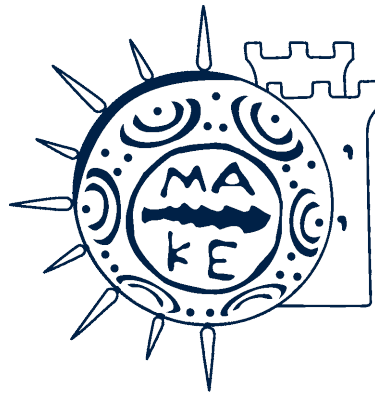


**FUTURES CONTRACTS AS HEDGES
IN EQUITY INVESTMENTS**



a Bachelor Thesis of

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Declaration

All sentences or passages quoted in this thesis from other people's work have been specifically acknowledged by clear cross referencing to author, work and page(s). I understand that failure to do this amounts to plagiarism and will be considered grounds for failure in this module and the degree examination as a whole.

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Abstract

In this work we try to identify, assess and evaluate the hedging performance of derivative contracts on equity portfolios that are available in the financial markets. We specifically focus on the use of future contracts, such as gold and oil futures, as hedgers on equity indices. We first present in brief theory and basics of equity investments and financial derivatives. We further focus on the concept of hedging and the uses and characteristics of future contracts. The thesis continues with a literature review on how the optimal hedge ratio is defined and how it can be estimated with the implementation of econometric models. We then employ multivariate GARCH BEKK models in order to estimate the dynamic conditional variance of the assets returns and evaluate the performance of their hedge ratios. Finally, we discuss the results and conclude with investment proposals.

Introduction

Every equity investment undergoes some risk. Even after international diversification is achieved, following the modern diversification theory of Markowitz (1952), there is always a form of undiversifiable risk. The type of risk, called systematic, is the exposure that each asset has on the course of the market that it belongs. Assets that are negotiated in the same market often show similar patterns in response to news that affect the entire market and create cross sectional relationships, while volatility spillovers between different stock markets might also occur. Therefore, in order for an equity investment to be more secure in shocks that are not related to the assets' performance, hedging is necessary.

Hedging is an investment procedure through which a position on a financial instrument is taken, in order to offset the potential losses of another. One of the ways to achieve hedging is with the use of financial derivatives. The financial derivatives are negotiated contracts that derive their price from the value of an underlying asset. Derivatives have many characteristics that allow them to be flexible and utilized for purposes such as hedging, arbitrage opportunities, as well as speculation. More specifically, there is plenty of literature that considers the futures contracts to be the most appropriate instruments for hedging. Their daily settlement, variety of contracts and security of default are only some of the characteristics that make futures more suitable, compared to other derivatives.

However, after deciding which of them can be used, an investor should also select the strategy to follow. For that reason, the optimality of the hedge ratio should be defined based on the investor's profile. Starting from the seminal work of Ederington (1979), the literature has proposed multiple optimal hedge ratios that can be selected based on the investor's preferences. There are different measures based on risk minimization, return maximization, adjusted return maximization criteria and others.

A final question that needs to be answered before making a hedge is how the optimal hedge ratio is calculated. The estimation includes decisions such as using ex-ante or ex-post econometric analysis that can be further divided into static and dynamic. Furthermore, the method for modeling the moments of the time series needs to be selected based on the properties of the sample. It appears that for the estimation of the hedge ratio dynamic models are the most effective. There is a vast literature on dynamic econometric methods for modeling financial time series, among which the GARCH models present important

advantages.

In this thesis, we review the literature aiming to find, which financial derivative is the most effective for hedging, how the optimal ratio is defined and which is the best fitting econometric model for estimating it. After that, we intend to assess and evaluate the performance of futures contracts on equity investments, by testing the hedging effectiveness of WTI oil and GCS gold futures on stock indices of large financial markets such as the S&P500, Eurostoxx, Nikkei 250 and the Shanghai Stock Exchange. The first part is theoretical and presents the main concepts regarding, equity investments, financial derivatives, and hedging, as well as the literature review. The second part reports the empirical analysis including the methodology, the results of BEKK models and discussion on the findings.

Chapter 1

Theoretical Review

This chapter aims to help the reader understand the key concepts concerning equity investments and futures, and review the existing literature. We consider appropriate to introduce these concepts in order to achieve a better transition to the next parts of this thesis. We first start with some definitions about equity investments and mutual funds; we then introduce the most common types of financial derivatives and analyze in detail the futures contracts. Hedging and its importance are later explained and a comparison among futures and the other financial derivatives is made. We then present part of the literature concerning the superiority of futures contracts as hedgers, the determination of the optimal hedge ratio and methods for estimating it.

1.1 Equity Investments and Financial Derivatives

1.1.1 Equity Investments

An equity is generally the ownership of an entity over an asset. In capital markets a stock, or any other security representing an interest of ownership, usually on a private company, can be referred as equity. The most common type of equity is stocks (common or preferred). Therefore, an equity investment can generally describe the purchase of shares of stock in anticipation of income from dividends and capital gains.

Every investment however, involves some risk. Although the types of risk differ among investments, there are two broad types of risk that every equity investment undergoes; the systematic and the non-systematic risk. The first is the risk that every asset is exposed to, by being a component of a market. One could also refer to systematic risk as market or undiversifiable risk because it actually includes the exposure that an asset has on the market operation and course. Systematic risk affects the overall market and not only a particular asset or industry. On the other hand, the non-systematic risk, which is also called unsystematic or diversifiable risk, refers to the uncertainty caused by the course of an asset, company or industry.

Around the middle of the previous century, Harry Markowitz (1952) introduced the “Modern Portfolio Theory” (MPT), that would later award him with a Nobel prize in Economics. The MPT introduced the concept of diversification which denotes that, having a portfolio of many different assets reduces the non-systematic risk at which the total investment is exposed to. As the number of assets within a portfolio tends to infinity, the non-systematic risk tends to zero. Stated differently, the more assets a portfolio has the less risk it faces. Diversification is even more effective when a portfolio consists of assets from different international markets (Solnik, 1995).

An efficient way for an investor to hold one or more portfolios is to invest in mutual funds or ETFs instead of buying all the assets separately. A mutual fund is an investment vehicle that is constituted of the funds of many investors that are willing to invest in multiple similar assets. There are mutual funds for example that are designed to mirror the movements of a stock market index. Mutual funds may include any type of asset and have reduced transaction costs for an investor that is willing to buy the same assets separately. Similar to MPT the “Mutual Fund Separation Theorem” denotes that the optimal portfolio of an investor might be constructed by holding multiple mutual funds positions, in appropriate ratios while the number of mutual funds will be less than the number of individual assets in the portfolio.

1.1.2 Financial Derivatives

A financial derivative product, as its name indicates, is a financial instrument whose value derives from the performance of an underlying entity (Hull, 2012) . This entity might have the form of an asset, a bond, an index, a commodity, an interest rate or an insurance contract and many more. The derivative can be a contract or an agreement between two counter parties. In most cases the underlying variable is the price of a traded asset, but it can depend on nearly any variable like the mean monthly temperature on a specific region, which is the case of weather derivatives. The derivatives of both forms derive their price from the value of the underlying asset. Another difference with spot investments, is that a derivatives transaction does not necessarily include an actual transaction of the ownership of the underlying asset at the moment that the contract is set up. The derivative represents an agreement or an obligation to transfer the ownership of the underlying asset at a specific price, time and place in the future that the contract indicates. In any derivative contract two counter parties are needed. The party that holds a contract to buy is said to have taken a long position, while the other side that agrees to sell holds a call position. Therefore, for any derivative contract the presence of both sides is necessary. In other words, for every long position an opposite short position is necessary. The need for both supply and demand sides in financial derivatives, results their price to depend not only on the value of the underlying asset but also on the interest and the creditworthiness of the parties (Kolb

and Overdahl, 2007).

Some derivatives such as forward contracts are negotiated over-the-counter (OTC) like any other private contract, but others such as futures and options are traded in organized exchanges like the Chicago Mercantile Exchange (CME). As we mentioned, there are many different financial derivatives that vary based on the nature of their underlying asset. We can categorize them in financial and commodity derivatives. Financial commodities include derivatives in indices, bonds, interest rates, foreign exchange and others, while commodity derivatives may include agricultural products, metals, energy products such as oil and natural gas or commodity indices. As the integration of the financial markets and the financialization of commodities continue to develop, new instruments will be available to the investors in order to diversify, and hedge their investments.

The importance of financial derivatives can be found in both their theoretical applications and benefits, as well as on evidence of their widespread usage. Some of the benefits of financial derivatives according to Kolb and Overdahl (2007) are the following.

Market completeness A complete financial market is one where there is perfect information, negligible transaction costs and the price of every asset is the same in every possible state traded, so there are no arbitrage opportunities. Even though market completeness is an idealized state, derivatives markets help financial markets approach it. The bilateral nature of all derivatives transactions leads to price discovery, the process that determines the price of an asset in a market through the interaction of buyers and sellers. Also, when investors discount the impact of future events in the value of the underlying asset in the day of the delivery, they discount the same events before the spot market does and therefore lead to a more efficient use of the information. Finally, the simultaneous operation of financial and derivatives markets increase the risk and return perspectives of investors portfolios increasing their welfare.

Risk management Derivative instruments can effectively help investors hedge their risk exposures. This is achieved by transferring the risk from the components of the market that don't want it to those who are willing to accept it for a premium. Even though they can be risky in the sense that their price fluctuates much, if they are used parallel to equity investments they can absorb and reduce the systemic risk of those investments. The variety of derivatives contracts allows for many types of risk to be hedged, like currency, credit, interest rate, equity risk and others. We further analyze both hedging with financial derivatives later in the next part of this chapter.

Speculation Even though financial derivatives were introduced with the opposite purpose, if properly used, they can present significant speculative opportunities. Products

such as options can lead to excessive returns with only slight movements in the underlying asset price as we will later show.

Trading efficiency Positions in financial derivatives can work as substitutes for spot positions directly in the underlying assets. The derivatives have the ability to mimic the movements in the price of the underlying asset. At the same time the futures contracts have lower transactions costs. They can also be used in the case where an investor wants to exempt his position in an equity investment for only a specific period of time. He may buy an opposite position in a derivative product of the same underlying asset to offset the returns of that period, instead of selling and repurchasing the same portfolio. In this way, many investors prefer the derivatives to the conventional investments on the underlying security. Financial derivatives are also more liquid products, because derivatives markets have much more trading activity. An illustration of this difference in liquidity is the case of October 1987 stock market crash, when trading was interrupted in the otherwise highly liquid New York Stock Exchange (NYSE), because of the huge imbalances between sell and buy orders in most stocks. This illiquidity in the spot markets did not spill over into the futures markets that continued their operation normally.

All the benefits mentioned above have lead to a great interest in derivatives markets (Malliaris, 1997) that can be seen in the following statistics that were derived from the IFA annual report 2018.

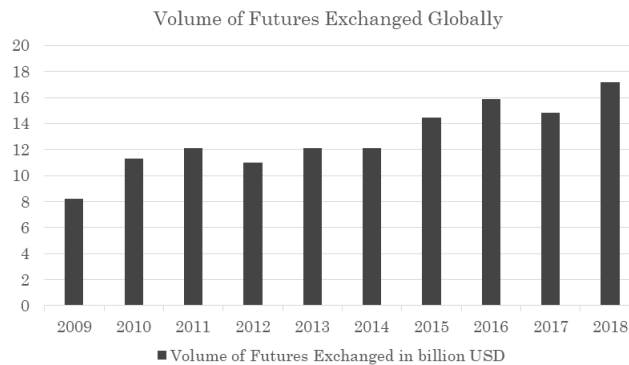


Figure 1.1: Global Volume of Futures Contracts Exchanged (in billion USD)

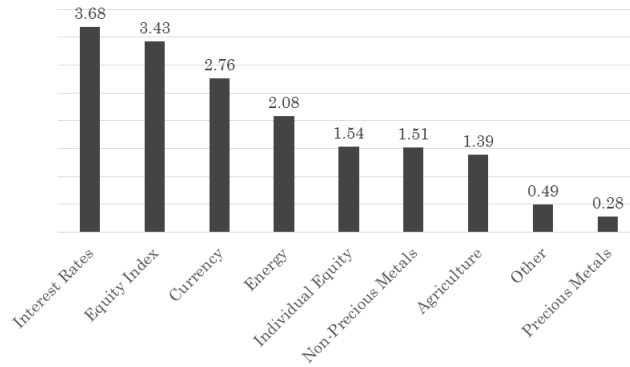


Figure 1.2: Global Volume of Future Contracts per type (in billion USD)

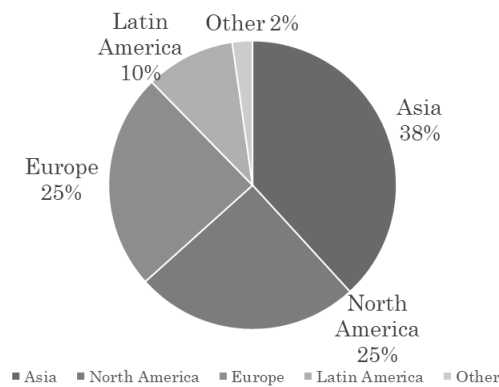


Figure 1.3: Share of traded Futures on global volume by region

There are several types of financial derivatives all of which have different characteristics in terms of negotiation and obligations for the counter parties. The basic types of financial derivatives are forward contracts, futures, options, swaps and quantos. Each type has different benefits and limitations creating many investment opportunities for all types of market participants. The most traded derivatives and the most liquid derivative markets are presented in the tables below.

Table 1.1: Most traded Derivative Contracts per type

Type	Contract	Jan-Dec 2018 Vol
Equity	Bank Nifty Index Options, National Stock Exchange of India	1,587,426,222
Rates	Eurodollar Futures, Chicago Mercantile Exchange	765,208,581
FOREX	US Dollar/Indian Rupee Options, BSE	559,489,717
Energy	Brent Oil Futures, Moscow Exchange	441,379,480
Agricultural	Soybean Meal Futures, Dalian Commodity Exchange	238,162,413
Metals	Gold Futures, Tokyo Commodity Exchange	8,090,879

Source: *Futures Industry Association Annual Report 2018*

Table 1.2: Most liquid Derivatives Exchanges based on volume of total trades

Rank	Exchange	Volume
1	CME Group	4,844,856,880
2	National Stock Exchange of India	3,790,090,142
3	B3	2,574,073,178
4	Intercontinental Exchange	2,474,223,217
5	CBOE Holdings	2,050,884,142

Source: Futures Industry Association Annual Report 2018

Forward contracts

A forward contract is an agreement between two parties to exchange an asset for a certain price, in a certain time in the future. The two sides of a forward contract include a buyer and a seller, who together are called counterparties. The side that agrees to buy the underlying asset has a long position, while the seller of that enters a short position (Hull, 2012). The difference between forward contracts and spot transactions is that, in forwards the actual transfer of the ownership on the underlying asset doesn't take place the time the contract is issued but in the future. There are certain terms that every contract specifies such as the time period, the price, details on the delivery of the asset etc., and are all decided solely by the two parties.

The underlying asset can be anything of interest for the two parties, however, the most usual contracts include foreign exchange and physical commodities. These two types of forwards include the physical delivery of the underlying assets at maturity of the contract, while many other forward types are cash-settled. In this case, if at maturity date the spot price of the underlying asset is higher than the one specified in the contract, the short side has to make a cash payment. Similarly, if the spot price is lower, the holder of the long position will have to make the payment.

Forwards are traded over-the-counter but while the absence of a clearing house is an advantage in terms of transaction costs, it also makes the two sides exposed to the default risk of the other. The simplicity of forward contracts makes them very flexible and useful in resolving the risk related with the course of a price. If we express the strike price of an asset as S_t and the agreed delivery price of the contract as F_0 the payoff for the holder of a long position will be

$$S_t - F_0$$

This means that as long as the spot price of the asset at the delivery date is higher than the price agreed in the contract it will be true that $S_t > F_0 \implies S_t - F_0 > 0$, it is beneficial for the holder of the contract because he will be able to acquire the asset at a lower cost. In any other case the payoff will be negative. On the other hand the payoff for the seller

of a forward contract will be determined by

$$F_0 - S_t$$

and profit is made when $F_0 > S_t \implies F_0 - S_t > 0$. The positive and negative payoffs for both sides can be seen in figure 1.4 In this way the maximum profit and loss for a forward

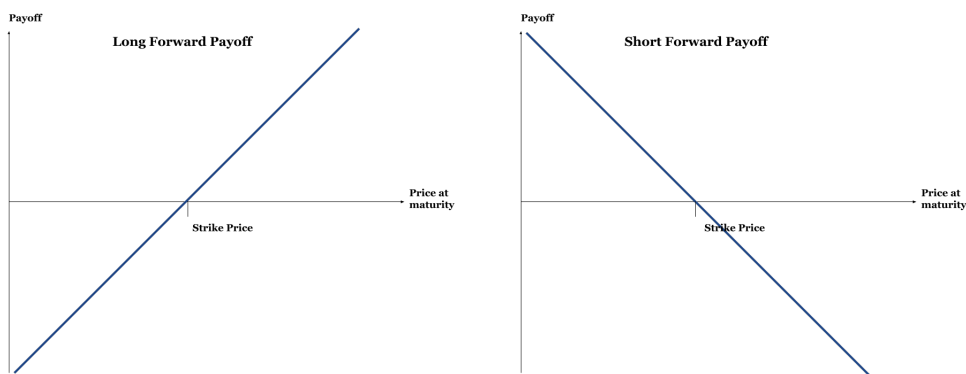


Figure 1.4: Long and Short Forward payoffs

contract holder can be shown in Table 1.3

Table 1.3: Forward positions and payoffs

	If $S_t > F_0$	If $S_t < F_0$
Long Forward	Profit $S_t - F_0$	Loss $S_t - F_0$
Short Forward	Loss $F_0 - S_t$	Profit $F_0 - S_t$

Options

The options are contracts that give to their holder the right and choice to either buy or sell an underlying asset in a predetermined price in the future. There are two classes of options, the call options which give their holder the right to buy the underlying asset and put options that give him the right to sell this asset. Similar to other derivatives, options also require two opposite sides in order to build a contract. The buyer or holder of an option has the option or choice to exercise his contract and buy (holder of call option) or sell (holder of put option) the underlying asset in the agreed price. To acquire these rights, owners of options buy them by paying a price or premium to the sellers of the contracts. The seller or issuer of an option on the other hand, has the obligation to either sell (issuer of call option) or buy (issuer of put option) the underlying asset that was agreed. In the case of European options the right can be exercised by the holder at maturity date, while for American options the owner can exercise it at any moment prior to maturity. The underlying asset of an option might be an individual stock or bond, stock index, foreign

currencies, exchange traded funds (ETFs) and futures contracts. Options are traded in both over-the-counter and organized markets. The choices of put and call options give to the two sides of investors, buyers and sellers, four possible positions in option contracts. The owner of an option will always have the right to decide whether to exercise or not his position, while the seller will always have an obligation to meet the holder's demand. The four different possible positions that an investor might take in an option result to

Table 1.4: Options types and their characteristics

	Buyer	Seller
Call	Long Call	Short Call
Put	Long Put	Short Put

four different cases of payoff, as shown in table 1.4. There is also a specific category of options that has future contracts as underlying assets. These options are referred as futures options. They differ from the other options that include the delivery of a physical asset or equity in that, if the holder of a call option exercises his right he will receive a long position in a futures contract at the option's strike price. For put options, the holder of the option would enter into the short side of the contract and would sell the underlying asset at the option's strike price (Kolb and Overdahl, 2007). It is also important to note that future options are derivatives on a derivative instrument, or second derivatives. This means that in order to specify their terms one should take into consideration the expiration dates and the different supply and demand profiles of both products.

