Mathematical modeling of tax evasion:A case study for Greece

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Abstract

The aim of this thesis is to study how taxpayers decide to evade part of their income under certain constraints specific to the nature of their occupation. We further develop a model on taxpayer's compliance decision based on the Standard Model of Yitzhaki by introducing a threshold defined as the fraction of the actual income that is declared. We also make computations to examine taxpayer's response to changes in policy parameters such as the tax penalty coefficient, the tax rate and the probability of tax audits to take place, with respect to the income they decide to hide. We find that taxpayers are motivated to declare a higher portion of their income when policy makers increase the tax penalty coefficient and arrange some more frequent tax audits.

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Contents

1 Introduction **5**

2 Previous Literature **9**

3 Modeling **12**

3.1 Effect on a_i* . . . 14

3.1.1 Effect of changes in tax rate t . . . 15

3.1.2 Effect of changes in tax penalty coefficient f . . . 16

3.1.3 Effect of changes in the probability of tax audits p . . . 17

3.1.4 Effect of changes in h . . . 17

3.2 Effect on tax revenues . . . 17

4 Simulation Results **18**

4.1 Changing the f . . . 20

4.1.1 First case: $f = 0.5$, $t = 0.29$, $p = 0.05$ . . . 20

4.1.2 Second case: $f = 2$, $t = 0.29$, $p = 0.05$ . . . 22

4.2 Changing the p . . . 23

4.2.1 First case:$f = 0.5$, $t = 0.29$, $p = 0.1$ . . . 23

4.2.2 Second case:$f = 2$, $t = 0.29$, $p = 0.1$ . . . 23

4.3 Changing the t . . . 24

4.4 Changing the h . . . 26

5 Conclusion **28**

List of Tables

1 Values of λ_i, h_i, a_i^* and T_i for each taxpayer group i corresponding to $f = 0.5$. . . . 21

2 Values of λ_i, h_i, a_i^* and T_i for each taxpayer group i corresponding to $f = 2$. 22

3 Values of λ_i, h_i, a_i^* and for each taxpayer group i corresponding to $p = 0.1$. 23

4 Values of λ_i, h_i, a_i^*, X_i and T_i for each taxpayer group i corresponding to $p = 0.1$. . 24

5 Values of λ_i, h_i, a_i^* and T_i for each taxpayer group i corresponding to $t = 0.29$. . . . 25

6 Values of λ_i, h_i, a_i^*, X_i and T_i for each taxpayer group i corresponding to $t = 0.3$. . 26

7 Values of λ_i, h_i, a_i^*, X_i and T_i for each taxpayer group i before h_i changes. 27

8 Values of λ_i, h_i, a_i^*, X_i and T_i for each taxpayer group i after h_i changes. 28

1 Introduction

Tackling tax evasion has always been one of the challenges that governments face. Tax evasion is the deliberate underreporting of income derived from a taxable activity and it is considered to be part of the "shadow economy" and thus an illegal activity. It can be blamed for deficits and decreased revenues which are necessary to support various sectors of the economy such as education, health systems, infrastructure and provide the necessary goods and services to the individuals. In countries where the tax rate is relatively high and individuals are unsatisfied from public goods and services, tax evasion may be a more serious problem, because in this case individuals care more about maximizing their own utility by hiding income, ignoring the implications for social welfare. When policy makers cannot collect enough tax revenues to have available for public spending, they may take stricter measures such as increasing the tax rate to obtain the necessary revenues. This, however, could be detrimental to honest taxpayers who reveal all of their income as they would be obliged to pay for dishonest taxpayers as well, leading to a decrease in their utility and wealth and finally to a welfare loss. In addition, raising tax rates becomes counterproductive after a point, because it encourages tax evasion and discourages economic activity. Greece is an example where tax rates continued to increase within the years as a response to tax evasion. Greece has suffered from tax evasion persistently throughout the years starting long before the economic crisis of 2009. The size of the Greek shadow economy was estimated to be up to 40% (Artavanis et al., 2016)[2]. This is an indication of how challenging it has been for the government to finance its debt. Both the EU and the IMF have pointed to the need for increasing tax revenues and decreasing tax evasion. As a result, policy makers proceeded to tax rate increases. However, these tax rate increases did not contribute to collecting the estimated amount of tax revenues because of the persistence of some taxpayer groups to hide their income. For example, Pappada Zylberberg (2015)[12] have shown that the implementation of the austerity plan in 2010 in Greece which allowed for an increase in the tax rate was not efficient as tax revenues raised by a lower percent than was expected because small and middle firms decreased their declared income. Thus, it is vital for policy makers to find effective ways of combating the problem of tax evasion.

One way to achieve this goal is to increase tax penalty rates. An increase in the tax penalty rate will not guarantee though positive results for the policy makers, because some occupations are characterized by high opportunities for tax evasion and thus taxpayers can hide their income more easily, because either tax audits may be less frequent or unreported income may be difficult to detect. Besides adjusting tax penalty rates, policy makers could potentially combat tax evasion by raising the probability of tax audits too. In the context of the Greek economy, it was found that higher probabilities of tax audits on risk-averse firms may be a more effective method to deter tax evasion than raising tax penalties (Goumagias, Hristu-Varsakelis, 2012)[6]. However, increasing the frequency of tax audits may be costly for the government as policy makers would spend more resources needed to make proper and careful tax audits such as hiring experts. In addition, it should be taken into account that the desired outcome of minimizing tax evasion problem by both from rising tax penalty rates and the frequency of tax audits may be outweighed by an increase in tax rates. Policy makers could impose increases in tax rates in order to raise tax revenues. In this case, it is not certain how taxpayers will respond to this increase and it may be possible to become motivated to hide their income when they face an increasing tax rate as we will show in the next sections of the thesis. Clotfelter (1983)[4] also showed that an increase in US tax rates tends to encourage tax evasion.

