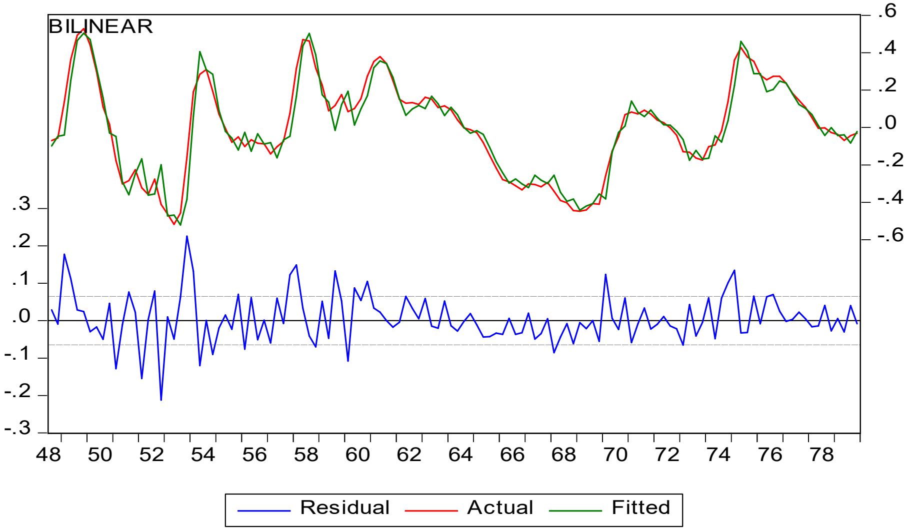
Table 6.2 Estimated models of the replication for quarterly log-linear detrended US unemployment rates, 1949:1-1979:4

Replicated Results (1949:1-1979:4)		
Model	Estimated Model	$s^2/s^2{}_L$
AR(2)	$U_t = 1.562 \cdot U_{t-1} - 0.670 \cdot U_{t-2} + \hat{\varepsilon}_t$ (23.46) (-10.06)	
SETAR	$U_t = 0.008 + 1.521 \cdot U_{t-1} - 0.672 \cdot U_{t-2} + \hat{\varepsilon}_{1t}$ if $U_{t-2} \geq -0.276$ (1.25) (22.11) (-9.39)	0.941
	$U_t = -0.234 + 1.630 \cdot U_{t-1} - 1.321 \cdot U_{t-2} + \hat{\varepsilon}_{2t}$ if $U_{t-2} < -0.276$ (-3.26) (9.26) (-6.04)	
ESTAR	$U_t = 0.978 \cdot U_{t-1} + 0.081 \cdot U_{t-2} + (0.640 \cdot U_{t-1} - 0.855 \cdot U_{t-2}) \times [1 - exp(-79.062 \cdot (U_{t-2} + 0.285)^2] + \hat{\varepsilon}_t$ (3.54) (0.27) (2.22) (-2.84) (1.93) (-11.36)	0.926
EAR	$U_t = -0.001 + 1.782 \cdot exp(-U^2{}_{t-1}) - 0.665 \cdot U_{t-2} + \hat{\varepsilon}_t$ (-0.20) (23.14) (-9.89)	1.007
GAR	$U_t = 1.547 \cdot U_{t-1} - 0.544 \cdot U_{t-2} - 0.800 \cdot U^3{}_{t-2} + \hat{\varepsilon}_t$ (23.63) (-6.61) (-2.50)	0.975
BILINEAR	$U_t = 1.565 \cdot U_{t-1} - 0.675 \cdot U_{t-2} - 0.095 \cdot U_{t-1} \cdot \hat{\varepsilon}_{t-3} + \hat{\varepsilon}_t$ (23.60) (-10.17) (-1.45)	0.987

Figure 6.2 Actual, Fitted, Residual graph of the estimated models for the same timeline, 1949:1-1979:4




GAR
Residual
Actual
Fitted


BILINEAR
Residual
Actual
Fitted

6.2. Extended Timeline

Having analyzed how and to what extent our estimated models differ from the ones of the original work, it is time we carry on with our analysis and find how these correspond to the more recent data of the unemployment rates. With the starting point unchanged, we add observations on unemployment up to the second quarter of 2008. Having our dataset larger by 114 observations, we follow the same methodology and examine how the models relate to the more modern data including the graph of each model at the end, with the shaded areas corresponding to the extended period. We also perform out of sample forecasts for 13 years, the same way it was carried out by Rothman (1998).

6.2.1. Estimated models

Starting once again from the top we find the basic AR model to be extremely similar with the previous one and to have its lags barely affected. From the Figure 6.4 we notice that the model before 1980 fails to estimate successfully the depth of the shock, although this problem is solved afterwards.

Proceeding with the residuals analysis, we perceive with the help of Figure 6.3 that many lags have either significant autocorrelation or partial autocorrelation or even both. This leads to the conclusion, that the error term contains significant information, so there is major structure left unexplained by the autoregressive process. Having said that, our main goal now becomes to solve the above problem with the use of nonlinear ARMA models.

Figure 6.3 Residual Correlogram and Q-statistics for the extended timeline, 1949:1 - 2008:2

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.052	0.052	0.6605	0.416
		2	0.013	0.011	0.7035	0.703
		3	0.020	0.019	0.8032	0.849
		4	-0.145	-0.147	5.9656	0.202
		5	-0.076	-0.063	7.4102	0.192
		6	0.089	0.102	9.3861	0.153
		7	0.113	0.117	12.555	0.084
		8	-0.111	-0.151	15.652	0.048
		9	0.137	0.126	20.354	0.016
		10	0.089	0.108	22.347	0.013
		11	0.010	0.047	22.373	0.022
		12	-0.098	-0.167	24.827	0.016
		13	0.078	0.101	26.383	0.015
		14	-0.114	-0.061	29.706	0.008
		15	-0.055	-0.023	30.495	0.010
		16	0.097	-0.005	32.944	0.008
		17	0.114	0.170	36.307	0.004
		18	0.090	0.080	38.408	0.003
		19	0.046	-0.012	38.954	0.004
		20	0.007	-0.060	38.968	0.007
		21	-0.090	0.055	41.138	0.005
		22	0.025	0.043	41.310	0.008
		23	-0.005	-0.043	41.317	0.011
		24	-0.022	-0.066	41.453	0.015
		25	-0.061	-0.021	42.444	0.016
		26	0.095	0.088	44.893	0.012
		27	0.009	-0.040	44.913	0.017
		28	0.017	-0.027	44.997	0.022
		29	0.048	0.037	45.621	0.026
		30	-0.118	-0.066	49.455	0.014
		31	-0.022	-0.001	49.596	0.018
		32	-0.050	-0.054	50.282	0.021
		33	0.005	0.012	50.288	0.027
		34	-0.029	-0.027	50.528	0.034
		35	0.092	0.037	52.926	0.027
		36	0.052	0.023	53.687	0.029

Utilizing the SETAR model with the extended dataset we come to the conclusion that it is inexpedient for the interpretation of the unemployment rates and is not included in the table 6.4, due to the fact that both the first and the second lag as a threshold fail to yield lower AIC value than the pure autoregressive model. This result is opposed to the previous analysis as well as Rothman's findings where it is one of the most significant models.

Moving to the other Threshold Autoregressive model, the estimated ESTAR model has only significant nonlinear part as both of the linear lags cannot be characterized statistically significant. Moreover we find the threshold to be also insignificant although we have a significant slope. So our model will have the form as shown below:

$$U_t = (c_1 \cdot U_{t-1} + c_2 \cdot U_{t-2}) \times [1 - exp(-\gamma \cdot {U_t}^2)] + \hat{\varepsilon}_t$$

The statistical insignificant threshold means that as U_t approaches 0 the term $[1 - exp(-\gamma \cdot {U_t}^2)]$ will also approach 0 so the behaviour of the unemployment rates will be explained solely by the error term $\hat{\varepsilon}_t$. On the other hand, as U_t gets further from 0, the term $[1 - exp(-\gamma \cdot {U_t}^2)]$ will approach 1 and as a result the behaviour of the unemployment rates will be explained by the nonlinear part as well as the error term. In short, the distance between U_t and 0 shows the intensity of the nonlinear autoregressive effect.
The coefficients of the nonlinear part are very similar to those of the standard autoregressive model, so the ESTAR model helps in the explanation of the asymmetric behaviour of the unemployment rates by having a completely random process for low values of the dependent variable and the standard autoregressive process, that slowly restores the equilibrium after a shock has hit, for high values. A behaviour that fits well the unemployment rates. The ratio $s^2/{s^2}_L$ considers this to be the best among all the other models, as it yields the lowest value of 0.914. It should also be noted that when we look at the correlogram of its residuals, we still find some significant correlations and autocorrelations, therefore even though it is the best model, it still fails to leave no structure unexplained in the relationship between our variables. The correlogram can be found in Appendix 3.

Regarding the EAR model, although we find all the variables, except the constant term, being statistically significant, the standard deviation of the residuals is actually greater than the linear model, as $s^2/{s^2}_L$ is higher than 1. This means that it has worse performance. Looking at the Figure 6.4, it is clear that the model assumes the unemployment to have a smoother behaviour than it actually does. It fails to estimate the true effect of the shocks, as we have sizeable residuals at the peaks of the actual data.

Concerning GAR model, we observe that due to the second lag to the power of three being insignificant, it cannot improve our analysis more than what the pure autoregressive does.

Finally, we examine the Bilinear model, even though we have already come to the conclusion that our methodology is not suitable for its estimation. The estimated model confirms the latter as we find the bilinear term to be insignificant, thereby it constitutes a standard autoregressive process. Another evidence of this, is the residual standard deviation ratio that is very close to 1 meaning that the residuals standard deviation of the Bilinear and the Autoregressive models are equivalent. The Bilinear graph in Figure 6.4 is included to show its homogeneity with the autoregressive one.

