VOLATILITY MODELING AND FORECASTING IN ENERGY MARKETS

Bachelor Thesis

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Abstract

In this work, we test several GARCH models; linear, non-linear, univariate, and multivariate models are constructed to find which one best models volatility characteristics. Another important aspect of this study is the best models for forecasting, according to loss functions. In addition, volatility spillover effects and covolatility spillover effects are also studied in our series using univariate and multivariate models. Lastly, optimal hedge ratios are discussed, and their capabilities are tested using hedging effectiveness index. Through this paper, GARCH, EGARCH and GJR models are being used with three distributions (Normal, t-student and GED), while for multivariate models we use diagonal GARCH and DCC, with the latter being used for the optimal hedge ratios mostly.

1. Introduction

1.1. History

In United States, utilities were considered monopolies, and many customers did not have many choices choosing their utility, depending on the area. Without many competitors in particular areas, the markets functioned as a monopoly, controlling prices, keeping high retail prices for consumers. When energy markets deregulated, retail firms were created, producing, and selling utilities, increasing competition, lowering prices, and benefiting consumers. To better understand the deregulation in U.S., we must begin from the Great Depression. When the stock market crashed in 1929, energy industry was affected severely as well. Many energy firms closed, and the markets became an oligopoly, where entities could affect prices. Government had to step in and regulate the markets, to prevent aggressive pricing that would affect consumers. That's why in 1935 the Congress passed the Public Utilities Holding Company Act (PUHCA), in their first attempt to regulate the energy market. The act (PUCHA) was passed by Congress with new rules on how the energy could be sold.

1.1.1. Energy Crisis 1970

The energy crisis of 1970 occurred when United States, Australia, and Western Europe experienced shortages in petroleum. The worst and most famous in history periods of the crisis, were the petroleum crisis of 1973 and the energy crisis of 1979. The main reason behind the 1973 crisis was the Yom Kippur War. Yom Kippur war was a conflict between Israel and Arab states led by Egypt and Syria. United States helped Israel, by providing it with ammunition. Arab States had warned western countries, and especially United States to stop supporting Israel. However, the support was not halted, and as a result OAEPC (Organization of Arab Petroleum Exporting Countries) proclaimed an oil embargo and stopped exporting oil to U.S. and countries that helped Israel, and therefore, supply dropped and prices increased, causing the petroleum crisis of 1973. In the end, U.S. negotiated with Arab States, Egypt, and Syria, and Israel pulled back from Sina. OPEC observed that it had a major role in the energy markets and in the world economy. The energy crisis of 1979 started with the Iranian revolution, when

Ayatollah Khomeini took control, after Mohammad Raza Pahlavi was forced to fly away because of the protests. While the country was still producing and exporting oil, it did it in a much lower volume, lowering production and supply, and therefore increasing prices. The energy crisis of 1979 created the Public Utility Regulatory Policies of Act (PURPA). The act was enacted to encourage cogenerate and renewable resources and promote competition for electric generation.

1.1.2. Energy policy from 1990 through 2000

The decade started with the invasion of Kuwait from Iraq, later leading to the Gulf War that lasted for 1 month, 1 week and 4 days (17 January – 28 February 1998). Margaret Thatcher and George H. W. Bush deployed forces in Saudi Arabia, urging other countries to send troops as well. This was the biggest troop deployment since World War II, where 35 countries in total provided troops. But the war remained between 7 countries: Saudi Arabia, Kuwait, United States, United Kingdom, France, Egypt against Iraq. To prevent United States troops strike from the Persian Gulf, Iraqi soldiers dumbed approximately 4,000,000 of US barrels, creating a massive spill, named "*The Gulf War oil spill*". There are some disagreements, whether this oil spill caused environmental damage in the gulf, with most opinions yielding that it indeed had a significant long-term environmental damage. The spill caused WTI prices to increase in January 22 from 21.63 to 24.91. However, this is not as serious to the WTI prices as the first day of US and its allies attack on Iraq, on 17 January, where WTI prices plunged to $21.48 from 32.25. This worried many experts that with the War, the oil prices could reach as high to $60 per barrel. Many oil companies such as Exxon, America Petrofina Inc., Marathon Oil lowered their wholesale gasoline prices. The following graph X.X shows the implications of Gulf War on the crude WTI prices and that the worrying expectations of crude oil prices after the war never happened.

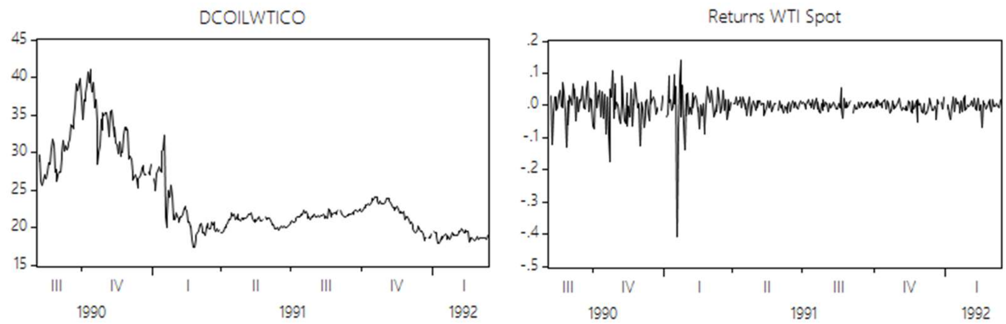

Figure 1. 1: Implications of Gulf War on crude oil prices, Source: Fred St. Louis

The department of Energy presented to President Bush some policy options, to avoid in the future political mismanagements and involvements in Wars that would again hurt energy prices and cause fear in the energy markets. In February 1991, Bush administration proposed to Congress energy policies that would increase oil, natural gas, and nuclear power production, including oil and gas exploration in the Arctic National Wildfire Refuge. But democrats and environmentalists opposed those policies. In June 1991, Congress rejected those proposals. After the end of Gulf War, that took with it the fears of oil prices increase, the Congress passed the Energy Policy Act of 1992 (EPAct92). The Act focused on tax creation and subsidies for energy efficiency and renewable energy technologies. It also changed Federal Power Act (FPA) and Public Utility Holding Company Act (PUHCA) that helped restructure the electricity industry.

During their presidential election, Clinton and Gore (later president and vice president), were supporters of environmental improvement and were afraid of the climate change. His energy policies, as a president, where to implement higher taxes based on energy consumption, to encourage energy conservation, reduce the country's deficit and reduce pollution associated with the combustion of fossil fuels. However, Congress was against those proposals, and only passed a higher tax in gasoline prices. Until the end of the decade, the Congress passed no act, but Clinton signed many environmentally friendly orders such as "*Executive Order 12856*", requiring federal agencies to reduce pollution as much as possible and to report to the community any toxic chemicals that are related into the environment. Those orders influenced the department of energy policy, and the consequence was a decrease in funding for coal and nuclear technology and an increase in funding for renewable and energy efficiency technologies increased.

1.1.3. 9/11 Terrorist attacks

The 9/11 Terrorist attacks had serious implications for the stock market. But oil did not receive a significant shock. More precisely, market chaos, panic selling, and disastrous loss of value prevailed in the day of the attack and NYSE and NASDAQ remained closed until September 17th. In addition, many brokerage firms were unable to function since they had their offices in the World Trade Centre. In the day of the attack, the Dow Jones fell 684 points. At the close of trading that Friday, Dow Jones was down more than 14%, S&P 500 index lost 11.6% and NASDAQ plunged 16%. Gold saw an increase, increasing

6%, confirming the financial uncertainty. Oil and gas also soared. Crude oil WTI, reached after the attack $29.59 from $27.65 on September 11th, not a significant price increase, but a consecutive price increase, 4 days in a row. It seemed that the America – Middle East relationships will never be the same, and that the price increase of oil will be permanent. However, that was not the case. The oil prices reached their pre-attack levels approximately a week after.

1.1.4. International Oil Markets History

Oil was discovered by Colonel Drake and William A. Smith in 1859. From then it had undergone four distinct phases between 1859 and 1960, when OPEC was formed. The first pre-OPEC phase was named "period of gold rush". As soon as oil was discovered, it triggered an oil rush, were the owners of the lands tried to pump as much oil as possible. At that time oil had limited use. In the second phase, named "the phase of standard oil domination", John D. Rockefeller changed and dominated the petroleum markets. He entered the refining business in 1863 in Cleveland Ohio and with Henry Flagler set up the Standard oil Company in 1870. Rockefeller took control of refineries, transportation and distribution segments, controlling 90-95% of refineries in United States through aggressive mergers and acquisitions between 1870 and 1880. Through economies of scale, Rockefeller suppressed his competitors and established a physical monopoly. His empire got threatened, as soon as other countries discovered they had resources of the "black gold", such as Russia. Texas region had plenty of resources as well, and new companies such as Texas Oil Company and Gulf Oil Company became competitors as well. Lastly, new legal and regulatory frameworks in 1890 threatened Standard oil more. During the early 1910's, oil started replacing coal as the dominant fuel in the world. The automobile sector increased demand for oil, so did the breakout of World War I. Industrialized economies realized the need of oil in the future and started investing in crude oil production. In 1928, this increased production led to a decrease in prices. The fourth phase was initiated by a series of major developments and situations. First, Middle East made it's presence in the oil market, due to it's huge reserve potential. Venezuela became a major producer in South America. Oil exporting countries demanded bigger share in oil profits. The importance of oil grew more with the breakout of World War II. In addition, seven oil companies at that time controlled the entire supply chain and had influenced the market and lastly, the USA became a net importer of oil from the oil pioneer and requested changes in the pricing policy, since now they were mostly importing oil from Middle East and were not net exporters. USA faced an increase in the production costs at that time, but policies of market expansion resulted in lower prices in the international markets and affected the revenue income of host governments from royalty payments. Competition from Middle East brought new issues as well. America administration became concerned with a big proportion of imports coming from Middle East and imposed some import quotas to increase the price of importing and try to increase the domestic production of oil. Finally, in 1960 the Organization of Petroleum Exporting Countries was established in Iraq in September 1960, by five leading oil producing states: Iran, Iraq, Kuwait, Saudi Arabia and Venezuela. OPEC's main goal was to coordinate petroleum policies of states to secure a fair and stable remuneration for their outputs. The organization was established in a time, where the oil production was rising, and prices were decreasing.

1.2. Energy Market, Supply and Demand

Energy demand usually refers to any kind of energy used to satisfy energy needs. For example, for cooking, using electronic devices, washing, etc. Industrials and factories also have demand for energy, either for electricity uses, as the previous mentioned, or exploiting energy as a raw material. For example, a petrochemical company uses energy to exploit it's chemical properties, rather than it's electricity output. In other words, individuals, companies, countries have a different "equation" for energy demand, depended on their needs and utility functions. To derive energy demand, we have to make a three-stage analysis. First, will the household or factory switch or not to commercial energies? The switching choice is affected by the available income and willingness to spend money for commercial energies. Higher willingness to spend money for commercial energy means higher demand for commercial energy. The second stage is to decide for the appliances to be used, if chooses of course to use commercial energy. In this stage, two important parameters need to be decided. If there are other available fuel choices and type of appliance should be used for this appliance. The last stage is the level of utilization. High use or low use? This leads us to the classic microeconomic concept of utility maximization, that derives from consumer (entity) to consumer (entity). Since the utility maximization problem is a classic microeconomic concept, we are not going to cover this. But we need to explain how this created demand is going to get covered and bring equilibrium in the energy markets. Supply is created, with respect to the costs and the benefits that will derive from an investment in energy production. If the value of benefits is bigger than the costs, then investments in the energy sector will be executed to cover the vast energy demand. Of course, in real life, when costs are higher that does not mean the energy production will stop, but the sector, either private or public, responsible for the energy production, tries to find ways to minimize costs, seek for financing, maximize benefits for the investment of energy production. In addition, energy sector assets tend to be capital intensive. Often capital cost accounts for a large part of the average cost, and consequently, per unit costs fall with higher sizes, showing economies of scale. An implication of such capital intensiveness and economies of scale, is that the marginal costs tend to be low compared to average costs and as a result any pricing in marginal costs (perfect competition) would lead to losses. Capital intensives need big investments to keep marginal costs in a level beyond average costs, and as bigger installations provide economies of scales, few suppliers tend to control the market share creating a monopoly environment. But suppliers could create natural monopolies, that usually don't face legal issues, by finding a way to decrease costs, and especially average cost, lowering marginal costs as well. In general, however energy markets could be characterized as competitive markets with some assumptions, with marginal costs and average costs equal in the long run. An implication of such capital intensiveness and economies of scale is that the marginal costs tend to be lower than the average costs, and a competitive pricing result in financial losses. Therefore, there needs to be a

Target	Acquirer	Announcement Day
Amoco	BP	08/11/1998
PetroFina	Total	12/01/1998
Mobil	Exxon	12/01/1998
Arco	BP	04/01/1999
Elf Acquitaine	TotalFina	07/05/1999
Texaco	Chevron	10/16/2000
Tosco	Phillips	02/04/2001
Gulf Canada	Conoco	05/29/2001
Conoco	Phillips	11/18/2001

Table 1. 1: M&A's in the oil sector during the 4th merger wave

premium in price, to make the pricing equal at least to the minimum average cost. It would not be viable for the firms operating in the energy sector to keep producing in their maximum capacity, treating fixed costs as sunk costs. This procedure would in fact, not bring the firm in it's break-even point and the excess supply would have created storage problems. Through time, the problem of excess capacity and capital intensiveness has been resolved, with horizontal integrations in the oil industry, and regulation in the electricity industries. Horizontal integration implies linking with firms in the industry, usually at the same stage of value chain and approximate size, either through mergers and acquisitions or cartels. During the 4th merger wave, many firms in the oil industry, either integrated horizontal through M&A's, or attempted to. The table above presents some important M&A's during the 4th merger wave, and some years after it, in the oil industry.

1.3. Energy Market Outlook 2021 and Forward

The previous and current year could be characterized as "odd years". Crude oil spot and future markets reached below zero levels for the first time, bewildering the markets. On a different market, the gas market, prices soared significantly, even 800% in some areas. The main cause behind that, were the cold months of January and February. Temperatures were so low that caused freeze-offs, a situation when wells shut down because of liquids freezing pipeline. In addition, demand for electricity, gas and oil kept rising. From one side there was lower gas supply and from the other increase in gas demand, driving prices high. According to EIA energy outlook 2021 and its projections, the gas consumption will keep rising while the consumption of petroleum and other liquids will slow. Renewable energy consumption will increase rapidly, so will it's production. For year 2021, petroleum and other liquids remain number 1 source of energy consumption. That is because the transportation sector is the one that consumes the most petroleum and other liquids. But petroleum is essential for retail products and industrial uses, as a production input, as well. Projection show that renewable energy consumption will increase and that is because federal policies have encouraged investments in renewable sources, increasing supply. In addition, new technologies have also driven down costs for installation of solar and wind panels. The decreased costs and profit margins had consequently the entrance of many investors, companies, individuals, and as a result transformed the market environment into a more competitive than before. Before natural gas reach their rapid increase in prices in 2021, in 2020 there was oversupply of natural gas, and as a result prices dropped, reaching their lowest prices since 1990. In addition, mines close to limit the spread of COVID-19, and therefore supply dropped. EIA projects that electricity generation from gas will have an upward trend, while coal generation will increase in 2022, as electricity demand, until it returns to its long term and constant decline structure. On the petroleum markets, EIA projects that onshore crude oil production, will continue with it's current trend that started from 2010, and Southwest will be the region that will dominate the oil production for the upcoming years, while the West coast will face a decline in the oil production. A quick scenario analysis shows that, if oil prices reach their projections and be high, the exports would be face an upward slope, reaching it's maximum exports in 2040 approximately, where US will export 10 million barrels per day. On the other hand, the worst-case scenario, where the prices of oil are low, shows that US would import every year more barrels of oil.

1.4. Risk in Energy markets

The value of correctly measuring risk in energy markets is becoming essential day by day. Several legislations, the deregulation of the energy market converting it from monopoly to competitive marketplace, the environmental concerns, and the financial markets do have significant impact in energy markets. Energy and environmental financial markets are rising but they are still immature financial markets. When we talk about risk, companies bear the brunt. According to Markus Burger, Bernhard Graeber and Gero Schindlmayr (Markus Burger), first thing we need to do is to identify risk and the result we get is a risk map for a specific company, by which factors is the total firms risk associated with energy risk derived from (Markus Burger). Among the six the most important and hard to manage

Figure 1. 2 : Risk Map, source: Managing energy risk, a practical guide for risk management in power, gas, and other markets

is credit risk. Unlike market risk, credit risk can not be observed so easily. The correct management and identification of credit risk needs an exhaustive audit and check, as well as significant information. The corresponding Lehman Brothers in energy markets (which also had implications in the energy market) could be Enron, because it caused firms to re-evaluate the way they were managing credit risk. Enron was a famous electricity and gas company based in Houston, and while they were reporting significant revenues (nearly $101 billion during 2002) they were hiding their losses in offshore accounts. Therefore, the economic results of the year showed significant profits and an appreciation of the stock price, while in reality the firm may had losses and the stock was way too overpriced. Clear policy and no conflicts of interest are also vital for a minimization of a firm's exposure to energy and total risk. The market risk refers more to the risk that arise between the contracts purchased and the contracts sold, the imbalance between those two that can cause risk to one counterpart. Liquidity risk can be categorized in market liquidity risk, which refers to the lack of marketability of a contact and it's difficulty to be converted in cash, and funding liquidity risk, where liabilities cannot be met, or they do so in damaging prices.