Because options create a greater number of outcomes in terms of profit for both sides of an option, compared to the other types of derivatives, the concept of moneyness was developed in order to distinguish the cases that are profitable to each side if the option is immediately exercised. If we express the spot price of the underlying asset as S_t and the agreed delivery price or strike price agreed for the underlying asset as K , and p the premium payment that the holder of the option has to pay to the issuer then, for a call option if $S_t > K$ we say that the option is in-the-money. Similarly, if $S_t < K$ the call option is out-of-the-money since for the holder of such option it is preferable to buy the underlying asset directly from the spot market rather than exercising his call option. In the case of zero profit, we say that the option is at-the-money. It can be clearly seen that the state of the option is not profitable for both sides. For example the case $S_t > K$ is profitable for the holder of a call option, but not for the holder of a put option that has the same strike price and conditions. The issuer of both sides makes profit only when the option is not exercised by the holder as he keeps the premium p that was paid. In order to calculate the profit for a holder of an option that is in-the-money we need to subtract the cost that he had in order to enter the position, which is the premium. It can be shown that if for the holder of an option it is preferable to exercise the price when the option

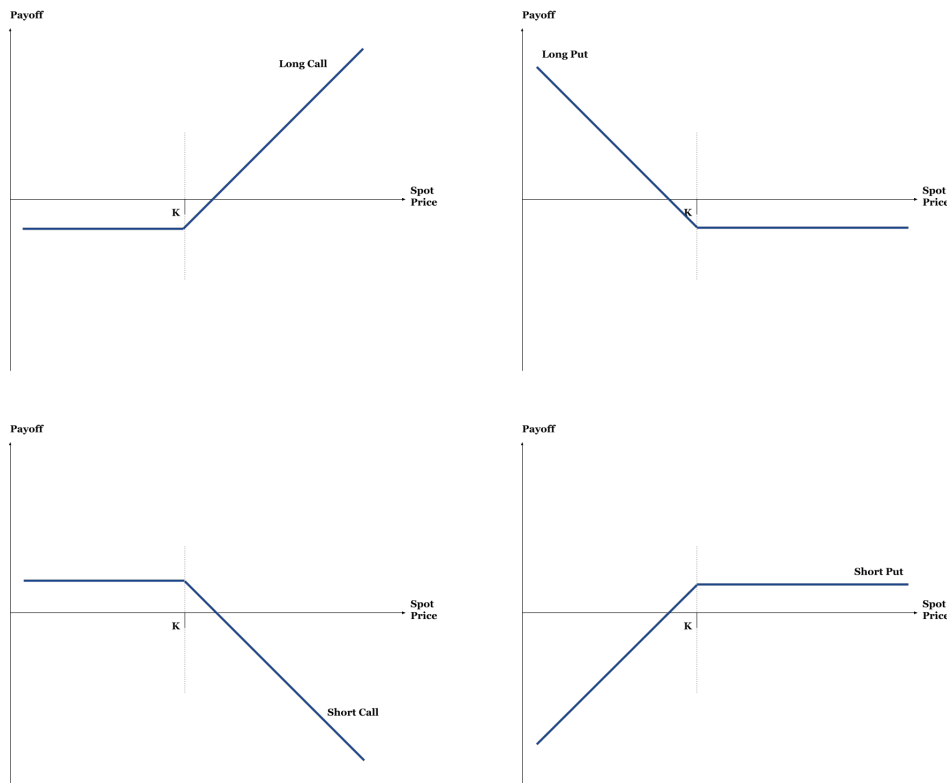


Figure 1.5: Option positions payoffs

is in-the-money, the issuer of the same option will make profit when the holder doesn't exercise his right, which is when the option is at-the-money or out-of-the-money. In this way

Table 1.5: Option types and exercise cases

	Calls	Puts
In-the-money	$S_t > K$	$S_t < K$
At-the-money	$S_t = K$	$S_t = K$
Out-of-the-money	$S_t < K$	$S_t > K$

the profit or loss of holders and issuers is shown in table 1.6. The European options are usually described in terms of payoff for their purchaser. The holders of long call and short put options benefit from increases in the spot price S_t above the strike price K as the first will exercise the option and the second will win the premium. On the other side, the holders of long put and short call will profit in the case that S_t falls below K because in this case the first will exercise the option and the later will win the premium from the option that will not be exercised.

There are plenty of strategies for combinations of option contracts that result to different risk and return characteristics and allow for speculation in any case of spot price movement. There are many benefits with the use of options. The benefits derive from the

Table 1.6: Option positions payoffs

	$S_t > K$	$S_t < K$
Long Call	$(S_t - K) - p$	$-p$
Long Put	$-p$	$(K - S_t) - p$
Short Call	$(S_t - K) + p$	p
Short Put	p	$(K - S_t) + p$

fact that options replicate the behavior of the underlying asset price, an investor can trade options to speculate on the price movements of the underlying asset. One of the reasons to trade an option instead of the original asset is that call options are always and put options most of the times, cheaper than the underlying asset. Another thing is that the option price is more volatile and can yield higher return for the same investment. In this way, small movements in the spot price of the underlying asset can lead to large returns in options and large movements to even larger returns.

Apart from speculation, hedging can also be achieved with the use of options if they are traded in combination with portfolios. Investors can in this way use options to increase or decrease risk of their existing portfolios for a very small price, which is the premium. Options have also significantly lower transaction costs and taxation. Finally, by trading options an investor can avoid some stock market restrictions. For example, systematically shorting a stock is highly restricted in most exchanges. By trading an option it is possible for an investor to replicate a short sale of stock. By combining an investment in equity, with one of all the possible positions in options the resulting profit for the aggregate investment is shown in figure 1.6.

Swap

A swap is a contract in which the two parties usually agree to exchange cash flows in the future, usually based on a notional principal amount. The swaps are private contracts very similar to forwards but they differ because the principal amount usually doesn't change hands and because the exchanges of the cash flows take place multiple times. The cash flows that are exchanged by the counterparties are most of the times to the value of interest rates, debt instruments or foreign currencies. The terms of the agreement are decided solely by the two counterparties and their objectives, so that each swap may vary in terms of principal capital, interest rate, time and frequency of payments.

In the most common and simple swap type, called plain vanilla, the one counterparty agrees to pay a constant rate on the agreed capital, in exchange of a floating rate payment on the same capital paid by the other counterparty. The first party is said to have the pay-fixed side of the deal, while the other has the receive-fixed side of the deal. An example of plain vanilla can be shown in figure 1.7. The time for which the cash flows will be

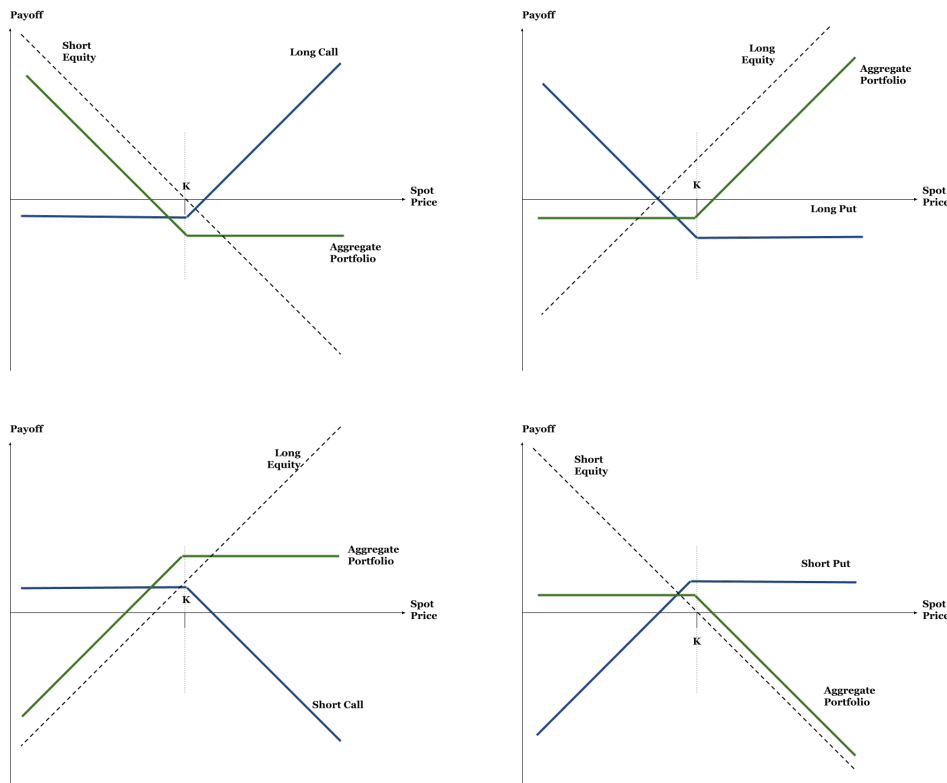


Figure 1.6: Combined Equity and Options positions payoffs

exchanged is usually referred to as the tenor of the swap and the amount upon which the rates are specified is called notional principal. Usually the swap uses existing interest rates such as the London Inter-bank Offered Rate (LIBOR). If the interest rate agreed is exactly LIBOR the swap is called LIBOR flat.

Based on the type of the underlying asset of the swap, they can be categorized in

- Interest rate swaps, that exchange different types of rates such as fixed for flexible,
- Equity swaps that make payments based on the price of a specified equity,
- Commodity swaps that are similarly based on the value of a commodity,
- Credit swaps where the payoff is linked to the credit characteristics of a reference asset.

Swaps also differ in terms of how the notional capital is determined. These swaps are usually called flavored swaps and some of them are

- Amortizing swaps in which the notional principal is reduced with time,
- Accreting swaps are the exact opposite because the notional principal is increasing,
- Seasonal swaps in which the notional principal varies.

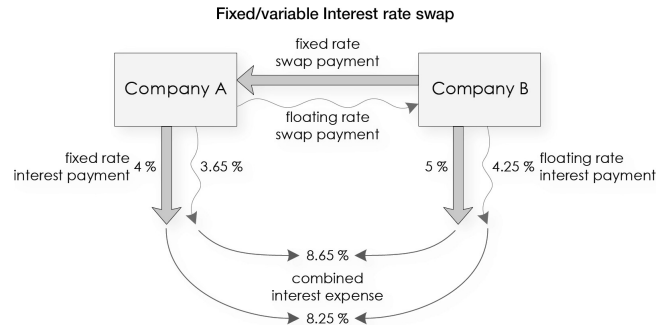


Figure 1.7: Example of Swap contract
Source: International Swaps and Derivatives Association

Swaps are very useful in changing the nature of assets or liabilities. With a swap an investor can transform a floating rate asset with volatile cash flows into a fixed-rate asset with constant payments, while this can happen with debt as well. The fact that swaps are not settled in organized exchanges creates some benefits in their use, but also some disadvantages. They are very flexible, have low transaction costs and afford privacy (Kolb and Overadhl, 2007). However, the absence of a regulator, to guarantee that the payments will be made, is a disadvantage as the counterparties undergo the default risk of the other. Another disadvantage is that in order for one party to enter a swap it must find another counterparty that is willing to enter the agreement under the same terms of maturity and cash flow pattern. Swaps also cannot be altered or terminated early unless both sides agree.

Future contracts

A futures contract is a standardized contract or legal agreement, to buy or sell an asset at a specified time in the future. The seller of a future contract is committed to deliver the asset in a predetermined day in the future, in exchange of a payment that occurs also in the future. The buyer will take delivery of the underlying asset and will pay the agreed-upon price. It can be said that futures are a type of forward contracts that have highly standardized and precise contract terms. The price of the asset on the delivery, is determined at the time that the contract is exchanged by the forces of demand and supply, while the payment is made when the contract expires.

Futures contracts are traded in organized exchanges in which a clearing house operates as middleman, is responsible for executing the exchanges, and decides the terms of the contracts. The terms include details such as the type of the underlying asset, the delivery date, the contract size, the currency, the hours that the contract can be traded and others. Especially for futures contracts of physical commodities the rules are very strict and include many more details such as the quality of the asset, the process of product delivery and others (Hull, 2012).

The holder of a futures contract can close his position by exchanging it before this expires. In this way, an investor may enter a futures position without a need for delivery. Most of futures contracts are closed before expiration and physical delivery takes place usually in cases when a corporation needs the commodity as input for its production process. A hedge fund for example may enter a long position in oil futures in order to take advantage of a possible increase in its price and not because it needs to use barrels of oil. In this thesis we consider the use of futures as hedgers on equity investments and therefore, we don't focus on the use of futures for hedging on physical production or corporate hedging.

Unlike the case of other financial derivatives, in futures contracts the clearing house minimizes the counterparty risk to traders, by becoming the buyer to each seller, and seller to each buyer, and assumes the risk of loss if a counterparty defaults. The clearing house also sets a minimum price fluctuation, called tick size, and a maximum price fluctuation which restricts the price movement of a contract in a single day.

The plethora of rules may at first seem very restrictive, however, they actually promote liquidity as all participants in the market know the exact terms of the transactions and trading tends to be more efficient (Kolb and Overdahl, 2007).

Another important factor in futures trading is margins. Margins act as a safeguard by requiring traders to deposit funds with a broker, before entering in futures contracts. These funds are used to ensure that the traders will perform their contract obligations and continuously adjust based on the value of the contracts the trader holds. If a trader has not enough margin to meet the obligations of his open positions, he receives a call from the clearing house to add more funds in his account. Margins restrict the activity of traders from taking very risky positions that could lead to default, and their rate vary between contracts and positions. Table 1.7 presents the margin requirements that CME had for some index equity futures on December 2018. For these reasons, futures are very uniform

Table 1.7: Margin requirements of various Futures Contracts

Product Name	Code	Start Period	End Period	Maintenance	Currency
NIKKEI 225 YEN FUT	N1	1/12/2018	1/12/2021	560,000	JPY
S&P500 FUTURES	S&P500	1/12/2018	1/12/2018	30,000	USD
BITCOIN FUTURES	BTC	1/12/2018	1/12/2018	7,515	USD
E-MINI NASDAQ 100 FUT	NQ	1/12/2018	1/12/2018	7,000	USD
FTSE EMERGING INDEX FUT	EI	1/12/2018	1/12/2019	2,600	USD

Source: CME Group

and their well-specified terms provide a good guarantee that the asset will be delivered on time and in an appropriate manner.

There are many types of futures based on the nature of the underlying asset.

- Physical commodity futures, that include the future delivery of agricultural products,

such as corn or soy bushels, metallurgical products like gold and silver, and energy commodities such as barrels crude oil and natural gas.

- Foreign currency futures, that include the delivery of a quantity on foreign exchange. Interest-earning assets futures, in which the underlying asset might be treasury notes, bonds, Eurodollar deposits, interest rate swaps and other interest paying instruments.
- Index futures, are actually stock index futures that are linked to the course of indices such as S&P500 or Russell 2000. This type of futures does not include the physical delivery of a portfolio but it is settled with a reversing trade or cash payment instead.
- Individual stock futures, include the delivery of ownership on stocks of private enterprises.
- Cryptocurrency futures, CME has recently introduced Bitcoin futures contracts and National Association of Securities Dealers Automated Quotations (NASDAQ) also plans to do so.

In our analysis we will use physical commodity, asset-type futures, namely oil and gold.

Trading futures contracts is very similar to trading assets in the spot markets. When an investor buys a future contract it is said to have a long position, while when he sells a contract he enters a short position. When one trader buys a future contract from another one that sells it, the transaction results to one contract of trading volume. The number of futures contracts obligated for delivery each moment is called open interest. The volume and open interest of various futures types that were traded on CME on 19th December 2018 are presented in Table 1.8. Similarly to spot exchanges, an increase in the price of a

Table 1.8: Volume and Open Interest of Futures that are traded on CME

Type	Volume	Open Interest
Agriculture	857,885	4,692,949
Energy	2,233,633	11,785,118
Equities	5,582,760	5,179,170
FOREX	833,498	1,586,209
Interest Rate	11,125,416	29,476,359
Metals	490,393	1,040,999

Source: CME Group

futures contract above the delivery price generates a profit for a long position and a loss for a short position. On the other hand, a decrease in the price of a futures contract below the delivery price results to a loss for a long position and profit for a short position.