In this thesis, we study taxpayers' decision to hide income under certain constraints specific to their occupation in the context of the Greek economy. In particular, we will examine taxpayers' response to changes in tax penalty coefficient and probability of a random tax audit to take place in the context of tackling tax evasion. We will also examine how these changes increase or decrease the expected amount of tax revenues. By studying taxpayers' decision to hide income and the risk they face, governments not only should be able to design policies which restrict this illegal activity by targeting specific groups of taxpayers, but also to collect more tax revenues. This study offers a computational tool needed to explore the taxpayers' decision to hide income under constraints which are specific to their occupation. Some occupations provide the opportunity to taxpayers to hide more income such as large enterprises, whereas other occupations such as salaried employees do not make it easy to engage in tax evasion as their wages and salaries are known by the government and therefore it is more probable for salaried employees to get caught by tax authorities. Because there is a large set of occupations in the job market, we will study compliance decision in terms of occupational group to which different taxpayers belong. Thus, we assume that each taxpayer group faces constraints regarding tax evasion which is the same for all taxpayer members belonging to the same group.

In particular, we extend the Standard Model of Yitzaki (Yitzaki, 1974) by introducing in the utility maximization problem a threshold a_i which is defined as the fraction of the income that a taxpayer group i declares and therefore it reflects the tax evasion opportunities of a specific occupation category. This threshold a_i though takes a minimum value h_i for each taxpayer group such that $a_i \geq h_i$ depicting the lower bound on the fraction of income that a taxpayer group could declare. This value h_i will be low for some occupations allowing taxpayers to declare only a small portion of their actual income, and high for other occupations where taxpayers cannot avoid to declaring a larger portion of their income. However, the amount of income that will be finally declared depends on the self-interested optimal level a^* that maximizes each taxpayer group's utility. It is possible as we show in the section 4 that a taxpayer group's utility is maximized at the point where taxpayers report a higher level of income rather than the lowest possible income level which they could have declared it. The addition of the threshold a to the model corrects for the assumption in (Yitzhaki, 1974) that all taxpayers are able to evade to the same degree and will allow us to observe how digression from the self-interested optimal level of tax evasion opportunities that maximizes taxpayer's utility affects his or her "current" utility.

We also extend the model by using constant relative risk-aversion utility function as Yitzhaki's model assumes absolute risk-aversion which implies that an increase in the tax rate decreases the amount of hidden income, a notion that might not be in accord with empirical data. Using MATLAB, we will calculate the optimum level of threshold a that maximizes the utility for several occupation groups taking into account different levels of tax penalties, tax rates and probabilities of random tax audits. Therefore, we will know the maximum fraction of the income that each taxpayer group on average will declare and compare it to the lower bound h on the fraction of the income that taxpayers could declare given the values of the other parameters. If it is in the interest of taxpayers to declare the lowest possible income because at that amount of declared income their utility is maximized, then policy makers should use specific tools such as increasing tax penalty coefficients or organizing more frequent tax audits to motivate taxpayers to behave more compliantly.

For that reason, we will examine different scenarios where in each of them we will observe taxpayer group decision-making regarding tax evasion when the tax penalty varies or when the probability of a random tax audit is high. These scenarios depict how taxpayers' decision-making with respect to hiding income changes when policy makers tackle tax evasion problem. We expect that high tax penalties will change taxpayers' decision towards tax compliance, whereas a higher probability of a tax audit will leave taxpayers' decision unchanged.

Apart from tackling tax evasion, policy makers take measures to increase tax revenues. More specifically, we will examine how changes in the tax penalty coefficient f, the probability of a tax audit p, the tax rate coefficient t or the lower bound on the fraction of taxpayers' income h may affect the collection of tax revenues. We expect that either giving emphasis on tax tax penalty by increasing highly the tax penalty coefficient or developing mechanisms to make tax declaration more transparent and thus increasing the lower bound of the portion of the actual income, tax revenues will increase.

In conclusion, the thesis is structured as follows. In chapter 2 there will be a brief discussion on the literature review. In chapter 3 there will be a description on the structure of our model under the specific constraints that taxpayer groups deal with. In chapter 4 simulation results are presented regarding the taxpayer group decision to hide income when the tax penalty, the probability of a tax audit, the tax rate and the lower bound on the fraction of the income change for specific values of the risk-aversion coefficient which we assume they are constant for all the scenarios.

2 Previous Literature

In the relevant literature we can distinguish between the neoclassical approach of modelling taxation and that of behavioral economics which attempts to replace the assumptions of the former approach with more realistic ones. Allingham and Sandmo(1972)[1] were the first to establish the neoclassical account of taxation by introducing a theoretical model of individual taxpayers' decision with respect to the size of tax evasion. They assumed that taxpayers are homogeneous and rational and that they face two decisions under risk: a) to declare all of their income and b) to declare less than their income with the risk of being detected. Yitzhaki(1974)[15] introduced modifications to the standard model of Allingham-Sandmo (A-S), where the punishment for tax evasion is calculated as a fine levied on unpaid tax rather than on undeclared income.

