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NON-LINEAR AND CHAOTIC DYNAMICS IN THE VIETNAMESE STOCK MARKET

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Abstract

This paper examines if the returns in the Vietnamese stock market are generated by a nonlinear dynamic system. The daily data between 2008 and 2018 for six indices from Ho Chi Minh City Stock Exchange, the largest market in Vietnam, are used. We test our data for IID or non-IID behavior such as linear dependence and nonlinear stochastic process using a set of linear and non-linear tests. The results suggest strong evidence of non-linear structure in stock returns. Furthermore, we analyze the stock returns for the presence of chaotic structure (non-linear deterministic process) using the max Lyapunov exponent. The results show negative signs of chaos for all indices.

1. Introduction

Non-linear dependence and chaos theory in stock returns has captured the attention of many financial analysts and economists in these years as it indicates possibility of predictability. The dominant Efficient Market Hypothesis (EMH) was challenged in many studies in the past using many conventional tests in order to find patterns in returns series. The presence of patterns in the returns provides opportunities for the investors to make excess profits. The kind of dependence in the return series can be linear, non-linear or chaotic. The use of linear models in most of the past studies in non-linear or chaotic conditions may give wrong inference of unpredictability and thus conclude that markets are efficient even though they are not. Linear models have poor forecasting power when it comes to financial data and this is why the results in the literature about the behavior of the markets are not exclusive. Some studies, depending on the financial model they use, suggest that the dynamics are actually chaotic and not efficient and others have found that the dynamics are stochastic.

The main aim of this study is to investigate the presence of nonlinear dependence and deterministic chaos in daily returns on the Vietnamese stock market indices by using a number of tests for dependence, non-linearity and chaotic behavior.

2. Literature Review

The EMH theory had been the dominant one the last decades as it had grand success and it was commonly accepted by financial investors. According to the Fama's Efficient Market Hypothesis (1965), an efficient market is one in which returns cannot be exploited by trading in a specific pattern. No investor can affect the prices and make excessive profits. Market efficiency suggests that all information is reflected in the prices and they do not follow any trend. The efficient market hypothesis is linked with the notion of random walk (RW), which in finance is reflected as random changes in prices of stocks such that the future prices cannot be predicted from previous prices. The are three kinds of efficiency hypothesis namely the strong, the semi-strong and the weak.

Many studies have been conducted to test the theory. One of the first empirical challenges of the EMH questioning the theory showed that market prices are much more volatile than they should be. (Shiller, 1981). Since then, the idea that stock prices are generated by a random process with no long-term memory started to be doubted. A lot of arguments against the EMH were raised, questioning the assumptions of rationality (Barberis and Thaler, 2003) and a new era of empirical researches commenced that showed that stock prices do not follow the EMH, or otherwise, that stock returns are not independent and identically distributed (IID) random variables. Neiderhoffer and Osborne (1966) show that NYSE specialists use their monopolistic access to the book of orders to generate trading profits, which is a sign of market inefficiency in the strong form. Also, Lo and MacKinlay, (1988) show that returns are more predictable for small-stock portfolios. Those studies suggest that stock returns do not follow a random walk and that there are some ways to predict future returns and make excessive profits.

Other studies have shown that the hypothesis is only valid in developed stock markets. Traditionally markets of developed economies are more efficient as compared to emerging markets (Gupta, 2006). As a result, a lot of interest has been instigated about the validity of the efficient market hypothesis in the emerging markets by researched like D'Ambrosio (1980), Harvey (1993), Balaban (1995) and Kawakatsu and Morey (1999). The idea that stock returns can be predicted in emerging market economies has attracted the attention of investors since it allows them to diversify their portfolios including assets from those markets in order to enhance their returns (Harvey, 1993 as well as Pandey, Kohers & Kohers, 1998). The Vietnamese stock market is considered to be an emerging one with a lot of new foreign capitals inflows and this is why our study will focus on it.

In any case, the results on the EMH in all markets have shown mixed results over the years. There are numerous tests designed to test the IID assumption (efficiency) against specific alternatives, such as but not limited to structural breaks, serial correlation, or autoregressive conditional heteroskedasticity. Those tests have special purposes and may leave out other possible structures in the data, such as deterministic process.

The new trend debate for the financials now is whether financial markets are primarily generated by stochastic or chaotic dynamics. This is why investors are more interested in the kind of dependence that exists in the markets. The traditional linear models that ruled in finance literature are replaced by non-linear models. The stochastic or deterministic non-linear dependence gives them even more possibilities for excessive profit and this is why the field of non-linearities has so grand success lately.

While non-linearity- stochastic or deterministic- is fairly well researched in developed economies, the evidence on emerging economies is scarce and even though there are some studies for the emerging markets widely available, so far not any study has focused on Vietnam. This is why we attempt to investigate the Vietnamese stock market for random walk, nonlinear and chaotic structure in the return series. Vietnamese stock market is among the emerging markets with its first transaction in 2000. There are some experimental studies made for Vietnamese stock market indicating that VN-index does not follow the random walk, which implies the fact that stock prices are predictable (TN My and Truong 2011, Dong Loc et'al 2010). However, there is not much study on this topic particularly in Vietnam compared with other countries suggesting any kind of chaotic or nonlinear dependence in the return series. Our analysis will try to fill these gaps in the literature relatively in Vietnam that are open by other studies. For example, Khoa Cuong Phan and Jian Zhou (2014) showed that because of physiological factors stock prices are predictable, Nguyen Viet Dung (2010) showed that financial statement information is reflected in the stock prices indicating possibility of predictability.

We will start our analysis by checking for random walk in the return series, then we will conduct more advanced tests to check what kind of dependence exist between the returns, if the dependence hypothesis will be accurate. The dependence could be linear, non-linear or chaotic. A big number of recent studies have used chaotic and non-linear estimation techniques for modeling financial data and have found strong evidence of nonlinear deterministic behavior of stock prices indicating that prices are even more predictable than it was previously thought under the random walk assumption. Mentioning Frank & Stengos (1989) examined the returns of gold and silver, while Hinich & Patterson (1985) estimated the returns of 15 common stocks. These studies have changed our perspective in analyzing financial data and have caused the need for further research as it concerns the dependence of the return series in the stock markets.

Against this background, in the present paper we endeavor to investigate nonlinear and chaotic structure in returns series of Vietnamese stock market indices. To our knowledge, this paper will offer several contributions to the existence literature since no similar tests have been made before for Vietnam, an emerging stock market. In the last 10 years the performance of Vietnamese economy has been impressive with the gross domestic product clocking on average around 9.8% percent (Trade Economics). As a result, a lot of investors gather their money in the Vietnamese market. The dependence in the indices' returns and the possible predictions are very important for them in order to make their optimal trading strategies. To find evidence, instead of performing a direct test for chaos, we apply a variety of recently developed tests to investigate the underlying data generating process. These tests will help investigate the adequacy of generally applied linear or nonlinear econometric models for forecasting these financial time series. At last, the test of chaotic dynamic will help determine the level of predictability and consistency in the Vietnamese stock market.