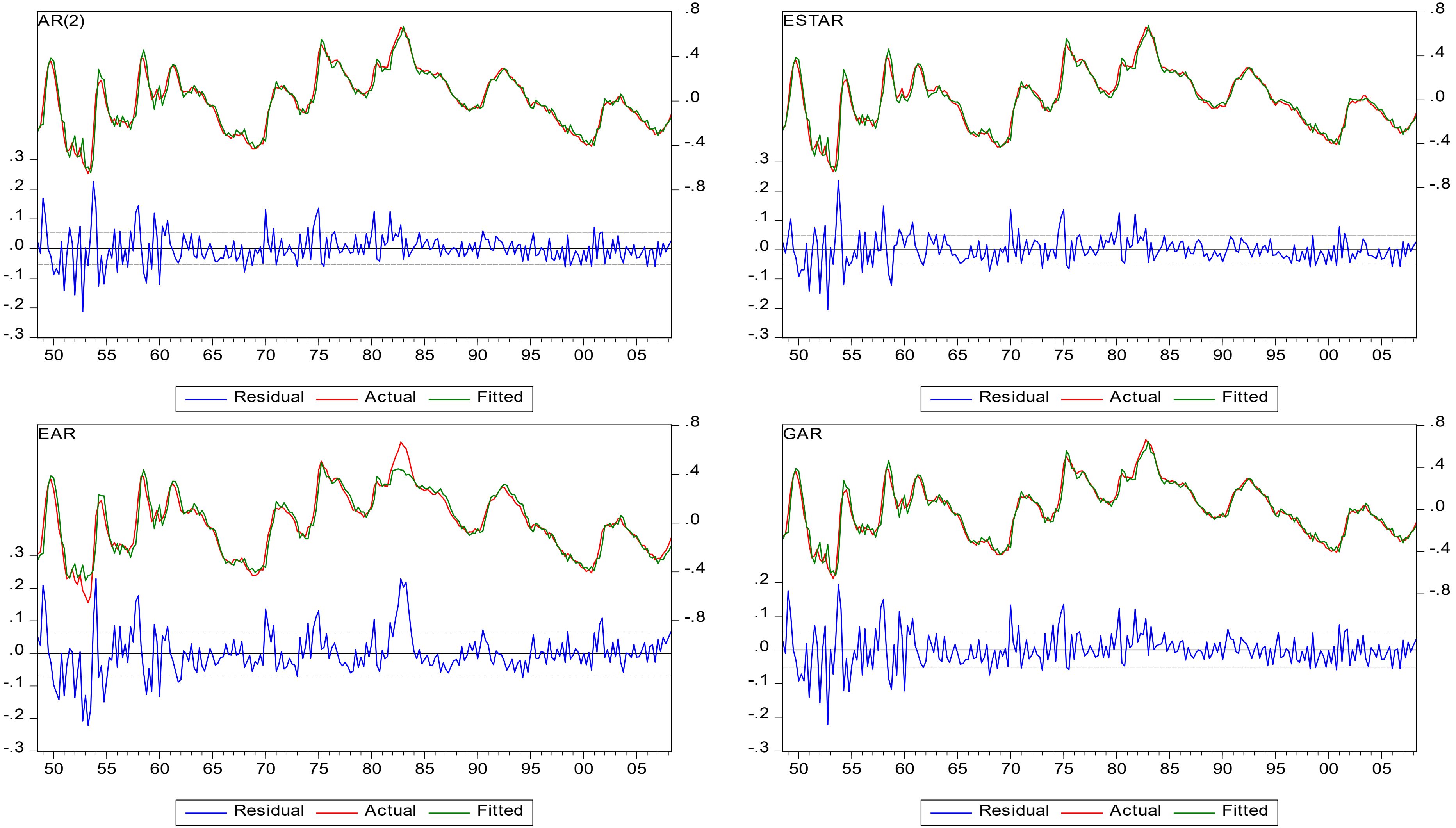
Table 6.3 Estimated models of the original work for quarterly log-linear detrended US unemployment rates, 1949:1-1979:4

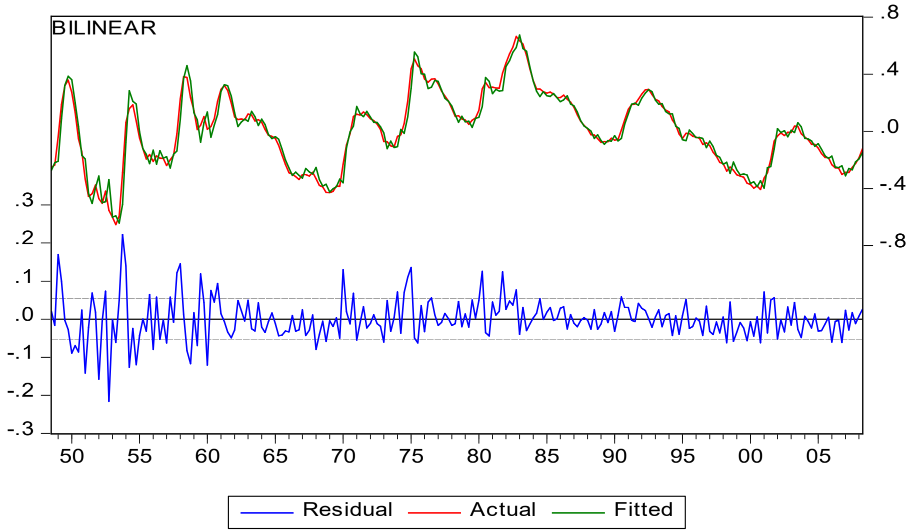
	Rothman's Results (1949:1-1979:4)	
Model	Estimated Model	s^2/s^2_L
AR(2)	$U_t = 1.563 \cdot U_{t-1} - 0.670 \cdot U_{t-2} + \hat{\varepsilon}_t$ (22.46) (-10.06)	
SETAR	$U_t = 0.0529 + 1.349 \cdot U_{t-1} - 0.665 \cdot U_{t-2} + \hat{\varepsilon}_{1t}$ if $U_{t-1} \geq 0.062$ (3.46) (16.03) (-9.37) $U_t = 1.646 \cdot U_{t-1} - 0.733 \cdot U_{t-2} + \hat{\varepsilon}_{2t}$ if $U_{t-1} < 0.062$ (14.27) (-6.37)	0.942
ESTAR	$U_t = 0.325 \cdot U_{t-1} - 1.771 \cdot U_{t-2} + (1.219 \cdot U_{t-1} + 1.124 \cdot U_{t-2}) \times [1 - exp(10.230 \cdot (-200) \cdot U^2{}_{t-1})] + \hat{\varepsilon}_t$ (2.64) (-3.97) (2.34) (2.51) (83.10)	0.953
EAR	$U_t = 0.937 + 0.729 \cdot exp(-U^2{}_{t-1}) - 0.680 \cdot U_{t-2} + \hat{\varepsilon}_t$ (2.89) (1.97) (-10.30)	0.977
GAR	$U_t = 1.500 \cdot U_{t-1} - 0.553 \cdot U_{t-2} - 0.745 \cdot U^3{}_{t-2} + \hat{\varepsilon}_t$ (23.60) (-6.72) (-2.33)	0.965
BILINEAR	$U_t = 1.591 \cdot U_{t-1} - 0.690 \cdot U_{t-2} - 0.585 \cdot U_{t-1} \cdot \hat{\varepsilon}_{t-3} + \hat{\varepsilon}_t$ (24.11) (-10.55) (-2.08)	0.936

Table 6.4 Estimated models of the extended dataset for quarterly log-linear detrended US unemployment rates, 1949:1-2008:2

Extended Timeline Results (1949:1-2008:2)		
Model	Estimated Model	s^2/s^2_L
AR(2)	$U_t = \underset{(31.72)}{1.579} \cdot U_{t-1} - \underset{(-12.89)}{0.641} \cdot U_{t-2} + \hat{\varepsilon}_t$	
ESTAR	$U_t = \underset{(0.23)}{0.078} \cdot U_{t-1} - \underset{(-0.38)}{0.074} \cdot U_{t-2} + (\underset{(4.32)}{1.494} \cdot U_{t-1} - \underset{(-2.60)}{0.546} \cdot U_{t-2}) \times [1 - exp(-\underset{(3.11)}{232.81} \cdot (U_t - \underset{(0.81)}{0.006})^2)] + \hat{\varepsilon}_t$	0.914
EAR	$U_t = \underset{(0.21)}{0.0009} + \underset{(23.91)}{1.625} \cdot exp(-U^2{}_{t-1}) - \underset{(-7.74)}{0.448} \cdot U_{t-2} + \hat{\varepsilon}_t$	1.237
GAR	$U_t = \underset{(31.73)}{1.575} \cdot U_{t-1} - \underset{(-11.20)}{0.605} \cdot U_{t-2} - \underset{(-1.68)}{0.188} \cdot U^3{}_{t-2} + \hat{\varepsilon}_t$	0.994
BILINEAR	$U_t = \underset{(31.66)}{1.579} \cdot U_{t-1} - \underset{(-12.87)}{0.641} \cdot U_{t-2} - \underset{(-0.30)}{0.010} \cdot U_{t-1} \cdot \hat{\varepsilon}_{t-3} + \hat{\varepsilon}_t$	0.999

Figure 6.4 Actual, Fitted, Residual graph of the estimated models for the extended timeline, 1949:1-2008:2




BILINEAR
.8
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.0
-.4
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.3
.2
.1
.0
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60
65
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75
80
85
90
95
00
05
Residual
Actual
Fitted

6.2.2. *Out of sample forecast*

As we have already mentioned, we follow the original work's methodology and perform an out of sample forecast for 13 years, essentially forecasting the period 2008:3 - 2021:3. The forecast is performed on 3 types of data, these being, the log linear detrended unemployment rates which have been already analyzed in the previous sections, the log level unemployment rates which are basically the logarithmic form of our data and the HP-filtered unemployment rates, in other words the smoothed data yielded by the Hodrick-Prescott filter.

The models estimated for the first two new types of data did not vary too much from the ones we already saw for the latter, although they were some particularities, that we are going to briefly present and can be found in more detail in Appendix 4. In both cases the ESTAR model's coefficients could not be estimated and the EAR model for the HP filtered data had lower explanatory ability than the standard autoregressive. Last but not least in both bilinear models, their distinctive variables were found statistically significant. Having said that, we have already shown that there is a problem when trying to estimate the bilinear models using non linear least squares, so one should be extra cautious when examining this particular case.

At this point, we can proceed with the results of the forecasts. They are presented in table 6.5,which is divided in half.
The first half shows the estimated bias, which is the constant error of the forecast, estimated by running a regression with the residuals from the forecast as dependent variable and the constant term as the only independent variable. The value is shown in the first row and below is the t-statistic for the constant, inside the parenthesis. Looking at the table, we can see that no model can achieve unbiased predictions, as all of them have statistically significant prediction errors with large t-statistics. The only exception is the bilinear model using the log level unemployment rates where the prediction error is barely insignificant. All in all, our forecasts present some weak forecasts. The main reason for this should be the extremely unstable economic state that dominated this time period. Specifically in just 15 years the world experienced two major economic crises. The first was the Great recession starting from the first quarter of 2008 and lasting up until the second quarter of 2009, characterized as the most extreme downturn since the great depression, the breakdown of the financial system and the uncertainty that engulfed the economy, resulted in extreme unemployment that lasted for a long time. The COVID-19 outbreak followed, which started from the first quarter of 2020 and its effects last until the present day that is of the fourth quarter of 2021. Even though it began as a public health crisis, it did not take long to be converted to an economic crisis as well. Following the harsh lockdown imposed by most of the governments, the unemployment rates rose sharply.
The other half, presents the ratio of mean squared prediction error (MSPE) of each nonlinear model respectively, divided by the MSPE of the linear autoregressive model. This ratio is similar to the one used when we examined

the estimated models, with their common purpose being to show if these models-extensions can further improve the interpretation of the asymmetric unemployment rates offered by the pure autoregressive model. Most of the models do not offer great improvement. The most interesting results are found in the HP-Filtered data forecasts, where we find great reductions in the MSPE by the GAR and Bilinear models. This can serve as a first indication for their potential use on forecasting, notwithstanding the significance of these results should be confirmed by the P-values for the Mizrach robust forecast comparison statistic, as proposed by Rothman (1998)

Table 6.5 Estimated bias and MSPE ratios for the out of sample forecasts of the unemployment rates, 2008:Q3–2021:Q3

Estimated bias for log-linear detrended unemployment rates forecasts					MSPE ratios for log-linear detrended unemployment rates forecasts				
AR(2)	EAR	GAR	ESTAR	BILIN	AR(2)	EAR	GAR	ESTAR	BILIN
0.05 (105.62)	0.06 (166.87)	0.05 (100.65)	0.002 (4.08E+16)	0.13 (2.41)	1	0.64	0.99	0.98	1
Estimated bias for log-level unemployment rates forecasts					**MSPE ratios for log-level unemployment rates forecasts**				
AR(2)	EAR	GAR	ESTAR	BILIN	AR(2)	EAR	GAR	ESTAR	BILIN
0.05 (105.1)	0.09 (424.85)	0.05 (104.71)	-	7.67 (1.9)	1	0.82	0.98	-	0.96
Estimated bias for HP-filtered unemployment rates forecasts					**MSPE ratios for HP-filtered unemployment rates forecasts**				
AR(2)	EAR	GAR	ESTAR	BILIN	AR(2)	EAR	GAR	ESTAR	BILIN
0.01 (1281.59)	0.15 (1699.55)	0.008 (1020.57)	-	0.009 (117.38)	1	257.54	0.44	-	0.46

6.3. Estimations for Greece

The third part of our empirical results is dedicated on estimating these models using the unemployment rates of the Greek economy, after the log linear detrendation, in order to see whether and to what extent they can be used. With our data ranging from the first quarter of 2001 to the last quarter of 2021, we have in our availability 84 observations. They methodology we will follow and the presentation of the result remain unchanged.