There are of course limitations to Yitzhaki's model. The model predicts that tax evasion becomes less attractive as we increase the probability of being detected as well as with an increasing tax rate because of the absolute risk-aversion hypothesis. This does not agree with empirical data as taxpayers tend to overestimate the probability of a tax audit being conducted. Second, the standard models of A-S and Yitzhaki assume that *all* individuals are involved in tax evasion; this is also unrealistic, as some taxpayers face obstacles in hiding their income because of the nature of their job. Finally, Yitzhaki's Model assumes absolute risk-aversion on behalf of taxpayers. However, taxpayers' decisions with respect to the amount of income conceal changes when relative risk-aversion is assumed. "Hidden" income could be reduced when the level of risk-aversion of the average taxpayer increases . In terms of risk-aversion, Bernasconi (1998)[3] uses a model of tax evasion with two different orders of risk averse preferences and show that the excess degree of risk aversion is not required to explain tax compliance, as implied by the portfolio choice approach. By calibrating the preferences of a taxpayer whose attitude towards risk is of order one with parameters obtained from empirical and experimental studies and using representative values of the tax rate and tax penalty, the paper shows that there is not an excess rate of tax compliance. Although it is difficult to alter the taxpayer's attitude towards risk, it is possible for the government to change and optimize the allocation of tax audits with the more frequent ones to take place for the less risk-averse taxpayers (Goumagias, Hristu-Varsakelis, Assael, 2017)[7].

More recent literature has introduced some behavioral features to the A-S model to make it more realistic. Behavioral economic models make use of non-expected utility models based on the Prospect Theory Model which can reverse the taxpayer's decision to evade taxation, if social factors entered models. In particular, Quiggin(1982)[14] formalized a so-called Anticipated Utility Theory by suggesting that decision weights should depend on all individual probabilities, thus correcting the assumption that taxpayers "overweight" the possibility of extreme outcomes. In addition, Piolatto and Rablen (2013)[13] examined the role of the Prospect Theory in reverting the Yitzhaki puzzle which predicts a negative relationship between tax rates and tax evasion. However, their results showed that the diminishing sensitivity of the utility function is not a necessary condition for the dependent model to overturn Yitzhaki puzzle, as the puzzle can be reversed by endogenizing the reference level of wealth. The loss aversion and the probability weighting are irrelevant to the puzzle too. Therefore, for the aforementioned elements, they found that the Prospect Theory failed to reverse Yitzhaki puzzle. Taking into account behavioural factors, Loomes and Sugden(1986, 1987)[10] proposed that the disappointment derived from a poor outcome, such as in the case of a tax audit being conducted, will reduce taxpayer's utility and therefore this sentiment represents the taxpayer from making risky decisions to some extent (Myles, Tran-Nam, 2013)[8]. Furthermore, the introduction of social factors in the taxpayer's tax compliance model may reverse the tax effect. By social factors, we refer to psychic costs, social customs, perception of fairness and tax morale. Lee (2016)[9] considers the role of tax evasion with morality in determining equity of the tax system by using a standard model of tax evasion which incorporates in taxpayers expected utility function a parameter capturing the moral costs proportional to evasion. The analysis has shown that an increase in the moral costs decreases evasion. In addition, the equity of the tax system depends on the degree of morality. With low moral costs, evasion makes the tax system regressive, whereas with high moral costs, evasion makes the tax system progressive.

Empirical literature has also studied the nature and the extent of tax evasion in the Greek economy. Matsaganis, Leventi and Flevotomou (2012)[11] examine the distributional aspects of tax evasion in Greece by combining an estimation of non-compliance patterns in terms of under-reporting income with an estimation of the distribution gains from tax evasion. They employ a direct approach to tax evasion by comparing data form tax returns with survey data, and assume that taxpayers hide part of their income from fiscal authorities, but they reveal it to an anonymous reviewer. Their results show that under-reporting income depends on the level of income. In addition, tax evasion increases average disposable income and reduces tax yield, thus increasing inequality and relative poverty. It also reduces tax receipts, and the tax system becomes less progressive. The sovereign debt crisis has not significantly changed the distribution patterns of under-reporting income. In addition, studies show the extent of tax evasion problem due to the inefficiencies of the tax system.

Pappada Zylberberg (2012) have shown that the implementation of the austerity plan in 2010 in Greece which allowed for an increase in the tax rate was not efficient as tax revenues raised by a lower percent than was expected because small and middle firms decreased their declared income. Georgakopoulos (2016)[5] analyses available data to map out the problem of tax evasion in Greece and provides policy recommendations. The revenue losses are due to tax evasion associated with both direct and indirect taxes, and the scale of evasion ranges from 11 to 16 billion euro a year. The revenues lost are mainly associated with personal income tax evasion, ranging from 1.9-4.7% of GDP, and with VAT evasion, amounting to 3,5% of GDP. In 2011, 8% of taxpayers, representing highly paid workers, and 0.4% of taxpayers, representing large firms, paid 69% and 61% of personal income tax and legal entity income tax, respectively. The self-employed are the taxpayers who hide a large share of their income, from 57-59%. The shadow economy, amounting to 20-30% of GDP, constitutes a part of the economy which avoids paying taxes. A reduction in tax rates, electronic tax administration, penalties, simplification of the tax system are some of the policy recommendations.

3 Modeling

This study pursues a rigorous approach of a taxpayer's utility maximization constrained by a threshold which puts an upper bound to the degree to which he or she can evade taxes. Because in the real world there is a large set of occupations, we study taxpayer behaviour in terms of occupational group to which they belong. Thus, we consider four different occupation groups defined by ISCO in order to create taxpayers group. Specifically, taxpayer group i consists of all taxpayers belonging to the same occupation group, consisting of all jobs with a high degree of similarity, such as skill specialization (ISCO-08 Resolution).