2.1. Introducing Chaos in Financial data

Deterministic non-linear behavior in financial data, namely chaos, has received great attention from researches. Chaos has raised the possibility of short-term predictions in return series and thus the possibility of excessive profits. It has the ability to explain how small changes can cause large different outcomes that would appear to be unrelated. It belongs to the class of deterministic dynamical systems that are highly sensitive on the initial conditions (Eckmann and Ruelle, 1985). The difference between stochastic and deterministic system is that in stochastic systems the fluctuations are caused by external shocks. Whereas in deterministic systems the fluctuations are caused internally (Gilmore, 1996). The endogenous fluctuations give space for possible predictions, even though predicting the system's behavior is not possible. This is why chaotic behavior can explain movements in returns that otherwise would appear to be random and it is only predictable in the short run.

The rest of the paper is organized as follows:

Section 3 presents a brief review of the Vietnamese stock market and the indices we used. Section 4 discusses and presents empirical methodologies and provides a brief account of tests used in the study. Section 5 presents the results and the findings of our study. Finally, Section 6 provides the conclusion of this paper.

3. Data and Empirical Analysis

3.1 Data

The data used in this study consist of the daily returns of 6 indices that are listed in the Ho Chi Ming City Stock Exchange in Vietnam. The Vietnam Stock Index or VN-Index is a capitalization-weighted index of all the companies listed on the Ho Chi Minh City Stock Exchange. The index was created with a base index value of 100 as of July 28, 2000. VN-all shares index covers the top 92% of the full market capitalization from Ho Chi Minh Stock Exchange and Hanoi Stock Exchange. VN- 30 is a market-capitalization weighted index which measures the performance of 30 large cap and high liquidity stocks from VN- all shares index. Like VN- 30 index, the VN- 100 index measures the performance of 100 large cap stocks from VN- all shares index. The VN- mid cap index includes the medium capitalization assets from Ho Chi Minh Stock Exchange and the VN- small cap the small capitalization assets.

The period of the data differs, with the VN-INDEX to be the oldest index and the VN100 the latest. The starting point of each series was chosen by going back as far as the data was available from publicly available official sources.

Table 1: Descriptive Statistics of the returns

DESCRIPTIVE STATISTICS						
	VNI	VN30	VN100	VNSMALLCAP	VNMEDIUMCAP	VNALLSHARES
Period	1/2/08-12/21/18	1/5/09-12/27/18	11/5/2014 -12/28/2018	11/7/2014-1/2/2019	11/20/2014-10/2/2019	11/4/14 -1/2/19
Observations	2733	2491	1038	1037	1028	1040
Mean	-3,52E-06	0,000411	0,000334	5,59E-05	0,000307	0,000315
Median	0,000723	0,000916	0,001075	0,000435	0,000997	0,000976
Maximum	0,046468	0,046429	0,035839	0,028316	0,030084	0,034542
Minimum	-0,060512	-0,057746	-0,050973	-0,041596	-5,057555	-0,049861
Std. Dev.	0,014207	0,013121	0,009833	0,007754	0,00938	0,009545
Skewness	-0,305781	-0,214573	-0,684169	-0,836016	-1,058025	-0,706505
Kurtosis	4,61022	4,953205	6,331686	6,40542	7,954448	6,370737
Jarque-Bera	337,846	415,081	561,0598	621,8791	1243,204	578,867
P-value	0	0	0	0	0	0

We downloaded the data from the site Investing.com (www.investing.com).

Table 1 refers to the descriptive statistics of the returns of the indices. The mean value of all the sample indices except VN-index was positive, which means that all the indices prices' have increased over time. In all cases kurtosis is high and the Jarque-Bera tests clearly reject the null hypothesis of normality. The value of kurtosis greater than 3 indicates fatter tails than the normal distribution. In terms of unpredictability as measured by the std. deviation, the VN-index has the highest risk value, whereas the VN-small cap index the lowest. The skewness is negative in all the series which is a sign of leverage for the investors of the Vietnamese stock market, meaning that they have higher probability of earning more than the mean when the market goes down. Together with kurtosis, it means that the left tail is particularly extreme. These statistics point towards the possibility of dependence in the data.

Figure 1: Returns Series of VN-index

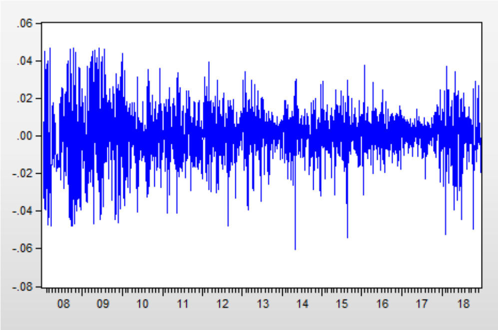


Figure 2: Return Series of VN-30

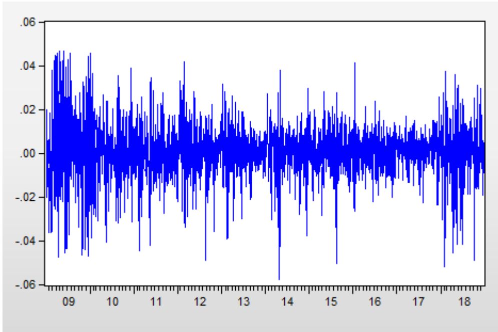


Figure 3: Returns Series of VN-100

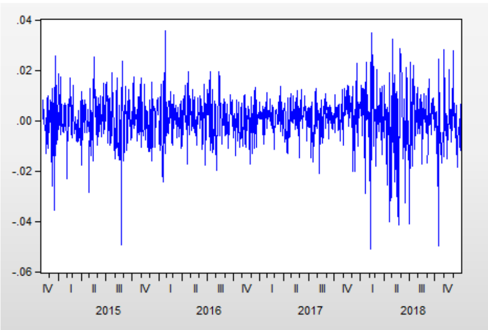


Figure 4: Return Series of VN-smallcap

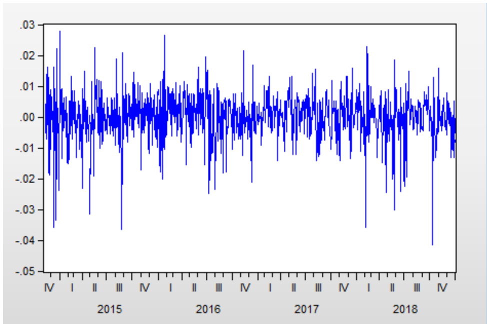


Figure 5: Return Series VN-mediumcap

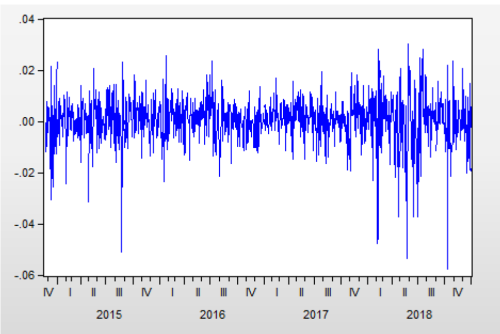
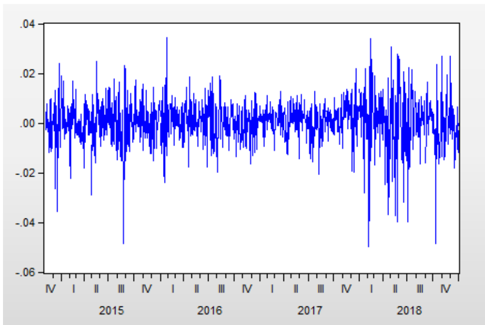


Figure 6: Return Series VN-allshares



3.2 Empirical Analysis

We apply the following transformation to the raw data before conducting statistical tests

$$Z_t = \log \frac{P_t}{P_{t-1}} \quad (1)$$

where Pt is the price at date t and the transformed data (z_t) are rates of returns. This transformation reduces the variation of the time series and implements an effective detrending of the series. This method also provides an effective way to measure the continuously compounded rates of returns.