2. Literature Review

Mitigating and Quantifying volatility in energy markets is of great interest for various types of investors, businesses, governments. For example, an oil company could see its revenues shrink in the balance sheet, if a sudden drop in prices occurred. Forecasting that price fluctuation could have prevented the profit decline. The administration for example could have hedged the company's position. In addition, a speculation about a worldwide commodity, such as crude oil, could be proved a lucrative investment. Oil prices have crucial part on macroeconomy and inflation and forecasting them is important for the appropriate monetary policy decisions. But oil dynamics vary substantially over time, being affected not only by their fundamentals, but also from many factors in the macro-finance sphere, and overall, that is what makes difficult the prediction. This uncertainty in oil markets do indeed affect in general energy markets. An appropriate model and method should be used. In their paper, Wei et al (2010) (Yu Wei) found by comparing three linear GARCH models and six non-linear GARCH models that there is no superior model, but non-linear models tend to have better forecasting power than linear models. Therefore investors, companies or government entities for monetary policy decision making must not relay on standard models, but they must test several, and which one fits the best and more accurate according to their data. In their paper, Manera et. al. (2014) (Matteo Manera) found that energy futures markets are procyclical with the market (S&P 500), and that speculation stabilize prices in energy futures markets, not destabilizing them, as Stein (1987) (Stein) and Hart and Kreps (1986) (Hart) had suggested. In their study, Kang & Yoon (2013) (Sang Hoon Kang) sought to identify a good model for forecasting volatility in petroleum futures. Out of sample analysis indicated that results differ relative to the data, and that investors should be cautious when estimating volatility in petroleum futures markets. They also found that FIGARCH models with ARFIMA models capture better long-memory features than GARCH and IGARCH models. In a different approach and paper, Nomikos and Pouliasis (2011) (Nikos K. Nomikos) showed that Mix distribution GARCH and Markov Regime Switching GARCH (MRS GARCH) models obtain better accuracy in out of sample analysis than conventional GARCH models for petroleum futures and propose that financial analysts use these models in the volatility modelling process of oil prices. They also quantify risk with Value at Risk approaches, finding that GARCH-X model is the most consistent while VaR based on Extreme Value Theory indicates results suitable for risk-averse investors. Hung et. al. (2008) (Yuan-Hung Hsu Ku) found that VaR with Heavy Tail

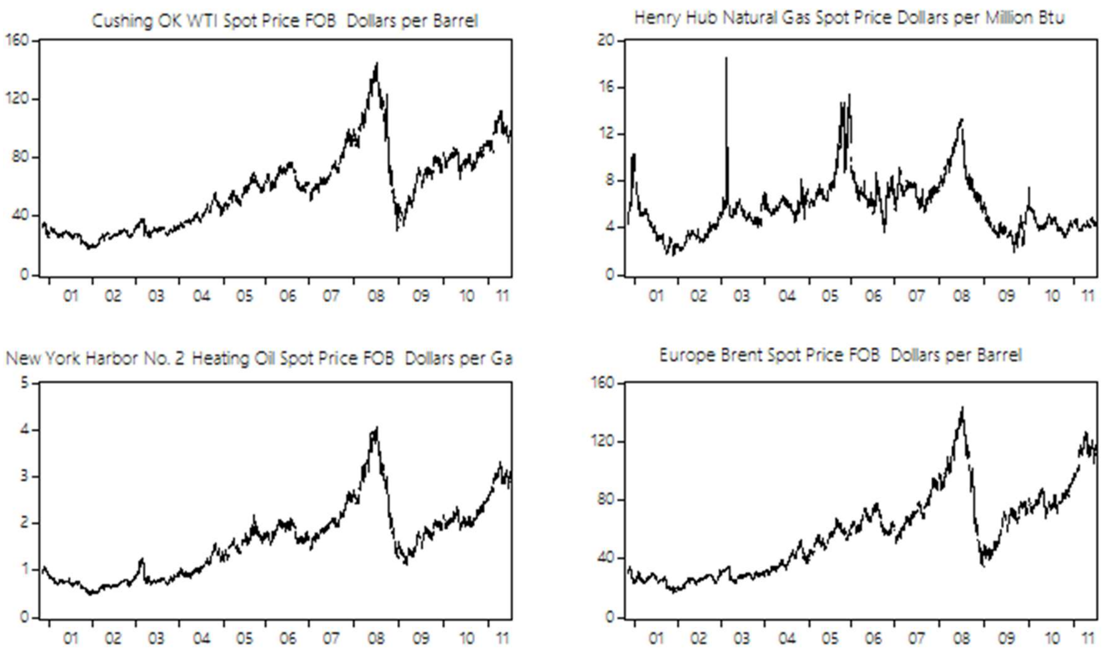

Figure 2. 1 : WTI, Brent, Henry Hub and NY Harbor Heating oil spot prices, Source EIA

distribution GARCH models, outperform GARCH models with normal and student distribution, in terms of failure rate. In their study the fractional integrated model indicates that shocks in volatility of heating oil and unleaded gasoline futures returns scatter exponentially, converging to a GARCH model, having serious implications in for value at risk estimations, pricing of oil derivatives and hedge ratios. In each case, accuracy of the results is biased. But oil spot and future prices can not only be affected from the markets of their refining products, but also from other energy markets, such as the natural gas markets and electricity markets. A look in the graph (above) indicates that between 2002 and 2008, before the financial crisis, oil prices kept an upward trend. In the same period, heating oil and natural gas seemed to follow the same trend, with natural gas being more volatile, indicating in general that markets may affect prices from one market to other. Efimova and Serletis (2014) (Serletis) found that there is unidirectional volatility spillover effects, and volatility from one market affects volatility in a different market, with hierarchy from oil to gas to electricity markets. Results also indicate that univariate models perform more accurate forecasts and the conditional correlation for oil-gas and oil-electricity decreases dramatically in periods of slow economic growth or recession, indicating a procyclical move, while correlation for gas-electricity increases for the same period. Since 2011, dynamic correlations for all pairs of commodities decreased, while the reason has not been investigated. Wang et. al. (2012) (Yudong Wang) showed that the relationship between crude oil and other refining products is characterized by volatility asymmetry and spillover effects, meaning that the returns and (or) the variance of one market is affected by another market, in that case oil is affected by its refineries. They additionally show that multivariate GARCH models perform better forecasts than univariate models. Karali and Ramirez (2014) (Berna Karali), found indirect volatility spillover effects between natural gas and crude oil markets and between natural gas and the heating oil markets, and found direct volatility spillover effects from natural gas returns to crude oil returns. Several major effects that caused macroeconomic shocks, such as Hurricane Katrina, Asian financial crisis, and U.S. invasion in Iraq created a volatile situation in oil spot and future markets, while terrorist attacks on September 11 seemed it would change West's relationships with Middle East. Lehman Brothers did not affect crude oil, but it did affect heating oil. They also found that crude oil markets exhibit much more volatility from March through November, heating oil has less seasonality, and natural gas does not have seasonality at all. On April 20th, 2020, crude oil in US has reached negative prices for the first time in history, after oil producers left out of space to store the oversupply of oil, with demand being insignificant and decreased. The losses for investors, producers and the economy in whole were massive, and even though this might be an unpresented and not suitable example, it can be assumed that market participants try to avoid such losses and hedge their positions. The most common way to hedge a position, either in a financial product or commodity, is through derivatives. The most used are futures, forwards, and options. For example, an oil producer fears that the price of oil would decrease next month, and he will see a decline in profits. He can take short position in a future or forward contract, or a long position in a put option (or short position in a call option) and hedge his position. This procedure however needs to be completed with an appropriate number of derivatives contracts, relative to the initial position. Optimal portfolio weights and optimal hedge ratios is of great importance in the academic and real world as well. Chang et. al. (2011) (M. M. Chian-Lin Chang) found that optimal portfolio ratios for brent crude oil, more futures hold are needed in a portfolio than spot markets, while for WTI crude oil, BEKK models showed that larger proportion in spot markets than future markets is needed. CCC, VARMA-GARCH and DCC models showed holding bigger proportion in futures than spot. In addition, they found that optimal hedge ratios recommend short hedgers to short in crude oil futures with a higher proportion of one dollar long in crude oil spot. With the construction of hedging effectiveness indices, they concluded that variance is hedged better in WTI, and that the best model is diagonal BEKK. The exact same authors in a different paper, Chang et. al. (2010) (Chia-Lin Chang), found volatility asymmetries in both positive and negative oil shocks. Volatility spillover effects were presented from WTI and Brent to Dubai and Tapin oil markets, confirming that WTI and Brent are the world benchmarks. Lastly, they found that the optimal portfolio holds WTI and Brent in a bigger proportion than Dubai and Tapin. Additionally, in a paper already mentioned, Wang et. al. (2012) (Yudong Wang) showed that for hedging crude oil with heating oil or conventional gasoline, scalar BEKK models do give better results compared to other multivariate models, but if jet fuel is used, diagonal BEKK is the optimal.

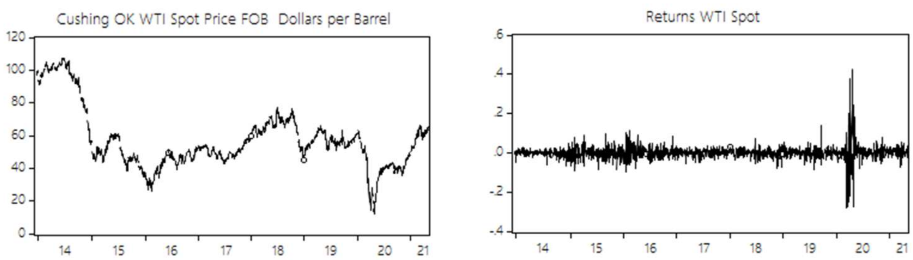

Figure 2. 2 : WTI spot prices and returns 2015-2021 (not negative values) , Source Fred St Louis

Authors (year)	Univariate GARCH	Multivariate GARCH	Spot/Futures	Results
Olga Efimova & Apostolos Serletis (2014)	✓	✓	Spot	Univariate and Multivariate GARCH yield similar results. Undirectional volatility spillover effects.
Yudong Wang, Chongfeng Wu , (2012)	✓	✓	Spot	Multivariate models indicate better results than univariate models. Relationship between crude oil and refining products can be characterized by volatility spillover and timevarying correlation.
Jui-Cheng Hung , Ming-Chih Lee , Hung-Chun Liu (2008)	✓	✗	Spot	VaR with Heavy Tail distribution outperforms GARCH models with normal and student distribution.
Yu Wei , Yudong Wang b , Dengshi Huang (2010)	✓	✗	Spot	Not a model superior than the others, but non-linear models tend to perform better for volatility forecasting
Sang Hoon Kang , Seong-Min Yoon (2013)	✓	✗	Futures	Model with mean model ARFIMA and variance FIGARCH can capture better long-memory features, than models with the same mean equation and variance model GARCH and IGARCH.
Berna Karali , Octavio A. Ramirez (2014)	✗	✓	Futures	Indirect volatility spillover effects between natural gas and crude oil and between natural gas and heating oil. Direct volatiltiy spillover effects from natural gas to crude oil returns. Major effects affected in general the crude oil markets.
Nikos K. Nomikos , Panos K. Pouliasis (2011)	✓	✗	Futures	MRS GARCH models and MIX distribution GARCH models obtain better accuracy in out of sample analysis than conventional GARCH models for petroleum futures.
Matteo Manera , Marcella Nicolini , Ilaria Vignati (2014)	✓	✗	Futures	Energy futures markets are pro-cyclical with the market and speculation stabilizes prices in energy futures markets
Chia-Lin Chang , Michael McAleer , Roengchai Tansuchat (2010)	✓	✓	Both	Volatility assymetries in both positive and negative oil shocks. Volatility spillover effects from WTI to Brent to Dubai to Tapin oil markets. Optimal portfolios holds WTI and Brent in a bigger proportion than Dubai and Tapin.
Chia-Lin Chang , Michael McAleer, Roengchai Tansuchat (2011)	✓	✓	Both	For brent, nore futures are needed than spot according to optimal portfolio ratios, while for WTI mix results depending on the model. Hedging effectivness indeces showed that variance is hedged better in WTI.

Table 2. 1: Literature Review summary

3. Data

In this paper, we use daily price data (in US dollars per barrel) of Brent spot, WTI (West Texas Intermediate) crude oil spot and future markets, Henry Hub spot and future markets and New York harbor heating oil, from January 7, 1998, to May 10, 2021. Data were obtained from Fred St Louis, EIA, and S&P Global Market Intelligence. On 20 April, WTI spot and future prices reached negative levels. The estimated returns calculated later, gave us a significant number, with significant deviation from the mean. Since we care for the long-run effects of the energy markets, it may be better to exclude these two outliers. All sample prices are converted to log returns, to make the series stationary. We take the first logarithmic differences of every series.

$$returns = log\left(\frac{prices_t}{prices_{t-1}}\right) \quad (1)$$

After that, stationarity for each series was tested and ensured. The table below, presents the descriptive statistics of our series returns. While WTI spot and futures markets have similar values in mean and standard deviation, we can see that the spot market tends

	WTI Spot	WTI Futures	Brent	Henry Hub Spot	Henry Hub Futures	Heating oil Spot	Heating oil Futures
Mean	0.000378	0.000354	0.000418	-0.000531	0.000266	0.000414	0.000432
Median	0.000901	0.000915	0.000659	0.00000	-0.000375	0.000362	0.000349
Standard Deviation	0.028955	0.027137	0.025545	0.049733	0.033812	0.025608	0.02104
Minimum	-0.281382	-0.282206	-0.256389	-1.025103	-0.198993	-0.470117	-0.171669
Maximum	0.425832	0.319634	0.412023	0.745632	0.324354	0.229538	0.1039
Skewness	0.64459	0.088531	0.541131	-0.753782	0.633573	-1.3057	-0.14231
Kurtosis	27.91481	18.88566	28.35744	64.40521	8.932903	37.57155	6.315197
Jarque-Bera	142480.0	57775.23	147461.5	863673.3	8425.277	275160.5	2534.351
ADF	-77.30188	-75.23034	-20.56864	-48.22465	-79.38609	-40.58331	-76.56908

Table 3. 1: Descriptive Statistics of series returns

to be more volatile than future markets. Compared to all other markets, gas market presented to be riskier than the other. This is attributed to the significant increase in prices in February, because of the pipelines freeze of, that decreased supply, while demand was rising, and consequently, gas prices experienced an unpresented aggressive inflation in prices.

4. Methodology

4.1. Linear Univariate GARCH models

GARCH models are the most used method to interpret risk in many markets, such as the financial markets and in this case the energy markets. Tim Bollerslev (1986) (Bollerslev) introduced the GARCH models, as a generalization of the ARCH models of Engle (1982) (R. F. Engle). In a similar way as the ARMA models, Bollerslev embedded the autocorrelations and partial autocorrelations for the squared process to identify variance behaviour. The simplest GARCH model, is the GARCH(1,1) model. For simplicity in this paper, we will use this process and not try to identify the best GARCH, linear and non-linear (presented later) models with the use of Akaike Information. The equation for GARCH(1,1) is the following:

$$\sigma^2 = \alpha_0 + \alpha_1 \varepsilon_{\tau-1}^2 + \beta_1 \sigma_{\tau-1}^2 \tag{2}$$

Term α_0 is the constant term, which shows how much the volatility will increase or decrease with all the other terms constant. Term α_1 is also known as ARCH term and measures the extent to which a volatility shock today feeds through into next period's volatility. Term β_1 also known as GARCH term, indicates how past volatility affects today's volatility, the volatility dependence. ARCH term usually tends to be low, while

GARCH term tends to be high. A GARCH model with high α and low β tends to be spikier than a series with a low α and a high β. Adding together the ARCH and GARCH term we get the rate at which the volatility shock today feeds through into next period's. When the sum is equal to 1, we have infinite volatility persistence, and the series is a non-stationary process. In that case, our GARCH models is called Integrated GARCH (IGARCH). The closer the sum to 1, the higher the volatility persistence. In our paper, we do use GARCH(1,1) model for our univariate analysis, but we do examine and test the best mean equation for each series. The selection for the best mean equation is made according to the lowest information criteria, in our case, AIC. We also add the returns of S&P 500 to increase the explanatory power of the model, if applicable. There are instances where the returns of the market make some coefficients statistically insignificant and therefore it is better to not use the returns of the market in the model. There are also cases where we include insignificant lags of the ARMA process, because the AIC indicates that this is the best model. Last, the model must satisfy the residual diagnostics. The autocorrelation is an issue in this paper where different models could not fix. After the best mean equation is selected, if we do have ARCH effects, the creation of GARCH models is appropriate. We test both linear and nonlinear models with 3 distributions: Normal, t-student and GED distribution. Among the 3 types of univariate GARCH models that will be examined the one with the lowest AIC and that satisfies the residual diagnostic test will be selected as the best model for volatility modelling. As already mentioned, our data is WTI spot and futures market, excluding the day of negative prices to gain more robust results, Henry Hub gas spot and future prices, New York Harbor Heating oil spot and future prices and Brent oil spot market. Starting with Henry Hub gas spot prices, our results showed that the best model mean equation to use is ARMA(3,5), where the MA(4) term is not statistically significant, but we are still going to keep it, as our AIC indicates. Constant term and market return where statistically insignificant when added. Ljung Box test showed that we have autocorrelation for 20 lags, so did Q^2. Breusch Godfrey LM test for serial correlation showed no presence of serial correlation for lag 2, the one with the lowest AIC. The model was found to have ARCH effects and GARCH models were necessary.

$$r = \varphi_1 AR(1) + \Phi_2 AR(2) + \varphi_3 AR + \Theta_1 MA(1) + \Theta_2 MA(2) + \Theta_3 MA(3) + \Theta_4 MA(4) + \Theta_5 MA(5) \quad (3)$$

The same mean equation implies as well for gas futures, but with 4 MA terms, and the returns of the market added. Thus, our mean equation for gas futures returns in ARMA(3,4) with the returns of the market. All terms are statistically significant, and we do have autocorrelation according to Q stat and Q^2 for 20 lags. According to Breusch Godfrey LM test, we do not have serial correlation, but we do have ARCH effects. For WTI crude oil spot markets, the best mean equation to use is ARMA(5,5) with S&P 500 returns. Terms AR(1), MA(1), AR(5) and MA(3) are statistically insignificant, while all other were significant. Running a Wald coefficient test, we obtain p-value=0.000 and reject the null hypothesis that the coefficients are statistically insignificant. The estimation may have given as the insignificant results, due to multicollinearity. The returns of the S&P 500 show that if the market returns increase by 10%, WTI spot returns will increase by 3.85%. There is autocorrelation but not serial correlation, and construction of GARCH models is needed since we do have ARCH effects. For WTI futures, the optimal mean equation is ARMA(4,4) with returns of the market added. All variables are statistically significant, there is presence of autocorrelation and ARCH effects, but not of serial correlation according to Breusch Godfrey LM test.

$$r = \varphi_1 AR(1) + \varphi_2 AR(2) + \varphi_3 AR(3) + \Phi_4(4) + \theta_1 MA(1) + \theta_2 MA(2) + \theta_3 MA(3) + \theta_4 MA(4) + \beta rsp500 \quad (4)$$

In this case, if market returns increase by 10%, WTI futures returns will increase by 4.11%, indicating a stronger affection that the market returns have in futures markets than spot. For heating oil markets, spot and futures, the best models to use as mean equation are ARMA(4,4) with the market and ARMA(4,4) without the market respectively. For spot markets, MA(2) and AR(2) term were found to be insignificant, and that is something Wald test coefficient could not fix. According to Q-stat there is no autocorrelation for 20 lags, while Q^2 showed presence of autocorrelation. For our futures market model, every coefficient is statistically significant and there is no autocorrelation. For both markets, there is no serial correlation, but there are ARCH effects. Lastly, for our brent markets, our best model was ARMA(5,0) with only AR(1) term statistically significant. By removing all variables and resulting in a AR(1) we would get a significant larger AIC, so we kept AR(5) as our model, and kept a curiosity about the 4 insignificant terms and what they would mean in our analysis, since the lags denote days of the week. Market returns coefficient, seemed to be statistically significant and was added, where a 10% in S&P 500 returns would increase brent spot market returns by 3%. There is autocorrelation, but not serial correlation and GARCH models are needed to interpret risk.

4.2. Non-Linear Univariate GARCH models

However, GARCH(1,1) models are simple, and since then many variants and extensions have been developed to solve 2 important issues that arise with GARCH models: i) Non-negative constraints may still be violated ii) GARCH models do not account for asymmetry and leverage effects. Two famous models to resolve this issue and specially to capture leverage effects are the exponential GARCH model, also known as EGARCH, and the GJR GARCH model, taken its abbreviations from Glosten, Jaganathan and Runkle. The EGARCH model, was suggested by Nelson in 1991. The variance equation is given by:

$$\ln(\sigma_\tau^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \tag{5}$$

Where the first term is constant, the second is the GARCH term in natural logarithm , the third is the leverage term or asymmetry term and the fourth is the ARCH term. Since we now have logarithms in our models, even if the parameters are negative σ^2 will be positive. We can account now for leverage effects when γ is negative. That indicates a negative relationship between risk and returns. If γ is positive there is still asymmetry, since good news have higher impact on volatility than bad news. If γ is 0 there is no asymmetric volatility. When we have leverage effect, bad news affect volatility more than good news. If leverage effects occur that can be visualized with the news impact curve. According to Baillie and DeGennaro (1990) (DeGennaro) the risk-return relation depends on the error distribution, so again we will examine the normal, t-student and GED distribution. To ensure stationarity in EGARCH models, $\alpha+\beta+\gamma/2$ must be less than 1. GJR model are identical to GARCH, but the asymmetry term is also added.

$$\sigma_\tau^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{\tau-1}^2 + \gamma u_{t-1}^2 I_{t-1} \tag{6}$$

Here, I_{t-1} denotes a dummy variable, where it takes 1 as value, when yesterday's news is negative, otherwise it takes 0. For leverage effect we would see $\gamma>0$ and that means when negative news occur, volatility will increase by γ.