Let Y, X, E, a_i be the actual, declared, undeclared income, and the threshold of tax evasion opportunities respectively, for each taxpayer group i. Undeclared income E_i is defined as $E_i = Y_i - X_i$. Because the average taxpayer in the group i desires to maximize his/her utility which depends on the disposal income(?), each group i faces a choice on how much income could be hidden. The taxpayer's decision whether to declare all or partly of his/her income is constrained by the threshold a_i which allows each taxpayer group to evade tax up to a certain point. Thus, we define the declared income X_i as $X_i = a_i Y_i$, so that a_i is to be understood as the fraction of actual income which the taxpayer reveals. a_i is measured between 0 and 1. The closest to 1, the less the level of tax evasion it is. We will also assume that the tax T is proportional to declared income, i.e, $\mathcal{T}(x)= tx$, and that any tax penalties , F, levied on taxpayers when they are caught in a random tax audit are proportional to undeclared income, i.e, $\mathcal{F}(E)= ftE$. Taxpayer utility also depends on the probability p of a random tax audit taking place. Therefore, the decision problem of taxpayer group i can be formulated:

$$\max_{a_i} V = p_i U_i(Y_d) + [1 - p_i] U_i(Y_d) \tag{1}$$

where $U_i(Y_d)$ is the utility function for taxpayer's group i. We also assume constant relative risk-aversion utility function:

$$U_i(Y_d) = \frac{Y_d^{1-\lambda_i}}{1 - \lambda_i} \tag{2}$$

where λ_i is the risk-aversion coefficient for taxpayer group i, and Y_d is the level of disposable income. The disposal income takes into account two possible situations:: to tax evade and get away with it, and tax evade and be caught. Therefore the disposal income Y^d is equal to

$$Y_i^d = p_i[Y_i(1-t) - ftY_i(1-a_i)] + (1-p_i)[Y_i(1-t) + tY_i(1-a_i)] \tag{3}$$

Thus,the taxpayer's decision problem takes on the form:

$$\max_{a_i} V_i = p\frac{[Y_i(1-t) - ftY_i(1-a_i)]^{1-\lambda}}{1-\lambda} + (1-p_i)\frac{[Y_i(1-t) + tY_i(1-a_i)]^{1-\lambda}}{1-\lambda} \tag{4}$$

subject to $a_i \geq h_i$ where h_i is the minimum value of threshold a can allow a taxpayer group to declare its income.

If the problem is unconstrained, the necessary condition for a maximum with respect to the threshold a_i^* of tax evasion opportunities is:

$$\frac{dV_i}{da_i} = 0 \tag{5}$$

Solving with respect to a_i^* we obtain:

$$a_i^* = \frac{1 + (1-t+ft)(\frac{1-p}{fp})^{\frac{1}{\lambda}}}{t + ft(\frac{1-p}{fp})^{\frac{1}{\lambda}}} \tag{6}$$

Under the constraint $a_i \geq h_i$, the necessary condition for the optimum threshold a_i^* changes:

$$\frac{dV_i}{da_i}(a_i - h_i) = 0 \tag{7}$$

Therefore, there are four cases which are going to be tested using MATLAB.

3.1 Effect on a_i*

In the previous analysis, we examined how each taxpayer group i decides to hide part of the actual income in order to maximize utility given specific values of the other parameters. We determined mathematically the optimum level of the threshold a that determines the optimum fraction of income that each taxpayer group i desires to declare. Then, we are able to determine how changes in the amount of each of our parameters (t, f, p, h) may affect the optimum threshold a_i*. This would be helpful for policy makers as we show in the section 4 to know which measures to take with respect to tax penalty or tax audits so as to combat the problem of tax evasion. At the same time, policy makers care about raising more tax revenues. Tax revenues could be potentially increased through an increase in the tax rate, the tax penalty and the frequency of tax audits. However, these increases may not lead to an efficient collection of tax revenues since taxpayers may not respond to these increases compliantly and consequently tax evasion will still be an important obstacle for the collection of tax revenues. Therefore, it is vital to examine which specific tool that policy makers can use contributes to a higher expected amount of tax revenues.

3.1.1 Effect of changes in tax rate t

Based on (4), if the problem is unconstrained, a change in the optimal level of a_i^* because of a change in the tax rate is equal to:

$$\frac{da_i^*}{dt} = \frac{f(\frac{1-p}{fp})^{\frac{2}{\lambda}} - t(\frac{1-p}{fp})^{\frac{1}{\lambda}} - (\frac{1-p}{fp})^{\frac{1}{\lambda}} - 1}{(t + ft(\frac{1-p}{fp})^{\frac{1}{\lambda}})^2} \tag{8}$$

Because the denominator $t+ft(\frac{1-p}{fp})^{\frac{1}{\lambda}})^2$ is always positive, the relationship between a_i^* and t depends on the sign of $f(\frac{1-p}{fp})^{\frac{2}{\lambda}} - t(\frac{1-p}{fp})^{\frac{1}{\lambda}} - (\frac{1-p}{fp})^{\frac{1}{\lambda}} - 1$. Thus, an increase in the tax rate may increase or decrease the optimal level of a and consequently it may decrease or increase the evaded income respectively. In this case, it is not straightforward how a change in the tax rate will affect the threshold a and consequently the level of evaded income as this link depends on the parameters of f, t, p and λ. However, in the Yitzhaki's model assuming for absolute risk-aversion, an increase in the tax rate leads to a decrease in the level of evaded income, a conclusion which may not accord with reality When the tax penalty f increases, the ratio $\frac{da}{dt}$ becomes less negative which means that the amount of tax evasion decreases. Given the levels of fine f, tax rate t and the probability of a tax audit p, the optimal level of threshold a_i^* can be computationally calculated and thus we can find how the relationship between a_i^* and t is defined. a_i can never take the value zero, as every taxpayer is obliged to declare a certain amount of income.