In the first stage, we test for general dependence by using the autocorrelation test of Ljung-Box Statistic Q. From the descriptive statistics we can observe that the Jarque-Bera test indicates that stock returns are not normally distributed. Therefore, we also use non-parametric tests such as Runs test and the variance ratio test (VR) to enhance the results. The VR test is made for 32 lags and for both homoscedasticity and heteroscedasticity assumptions.

After testing for dependence, we conduct tests to find what kind of dependence exist between the returns of our indices. We use the BDS test in order to test for non-linearity and chaos in the return series. To be accurate, the BDS test does not provide a direct test for nonlinearity or chaos. It actually tests the null hypothesis of whiteness (IID), as the previous tests, against an unspecified alternative using a nonparametric technique. However, it is possible to use the BDS test to indirectly search for nonlinear dependence which is necessary but not sufficient condition for chaos.

We first apply the BDS test on raw data, our return series, to further confirm the results from the above tests. The BDS is a more powerful test for IID based on the concept of correlation integral. The null hypothesis of the test is that the data are random (IID). If the null hypothesis of randomness is rejected, the series may be either linear stochastic or deterministic. Next, we filter our series using the appropriate ARMA model and we apply the BDS test to the residuals of the ARMA series. It is worth mentioning that the residuals are filtered by the best linear ARMA(1,1) based on the Akaike information criterion (AIC). This filtering removes linear structure from the model and thus, if the results of the BDS test on residuals show rejection of IID, then it indicates that the dependence that is left, is for sure non-linear. Furthermore, we apply the BDS on the standardized residuals of the GARCH AND E-GARCH model. If the null hypothesis of IID is accepted, it means that GARCH AND E-GARCH models are sufficient enough to describe the movement of the return indices. On the other hand, if the hypothesis is rejected, chaos might exist.

The use of GARCH and E-GARCH model is to check if the non-linear dependence enters to our sample from the volatility. If the BDS rejects the null even in the third stage, that is from the standardized residuals of GARCH, then those models can not fit the data adequate and further investigation must be made. In the case that the null hypothesis in the third stage is rejected, the rejection of the previous hypothesis is probably due to conditional heteroscedasticity and it means that our return series behave as non-linear stochastic. In other words, those ARCH-type models can explain the behavior for the data series.

Lastly, we estimate the Lyapunov exponent to find out if the returns are generated by a chaotic process. If the results of Lyapunov exponent are negative it is an indication that the nature of our indices is consistent with a stochastic process and not with some deterministic chaotic dynamics.

4. Empirical Methodologies

4.1. Autocorrelation Test

Ljung-Box (1978) statistic Q tests the joint hypothesis that all autocorrelations are simultaneously equal to zero. It is a portmanteau test.

The Ljung–Box test may be defined as:

H_0: The data are independently distributed; no autocorrelation up to order k (ρ=0)

H_1: The data are not independently distributed; they exhibit serial correlation.

The test statistic is:

$$Q = n\,(n+2) \sum_{k=1}^{h} \frac{{\rho_k}^2}{n-k} \qquad (2)$$

where n is the sample size, ρ_k is the sample autocorrelation at lag k, and h is the number of lags being tested. Under H_0 the statistic Q follows a $X_{2,h}$ distribution

For significance level α=5%, the critical region for rejection of the hypothesis of randomness is: $Q > x^2{}_{,h}$

Where h: the degrees of freedom

4.2. Runs Test for Detecting Non-Randomness

Runs test (Bradley, 1968) is a statistical test that is used to check if there is randomness in a data series. It is an alternative way to test for autocorrelation in the data and define if a data series is produced randomly. It is a non-parametric test and is based on the run. A run can be defined as a sequence of upward values or a sequence of decreasing values. Basically, it is a series of one symbol such as + or -. The number of increasing, or decreasing, values is the length of the run. For daily data, in our case, a run is defined as a sequence of days in which the stock price changes in the same direction.

It assumes that the variance and the mean are not changing.

We will code values above the median as positive and values below the median as negative. A run is defined as a series of consecutive positive (or negative) values. The runs test is defined as:

Ho: The series was produced in a random way

H1: The series was not produced in a random way

The test statistic is:

$$Z = \frac{R - R\prime}{S_R} \quad (3)$$

Where R is the observed number of runs

R' is the expected number of runs

S_R is the standard deviation of the number of runs

The values of R' and S_R as measured as follows:

$$R' = \frac{2n_1 n_2}{n_1 + n_2} + 1 \quad (4)$$

$$s_R{}^2 = 2n_1 n_2 \quad (5)$$

with n_1 and n_2 denoting the number of positive and negative values in the series.

The runs test rejects the null hypothesis if $|\boldsymbol{Z}| > \boldsymbol{Z}_{1-\alpha/2}$

At the 5 % significance level a test statistic with an absolute value greater than 1.96 indicates non-randomness.

4.3 Variance Ratio Test

The Variance Ratio (VR) test has been developed by Lo and MacKinlay (1988). The variance ratio test is based on the premise that if the returns of a time series zt follow a random process then the variance of the k-differences will grow proportionally with the difference k. Meaning that the variance of its k-differences would be k times the variance of its first differences. Lo and

MacKinlay (1998) have introduced two tests under the hypothesis of homoskedasticity and the hypothesis of heteroskedasticity.

Supposing that z_t follows a random walk process: $z_t = \mu + z_{t-1} + \varepsilon_t$,

where φ is an unknown drift parameter and ε_t is IID Gaussian with variance σ2 [N ~ (0, σ2)] (only in the case of homoscedasticity)

The central idea of the variance ratio test is that if returns are uncorrelated over time then VR (k) = 1.

In order to test for random walk hypothesis, we perform a test by comparing the variance of the one period return with that of the k-period returns as follows:

$$\mathrm{VR(k)} = \frac{{\sigma_k}^2}{{\sigma_1}^2} \quad (6)$$

The estimators of the variances ${\sigma_k}^2$ and ${\sigma_1}^2$ are unbiased estimators of the k-period and 1- period variances.