4.3. Univariate GARCH models volatility spillovers

A well-known method and established in the literature for spillover effects using univariate GARCH models is that Christiansen (2007) (Christiansen) used for measuring volatility spillovers in European market bonds. This paper was based in Bekaert and Harvey (1997) (C. R. Geert Bekaert), A Michaelidis, Serena Ng (2000) (Alexander Michaelides) , and Bekaert et al. (2005) (H. C. Geert Bekaert), where all considered volatility spillover effects on the international bond markets. In our paper however, the 7 series will make more difficult this procedure that the author mentioned followed. Our approach will be to treat volatility as a mean equation. We will estimate the conditional variances and save them, from the best selected GARCH models that we found in result section 5.1. and then run a 7-variate VAR model with the conditional variances as our variables. The interpretation will be identical with that followed in section 5.4 for diagonal BEKKs mean equations. With this method we will count for long-term spillovers in the markets. Our 7-variate VAR(1) model will be:

$$Y_t = \begin{bmatrix} varws \\ varwf \\ vargs \\ vargf \\ varhs \\ vargf \\ varbs \end{bmatrix} \qquad M_t = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu\_7 \end{bmatrix} \tag{7}$$

$$A_t = \begin{bmatrix} a_{11} & \cdots & a_{17} \\ \vdots & \ddots & \vdots \\ a_{71} & \cdots & a_{77} \end{bmatrix} \qquad Y_{t-1} = \begin{bmatrix} varws(-1) \\ varwf(-1) \\ vargs(-1) \\ vargf(-1) \\ varhs(-1) \\ vargf(-1) \\ varbs(-1) \end{bmatrix}$$

,where ws stands for WTI spot, wf for WTI futures, gs for gas spot, gf for gas futures, hs for heating oil spot, hf for heating oil spot and bs for brent spot. More details about VAR models functionality are stated in section 4.5

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4.4. Univariate GARCH models Forecasting

So far, we have seen the history performance of our data. But market participants, governments, entities, investors etc, are interested in the future performance, the one that cannot be observed. That is why we need to find an appropriate model for forecasting. In papers of Wei et. al. (2010) (Yu Wei) and Nomikos and Pouliasis (2011) (Nikos K. Nomikos), the sample is separated into "in-sample" and "out of sample", when the first one is used for volatility modelling and the latter for forecasting. In our paper, we are going to remove the last 30 observations, and re-run our models to forecast the 30 last observations and find for each series the best one. That said, our sample for estimation will be from 7th January 1998 until 22nd March 2021, while our forecast period with 30 observations will be 23rd March 2021 until 10th May 2021. The best model will be selected according with the loss functions or accuracy statistics. We will use the Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE).

$$\text{RMSE}=\sqrt{n^{-1}(\sum_{t=1}^{n}(\sigma_t^2-\hat{\sigma}_t^2))^2} \tag{8}$$

$$\text{MAE}= n^{-1}\sum_{t=1}^{n}|\sigma_t^2-\hat{\sigma}_\tau^2| \tag{9}$$

$$\text{MAPE}=n^{-1}\sum_{t=1}^{n}|\frac{\sigma_t-\hat{\sigma}_\tau^2}{\sigma_t}| \tag{10}$$

In our Loss functions, n is the number of forecasts, σ^2_τ is the actual volatility and $\hat{\sigma}_\tau^2$ is the estimated volatility in day t. The results for our models, the forecasted returns, and the derived variance, are presented in section 5.2. The forecast method used, is the dynamic forecast, meaning that the forecasted value of the dependent variable is used.

4.5. Multivariate GARCH models, BEKK, DCC

While univariate models give us a picture of how the markets are affected by their own characteristics and risks, it is more important in economy and markets to search how each market affects one another. To do that, univariate models are not suitable to give us answers. Multivariate models for mean and variance equation need to be presented. For our mean equation we will be using a seven-variate VAR(1) model. VAR models have more than one-time dependent variable. Each variable depends not only in its past values, but also in other variables and their past values as well. In essence, multivariate time series help us understand the relationship between several variables. The advantages of using VAR as our mean equation is the dependence with other variables and that we do not need to specify which variables are endogenous and exogenous – all variables are endogenous. However VAR models are still a-theoretical. For simplicity, we will use only 1 lag to our variables, and that is how VAR(1) arises. One method for the optimal leg selection would have been the information criteria, while a different approach would have been the cross-equation restriction. Our variables that will be used are the returns of our series, WTI spot, WTI futures, gas spot, gas futures, heating oil spot, heating oil futures and brent spot. As already tested, the prices of our series require the logarithmic first differences to become stationary. We could have tested for cointegration among the prices, but since we have many variables, we eliminate the possibility for cointegration and for simplicity we instantly use our returns for our VAR model, as mean equation.

$$Y_t = M + AY_{t-1} + \varepsilon_t \tag{11}$$

Where Y_t, M, A and Y_{t-1} are all matrices, with Y_t being our dependent variables, M the constant terms, Y_{t-1} our lagged variables and A our coefficients for our variables, a 7x7 matrix as shown below.

$$\text{Yt} = \begin{bmatrix} dlogws \\ dlogwf \\ dloggs \\ dloggf \\ dloghs \\ dloghf \\ dlogbs \end{bmatrix} \qquad \text{M} = \begin{bmatrix} \mu 1 \\ \mu 2 \\ \vdots \\ \mu 7 \end{bmatrix} \tag{12}$$

$$\text{A} = \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{17} \\ \vdots & \ddots & \vdots \\ \alpha 71 & \cdots & \alpha 77 \end{bmatrix} \qquad \text{Y}_{\text{t-1}} = \begin{bmatrix} dlogws(-1) \\ dlogwf(-1) \\ dloggs(-1) \\ dloggf(-1) \\ dloghs(-1) \\ dloghf(-1) \\ dlogbs(-1) \end{bmatrix}$$

Where dlog for every case, indicates the logarithmic differences, while ws stands for WTI spot, wf for WTI futures, gs for gas spot, gf for gas futures, hs for heating oil spot, hf heating oil futures and bs for brent spot. Off-diagonal elements, contribute to return spillover effects among the markets. For our variance equation we will use 2 models, Diagonal BEKK models and Dynamic Conditional Correlation models (DCC).

4.5.1. Diagonal BEKK models

Baba et. al.(1985) (Baba) and Engle and Kroner (1995) (R. K. Engle) considered the BEKK models, a multivariate GARCH model, extension from the univariate GARCH models. BEKK models help us explain the volatility transmission between series.

$$Ht = C'C + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BH_{t-1}B' \tag{13}$$

The above, illustrates the simplest BEKK GARCH model specified with all orders set to 1. A and B are k x k matrices parameters and C is a triangular matrix that ensures positive definiteness as well. Model is estimated using Quasi Maximum Likelihood estimation using the following Likelihood function:

$$l = -\frac{TN}{2}log2\pi - \frac{1}{2}\sum_{t=1}^{T}(log\,|Ht| + B'_t HtB_t) \tag{14}$$

Volatility spillover effects are indicated by the coefficients of matrix A. Suppose the above BEKK model, a bivariate GARCH. Matrix A coefficients would have been:

$$\mathrm{AA}'=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{15}$$

The diagonal elements of the matrix would suggest volatility spillover effects from one market to another, meaning that a_{21} measures volatility spillover from market 2 to market 1 while a_{12} measures volatility from market 1 to market 2. For B matrix respectively, the off-diagonal elements help account for volatility persistence among the markets. However full BEKK models, recently have been criticized, since the "curse of dimension", meaning the high number of parameters to be estimated, makes the process difficult and by using QMLE, asymptotic and irregular properties in the parameters occur. To fix this, using diagonal BEKK GARCH models, McAleer et al. (2009) (McAleer.), showed that QMLE in diagonal BEKK produce consistent and asymptotically normal parameters. For Diagonal BEKK models, the off-diagonal elements are zero. The matrices A and B are given as.

$$\mathrm{A}=\begin{bmatrix} a_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & a_{mm} \end{bmatrix}, \quad \mathrm{B}=\begin{bmatrix} b_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & b_{mm} \end{bmatrix} \tag{16}$$

We can see that in this case we can't count for volatility spillover effects, since the off-diagonal elements are zero, but we can observe and test for the significance of partial covolatility spillovers. Spillover are defined in three categories : i) Full volatility spillover, where the returns shock from an asset k affects the volatility of a different asset i, ii) full covolatility spillover, when the return shock from an asset k affects the covolatility between two different assets i and j and iii) partial covolatility, when the returns shock from an asset i, affects the covolatility between two assets, i and j.

i) Full volatility spillovers : $\frac{dQ_{iit}}{de_{kt-1}}$, k≠i (17)

ii) Full covolatility spillovers : $\frac{dQ_{iit}}{de_{kt-}}$, i≠j, k≠i,j (18)

iii) Partial covolatility spillovers: $\frac{dQ_{iit}}{d\varepsilon_{it-1}}$, i≠j (19)

However, Chang et al. (2019) (C.-L. M. Chang) mentioned that the A matrix explains the weights for the return shocks, and does not display volatility spillover effects. In this paper, we are going to follow the method of Chang et al. (2019) (C.-L. M. Chang), the same as Zolfaghari et al.(2020) (Mehdi Zolfaghari) have used. We are going to calculate the partial covolatility spillover effects with the formula α_{ii} x α_{jj} x $\varepsilon_{j,\, t-1}$. By that we will be able to check if there is partial covolatility between the markets. Our BEKK model will be the simple BEKK(1,1) model, using our 7 series. We use t-student distribution for non-normality.

$$Ht = C'C + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BH_{t-1}B' \quad (20)$$

$$C=\begin{bmatrix} c_{11} & \cdots & c_{17} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & c_{77} \end{bmatrix} \quad A=\begin{bmatrix} a_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & a_{77} \end{bmatrix} \quad B=\begin{bmatrix} b_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & b_{77} \end{bmatrix} \quad (21)$$

4.5.2. DCC models

A different class of multivariate GARCH models is the Dynamic Conditional Correlation models, or DCC models in short. Proposed by Engle at 2002 (R. Engle), this model is related to CCC (Constant Conditional Correlation) but allows correlation to vary over time. The conditional covariance matrix in DCC, is defined as:

$$Q_t = D_t R_t D_t \quad (22)$$

Where D_t is diagonal matrix of conditional variances (D_t =diag ($h_1^{1/2}$,...., $h_m^{1/2}$)) that can be estimated with a univariate GARCH model. In our case, the conditional variances will be estimated using a simple GARCH(1,1) with normal distribution. In equation 22, R_t

denotes the dynamic conditional correlation. The symmetric positive definitive matrix Q_t is given by the equation below.

$$Q_t = \bar{Q}(1-\theta_1-\theta_2)+\theta_1(\varepsilon_{t-1}\varepsilon_{\tau-1}{}')+\theta_2 Q_{t-1} \qquad (23)$$

,where θ_1 and θ_2 are scalar parameters to be estimated in the 1st and 2nd stage respectively, and for stability condition to be satisfied, their sum must be less than 1. When θ_1 and θ_2 are equal to 0, $\bar{\boldsymbol{Q}}_t$ from equation X.X. is equivalent to CCC. The log likelihood function, used in the 2nd stage to maximize the conditional likelihood is:

$$l(\theta_1|\theta_2) = \sum_{t=1}^{T}(\log|R_t| + u'Rt^{-1}u_t) \qquad (24)$$

4.6. Multivariate GARCH models Forecasting

In this section, we are not going to forecast volatility directly, but we are going to forecast returns, using a multivariate mean equation, thus a VAR(1) model with our returns. The results from loss functions discussed in section 4.4. (equations 8,9,10) will be compared with the results from univariate models, and the derived variance from those returns is probably better forecasted with what the loss functions will suggest. The only model in our multivariate analysis that could be used for volatility forecasting is diagonal BEKK. However with DCC models we could forecast the dynamic conditional correlations for our series. We have results in Appendix for dynamic forecasts in WTI spot and futures, heating oil spot and futures, gas spot and futures and lastly between WTI and heating oil spot prices. The forecasted correlations could help us in an out of sample analysis for optimal hedge ratios between the markets.

4.7. Optimal Hedge Ratios

So far, we have modelled volatility (results in section 5) using univariate and multivariate models, we have seen how it moves during time and what are the characteristics of the series and we have also forecasted conditional variance and searched for the best model for forecast. But after those steps, when we already have find the presence and characteristics of the risk, how can market participants minimize the risk? In this section we will discuss the optimal hedge ratios between spot and future markets. More detailed, using diagonal BEKK and DCC, we will estimate the optimal hedge ratios, from a long position to spot market , to a short position in futures markets. In equation 25, the dynamic optimal hedge ratio at time t is given.

$$\dot{\delta}_t = Cov_{sf,t}/Var_{f,t} \tag{25}$$

To minimize the risk in a portfolio, for every dollar taken as a long position in spot market it should be hedged by a short position in futures markets equal to $\dot{\delta}$ times the product in futures market (Chang et al. 2010a) (Chia-Lin Chang). Having found the optimal hedge ratios, we can calculate the returns that would have been produced in a hedging portfolio and compare them with the actual returns. That would give us a good measure of the hedging effectiveness. The returns are presented in equation 26.

$$r_{H,t} = r_{s,t} - \dot{\delta}_t \times r_{f,t} \tag{26}$$

, where the negative sign in the equation displays the reverse position in the futures markets. The hedge will be effective, if the conditional variance of the adjusted returns is lower than the unhedged. Another way to measure the hedging effectiveness, is the "hedging effectiveness index" that Chang et al. (2011) ; Ku et al. (2007) (M. M. Chian-Lin Chang) used in their work, presented in Equation 27.

$$HE = \frac{var_{unhedged} - var_{hedge}}{va\ _{unhedged}} \tag{27}$$

,where var unhedged is the variance of the unhedged portfolio and var hedged is the variance of the hedged portfolio. Hedged variance will derive by the conditional variance of a GARCH(1,1) model, from the hedged returns, while the unhedged variance with the same model, from the original series. Hedging effectiveness will give us the final answer, whether the hedger is efficient and hedged series have lower variance or not. Contrary to literature review, we will estimate the optimal hedge ratio for our entire sample and not separate it into in-sample and out-of-sample. We will estimate optimal hedge ratios for WTI, heating oil and gas both with BEKK and DCC using as already mentioned the conditional covariances between the series and the variances from those two multivariate GARCH models and then we will compare the results.

5. Results

5.1. Results for Univariate GARCH models

GARCH(1,1) and GJR results showed that gas spot markets are the spikiest series, with the highest ARCH term close to 0.18 on average, indicating strong volatility clustering and that a shock today feeds through next periods volatility significantly. Volatility dependence was also significant, close to 0.8 on average, indicating strong dependence with previous periods volatility. However the series is non-stationary with normal distribution. Leverage effects where not present in gas spot markets, and there is ambiguity whether there is asymmetry, since in some cases the asymmetry term was not statistically significant. The best model for volatility modelling was selected according to the lowest AIC and the one that captures the ARCH effects, and in some instances also satisfies the residual diagnostics tests (Appendix). Our best model according to AIC is EGARCH with t student distribution but could not capture ARCH effects. Therefore, our model with the second lowest AIC that could capture ARCH effects was EGARCH with GED distribution, with a p-value in ARCH test equal to 0.0566, indicating that ARCH effects are captured in 5% significance level. GJR models do capture better ARCH effects, with p-values ranging from 0.26 to 0.3, depending on the distribution. Therefore we could either pick EGARCH with GED distribution, or GJR with GED distribution as our optimal models for volatility modelling, depending on whether give more weight in the information criteria, or the ability to capture ARCH effects. Table 5.1. gives a better view of the results. Gas futures are less spiky than spot markets, and that was also confirmed during the 2002 and 2020 gas prices surge. High gas spot prices occurred in winter of 2020 due to pipeline freeze-offs that decreased supply significantly. On the demand side, the cold winter increased rapidly demand and we saw a huge surge in gas spot prices. For GARCH models with normal and t student distribution in gas future markets we do get high volatility dependence and lower cluster, but when implementing GED distribution, the characteristics of our series change, and we do have low volatility persistence overall, meaning that volatility will

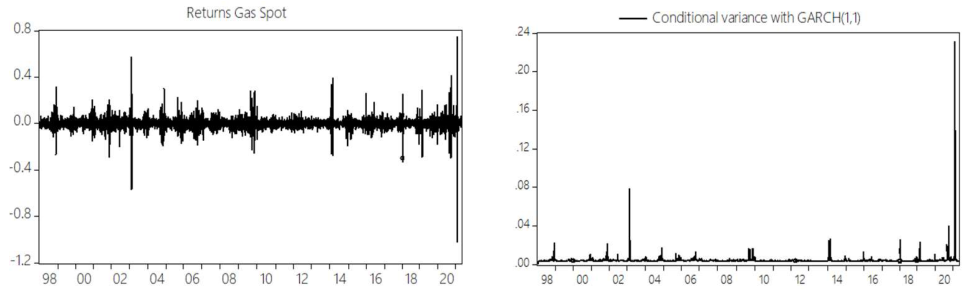


Figure 5. 1: Gas spot returns and gas spot conditional variance with GARCH(1,1)

fade away quicker when a shock occurs, but the degree of volatility feed is significant, since we have an ARCH term equal to 0.15. Non-linear GARCH models indicate that there are no leverage effects in gas futures markets since the asymmetry terms are not statistically significant. Our optimal model to use, was EGARCH with t distribution since it is the one with the lowest AIC and captures well ARCH effects according to LM ARCH test, with a p-value equal to 0.2686. In that case, gas futures markets have high volatility dependence, persistence and clustering and stationarity is questioned. Leverage effects are not present in gas futures markets, and we do not have serial correlation according to Q stat and Q^2 for 20 lags. For WTI spot markets, there is not a model that can capture ARCH effects. The best is GJR with GED distribution, where ARCH LM test shows that we have a p-value equal to 0.01 for lag 1, but 0.07 for lag 3 and keeps increasing by adding more lags. It also has the 3rd lowest AIC. The asymmetry term for that model, is statistically significant and equal to 0.07395, indicating that there are leverage effects and bad news have higher impact in volatility. Q stat shows that we do not have serial correlation, and so does Q^2. The series have high volatility dependence and persistence, while the lower ARCH term (0.045) indicates that is less spiky, and volatility does not feed in a significant manner future volatility. In WTI futures case, again we cannot capture ARCH effects in any model and increase of the lag number does not change the outcome. The model that captures best ARCH effects but still, not in significant way, is GJR with Normal distribution, where the F statistic for ARCH LM test is 7.69 and the AIC for the model is -4.82757. Both markets have high persistence and dependence, with high GARCH terms (0.906 for both spot and futures markets) and significant positive asymmetry terms for our GJR models (and negative for our EGARCH models), indicating that we do have leverage effects in WTI spot and future markets. Thus, bad news increase volatility more than only good news would have. The impact in oil prices during COVID-19 crisis, dipped the WTI spot prices, also reaching negative for the first time on 20th April 2020 (price -$37.63), affecting the May futures contract as well, were it fell 55.90%, to close at negative $37.62. Even before that, the WTI markets were facing uncertainty because of the pandemic outbreak and that can be seen in figures 5.2 and 5.4, where March was

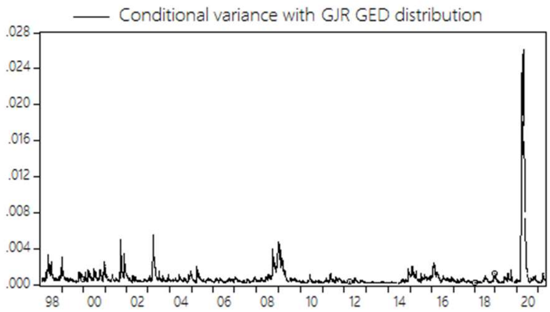


Figure 5. 2: Conditional Variance WTI spot GJR GED dist

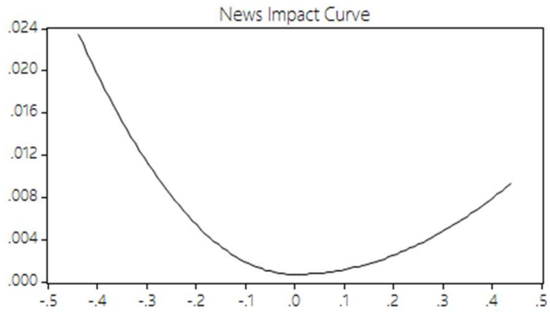


Figure 5. 3: News Impact Curve WTI spot GJR GED

a very volatile period for WTI markets. Figures 5.3 and 5.5 present the outcome of bad news on WTI spot and futures markets respectively, that the asymmetry term captured in GJR models. For heating oil markets only GJR models could capture ARCH effects with normal distribution after 1 lag for 5% and with GED distribution after 3 lags.