3.1.2 Effect of changes in tax penalty coefficient f

With respect to tax penalty coefficient f, a change in the optimum level a_i^* because of a change in the tax penalty coefficient is :

$$\frac{da_i^*}{df} = \frac{t(t-1)(\gamma^{\frac{1}{\lambda}} - \gamma^{\frac{2}{\lambda}}) + \delta((1-t)(1+f))}{t^2(1+f\gamma^{\frac{1}{\lambda}})^2} \tag{9}$$

where $\gamma = \frac{1-p}{fp}$ and $\delta = \frac{t(1-p)\gamma^{(\frac{1-\lambda}{\lambda})}}{f^2\lambda p}$. We observe that $t^2(1+f\gamma^{\frac{1}{\lambda}})^2$ is always positive for every values of t, f, p, λ. The sign of $\delta((1-t)(1+f))$ is also positive. But, for every $t \in (0,1)$ the sign of $t(t-1)$ is negative. Thus, in order to determine the relationship between changes in the optimum a_i^* and changes in the tax penalty coefficient we need to know the sign of $\gamma^{\frac{1}{\lambda}} - \gamma^{\frac{2}{\lambda}}$. If the condition $f < \frac{1-p}{p}$ holds for every $f \in (0,1)$ and $p \in (0,1)$, then $\gamma^{\frac{1}{\lambda}} - \gamma^{\frac{2}{\lambda}}$ is negative and $\frac{da_i^*}{df}$ is positive. That is, a potential increase in the tax penalty coefficient may also lead to an increase in the optimum threshold a_i^*, increasing the amount of income declared by each taxpayer group i. This positive relationship between the optimum threshold and the tax penalty coefficient was expected since we assumed that each taxpayer group consists of risk-averse taxpayers who are motivated to declared more of their actual income when the amount of tax penalty increases in case they are caught of tax evasion.

3.1.3 Effect of changes in the probability of tax audits p

Based on (4), if the problem is unconstrained, a change in the optimal level of a_i^* because of a change in the probability of a tax audit to take place is equal to:

$$\frac{da_i^*}{dp} = \frac{((\frac{1-p}{fp})^{\frac{1-\lambda}{\lambda}})(\frac{ft(1-t)}{fp^2} + \frac{t(1-t)}{fp^2})}{\lambda(t + ft(\frac{1-p}{fp})^{\frac{1}{\lambda}})^2} \tag{10}$$

We observe that for any given value of the parameters $\lambda, f, p \in (0,1)$ the denominator $\lambda(t + ft(\frac{1-p}{fp})^{\frac{1}{\lambda}})^2$ is always positive. For any given values of λ, f, p, t, $(\frac{1-p}{fp})^{\frac{1-\lambda}{\lambda}}$ is positive and $\frac{ft(1-t)}{fp^2} + \frac{t(1-t)}{fp^2}$ is positive.Thus, $\frac{da_i^*}{dp}$ is positive which implies that a potential increase in the probability of tax audits may lead to an increase in the optimum threshold a_i^* and consequently to an increase in the declared income of each taxpayer group. This positive relationship between the optimum threshold and the probability of tax audits was also expected since risk-averse taxpayers behave more compliantly and declare more of their actual income when more tax audits are to take place.

3.1.4 Effect of changes in h

As we mentioned above, h denotes the lower bound of the fraction of the actual income that is declared. Assuming for a concave utility function, if h is lower than the optimal threshold $a*$ then $\frac{da_i^*}{dh} > 0$ and if h is higher than $a*$ $\frac{da_i^*}{dh} < 0$.

3.2 Effect on tax revenues

Tax revenues are defined by the equation $T = t * Y$ where t is the tax rate and Y is the actual income of all taxpayer groups. However, because some taxpayers hide part of their income and some others do not the actual income Y may differ from with the declared income X for each taxpayer group i. Thus, we use the amount of declared income X to calculate tax revenues $T = t*X$ collected from all taxpayer groups. The total amount of tax revenues is defined as follows:

$$R = \sum_{i=1}^{n} t * X_i = \sum_{i=1}^{n} t * a_i * Y_i$$

We also need to measure how changes in either the tax rate t, or the tax penalty coefficient f, or the probability of tax audits p, or the lower bound h may affect tax revenues. In section 4 we show the results from calculating the marginal effect of each parameter on tax revenues for each taxpayer group seperately.

As we will discuss in the next section we will assume that the lowest level of a_i is 0.2. On the other hand, there may be taxpayers who are fully obedient to the law or their job does not allow them to tax evade ($a_i = 1$). Thus, the threshold a_i takes a minimum value which is greater than 0. If we know the actual a_i for each taxpayer group, we can compare it to the optimum level of a_i^* of the each group. Therefore, we will proceed to find changes in the utility of each taxpayer group. By examining the effect of a potential change in taxpayers' threshold a on their utility, we would be able to explore ways which will influence the threshold of tax evasion opportunities, such as an increased probability of a tax audit. That way our model would predict how taxpayer's behaviour may change in terms of utility due to a potential tax policy alteration.