The assumption of homoscedasticity (equal variances) assumes that different samples have the same variance, even if they came from different populations. Otherwise stated that the variance around a regression line is the same for all values of a predicted variable.

So if zt is IID under the assumption of homoscedasticity and the null hypothesis of the VR (k) = 1 then the test statistic Z or (M_1) is:

$$M_1(k) = \frac{VR(z;k)-1}{\left[\varphi|(k)\right]^{\frac{1}{2}}} \quad (7)$$

Which follows the standard normal distribution asymptotically. The asymptotic variance, φ(k) is given by:

$$\varphi(k) = \frac{2(2k-1)(k-1)}{3kT} \quad (8)$$

The heteroscedasticity assumption states that error terms of an estimated equation differ between the values of a variable.

For the heteroskedasticity hypothesis, which is a general phenomenon in financial time series, Lo and MacKinlay (1988) proposed a heteroscedasticity robust test statistic Z* or (M2):

$$M_2(k) = \frac{VR(z;k)-1}{[\varphi*(\mathrm{k})]^{\frac{1}{2}}} \quad (9)$$

Which follows the standard normal distribution asymptotically.

The φ*(k) is given by:

$$\Phi^*(\mathrm{k}) = \sum_{j=1}^{k-1}[\frac{2(k-j)}{k}]^2 * \delta_j \quad (10)$$

Where $\delta_j$is the heteroskedasticity- consistent estimator.

4.4 ARCH Models

ARCH-GARCH and E-GARCH Model

The ARCH type model introduced by Engle (1982) is used to measure and forecast volatility. In financial data, volatility is measured by variance and most of the times is not consider to be stable. In stock markets, some periods are riskier than others and thus, the expected value of error terms in some cases are greater than others. The variance of the error terms is not equal in our data sets and as a result our data series suffer from heteroskedasticity. To model the variance, this model uses weighted past values of the variance. The ARCH model allows the data to determined the best weights in order to forecast the variance.

GARCH model is the extension of the Autoregressive Conditional Heteroscedasticity (ARCH) model. The GARCH model proposed by Bolleslev (1986) has also weighted average of past squared residuals but with declining rate of weights that never goes to zero. The most common GARCH, GARCH(1,1) uses weighted average of the long-run average variance, the variance predicted for this period and the most recent squared residuals. The GARCH(1,1) is described by:

$$logZ_j = \beta_0 + \sum_{j=1}^{p} \beta_j \, logZ_{t-j} + \varepsilon_t \qquad (11)$$

$$\sigma_t{}^2 = \alpha_0 + \alpha_1 \, \varepsilon_{t-1}{}^2 + \alpha_2 \, \sigma_{t-1}{}^2 \qquad (12)$$

In ARCH and GARCH models, the conditional variance depends linearly on past squared residuals and the lagged conditional variance. It ignores the direction of information and assumes that only magnitude matters. It allows the conditional variance to depend only on the magnitude of lagged information but not on their sign, since it uses only the squared values. In cases of negative skewness, the models are not adequate enough to capture and describe the leverage effect and they are considered to be inefficient. The E-GARCH model of Nelson (1991) is an asymmetric GARCH model and is used to overcome this issue. Unlike GARCH model, the E-GARCH is nonlinear in the parameters of the conditional variance equation.

$$\log \sigma_t^2 = w_0 + \sum_{i=1}^{p} \beta_i \log \sigma_{t-i}^2 + \sum_{j=1}^{q} \left(\alpha_j \left| \frac{\varepsilon_{t-j}}{\sigma_{t-j}} \right| + \gamma_t \frac{\varepsilon_{t-j}}{\sigma_{t-j}} \right) \qquad (13)$$

4.5 BDS Test

BDS test developed by Brock, Dechert, Scheinkman and LeBaron (1996) (taken from Modeling Stock Market Returns under Self-exciting Threshold Autoregressive Model: Evidence from West Africa) is used for finding the non-linear dependence in the return series. It tests the null hypothesis of independent and identically distributed (I.I.D.) returns against a non-IID returns.

The computations of BDS test follow the following procedures:

Given a time series with N observations, which should be the first difference of the natural logarithms of raw data in time series. In our case it is the daily return series.

$$xi = (x1, x2, x3,\ldots, xN)$$

We select a value of m (embedding dimension), embed the time series into m-dimensional vectors, by taking each m successive points in the series. This converts the series of scalars into a series of vectors with overlapping entries.

$$\begin{gathered} x_1^m = (x_1, x_2, \ldots, x_m) \\ x_2^m = (x_2, x_3, \ldots, x_{m+1}) \\ \vdots \\ x_{N-m}^m = (x_{N-m}, x_{N-m+1}, \ldots, x_N) \end{gathered}$$

We compute the correlation integral, which measures the spatial correlation among the points, by adding the number of pairs of points (i, j), where $1 \leq i \leq N$ and $1 \leq j \leq N$, in the m-dimensional space which are "close" in the sense that the points are within a radius or tolerance ε of each other.

$$C_{e,m} = \frac{1}{N(N-1)} \sum_{i \neq j} I_{i,j,\varepsilon} \qquad (14)$$

Where, $I_{i,j,\varepsilon}$ is an indicator function that takes the values:

$$= 1 \quad \text{if } \left\| x_i^m - x_j^m \right\| < \varepsilon$$

$$= 0 \quad \text{if } \left\| x_i^m - x_j^m \right\| >= \varepsilon$$

Where $|x_i{}^m$- $x_j{}^m|$ is the distance between points $x_i{}^m$ $x_j{}^m$ denoting the sup.norm.

Brock, Dechert and Scheinkman (1987) showed that if the time series is I.I.D.

$$C_{\varepsilon,m} \approx [C_{\varepsilon,1}]^m \qquad (15)$$

If the ratio $\frac{N}{m}$ is greater than 200, the values of $\frac{\varepsilon}{\sigma}$ range from 0.5 to 2 (Lin, 1997) and the values of m are between two and five (Brock et al., 1988). The quantity has an asymptotic normal distribution with zero mean and a variance $V_{\varepsilon,m}$ defined as:

$$V_{\varepsilon,m} = 4[K^m + 2\sum_{j=1}^{m-1} K^{m-j} C_\varepsilon{}^{2j} + (m-1)^2 C_\varepsilon{}^{2m} - m^2 K C_\varepsilon{}^{2m-2}] \qquad (16)$$

Where $\quad K = K_\varepsilon = \frac{6}{N_m(N_m-1)(N_m-2)} \sum_{i<j<N} h_{i,j,N;\varepsilon}$

$$h_{i,j,N;\varepsilon} = \frac{[I_{i,j;\varepsilon}I_{j,N;\varepsilon}+I_{i,N;\varepsilon}I_{N,j;\varepsilon}+I_{j,i;\varepsilon}I_{i,N;\varepsilon}]}{3} \quad (17)$$

The BDS test statistic can be stated as:

$$BDS_{\varepsilon,m} = \frac{\sqrt{N}[C_{\varepsilon,m} - (C_{\varepsilon,1})^m]}{\sqrt{V_{\varepsilon,m}}} \quad (18)$$

BDS test is a two-tailed test, we should reject the null hypothesis if the BDS test statistic is greater than or less than the critical values (e.g. if α=0.05, the critical value = ±1.96).