From table 6.6 we observe the autoregressive process to have once again small deviations in relation to the previous subsections. The effects of the lags now become weaker, with smaller values both for the positive and the negative coefficient, having said that they are still statistically significant.

In the following analysis of the residuals we perceive from the figure 6.5 that nearly all autocorrelations and partial autocorrelations are statistically significant and as a result there is significant deviation from a white noise process so there is a structure in the relationship between the log linear unemployment rates and its lag that the autoregressive model fails to explain, thence we will have to examine the other autoregressive extensions to see whether another model can explain every major structure.

Figure 6.5 Residual Correlogram and Q-statistics for the Greek unemployment rates, 2001:1-2021:4

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.057	0.057	0.2760	0.599
		2	-0.321	-0.325	9.1547	0.010
		3	0.094	0.153	9.9250	0.019
		4	0.634	0.581	45.431	0.000
		5	0.073	0.097	45.909	0.000
		6	-0.274	-0.012	52.691	0.000
		7	-0.017	-0.165	52.718	0.000
		8	0.557	0.236	81.634	0.000
		9	0.109	0.051	82.751	0.000
		10	-0.377	-0.200	96.316	0.000
		11	-0.018	0.006	96.346	0.000
		12	0.425	-0.071	114.14	0.000
		13	0.008	-0.120	114.14	0.000
		14	-0.350	-0.055	126.54	0.000
		15	-0.115	-0.147	127.90	0.000
		16	0.348	0.063	140.51	0.000
		17	-0.029	-0.155	140.60	0.000
		18	-0.379	-0.025	156.02	0.000
		19	-0.143	-0.036	158.27	0.000
		20	0.345	0.097	171.51	0.000
		21	-0.054	0.016	171.83	0.000
		22	-0.366	-0.022	187.21	0.000
		23	-0.140	0.008	189.51	0.000
		24	0.307	0.042	200.72	0.000
		25	-0.040	-0.027	200.92	0.000
		26	-0.382	-0.067	218.84	0.000
		27	-0.123	-0.011	220.73	0.000
		28	0.278	-0.049	230.57	0.000
		29	-0.034	-0.063	230.72	0.000
		30	-0.343	-0.016	246.33	0.000
		31	-0.060	0.078	246.81	0.000
		32	0.242	-0.057	254.88	0.000
		33	-0.066	-0.132	255.48	0.000
		34	-0.310	-0.029	269.25	0.000
		35	-0.052	0.003	269.65	0.000
		36	0.207	-0.072	276.09	0.000

The SETAR model is absent from our results, yet again, as no threshold could achieve lower AIC value than the standard autoregressive process.

According to the $s^2/s^2{}_L$ ratio, we find once more the ESTAR model to be the best improvement of the autoregressive model. The form of the model is the same as the one from the extended analysis, with the linear part as well as the threshold being insignificant, another indication that this form is the most efficient when it comes to interpreting more contemporary unemployment rates. That being said, we still find the residuals to have significant difference from the white noise process, which means that there is still a lot of room for improvement. The corresponding correlogram can be found in Appendix 5.

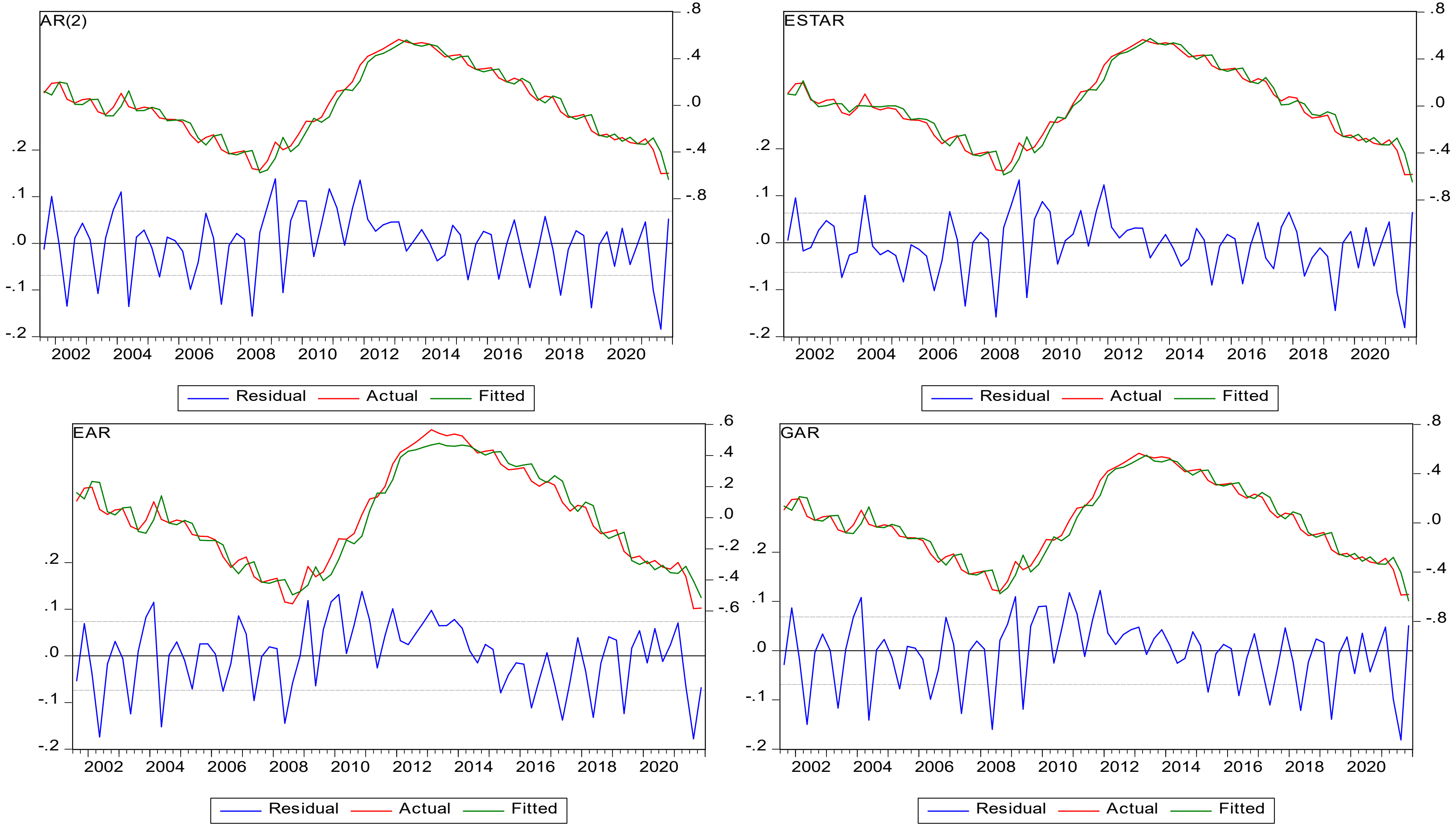
On the other hand, the $s^2/s^2{}_L$ ratio differentiates the EAR model from the others, for not only failing to improve the residual standard deviation of the autoregressive model but also increasing it, proving it to be useless. This result was also seen in the two previous subsections, 6.1 and 6.2 when using the U.S. dataset, although this time the augmentation of the residual standard error is much greater. This could be the result of the second lag now being statistical insignificant. From figure 6.6 it is clear that the model struggles to estimate successfully the actual values, having many significant residuals.

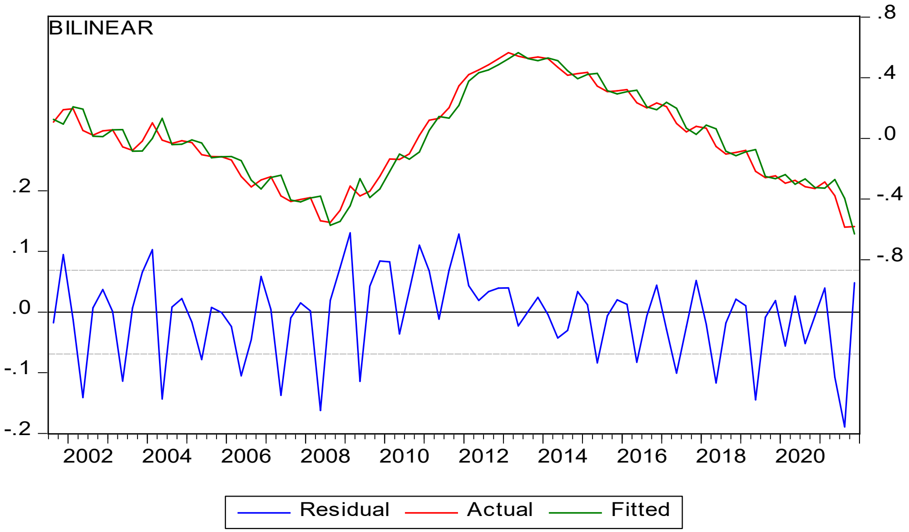
The last two models fail to achieve better results than the autoregressive, since we cannot reject the null hypothesis of statistical insignificance for their distinctive variables. This contradicts the results found on the estimation of the HP-filtered models, in which we found this two models to be among the best improvements of the autoregressive.

Table 6.6 Estimated models for quarterly log-linear detrended Greek unemployment rates, 2001:1-2021:4

Results for Greece (2001:1-2021:4)		
Model	Estimated Model	s^2/s^2_L
AR(2)	$U_t = 1.247 \cdot U_{t-1} - 0.266 \cdot U_{t-2} + \hat{\varepsilon}_t$ (11.64) (-2.43)	
ESTAR	$U_t =- 0.059 \cdot U_{t-1} - 0.017 \cdot U_{t-2} + (1.361 \cdot U_{t-1} - 0.288 \cdot U_{t-2}) \times [1 - exp(- 152.279 \cdot (U_t + 0.002)^2)] + \hat{\varepsilon}_t$ (-0.14) (-0.06) (3.06) (-0.90) (2.03) (-0.13)	0.896
EAR	$U_t =- 0.001 + 1.384 \cdot exp(- U^2{}_{t-1}) - 0.168 \cdot U_{t-2} + \hat{\varepsilon}_t$ (-0.23) (10.16) (-1.45)	1.070
GAR	$U_t = 1.237 \cdot U_{t-1} - 0.197 \cdot U_{t-2} - 0.316 \cdot U^3{}_{t-2} + \hat{\varepsilon}_t$ (11.53) (-1.58) (-1.12)	0.993
BILINEAR	$U_t = 1.247 \cdot U_{t-1} - 0.265 \cdot U_{t-2} - 0.001 \cdot U_{t-1} \cdot \hat{\varepsilon}_{t-3} + \hat{\varepsilon}_t$ (11.47) (-2.38) (-0.02)	1

Figure 6.6 Actual, Fitted, Residual graph of the estimated models for the Greek unemployment rates, 2001:1-2021:4