Very similar to forward contracts payoffs, if we express the futures price of an asset as

F_t and the agreed delivery price as F_0 the daily settlement will be

$$F_t - F_0$$

and every scenario when $F_t > F_0 \implies F_t - F_0 > 0$, is beneficial for the holder of a long position on the contract, because he can either acquire the underlying asset at a lower cost compared to the spot market (opportunity cost) or sell the contract before this expires and make a profit. The same case results to a loss of $-(F_t - F_0)$ for a short position holder of the contract. Profit will be made from a short position if the futures price is lower than the delivery price $F_0 > F_t \implies F_0 - F_t > 0$ and the payoff will be equal to

$$F_0 - F_t$$

and in this case the loss for a long position on that contract will be $-(F_0 - F_t)$. Futures

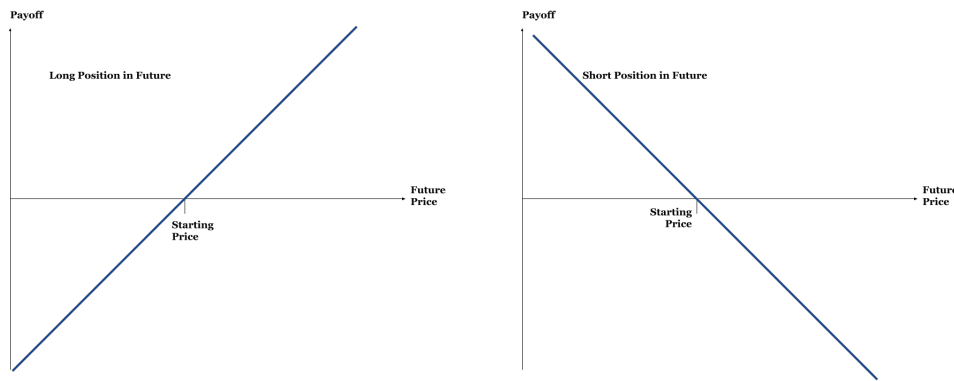


Figure 1.8: Long and Short Futures payoffs

have many different and common characteristics as other financial derivatives some of them are:

1. Similar to options, futures contracts are traded on organized exchanges as opposed to forwards and swaps that are over-the-counter private agreements between the counterparties.
2. Futures are standardized in that their terms are specified, restrictive and vary only between different types of futures. This is different with forwards and swaps where the terms might take any form creating unlimited possibilities of contracts.
3. Futures operate in the presence of a clearing house that is responsible for executing all the exchanges and guaranteeing the on time delivery of all products, something that does not exist in other derivatives markets.
4. Futures markets also rely on a system of margins that protects the financial integrity of the contracts. They therefore have zero credit risk.

5. Futures are settled daily. In this way, their prices are in continuous move and their value changes daily, in contrast with forwards whose value is determined only upon delivery date.
6. Holders of futures contracts can easily offset and close their positions prior to the expiration of the contracts.

As Tesler and Higinbotham (1977) mention, an organized futures market facilitates transactions and substitutes the trustworthiness of the exchanging parts. Similar to the other derivatives, futures can be used for speculation or hedging. However, there is one more purpose in futures markets, that of price discovery. While all other instruments derive their price from the spot price of the underlying asset, futures reflect the expectations of investors for the price of the asset the day of the delivery (Working, 1961; Sliber, 1981; Evans, 1978). Price discovery refers to the reveal of information about the future in cash markets through futures markets. The strong positive relationship (further analyzed in Chapter 1.2.1) between futures and spot prices is not only expected but it also appears very useful for predictions. Futures can even assist the price discovery of even spot markets if the later are not well developed. Also, according to Dale (1981) the greater the risk reduction comes from futures markets, the greater the demand for tradable goods in an economy is.

As we mentioned earlier, there are multiple different contracts for different delivery dates of the same underlying asset. The prices of more distant futures are usually higher than those of the nearby months, in what is called a normal market. If the distant futures cost less than those close to delivery, we say that there is an inverted market. This intracommodity spread should tend to zero as the expiration of one contract approaches the contract of the next negotiable delivery date. This enables an investor to roll forward a contract and extend its expiration time as the values of the expiring and new contract are equal.

Quantos

Finally, another financial instrument that uses derivatives is Quantos. A quanto is an instrument in which, while the initial price of the underlying asset of a derivative is valued in one currency, the instrument itself is settled in another currency at some rate. In this way, a quanto enables investors to arrange payments in different currencies other than the asset's pricing currency without being exposed to currency risk. The fee that is paid is guaranteed and doesn't fluctuate as the exchange rate does. There are quantos for all financial derivatives except for forwards. Quanto futures contracts for example can be used to purchase futures contracts in a European stock market index which is settled in US dollars. Quanto options, are used when the underlying and a fixed strike price are paid

in different currency. With quanto swaps one counterparty pays a non-local interest rate to the other, but the notional amount is in local currency.

1.2 Hedging

The multiple natures of financial derivatives allow them to be utilized by different type of investors and strategies. Derivatives might be used for speculation, arbitrage purposes or hedging. In this thesis we focus in this last practice of financial derivatives. The derivative markets facilitate hedging by allowing to transfer the risk of price changes to those who are willing to undertake it (Ederington, 1979).

A hedge is an investment procedure through which a position in a certain financial instrument is taken in order to offset the potential losses of a different initial investment. The hedging position can mitigate different types of financial risk such as currency risk, credit risk, interest rate risk, equity risk and more. Commodity hedging for example can be used by producers in order to protect themselves from fluctuations in their production price or from unfavorable weather. In our case however, we will focus on hedging against risks that are related with equity investments. In this scope, it is suitable to mention some aspects of market risks that altogether compose the systematic risk previously mentioned. Equity risk is the risk that the price of an asset might change due to the dynamics in the stock market, and not relative to the performance of the asset itself. Currency risk refers to the risk that foreign exchange rates will change and consequently the value of an asset held in this currency. Interest rate risk is the risk that the interest rates will change and can affect an investment that has positions in fixed income products. Finally, the commodity risk, involves the risk that the price of a commodity will change. We can infer that if a commodity price changes unfavorably, the profitability of a company on this sector will also change negatively and therefore the value of an asset on this company will decrease.

Generally, the aim of a hedging strategy is to reduce some type of risk of those that we mentioned above. A perfect hedge can be considered one that completely eliminates risk. This is however very uncommon, and therefore we study how closely different strategies tend to a perfect hedge. A perfect hedge can be estimated using ex-ante data but not applied in real investments as the information is not available at the time the hedging strategy is drawn. Therefore, in order to create a hedge strategy one should use ex-post data, which can be obtained from historical prices of the futures and equities investments. In technical terms there are many ways through which the performance or efficiency of a hedging strategy can be quantitatively evaluated on historical data. The most common measure, is the decrease of portfolio returns volatility. This can for example be achieved with minimum variance hedge ratio. We later cover many of the measures that have been used in the literature, in order to determine the appropriate hedge ratio.

These two objectives characterize the hedging strategies as positive or passive hedging (Gregoriou & Pascalau, 2011). A positive hedging strategy aims to maximize the revenues of the hedged investment. When an investor is facing systemic risk greater than usual, he can use the positive hedging to hedge the systemic risk of the portfolio. This would normally be a temporary choice and after the release of risk, he will close the position. A passive hedging strategy has an objective to reduce the risk regardless of the revenue that will decrease due to its operation.

Hedging can be also adopted in the case where an investor wishes to get-off a position for a time period without undergoing the transaction costs of selling and repurchasing this portfolio. If the investor finds a contract that acts as perfect hedge and offsets all the possible losses from the initial investment, then the overall return will be zero while the investor only purchased and then sold for example a number of future contracts instead of selling and repurchasing the entire portfolio. Figure 1.9, using the data sample that will be

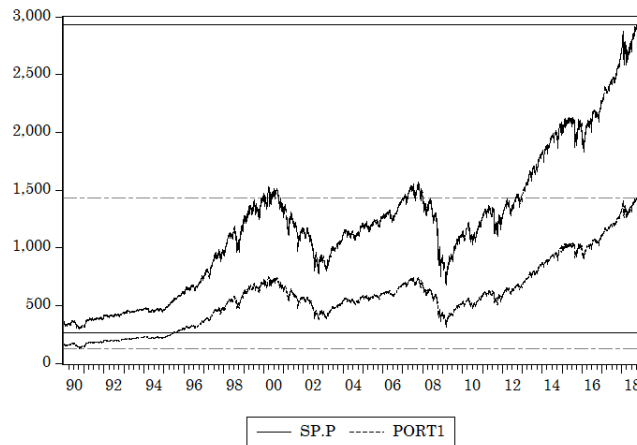


Figure 1.9: Hedged vs Unhedged portfolio value

analyzed in the next chapter, shows how the returns of an investment on S&P500 change when an equally weighted short hedge with oil futures is introduced. In the specific case the returns are lower but significantly less volatile. This can be also seen in figure 1.10 that compares the distribution of returns of the hedged and unhedged portfolio. More daily returns accumulate around zero for the hedged portfolio, but also to be more uniformly distributed.

1.2.1 Hedging with futures contracts

As we have previously mentioned hedging aims to reduce the price risk of an investment. One can achieve that by taking a position in futures contracts. This position should be opposite to the original so that the gain of the future contract will offset the loss of the initial position.

There are therefore, two forms of hedging long hedges and short hedges. A long hedge

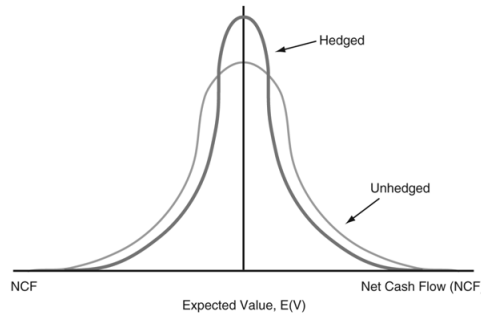


Figure 1.10: Hedged vs Unhedged portfolio returns distribution
Source: Shin Chan Business Repository

involves taking a long position in a futures contract as a counter-position on a shorted portfolio or assets. Similarly a short hedge includes a short position on a futures contract for a long position in the initial investment. The total return of the two portfolios is shown in Figure 1.11.

In our analysis, where equity investments are considered, an investor may have chosen a long position on a well diversified portfolio, but also wants to hedge against equity risk, being concerned about the performance of the market. In such case, he could offset the potential loss of a potential market fall by shorting a stock index futures contract that mirrors the movement of the stock market as a whole. There are certain characteristics

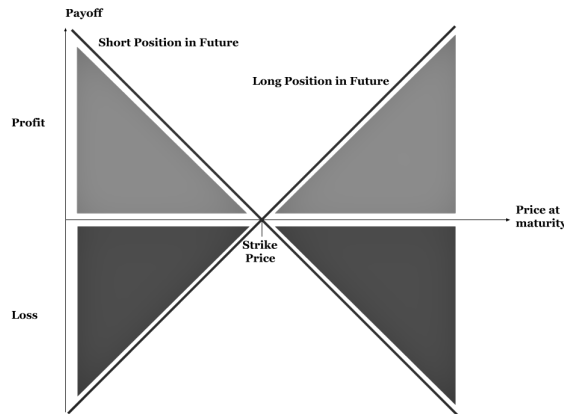


Figure 1.11: Combined Equity and Futures payoff
Source: Kolb and Overdahl (2007)

of the spot and futures markets that allow this function of hedging. First, both prices generally change in the same direction. This happens because, even though they are two separate markets, the economic environment and the factors affecting the prices in both markets are similar, so that futures markets lead to price discovery. Secondly, if the asset that is to be hedged is similar with the underlying asset of the future contract, then as the expiration date of the future contract approaches the spot and future price will tend to be equal. At delivery day any difference between the two prices should only be due to

transaction costs. If the price of the future contract starts higher than the spot price it will tend to decrease while if it starts from a lower level it will tend to increase. If the price of

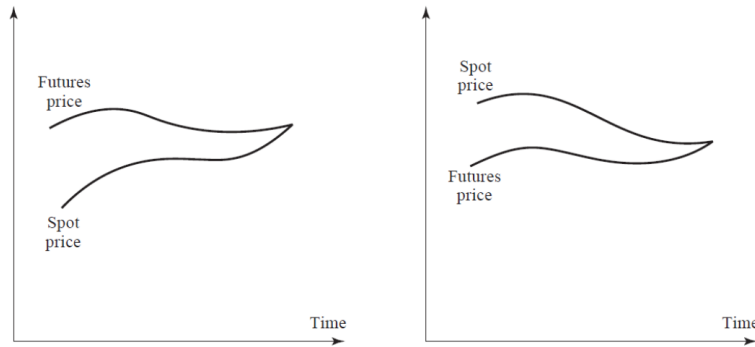


Figure 1.12: Spot and Future price convergence
Source: Hull (2012)

the future contract has not been equal to the spot price, as the expiration date approaches arbitrageurs will take advantage of the situation and immediately force the two prices into convergence. The measure of the spread between the spot and future price is mentioned as basis so that

$$Basis_t = S_t - F_t \quad (1.1)$$

where S_t represents the spot price of the asset to be hedged at time t and F_t the future price of the contract used in order to hedge. Therefore, if the hedged and hedging assets are the same, the basis will be zero at expiration date. The basis will be positive for every time that $S_t > F_t$ and negative when $S_t < F_t$. When the rate of change between two periods is positive and higher for the spot price then the basis increases and we refer to that as “strengthening of the basis”. When on the other hand, the future price increases more than the spot price then the basis declines and there is a “weakening of the basis”.

In the real market however, there are many reasons why hedging with futures is not so straightforward. First, the asset that is to be hedged may not be the same with the asset that is underlied by the future contract. There are only few future contracts compared to the total of assets that are traded globally. Therefore, it is very common for an investor not to be able to use a future contract that exactly includes the underlying assets of the initial investment. In this case the investor will choose a future contract that will be highly correlated with the equity investment and will therefore have the desired properties. A hedge of this type is referred to as a “cross hedge”. Another reason is that the investor may not know the exact date that the asset will be purchased or sold. Also, the hedge may require the futures contract to be closed out before its delivery month. The contract may be rolled to the future but still it would be very rare for the delivery date to be the same with that of the transaction needed. In this case we refer to a “stack hedge” which in contrast with a “strip hedge” at includes different contracts for each delivery date. These

problems result to what is referred as basis risk. If we set $b_{t_1} = S_{t_1} - F_{t_1}$ as the basis at the first moment of the investment and $b_{t_2} = S_{t_2} - F_{t_2}$ the basis in every next period, the uncertainty of b_{t_2} determination is considered as the basis risk. This risk might lead to either worsening or improvement of the hedging strategy. For example for a long hedge, if the basis strengthens unexpectedly the hedged position worsens, but if the basis weakens then the same position improves. Usually when the underlying asset of the future contract is not the same with the initial position the basis risk would normally be greater. Finally, the investor will also choose the contract to use based on the liquidity.

In any case, a cross hedge with future contracts is the most widely used way to hedge an equity investment. In this thesis we don't get far from this scope and we focus on the employment of future contracts as hedgers for many reasons. Firstly, the future contracts are standardized. This means that they are traded on exchanges under specific conditions and terms. For example each contract represents always a specific quantity of the underlying asset while in a forward contract, which is a private agreement, the terms may vary. Therefore, it is very difficult to use and compare different forward contracts as they are not negotiated under similar terms. Secondly, the future contracts are exchanged in organized markets. This implies that their prices are continuously recorded and are publicly available; in contrast with forward contracts that are traded over the counter. An other reason that results from the fact that futures are traded in organized markets is that they don't involve counterparty risk. The parties of all sides are obligated to pay margins to the clearing house and therefore there is no risk that the other party might default and won't fulfill the agreement. In other derivatives that do not require margins the systemic risk that the investor intends to hedge may be transferred to default risk.

What is of the highest importance for the investor, however, is to decide the appropriate number of future contracts to be used. If an investor buys exactly the same amount of contracts as his positions in equities, then we refer to a full hedge which is not necessarily optimal. More specifically, the ratio of the number of futures relative to the number of assets in the initial investment that specifies the appropriate number of futures needed. This ratio is called "hedge ratio". If we are able to answer what is the optimal ratio for an equity investment, we are then able to specify the exact amount of future contracts needed to hedge the equity investment. There has been a great research regarding the calculation of the optimal ratio. We deal with it extensively later on in the Methodology section.

The efficiency of futures contracts in hedging was first introduced by the seminal work of Ederington (1979). He started studying interest rate futures and found that the recently-introduced at the time, Government National Mortgage Association (GNMA) futures market was more effective in reducing risk than the Treasury Bills (T-Bill) market, especially in short-term hedging periods. He also found that futures hedging performance is even better in long term periods.

Since that time, many researchers have been interested in both evaluating the hedging performance of futures contracts and comparing it with that of other financial derivatives.

Baillie et al. (1991) for example find that the use of futures contracts on commodities significantly reduces the fluctuations of portfolio prices compared to cash positions only, but with different significance for each commodity. Benet (1992) finds that the use of constant cross-hedge with both commodity and currency futures are sufficient but perform better under shorter horizons. Also, Lien and Wilson (2001) showed that the conventional hedging strategy is sufficient to reduce the risk of an investment using crude oil futures. Lien et al. (2002) proved that the conventional hedging model is also sufficient to reduce the risk of an investment by using ten different futures contracts.

Cotter and Hanly (2006) using stock index futures proved that there is a difference in the hedging performance for short compared to long hedgers, suggesting that investors who are interested in opposite tails of the return distribution can benefit if they use hedging performance metrics that differentiate. Lien and Yang (2008) have found that the performance of futures contracts to reduce risk is even more effective when asymmetric effect is taken into account by conditional models.

As forwards (Giddy, 1976) and swaps are private agreements that are not flexible, for all the reasons we have already mentioned, most of the literature has focused on comparing the hedging performance of futures and options.

Paroush and Wolf (1986) show that the concurrent use of forwards and futures contracts enables the complete separation of production and hedging decisions in the framework of utility maximization. Futures are proposed for hedging purposes even in the presences of basis risk in futures markets.

Chang and Shanker (1986) after testing mean-variance criteria, concluded that currency futures are better in hedging than currency options are. Benet and Luft (1995) also revealed that S&P500 futures better reduce the variance of returns than S&P500 options do. Battermann et al. (2000) based their analysis on expected-utility maximization and proposed that futures are better instruments than options in the production and hedging framework.

Ware and Winter (1988) challenged the hypothesis that contingent exposures favor the use of options in hedging. A more analytical framework by Steil (1993) also rejected this argument and both papers concluded that options play no significant role in hedging transaction risk exposures.

Ahmadi et al. (1986) rejected the argument that that options are better than futures as hedging instruments, as they can eliminate downside risk associated with negative target returns. They specifically found that currency futures provide significantly more effective hedging than currency options for the British pound, the Deutsche mark and the Japanese yen when the target return is zero.

Also according to Lapan et al. (1991), in order to achieve an optimal hedge, when their prices are considered unbiased, futures are only required and options are redundant. Instead, they find that options are used as an alternative instrument to futures for speculation when market prices are perceived as biased.

Similar results were found by Lien and Tse (2001), when they examined the hedging effectiveness of the futures and options for three major currencies. They confirmed that currency futures outperform currency options in hedging, with only exception a situation in which a hedger will be optimistic and not very concerned about potential large losses. Cheung et al. (1990) suggest that in both minimum variance and minimum mean-Gini approaches futures are better hedging instruments than options.

Finally, Adams and Montesi (1995) provide evidence that in the real world, corporate managers prefer to hedge the downside risk using futures instead of options, mainly due to the large transaction costs occurred in option trading.