4.6 Lyapunov Exponent

In chaos theory more complex approaches have to be used to determine and study the time series. We use the dominant Lyapunov Exponent, which provides both qualitative and quantitative information of dynamical behavior. The characteristic of chaos of sensitive dependence on the initial conditions can otherwise be described as divergence of trajectories with similar initial conditions. The Lyapunov Exponent is an important tool for finding the presence of sensitive dependence. (Sprott, 2013). The L. exponents describe somehow the exponentially divergence or convergence of nearby orbits in the space. They can be thought of as the average exponential rate of divergence or convergence between trajectories that differ minuscule in their initial conditions (Wolf et al., 1985). A system with one or more positive Lyapunov exponents is defined to be chaotic.

The Lyapunov exponent can be described in mathematics as a quantity that characterizes the rate of separation of infinitesimally close trajectories. Considering two points in space with initial separation "ε" that diverge. The first point is $x_o$and the second differs in the initial conditions by Δx_o , that is x_o + Δx_o. The separation is defined as:

$$\varepsilon = || \; x_o - x_o + \Delta x_o || \qquad (19)$$

Those two points will generate two paths in the space, namely trajectories. The trajectories will be a function of the initial difference and time.

The difference between the two trajectories after time can be estimated by the Lyapunov exponent.

$$\lambda = \lim_{t \to \infty} \frac{1}{t} \ln \frac{|\Delta x \; (x_0,t)|}{|\Delta x_0|} \qquad (20)$$

A positive Lyapunov Exponent indicates a chaotic system in the sense that two trajectories which start from similar states, will diverge exponentially. The bigger the value of the positive exponent, the more chaos exists in the system and the shorter the available time for predictions. Wolf et al. (1985) estimate the Lyapunov Exponent by averaging the observed orbits divergence rates.

4.7 Robustness Tests

4.7.1 Augmented Dickey Fuller

Rejection of the null hypothesis in all cases of BDS is neither sufficient nor exclusive evidence to say that the series follow nonlinear dynamics. It is possible that structural changes in the data series can lead to the false rejection of the null hypothesis (Pandey V., Kohers T. & Kohers G. (1998), Hsieh (1991)). The issue of non-stationarity in time series is highly likely in the emerging markets like Vietnam. To rule out any possibility of unit root we conduct the Augmented Dickey Fuller (ADF) test. The test is applied to the return series to find out the presence of unit root in order to check whether there is any non-stationarity in the series or not. The rejection of the null hypothesis of unit root implies that all series are stationary and the rejection of the null IID of BDS is accurate and was not because of non-stationarity. The ADF test is based on the estimation of the following regression:

$$\Delta_{Y_t} = \alpha + \beta t + \gamma^* y_{t-1} + \delta_1{}^*\Delta y_{t-1} \text{ .... } + \delta_{p-1}{}^*\Delta y_{t-p+1} + \varepsilon_t \quad (21)$$

Where a is the constant and βt is the trend (deterministic terms)

$\delta_1{}^*\Delta y_{t-1}$ .... + $\delta_{p-1}{}^*\Delta y_{t-p+1}$ with p lagged difference terms is used to estimate ARMA structure of errors

Values of p are set such that the error ε_t is serially uncorrelated and the error term is implicitly assumed to be homoscedastic.

The DF test statistic:

$$\text{DFt} = \frac{\gamma}{SE(\gamma)} \qquad (22)$$

Where γ is the estimated coefficient of the equation.

H_0: There is a unit root

H_1: The series is stationary (or trend-stationary)

4.7.2 Phillips Perron test for stationarity

The Phillips-Perron (1988) (PP) unit root test is a non-parametric test based on asymptotic theory. It differs from the Augmented Dickey Fuller (ADF) test mainly in the fact that it is not required to select serial correlation and that it takes into account the heteroskedasticity of error terms. The test regression for the PP test is given by:

$$\Delta_{Y_t} = \alpha + \beta t + \gamma^* y_{t-1} + \mu_t \quad (23)$$

Where μ_t is stationary and might be heteroskedastic in nature. The advantage of using PP test over ADF test is that PP test is more robust to general forms of heteroskedasticity of the error term and also that no specification is needed for the lags length. Those are called HAC type corrections.

5. Empirical Results

5.1 Autocorrelation Ljung-Box statistic Q

Table 2 : Autocorrelation

AUTOCORRELATION Q-stat												
	VNI		VN30		VN100		VNMIDCAP		VNSMALLCAP		VNALLSHARES	
LAG	AC	Q-stat	AC	Q-stat	AC	Q-stat	AC	Q-stat	AC	Q-stat	AC	Q-stat
1	0,216	128,17	0,139	47,846	0,038	1,522	0,086	7,5904	0,117	14,171	0,04	1,6817
2	0,062	138,63	0,039	51,644	0,097	11,404	0,062	11,508	0,029	15,025	0,094	10,87
3	0,035	142,05	-0,002	51,656	0,008	11,465	0,009	11,599	0,008	15,087	0,01	10,97
4	0,067	154,39	0,035	54,784	0,038	12,946	0,024	12,203	0,021	15,567	0,038	12,45
5	0,03	156,78	0,023	56,748	-0,047	15,234	-0,034	13,392	-0,016	15,826	-0,045	14,616
6	0,007	157,19	0,011	56,145	-0,005	15,626	-0,004	13,411	0,031	16,799	-0,005	14,641
7	0,01	158,15	0,037	56,441	0,005	15,289	0,016	13,679	-0,038	18,274	0,003	14,684
8	0,019	160,65	0,001	59,832	0,008	15,36	0,015	13,9	0,039	19,843	0,009	14,733
9	-0,03	161,22	-0,045	59,835	-0,035	16,657	0,016	14,173	0,045	21,93	-0,033	15,883
10	-0,014	161,52	-0,038	64,889	-0,007	16,705	0,043	16,121	0,013	22,101	-0,005	15,911
11	-0,011	164,24	-0,012	68,864	-0,049	19,215	-0,054	19,158	-0,036	23,469	-0,051	18,648
12	0,031	174,61	0,004	68,897	0,002	19,22	0,029	20,015	-0,022	23,991	0	18,648
13	0,061	176,25	0,039	72,76	0,034	20,465	-0,009	20,098	-0,025	24,628	0,034	19,844
14	0,024	176,99	0,007	72,872	-0,03	21,404	-0,034	21,33	0	24,628	-0,029	20,705
15	-0,016	177,63	0,002	72,883	-0,011	21,532	-0,017	21,627	-0,018	24,958	-0,012	20,863
16	-0,015	177,71	0	72,883	0,071	26,791	0,086	29,411	0,027	25,743	0,072	26,31
17	0,005	177,71	0,014	73,388	0,001	26,792	-0,019	29,794	0,001	35,2	0,001	26,311
18	-0,001	177,9	-0,015	73,94	-0,023	27,354	-0,05	32,406	-0,095	35,528	-0,026	27,032
19	-0,008	178,76	-0,001	73,943	-0,061	29,324	-0,059	36,013	-0,018	36,885	-0,045	29,149
20	0,018	181,95	0,03	76,169	-0,056	29,874	-0,008	36,072	0,036	40,701	0,023	29,715
21	0,034	186,42	0,022	77,415	0,018	33,805	-0,054	39,101	-0,06	42,464	-0,062	33,828
22	0,04	187,14	0,01	77,688	0,008	37,135	-0,039	40,686	-0,041	42,709	-0,057	37,238
23	0,016	187,17	0,029	79,758	0,017	37,498	-0,017	41,008	-0,015	42,796	0,017	37,55
24	-0,003	187,36	-0,014	80,286	0,062	37,567	0,016	41,289	0,009	44,947	0,009	37,632
25	0,008	187,38	0,012	81,002	-0,017	37,885	0,014	41,505	0,045	44,951	0,02	38,061