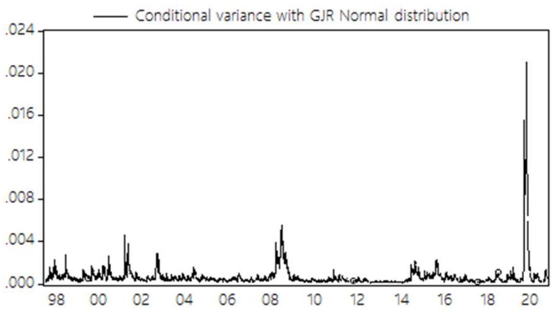


Figure 5. 4: Conditional Variance WTI futures GJR normal dist

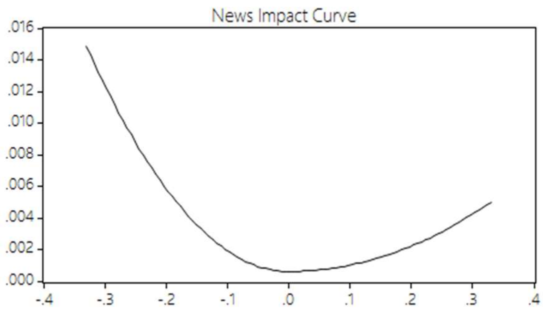


Figure 5. 5: News Impact Curve WTI futures spot GJR normal

Normal distribution captures better ARCH effects, but AIC is the 2nd highest. Therefore if we had to pick a model, we would pick either GJR with normal distribution ,where ARCH effects are better captured, or with GED distribution, where we have a lower information criteria. Asymmetry term is not statistically significant, indicating that heating oil spot markets do not have leverage effects. In addition, we do not have serial correlation in any distribution for 5% significance level. For both cases, our series have high volatility dependence and persistence, while volatility clustering is higher compared to WTI spot markets with GJR models. Therefore volatility in heating oil markets, do feed in higher degree future volatility, while WTI markets may present a more complex market, being affected by other variables and having a closer relationship with the world of financial markets. Further research would give us a better view. On the other hand, for heating oil futures markets, our best model is EGARCH with t distribution, where p-value of ARCH LM test is 0.1859, and it also has the lowest AIC. Q-stat shows serial correlation while Q^2 implies no serial correlation for 5%. Heating oil futures have high volatility persistence and dependence, with a high GARCH term while the asymmetry term in EGARCH model is statistically significant and negative with a coefficient -0.040611. The negative γ coefficient shows a negative risk-return relationship and counts for leverage effects. Figure 5.7 show the impact that negative news has on conditional variance. Lastly for brent spot markets, EGARCH with t student distribution is the model

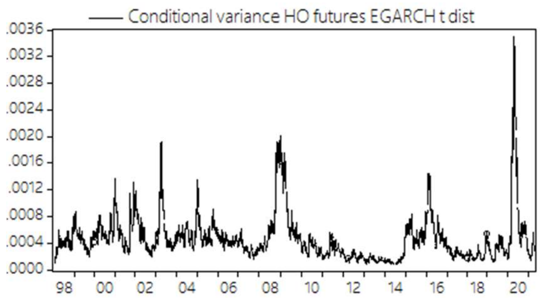


Figure 5. 6: Conditional variance HO futures EGARCH t dist

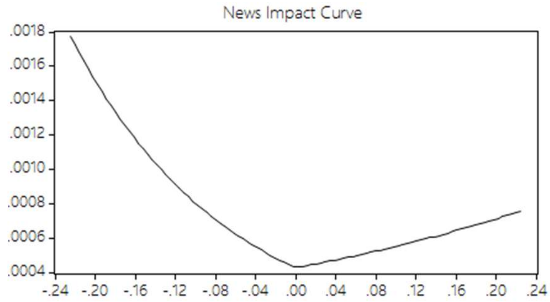


Figure 5. 7: News Impact Curve HO futures EGARCH

model that has the lowest AIC but does not capture well ARCH effects. The optimal model with low AIC (4th lowest) and that captures ARCH effects, is GJR with GED distribution. Brent spot markets, do have low ARCH term, indicating that they do have lower volatility clustering compared to other series we saw, while there is strong volatility dependence. The asymmetry term for our GJR model with GED distribution, is statistically significant and positive, equal to 0.070. That means we do have leverage effects and if negative news occurs, the conditional variance will increase more by 0.070. Q stat shows that we do have serial correlation while Q^2 shows that we do not have for 5% significance level. In summary, among all the series, GARCH models show that gas spot markets are the spikiest series, with significant ARCH term and volatility clustering, as well as persistence and dependence. Normal distribution could not fit well, and the process seemed to be non-stationary. Through several re-estimates, GED distribution seemed to be the more robust with the same results. The best model to use was either EGARCH with GED distribution, or GJR with GED distribution, depending whether our attention is more on the lowest information criteria or the ability to capture ARCH effects. On the other hand, gas futures markets seemed to have less cluster and not be as spiky as the spot market. Leverage effects where not present for those two markets.

WTI markets, both spot and futures, had significant volatility dependence and persistence, while ARCH term was not high according to GJR models, but it was significant according to GARCH models. The asymmetry term according to GJR GED for spot markets and GJR normal for futures markets showed presence of leverage effects, where bad news affect volatility more than only good news would have done. Heating oil spot markets do not have leverage effects and asymmetries, but GJR models indicated that they have higher volatility clustering from the WTI spot markets. On the other markets side, the heating oil futures market, leverage effects where present. Lastly for brent oil spot markets, the optimal model, GJR with GED distribution, showed very low volatility clustering, while linear GARCH models presented an ARCH term between 0.6 and 0.8. Volatility dependence among previous periods volatility was significant and leverage effects as well.

		GARCH(1,1)			EGARCH(1,1)			GJR-GARCH(1,1)		
		Normal	t-student	GED	Normal	t-student	GED	Normal	t-student	GED
	ω	3.98E-05	8.15E-05	1.05E-05	-0.421807	-0.486982	-0.472517	4.07E-05	7.89E-05	7.19E-05
	α	0.184814	0.180946	0.186563	0.31083	0.289345	0.295565	0.205392	0.185537	0.196268
gas spot	β	0.818269	0.786348	0.788452	0.969926	0.957707	0.960824	0.817775	0.787165	0.787973
	γ	--	--	--	0.021927	0.001775	0.006542	-0.041844	-0.007878	-0.019139
	ω	1.41E-05	1.76E-05	0.001171	-0.237246	-0.236948	-0.239172	1.41E-05	1.77E-05	1.67E-05
	α	0.083271	0.071999	0.15	0.17465	0.154845	0.162725	0.085086	0.076818	0.079947
gas futures	β	0.90887	0.913906	0.6	0.984872	0.982822	0.983385	0.909249	0.914426	0.911484
	γ	--	--	--	0.006515	0.012466	0.00918	-0.004728	-0.011597	-0.008656
	ω	5.87E-06	4.74E-06	5.28E-06	-0.284282	-0.229418	-0.258712	5.83E-07	4.7E-06	5.42E-06
	α	0.095558	0.080083	0.08645	0.204919	0.164113	0.184421	0.090953	0.074442	0.083663
ho spot	β	0.899219	0.914783	0.907762	0.983258	0.986382	0.984636	0.899084	0.915184	0.905378
	γ	--	--	--	-0.014447	-0.017244	-0.016458	0.009977	0.010952	0.010326
	ω	0.000116	1.56E-06	2.01E-06	-0.147568	-0.123155	-0.131707	2.21E-06	1.53E-06	1.66E-06
ho futures	α	0.149816	0.042036	0.047973	0.109906	0.095519	0.10031	0.02879	0.025664	0.023391
	β	0.599816	0.955595	0.948822	0.991882	0.993584	0.992985	0.945184	0.955743	0.953801
	γ	--	--	--	-0.048044	-0.040611	-0.043459	0.044579	0.033185	0.040859

	ω	1.12E-05	8.72E-06	9.42E-06	-0.269655	-0.205201	-0.225652	1.02E-05	7.71E-06	8.46E-06
wti spot	α	0.105381	0.085988	0.092275	0.173914	0.14096	0.150817	0.054087	0.04184	0.045066
	β	0.882199	0.903306	0.895481	0.98166	0.986922	0.985275	0.894679	0.913102	0.906463
	γ	--	--	--	-0.06558	-0.06006	-0.063742	0.07778	0.07021	0.07395
	ω	9.89E-06	6.36E-06	7.36E-06	-0.228697	-0.171819	-0.179668	7.76E-06	5.62E-06	6.34E-06
wti futures	α	0.101808	0.076794	0.085321	0.150807	0.122337	0.12674	0.03984	0.036167	0.036395
	β	0.886314	0.914854	0.905035	0.985052	0.989698	0.989178	0.905953	0.923848	0.916277
	γ	--	--	--	-0.075783	-0.059434	-0.068582	0.088819	0.064251	0.077312
	ω	4.29E-06	4.17E-06	4.22E-06	-0.189115	-0.153801	-0.164777	4.10E-06	3.42E-06	3.61E-06
brent spot	α	0.081162	0.066467	0.072262	0.133916	0.109456	0.117923	0.025983	0.022173	0.023718
	β	0.91609	0.928833	0.923306	0.988454	0.990687	0.990103	0.930212	0.941036	0.936814
	γ	--	--	--	-0.066927	-0.055381	-0.060301	0.079845	0.065299	0.070588

Table 5. 1: Univariate GARCH results

5.2. Results for Univariate GARCH spillover effects

In the section above, we modelled volatility using univariate GARCH models and found the best univariate model for volatility modelling. To estimate the conditional variance, we will use the best models for volatility modelling found previously. That said, for WTI spot we will use GJR with GED distribution, for WTI futures GJR with normal distribution, for gas spot we will use GJR with GED distribution since it captures better the ARCH effects, for gas futures EGARCH with t distribution, for heating oil spot markets we will use GJR with GED distribution, for futures markets EGARCH with t distribution and for brent markets GJR with GED distribution. Lag length criteria suggest a VAR(8) system with AIC -98.36302. However we will use a simple VAR(1) model for simplicity, even though it is not the optimal model, with AIC -96.98788 where there is not significant difference. The coefficients between the own series (e.g. wtispot(-1) wtispot) are very close to one and that may indicate that our series are not stationary. To ensure stationarity we are going to get the first differences on our conditional variance and run the VAR model. Granger causality test showed in Appendix show that heating oil futures and brent spot markets granger causes WTI spot and future markets. In addition, WTI markets and brent spot markets granger causes heating oil future markets and help us explain and predict heating oil future markets values. Lastly, WTI markets also granger causes brent spot markets. All inverse roots of the AR polynomial have modulus less than 1 and lie inside the unit circle, implying that the VAR is stable.

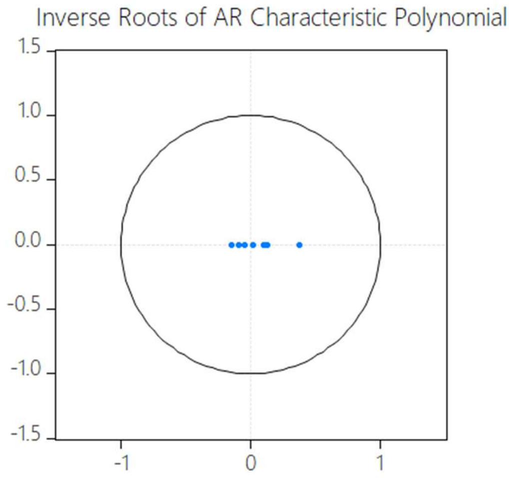


Figure 5. 8 : Inverse Roots of AR Characteristic Polynomial

The problem of autocorrelation and serial correlation could not be fixed with neither lag addition in our model. Therefore, even our optimal VAR(8) model, according to MAIC, is not sufficient to model the dynamics and the interpretation of the VAR results will be conducted with our VAR(1) model. Table 5.2. present the results for our VAR(1) model. We can see that WTI futures affect volatility of spot market, where an increase of 1 point in futures variance difference will increase spot variance difference by 0.095 points. However the spillover from heating oil futures is more significant, with a coefficient equal to 0.46. Negative spillovers occur from WTI spot markets to futures variance, where an increase in spot markets variance will decrease futures market variance. Again heating oil futures markets, are the market with the highest spillover in WTI market, this time in futures markets, where an increase of 1 point in variance difference in heating oil futures will increase WTI futures variance difference by 0.55 points. A negative spillover occurs in brent spot markets, where an increase in brent's spot variance difference will decrease WTI futures variance difference. In gas markets and Heating oil spot markets, spillovers occur only with their product in different markets. That means there are spillovers from spot to futures and reverse. However, had heating oil futures markets increase in variance difference by 1%, heating oil spot markets variance difference would drecrease by 0.22 points. Finally, spillovers for brent markets and heating oil futures arise from WTI markets and brent and heating oil futures markets correspondingly.

	WTI spot variance	WTI futures variance	Gas spot variance	Gas futures variance	HO spot variance	HO futures variance	Brent spot variance
WTI spot variance (-1)	-0.140487	-0.100631	-0.097788	-0.014866	-0.053958	-0.017649	-0.134443
	(-5.85300)	*(-4.65643)*	*(-0.37835)*	*(-1.02945)*	*(-1.02945)*	*(-4.20320)*	*(-8.60848)*
WTI futures variance(-1)	0.095428	0.063403	0.178783	0.022608	0.031546	0.016349	0.071444
	(3.74109)	*(2.76066)*	*(0.65090)*	*(1.47114)*	*(0.99020)*	*(3.66375)*	*(4.30462)*
Gas spot variance (-1)	-9.03E-05	-0.000288	0.37545	0.001173	-5.66E-05	-0.000133	-0.000163
	(-0.07880)	*(-0.27890)*	*(30.4207)*	*(1.69890)*	*(-0.03952)*	*(-0.66371)*	*(-0.21860)*
Gas futures variance(-1)	-0.017559	-0.014503	0.714728	-0.04557	0.014598	-0.003069	-0.015414
	(-0.79094)	*(0.01999)*	*(2.98988)*	*(-3.40623)*	*(0.52652)*	*(-0.79024)*	*(-1.06710)*
HO spot variance(-1)	-0.011794	-0.001382	0.02351	0.002782	0.134138	0.002698	-0.008172
	(-1.08584)	*(-0.14127)*	*(0.20100)*	*(0.42507)*	*(9.88784)*	*(1.141962)*	*(-1.15621)*
HO future variance(-1)	0.464757	0.54921	-0.708409	-0.075755	-0.229742	-0.093786	0.22255
	(4.99998)	*(6.56238)*	*(-0.70777)*	*(-1.35278)*	*(-1.97900)*	*(5.76769)*	*(3.67974)*
Brent spot variance (-1)	-0.091929	-0.0792979	0.038277	0.009098	-0.005024	0.016479	0.054581
	(0.02578)	*(-3.14456)*	(0.13791)	*(0.58584)*	*(-0.15607)*	*(3.65855)*	*(3.67974)*
M	-9.07E-09	1.30E-08	5.06E-08	9.32E-09	1.03E-08	2.85E-08	5.68E-09
	(-0.00243)	*(-0.00388)*	*(-0.00126)*	(0.00415)	*(0.00222)*	*(0.04370)*	*(0.00234)*

Table 5. 2 : Long term spillovers using VAR model with conditional variances from Univariate GARCH models

5.3. Results for Univariate GARCH models Forecasting

Our results in summary show that the appropriate model for volatility modelling, is not the same with the model for forecasting. However, both in modelling and forecasting, ambiguity arises about the most efficient model to use. With the use of our 3 loss functions, different efficient models arise for the same timeseries. To pick the best, we will see how the forecasted returns are compared with the actual. For gas spot markets Root Mean Squared Error suggest GJR with t distribution as the best model, while Mean Absolute Error suggests GJR with GED distribution. For gas futures, the RMSE suggest using GARCH with GED distribution, while MAE and MAPE are minimized with GJR GED distribution. In Heating oil markets, all three-loss function agree that GJR with normal distribution is the optimal model to use, while for the futures markets there is ambiguity. RMSE shows that it is minimized using GJR with t-student distribution, while MAE suggest GARCH(1,1) with GED distribution. For WTI spot markets according to RMSE the best model is to use EGARCH with normal distribution, while according to MAE and MAPE, GJR with normal distribution is the best model for forecasting. For futures market, EGARCH with t distribution is the best model according to MAE and MAPE, while according to RMSE is EGARCH with normal distribution. Lastly for Brent markets, RMSE and MAE agree that the best model to use is GJR with Normal distribution, while MAPE suggests EGARCH with normal distribution. As we can see, our best models differ depending on the loss function, but there is an agreement for the best model for forecasting in heating oil spot markets. A more advanced method to decide which model to use and which loss function is the best, is to run a SPA test. By trying to make a conclusion about the best model visually, to see how the values converge to the actual, no clear deduction can be made. For WTI futures case only, it is possible that the fitted values come closer to the actual as presented in Appendix

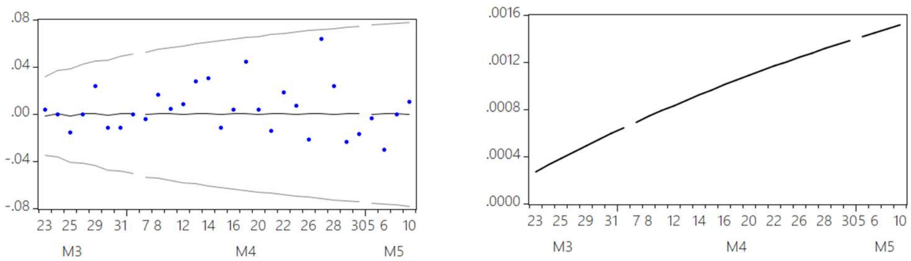


Figure 5. 9: Forecasted returns and variance, gas spot GJR t distribution (RMSE)

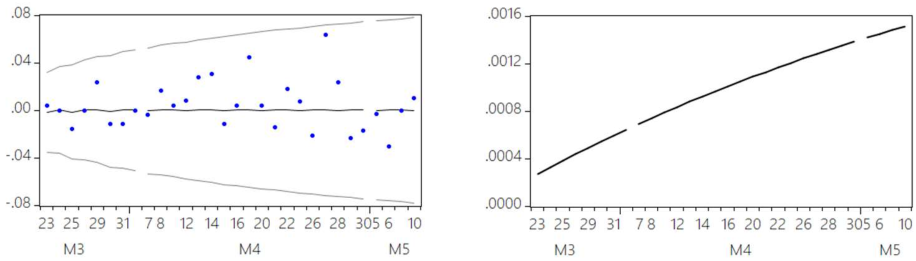


Figure 5. 10: Forecasted returns and variance, gas spot GJR GED distribution (MAE)

, even though some actual values are outside the confidence interval. Still this is a very abstract conclusion. Figures for forecasted variance and returns according to our loss functions are displayed in Appendix, so are results for our series and loss functions. In summary, in most cases optimal models for forecasting are not the same with optimal models for volatility modelling. In addition, while t and GED distribution seemed as the best to use in volatility modelling, for forecasting, normal distribution give better results in minimizing the loss functions most times.

5.4. Results for Multivariate GARCH models, BEKK

5.4.1. Partial covolatility spillovers – Diagonal BEKK model

As already mentioned, according to Chang et al. (2019) (C.-L. M. Chang), matrix A in BEKK models is not capturing spillover effects. What they represent is the weights or multipliers for the returns shocks. In our paper, following Chang et al. (2019) (C.-L. M. Chang) method, to calculate partial covolatility spillover effects, as we use diagonal BEKK model, we multiply the weight of asset i times weight of asset j times the mean returns shock of asset j. As presented below in the results for matrix A and the mean returns shocks for our series, the highest weight for return shock and the highest mean return shock is for gas spot, confirming that it is probably the spikiest series with the highest volatility clustering. Year 2020 and the ongoing crisis of 2021 (no data) can confirm this, where gas prices maintain an aggressive upward trend in prices, with significant increases and returns. On the other hand, WTI, heating oil and brent markets, have weights that are very close to each other, ranging from 0.199409 to 0.231388. The same implies for their mean returns shock as well, while gas futures is the only series with negative mean return shock and lowest weight for return shock among the series. In Appendix tables 5.14 and 5.15, correlation for returns and prices respectively are presented.