4 Simulation Results

Let us assume we have four out of ten different occupations according to Independent Authority for Public Revenue (IAPR) : Enterprises, Rentiers, Agricultural workers, Salaried workers. For each group we assign a minimum threshold of tax evasion opportunities under which they cannot tax evade, where $h_i = [0.2, 0.4, 0.6, 0.8]$ respectively. For example, we assume a minimum threshold $h_i = 0.2$ for enterprises which means that they could declare up to 20% of their income since they may be more flexible than other occupation categories to hide income derived from the company's profits. In contrast, salaried employees who work in the public sector get their wages directly from the government and therefore they are restricted regarding tax evasion opportunities ($h_i = 0.8$). Thus, taxpayer groups are divided into four job categories. In this case, we assume that the minimum level of tax evasion opportunities is $h_i = 0.2$. By assigning each profession to a tax evasion threshold and given the values of f, t, p_i and X_i, which is the declared income, we are able to calculate (using MATLAB) the optimal level of threshold a^* corresponding to each taxpayer group and then compare it to the minimum level h_i. We assume that the levels of f, t, p are the same for all taxpayer groups. The risk-aversion coefficient λ_i differs across taxpayer groups but we assume that it is between 2 and 4. We will also need to the actual taxpayer's income Y_i for the calculation of each taxpayer's group average utility. The actual income Y_i will be approximately estimated through the equation $X_i = a_i Y_i$, where X_i is obtained from the IAPR database for each occupation category for the year 2016. For Enterprises, Rentiers, Agricultural workers and Salaried workers the average declared income is approximately 18,300, 4300, 9500 and 14200 respectively. Then we can proceed to the calculation of the average utility for each taxpayer group through the following utility function

$$U_i = \frac{Y_d^{1-\lambda_i}}{1-\lambda_i} \tag{11}$$

In practice, it is possible that taxpayers may not be able to hide the greatest amount of income that maximizes their utility because as we mentioned earlier they are constrained by frequent tax audits. Thus, we should take into account that the minimum level of income that taxpayers can declare defined by threshold a may differ from the level of income that they can declare at which level their utility is maximized and a_i^* holds. In some cases though, taxpayers' utility maximizes when they can declare the minimum level of income or in other words when $a_i^* = h_i$. In order to make these taxpayer groups to declare more than the minimum, policy makers should find tools to "move" the point where their utility maximizes away from the minimum level of threshold a. Therefore we can observe the changes in the utility of these groups by calculating the utility function and then the optimum level of utility U_i^* using the optimum level a_i^* of each occupation category. Finding the utility function, we are able to observe how digressing from the optimum threshold level a_i^* each taxpayer's group utility changes. Thus, the government could change tax rate or fine level to target this group mostly benefit from tax evasion opportunities.

As we mentioned earlier, policy makers can either increase the tax penalty coefficient or make more frequent tax audits to tackle tax evasion problem. We will examine four scenarios. In the first three scenarios we will study how increases in the tax penalty coefficient f affects the optimum threshold a^* holding all the other parameters of the model constant and in the fourth scenario we will study the effect of increases in the probability of tax audits to take place holding all the other parameters again constant. We will start assuming that the probability for a tax audit to take place is $p = 0.05$, the tax penalty coefficient equals $f = 0.5$ and the tax rate is $t = 0.29$. The risk-aversion coefficient is assumed to be different among taxpayer groups, as taxpayers differ across groups according to their occupation category. We will also assume that taxpayer groups with a lower threshold a_i deal with a lower risk-aversion coefficient since taxpayers who are aware of the fact that their threshold of tax evasion opportunities is low and thus they can declare a lower percent of their actual income they may take more risk to evade taxes. Empirical literature has shown that risk-aversion coefficient is around $\lambda = 2$. Goumagias & Varsakelis have estimated the risk-aversion coefficient around $\lambda = 3, \lambda = 4$. Thus, $\lambda \in [2, 4]$.

4.1 Changing the f

4.1.1 First case: $f = 0.5$, $t = 0.29$, $p = 0.05$

Table 1: Values of λ_i, h_i, a_i^* and T_i for each taxpayer group i corresponding to $f = 0.5$.

Parameters for group $i = 1, 2, 3, 4$				
λ_i	3	4	6	9
h_i	0.2	0.4	0.6	0.8
a_i*	0.2	0.4	0.6	0.3
T_i	5307	1247	2755	4147

For a tax penalty coefficient $f = 0.5$, a tax rate $t = 0.29$ and a probability of a random tax audit $p = 0.05$, we have calculated using MATLAB the optimal threshold a_i^* for each taxpayer group i. We also assumed that the risk-aversion coefficient is 3, 4, 6 and 9 for each group respectively with the enterprises being the least risk-averse group from all the others (λ_i). Table 1 shows that the optimal level of a_i^* given the values of all the other parameters is equal to the minimum value h_i for each group i. This means that the threshold a_i that maximizes the average utility for each group i is equal to the minimum value that threshold a_i can take. Thus, taxpayers will be motivated to declare the lowest possible income they can as their utility is maximized at the minimum level of declared income. As a result, in this case, policy makers should take specific measures to make taxpayers become more obedient and declare more of their actual income. This could be possibly done by "moving" the maximum level of their utility function away from the point where each taxpayer group i declare the lowest possible level of income. In other words, policy makers should either increase the tax penalty f or arrange more frequent audits leading to a higher probability p of a tax audit to take place in order to make a_i^* higher than the minimum level h_i.