The estimated results from Table 2 show that autocorrelation coefficients of the returns are significant. Additionally, based on the Q-statistics, the null hypothesis of absence of autocorrelation in the index returns for all lags selected is strongly rejected at the one percent significance level. The Q-statistics fails to support the joint null hypothesis that all autocorrelation coefficients observed (from lag 1 to lag 25) are equal to zero. This is the first sign of dependence in our return series. It also worth noticing that the 1st A/C of the VN-index has the highest value (0,216) compared to the other indices, meaning that the autocorrelation for VN-index is more intense.

5.2 Variance Ratio Test

Table 3 presents results for both null hypotheses of the variance ratio test, namely the homoscedasticity and heteroscedasticity random walk.

Table 3: Variance Ratio

	2	4	8	16	32
VNI					
VR	0,599105	0,298158	0,156919	0,081081	0,038694
Z	-20,95419	-19,60856	-14,89723	-10,91181	-7,877192
Z*	-13,4370400	-13,0661200	-10,3881700	-7,9881700	-6,0177940
VN30					
VR	0,557921	0,280283	0,145435	0,073039	0,036312
Z	-22,059720	-19,196730	-14,415860	-10,508490	-7,538856
Z*	-14,265110	-12,911435	-10,189410	-7,814162	-5,839037
VN100					
VR	0,469877	0,251276	0,130412	0,061585	0,032301
Z	-17,07129	-12,88776	-9,466706	-6,865364	-4,885391
Z*	-9,589148	-7,703188	-6,1686	-4,888422	-3,693359
VNMIDCAP					
VR	0,514144	0,267906	0,135829	0,063027	0,032542
Z	-15,57014	-12,54059	9,362261	6,821685	4,860564
Z*	-7,958097	-6,940449	-5,8086	-4,771181	-3,690046
MDSMALLCAP					
VR	0,550682	0,277955	0,136892	0,069559	0,034723
Z	-14,46219	-12,42253	-9,391635	-6,803745	-4,870812
Z*	-7,926307	-7,363288	-6,21078	-4,89939	-3,817818
MDALLSHARES					
VR	0,472525	0,251738	0,130489	0,061565	0,032253
Z	-17,00236	-12,8922	-9,47499	-6,872124	-4,890341
Z*	-9,570989	-7,710385	-6,177245	-4,898265	-3,705902

Note: 1 Z is the statistic for the homoscedasticity assumption, whereas Z the statistic for the heteroscedasticity assumption.*

From the estimation results we can observe that the null hypothesis of random walk behavior is strongly rejected for all the data series for all the lags tested. The Z statistics for both hypotheses are greater than the conventional critical value (1.96 for a=5%). The results of variance ratio test further confirm the rejection of IID and enhance the dependence between the series and the rejection of the random walk.

5.3 RUNS TEST

Table 4: Runs Test

RUNS TEST	
VNINDEX	
Standard Normal	54.471
p-value	5.119-e08
alternative hypothesis	two sided
vn30	
Standard Normal	-29.098
p-value	0.003616
alternative hypothesis	two sided
VN100	
Standard Normal	-16.121
p-value	0.1069
alternative hypothesis	two sided
smallcap	
Standard Normal	-38.049
p-value	0.0001419
alternative hypothesis	two sided
mediumcap	
Standard Normal	-21.149
p-value	0.03444
alternative hypothesis	two sided
allshares	
Standard Normal	-14.756
p-value	0.1401
alternative hypothesis	two sided

We can observe from the results of the table 4 that for the indices VN-index, VN30, VN-small cap and VN-medium cap the null hypothesis of randomness is rejected. For the other two indices, namely the VN-100 and the VN-all shares, the null is accepted, meaning that for the runs test the two indices are produced randomly and there is no dependence between the returns.

5.4 GARCH Model

Table 5: GARCH

	GARCH			
	B	C	RESID	GARCH
	VN-INDEX			
coef	0,000484	4,10E-06	0,146052	0,836565
p-value	0,0107	0	0	0
	VN30			
coef	0,000515	5,08E-06	0,131662	0,839648
p-value	0,0105	0	0	0
	VN100			
coef	0,000828	4,05E-06	0,118564	0,8435
p-value	0,0016	0	0	0
	VNMIDCAP			
coef	0,001031	1,35E-05	0,263684	0,607353
p-value	0,0001	0	0	0
	VN-SMALLCAP			
coef	0,000427	1,56E-04	0,243338	0,510239
p-value	0,0665	0	0	0
	VN-ALLSHARES			
coef	0,000797	4,05E-06	0,122899	0,836735
p-value	0,0017	0	0	0

Since all of the above tests have rejected the IID hypothesis of our data series, we want to check whether the non-linear dependence enters to our data through volatility. ARCH type models are sufficient to capture and model the volatility and correct our data series. We will use the residuals after filtering for GARCH and E-GARCH model to apply the more advance BDS test.

We can observe from the table 5 that for all indices the GARCH model is statistically significant.

$$\sigma_t{}^2 = \alpha_0 + \alpha_1\ \varepsilon_{t-1}{}^2 + \alpha_2\ \sigma_{t-1}{}^2 \qquad (12)$$

The first Column (B) refers to the constant of the mean equation, whereas the rest three columns to the variance equation (12). The second column, named with C is the constant of the variance equation (α_0), the third and fourth column are the coefficients of the GARCH model. Third column refers to α_1 and the fourth column to α_2.

It is worth noting that the values of residuals and GARCH are very close but less than 1 for all market indices, which means that the process reverts slowly. The null hypothesis of $\alpha_1 + \alpha_2 = 1$ (no volatility among the indices) is rejected. This indicates that the returns of all indices were highly volatile during the period we study. The coefficients of residuals show the sign of the volatility whereas GARCH coefficient shows the persistence.