BILINEAR
.8
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-.4
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-.1
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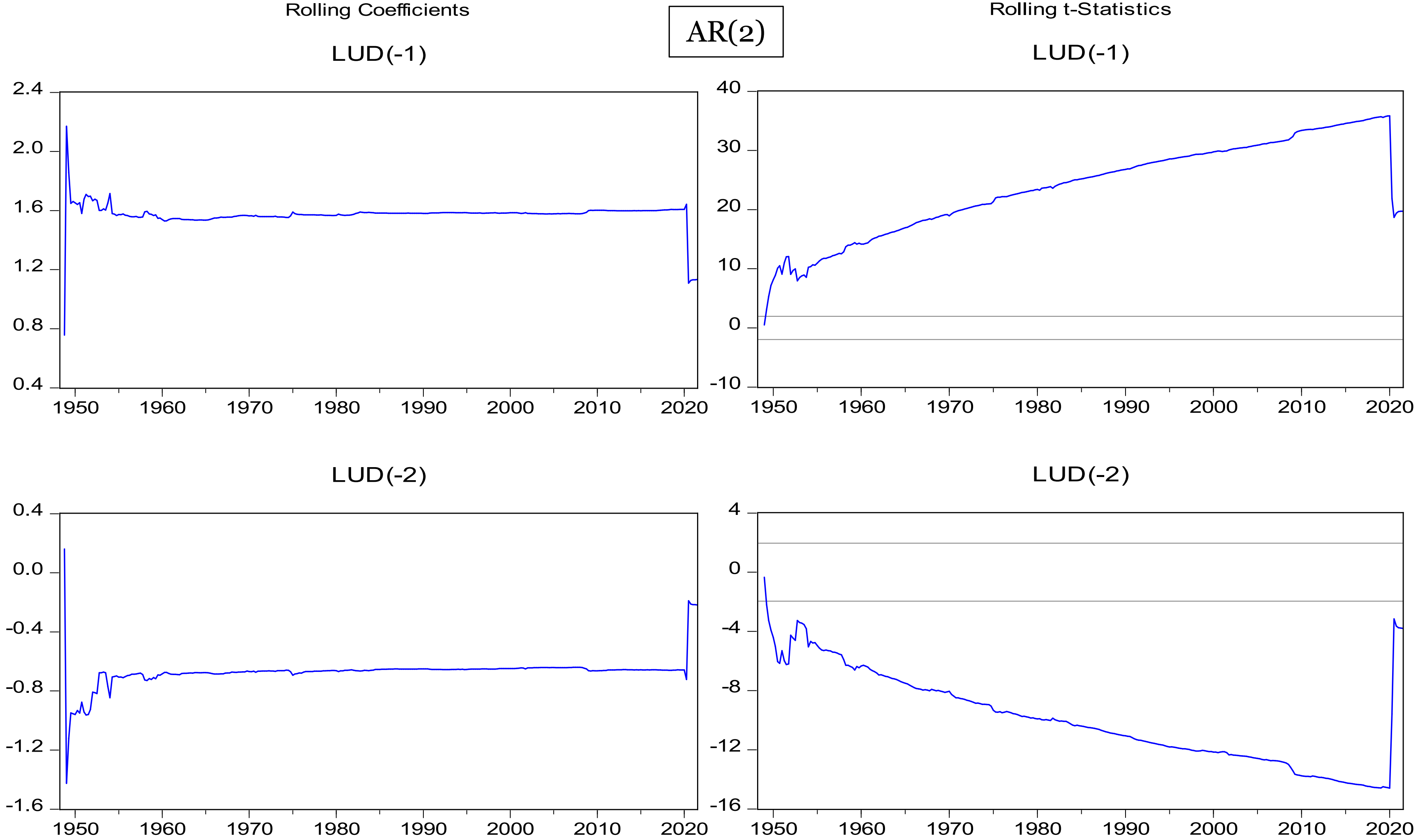
2002
2004
2006
2008
2010
2012
2014
2016
2018
2020
Residual
Actual
Fitted

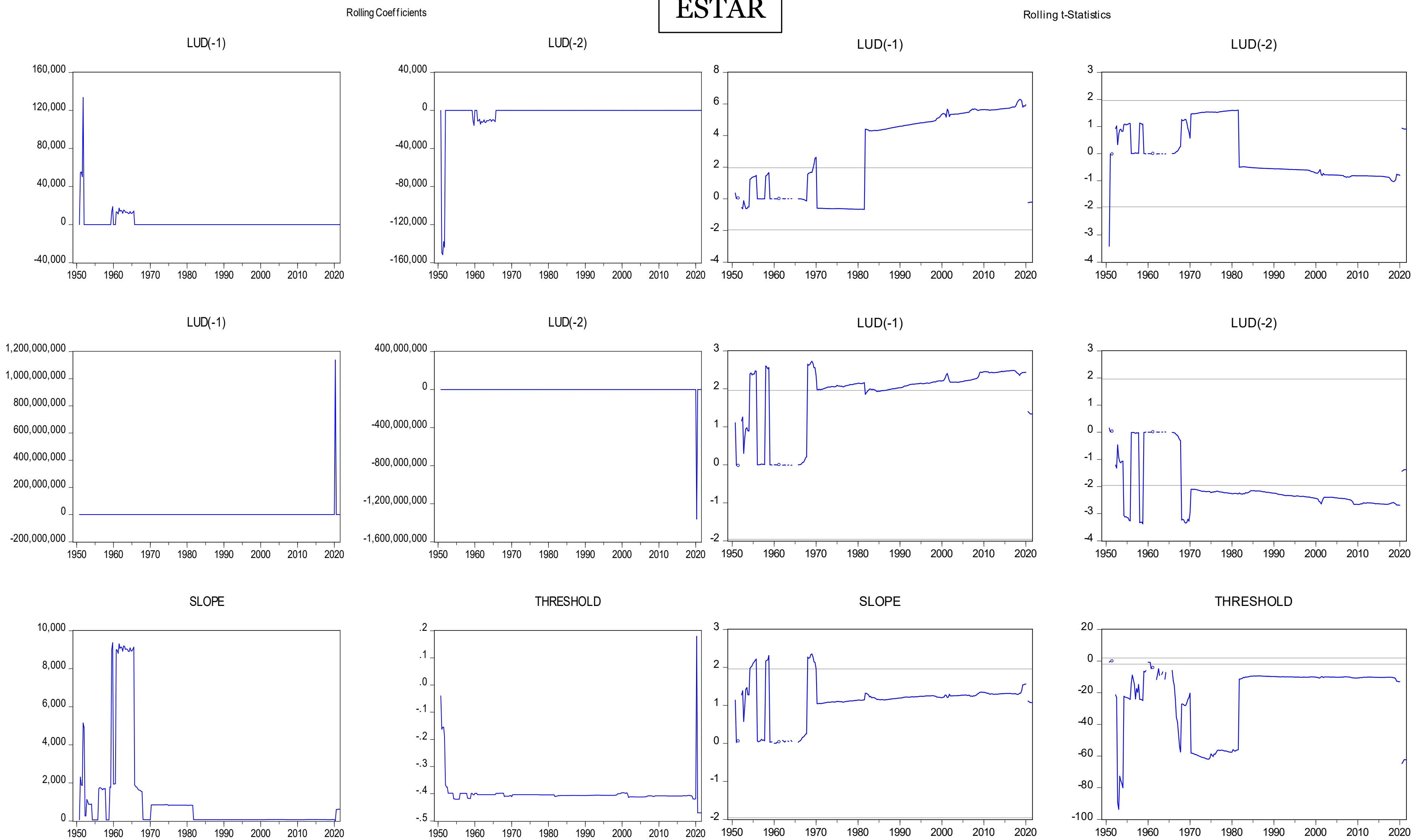
6.4. Model evolution over time

Before we conclude with our work, we present graphs that show how the models of our work evolve over time. In particular, they show how the coefficients change over time as well as the periods, in which they are statistically significant. This analysis is done both for the U.S. dataset, the time period being 1949:3-2021:3, in figure 6.7 and the Greek dataset, for the time period of 2001:3-2021:4, in figure 6.8, with the respective order. Each page is divided into two parts. In the left part, the graphs show the course of the coefficients values of each variable over time and in the right part, the graphs show their respective t-statistic course. In the latter, we also include horizontal lines for the values of 1.96 and -1.96, in order for the significance of the variable to be more clear. It should be noted that the SETAR model is not included, for the reason that its complex form made this analysis impossible to be performed.We will not go into detail with this analysis, as this would be very extensive and exhausting. Nonetheless, the findings should be clear for anyone looking at the Figure 6.7 and Figure 6.8. We highly encourage the reader to look at the graphs. Of high interest is the way the coefficients and t-statistics react to the aftermath of the COVID-19 outbreak, showing a clear struggle for the models to adapt in this extreme situation.

Figure 6.7 Graphs of the over time evolution of the models coefficients and t-statistics for the U.S. unemployment rates, 1949:3-2021:3