4.1.2 Second case: $f = 2$, $t = 0.29$, $p = 0.05$

In this case, we assume that policy makers take stricter measures and increase the tax penalty coefficient to 200%, while keeping constant the tax rate at $t = 0.29$ and the probability at $p = 0.05$. We observe now that a large penalty coefficient above 1 is able to change the optimum threshold a_i^* for all the categories. In particular, for the first taxpayer group with minimum thresholds of 0.2, its optimum level of tax evasion opportunities increase by a larger percent comparatively to the other groups. Enterprises face now an optimum threshold $a_i^* = 0.4773$ which corresponds to an increase of 139%, while rentiers face an optimum threshold $a_i^* = 0.5899$ corresponding to an increase of 47% less than 100%. This could be possibly explained by the fact that before the tax penalty increase, enterprises had the incentive to declare only 20% of their actual income, but now their utility is maximized when they declare a larger portion of their income as they are more "afraid" to be caught and pay a large penalty.

Table 2: Values of λ_i, h_i, a_i^* and T_i for each taxpayer group i corresponding to $f = 2$.

Parameters for group $i = 1, 2, 3, 4$				
λ_i	3	4	6	9
h_i	0.2	0.4	0.6	0.8
a_i*	0.4773	0.5899	0.7150	0.8050
X_i	43737	6321	11305	14389.37
T_i	12683.73	1833.09	3278.45	4172.81

4.2 Changing the p

4.2.1 First case: $f = 0.5$, $t = 0.29$, $p = 0.1$

Table 3: Values of λ_i, h_i, a_i^* and for each taxpayer group i corresponding to $p = 0.1$.

Parameters for group $i = 1, 2, 3, 4$				
λ_i	3	4	6	9
h_i	0.2	0.4	0.6	0.8
a_i*	0.2	0.4	0.6	0.4512

Table 3 shows that when the tax penalty coefficient is constant at $f = 0.5$ and policy makers decide to make more frequent audits increasing the probability of a tax audit at $p = 0.1$ each taxpayers' group decision regarding tax evasion does not change. Even though now the probability of being caught tax evading is higher than in the previous cases, taxpayers may not feel "afraid" enough because the penalty they face in case they get caught is still small enough to persuade them to become more obedient.

4.2.2 Second case: $f = 2$, $t = 0.29$, $p = 0.1$

However, when the probability of a tax audit to take place increases at $p = 0.1$ and at the same time the tax penalty coefficient increases at $f = 2$, we observe that each taxpayer group changes its behaviour and declares a larger portion of income. This combined increase leads to a higher level of optimum threshold than in the cases when separately the tax penalty increases and tax audits become more frequent. Thus, taxpayers are more motivated to behave compliantly and declare a larger amount of their actual income.

Table 4: Values of λ_i, h_i, a_i^*, X_i and T_i for each taxpayer group i corresponding to $p = 0.1$.

Parameters for group $i = 1, 2, 3, 4$				
λ_i	3	4	6	9
h_i	0.2	0.4	0.6	0.8
a_i*	0.6295	0.7144	0.8046	0.8676
X_i	57462	7654	12730	15501
T_i	16663.98	2219.66	3691.7	4495.29

4.3 Changing the t

Increases in the tax rate is the common measure which policy makers take to raise more tax revenues. However, the final amount of tax revenues may differ from the expected one. This could be explained by the fact that some taxpayers hide a part of their income. In this section we examine how a small increase in the tax rate affects the taxpayers' decision to hide income and consequently the collection of tax revenues. Initially we assume that the tax penalty coefficient is set to $f = 0.5$ and the probability of tax audits to $p = 0.05$ and that policy makers decide to increase the tax rate by 3.4% which corresponds to a tax rate $t = 0.3$. Again we assume that the level of the actual income is given and constant but unknown.

Table 5 shows that at these given levels of our parameters, each taxpayer group does not have the incentive to declare a larger portion of income and thus the increase in tax revenues will come only from the increase in the tax rate and not from any potential increase in the declared income. In particular, the increase in tax revenues coming from each taxpayer group will be 622.2, 146.2, 323, and 486.2 monetary units respectively with the enterprises contributing more to the tax revenues.

Table 6 shows the computational results of another scenario. Let us now assume that policy makers increase the tax rate by 3.4% ($t = 0.3$), but the tax penalty coefficient is now $f = 2$ and the probability of tax audits still at $p = 0.05$. At these given levels of the parameters, taxpayers' decision to hide income change and they have a higher incentive to declare a larger amount of their income because of a higher punishment ($f = 2$). Each taxpayer group deals with a higher optimum threshold a_i^* and thus they deal with a higher declared income X_i. In this case, we observe that the increase in tax revenues is higher than in the previous case (Table 5) since not only the increase in the tax rate contributes to more tax revenues, but also a higher rate of tax penalty which makes taxpayers feel more "afraid" to act non-compliantly by hiding income.

Table 5: Values of λ_i, h_i, a_i^* and T_i for each taxpayer group i corresponding to $t = 0.29$.

Parameters where $f = 0.5$, $p = 0.05$, $t = 0.29$				
λ_i	3	4	6	9
h_i	0.2	0.4	0.6	0.8
a_i*	0.2	0.4	0.6	0.8
X_i	18300	4300	9500	14300
T_i	5490	1290	2850	4290

Table 6: Values of λ_i, h_i, a_i^*, X_i and T_i for each taxpayer group i corresponding to $t = 0.3$.

Parameters where $f = 2$, $p = 0.05$, $t = 0.3$				
λ_i	3	4	6	9
h_i	0.2	0.4	0.6	0.8
a_i*	0.5018	0.6092	0.7283	0.8142
X_i	45923.85	6510.63	11508.5	14533.26
T_i	13777.155	1953.19	3452.55	4359.9

4.4 Changing the h

In this section we examine how an increase in the lower bound on the fraction of the actual income of each taxpayer group may affect tax revenues. Practically, the increase in the h could be potentially achieved though the development of a more transparent tax system. For example, the provision of receipts could contribute to a higher h. That is, each taxpayer group will be forced to declare a higher amount of the actual income. Let us assume that the required provision of receipts increase the lower bound h by 10%. According to the equation $X_i = a_i * Y_i$, this increase results in a higher level of declared income by 10% as we assume that the actual income Y_i is constant for each taxpayer group i.