Generally, it is true that options lead to a larger excess return per unit risk than futures. However, this is true only when we don't consider the transaction costs. The above conclusion is reversed when transaction costs are taken into account and empirical results tend to be mostly in favor of futures (Lien et al., 2002).

1.3 Literature Review

1.3.1 Determining the optimality conditions for hedge ratio

The determination of the optimal hedge ratio has long been a concern for the scientific community of finance. After the introduction of the first financial derivatives in organized exchanges, economists have tried to define the optimal hedge ratio. Based on the neoclassical economic paradigm, the investor will choose the best hedge based on the maximization of his expected utility and on indifference curves between different investments (Johnson, 1960; Rutledge, 1972).

Fishburn (1977) introduced first a mean-variance analysis for financial derivatives in which he associated risk with target returns, while Ederington (1979) was the first to evaluate the hedging performance of the newly introduced at the time futures contracts. In the same framework of utility maximization Benninga et al. (1984) and Cecchetti et al. (1988) improved the research specifically on hedging with futures contracts.

As the research continued the interest was turned to a more technical level and many methods were developed in order to determine the exact number of contracts to build a hedging strategy. It was made clear that in order to achieve that, one should decide both, how the optimal hedge ratio should be defined and then how it can be estimated.

The first question depends on the theoretic assumptions regarding the investor's preferences towards risk and on practical issues that emerge, such as the transaction costs.

Different approaches include minimization and the minimization of risk conditioned to the total returns, the maximization of the investor's expected utility. The first and more popular definition of hedging is that of Minimum-variance (MV) hedge ratio. This method, first introduced by Johnson (1960) and Ederington (1979), states that the optimal hedge ratio is the one that minimizes the variance of portfolio returns. Risk is quantified by variance and therefore the minimization of variance, leads to minimization of the risk that an investment undergoes. Less exposure to risk leads to higher utility levels for the investor. This method has also been widely used by Figlewski (1984) and Howard & D'Antonio (1984), as well as in papers of Cecchetti et al. (1988), Alder and Detemple (1988) and Myers & Thompson (1989).

However, after implementing a minimum-variance hedge ratio, a practical issue arises. Hedge ratios that aim only at the reduction of risk can lead to minimal or zero total returns for the overall investment. The need for conditional hedge ratios led Howard and D'Antonio (1984) and Chen et al., (2001) to use the Sharpe ratio in order to define the optimal hedge ratio. The Sharpe ratio subjects the excess return of a portfolio to its risk by dividing the first with the later. A different process was used by Cecchetti et al. (1988) and Lence (1995, 1996) who define the optimal hedge ratio as the one that maximizes the expected utility of the investor based on both the risk and returns of his potential investments.

An alternative approach is the use of a Minimum mean Extended-Gini (MEG) coefficient hedge ratio. First introduced by Kolb & Okunev (1992, 1993), the optimum MEG hedge ratio involves the minimization of a coefficient that is based on a cumulative probability density function that is in turn estimated by ranking the observed return on the hedged portfolios. Several variations of the model have also been created by Cheung, et al. (1990), Lien & Shaffer (1999), Shalit (1995) who tests instrumental variables and Lien & Luo, (1993b) that propose a non-parametric kernel function instead of a rank function.

Another measure that can be adopted to define the optimal hedge ratio is the Generalized Semi-Variance (GSV) method. This can be implemented to give either the minimum GSV hedge ratio (Fishburn, 1977; Bawa, 1978; Crum et al. 1981, De Jong et al., 1997; Lien & Tse, 1998; 2000; Chen et al., 2001), or the maximum-mean GSV hedge ratio that can be calculated following Chen et al. (2001). Finally, the minimum value-at-risk (VaR) hedge ratio over a time period was proposed by Hung et al. (2006).

1.3.2 Estimation methods for the optimal hedge ratio

The second question is how the optimal hedge ratio, should be estimated. Differently stated, one should determine how the variables in any of the proposed formulas should be calculated. As we have mentioned earlier, the variance of both futures and spot prices have an important role in the determination of the optimal hedge ratio. Therefore, it is

essential to determine the way that the variance and the other necessary measures must be estimated from a statistical and econometric point of view.

1.3.2.1 Static Models

The first methods considered the variance and covariance to be constant with time and therefore resulted in single-value static optimal hedge ratios. The conventional method for calculating the variance is that of the Ordinary Least Squares (OLS). First utilized by Ederington (1979) and Howard & D'Antonio (1984) the simple regression method became very popular and was used in many papers mostly due to the absence of more sophisticated econometric models. For instance, the method was incorporated in many papers such as those of Malliaris & Urrutia (1991), Benet (1992), Kolb & Okunev (1992), Ghosh (1993), Kuo & Chen (1995), Lence (1995), Vähämaa (2003), Lien (2005), Deng et al. (2012). Similar to the OLS static hedge ratios, other static hedge ratios have also been proposed by Grammatikos & Saunders (1983) and Wang et al. (2015) who employed a Random Coefficient Method in order to estimate the parameters.

However, the application of a single-value hedge ratio over a sustained period will most probably not be optimal as market conditions and the relationship between the spot and future prices continuously change. As Alder and Detemple (1988) underline if the regression coefficients depend on exogenous state variables, OLS procedures provide only approximations and more complex statistical techniques are required. A constant hedge ratio may be optimal only in the case where there is no quantity uncertainty and a perfect hedge is possible. Generally, minimum-variance hedges must be continuously rebalanced as in any occurring event, the minimum or zero-variance hedges will not be optimal. The need for a time-varying hedge ratio therefore emerges and a method that will estimate time-varying estimations for the variables is necessary. Working (1961) states that “hedging is done in expectation of a change in spot-futures relations” and not by moving together.

Lien et al. (1996) have found that if there is a cointegration relationship between the spot and futures prices and one omits this relationship, then the optimal futures position will be smaller, and the hedging performance relatively poor. They also showed that spot and futures prices can be expressed by a complete cointegration system. If cointegration relationship between spot and futures is not taken into account there will be a misspecification of their pricing behavior and result will be underhedging (Gosh 1993; Lien 1996). In the case that a cointegration relationship is found, Error Correction Models (ECM) should be constructed before the hedge ratio is estimated. Using index futures Ghosh, (1993) proved that the out-of-sample performance of a hedge ratio obtained from ECM is better compared to the conventional static hedge ratio of the Ederington model, and the same result was found from an intertemporal ECM (Ghosh & Clayton, 1996). Chou et al. (1996), similarly showed that the error correction models are superior to the conventional models

based on likelihood ratio statistics. They also found that the out-of- portfolio variances of error correction hedging models outperformed the conventional method models by an average of 2%. Finally, Li (2010) supported the superiority of the threshold Vector Error Correction Model (VECM) in enhancing hedging effectiveness for emerging markets, while Lien & Luo (1993a) also confirm the presence of cointegration relationships between spot and futures prices of indices and currencies using multi-period hedge ratios.

1.3.2.2 Dynamic Models

In 1986, Bollerslev (1986) and Taylor (1986) developed the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model that quantifies volatility in a time varying framework. Also, as Park and Bera (1987) indicate heteroscedasticity appears to be a serious problem in cross-hedging strategies. The model was immediately applied for the determination of time-varying variance and the determination of the optimal hedge ratio in many papers. Cecchetti et al. (1988) showed that Autoregressive Conditional Heteroskedasticity (ARCH) procedures can allow the hedge ratio to change over time and result in significantly lower ratios than conventional static models. Similarly, Sephton (1993a) proved that the GARCH-based hedge ratio performs better compared to the conventional minimum-variance hedge ratio using commodity futures. Lien & Luo (1993a) also discovered strong GARCH effects in cointegrated markets and that the parameters estimated from the GARCH processes differ much compared to those of simple error-correction models and are more likely to be statistically significant. Park & Switzer (1995b) also showed that the GARCH hedge is more economically useful in improving the utility function of an investors as opposed to the OLS hedge.

1.3.2.3 Univariate GARCH models Tong (1996) stated that GARCH-modeled dynamic hedging reduces risk more than static hedging with an in-sample improvement of 6 percent, and an out-of-sample improvement of 2 percent, while more complex hedging methods didn't seem to improve much the performance. The inferiority of the Ederington method for static hedge ratio was also proved by Lien (2005). The conclusion that the dynamic hedging methods outperform the conventional method is also shown by Baillie & Myers (1991), Lien & Tse (2001; 2002), Lien (2009), Lee & Yoder (2007a), as well as Zantotti et al. (2010), Moon et al. (2009) and Ewing & Malik (2013) that all used univariate GARCH models for different classes of futures in order to estimate the optimal hedge ratio and evaluate the performance of such hedges.

1.3.2.4 Multivariate GARCH models The GARCH models have many variations that can be used based on the objectives of the research. More complex methods of GARCH models include bivariate and multivariate GARCH models. Park & Switzer (1995a), have

showed that a dynamic hedging strategy based on the estimation of a bivariate GARCH improves the hedging performance of a conventional constant hedging strategy. Olgun et al. (2011) also used bivariate GARCH frameworks to reveal that the dynamic hedge strategy outperforms the static and traditional strategies. The superiority of multivariate GARCH models was also proved by Park & Switzer (1995b) and Yang & Allen (2005) for stock index futures and Chang et al. (2010) for energy futures. Bivariate GARCH models were also suggested by Lien & Luo (1994), Lien & Yang (2008), Salvador & Aragón (2014).

CCC GARCH The use of Constant Conditional Correlation (CCC) GARCH has been widely accepted using exchange rates (Hsin et al., 2007), energy (Arouri et al., 2012), and stock index futures (Basher & Sadorsky, 2016). Yang et al. (2004) found that all the approaches favor the CCC-GARCH hedge ratio estimates to the conventional hedge ratios in all out-of-sample analyses.

DCC GARCH However, as CCC GARCH does not model the stochastic behavior of the correlation, an improved version was necessary. Dynamic Conditional Correlation (DCC) GARCH models that allow for the correlation to vary and have been preferred by the most researchers from that time. DCC GARCH models are proved to provide hedge ratios with superior hedging performance in the works of Lien & Tse (2002), Chang et al. (2010) that use energy futures, Park & Jei (2010) who implement commodity futures and Chang et al. (2013) who use currency futures hedges. Similarly, Chang et al. (2010) showed that the optimal portfolio weights of multivariate volatility models for Brent and West Texas Intermediate (WTI) suggest holding crude oil futures in larger proportions than spot. Basher et al. (2016) have tried to hedge emerging market stock prices with oil, gold, Volatility Index (VIX), and bonds futures using DCC GARCH models. They concluded that stock and oil prices display positive leverage effects and that hedge ratios vary considerably over different periods, proving that hedged positions should be updated regularly. The highest hedging effectiveness was achieved with oil futures.

There are also many variations of multivariate GARCH models that account for different specifications in the samples used. For example, Lai et al. (2009) proposed a new class of RV-based GARCH model that can estimate risk-minimizing hedge ratios and they proved once more the return-based GARCH to have many benefits relative to OLS models. Bivariate GARCH (BGARCH) are applied by Sim & Zurbrugg (2001) as well as, Park et al. (2010) and Exponential GARCH (EGARCH) models have been tested by Lien & Tse (2002) who found that GARCH strategies may be better in terms of variance reduction than the strategies provided by Stochastic Volatility (SV) models and Xu & Li (2017). Arouri et al. (2012) also test Vector Autoregressive GARCH (VAR-GARCH) and Hsin et al. (2007) and Chang (2012) try Vector Autoregressive Mean Average GARCH

(VARMA-GARCH) models.

1.3.2.5 Extended multivariate GARCH models More specialized GARCH are also implemented by Lee & Protter (2008) and Hsu et al. (2008) that examine index futures and suggest that Regime Switching Volatility Spillover GARCH (RSVSG) have higher hedging effectiveness. AFRIMA GARCH were used by Lien & Tse (1999), Lee & Yoder (2007) and Chang (2012) and Exogenous Variables GARCH (X-GARCH) by Sim & Zurbruegg (2001) and Sultan & Hasan (2008). In some of the works mentioned above BEKK models have also been tested and their performance was evaluated (Hsin et al., 2007; Arouri et al., 2012).

Another well-established method is the Copula-based GARCH. Examining Asian stock market indices, Lai et al. (2009) showed that Copula-Threshold GARCH (T-GARCH) time-varying hedge ratios are more effective in reducing risks in portfolio returns than OLS and DCC hedge ratios do. They further presented that even though DCC and copula do not reduce the risk significantly more than OLS hedge ratios in stable markets like those of Japan and Singapore, they provide higher returns. The effectiveness of copula-based GARCH was again proved by Hsu et al. (2008) that examined stock index and currency futures, and Ghorbel & Trabelsi (2012) that did so with oil futures. Chang (2012) employed a time varying asymmetric copula-based model to account for leverage effects.

The most recent paper published concerning the estimation of the optimal hedge ratio for future contracts, by the time this thesis was written, is that of Lai (2018) who implements a Realized-beta GARCH model.

Koutmos et al. (1996) have already proved that stock return volatility is an asymmetric function of past innovations, which is the leverage effect. They noticed that equilibrium models, which rely on contemporaneous relationships, may be miss-specified. Moreover, the hedging strategies that ignore the time varying covariance structure of the two markets, are not likely to be optimal. The importance of such leverage effects was taken into consideration by Lien et al. (2007) who developed dynamic minimum variance hedge ratios (MVHRs) using BGRACH bivariate models. Their performance revealed that the models with asymmetric effects provide a more effective reduction of the risk. Similar results were found by Lien et al. (2008) who observed in both in-sample and out-of-sample results that incorporating the asymmetry basis effect into the hedging strategy leads to a better risk reduction. They similarly showed that the dynamic hedging strategy generated from the asymmetric model outperforms the conventional strategies even after considering the transaction costs. Ghorbel (2012) concluded that a precise specification of the joint distribution of risk factors can more effectively hedge the risk exposure of portfolios and he also suggested that the use of GARCH Regime Switching models that differentiate the ratios between crises and more quiet periods can provide superior hedging strategies. Lee

and Chien (2010) revealed that state-dependent IS-DCC outperforms state-independent DCC GARCH, while the three-state IS-DCC has the best hedging effectiveness, showing importance of modeling higher-state switching correlations in dynamic futures hedging.

Salvador et al. (2012) showed that introducing nonlinearities through a regime-switching model, leads to more efficient hedge ratios and superior hedging performance compared to the other methodologies (constant hedge ratios and linear GARCH). Lee et al. (2007b) have employed Markov Regime Switching Time-Varying Correlation GARCH to show that this model outperforms the CCC GARCH, and later Lee & Protter (2008) developed a Markov Regime Switching Generalized Orthogonal GARCH model with Conditional Jump Dynamics (JSGO) which was proven to improve the hedging effectiveness both in reducing the variance and maximizing the utility.

1.3.2.6 Other dynamic models The Markov Regime Switching method was found to be inferior in performance compared to a Random Coefficient Autoregressive Regime Switching (RCARRS) as this was developed by Lee et al. (2006).

Lai (2016) introduced the use of High-Frequency-Based Volatility (HEAVY) hedge ratios as he found that noise-free predictions are superior, substantially increasing the utility hedgers with pronounced risk aversion. The importance of removing micro-structure noise and asynchronous trading from covariance estimation is raised for the prediction of the hedge ratio. Later Lai et al. (2017) showed that high-HEAVY hedge ratios perform more effectively than GARCH hedge ratios do in shorter hedging horizons. Momentum effects have some properties of short-time response that considered important for hedge ratio estimation are revealed only with such models.

Finally, Wang et al. (2015) proposed that under the minimum variance framework the Naïve hedging strategy is consistently and significantly the best performing. Wei et al. (2011) proposed that copula–Multifractal Volatility (MFV) models obtain better hedging effectiveness, than copula–GARCH type models and involve fewer transaction costs.

Moosa (2003) using stock and currency futures found that the model specification does not change the performance of a hedging instrument. Instead he finds that the correlation between the prices of the unhedged position and the hedging instrument is what matters the most for the success of a strategy. Low et al. (2002) developed a variant cost of carry model that using stock index and energy futures outperformed all other hedging strategies on ex-ante basis. Finally, other estimation methods have been used by Lien and Shrestha (2010) that employ a Multivariate Skew-Normal Distribution Method, Lence (1995, 1996) that uses a Coefficient of Absolute Risk Aversion function (CARA) and again Lien and Shrestha (2007) this time with a Wavelet analysis.

Chapter 2

Empirical Application

2.1 Methodology

2.1.1 Optimal hedge ratios

Since risk is usually measured by the volatility of portfolio returns and the hedging aims to reduce such risks, a possible solution to the problem of defining the hedge ratio is to choose the ratio that will minimize the variance of portfolio returns containing equity investment and futures positions. According to Johnson (1960) this optimal hedge ratio can be calculated in the following way. Suppose that, ΔS is the change in the spot price or differently said the return of the equity investment, ΔF is the change in the futures price during the same period, σ_S, σ_F the standard deviations of the spot and future returns $(\Delta S, \Delta F)$, ρ is the correlation coefficient between ΔS and ΔF , and finally h is the hedge ratio. If we take a total of h positions in futures, the total return of the hedged portfolio will be equal to $R_{portfolio} = h\Delta F - \Delta S$ for a long hedge and $R_{portfolio} = (\Delta S - h\Delta F)$ for a short hedge. The variance of the two portfolios $Var(R_{portfolio})$ will be the same for both long and short hedges and is obtained as $Var(\Delta S - h\Delta F)$. On the basis of this expression using the properties of the variance formula we can show that

$$\begin{aligned} Var(R_{portfolio}) &= Var(\Delta S) + Var(h\Delta F) - 2Cov(\Delta S, h\Delta F) \\ &= Var(\Delta S) + h^2Var(\Delta F) - 2hCov(\Delta S, \Delta F) \\ v &= \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F \end{aligned} \tag{2.1}$$

If equation 2.1 is minimized with respect to h we will get

$$h = \rho \frac{\sigma_S}{\sigma_F} \tag{2.2}$$

or differently stated

$$h = \frac{Cov(\Delta S, \Delta F)}{Var(\Delta F)}$$

At this stage, the hedge ratio is not a time-varying value. Equation 2.2 is known as “static optimal hedge ratio”. The standard deviations and the correlation are considered constant during the life of the hedge. Another way for defining the optimal hedge ratio is that proposed by Howard and D’Antonio (1984), where the criterion incorporates the portfolio return in the hedging strategy. The return and the variance are used in a risk-return trade-off as in the Sharpe measure. So the optimal level of futures contracts is calculated by maximizing the ratio of the portfolio’s excess return with respect to its volatility.

$$max\ Sharpe\ Ratio_n = \frac{E(R_{portfolio}) - R_f}{\sigma_{portfolio}} \quad (2.3)$$

Where n is the optimal number of future contract units and R_f the risk-free interest rate. The earlier discussion would be appropriate for only one-period hedging strategies. However, this assumption is not realistic (Lien and Luo, 1994) as the settlement of futures is daily and the hedger’s horizon includes multiple periods. We can relax these assumptions by estimating time-varying standard deviations and correlations. The GARCH models are considered appropriate by the scientific community for this purpose and equation 2.2 will be transformed respectively to

$$h_t = \rho_t \frac{\sigma_{S,t}}{\sigma_{F,t}} \quad (2.4)$$

There are many methods to model volatility of equation 2.4 such as using historical or implied volatility. The hedge ratio can work as an information transmission mechanism that incorporates information from the futures market into the spot investment. However, it has been observed that financial data, such as asset returns, are usually nonlinear and therefore linear models would most probably fail to capture some of the properties of the sample (Brooks, 2014). These characteristics are

1. Leptokurtosis, which is the tendency of the returns not to follow the normal distribution, but rather to exhibit distributions with fat tails and more concentration in the mean.
2. Volatility Clustering, which is the tendency of the volatility to appear in groups. Generally, large returns either positive or negative are followed by large returns while small returns are followed by small returns. This happens because the presence of information which drives price changes is not evenly spaced in time.
3. Leverage, which is the tendency of volatility to be larger when the returns are negative, compared to that when returns are positive.