Diagonal BEKK weights and mean returns shocks

	A(i,i)	$\hat{\varepsilon}_{\tau-1}$
WTI spot	0.219087	0.002753
WTI futures	0.219225	0.003205
Gas Spot	0.434516	0.064726
Gas futures	0.173724	-0.00745
Heating oil spot	0.231388	0.003205
Heating oil futures	0.212637	0.002371
Brent spot	0.199409	0.030903

Table 5. 3: A(i,i) matrix and mean return shocks

While there are strong correlations between spot and futures markets in prices, that changes dramatically when we give attention to returns. The same implies to oil markets, where correlations among brent, WTI spot and futures markets, and heating oil spot and futures markets decreases significantly in several combinations. However WTI spot and futures markets maintain strong correlation (but decreased) for returns as well. Table 5.4 gives us the results for our mean equation. Matrix A is sufficient to explain for return spillover effects among the markets. Statistical significance is not available, due to failure of likelihood to continue after a few iterations. We are going to assume that all are statistically significant for explanatory reasons. There is significant spillover from WTI future markets returns to WTI spot market returns, where a 1% increase in WTI futures returns in the previous day, would increase by 0.17% WTI returns the next day. The impact from other markets is not even close to the impact the futures market has to spot, which is a conclusion that makes sense. For WTI futures markets, surprisingly heating oil future markets affect more WTI futures than WTI spot does. If heating oil futures returns increase by 1%, WTI futures returns will decrease by 0.08799% . For gas spot, the biggest spillover is from gas futures, where an increase of 1% in gas futures returns the previous day, would increase gas spot by 0.689%, while for gas futures, the most significant spillover comes from WTI futures markets. For heating oil markets, spot and futures and brent, do have the highest impact on the returns spillovers. Specifically, would brent returns increase by 1%, heating oil spot would have increased by 0.033%, while heating oil future returns would have increased spot returns by 0.17%, where it is the dominant spillover (excluding its own market effect). Returns correlation among the two heating oil markets present a strong and positive correlation, however the negative impact from spot to future returns is significant. If heating oil spot markets increased by 1%, heating oil futures markets would have increased by 0.076%. Lastly, brent market is affected significantly, excluding its own market, from WTI and heating oil futures, with the latter having the highest spillover effect between the two. Returns correlation among heating oil futures, WTI futures and brent spot is also high, positive, and significant. There is a quite strong linkage between those markets. Table 5.5 presents the results for mean partial covolatility spillovers. Results for significance where not available, however we will assume the values as statistically significant. It can be observed that some values are very small and not significant all. The highest variance to other markets arises from gas spot markets, as can been seen in the 3rd column of series. The highest and most significant covolatility spillover is from heating oil spot markets to gas spot markets covolatility with heating oil spot. In gas futures, in every case, the variance is decreasing slightly and not increasing. In any other case, positive spillovers occur, but the significance of the results is not available. Covolatility spillovers from futures to spot and inversely is negligible. In summary, there are partial covolatility spillover, with the most volatile coming from gas spot markets, while in the gas futures, the spillovers are decreasing the volatility, but the values are extremely close to zero. WTI future and spot markets don't have significant covolatility spillover effects, contrary to what we may have suspected. In our sample, gas spot markets tend to have more effect in those two markets. The same implies for heating oil spot and futures markets.

Diagonal BEKK mean equations							
	DLWS	DLWF	DLGS	DLGF	DLHS	DLHF	DLBS
C	0.000687	0.000668	-0.00021	-0.00022	0.000604	0.000648	0.000439
DLWS(-1)	-0.143365	0.068598	0.065647	0.009584	-0.0149	0.017356	0.088995
DLWF(-1)	0.170097	-0.03992	-0.06114	-0.04282	-0.00232	-0.00296	0.152738
DLGS(-1)	-0.003406	-0.00297	-0.16087	0.011718	0.004779	0.00142	0.002508
DLGF(-1)	0.015786	0.014659	0.689064	-0.01374	0.021586	0.020553	0.007457
DLHS(-1)	0.043665	0.043933	0.020376	-0.03209	-0.18995	0.075995	0.051988
DLHF(-1)	-0.093982	-0.08799	-0.04336	0.026439	0.170009	-0.11638	0.160452
DLBS(-1)	0.030474	0.02612	-0.05663	-0.01128	0.033232	0.020616	-0.24782

Table 5. 4 : Diagonal BEKK mean equation

average partial covolatility spillovers								
	i	WS	WF	GS	GF	HS	HF	BS
j	WS	--	0.000132211	0.00026	0.0001048	0.00014	0.000128238	0.00012026
	WF	0.00015	--	0.00031	0.0001221	0.00016	0.000149402	0.000140108
	GS	0.00616	0.006165617	--	0.0048859	0.00651	0.005980332	0.0056083
	GF	-0.0003	-0.00028389	-0.0006	--	-0.0003	-0.00027536	-0.00025823
	HS	0.00016	0.000162562	0.00032	0.0001288	--	0.000157677	0.000147868
	HF	0.00011	0.000110512	0.00022	8.757E-05	0.00012	--	0.000100522
	BS	0.00135	0.001350932	0.00268	0.0010705	0.00143	0.001310335	--

Table 5. 5: Average partial covolatility spillovers

For our series, expect gas spot, our ARCH terms range between 0.03018 and 0.05354, while GARCH terms are above 0.9, indicating that the series (expect gas spot) have low clustering, and high dependence and persistence. However that is not the case for gas spot markets. They produce the highest constant among all other series, showing the risk these markets have, they have ARCH term equal to 0.188, indicating the significant reaction in shocks and the clusters, and they do have lower GARCH term. Concluding for gas spot markets, BEKK models show that they are the spikiest series among all, they have high persistence but not dependence, and they do produce the most significant volatility spillovers among the other markets.

Conditional Variance Equation			
	C	A	B
DLWS	2.28E-05	0.047993	0.926108
DLWF	2.14E-05	0.048059	0.928706
DLGS	1.08E-04	0.188804	0.771077
DLGF	3.48E-05	0.03018	0.95072
DLHS	1.99E-05	0.05354	0.925443
DLHF	1.54E-05	0.045214	0.935795
DLBS	1.81E-05	0.03976	0.93805

Table 5. 6 : Conditional Variance Equations BEKK model

In our analysis, it is a huge disadvantage that we do not have values for statical significance and residual diagnostics. We assumed that all our results are statistically significant for explanatory reasons but a conclusion about the series modelled with BEKK is a little bit abstract.

5.4.2. Comparison – Univariate and Multivariate Results

Both Univariate and Multivariate models confirm that gas spot markets are the spikiest series, with high volatility clustering. That means big changes in returns will follow big changes and small changes will follow small changes in a high degree. Univariate models showed that there are no leverage effects in both spot and future markets for gas. For BEKK models, all other series except gas spot have low ARCH terms and high GARCH terms, indicating that series are less spiky, but with high dependence on previous periods volatility. While there were small deviations between univariate and multivariate models in the GARCH term, the ARCH terms were very close. However multivariate models could not account for leverage effects since we did not incorporated the asymmetry term in the models. But results were not identical with gas futures. The univariate models showed a series with high ARCH term in the EGARCH models, whereas multivariate models show low volatility clustering. However, issues arise for several series in univariate models, where the ability to capture the ARCH effects is not significant. Investigating the WTI markets, no univariate model was able to well capture the ARCH effects and we had to use the one that has the highest p-value but still was negligible. Leverage effects were found in the markets. Identical situation is on the heating oil spot markets as well, where the two best models for volatility modelling could still not capture ARCH effects. While leverage effects were not present in spot markets, they were in heating oil futures markets, a market with high dependence and persistence and low volatility clustering according to univariate and multivariate models. ARCH effects are well captured with univariate models and they are sufficient for volatility modelling. Univariate models capture ARCH effects on brent markets, and they find asymmetries. Coefficients agree with the results of the multivariate models. Conditional variances from the univariate models, embedded into a VAR model to capture spillover effects found that heating oil futures have the highest impact in WTI spot and future markets. An increase in spot markets WTI volatility would decrease futures markets volatility. Negative spillover occurs as well in brent markets, where an increase in brent markets will decrease variance in WTI futures. Gas markets and heating oil spot markets are affected from spot to future market, while would heating oil futures variance first differences increase by 1%, heating oil's variance differences would decrease by 0.22 points. Spillover for brent markets and heating oil futures arise from WTI markets and brent and heating oil futures markets correspondingly. Partial covolatility spillovers showed that gas markets produce the most significant covolatility spillovers, confirming again the significant risk of the markets. In addition, strong covolatility spillovers from heating oil futures too. WTI markets have negligible spillover effects between them and with all other markets. Lastly, forecasts for univariate results showed ambiguity for the optimal model to use, since different loss functions suggested different models. In addition, best model for volatility modelling was not pared with the optimal model for forecasting. In forecasting normal distribution in most cases was the best for forecasting, in contrast to volatility modelling.

5.5. Results for Multivariate GARCH models Forecasting and comparison with Univariate GARCH models

As already mentioned, direct volatility forecasting was not possible, and the method we will follow is an indirect approach, by forecasting the returns to see how close they are to their actual. The comparison will not be based on the models, but whether if univariate or multivariate models are better in forecasting. Our results are very close with the results in the univariate analysis. Table 5.7 present the loss functions for our series, the same as those with the univariate analysis. Values are very close to what we had in our univariate analysis. Between them, the one with the lowest loss function, does better job in forecasting returns and therefore, probably variance as well. Table 5.8

Multivariate Forecasts			
	RMSE	MAE	MAPE
GS	0.02081	0.01534	NA
GF	0.01500	0.01250	99.89523
WS	0.02429	0.01718	101.88390
WF	0.02443	0.01736	97.11430
HS	0.02099	0.01430	97.26462
HF	0.01970	0.01345	NA
BS	0.02334	0.01562	98.46355

Table 5. 7 : Results for multivariate forecasts – returns

give us results for the best model to forecasted returns, and derived variance. We can see that MAE suggests in every case univariate models as the best models to use, while RMSE suggest for WTI futures multivariate models. MAPE in all cases except gas futures, suggests as optimal model multivariate models, contrary to what the other two loss functions proposed. Overall, we could agree with what Serletis and Efimova (Serletis) have found , and conclude that univariate models forecast better volatility, even though we did not follow a direct approach and our result may be abstract, while Serletis and Efimova (Serletis) did. A further direct approach would give more robust results.

Comparison with Univariate models				
		RSME	MAE	MAPE
Gas spot	->	Univariate	Univariate	--
Gas Futures	->	Univariate	Univariate	Univariate
WTI spot	->	Univariate	Univariate	Multivariate
WTI futures	->	Multivariate	Univariate	Multivariate
HO spot	->	Univariate	Univariate	Multivariate
HO futures	->	Univariate	Univariate	--
Brent Spot	->	Univariate	Univariate	Multivariate

Table 5. 8: Comparison between multivariate and univariate models

5.6. Results for Optimal Hedge Ratios

5.6.1. BEKK models

Figure 5.6.1 presents the results of the dynamic hedge ratios, as equation X.X. showed. WTI hedged position of spot market with futures, is presented with the blue line. The maximum price is 1.3056, while the minimum is 0.3920. Values above 1 represent that more futures are needed than spot, already hold in portfolio. Our lowest values for the dynamic optimal hedge ratios for WTI are presented for December of 2008. During that period prices were lowered, due to Lehman brothers collapse, however returns were volatile. In both cases the correlation is significant between prices and returns, and the low dynamic optimal hedge ratio may indicate that the very close relationship between those 2 series would have negligible impact on hedging.

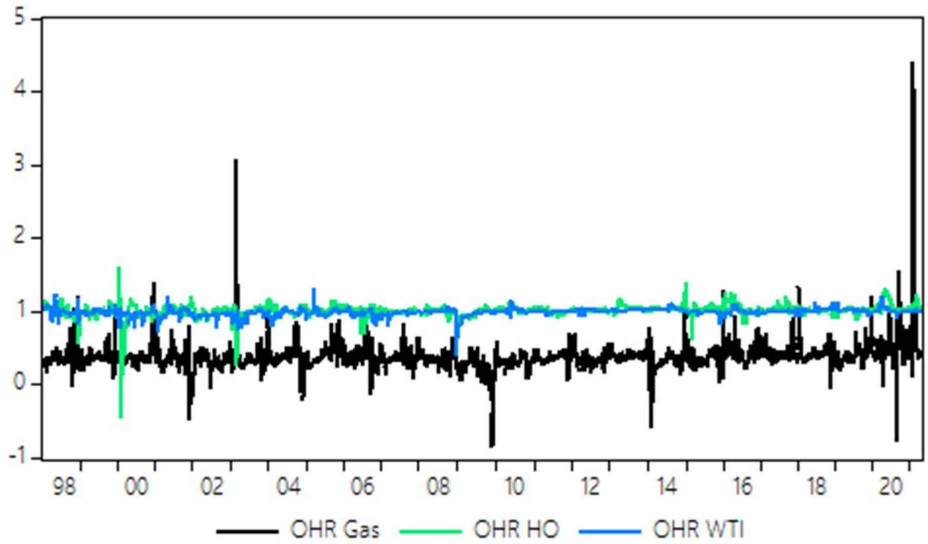


Figure 5.6. 1: Optimal Hedge ratios for WTI, heating oil and gas markets

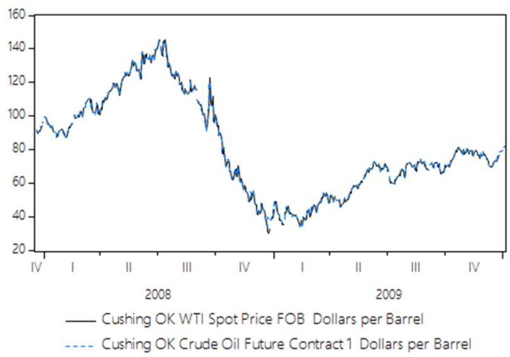


Figure 5.6. 2: WTI spot and futures prices for 2008 period

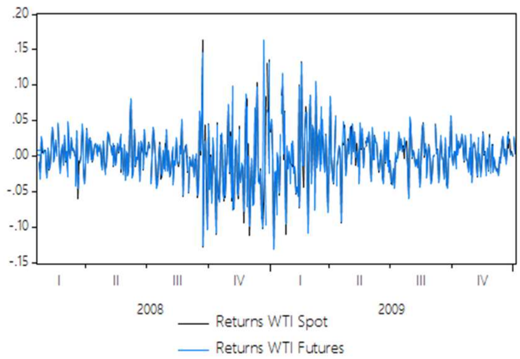


Figure 5.6. 3: WTI spot and futures returns for 2008 period

On the other hand, what we observe for heating oil optimal hedge ratios, is negative prices for period 2000. This does not mean necessarly that the futures of heating oil is not a good hedger, but indicates that it may have been better for that period to take a long position in futures markets and not spot markets. Lastly, the highest values for the optimal hedge ratios are taking place in the gas markets, where the maximum price for the dynamic hedger is 4.3890 and the minimum is -0.8513. By looking at the below two figures, 5.6.4 and 5.6.5, where values of the left axis is for the optimal hedge ratios and the right axis for the returns, the highest values for the optimal hedge ratios occur during the years 2002 and on the rough winter of 2020, where prices (and therefore returns as well) of spot and futures differ significantly. When a market participant would like to hedge his position, the ammount of futures he would need would have been significant. In our case in the gas markets, the futures contracts would have been in the range of something more than 3 and more than 4 dollars for every dollar in the spot markets, for those two increased volatile periods.

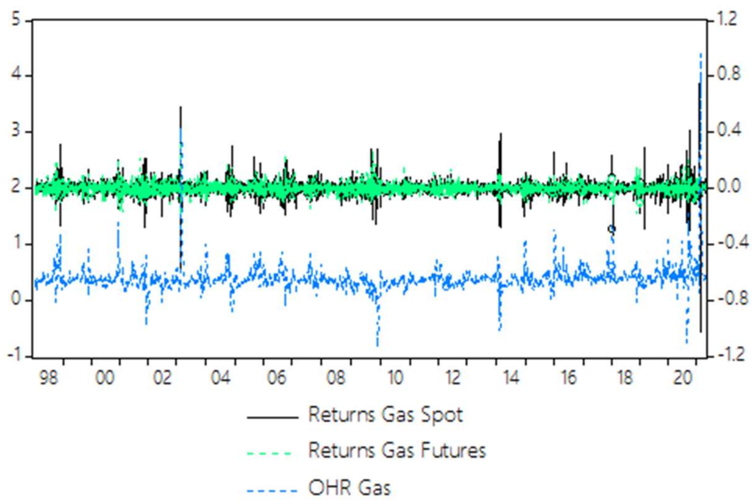


Figure 5.6. 4: Optimal hedge ratio, and returns fors gas markets.

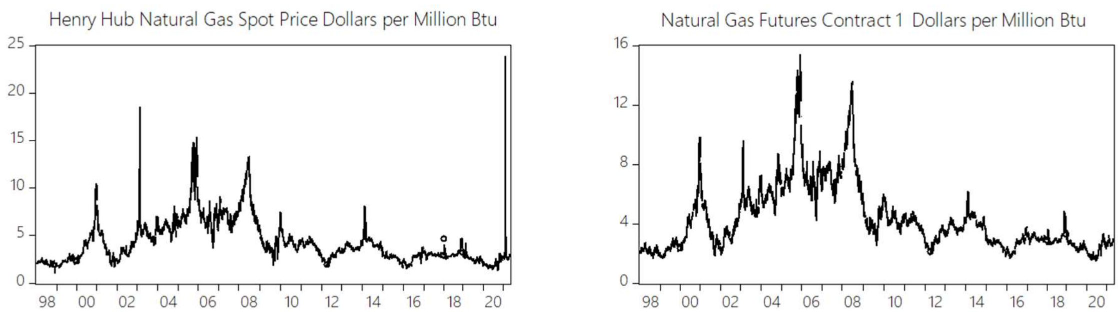


Figure 5.6. 5: Gas markets prices

However, in order to test for the effectivness of the hedge ratios, we must test the difference on the returns with the hedger. According to Equation 26 the hedged series is the difference between the returns of the spot market of a series minus the adjusted returns for the spot market with the optimal hedge ratio for that particular market. For the hedger to be effective, the derived variance in the adjusted series must be lower than the original. Testing for WTI spot markets, the new adjusted variance of returns for

WTI spot must be lower than the original. In order to test that, we will model volatility using ARMA(0,0) and run a simple GARCH(1,1) model with normal distribution to obtain the conditional variances for our simple and adjusted returns for our three markets.