5.5 E-GARCH model

Table 6: E-GARCH

	EGARCH			
	ω	α	γ	β
	VN-INDEX			
coef	-0,5131	2,57E-01	-0,036895	0,964646
p-value	0	0	0,0001	0
	VN30			
coef	-0,534902	2,52E-01	-0,040951	0,961941
p-value	0	0	0	0
	VN100			
coef	-1,061354	2,35E-01	-0,146137	0,90636
p-value	0	0	0	0
	VNMIDCAP			
coef	-1,83484	2,66E-01	-0,195055	0,828987
p-value	0	0	0	0
	VN-SMALLCAP			
coef	-2,843899	3,13E-01	-0,177481	0,735539
p-value	0	0	0	0
	VN-ALLSHARES			
coef	-1,085628	2,34E-01	-0,147752	0,9049
p-value	0	0	0	0

The table 6 from E-GARCH estimation can reveal that for all the indices the asymmetric and the leverage effect exists. Only the results from the variance equation are reported.

$$\log \sigma_t^2 = w_0 + \sum_{i=1}^{p} \beta_i \log \sigma_{t-i}^2 + \sum_{j=1}^{q} \left(\alpha_j \left| \frac{\varepsilon_{t-j}}{\sigma_{t-j}} \right| + \gamma_t \frac{\varepsilon_{t-j}}{\sigma_{t-j}} \right) \tag{13}$$

All γ are negative with p-value equal to zero. This is a result that we were expecting since the values of skewness from the descriptive statistics had also given us a sign. The higher leverage effect comes from the VN-midcap index, which is a sign that a negative shock in the conditional variance has more effect than a positive shock. The highest persistent of volatility (β) comes from the VN-index. It is also worth noting that all the indices have positive results for the ARCH effect (volatility clustering).

5.6 BDS Test

Since we find strong evidence from the previous tests that all of our return series do not follow random walk and the returns are dependent, we conduct the BDS test to find what kind of dependence exist between the returns of our indices.

We use different values of ε for the test, specifically at 0.5σ, 1σ, 1.5σ and 2σ, where σ is the standard deviation.

We assume ad hoc that the delays are 5 to include information with no meaning. Thus, the value of m is taken from 2 at the lower end to 5 at the upper end.

The null hypothesis is rejected for all the indices on raw returns. A level of 5% significance is taken in this hypothesis testing. Table 7 presents the test statistic for a) the returns series b) the residuals of the returns after filtering for AR c) the standardized residuals of GARCH and d) the standardized residuals of EGARCH. Even though the test is the same, the test statistics in every stage test different hypotheses.

For the first phase, the BDS is applied on the raw returns and the hypothesis of the test is as follow:

H_0: The data are independently and identically distributed (I.I.D.)

H_1: The data are not I.I.D. / General Dependence

We can clearly see that we reject the null hypothesis and accept the hypothesis of dependence. As the IID assumption is rejected in the first stage, the dependence that exists can be either linear or non-linear.

For the first reason (the serial or linear correlation), the second phase of the BDS test is done on the residual found by fitting the ARMA (p, q) model to the raw return series. The BDS statistics of the residuals of ARMA model are shown in the same table in column b.

As stated before, we fitted the best ARMA model based on the Akaike criterion for every index. The results show the rejection of the null hypothesis of IID for all indices in the case of residuals as well. Since serial correlation is not the cause of the rejection of IID then it can be because of non-stationarity or non-linearity.

Arriving in the last reason of possible rejection of IID in the first stage, the non-linearity, we apply GARCH and EGARCH model in our indices. The results are presented in the third and fourth column of the table.

In this case, only the VN-index rejects once again the null hypothesis for all dimensions tested. For the indices VN30 and VN100 only some z-values are greater than the rejection levels in some dimensions. So for all of the indices except VN-index, the null is rejected, meaning that conditional heteroskedasticity is the main cause of the initial rejection of the null hypothesis and that our data series behave as non-linear stochastic. Non rejection of IID in this stage suggests that low order ARCH type models like GARCH and E-GARCH are sufficient enough to capture all the potential nonlinearities in the returns of our data series. However, they cannot explain the behavior and the dynamics of the VN-index .

Table 5: BDS Test

ε	0.5σ				1σ				1.5σ				2σ			
m	RETURNS	AR	GARCH	EGARCH	RETURNS	AR	GARCH	EGARCH	RETURNS	AR	GARCH	EGARCH	RETURNS	AR	GARCH	EGARCH
VNINDEX																
2	18,06505	18,65322	3,01676	2,442546	19,83347	18,9692	3,270097	2,775191	20,60491	18,60088	3,136741	2,85452	20,53152	17,02564	2,551292	2,599355
3	24,53055	25,74576	3,331667	2,830776	25,55072	25,33749	3,494025	3,071749	25,62368	24,68446	3,192909	3,05644	24,6779	22,84858	2,580074	2,749771
4	31,64782	33,3731	3,952856	3,588604	30,52189	30,52532	3,632187	3,253935	23,12283	28,49161	3,091139	3,08749	26,98667	25,88831	2,438084	2,721608
5	40,37275	42,83859	4,166479	3,94055	35,36595	35,68704	3,814703	3,379346	32,03899	31,7102	2,912305	2,99211	28,96164	28,24315	2,190257	2,570604
VN30																
2	13,37338	13,91739	1,057270	0,677179	14,81021	15,31522	1,406556	1,075745	15,79083	15,87758	1,680198	1,305085	15,53795	15,07494	1,680959	1,582087
3	18,66904	19,23302	1,587198	1,185058	19,69248	20,41846	1,876206	1,564909	20,27584	20,90694	1,967023	1,622251	19,33684	19,68851	1,837521	1,791573
4	22,87348	23,42662	1,955125	1,393773	22,88408	23,5985	2,015361	1,680032	23,01524	23,59161	1,993748	1,743624	21,72507	22,09875	1,778622	1,839588
5	27,05447	27,59256	2,474466	1,882692	25,59834	26,40746	1,918573	1,574159	25,01453	25,68255	1,765705	1,542501	23,44769	23,86446	1,513264	1,653996
VN100																
2	7,077435	7,560868	0,715659	-1,05698	7,99205	8,514782	0,597462	-1,0326	8,894934	9,393382	0,859193	-0,73252	8,819555	9,410673	0,46224	-0,55914
3	9,425836	9,784138	1,708865	-0,20325	10,50501	10,9068	1,207154	-0,37239	11,2012	11,61024	1,309476	-0,26684	10,97573	11,5774	0,57033	-0,23062
4	10,96964	11,31855	1,978139	-0,36841	11,96141	12,23374	1,309566	-0,37952	12,31489	12,66135	1,332063	-0,334	12,01525	12,55835	0,42896	-0,40932
5	12,52608	12,77593	2,320812	-0,41593	12,77126	13,02279	1,055277	-0,71089	12,69413	13,04599	1,002227	-0,63443	12,29347	12,82918	0,07187	-0,60071
STOCK	MID CAP															
2	8,115901	8,277716	-0,25966	0,265577	8,337326	8,463495	-0,026168	0,240319	8,446281	8,816935	0,111512	0,46745	8,306801	8,965892	0,065698	0,308394
3	9,520880	9,933121	-0,35664	0,257405	10,06918	10,23444	-0,289924	0,056609	10,49935	10,72841	-0,172366	0,386921	10,31813	10,74886	-0,191485	0,330615
4	10,24568	10,82446	-0,87314	0,07254	10,6734	11,02084	-0,832093	-0,27197	11,05556	11,27691	-0,823527	-0,09272	10,7578	11,10983	-0,839499	-0,14994
5	10,75128	11,14975	-1,35738	-0,30184	11,20393	11,60911	-1,240978	-0,63567	11,56054	11,77604	-1,269767	-0,37889	11,15117	11,51481	-1,169450	-0,37021
STOCK	SMALL CAP															
2	7,691539	7,423625	0,787743	1,188558	7,918423	7,891624	1,056463	1,429744	8,371615	8,523329	1,217711	1,502722	8,365278	9,28962	1,161535	1,328746
3	9,021495	8,914093	0,604282	1,309905	9,495468	9,656041	1,253843	1,776598	9,796619	10,16721	1,463208	1,852009	9,490741	10,40933	1,265633	1,631096
4	9,840867	9,543209	0,349519	1,360024	10,40802	10,47673	1,342688	1,832494	10,40397	10,79112	1,431801	1,890894	9,696058	10,61801	1,136354	1,559135
5	11,04429	10,9625	0,511821	1,631606	11,22733	11,30564	1,453748	1,928896	10,88165	11,30973	1,472281	1,933192	9,849859	10,81631	1,170228	1,547325
STOCK	ALL SHARES															
2	7,10486	7,249815	0,77362	-0,8268	8,280372	8,64575	0,915417	-0,90409	9,198684	9,625299	1,232170	-0,37033	9,043209	9,614684	0,893133	-0,2813
3	9,265892	9,46654	1,492875	-0,35065	10,51979	10,92622	1,285648	-0,56581	11,24529	11,68992	1,269615	-0,25273	11,04887	11,63056	0,567065	-0,29386
4	10,72017	10,89367	1,727551	-0,97847	12,03654	12,35039	1,454677	-0,54928	12,39766	12,75369	1,333391	-0,29675	12,10725	12,62927	0,468130	-0,41232
5	12,24368	12,50205	1,784871	-1,2665	12,98265	13,33587	1,359806	-0,81285	12,92600	13,27049	1,129244	-0,55728	12,46373	12,97155	0,272588	-0,51859