For the reasons stated above a simple OLS regression between spot and future returns in

order to calculate the correlation coefficient ρ or the standard errors σ_S, σ_F with historical data, is not considered appropriate, while models that are nonlinear in the mean, the variance or even both might be. Using more sophisticated dynamic hedging techniques can better prevent from excess volatility.

2.1.2 Volatility modeling

The importance of time-varying second-order moments has been widely recognized in applied finance. One available method developed by Roberts (1959) to model volatility non linearly, is with Exponentially Weighted Moving Average Models (EWMA), in which the variance of a series is calculated based on the sum of squared differences of each observation with the sample's mean, multiplied by a decay factor that distributes higher weights to the more recent observations. An alternative method is that of Autoregressive (AR) Volatility models. In this case the variance at every time period can be calculated as a function of its previous values.

$$\sigma_t^2 = \beta_0 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 + \varepsilon_t \quad (2.5)$$

At this moment it is important, for the construction of the models that we will use later in this chapter, to introduce the notion of conditional variance. Suppose that the error term of the regression that models the volatility is u_t , under the Classical Linear Regression Model assumptions the variance of this error term will be homoscedastic, or its variance will be constant through time $Var(u_t) = \sigma^2$. However, constant error-variance is extremely unusual to be found in financial time series. The variance of the errors appears to be heteroscedastic and can be effectively modeled with the use of an Autoregressive Conditional Heteroscedastic (ARCH) model. If we denote the conditional variance of u_t as σ_t^2 then

$$\sigma_t^2 = var(u_t | u_{t-1}, u_{t-2}, \dots) = E[(u_t - E(u_t))^2 | u_{t-1}, u_{t-2}, \dots] \quad (2.6)$$

if we also assume that $E(u_t) = 0$ then equation 2.6 can be rewritten as

$$\sigma_t^2 = var(u_t | u_{t-1}, u_{t-2}, \dots) = E[u_t^2 | u_{t-1}, u_{t-2}, \dots] \quad (2.7)$$

so that the conditional variance of a zero mean variable that follows the normal distribution is equal to the conditional expected value of the square of u_t . So the ARCH model allows for the conditional variance of the error term σ_t^2 to depend on the immediately previous value of the squared error u_t^2 . In our case the variables are the spot and futures returns and the random variables u_t are the error term that results from

$$\Delta S = \mu_{\Delta S} + u_{tS}$$

$$\Delta F = \mu_{\Delta F} + u_{tF}$$

Having said that, the conditional variance of the returns will be written in an ARCH(q) model as

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + a_2 u_{t-2}^2 + \dots + a_q u_{t-q}^2 \quad (2.8)$$

$$\sigma_t^2 = a_0 + \sum_{i=1}^q a_i u_{t-i}^2$$

From now on, we will refer to the conditional variance as h_t following the relevant literature i.e. $u_t \sim N(0, h_t)$. The conditional variance therefore depends on a set of information which we seek to incorporate.

2.1.2.1 GARCH models

Previous research seems to conclude that the best fitting model of variance of financial time series is the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model which was developed by Bollerslev (1986) and allows for the conditional variance to depend also upon its previous values. Equation 2.9 extends a ARCH(q) to a GARCH(q,p)

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + a_2 u_{t-2}^2 + \dots + a_q u_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + \dots + \beta_p \sigma_{t-p}^2 \quad (2.9)$$

$$\sigma_t^2 = a_0 + \sum_{i=1}^q a_i u_{t-i}^2 + \sum_{j=1}^p a_j \sigma_{t-j}^2$$

Information criteria most of the times propose that GARCH(1,1) is more parsimonious and captures volatility clustering in the returns. The GJR model, as proposed by Glosten et al. in 1993, is an extension of the a GARCH(1,1) model that accounts for possible asymmetries by using a indicator variable. This model can be used to indicate the presence of leverage effect in our sample's returns.

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} \quad (2.10)$$

where $I_{t-1} = 1$ if $u_{t-1} < 0$ and $I_{t-1} = 0$ otherwise. The estimate γ reflects only the cases where the return is negative and if $\gamma > 0$ and statistically significant then there is presence of leverage effect in our series. If γ is found negative then model is still admissible if $\gamma + \alpha_1 \geq 0$. Up to now with these models we are able to estimate the time-varying variance in the spot prices of the equity portfolio and the prices of hedges.

2.1.2.2 Multivariate GARCH models

In order to estimate the time-varying ρ_t of equation 2.4, we need to obtain the time-varying variances, as well as, the time varying correlation of each pair. The calculation of

the moving hedge ratio though, can be alternatively estimated by using the time-varying covariance. As we mentioned in volatility modeling, implied covariance or EWMA models can be applied to do so. However, following the literature (Cecchetti et al., 1988; Baillie and Myers, 1991; Sephton, 1993a) we implement GARCH models that will result to both a conditional covariance and a conditional correlation for each pair of assets. In bilateral relationships the ARCH models are proved to be the most profitable (Engle, 1993), while research has found that the GARCH(1,1) model seems to be particularly useful to describe a wide variety of financial market data (Bollerslev et al., 1994).

The bivariate GARCH models require as inputs two returns series for every pair of assets and incorporate the information of one series to the other. In our case, this is done by adding the lagged variance of the futures returns in the variance modeling of the equity returns. In this way we can account for some stylized facts of the variance of real time, such as contemporaneous cross correlation and volatility spillovers. Among the multiple variations of multivariate GARCH models, we choose to estimate BEKK models as they provide time-varying correlations, contrary to CCC GARCH, and have by definition positive definite covariance matrix compared to the other models (Engle and Kroner, 1995).

The BEKK model (Baba et al., 1990) assumes that the conditional variance-covariance matrix H_t is positive definite. More specifically,

$$H_t = W'W + A'H_{t-1}A + B'\Xi_{t-1}\Xi'_{t-1}B \quad (2.11)$$

Where $H_t = \begin{bmatrix} h_{11t} & h_{12t} & \dots & h_{1nt} \\ h_{21t} & h_{22t} & \dots & h_{2nt} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n1t} & h_{n2t} & \dots & h_{nnt} \end{bmatrix}$ is a $n \times n$ conditional variance-covariance matrix

between n variables or portfolio assets, W is an upper triangular parameter matrix, A, B

two $n \times n$ matrices of estimated parameters and $\Xi = \begin{bmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{3t} \end{bmatrix}$ a disturbance vector so that

$$\Xi'\Xi = \begin{bmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{nt} \end{bmatrix} \begin{bmatrix} u_{1t} & u_{2t} & \dots & u_{nt} \end{bmatrix} = \begin{bmatrix} u_{1t}^2 \\ u_{2t}^2 \\ \vdots \\ u_{nt}^2 \\ u_{1t}^2 u_{2t}^2 \\ \vdots \\ u_{(n-1)t}^2 u_{nt}^2 \end{bmatrix} \quad \text{in regression term equation 2.11}$$

can be written as

$$h_{ij,t} = w_{ij} + \sum a_{ij}u_{i,(t-1)}u_{j,(t-1)} + \sum \beta_{ij}h_{ij,(t-1)} \quad (2.12)$$

for $i, j = 1, 2, \dots, n$

The CCC-GARCH was developed by Bollerslev (1990) and requires for the correlations between disturbances to be fixed through time. Although the conditional covariances are not fixed, they are very close to variances. If we write the correlations between the disturbances as ε_t , the conditional variances in the fixed correlation model, even though estimated together, take the form of a univariate GARCH as shown in equation 2.13. The diagonal elements of H_t , $h_{ij,t}$ for every $i \neq j$ are defined in equation 2.14 indirectly by the correlations ρ_{ij}

$$h_{ii,t} = c_i + a_i\varepsilon_{i,t-i}^2 + b_i h_{ii,t-1} \quad (2.13)$$

$$h_{ij,t} = \rho_{ij}h_{ii,t}^{1/2}h_{jj,t}^{1/2} \quad (2.14)$$

At this point, we need to mention that although the hypothesis of constant correlation through time, there is no evidence against it for stock returns series with the relative tests employed until now. However CCC-GARCH does not model the stochastic behavior of that correlation matrix at all. It is an artifact of the model whose results may be unreasonable.

The DCC-GARCH on the other hand, as its name (Dynamic Conditional Correlation) denotes, allows for the correlation between the variable to vary with time. There are several variations of DCC models but the most popular is that of Engle (2002). If we denote as D_t the diagonal matrix of the conditional standard deviations on the leading diagonal, and R_t as the conditional correlation matrix, then the variance-covariance matrix can be written as

$$H_t = D_t R_t D_t \quad (2.15)$$

Many variations, such as an exponential smoothing approach, of this model can be obtained based on the specification about R_t .

As we showed earlier, the BEKK model provides a richer dynamic structure and has the property of positive definite conditional covariance matrices. Also the diagonal version of BEKK economizes the number of parameters and as Bollerslev et al. (1988) indicate, if we assume the matrices A and B to be diagonal a more parsimonious representation is obtained as it will imply that each variance-covariance element depends only on its previous values and prediction errors. By taking equation 2.11, the general form of a BEKK model will be given by

$$H_t = M' M + B' H_{t-1} B + A' \varepsilon_{t-1} \varepsilon_{t-1}' A \quad (2.16)$$

where the coefficient of the matrices are given by

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, M = \begin{bmatrix} m_{11} & 0 \\ m_{12} & m_{21} \end{bmatrix} \quad (2.17)$$

with the formulation guaranteeing that H_t will be positive definite for all t and with the diagonal representation indicating that the conditional variances (H_t) are function of their lagged values (H_{t-1}) and lagged squared returns ($\varepsilon_{t-1}\varepsilon_{t-1}$), so that a BEKK(1,1) model will result to $N(5N+1)/2$ parameters. The parameters of such models are estimated by the maximum likelihood method (ML) that optimizes numerically the Gaussian log-likelihood function. If f denotes the normal density the contribution of each observation in time t to the log-likelihood l_t of the sample will be given by:

$$l_t = \ln \{f(\varepsilon_t|F_{t-1})\} = -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \ln(|\Sigma_t|) - \frac{1}{2} \varepsilon_t^T \Sigma_t^{-1} \varepsilon_t \quad (2.18)$$

2.2 Data Description

The data are daily closing prices for spot variables and daily closing continuous prices for futures contracts, that were derived from the database of Factset on 4th March 2019. The moments depend on the frequency, aggregation and seasonality of the sample. In our case we chose a daily frequency in order to have a greater sample and therefore obtain more robust interpretations. The sample is composed of about 7,300 observations spanning from the beginning of 1990 until the last trading day of 2018. The series of spot prices are the S&P500 index (S&P500), the Euro Stoxx index (ES), the Japan Nikkei 250 index (NIK) and the Shanghai Stock Exchange Composite index (SSE), all of which are expressed in index units. For the futures contracts, the Crude Oil West Texas Intermediate (WTI) that is negotiated in New York Mercantile Exchange (NYMEX), counted in US dollars per barrel and Gold (GCS) negotiated in the same exchange and counted in US dollars per oz. The plot of each of the variables can be seen in the following figure.

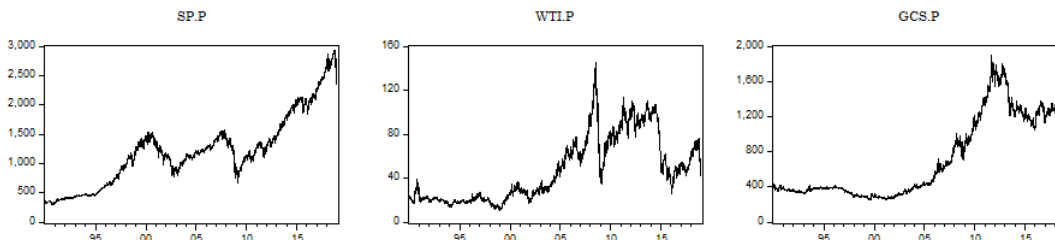


Figure 2.1: Daily closing values of S&P500 index, oil and gold futures

In our analysis we first focus only with the relation between the S&P500 and the two

hedgers WTI and GCS, that are all negotiated in the US markets. Then, we enrich the results by testing the same models on other international stock markets.

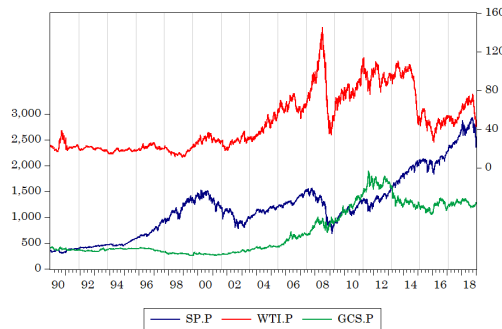


Figure 2.2: Simultaneous plot of the main variables S&P500, WTI, GCS
 * S&P500 and GCS (left axis) and WTI (right axis)

After that, daily returns of the series are calculated by subtracting each price from its previous value and dividing it with the same number as shown in equation 2.19.

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad (2.19)$$

The descriptive statistics of the returns are shown in table 2.1, and their plots in figure 2.3 respectively. From now on, we will use the returns in all the estimations that follow. The returns are symbolized in the tables and figures with the name of the asset following by the letter r i.e. for WTI the returns are WTIR.¹

Table 2.1: Descriptive Statistics of returns

	WTIR	GCSR	SPR	ESR	NIKR	SSER
Mean	0.000366	0.000209	0.000329	0.000218	6.70e-05	0.000525
Median	0.000551	0.00000	0.000503	0.000345	-2.97e-05	0.000199
Maximum	0.202542	0.092318	0.115809	0.115642	0.133973	1.109237
Minimum	-0.318917	-0.093446	-0.090352	-0.099760	-0.10585	-0.321892
Std. Dev.	0.022961	0.010183	0.011067	0.013379	0.015708	0.026253
Skewness	-0.309950	-0.079607	-0.078715	-0.015752	0.187291	12.14990
Kurtosis	12.34872	10.84204	11.91435	10.06486	7.465253	503.6600
Jarque-Bera	26,726.19	1,8731.17	24,201.50	15,733.05	5,965.077	68,8322.72
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations	7307	7307	7307	7565	7130	6575

¹ All the models that follow are estimated using the returns of the assets. Only for convenience, we refer to the variables with their full name in the text and descriptions of the figures and tables, and we use their returns in the estimations.

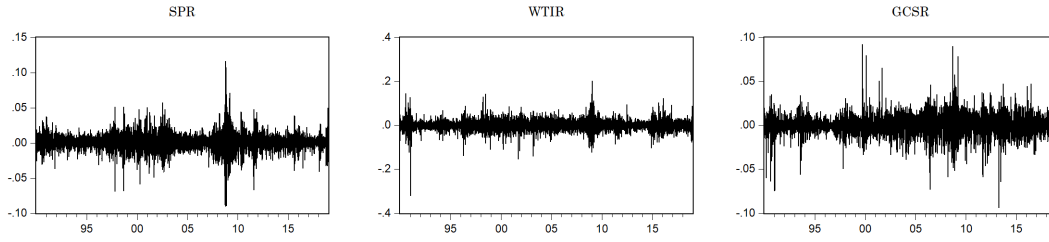


Figure 2.3: Daily returns of S&P500 index, oil and gold futures

We can clearly see that the non-stationary price series are now converted into a stationary process without trend. However, because we use data from five different stock markets that trade on different days, while the two futures contracts are traded in the US, the resulting samples have neither the same size nor negotiation days. For that reason, we exclude from the sample the dates that have at least one missing value in any variable. In that way the remaining sample is reduced as shown in table 2.2 but is continuous and have respective trading days in both stock indices and futures. The histograms of the rest variables are displayed in the appendix.

Table 2.2: Sample size alignment after excluding missing dates

	WTIR	GCSR	SPR	ESR	NIKR	SSER
Observations	7307	7307	7307	7307	6901	6360

2.3 Results

2.3.1 Static Analysis

As most financial series show clusters of high volatility in their returns, it can be seen from the squared returns that the variables of this analysis are not an exception. It seems that during some time periods the series are significantly more volatile usually around crisis events, when the traded volume is also increased. This can be due to the fact that in high frequency data the amount and quality of information is received by market participants in clusters that also delay to process it and react.

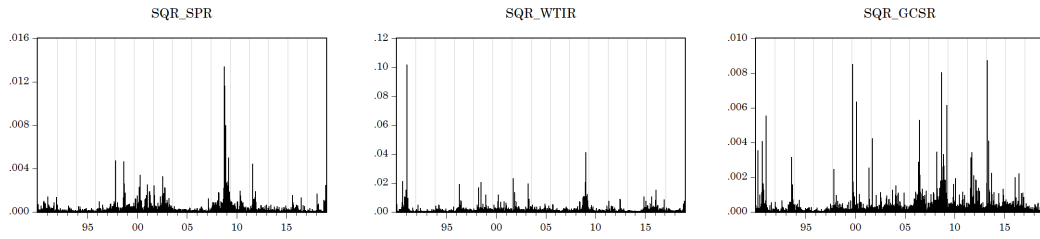


Figure 2.4: Squared returns of S&P500, WTI and GCS

This volatility clustering is also confirmed and more evident in a 25-step moving standard deviation plot of the series as shown in figure 2.5. The persistence in variance is a stylized fact for financial series and refers to the tendency of high conditional variance to be followed by high values.