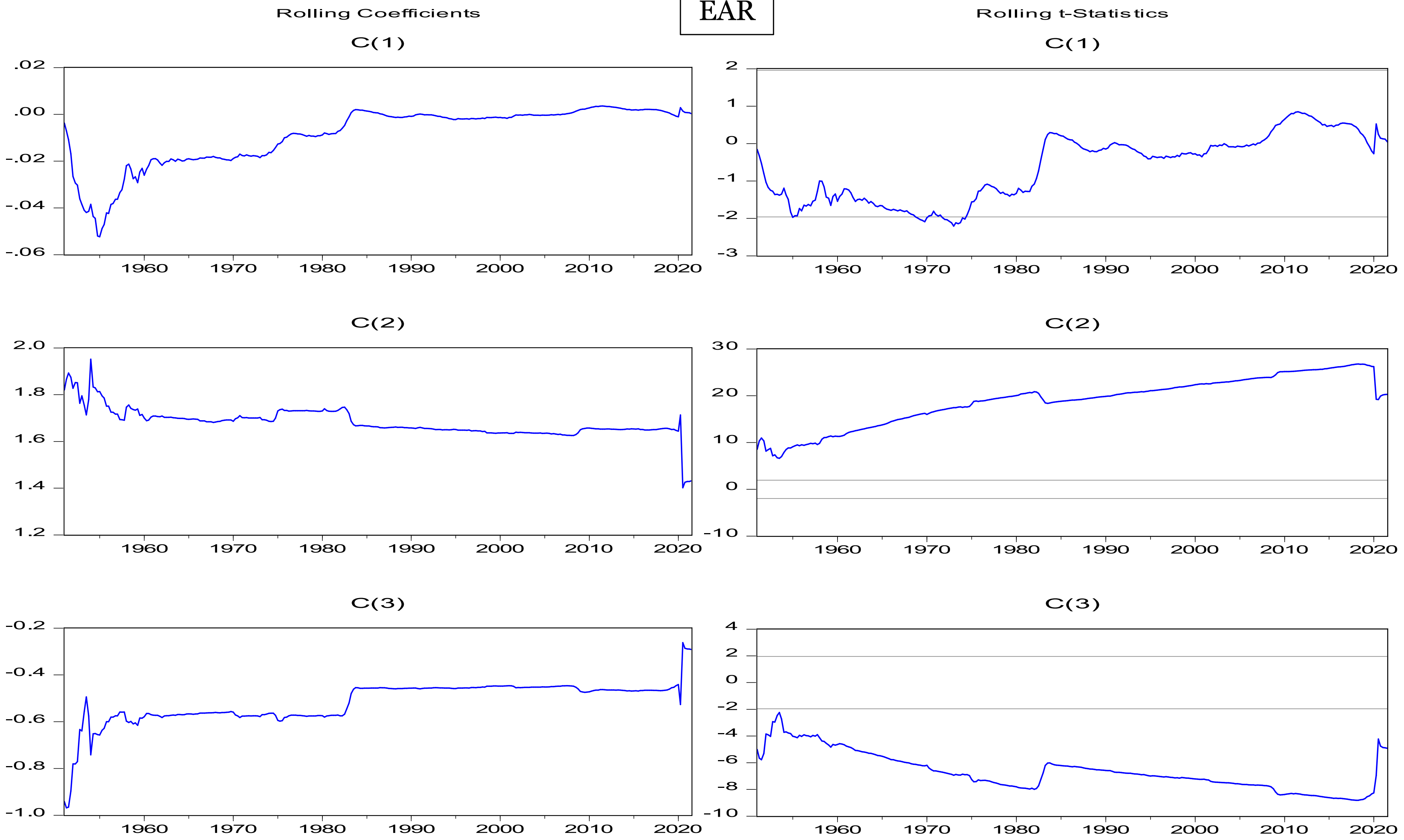


ESTAR



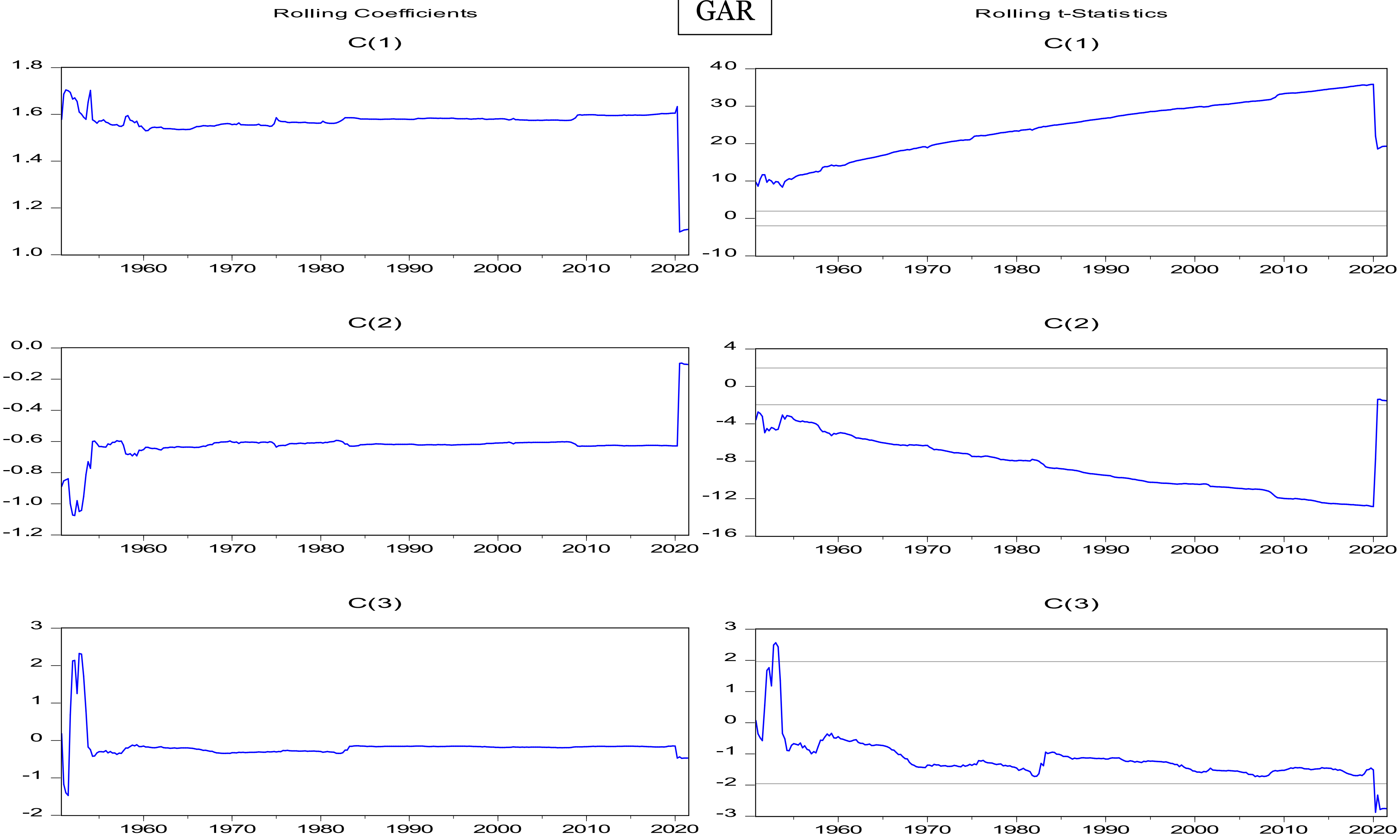
Rolling Coefficients
Rolling t-Statistics
LUD(-1)
LUD(-2)
LUD(-1)
LUD(-2)
LUD(-1)
LUD(-2)
LUD(-1)
LUD(-2)
SLOPE
THRESHOLD
SLOPE
THRESHOLD
160,000
120,000
80,000
40,000
0
-40,000
-80,000
-120,000
-160,000
1,200,000,000
1,000,000,000
800,000,000
600,000,000
400,000,000
200,000,000
-200,000,000
-400,000,000
-800,000,000
-1,200,000,000
-1,600,000,000
10,000
8,000
6,000
4,000
2,000
.2
.1
.0
-.1
-.2
-.3
-.4
-.5
1950
1960
1970
1980
1990
2000
2010
2020

EAR



Rolling Coefficients
Rolling t-Statistics
C(1)
C(2)
C(3)
1960
1970
1980
1990
2000
2010
2020

GAR



Rolling Coefficients
Rolling t-Statistics
C(1)
C(2)
C(3)
1960
1970
1980
1990
2000
2010
2020

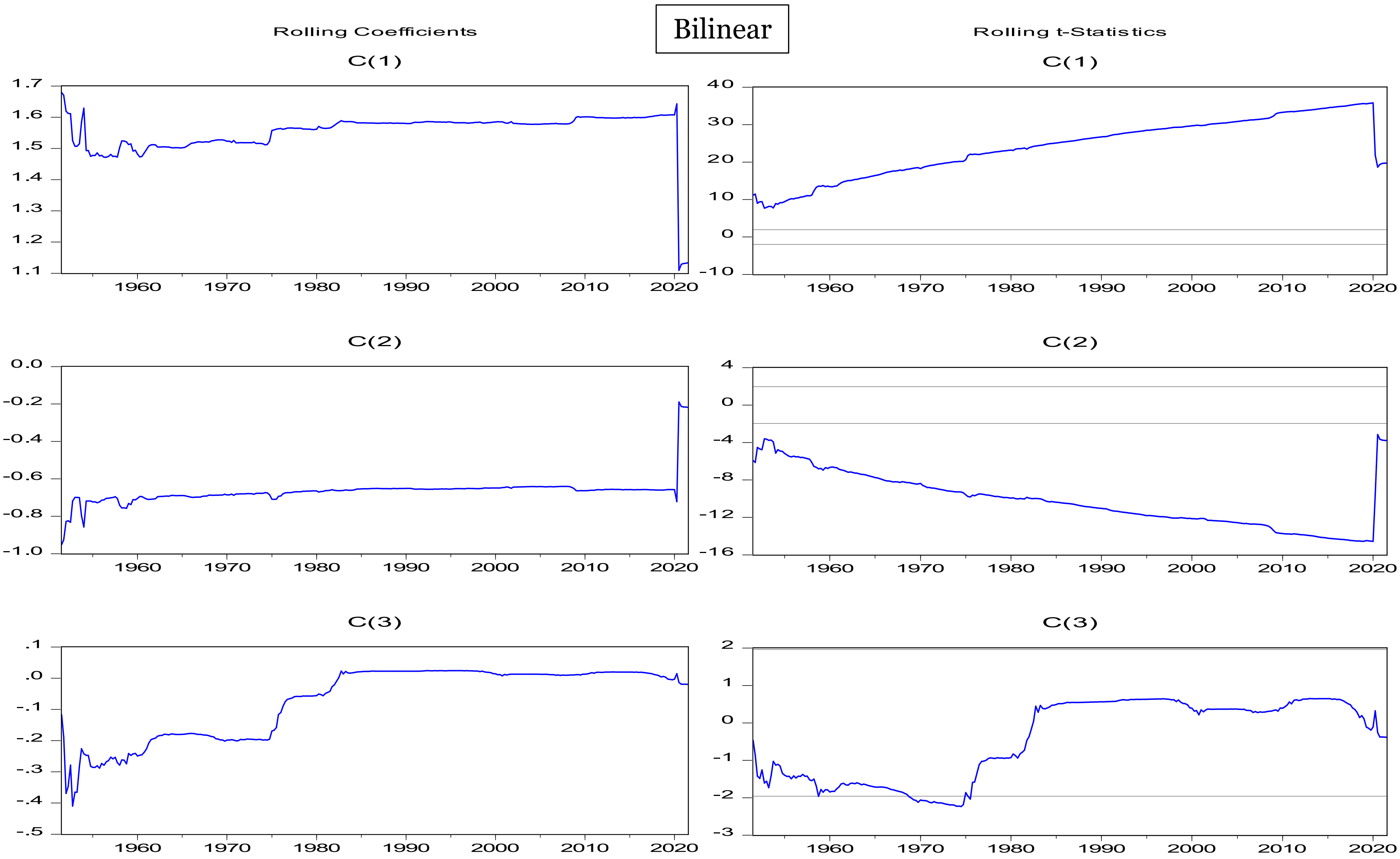
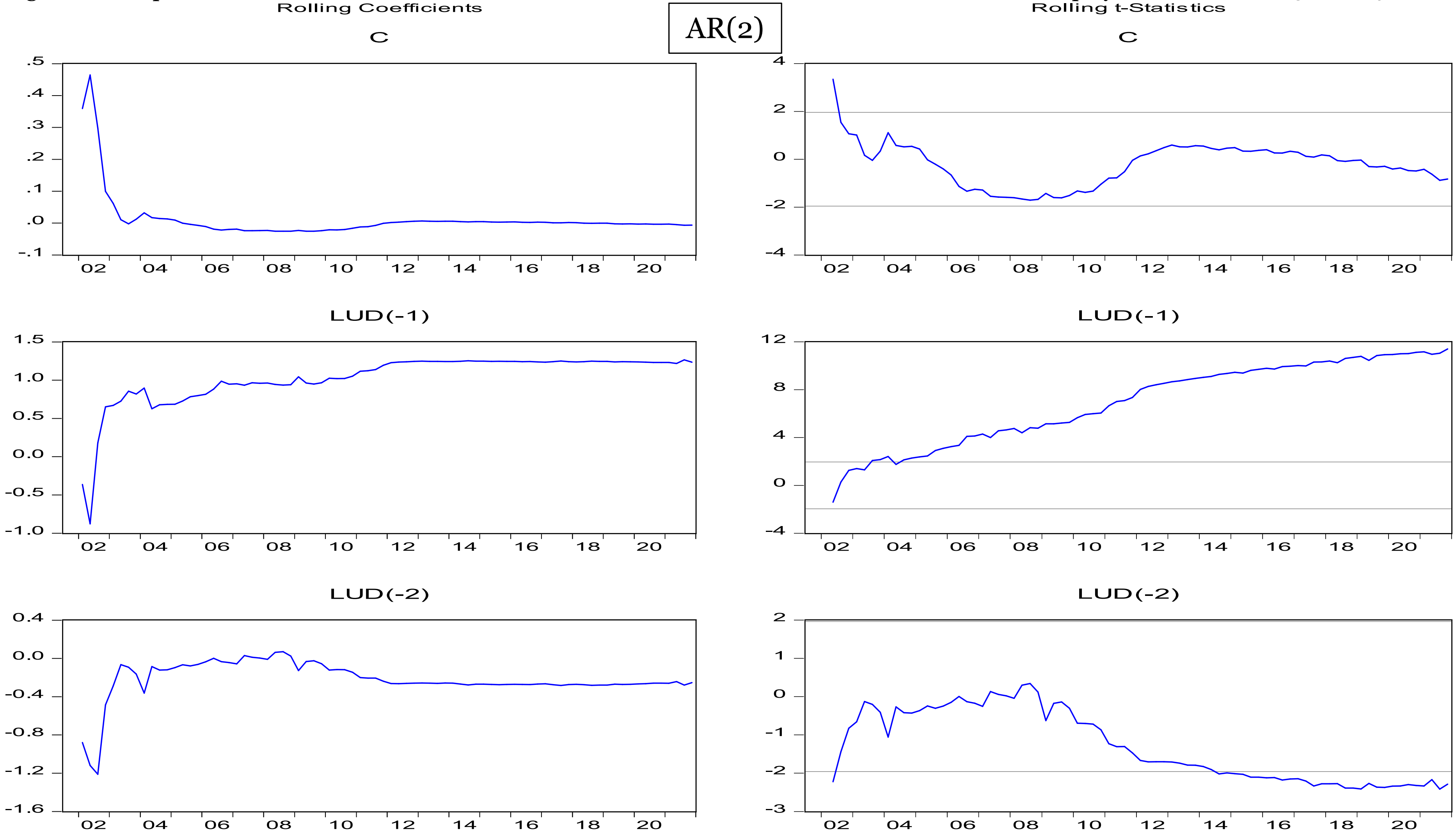