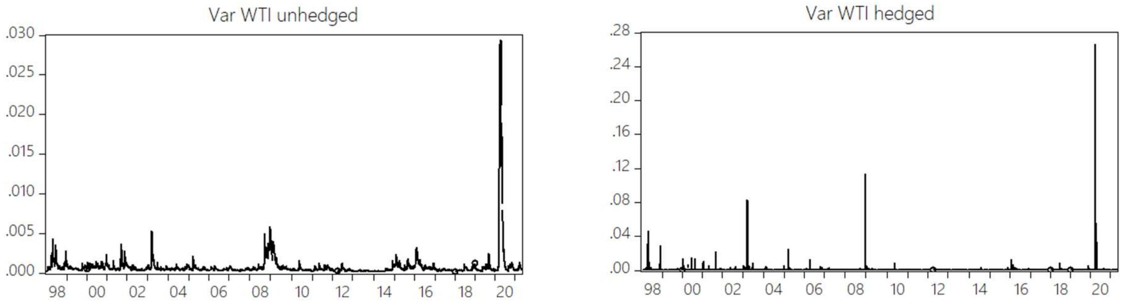


Figure 5.6. 6: unhedged vs hedged conditional variace for WTI markets

Figure 5.6.6, displays the unhedged (left graph) and the hedged (right graph) conditional variance of WTI. We cant properly conclude, whether the hedged series are better or not. In some cases the hedged returns do have lower conditional variance, especially in the crucial period of 2020, however in some other cases of market stability, we do have higher variance. We also have higher variance in the financial crisis period. We need to compute the hedging effectiveness index that Chang et al. (2011) ; Ku et al. (2007) (M. M. Chian-Lin Chang) used. Table 5.6.1 displays the hedging effectiveness index and OHR mean results. The mean optimal hedge ratio for WTI spot indicates that to hedge a one dollar long position in spot WTI, $0.9813 are required shorted in futures markets. For December 22 of 2008, we get an unusually high value compared to results from literature, equal to -26.2139. For that specific period our hedged returns are -22.9%

BEKK

Series	Mean OHR	Max OHR	Min OHR	Mean HE index	Max HE index	Min HE index
WTI Spot	0.9813	1.3057	0.0480	0.6601	0.9978	-26.2139
HO Spot	1.0040	1.5886	-0.4583	0.7783	0.9815	-7.5693
Gas Spot	0.3821	4.3890	-0.8513	0.0893	0.8558	-1.2166

Table 5.6. 1: Hedging Effectiveness and OHR table BEKK

,equal to approximately -25, or -2500%. For that specific period, our hedged returns are -23.09%, while the unhedged position is -6.44%. The optimal hedge ratios suggest for every $ in spot, hedging with $1,014 in futures markets, while the returns for WTI futures were 16.5%, and that is how the negative outlier in hedged returns occur. In other periods the same may imply and extreme values arise. However, even with those extreme negative values, the high kurtosis and negative asymmetry, the mean value of the index is 66%, indicating overall a decent hedging effectiveness. However those extreme negative values in certain periods of the sample are not persuasive for the effectiveness of the model, or the series at the certain period, to hedge at all. During the extreme negative price of 2008, conditional variance of the hedged returns, was

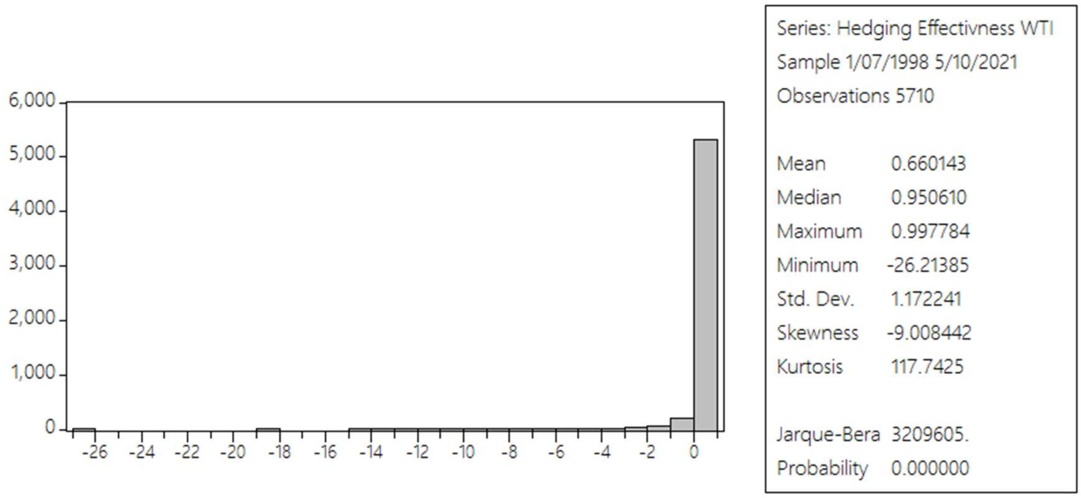
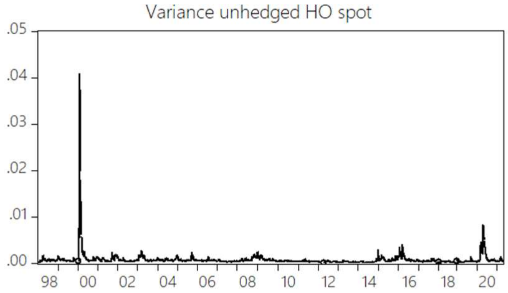


Figure 5.6. 7: Histogram and Descriptive Statistics for HE WTI

significantly higher than the unhedged position, resulting to this extreme negative hedging effectiveness value. According to Wang et al. (2012) (Yudong Wang), diagonal BEKK models are the optimal models to hedge crude oil price with jet fuel. Chang et al. (2011) (M. M. Chian-Lin Chang) found that diagonal BEKK models are the best models to hedge crude oil spot markets using WTI future markets. Similar results , as in WTI markets, we obtain for heating oil markets, with extreme volatility during early 2000. Hedged returns did not performed better at all instances than unhedged series. For the crucial period of 2000, hedged returns had significantly higher variance than unhedged. Further research would be useful to determine if at certain periods , like this one, the correlation among the two markets is close to negative, and therefore a short position in the futures markets makes our returns worse, and therefore increases our volatility. In Appendix , Table 5.15, overall the correlation between spot and futures returns is very high at 0.8. However, figure 5.6.9 shows that the derived dynamic correlation from BEKK model a negative correlation for that period, that derived from the dynamic correlation from BEKK model, confirming our assumption. The mean HE index shows a value equal to 77.83%, but as in the WTI case, there are many extreme negative negative values



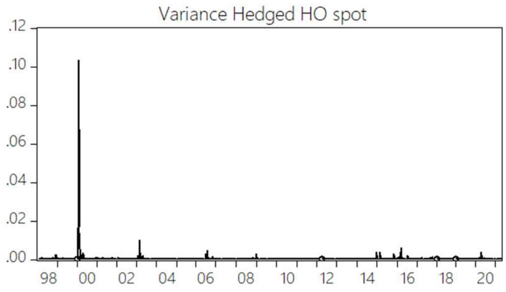


Figure 5.6. 8: unhedged(left) vs hedged (right) conditional variance for heating oil markets

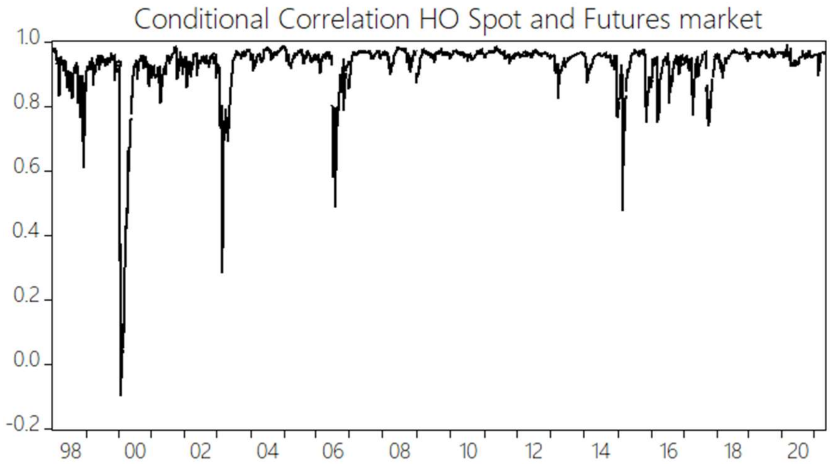


Figure 5.6. 9: Conditional Correlation HO spot and futures marekts diagonal BEKK

during our sample period, that questions the effectivness overall. Looking at our returns for spot and futures markets, there are specific days where the spot market had negative returns while the futures closed with positive. The hedging may make our returns even worse and that is how these extreme values arise. Lastly for gas markets not big deviations in the index occur. The minimum value we obtain is -1.2165 or -122% and the maximum value 0.8558 or 85.6%. In that case, futures markets do work efficeintly as a hedger in most cases. But as presented in the graph below, even in this case, the index shows again many negative values and still it is not persuasive that gas futures can be used to hedge spot positions in gas markets. In addition, the mean of the hedging effectivness index is low, equal to 8.9%. In summary, our symmetric diagonal BEKK finds mean hedging effectiveness prices that are above average. However, there are several extreme negative values that questions the effectivness in certain occasions. Gas markets do produce lower deviations compared to the other two markets.

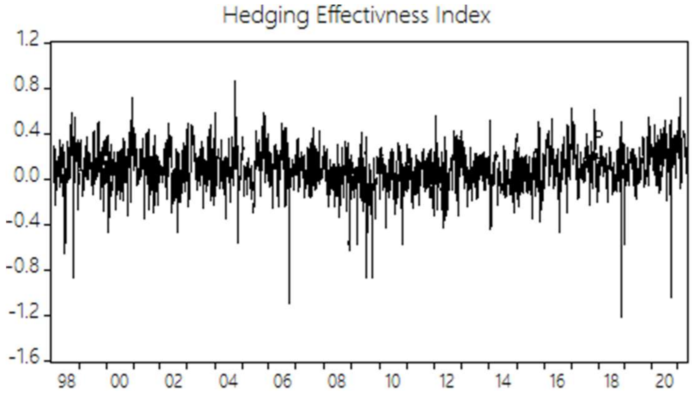


Figure 5.6. 10 : Hedging effectiveness index for gas markets

5.6.2. DCC Models

We have seen the optimal hedge ratios according to BEKK models. While for gas markets, there were no extreme outliers in the hedging effectiveness index, there are several significant outliers that questions the efficiency of hedging in the WTI and heating oil markets. The main reason for that is the difference in returns. In one market we had significant negative returns and in the other significant positive, in many periods through our sample. What the optimal hedge ratios should have indicated, is that the ratios should be negative, if the returns were negative in spot markets. That would imply, that is better if we would had positioned ourselves in futures markets, with significant positive returns. For WTI, heating oil markets and gas markets, we will use the DCC model to see the results we will get. We will run a DCC model using our 4 series together and a separate DCC model for the gas markets. That said, our DCC model will include WTI spot and future returns and heating oil spot and future returns in the 1st model, and gas spot and futures in the 2nd model. The optimal hedge ratios for our series are moving close to their conditional variances of the markets, contrary to what we saw in BEKK models, where the values where superior higher. That means that hedging will be negligible for small portfolio amounts, since for $1 in spot markets, approximately $0 would be needed for hedging. For WTI markets, the hedging effectivness index shows that the futures markets, with the current optimal hedge ratios from DCC model, are extremely sufficient to hedge the spot position. The mean HE idex shows 99.86% efficiency for hedging, with a maximum value of 1.0015 and a minimum value of 0.9665 for the COVID-19 period, as presented in Figure 5.6.11 below.

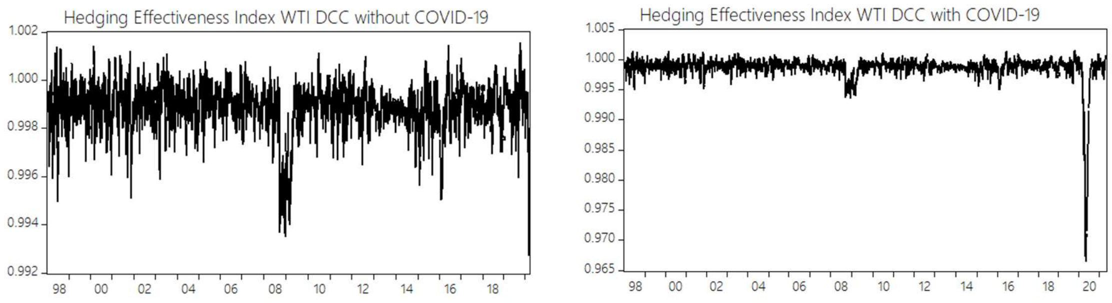


Figure 5.6. 11: Hedging effectivness, without covid (left chart) and with covid (right chart) in our sample

However, we can not say that the hedging is effective in heating oil markets. The hedging effectivness index produced mostly negative significant values, probably indicating that with DCC analysis we can't fing proper optimal hedge ratios to hedge our position, or that we should search for a different product to hedge our position, or even narrow and the change the sample of study. The mean HE index value was -25.17%, with a maximum value of 97.89%, and a minimum value of -1,235.94% for the COVID-19 crisis, with other significant negative outliers in years 98 2000-2002 and the financial crisis. Lastly for gas markets, results are quite similar with BEKK model, where no significant outliers arised and the efficiency of the index is slightly

DCC

Series	Mean OHR	Max OHR	Min OHR	Mean HE index	Max HE index	Min HE index
WTI Spot	0.0007	0.0263	0.0001	0.9986	1.0015	0.9665
HO Spot	0.0006	0.0093	8.6300E-05	-0.2517	0.9789	-12.3594
Gas Spot	0.0009	0.1392	-0.0034	0.0898	0.8984	-1.1808

Table 5.6.2 : Hedging Effetiveness and OHR table DCC

increased, from 0.0893 in BEKK model, to 0.898 in DCC model, with approximately the same minimum and maximum values.

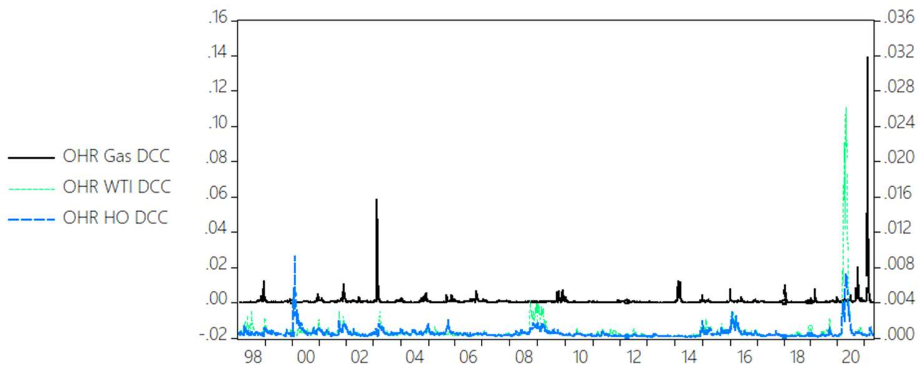


Figure 5.6. 12: Optimal Hedge ratios for WTI and Heating oil DCC (left axis HO&WTI, right axis Gas)

5.6.3. Comparison between BEKK and DCC for optimal hedge ratios

The optimal hedge ratios for BEKK models produced values close to 1 for WTI crude oil, and for heating oil and gas spot markets with bigger deviations, with the latter even reaching the maximum optimal hedge ratio value of 4.389. That indicates, for a market participant to hedge his spot position of $1, $4.389 would have been required in futures markets. On the other side, DCC models, estimate optimal hedge ratios closer to their conditional variance, thus closer to 0, with gas markets again deviating during the 2020 winter where gas prices surged. The hedging effectiveness index using BEKK model for WTI shows high mean value which resembels hedging efficiency. However the significant negative outliers does not persuade that the index is efficient at all. DCC models on the other side do give a very high hedging effectivness value close to 100%, with almost no outlliers. Clearly hedging is better in WTI markets using DCC models. On the contrary, heating oil markets do have better hedging efficeiny in BEKK models. But the negative outliers in certain periods of our sample arise ambiguity. A model to forecast these daily diffrences between spot and future prices is unlikely to be found. Finally for gas markets, BEKK and DCC produce identical results in hedging effectivenss, with the latter being sliglty better.

Series	Mean Optimal Hedge Ratio		Mean Hedged Variance		Mean Hedging Effectiveness Index	
	BEKK	DCC	BEKK	DCC	BEKK	DCC
WTI Spot	0.9813	7.31E-04	5.05E-04	7.87E-04	0.6601	0.9986
HO Spot	1.0040	5.50E-04	3.07E-04	2.01E-04	0.7783	-0.2517
Gas Spot	0.3821	9.29E-04	2.55E-03	2.55E-03	0.0893	0.0898

Table 5.6. 3: BEKK and DCC results

6. Conclusions

In this paper, we investigated the best GARCH models for volatility modelling, forecasting, checked for returns and variance spillovers among the markets and searched for the optimal hedge ratios from spot to future markets. In our sample, we found that gas spot markets are the spikiest energy market among the others, a statement that both univariate models and multivariate models can confirm. Leverage effects were not present in gas markets, but they were in WTI markets, brent spot markets and heating oil futures markets. Heating oil futures markets is a market that causes a lot of variance spillovers, either direct according to the univariate analysis, or partial covolatility spillovers, where it's variance affects the covariance between other markets. However, the most significant covolatility effects came from gas spot markets since they were the most volatile and also heating oil spot markets. Non-linear models did better work in modelling volatility and capturing ARCH effects. All other markets, excluding gas spot, have lower volatility clustering, and deviations do not feed in significant way future volatility. However, volatility dependence was higher in these series and persistence was strong in all energy markets. Therefore shock do fade away slowly in energy markets and significant deviations could have severe impact in the financial word and in the economies. Proper absorption must be achieved in countries and worldwide, when negative shocks occur in the energy markets, to prevent worst outcomes in world economy. Spillovers between WTI spot and future markets were negligible, while the biggest covolatility spillover comes from heating oil spot markets to gas spot markets. In terms of forecasting, different loss functions give different results for the optimal model to use, while in general univariate models are better in forecasting and our findings agree with the findings of Serleti and Efimova (Serletis). Finally, optimal hedge ratios using diagonal BEKK models present overall well mean hedging effectiveness values for WTI and heating oil markets, but they have significant negative values for certain periods, where the optimal hedge ratios suggested to have a long position in futures markets. For gas markets, during the 2020 surge, optimal hedge ratios even suggested hedging $1 position in spot markets with $4.39 dollars in futures markets. However, the hedging was not efficient overall. DCC models did a better job in hedging WTI markets with extreme efficiency, while for heating oil markets it is not the best model to use. In gas markets, the outcome and results were almost identical with the two models mentioned.

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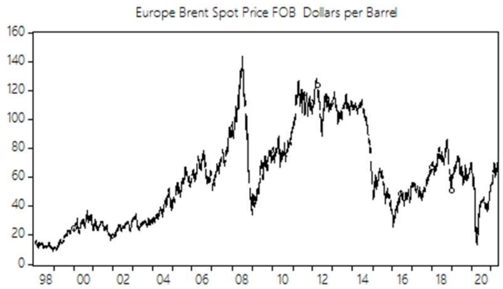
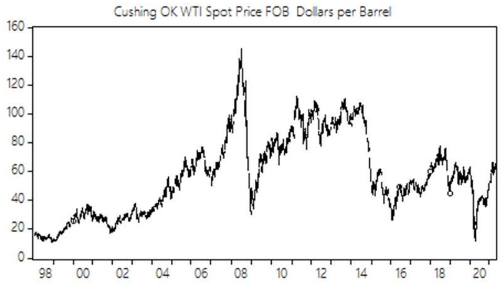
Appendix

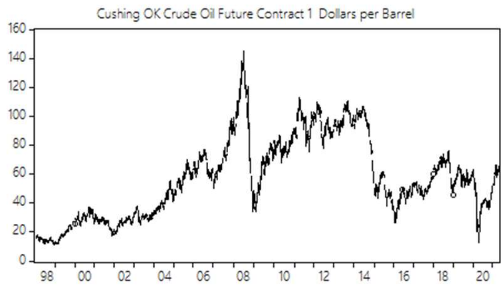
Brent spot prices



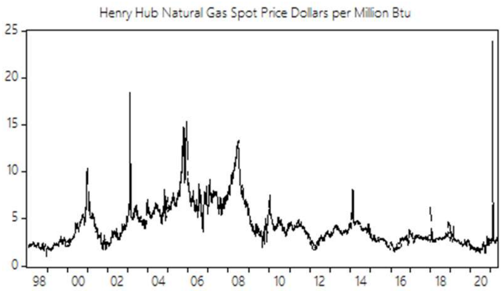
WTI spot prices



WTI Futures prices



Henry Hub Natural Gas spot prices



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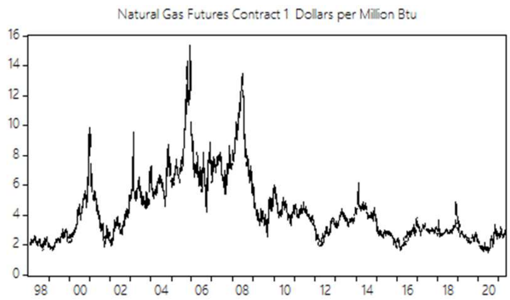
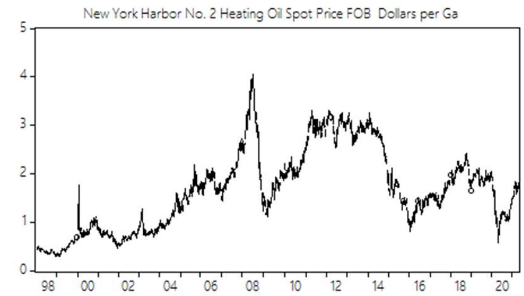


Heating oil spot prices New York Harbor



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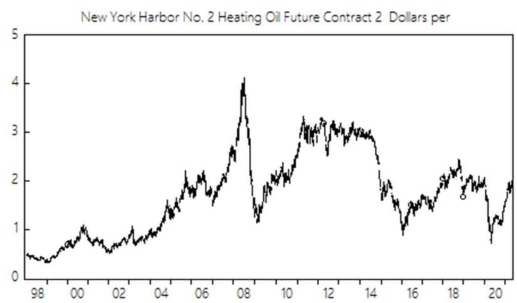