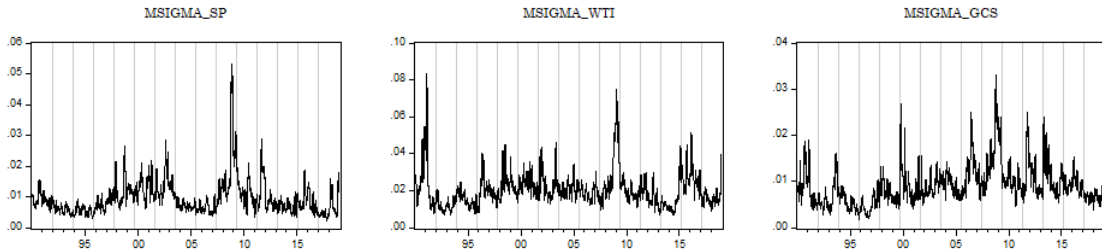


Figure 2.5: Moving Standard Deviations of S&P500, WTI and GCS returns

What we specifically seek to find is whether the relation between the futures and the index can be used to hedge a position on the later. After having confirmed a relationship, we will be able to model the volatility of the series and decide on how this relation can be optimally used to hedge. A static covariance analysis at first indicates statistically significant correlations with a stronger relation between the index and oil futures (correlation coefficient = 0.12), than between the index and gold futures (correlation coefficient = -0.05).

Table 2.3: Static Covariance analysis

	WTIR	GCSR
Covariance	3.27e-05	-5.96e-06
Correlation	0.128861	-0.052903
t-statistic	11.10629	-4.527952
Probability	0.0000	0.0000

Table 2.4: Static Covariance analysis II

		ESR	NIKR	SSER
WTIR	Covariance	6.42e-05	2.25e-05	1.97e-05
	Correlation	0.0217331	0.066676	0.033289
	t-statistic	17.29973	5.192145	2.587923
	Probability	0.0000	0.0000	0.0097
GCSR	Covariance	1.60e-05	1.93e-05	6.16e-06
	Correlation	0.118124	0.124249	0.022693
	t-statistic	9.242733	9.729304	1.763627
	Probability	0.0000	0.0000	0.0778

The simple Ordinary Least Squares regressions in which the returns of the index are used as a dependent variable on the independent oil and gold futures returns separately and simultaneously as shown in the following regressions:

$$SPr_t = C + \beta WTIR_t + e_t \quad (2.20)$$

$$SPr_t = C + \beta GCSR_t + e_t \quad (2.21)$$

$$SPr_t = C + \beta_1 WTIR_t + \beta_2 GCSR_t + e_t \quad (2.22)$$

The obtained results are presented in table 2.5 and 2.6 respectively

Table 2.5: Estimation results from equations 2.20 and 2.21

	SPR		SPR
C	0.000307	C	0.000341
β WTIR	0.062112	β GCSR	-0.057500
t-statistic	11.10	t-statistic	-4.527952
Probability	0.0000	Probability	0.0000
R-squared	0.01660	R-squared	0.00279

Table 2.6: Estimation results from equation 2.22

SPR	Coeff.	t-statistic	Probability
C	0.000323	2.5204	0.0117
WTIR	0.070514	12.3717	0.0000
GCSR	-0.090701	-7.0579	0.0000

A unit root analysis (results presented in the appendix) on the levels of prices and then on the returns reveals the existence of a unit root. The autocorrelation and partial correlation analysis also revealed little or no evidence of linear structure in the return series. Since no autocorrelation is found in the returns, there is no need for the conditional mean to be specified. Modeling of the conditional variance is only necessary.

However, the moving standard deviations have already suggested for a relatively unstable relation through time. If we compare the static measures with a 25-step moving correlation, it can be clearly seen that the relation between the spot and the futures is neither stable nor has the same direction during the entire sample, as the sign of the relation changes multiple times. This indicates that using a static measure does not fully reflect the dynamics between the assets and undermines the potential of implementing their changing relationship. In figure 2.6 the moving correlation (dark continuous line) is compared to the static value of the Pearson correlation coefficient (dashed faded line) for both pairs.

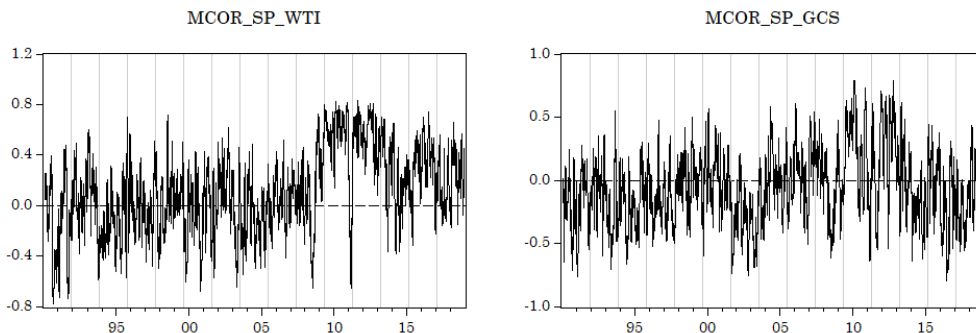


Figure 2.6: Moving Correlation for S&P500-WTI and S&P500-GCS pairs

If we use the results from tables 2.2 and 2.4, the equation 2.3 yields to a static hedge ratio of $h_{SP,WTI} = 0.06211$ for WTI, and $h_{SP,GCS} = -0.05749$ for GCS, which can be interpreted as, for every position taken in spot on the S&P500 index, 0.06 contracts of oil futures are needed to hedge it. But as the nature of the risk within the markets changes over time, the modeling of the hedge ratio should take into account the time-varying dimension.

2.3.2 Dynamic Analysis

In order to estimate a moving optimal hedge ratio that will incorporate the dynamic nature of the relations between the assets, we need to model the volatility in a time-varying context. The dynamic variance and correlation that will be conditional to their previous values, the volatility and the variance of the other asset can be obtained using a diagonal BEKK model, which in this case will be bivariate as we use pairs of spot and future contracts. The equations that were used are of the following form

$$H_t = M'M + B'H_{t-1}B + A'\varepsilon_{t-1}\varepsilon'_{t-1}A$$

or

$$H_t = MM' + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} +$$

$$\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}\varepsilon_{1,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}'$$

Engle and Kroner (1995) suppose that the diagonal elements of M and all a_{ii}, b_{ii} diagonal elements are restricted to be strictly positive. The resulting H_t is the variance-covariance matrix between each pair of equity index and future contract, and are estimated with the log-likelihood that is derived from

$$l_t = \ln \{f(\varepsilon_t|F_{t-1})\} = -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \ln(|\Sigma_t|) - \frac{1}{2} \varepsilon_t^T \Sigma_t^{-1} \varepsilon_t$$

S&P500 & WTI futures

Table 2.7: BEKK estimation coefficients for S&P500-WTI

Constants	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.000565	8.98e-05	6.296693	0.0000
C(2)	0.000342	0.000196	1.741418	0.0816
Equation Estimated				
GARCH=M+A1A1(RES(-1)^2)+B1B1GARCH(-1)				
Variance Equation Coefficients				
M(1,1)	1.23e-06	1.05e-07	11.72341	0.0000
M(1,2)	2.41e-07	1.03e-07	2.326808	0.0200
M(2,2)	3.00e-06	4.43e-07	6.758067	0.0000
A1(1,1)	0.282422	0.006312	44.74389	0.0000
A1(2,2)	0.230450	0.005244	43.94527	0.0000
B1(1,1)	0.954309	0.002075	459.9013	0.0000
B1(2,2)	0.971033	0.001460	665.2874	0.0000
Log likelihood	42385.27	Schwarz IC		-11.59032
Avg. log likelihood	2.900320	Hannan-Quinn IC		-11.59589
Akaike IC	-11.59882			

In this case, the sufficient condition for positive definiteness of H_t is true as the diagonal elements of M, A, B matrices are strictly positive and there are no other equivalent representations different than those produced by the diagonal BEKK model. The variances and covariance estimated with the BEKK model (figure 2.7) confirm the static measures results, that WTI is more closely related to the S&P500 and will therefore be a better hedger. The model produces in the time-varying conditional variance of each series, their conditional covariance and a conditional correlation. For the S&P500-WTI pair the conditional covariance seems to be stable through time with only one major exception in 2009, when it tends to 0.002 and some other minors in 1992, 2011 and 2016. On the other hand, the Conditional Correlation is moving in a range from -0.74 to 0.8 and is, for most cases, below zero until 2009, while for the period 2009-2018 the relation changes to positive and the coefficient moves around 0.4. The comparison with the static correlation coefficient (dashed faded line) in figure 2.9 clearly shows that the static measure fails to incorporate

the negative relationship that exists in the first third of the sample, as well as, the stronger positive relationship that is evident in the later part. This means that based on the static measures would not take advantage of these dynamics in the hedging strategy and the poor interpretation could probably lead to even extra losses.

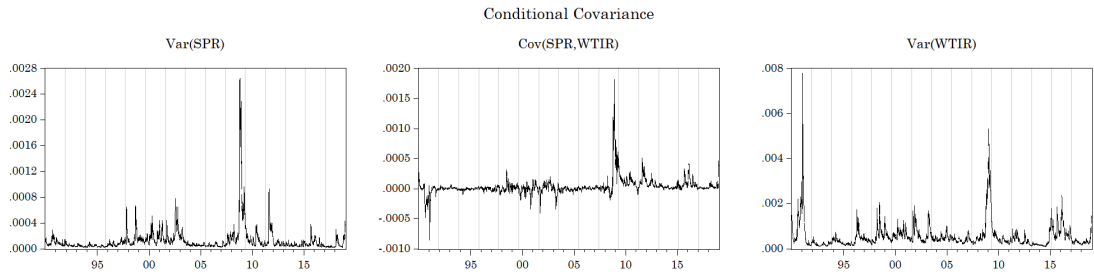


Figure 2.7: BEKK Conditional Covariance and Variance estimations S&P500-WTI

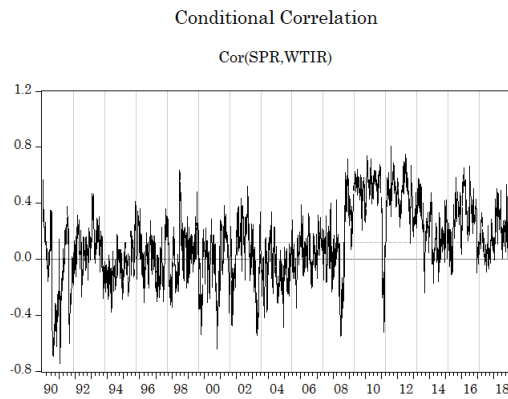


Figure 2.8: BEKK Conditional Correlation estimations S&P500-WTI

The dynamic hedge ratio obtained using the BEKK model takes values from around -0.4 to 0.8 positions of WTI futures for every single position taken in the spot index. It is once again of interest to compare the dynamic hedge ratio with the static one (black line in figure 2.9) that proposed holding less than 0.1 futures for every spot position.

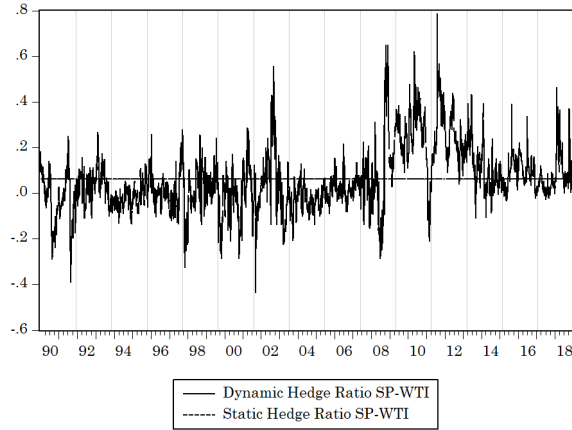


Figure 2.9: Static vs Dynamic Hedge Ratio with WTI futures

The negative values in the hedge ratio do not imply less efficiency for WTI as a hedger for that period, (this will be quantified later on with the use of the hedging effectiveness ratio) but instead they show how the inverted relation can be taken in advantage by changing the position on the future from short to long for that period. In order to be able to evaluate the effectiveness of the hedger, we first need to estimate the returns of the hedged portfolio employing the basis of spot minus futures returns and the estimated hedge ratio

$$R_{H,t} = R_{S,t} - h_{S,F,t}R_{F,t} \quad (2.23)$$

as indicated by Chang et al. (2011). Then, we calculate the conditional variance of the hedged and unhedged portfolio with a GARCH(1,1) model. The following formula quantifies the hedging effectiveness of a hedger based on variance reduction.

$$HE = \left[\frac{var_{unhedged} - var_{hedged}}{var_{unhedged}} \right] \quad (2.24)$$

The information from the future contract is incorporated into the hedged portfolio via the hedge ratio that is used in equation 2.23 and comes as a result of the BEKK model. Following these steps, figure 2.10 is obtained showing the hedging effectiveness of the oil futures. $HE \rightarrow 1$ is an indication of a more effective period.

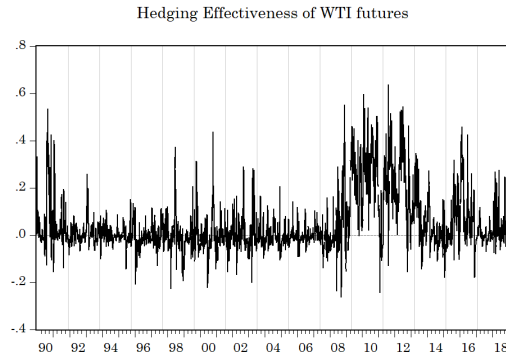


Figure 2.10: Hedging Effectiveness of WTI futures

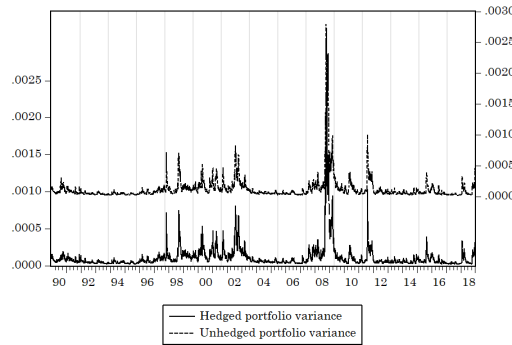


Figure 2.11: Hedged vs Unhedged Portfolio Variance S&P500-WTI

Considering that volatility is linked to information flow (Ross, 1989), we can assume that the amount of information from the futures markets, that is incorporated in the hedged portfolio variance, is increased after the financial crisis of 2007-2009. In an attempt to find which market events have an effect on the futures prices table 2.8 reports some events that affected the global markets and caused the returns of WTI futures to peak or plunge.

All of the shocks are related with major oil exporting countries and represent expectations that the supply flow of the oil will either be disrupted or suddenly increased. The variance of the hedged portfolio does not seem to be very reduced compared to the unhedged portfolio in the plot.

However, via hedging effectiveness, we can observe there is a sustained period of time during which WTI is more appropriate for hedging the S&P500 index. This period refers to the global financial crisis of 2007-2009 and the later recession 2009-2014 (Bureau of Economic Analysis). This period is also marked by increased volatility in the index as was previously shown, probably because of increased speculation, justifying why this specific period requires a hedging instrument to mitigate the turmoil. If next, the hedging effectiveness of the futures is compared with the variance of the hedged portfolio and the

Table 2.8: S&P500-WTI Hedge ratio extreme values and respective market events

Positive Peaks Dates	Event
April 2002	Venezuelan coup d'état attempt
July 2008	Global financial crisis
October 2008	
July 2010	Deepwater Horizon oil spill
May 2011	Serious Market Drop
Negative Bottoms Dates	Event
August 1991	Unsuccessful coup attempt against Soviet President Gorbachev.
November 1997	OPEC agrees to an increase in its production ceiling, to 27.5 million barrels per day.
September 2001	Major trading markets in the US, including the NYMEX, reopen for the first time since September 11.

crisis events, we can see that during periods that WTI has a higher hedging effectiveness, the variance of the hedged portfolio is very low and stable after S&P500 crises periods, as indicated in figure 2.12.

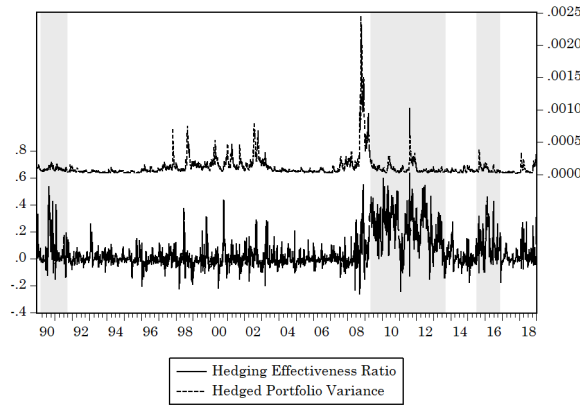


Figure 2.12: Hedging Effectiveness vs Hedged Portfolio Variance S&P500-WTI

These findings are consistent with the theory proposed by Andrew Lo (2004), namely the Adaptive Market Hypothesis (AMH). The theory is based on adaptivity as a characteristic of all living organisms in biology. According to this theory, market participants and therefore their actions as a whole, are not rational with the strict sense of the term. The investors instead base their investment decisions on heuristic rules that come as a result of adapting to their continuously changing environment. This seems to be true in our case as well, as the hedging effectiveness of the WTI contracts is significantly increased after market changing events.

The results are also confirmed in the case where we examine the same models in a

sub-sample that excludes the period after financial crisis, where the need for hedging was found to be more requisite, and leaves the time period from 1990 to 2006. The hedging effectiveness ratio moves in lower values that exceed the 0.4 threshold only once.