5.7 Augment Dickey Fuller and Phillips Perron test (Unit root test)

Another possible reason for rejection of IID in the BDS test is the presence of a unit root. To make sure and rule out any possibility of unit root we conduct the ADF and the PP test.

We compute the ADF and the PP test for our data series by checking whether the constant and the trend are statistically significant.

For the all the indices neither the constant nor the trend are significant.

Table 6: ADF and Phillips Perron TEST

	ADF					
	t-stat	Adj t-stat PP	1%	5%	10%	Probability
VNI	-41,9477	-42,70282	-2,565809	-1,94094	-1,616622	0
NV30	-43,36429	-43,66836	-2,565891	-1,940951	-1,616614	0,0001
VN100	-20,21449	-31,00513	-2,567196	-1,941129	-1,616494	0
VNALLSHA	-20,28237	-31,00209	-2,567192	-1,941129	-1,616494	0
VNMIDCA	-29,31205	-29,35451	-2,567216	-1,941132	-1,616492	0
VNSMALL	-28,59452	-2866209	-2,567196	-1,941129	-1,616494	0

The values at 1%, 5% and 10% stat. significant level for Phillip Perron test are not reported since they are very similar with the ADF test with changes only in the last digit. However, they are available on request.

The null hypothesis for the case of the non-stationary Augment Dickey Fuller and Phillips Perron test is the presence of a unit root in the returns. In table 8 it can be clearly seen that the null is rejected and therefore our returns are stationary. The t-stat refers to the results of the ADF test whereas the Adj t-stat to the PP test. Hence, the rejection of IID in the BDS test was not because of non-stationarity in our data.

5.8 Lyapunov exponent

Table 7: Lyapunov Exponent

index	max λ
VN-INDEX	-0,7404551
VN30	-0,7516026
VN100	-0,6729312
VNSMALL	-0,7011212
VNMED	-0,7208855
VNALLSHARES	-0,6917565

Table 9 presents estimates of the maximum Lyapunov Exponents of our series using the estimation method of Wolf et al. (1985). The Lyapunov Exponents were estimated with embedding dimensions up to four as in Wolf (1991). For all our indices, even the VN-index for which a GARCH model is not adequate enough to capture the all the information, the chaos assumption is invalid. The results (all λ are negative) show that no chaos exists in the Vietnamese stock market indices and that the nature of our indices is consistent with a stochastic process.

Like most of the studies in the emerging markets such as Ritesh KumarMishra et'al (2011) and Mattarocci, G. (2009) have not found any strong sign of chaotic behavior in their data. Especially Mattarocci, G. (2009) that has conducted an international comparison to find out what influence the returns dynamics of the stocks and the results were far from chaos for most of the countries he examined. Their surveys were also promising for positive sign of chaos but the negative λ in Lyapunov exponent did not allow them to accept deterministic behavior in their series.

6. Conclusion

The aim of this paper was to investigate if the six indices from the Vietnamese stock market in Ho Chi Minh City are governed by noisy dynamics, stochastic of deterministic. Non-linearity and chaos are important in financial markets, since their presence provides short term predictability and possible gains for investors. We try to detect non-linearity (including chaos) in the Vietnamese stock market because even though there are numerous empirical studies of non-linearity and chaos, those studies in emerging stock markets are scarce. Beginning with the descriptive statistics, we had the first sign that Efficient Market Hypothesis does not hold since the values of kurtosis and skewness were far from the efficient threshold. After checking for general dependence, the results showed strong positive signs of dependence. The ARCH-type models in most of the cases (5/6 indices) were able to adequate the information of our returns series resulting to the point that those indices are generated stochastically and chaos is not the reason of dependence. In one of the indices, namely VN-index, the ARCH-type models were not sufficient enough to capture all the information and thus the dependence could be a result of chaotic behavior. However, after applying the Lyapunov exponent test for chaos, for all of our indices the results were negative and close to -1, indicating that chaotic behavior does not appear in our indices. As a result, we conclude that the VN-index can be characterized by deterministic non-linear dependence, though not necessarily chaos.

Summarizing the findings of the paper, our results provide evidence for non-randomness in each of the return series index. In any case, since the results differ between the types of the indices we cannot assure and state that chaos or non-linear stochastic dynamics rule the Vietnamese stock market. Future implications considering other emerging Asian markets as well as the one examined here, the Vietnamese stock market, should take into consideration some corrections. The Vietnamese stock market may be one of the highly emerging markets in the world but that does not exclude the consideration of being known for its thin trading. In a thin market prices are more volatile and assets are less liquid. Corrections of the narrow-market phenomenon could change the results of Lyapunov exponent. Another future implication could be the search of chaotic dynamics not only in daily returns but also intra-day data. A lot of short-term investors look at intra-day data and try to predict the future returns in order to make fast gains.

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