S&P 500 prices

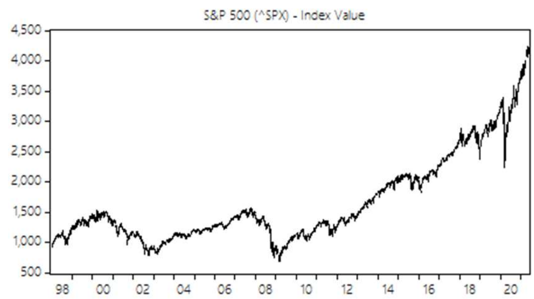
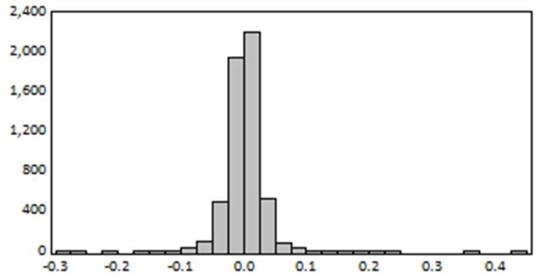
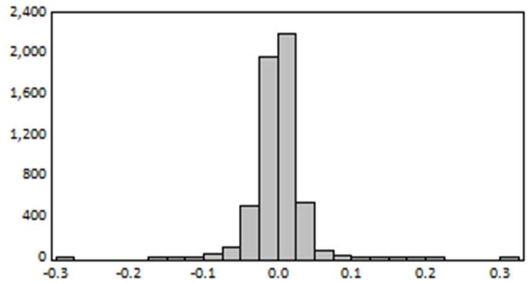


Figure 3. 1 : Plots of the series

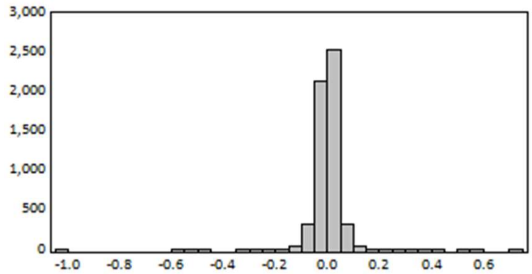
WTI spot returns descriptive statistics



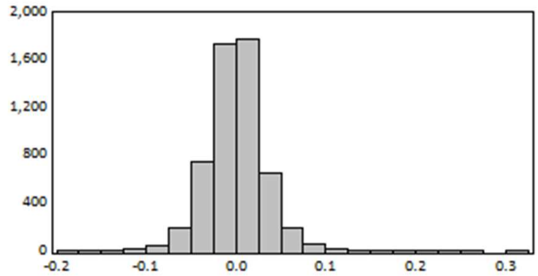
WTI futures returns descriptive statistics



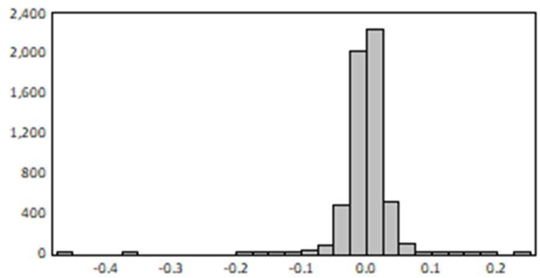
Henry Hub Natural Gas spot returns descriptive statistics



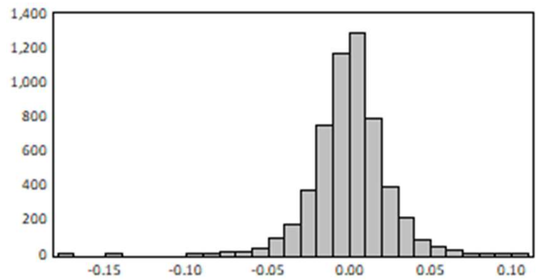
Henry Hub Natural Gas futures returns descriptive statistics



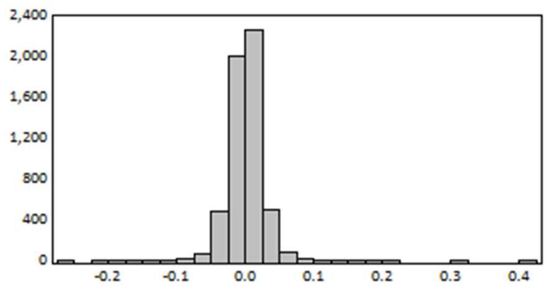
Heating Oil NY Harbor spot returns descriptive statistics



Heating Oil NY Harbor futures returns descriptive statistics



Brent spot returns descriptive statistics



S&P 500 returns descriptive statistics

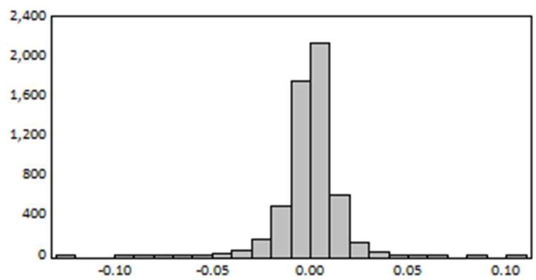


Figure 3. 2 : Histograms of the returns of our series

Table 3. 2: Unit root tests for the prices of our series

Null Hypothesis: WTI spot has a unit root

Exogenous: Constant			
Lag Length 1 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-2.097044	0.2460
Testcritical values:	1%	-3.43136	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: WTI futures has a unit root

Exogenous: Constant			
Lag Length 1 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-2.066268	0.2587
Testcritical values:	1%	-3.43135	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: gas spot has a unit root

Exogenous: Constant			
Lag Length 6 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-3.976629	0.0016
Testcritical values:	1%	-3.43136	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: gas futures has a unit root

Exogenous: Constant			
Lag Length 0 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-3.057574	0.0299
Testcritical values:	1%	-3.43135	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: ho spot has a unit root

Exogenous: Constant			
Lag Length 0 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-2.406158	0.2671
Testcritical values:	1%	-3.43136	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: ho futures has a unit root

Exogenous: Constant			
Lag Length 0 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-1.868763	0.3474
Testcritical values:	1%	-3.43135	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: brent spot has a unit root

Exogenous: Constant			
Lag Length 0 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-1.874504	0.3447
Testcritical values:	1%	-3.43136	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: S&P 500 has a unit root

Exogenous: Constant			
Lag Length 9 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		1.976503	0.9999
Testcritical values:	1%	-3.43135	
	5%	-2.861851	
	10%	-2.566978	

Table 3. 3: Unit root tests for the returns of our series

Null Hypothesis: WTI spot returns has a unit root

Exogenous: Constant			
Lag Length 0 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-78.54057	0.0001
Testcritical values:	1%	-3.43136	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: WTI spot returns has a unit root

Exogenous: Constant			
Lag Length 1 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-55.86621	0.0001
Testcritical values:	1%	-3.43135	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: WTI spot returns has a unit root

Exogenous: Constant			
Lag Length 0 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-78.54057	0.0001
Testcritical values:	1%	-3.43136	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: WTI futures returns has a unit root

Exogenous: Constant			
Lag Length 1 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-55.86621	0.0001
Testcritical values:	1%	-3.43135	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: ho spot returns has a unit root

Exogenous: Constant			
Lag Length 3 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-40.53335	0.0001
Testcritical values:	1%	-3.43136	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: ho futures returns has a unit root

Exogenous: Constant			
Lag Length 0 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-78.40294	0.0001
Testcritical values:	1%	-3.43135	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: brent spot returns has a unit root

Exogenous: Constant			
Lag Length 13 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-17.05061	0.0000
Testcritical values:	1%	-3.43136	
	5%	-2.861851	
	10%	-2.566978	

Null Hypothesis: S&P 500 returns has a unit root

Exogenous: Constant			
Lag Length 0 (Automatic - based on SIC, maxlag=32)			
		t-statistic	Prob*
Augmented Dickey Fuller test statistic		-83.99358	0.0001
Testcritical values:	1%	-3.43135	
	5%	-2.861851	
	10%	-2.566978	

Table 5. 9: Mean Equations of our series

	Mean Equations						
	Gas Spot	Gas Futures	Heatingoil Spot	Heatingoil Futures	WTI spot	WTI futures	Brent Spot
C	--	--	--	--	--	--	--
RSP500	--	0.126099	0.382473	--	0.38522	0.410968	0.300503
		(0.0005)	(0.0000)		(0.0000)	(0.0000)	(0.0000)
AR(1)	0.01992	0.117027	-0.139484	-0.621141	0.38522	0.734522	0.05381
	(0.5569)	(0.0000)	(0.0063)	(0.0000)	(0.0014)	(0.0000)	(0.0000)
AR(2)	-0.1433	-0.07458	-0.035042	-0.698716	1.00845	0.131821	-0.01895
	(0.0000)	(0.0054)	(0.4552)	(0.0000)	(0.0000)	(0.0000)	(0.1536)
AR(3)	0.86451	0.939509	-0.235131	-0.638831	0.93187	0.868553	-0.01293
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.3298)
AR(4)	--	--	-0.796538	-0.888368	-0.3629	-0.868546	-0.00654
			(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.6219)
AR(5)	--	--	--	--	-0.7857	--	0.009286
					(0.0000)		(0.4832)
MA(1)	0.00613	-0.18267	0.12205	0.597297	0.16566	-0.758871	--
	(0.8672)	(0.0000)	(0.0319)	(0.0000)	(0.0144)	(0.0000)	
MA(2)	-0.0055	0.061356	0.04985	0.69984	-1.0198	-0.141565	--
	(0.8775)	(0.0283)	(0.3377)	(0.0000)	(0.0000)	(0.0000)	
MA(3)	-0.9031	-0.9441	0.258086	0.648226	-0.9386	-0.86345	--
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
MA(4)	-0.0564	0.072641	0.737431	0.885834	0.41642	0.901725	--
	(0.0006)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
MA(5)	0.10339	--	--	--	0.77393	--	--
	(0.0000)				(0.0000)		

Table 5. 10: Residual Diagnostics Mean Equation of our series

	Gas Spot	Gas Futures	Heatingoil Spot	Heatingoil Futures	WTI spot	WTI futures	Brent Spot
AIC	-3.1468	-3.919265	-4.508635	-4.841622	-4.31169	-4.474918	-4.5074
Q(20)	AC	AC	NO AC	AC	AC	AC	AC
Q^2(20)	AC	AC	AC	AC	AC	AC	AC
LM Test Stat	0.10677	1.650567	0.006973	2.132069	0.00057	1.505252	6.32785
	(0.7439)	(0.1989)	(0.9334)	(0.1443)	(0.9809)	(0.2220)	(0.0000)
ARCH(5)	828.046	48.88524	288.0155	46.22781	267.507	280.2169	145.494
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Table 5. 11: Residual Diagnostics for variance Equation

	GARCH			EGARCH			GJR		
	Normal	T	GED	Normal	T	GED	Normal	T	GED
WI SPOT									
AIC	-4.755391	-4.82781	-4.819447	-4.772517	-4.840265	-4.828652	-4.766696	-4.832928	-4.823232
Q(20)	NO serial cor	NO serial cor	NO serial cor	No serial cor	No serial cor	No serial cor	No serial cor	No serial cor	No serial cor
Q^2(20)	NO serial cor for 5% and probably for 10% as well	No serial cor for 5%	No serial cor for 5%	No serial cor for 5%	Serial cor	Serial cor	No serial cor	No serial cor for 5%	No serial cor
ARCH(1)	6.017094	11.34791	9.532843	10.01435	16.73211	14.48107	4.527256	8.175687	6.678571
	(0.0142)	(0.0008)	(0.002)	(0.0016)	(0.0000)	(0.0001)	(0.0334)	(0.0043)	(0.0098)
WTI futures									
AIC	-4.813295	-4.872257	-4.86298	-4.835336	-4.881044	-4.873127	-4.82757	-4.877726	-4.865474
Q(20)	NO serial cor	NO serial cor	NO serial cor	NO serial cor	NO serial cor	NO serial cor for 5%	NO serial cor	NO serial cor	NO serial cor
Q^2(20)	NO serial cor	Serial cor	NO derial cor 5%	Serial cor	Serial cor	Serial cor	NO serial cor for	Serial cor	NO serial cor for 5%
ARCH(1)	5.646183	13.57255	10.80612	17.70783	27.41919	27.02292	7.698114	12.29062	10.62633
	(0.0175)	(0.0002)	(0.0001)	(0.0000)	(0.0000)	(0.0000)	(0.0055)	(0.0005)	(0.0011)
HO Spot									
AIC	-4.873324	-4.922074	-4.911172	-4.876511	-4.929888	-4.917558	-4.873128	-4.920593	-4.913389
Q(20)	NO serial cor	NO serial cor	NO serial cor	NO serial cor	NO serial cor	NO serial cor	NO serial cor	NO serial cor	NO serial cor
Q^2(20)	NO serial cor	NO serial cor for 5%	NO serial cor	Serial Cor	Serial Cor	Serial Cor	NO serial cor	Serial Cor	NO serial cor
ARCH(1)	4.80858	9.695506	7.594376	4.303865	31.45263	20.93415	4.924707	9.976234	6.978809
	(0.0284)	(0.0019)	(0.0059)	(0.0002)	(0.0000)	(0.0000)	(0.0265)	(0.0016)	(0.0083)
Gas Spot									
AIC	-3.748373	-3.862348	-3.865032	-3.756301	-3.875723	-3.875223	-3.748729	-3.86365	-3.864784
Q(20)	Serial Cor	Serial Cor	Serial Cor	Serial Cor	Serial Cor	Serial Cor	Serial Cor	Serial Cor	Serial Cor
Q^2(20)	No serial Cor	No serial Cor	No serial Cor	No serial Cor	No serial Cor	No serial Cor	No serial Cor	No serial Cor	No serial Cor
ARCH(1)	1.231093	1.439121	1.129356	2.11837	4.125914	3.63508	1.089654	1.23234	1.049698
	(0.2672)	(0.2303)	(0.288)	(0.1456)	(0.0423)	(0.0566)	(0.2966)	(0.2669)	(0.3056)
Gas Futures									
AIC	-4.088766	-4.139681	-3.5321	-4.0921	-4.142933	-4.134087	-4.088463	-4.139399	-4.129438
Q(20)	No serial cor	No serial cor	Serial cor	No serial cor	No serial cor	No serial cor	No serial cor	No serial cor	No serial cor for 5%
Q^2(20)	No serial cor	No serial cor	Serial cor	No serial cor	No serial cor	No serial cor	No serial cor	No serial cor	No serial cor
ARCH(1)	0.181828	0.830672	9.807273	0.651314	1.224192	1.037806	0.141083	0.664068	0.379592
	(0.6690)	(0.3620)	(0.0017)	(0.4196)	(0.2686)	(0.3083)	(0.7072)	(0.4151)	(0.5378)
HO Futures									
AIC	-4.936243	-5.066477	-5.057728	-5.036542	-5.076869	-5.064961	-5.029189	-5.068585	-5.059187
Q(20)	No serial cor for 5%	No serial cor for 5%	No serial cor	No serial cor for 5%	Serial cor	Serial cor	No serial cor	No serial cor	Serial cor
Q^2(20)	No serial cor	No serial cor	No serial cor	No serial cor	No serial cor for 5%	No serial cor for 5%	No serial cor	No serial cor	No serial cor
ARCH(1)	0.982139	1.915114	1.186665	1.274356	1.749518	0.765451	0.546329	1.155003	0.50257
	(0.3216)	(0.1664)	(0.2760)	(0.2590)	(0.1859)	(0.3816)	(0.4598)	(0.2825)	(0.4783)
Brent Spot									
AIC	-4.825248	-4.875799	-4.872901	-4.842695	-4.889285	-4.885187	-4.840394	-4.883437	-4.88127
Q(20)	Serial Cor	Serial Cor	Serial Cor	Serial Cor	Serial Cor	Serial Cor	Serial Cor	Serial Cor	Serial Cor
Q^2(20)	No serial cor	No serial cor for 5%	No serial cor	Serial Cor	Serial Cor	Serial Cor	No serial cor	Serial Cor	No serial cor for 5%
ARCH(1)	1.78458	5.364277	3.666644	2.731944	6.556842	5.083927	0.878892	2.864058	1.952878
	(0.1816)	(0.0206)	(0.0555)	(0.0984)	(0.0105)	(0.0242)	(0.3485)	(0.0906)	(0.1623)

Table 5. 12 : Granger causality tests

Depedent Variable: diffvariancewtispot

Excluded	Chi-sq	df	Prob
diffvariancewtifutures	13.99575	1	0.0002
diffvariancegasspot	0.006209	1	0.9372
diffvariancegasfutures	0.625593	1	0.429
diffvariancehospot	1.179042	1	0.2776
diffvariancehofutures	24.99982	1	0.0000
diffvariancebrentspot	12.71955	1	0.0004
All	53.02912	6	0.0000

Depedent Variable: diffvariancewtifutures

Excluded	Chi-sq	df	Prob
diffvariancewtispot	21.68232	1	0.0000
diffvariancegasspot	0.077786	1	0.7803
diffvariancegasfutures	0.526499	1	0.4681
diffvariancehospot	0.019957	1	0.8877
diffvariancehofutures	43.06477	1	0.0000
diffvariancebrentspot	9.888252	1	0.0017
All	72.40271	6	0.0000

Depedent Variable: diffvariancehofutures

Excluded	Chi-sq	df	Prob
diffvariancewtispot	17.6669	1	0.0000
diffvariancewtifutures	13.42307	1	0.0002
diffvariancegasspot	0.440514	1	0.5069
diffvariancegasfutures	0.624474	1	0.4294
diffvariancehospot	2.015319	1	0.1557
diffvariancebrentspot	13.38497	1	0.0003
All	30.61765	6	0.0000

Depedent Variable: diffvariancehofutures

Excluded	Chi-sq	df	Prob
diffvariancewtispot	74.1059	1	0.0000
diffvariancewtifutures	18.52978	1	0.0000
diffvariancegasspot	0.047788	1	0.8270
diffvariancegasfutures	1.138695	1	0.2859
diffvariancehospot	1.336823	1	0.2476
diffvariancehofutures	13.54048	1	0.0002
All	88.50738	6	0.0000

Table 5. 13 : Loss Functions for Forecasting by distribution

Normal Distribution

	GARCH(1,1)			EGARCH(1,1)			GJR(1,1)		
	RMSE	MAE	MAPE	RMSE	MAE	MAPE	RMSE	MAE	MAPE
GS	0.02305	0.017921	NA	0.02265	0.017526	NA	0.020764	0.015206	NA
GF	0.015103	0.012567	104.492	0.01518	0.012707	102.4506	0.015105	0.012568	104.6658
HS	0.020494	0.013828	107.098	0.02068	0.014053	112.7605	**0.020325**	**0.013681**	**104.4493**
HF	0.019848	0.013615	NA	0.01974	0.013492	NA	0.019843	0.013603	NA
WS	0.024618	0.017429	107.662	**0.0236**	0.01677	119.4346	0.024419	**0.01646**	**107.016**
WF	0.024647	0.017572	181.456	**0.0246**	0.017523	108.644	0.025353	0.0182	138.7348
BS	0.023318	0.015561	122.873	0.02331	0.015542	**119.0543**	**0.023286**	**0.015541**	119.5027

T-student Distribution

	GARCH(1,1)			EGARCH(1,1)			GJR(1,1)		
	RMSE	MAE	MAPE	RMSE	MAE	MAPE	RMSE	MAE	MAPE
GS	0.020741	0.015255	NA	0.02039	0.015882	NA	**0.019933**	0.015361	NA
GF	0.015197	0.012664	116.087	0.01504	0.012521	101.149	0.015177	0.012647	117.0021
HS	0.020633	0.014018	113.795	0.02063	0.014009	115.5541	0.020898	0.014283	119.5596
HF	0.019979	0.01359	NA	0.01968	0.013501	NA	**0.019625**	0.01345	NA
WS	0.024804	0.017518	110.12	0.0.24816	0.017491	107.9649	0.024791	0.017485	108.9052
WF	0.025667	0.018316	121.296	0.24154	**0.016975**	**97.6794**	0.025683	0.018324	121.5537
BS	0.023404	0.015586	122.871	0.02341	0.015581	121.5735	0.023387	0.015577	121.3193

GED Distribution

	GARCH(1,1)			EGARCH(1,1)			GJR(1,1)		
	RMSE	MAE	MAPE	RMSE	MAE	MAPE	RMSE	MAE	MAPE
GS	0.020089	0.015276	NA	0.02018	0.015402	NA	0.020693	**0.01517**	NA
GF	**0.014971**	0.012486	101.607	0.01498	0.012489	105.3182	0.014983	**0.01249**	**96.7740**
HS	0.020506	0.013916	112.373	0.02054	0.013925	112.3459	0.020506	0.013917	112.3675
HF	0.019633	**0.013369**	NA	0.01974	0.013492	NA	0.01966	0.013529	NA
WS	0.024775	0.017512	111.443	0.0243	0.017312	136.6864	0.024735	0.017474	110.4147
WF	0.024857	0.017445	98.7055	0.26324	0.018671	142.5181	0.024855	0.017448	99.47044
BS	0.023334	0.015557	121.93	0.02334	0.15551	119.9749	0.023312	0.015544	119.9575

Forecasted returns and variance for the optimal univariate GARCH models for every series (left graph are returns and actual, right graph variance, forecast period: 3/23/2021-5/10/2021 , 30 observations).