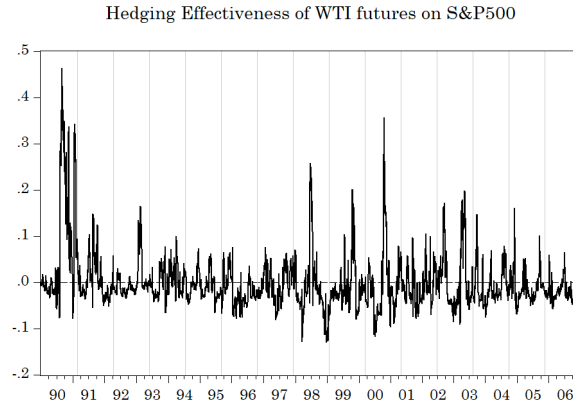


Figure 2.13: Hedging Effectiveness of WTI futures in sub-sample

S&P500 & GCS futures

Table 2.9: BEKK estimation coefficients for S&P500-GCS

Constants	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.000584	9.16e-05	6.376340	0.0000
C(2)	9.11e-06	9.11e-05	0.100007	0.9203
Equation Estimated				
GARCH=M+A1A1(RESD(-1)^2)+B1B1GARCH(-1)				
Variance Equation Coefficients				
M(1,1)	1.36e-06	1.18e-07	11.50310	0.0000
M(1,2)	-8.17e-08	3.92e-08	-2.086647	0.0369
M(2,2)	2.16e-07	3.14e-08	6.864373	0.0000
A1(1,1)	0.291883	0.007266	40.16872	0.0000
A1(2,2)	0.180387	0.002410	74.86404	0.0000
B1(1,1)	0.950771	0.002471	384.7236	0.0000
B1(2,2)	0.983372	0.000400	2457.867	0.0000
Log likelihood	48119.00	Schwarz IC		-13.15970
Avg. log likelihood	3.292665	Hannan-Quinn IC		-13.16527
Akaike IC	-13.16820			

Once more, the sufficient condition for positive definiteness of H_t is true as the diagonal elements of M, A, B matrices are strictly positive ($M(1, 1) = 1.36e - 06, M(2, 2) = 2.16e - 07$). Following the same procedure for the Gold futures, we first estimate the BEKK conditional measures, the hedged portfolio returns and variance and then the optimal hedge ratio. In this example, the results differ as the covariance between the two assets is lower

and the conditional correlation much more unstable. Compared to WTI the conditional variance of GCS has more extreme values, as the variance surpasses both the 0.0002 and 0.0004 threshold in more occasions, and the conditional covariance shows multiple jumps most of which are negative.

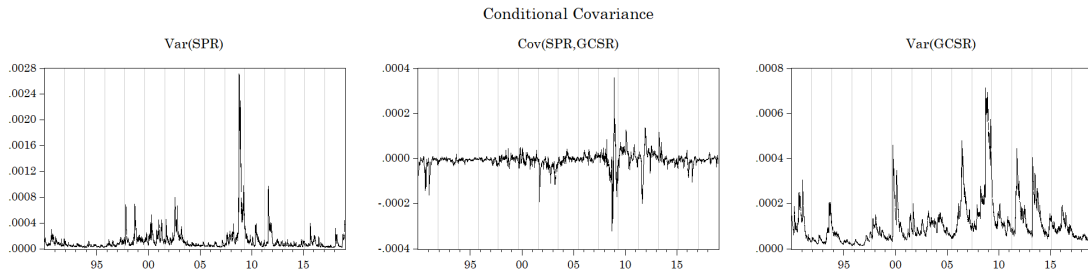


Figure 2.14: BEKK Conditional Covariance and Variance estimations S&P500-GCS

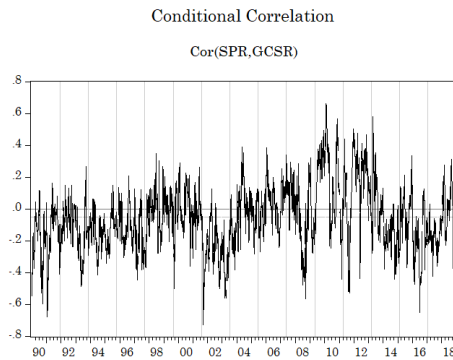


Figure 2.15: BEKK Conditional Correlation estimations S&P500-GCS

The conditional correlation between S&P500 and GCS also shows how the static measure (dashed, faded line) fails to entail some periods of stronger negative correlation, that reaches to -0.73, but also some periods that the relation changes to a positive one. The peak in positive relationship is observed just after the financial crisis on 5th February 2010, approaching 0.66 but it is not sustained as it falls to negative values in just 3 months later. The aforementioned results would make us expect that the more weak relationship will lead to lower hedging effectiveness for the certain asset. The hedge ratio in this case, as presented in figure 2.16, continuously moves around zero and gets bottom values after crises events. In our sample these moments arise after the dot-com bubble, around the end of 2002, and after the financial crisis of 2007-2009, around the middle of 2011. This means that gold returns are significantly more volatile posterior to financial crises.

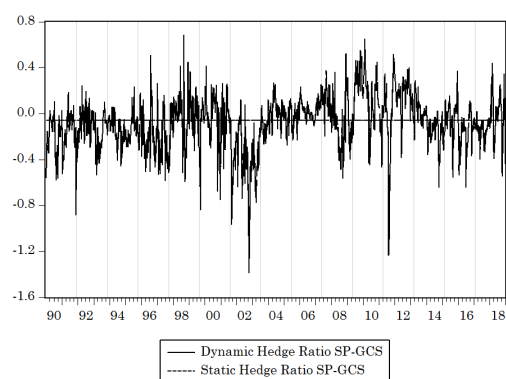


Figure 2.16: Static vs Dynamic Hedge Ratio with GCS futures

Table 2.10: S&P500-GCS Hedge ratio extreme values and respective market events

Positive Peaks Dates	Event
February 1998	Concern that uncoordinated central bank gold sales had destabilized the gold market.
February 2009	Global financial crisis. Gold rises back above \$1,000 an ounce to a peak of \$1,005.40 during the financial crisis.
May 2010	Fears over the contagion of debt problems in the Eurozone fuel safe-haven buying.
Negative Bottoms Dates	Event
September 2001	Major trading markets in the US, including NYMEX, reopen for the first time since September 11.
September 2011	The “August 2011 stock markets fall”

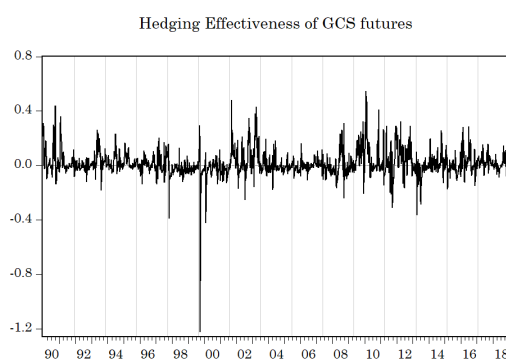


Figure 2.17: Hedging Effectiveness of GCS futures

Indeed, the hedging effectiveness fluctuates mostly around zero, having a mean of 0.013 and a median of -0.0013. Except one sole occasion, that of 5th February 2010, where the

effectiveness peaked 0.54, the ratio strongly indicated that the asset is not effective in hedging the S&P500. Even though the conditional variance of the hedged portfolio with GCS futures seems to reduce the variance of the initial position in S&P500, as indicated in figure 2.18, the conditional variance of the portfolio seems to be uncorrelated with the hedging effectiveness ratio of the same asset, indicating that the hedger is not responsible for the periods where the risk is reduced.

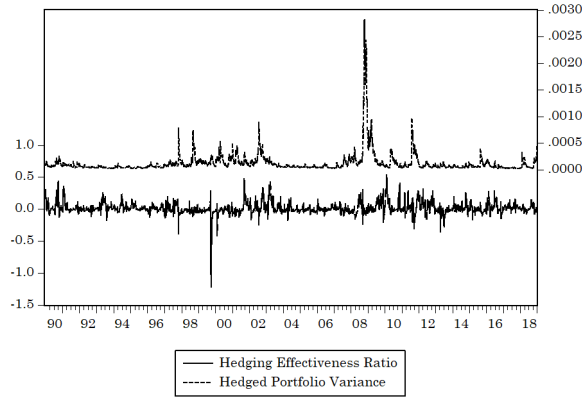


Figure 2.18: Hedging Effectiveness vs Hedged Portfolio Variance S&P500-GCS

The results are also confirmed when tested in the same sub-sample of 1990-2006. The properties of GCS, that is found not to be a good hedger. The Hedging Effectiveness ratio passes the 0.2 threshold only a few times, with most of the values around zero.

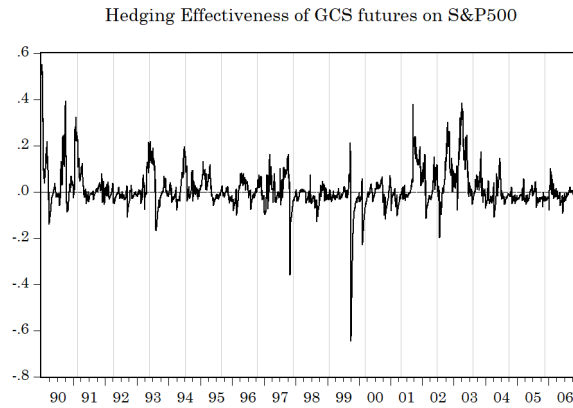


Figure 2.19: Hedging Effectiveness of GCS futures in sub-sample

Methodological comparison

After having tested those two contracts, we are able to compare the results based on the methodology used. We estimate a rolling hedge ratio with 25-day step, so that the hedging

decision is adjusted each trading month. We then proceed with a comparison of the hedging effectiveness in two different scenarios:

- Dynamic Conditional Hedge Ratio vs Unconditional (Static) Hedge Ratio
- Dynamic Conditional Hedge Ratio vs Rolling Hedge Ratio

The conditional hedge ratio as we showed is a result of bivariate BEKK variance-covariance estimations, the static hedge ratio is the beta coefficient of an OLS regression between the two variables of each pair, while the rolling hedge ratio is a result of a moving beta coefficient estimated from moving variance and moving covariance. The hedge ratio is multiplied by the returns of the hedging contract and their product is then subtracted from the initial S&P500 return (that is considered as the unhedged portfolio). The obtained series leads to the returns of the hedged portfolio. The time-varying variance of the hedged portfolio is taken using a GARCH(1,1) process in all cases, to produce the variance of the hedged portfolio. The variance of the unhedged portfolio is similarly estimated from a GRACH(1,1) process of S&P500 returns.

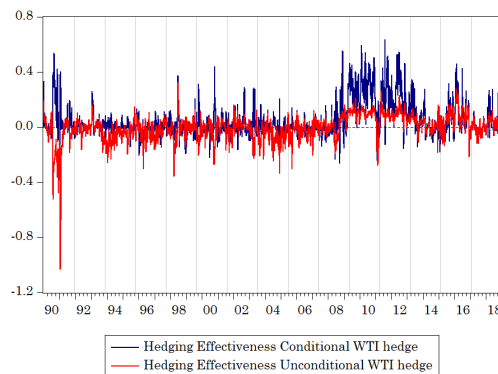


Figure 2.20: Hedging Effectiveness Conditional vs Unconditional WTI Hedge

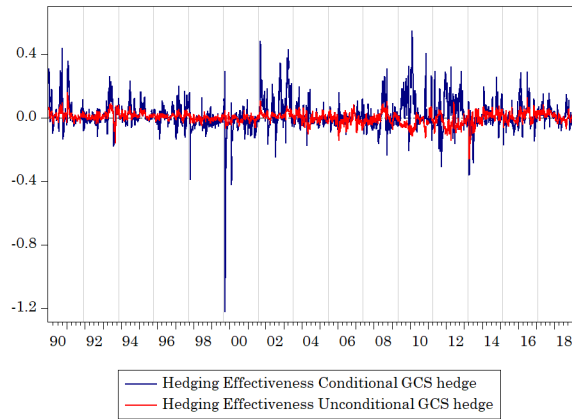


Figure 2.21: Hedging Effectiveness Conditional vs Unconditional GCS Hedge

What we observe once more, though formally quantified this time, is that the dynamic conditional hedge outperforms the static one for both assets. The hedging effectiveness of the static hedge ratio for WTI futures never exceeds 0.33 and is generally moving around zero. This effectiveness has also a skewness of -2, compared to 1.6 of the dynamic, implying that most of its values are negative. Similar results are found in the comparison of the two ratios for GCS futures as well. The hedging effectiveness of the static hedge never exceeds 0.16, while the dynamic reaches 0.54. The effectiveness of the static is more negatively skewed and has a kurtosis of 5.62 compared to 24.74, implying that most of its values are found close to the mean which in this case is almost zero. The descriptive statistics of the hedging effectiveness for each methodology can be found in the Appendix. It can be seen graphically that the conditional hedge is generally more effective than the unconditional one for the entire sample, but this is even more evident in incidents of financial crises when the amount of information that needs to be incorporated is increased. The unconditional model also fails to take advantage of the negative relationship between WTI and S&P500 in the end of 1990, when it is less effective.

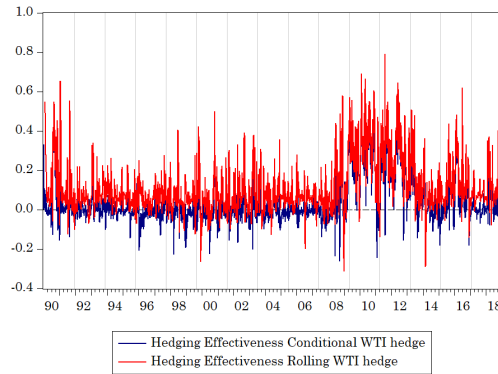


Figure 2.22: Hedging Effectiveness Dynamic Conditional vs Rolling WTI Hedge

We proceed with the comparison of the conditional model to the other alternative methodology, namely the rolling hedge. In this case however, the results are not so evident. In figure 2.22 it can be seen that the hedging effectiveness of the rolling hedge is higher than that of the conditional one, especially for the less volatile period of 1992 to 2008. During the 2007-2014 financial crises the two measures seem to be equivalent, but this changes again following 2014. With the exception of periods with high information flow, the rolling hedge seems to outperform the conditional one. However, if we compare the difference of the hedging effectiveness of the two methodologies we will realize that their difference is probably not significant. We therefore define the difference of the hedging effectiveness as:

$$d = he.conditional - he.rolling$$

Where the he denotes the hedging effectiveness ratio of the hedged portfolio will be increasing estimated with each method. When d takes positive values, it implies that the conditional hedge is more effective. In figure 2.23, the difference is of small scale as the mean is around zero and that the conditional model outperforms the rolling only in cases of demand shocks in 1999, 2008 and 2014.

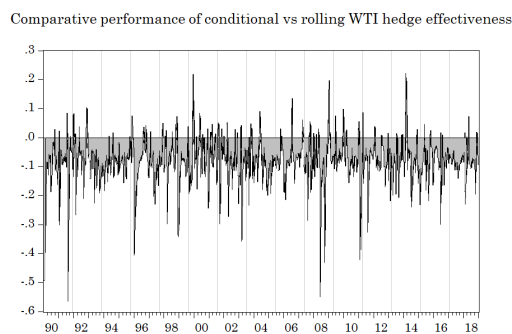


Figure 2.23: Difference in Hedging Effectiveness Dynamic Conditional vs Rolling WTI Hedge

Similar results are obtained for the GCS hedge. As it can be seen in figure 2.24, the hedging effectiveness of the rolling hedge is almost always slightly higher than the conditional one with the later having some negative spikes.

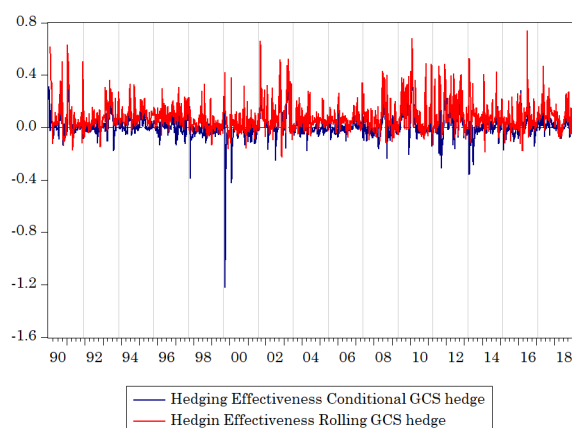


Figure 2.24: Hedging Effectiveness Dynamic Conditional vs Rolling GCS Hedge

We reestimate the difference of the two hedging effectiveness measures and plot the resulting series in figure 2.25. Their difference seems not to be significant for most of the sample, however there are incidents where the rolling hedge is much more efficient. These cases can be seen as the extreme negative spikes that exceed -0.4 and occurred in 1990, around the dot com bubble in 2000 and during the 2014 oil price drop.

Comparative performance of conditional vs rolling GCS hedge effectiveness

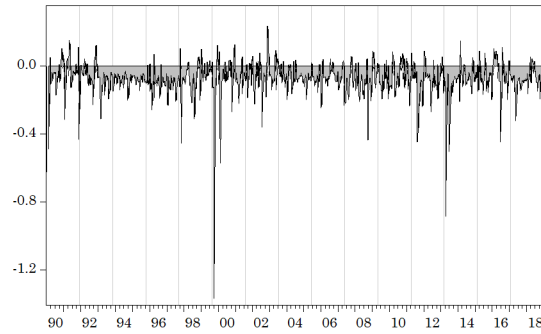


Figure 2.25: Difference in Hedging Effectiveness Dynamic Conditional vs Rolling GCS Hedge

These observations lead us to the conclusion that the conditional hedge clearly outperforms the unconditional one, and that the rolling hedge in some cases might outperform the conditional. This probably refers to events, when there is no considerable information flow affecting the cross sectional relationship between the spot and the futures market, and the simple adaptation based on their past values is sufficient to hedge the portfolio.

Eurostoxx & WTI, GCS futures

We move on by testing the hedging effectiveness of the two futures contracts on indices that are negotiated in other stock markets. In such case, the cross section relationships will either be altered or there might be evidence of volatility spillover from one stock market to another. More specifically, we test if there are information flows from the US futures market into spot markets around the globe. In this section, we present only the plots of the series obtained from the BEKK estimations, while the estimated coefficients can be found in Appendix.