Bilinear
Rolling Coefficients
Rolling t-Statistics
C(1)
1.7 1.6 1.5 1.4 1.3 1.2 1.1
40 30 20 10 0 -10
C(2)
0.0 -0.2 -0.4 -0.6 -0.8 -1.0
4 0 -4 -8 -12 -16
C(3)
.1 .0 -.1 -.2 -.3 -.4 -.5
2 1 0 -1 -2 -3
1960 1970 1980 1990 2000 2010 2020

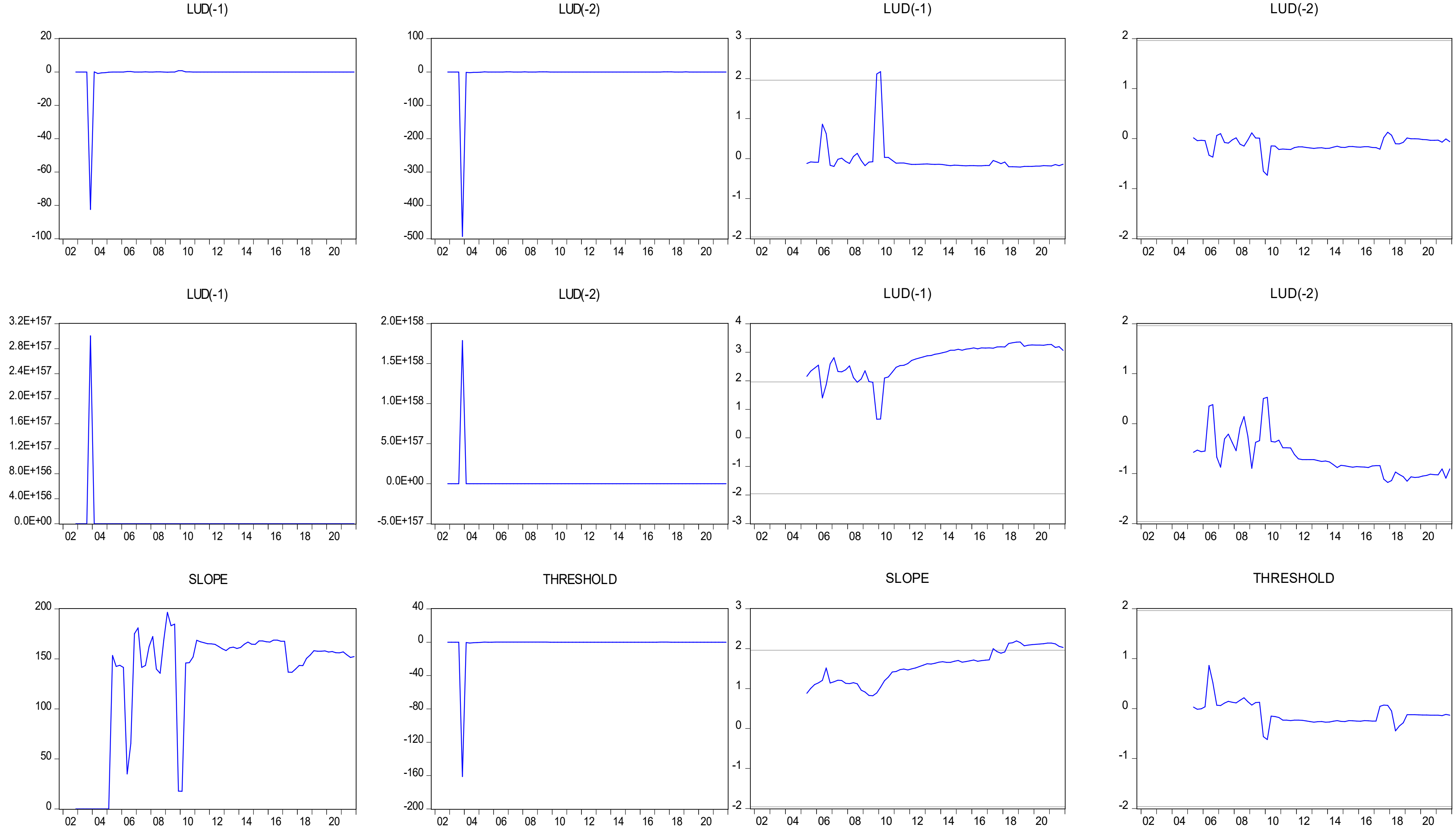
Figure 6.8 Graphs of the over time evolution of the models coefficients and t-statistics for the Greek unemployment rates, 2001:3-2021:4