Table 7 shows the initial values of the lower bound h_i, the optimal level a_i^*, the declared income X_i and tax revenues T_i for each taxpayer group i before the implementation of the receipts provision. We also assume that in this initial situation that the tax penalty coefficient is equal to $f = 0.5$, the probability of a tax audit is $p = 0.05$ and the tax rate holds for $t = 0.3$. At this initial situation given the values of the parameters it is in the interest of each taxpayer group to declare the lowest possible income they can as in all groups $h_i = a_i^*$ as we mentioned in the section 4.1.1.

Table 7: Values of λ_i, h_i, a_i^*, X_i and T_i for each taxpayer group i before h_i changes.

Parameters where $f = 0.5$ and $t = 0.29$				
λ_i	3	4	6	9
h_i	0.2	0.4	0.6	0.8
a_i*	0.2	0.4	0.6	0.8
X_i	18300	4300	9500	14300
T_i	5307	1247	2755	4147

Now table 8 shows the values of h_i, a_i^*, X_i after the implementation of the obligatory provision of receipts. In this table we also include the change in tax revenues after this implementation. As we mentioned above, we assume that the required provision of receipts results in an increase of the lower bound h on the fraction of the actual income by 10%. Using computational calculation, we observe that after the increase in the lower bound h_i by 10% given the values of the other parameters, it is in the interest of each taxpayer group i to declare the lowest possible income they can but higher than in the case without the required provision of receipts. Since the optimal level a_i^* has increased relatively to the initial situation, each taxpayer group will declare a higher amount of the actual income. That is the declared income for enterprises, rentiers, agricultural workers and salaried workers is now 20130, 4730, 10450 and 15730 respectively. Given the same tax rate of $t = 0.3$, this increase of the lower bound h_i which led to a higher declared income level X_i results in an increase in the amount of expected tax revenues with the higher increase coming mostly from enterprises and the salaried workers.

Table 8: Values of λ_i, h_i, a_i^*, X_i and T_i for each taxpayer group i after h_i changes.

Parameters where $f = 0.5$ and $t = 0.29$				
λ_i	3	4	6	9
h_i	0.22	0.44	0.66	0.88
a_i*	0.22	0.44	0.66	0.88
X_i	20130	4730	10450	15730
T_i	5890.77	1384.17	3058.05	4603.17
dT_i	583.77	137.17	303.05	456.17

5 Conclusion

In conclusion, tax evasion will continue to become a challenge for policy makers since every taxpayer group cares about maximizing its utility by declaring a smaller amount of income than the actual one. However, each taxpayer group faces some constraints with respect to the amount of income they desire to hide. By nature, some taxpayers deal with higher constraints than others because they occupied in the public sector and thus their income comes mostly from wages which wages are known to policy makers. As a result, a small level of declared income can easily raise doubts about the honesty of those taxpayers' tax declaration. Since those taxpayers know the fact that they are highly constrained by their occupation to hide a large portion of their income, they tend to behave more compliantly.

We also solved for the threshold of tax evasion opportunities to find the optimum fraction of the actual income that is in the interest of each taxpayer group to declare. After finding the threshold level that maximizes each group's utility function, we proceeded to compare it with the lower bound which defines the lowest possible income level each group can declare. Then, we examined different scenarios where each them depicts taxpayers' decision to hide income under certain changes which policy makers adopt.

In particular, we observe that taxpayers could be potentially motivated to declare a larger portion of their actual income if policy makers adopt proper and efficient measures. These measures should target to a higher utility level of taxpayers combined with a higher amount of declared income. As we examined in section 4, policy makers raise more tax revenues when they impose a high tax penalty coefficient and arrange some slightly frequent tax audits. We saw that for specific values of risk-aversion a higher tax penalty coefficient is the driving force of minimizing tax evasion since taxpayers become more "afraid" to hide a large portion of income when punishment is strict. At low levels of tax penalty, an increase in the probability of tax audits is not enough to intimidate taxpayers to declare more income. We also observed that a small increase in the tax rate can raise tax revenues but not as much as a high tax penalty coefficient combined with a high probability of tax audit can do. Finally, we saw that even the implementation of required provision of receipts which is reflected in a higher lower bound fraction of income for all taxpayer groups can contribute to a higher expected amount of tax revenues, but a smaller one than a high tax penalty along with more frequent tax audits.

These findings will be helpful for policy makers to design effectively tax policy in order to raise the expected amount of tax revenues and to minimize as much as they can the extent of tax evasion activity.

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$$N = 4;$$

$$\lambda(i) = [3, 4, 6, 9];$$

$$f = 0.5;$$

$$t = 0.29;$$

$$p = 0.05;$$

$$X(i) = [18300, 4300, 9500, 14300];$$

$$h(i) = [0.2, 0.4, 0.6, 0.8]^{';}$$

$$a(i) = zeros(N, 1);$$

$$j = 0$$

for i=1:N

$$a(i) = \frac{1+(1-t+ft)(\frac{1-p}{fp})^{\frac{1}{\lambda(i)}}}{t+ft(\frac{1-p}{fp})^{\frac{1}{\lambda(i)}}};$$

if $a(i) > 1$

$$a(i) = 1;$$

end

if$a(i) < h(i)$

$$a(i) = h(i);$$

end

for i=1:N

$$T(i) = t * X(i);$$

end

end