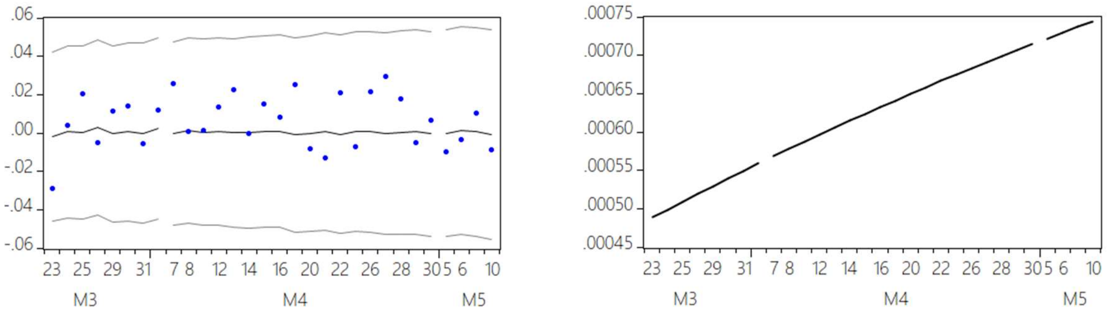


Figure 5. 11: Forecasted returns and variance, gas futures GARCH GED distribution (RMSE)

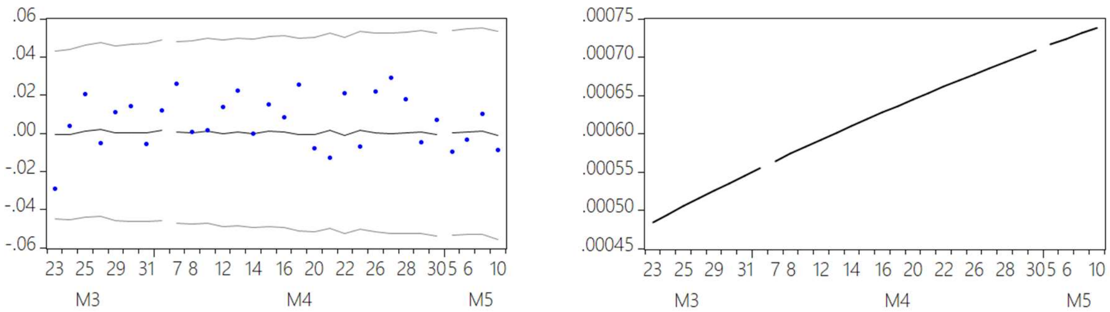


Figure 5. 12: Forecasted returns and variance , gas futures GJR GED distribution (MAE & MAPE)

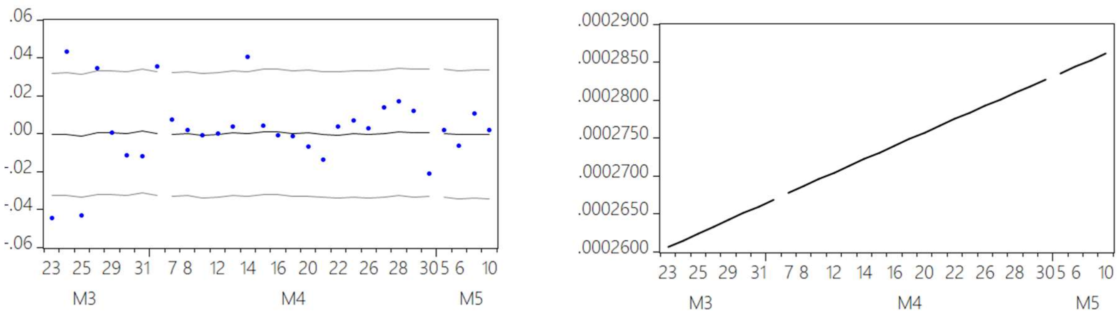


Figure 5. 13: Forecasted returns and variance, heating oil futures GJR t distribution (RMSE)

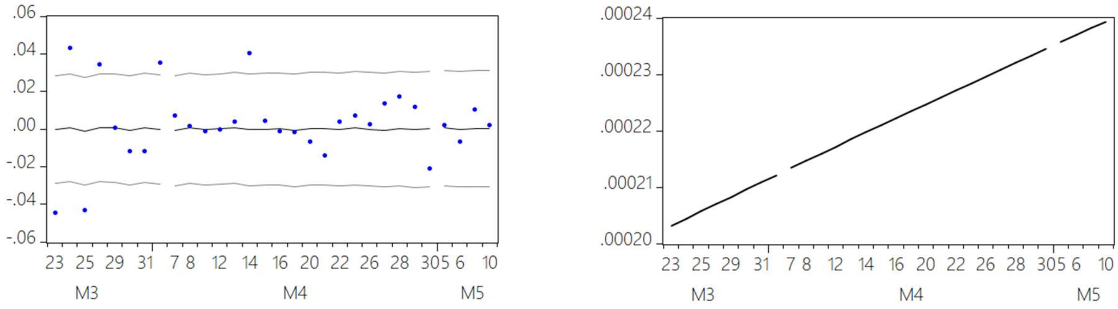


Figure 5. 14: Forecasted returns and variance, heating oil futures GARCH GED distribution (MAE)

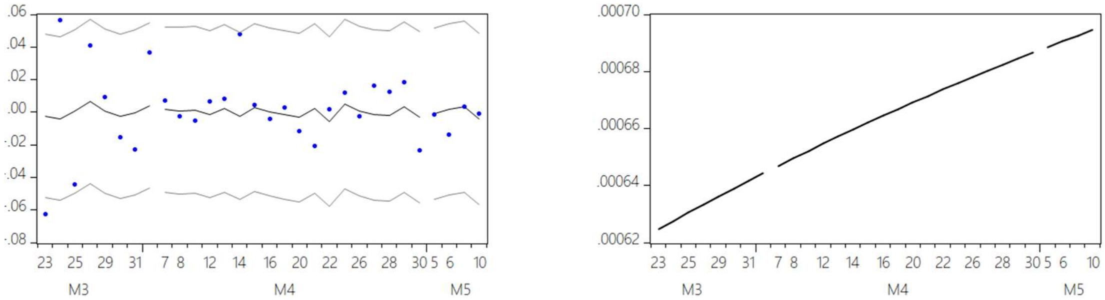


Figure 5. 15: Forecasted returns and variance, WTI spot EGARCH normal distribution (RMSE)

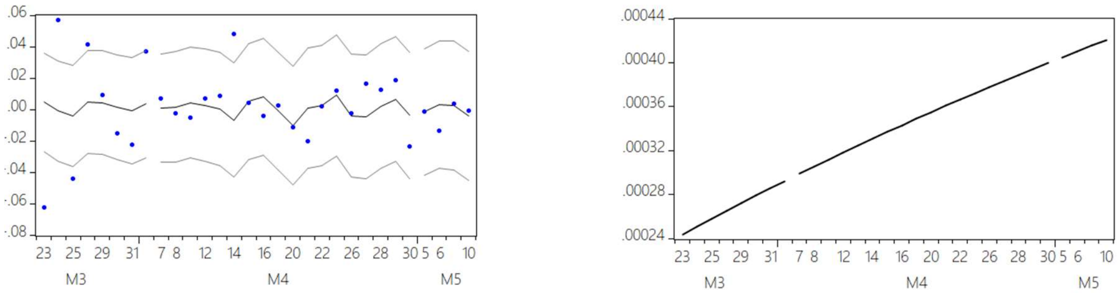


Figure 5. 16: Forecasted returns and variance , WTI spot GJR normal distribution (MAE&MAPE)

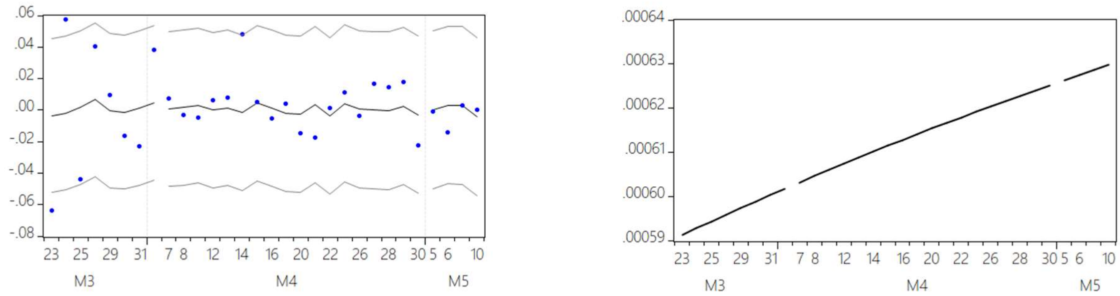


Figure 5. 17 :Forecasted returns and variance, WTI futures EGARCH normal distribution (RMSE)

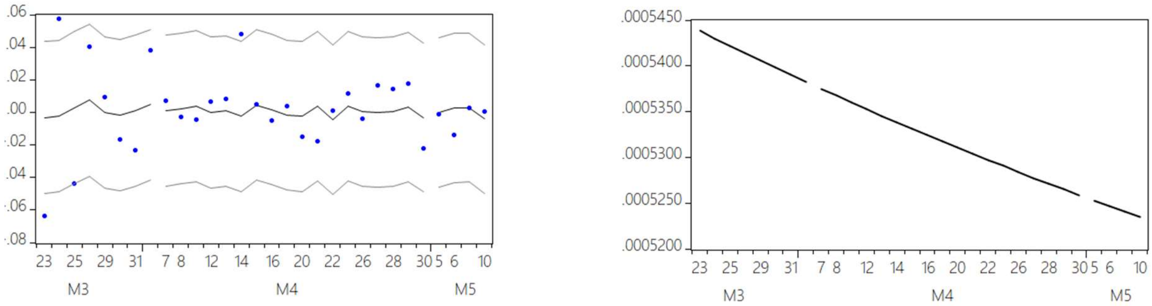


Figure 5. 18 : Forecasted returns and variance, WTI futures EGARCH t distribution (MAE&MAPE)

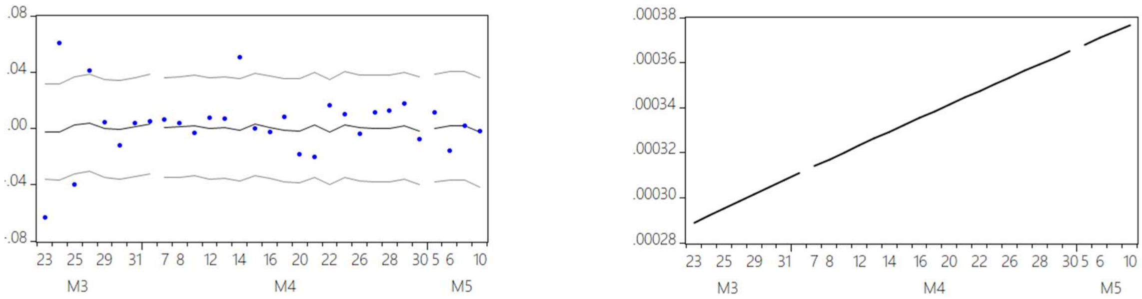


Figure 5. 19 : Forecasted returns and variance, brent spot GJR Normal distribution (RMSE&,MAE)

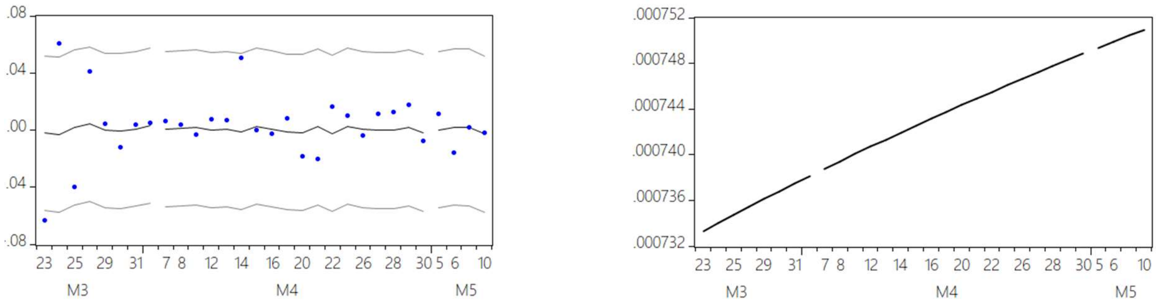


Figure 5. 20: Forecasted returns and variance, brent spot EGARCH normal distribution (MAPE)

Price and Returns Correlations

Correlation Probability	WS	WF	GS	GF	HS	HF	BS
WS	1.0000						
	--						
WF	0.99987	1.0000					
	0.0000	--					
GS	0.33104	0.33085	1.0000				
	0.0000	*0.0000*	--				
GF	0.32953	0.32935	0.97895	1.0000			
	0.0000	*0.0000*	*0.0000*	--			
HS	0.98103	0.98138	0.27079	0.26482	1.0000		
	0.0000	*0.0000*	*0.0000*	*0.0000*	--		
HF	0.98174	0.98213	0.25219	0.25219	0.99738	1.0000	
	0.0000	*0.0000*	*0.0000*	*0.0000*	*0.0000*	--	
BS	0.98494	0.9852	0.98519	0.23031	0.99223	0.9921	1.0000
	0.0000	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	--

Table 5. 14: Price Correlation

Correlation Probability	RWS	RWF	RGS	RGF	RHS	RHF	RBS
RWS	1.0000						
	--						
RWF	0.91423	1.0000					
	0.0000	--					
RGS	0.04615	0.0558	1.0000				
	0.0000	*0.0000*	--				
RGF	0.19215	0.2082	0.25988	1.0000			
	0.0000	*0.0000*	*0.0000*	--			
RHS	0.59823	0.64172	0.07896	0.235	1.0000		
	0.0000	*0.0000*	*0.0000*	*0.0000*	--		
RHF	0.71133	0.77956	0.08817	0.26279	0.81145	1.0000	
	0.0000	*0.0000*	*0.0000*	*0.0000*	*0.0000*	--	
RBS	0.59077	0.6491	0.113	0.12973	0.47117	0.56873	1.0000
	0.0000	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	--

Table 5. 15 : Returns Correlation

Dynamic Conditional Correlations DCC models

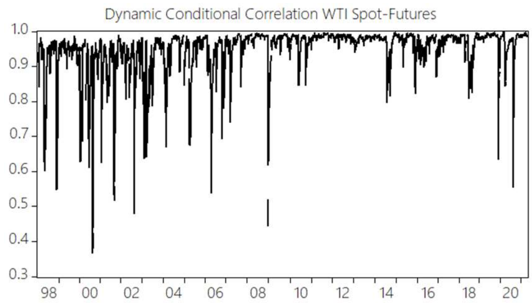


Figure 5.6. 13: DCC WTI spot-futures

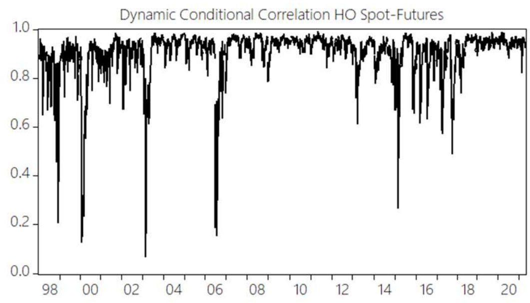


Figure 5.6. 14: DCC HO spot-futures

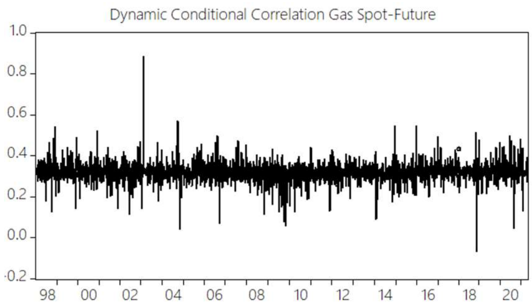


Figure 5.6. 15: DCC Gas Spot-Future

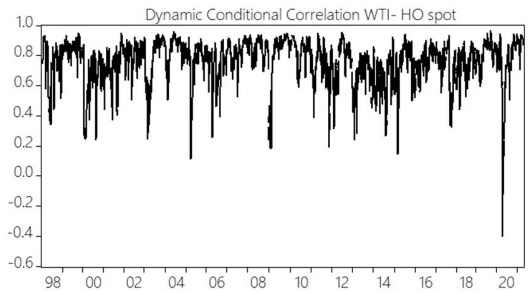


Figure 5.6. 16: DCC WTI-HO Spot

Forecasted Dynamic Conditional Correlations DCC models (30 obs)

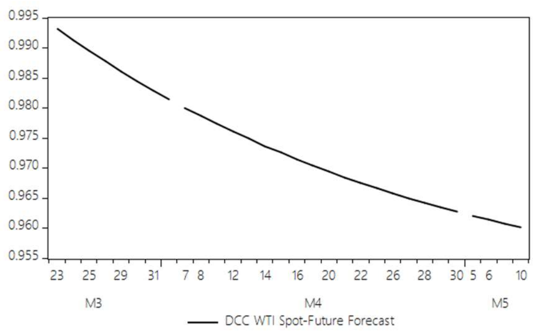


Figure 5.6. 17: DCC WTI Spot-Future Forecast

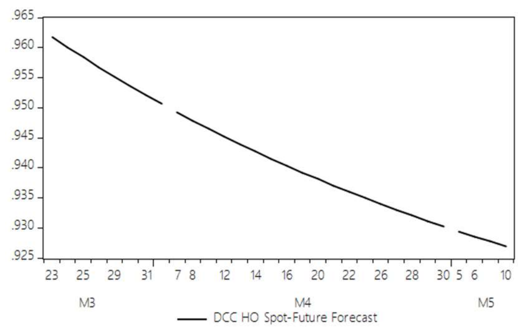


Figure 5.6. 18: DCC HO spot-Future Forecast

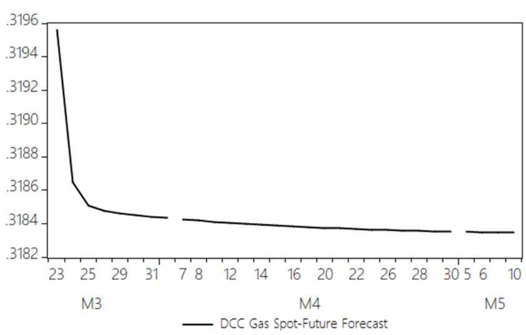


Figure 5.6. 19: DCC Gas Spot-Futures Forecast

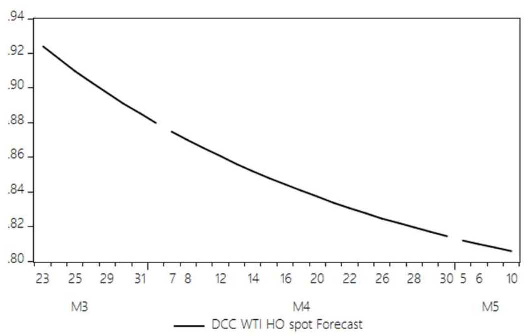


Figure 5.6. 20: DCC WTI HO spot Forecast