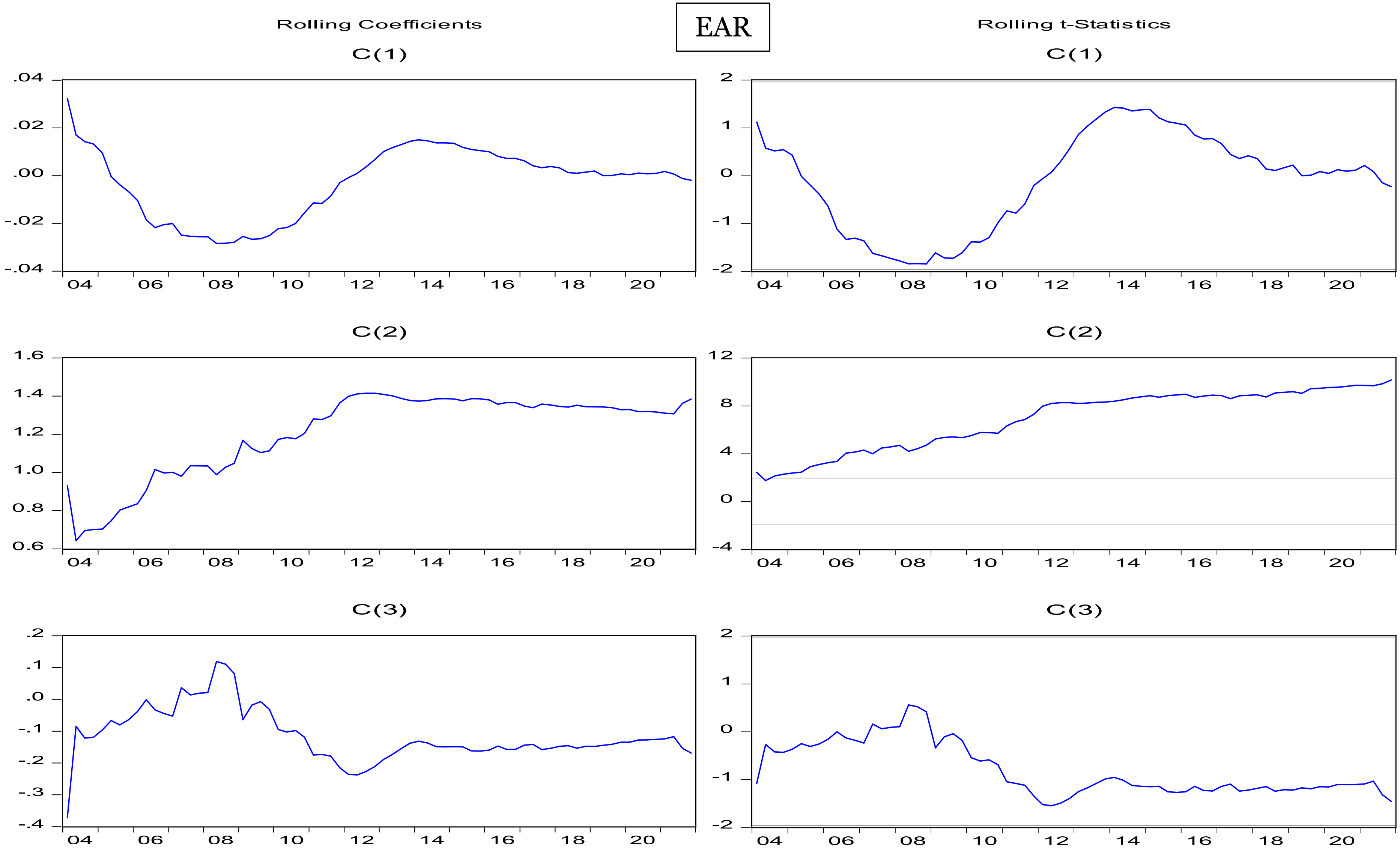
ESTAR

Rolling Coefficients

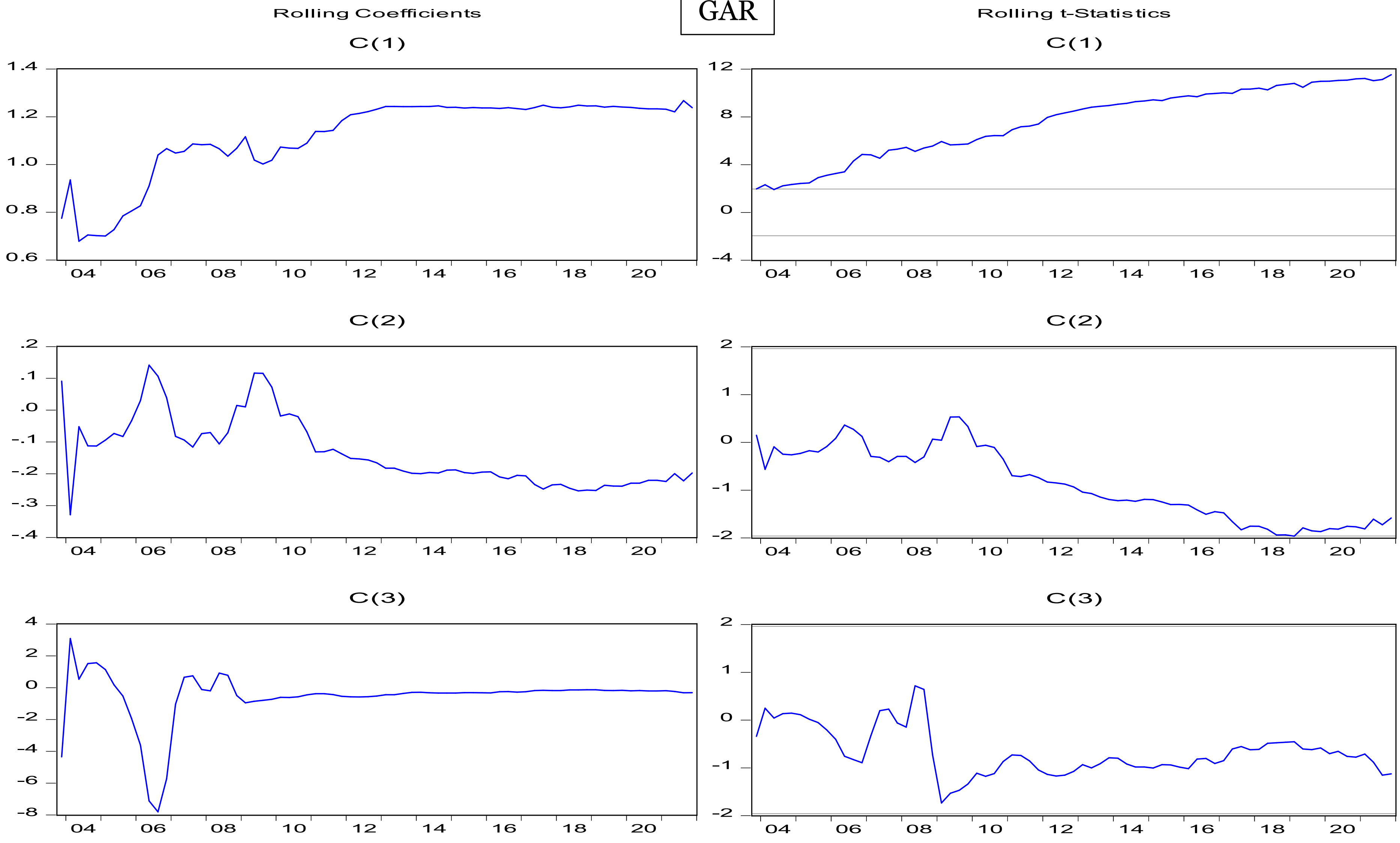
Rolling t-Statistics



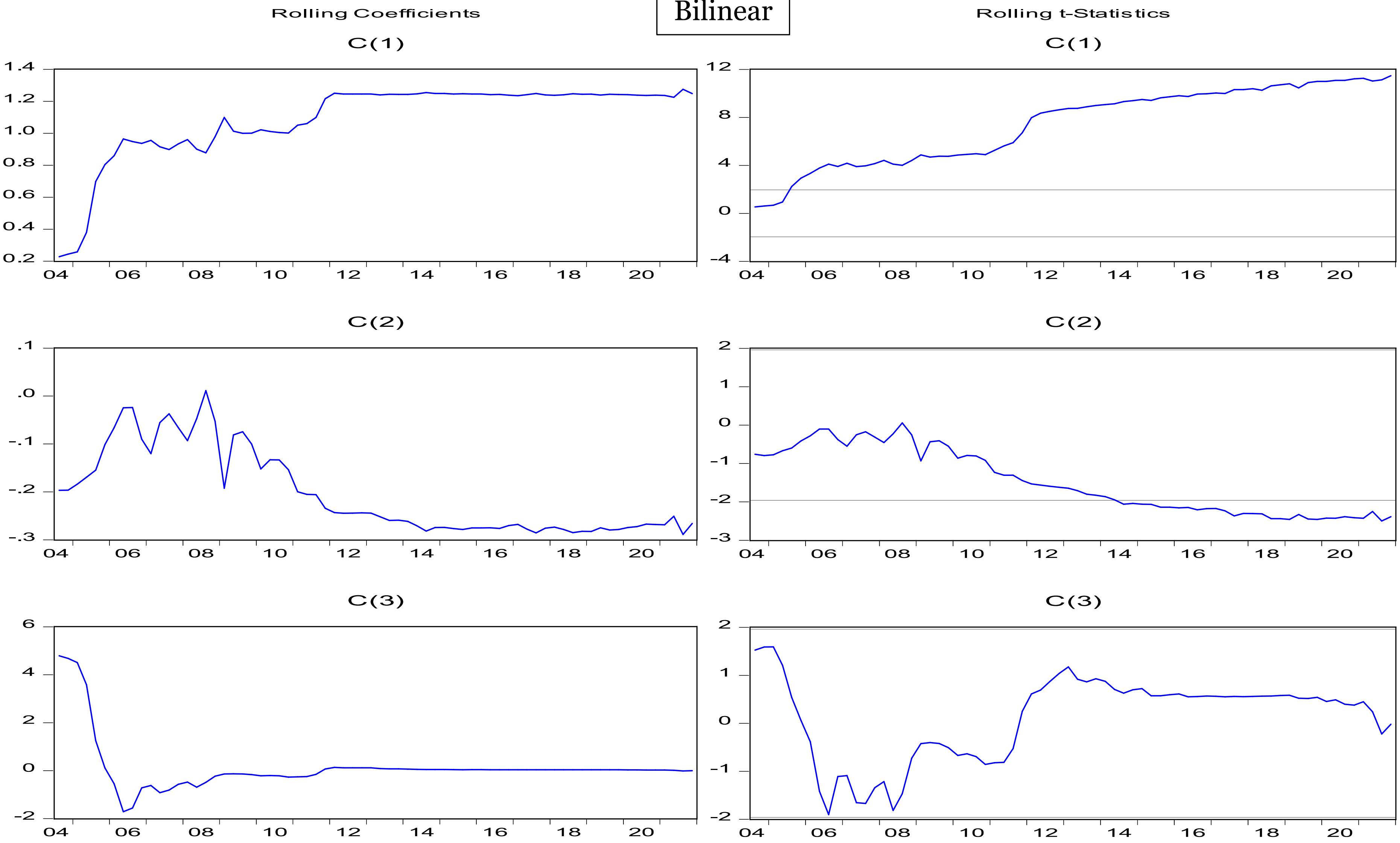
EAR


Rolling Coefficients
Rolling t-Statistics
C(1)
C(2)
C(3)
.04
.02
.00
-.02
-.04
1.6
1.4
1.2
1.0
0.8
0.6
.2
.1
.0
-.1
-.2
-.3
-.4
2
1
0
-1
-2
12
8
4
-4
04
06
08
10
12
14
16
18
20

GAR


Rolling Coefficients
Rolling t-Statistics
C(1)
C(2)
C(3)
1.4
1.2
1.0
0.8
0.6
.2
.1
.0
-.1
-.2
-.3
-.4
4
2
0
-2
-4
-6
-8
12
8
4
0
-4
2
1
0
-1
-2
04
06
08
10
12
14
16
18
20

Bilinear


Rolling Coefficients
Rolling t-Statistics
C(1)
C(2)
C(3)
04 06 08 10 12 14 16 18 20

7. Conclusion

The paper reviews a major work conducted by Rothman(1998) and attempts to reestimate the models that were originally used to interpretate the US unemployment rates. This is done for both the U.S. and Greek unemployment rates, this time including more recent data, aiming to analyze any dissimilarities between the periods as well as the countries and show the estimations for the present day. For the U.S. unemployment rates we also analyse the quality of the forecasts that these can offer. The whole analysis is conducted in four parts.

The results of the first part, when the same timescale is used, present several dissimilarities. Firstly, the threshold autoregressive models now yielded the lowest AIC for the second lag instead of the first, a result also found in the process of forecasting by Rothman (1998). Regarding the latter, we also found a negative threshold, in contrast to the positive one of the original estimates. The residual standard deviation ratio showed that the ESTAR model can offer the greatest improvement. Another interesting result was found in the estimated EAR model which had higher residual standard deviation than the standard autoregressive model and as a result lower explanatory ability.

In the second part we replicate the models while extending the dataset including more recent observations. With this being the main point of the paper, we attached a great importance to the results, in which we found even more dissimilarities. The most interesting of them, had to do with the SETAR model, in which no threshold could be found to improve the explanatory ability of the pure autoregressive. On the contrary, the other Threshold Autoregressive model was once again estimated to have the best explanatory ability. Moreover neither in this analysis could the EAR model improve the linear model. Similar results were now found for the GAR model. When we examine the forecasts offered from the models we find interesting results for the GAR and Bilinear models using the HP filtered data, although as we have already mentioned, the significance of these results should be further examined.

In the third part, we follow anew the same methodology, this time with unemployment rates for Greece. Interestingly enough, the results found are extremely similar to these of the previous analysis.

In the last part, we include graphs that show the how the models change during the whole time period offered by both the U.S. and Greek dataset.

In broad outline, our results suggest that the ESTAR model is undoubtedly the best model, that can account for the asymmetric behaviour of the unemployment rates and improve the explanatory ability of the autoregressive model. Having said it still fails to yield residuals that have no significant deviation from the white noise process, meaning that there is still major structure left unexplained by the model. Furthermore the remaining models prove to be incompetent for this analysis, as no threshold can be used by the

SETAR model to improve the pure autoregressive process and both the EAR, GAR models have insignificant distinctive variables. We also find that they are unable of forecasting, with the only possible exceptions being the GAR and Bilinear models using the HP-Filtered unemployment rates. We mainly attribute this incompetence to the instability that engulfed the economic system, at the last years of our examined time period.

Finally, we encourage future researches to examine if the Bilinear models can help the interpretation of the asymmetric variable, when they are properly estimated, as well as demonstrate their estimation process.

8. Appendices

Appendix 1

Table 8.1 Tests of stationarity in quarterly logarithmic unemployment rates, 1949:1 - 2021:3

Test	Test Statistic	Level of significance	Critical value
Dickey-Fuller	-3.92	1% level	-3.45
		5% level	-2.87
		10% level	-2.57
KPSS	0.36	1% level	0.73
		5% level	0.46
		10% level	0.34
Phillips-Perron	-3.75	1% level	-3.45
		5% level	-2.87
		10% level	-2.57
ADF-GLS	2.36	1% level	1.94
		5% level	3.21
		10% level	4.40

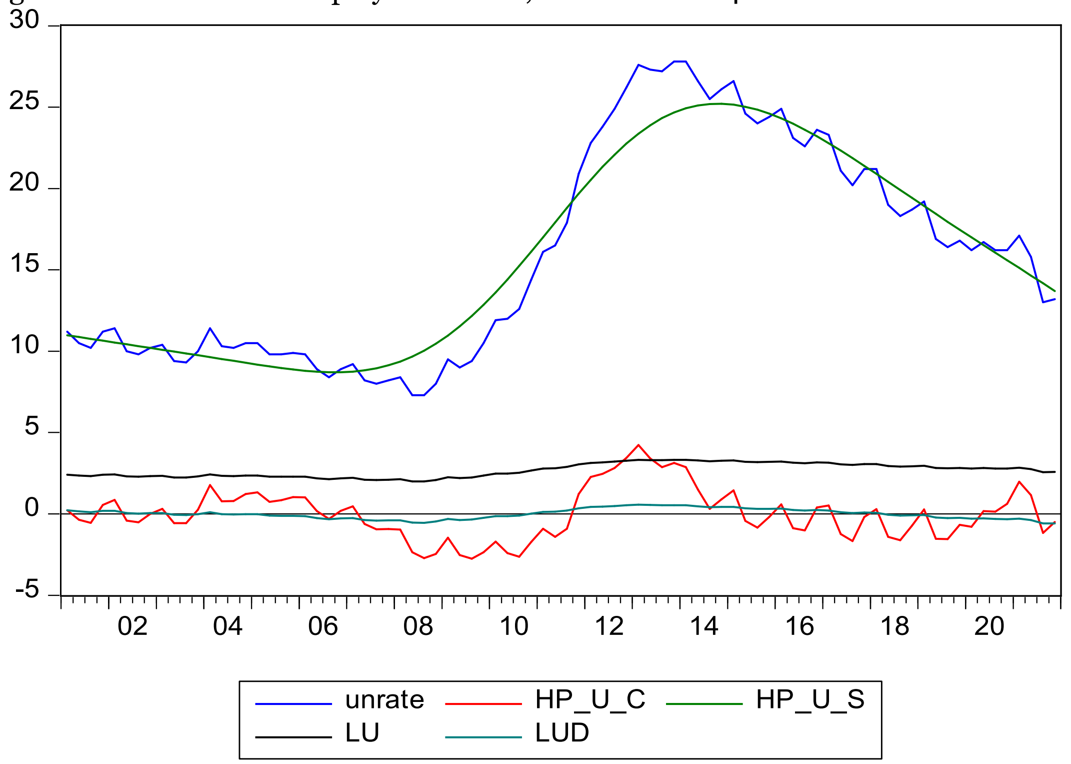
Table 8.2 Tests of stationarity in quarterly Greek unemployment rates, 2001:1 - 2021:4

Test	Test Statistic	Level of significance	Critical value
Dickey-Fuller	-2.45	1% level	-3.51
		5% level	-2.89
		10% level	-2.58
KPSS	0.15	1% level	0.21
		5% level	0.14
		10% level	0.11
Phillips-Perron	-0.26	1% level	-2.59
		5% level	-1.94
		10% level	-1.61
ADF-GLS	0.89	1% level	1.92
		5% level	3.06
		10% level	4.08

Table 8.3 Tests of stationarity in quarterly log-linear detrended Greek unemployment rates, 2001:1 - 2021:4

Test	Test Statistic	Level of significance	Critical value
Dickey-Fuller	-2.45	1% level	-2.59
		5% level	-1.94
		10% level	-1.61
KPSS	0.15	1% level	0.73
		5% level	0.46
		10% level	0.34
Phillips-Perron	-0.56	1% level	-2.59
		5% level	-1.94
		10% level	-1.61
ADF-GLS	0.26	1% level	1.92
		5% level	3.06
		10% level	4.08

Figure 8.1 Time series plot for Greek unemployment rates, log unemployment rates, log-linear detrended unemployment rates, 2001:1 - 2021:4



Note: unrate - Standard unemployment rates, LU - Logarithm of the unemployment rates, LUD - Log linear detrended unemployment rates, HP_U_C - HP filtered unemployment rates - cycle series, HP_U_S - HP filtered unemployment rates - smoothed series

Appendix 2

Table 8.4 Estimated TAR models, with the first lag set as threshold, 1949:1-1979:4

Model	Estimated Model	
SETAR	$U_t = \underset{(20.86)}{1.527} \cdot U_{t-1} - \underset{(-10.23)}{0.705} \cdot U_{t-2} + \hat{\varepsilon}_{1t}$	if $U_{t-1} \geq -0.173$
	$U_t = \underset{(-2.70)}{-0.135} + \underset{(4.55)}{1.017} \cdot U_{t-1} - \underset{(-2.60)}{0.447} \cdot U_{t-2} + \hat{\varepsilon}_{1t}$	if $U_{t-1} < -0.173$
ESTAR	$U_t = \underset{(12.67)}{1.721} \cdot U_{t-1} - \underset{(-6.74)}{0.706} \cdot U_{t-2} + (\underset{(-0.66)}{-0.416} \cdot U_{t-1} - \underset{(0.20)}{0.079} \cdot U_{t-2}) \times$	
	$[1 - exp(\underset{(0.42)}{-3.477} \cdot (U_{t-1} + \underset{(-1.25)}{0.066})^2] + \hat{\varepsilon}_t$	

Appendix 3

Figure 8.2 Residual Correlogram and Q-statistics of the ESTAR model for the extended timeline, 1949:1 - 2008:2

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.091	0.091	2.0069	0.157
		2	-0.006	-0.015	2.0167	0.365
		3	0.028	0.031	2.2149	0.529
		4	-0.094	-0.100	4.3801	0.357
		5	-0.083	-0.065	6.0748	0.299
		6	0.067	0.079	7.1996	0.303
		7	0.063	0.056	8.1948	0.316
		8	-0.064	-0.080	9.2296	0.323
		9	0.172	0.173	16.677	0.054
		10	0.093	0.067	18.862	0.042
		11	-0.005	0.007	18.869	0.063
		12	-0.023	-0.046	19.008	0.088
		13	0.085	0.108	20.851	0.076
		14	-0.105	-0.086	23.696	0.050
		15	-0.048	-0.035	24.294	0.060
		16	0.018	-0.025	24.379	0.082
		17	0.124	0.179	28.416	0.040
		18	0.067	0.018	29.595	0.042
		19	-0.009	-0.081	29.618	0.057
		20	-0.016	-0.034	29.683	0.075
		21	-0.131	-0.055	34.236	0.034
		22	0.030	0.039	34.479	0.044
		23	0.029	0.010	34.698	0.056
		24	0.015	0.013	34.758	0.072
		25	-0.030	-0.019	34.999	0.088
		26	0.099	0.060	37.676	0.065
		27	0.038	0.017	38.079	0.077
		28	0.017	0.031	38.156	0.095
		29	-0.010	-0.038	38.186	0.118
		30	-0.056	-0.034	39.046	0.125
		31	0.011	0.058	39.077	0.151
		32	-0.085	-0.083	41.116	0.130
		33	-0.026	-0.044	41.309	0.152
		34	0.002	0.012	41.310	0.182
		35	0.037	-0.008	41.707	0.202
		36	0.036	0.016	42.073	0.225

Appendix 4

Table 8.5 Estimated models of the quarterly log U.S. unemployment rates, 1949:1-2008:2

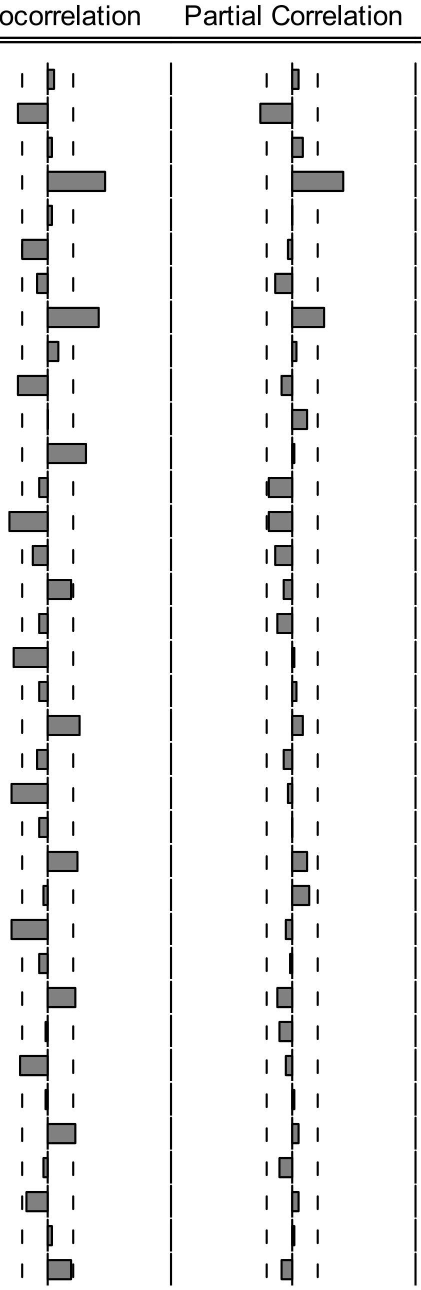
Log level Results (1949:1-2008:2)		
Model	Estimated Model	s^2/s^2_L
AR(2)	$U_t = 1.611 \cdot U_{t-1} - 0.612 \cdot U_{t-2} + \hat{\varepsilon}_t$	
	(31.34) (-11.90)	
EAR	$U_t = 1.689 - 2.620 \cdot exp(-U^2{}_{t-1}) + 0.177 \cdot U_{t-2} + \hat{\varepsilon}_t$	1.738
	(12.82) (-11.84) (2.78)	
GAR	$U_t = 1.587 \cdot U_{t-1} - 0.560 \cdot U_{t-2} - 0.008 \cdot U^3{}_{t-2} + \hat{\varepsilon}_t$	0.973
	(31.44) (-10.79) (-3.70)	
BILINEAR	$U_t = 1.666 \cdot U_{t-1} - 0.622 \cdot U_{t-2} - 0.025 \cdot U_{t-1} \cdot \hat{\varepsilon}_{t-3} + \hat{\varepsilon}_t$	0.982
	(30.98) (-12.28) (-3.00)	

Table 8.6 Estimated models of the quarterly HP-filtered U.S. unemployment rates, 1949:1-2008:2

	HP-filtered Results (1949:1-2008:2)	
Model	Estimated Model	s^2/s^2_L
AR(2)	$U_t = 2.007 \cdot U_{t-1} - 1.007 \cdot U_{t-2} + \hat{\varepsilon}_t$ (243.32) (-121.97)	
EAR	$U_t = 0.092 - 52346.48 \cdot exp(-U^2{}_{t-1}) + 0.987 \cdot U_{t-2} + \hat{\varepsilon}_t$ (1.61) (-1.03) (100.98)	15.789
GAR	$U_t = 1.992 \cdot U_{t-1} - 0.988 \cdot U_{t-2} - 8.05E^{-05} \cdot U^3{}_{t-2} + \hat{\varepsilon}_t$ (342.39) (-169.35) (-14.01)	0.928
BILINEAR	$U_t = 2.011 \cdot U_{t-1} - 1.007 \cdot U_{t-2} - 0.00006 \cdot U_{t-1} \cdot \hat{\varepsilon}_{t-3} + \hat{\varepsilon}_t$ (275.34) (-138.05) (-8.24)	0.885

Appendix 5

Figure 8.3 Residual Correlogram and Q-statistics of the ESTAR model for the Greek dataset, 2001:1 - 2021:4



	AC	PAC	Q-Stat	Prob
1	0.062	0.062	0.3303	0.566
2	-0.261	-0.266	6.2117	0.045
3	0.039	0.082	6.3446	0.096
4	0.484	0.440	27.063	0.000
5	0.039	-0.005	27.201	0.000
6	-0.216	-0.042	31.429	0.000
7	-0.090	-0.140	32.168	0.000
8	0.433	0.267	49.592	0.000
9	0.097	0.028	50.478	0.000
10	-0.255	-0.085	56.708	0.000
11	0.005	0.130	56.710	0.000
12	0.336	0.019	67.823	0.000
13	-0.072	-0.197	68.340	0.000
14	-0.332	-0.194	79.522	0.000
15	-0.128	-0.148	81.215	0.000
16	0.197	-0.066	85.260	0.000
17	-0.079	-0.128	85.913	0.000
18	-0.281	0.019	94.384	0.000
19	-0.074	0.032	94.988	0.000
20	0.270	0.098	103.07	0.000
21	-0.098	-0.081	104.15	0.000
22	-0.300	-0.045	114.48	0.000
23	-0.069	0.005	115.04	0.000
24	0.260	0.117	123.04	0.000
25	-0.032	0.138	123.17	0.000
26	-0.318	-0.058	135.60	0.000
27	-0.076	-0.012	136.32	0.000
28	0.235	-0.115	143.36	0.000
29	-0.010	-0.110	143.37	0.000
30	-0.243	-0.052	151.17	0.000
31	-0.012	0.018	151.19	0.000
32	0.238	0.050	158.96	0.000
33	-0.040	-0.109	159.19	0.000
34	-0.188	0.043	164.28	0.000
35	0.025	0.020	164.37	0.000
36	0.193	-0.083	169.94	0.000

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