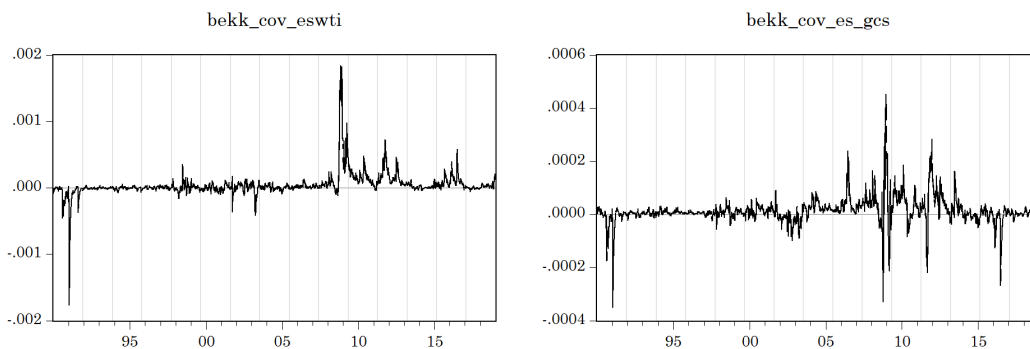


Figure 2.26: BEKK result Covariance between ES-WTI (left) and ES-GCS (right)

Starting with the European index Eurostoxx, the estimated covariances presented in figure 2.26 imply that there is little, if no significant covariance between the European index and the futures. The covariance moves around zero for both pairs and is lower in the case of GCS. The scale of the covariance is below 0.1 during the entire period examined, and even though both futures display some spikes in their covariance with the index, it is still not significant as it nearly passes 0.02.

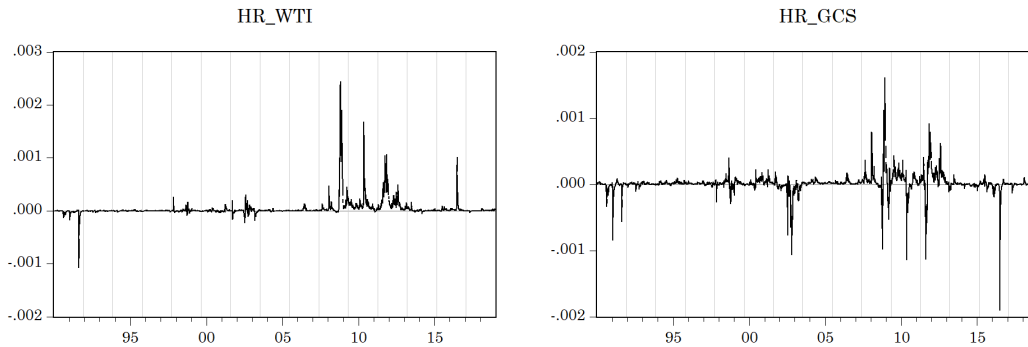


Figure 2.27: BEKK dynamic hedge ratio between ES-WTI (left) and ES-GCS (right)

Considering the absence of a strong relationship in both pairs, we expect that the hedge ratio will not have significant values. This is indeed found in figure 2.27, where the hedge ratio cannot exceed 0.003.

The weakness of the relationships is again confirmed by the hedging effectiveness ratio, getting values that are closer to zero than to the unit that would indicate a good hedge. Despite the three positive peaks in WTI hedging effectiveness during 2007-2014 and three negative bottoms in GCS the ratios are moving around zero and indicate that the assets do not constitute suitable hedgers for this index. One possible reason, might be the fact that European firms listed in Eurostoxx are not dependent on the US oil exporting markets and have stronger links to BRENT oil products. As for gold futures, the result may be expected as we previously found that they are relatively not effective even in hedging the US index.

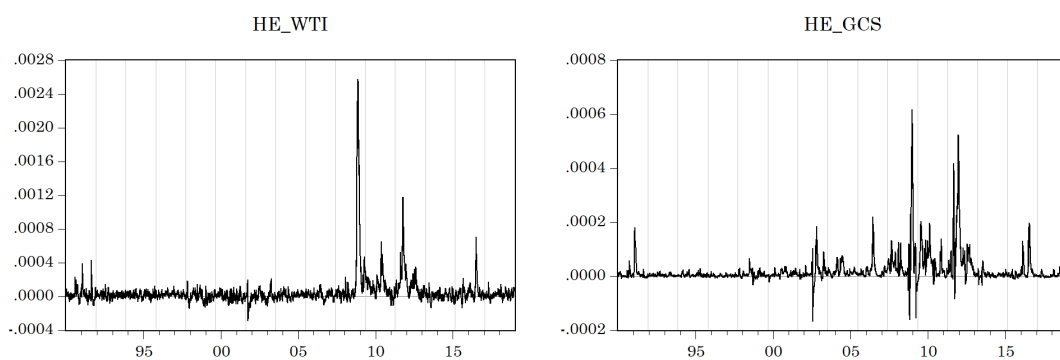


Figure 2.28: Hedging Effectiveness of WTI (left) and GCS (right) on Eurostoxx index

Nikkei & WTI, GCS futures

In this section we test the hedging effectiveness of the US oil and gold futures for Nikkei 250 index. As it can be seen in figure 2.30 the results from the BEKK estimations are similar to those of the European index. The obtained covariances are not significant in both cases, and despite one negative shock in 1991 and a positive in 2008, the covariances are generally moving around zero. The scale of the measures does not pass 0.0025 in WTI and 0.0006 in GCS, implying that there is no significant relationship between the US futures and the Japanese stock index.

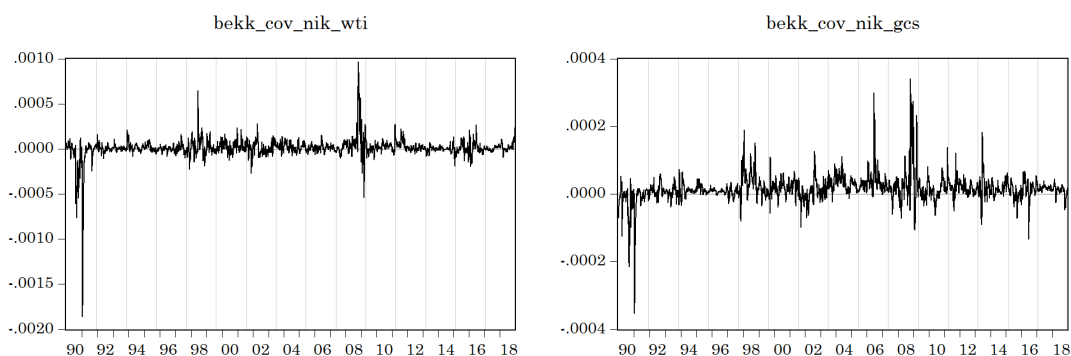


Figure 2.29: BEKK estimated Covariance between NIK-WTI (left) and NIK-GCS (right)

These low covariances lead to similarly low hedge ratios as they can be seen in figure 2.30. The absence of a durable and significant relationship in the pairs gives hedge ratios that equal zero and have only few shocks that are short in duration and trivial in impact. More specifically, the peaks do not surpass 0.0015 in WTI and 0.005 in GCS hedge ratio.

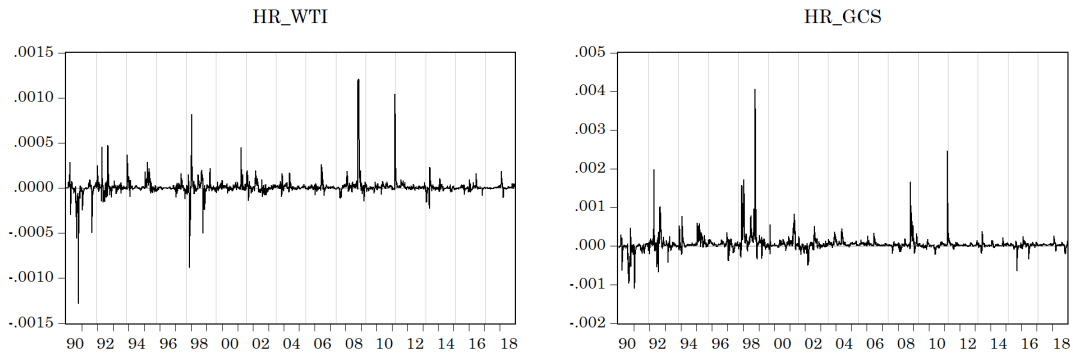


Figure 2.30: BEKK dynamic hedge ratio between NIK-WTI (left) and NIK-GCS (right)

In a similar way, the hedging effectiveness of both pairs is moving around zero as the hedged portfolio returns are not very different to those of the initial investment in solely the Nikkei index. The estimates clearly indicate that the Japanese spot markets do not significantly receive information flows from US futures markets. It is more plausible that investors acting in Japan prefer to trade on futures markets that are geographically closer, while at the same time Japan is not dependent on oil imports from US. This finding is specifically in line with the phenomenon described as “home equity bias” by Coval and Moskowitz (1999). It is argued that investors and investment funds tend to be biased towards asset proximity when issues such as information asymmetry arise (Gehrig, 1999; Brennan and Cao, 1997; Coval and Markowitz, 2001).

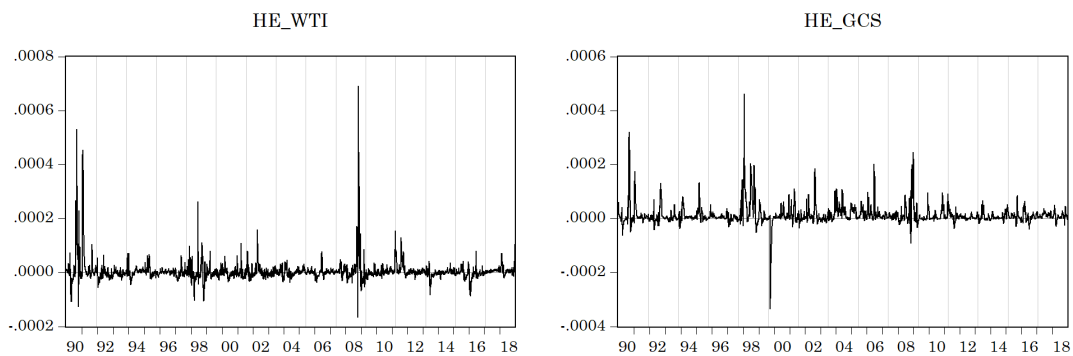


Figure 2.31: Hedging Effectiveness of WTI (left) and GCS (right) on Nikkei index

Shanghai Composite index & WTI, GCS futures

The last stock market is the Chinese Shanghai Stock Exchange. This time the sample is smaller starting from 1992. There is still a special regime as both private and public companies are listed on it. The trading of the index was much volatile during the first years but later fell in very bear periods from 1994 to 2006 and 2009 to 2014. The BEKK

estimated covariances are again very low with a mean close to zero. The scale is lower than 0.001 for WTI and 0.0002 for GCS as well.

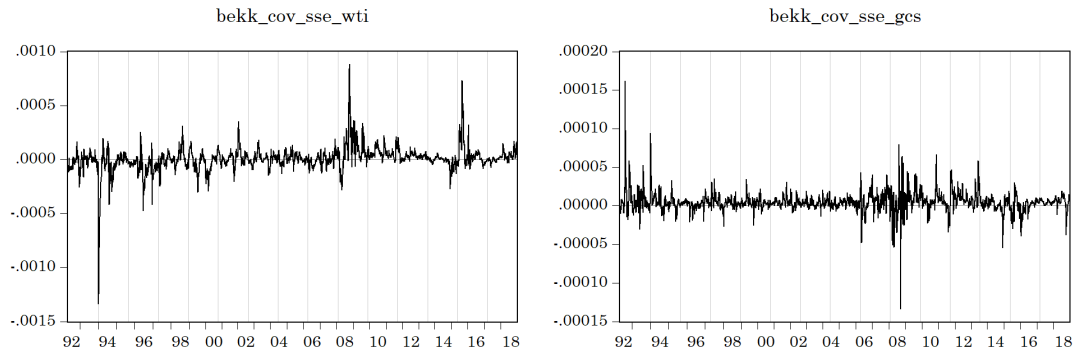


Figure 2.32: BEKK result Covariance between SSE-WTI (left) and SSE-GCS (right)

The obtained hedge ratio for WTI hedge approaches one in 1993 and 2014 but has three negative bottoms that reach -3 in the period 1992-1994. The GCS hedge is close to the unit more than four times and even reaches the value of 2 during the same period of 1992-1994. However, during the period 1994-2018 the hedge ratio of both assets is moving around zero.

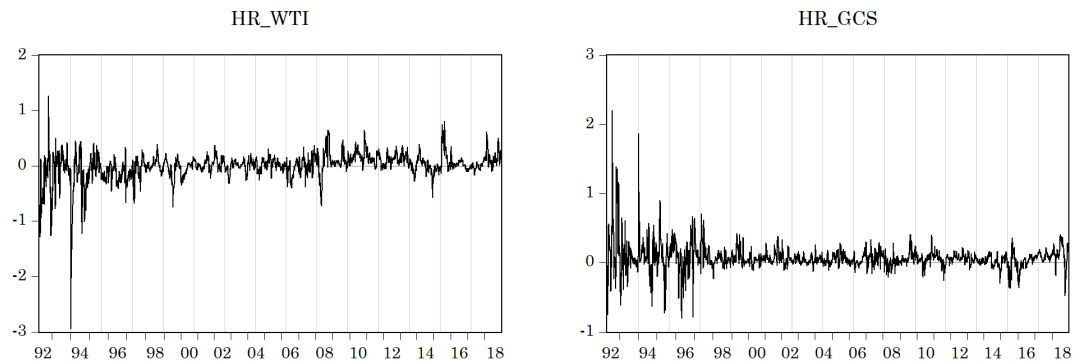


Figure 2.33: BEKK dynamic hedge ratio between SSE-WTI (left) and SSE-GCS (right)

The hedging effectiveness of the futures is more significant on the Chinese index compared to the European and Japanese, with a scale from -0.4 to 0.4 for WTI and -0.1 to 0.2 for GCS. However, even the highest values are not persistent as they last for only few days and the shocks do not seem to be caused by the same factors.

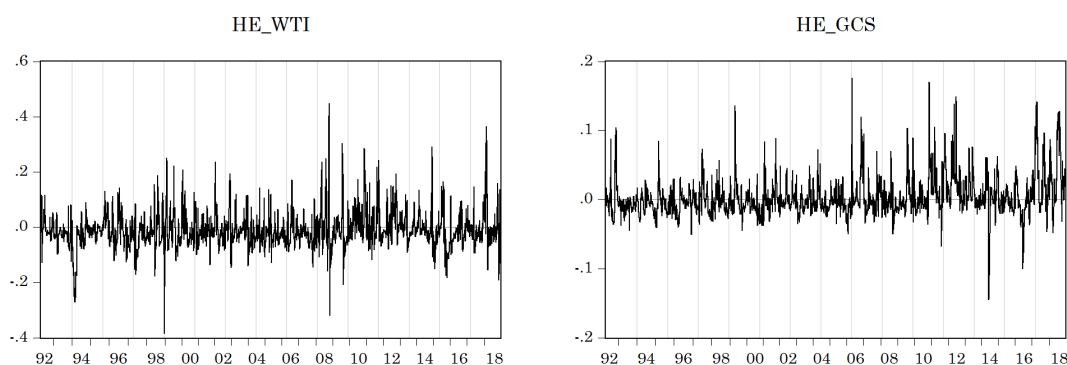


Figure 2.34: Hedging Effectiveness of WTI (left) and GCS (right) on Shanghai Composite index

2.4 Discussion

Our first finding concerns volatility clustering in all series except the SSE. This stylized fact was confirmed in our sample and therefore a method that models volatility seem to be appropriate.

In a second stage, we compared the hedging effectiveness of the futures using both conditional and unconditional models, with the former being superior. The dynamic modeling of the series in the case of the conditional model can better incorporate information from the futures market to the spot equity indices and take advantage of the moving relationship that exists between the assets. On the other side, the unconditional model fails to take into account the moving relationships and misses periods that the relationship is inverted.

Furthermore, the superiority of the conditional model is notably increased during crises episodes such as the dot com bubble, the 2007-2009 financial crisis, as well as oil price shocks. This can be explained by the fact that during such periods the volume of trades and information increases significantly. The conditional model that uses the information from the futures market as input is even more effective, when there are considerably more information flows.

Regarding the evaluation of the contracts, the WTI futures were proved to hedge better in the S&P500 index, compared to GCS gold futures. One of the reasons might be the properties that the assets obey. Oil futures are one of the most volatile futures contracts, while gold is considered a safe haven and is generally less fluctuating. This behavior of the assets generates respective amounts of information and affects the relationships between the assets. Another reason for the WTI-S&P500 pair might be the fact that many oil companies are listed in the S&P500. According to Factset, among the companies listed in S&P500, 5% are oil and gas refiners, while others concern large capitalization firms such as Exxon Mobil, Phillips 66, Marathon Petroleum and so on. Therefore, the prices of the

WTI futures play a significant role in their revenue and operational activity.

Finally, we provided evidence that the US traded futures are not effective hedgers for equity indices, traded in other countries. Their covariance and hedging effectiveness was insignificant for Europe and Japan indices and even in the case of China, the results were not persistent. It seems that “home equity bias” prevails and investors in the various stock exchanges seek to hedge their positions using products from markets close to them. Furthermore, prices of assets that are negotiated on the same market often show similar patterns as a response to news that are important for the market as a whole (Hafner and Herwartz, 1998). For that reason the two futures can effectively hedge only the US index.

Conclusion

In this work we tried to assess and evaluate the ability of financial derivatives to hedge equity investments. We were primarily interested in finding how futures contracts can be used to hedge diversified portfolios in the form of equity indices. We specifically examined the effectiveness of WTI oil and GCS gold futures to hedge stock market indices of major stock markets such as the S&P500, Eurostoxx, Nikkei 250 and the Shanghai Stock Exchange. In this aim, we first reviewed the literature and then proceeded to the empirical application of econometric models on historical data.

In chapter one, we investigated the existing bibliography in order to find how the academic community has dealt with the question of hedging with financial derivatives. In first place, we found that futures have many properties that allow them to be more flexible and convenient for hedging compared to other instruments such as forwards or options. Second, there is plenty of literature on how the optimal hedge ratio is defined. Among the multiple optimum conditions, the minimum variance criterion is the most common, but the optimal hedge ratio should be selected based on the investor's preferences. Last, it arises that the most appropriate method for estimating optimal hedge ratio is to take into account the time-varying variance with ARCH-type models.

In the empirical application, evidence is provided about volatility clustering in returns series, while bivariate diagonal BEKK GARCH model is applied prior to the computation of dynamic optimal hedge ratio. Estimation results indicate that the conditional models are notably more efficient during periods of economic crises, when the incorporated information flows are increased. Furthermore, the WTI oil futures turned out to be more effective instruments in hedging the S&P500 index, compared to the GCS gold futures probably due to the individual characteristics of each asset. Finally, none of the contracts was proved to be significantly efficient in hedging the equity indices of other stock markets, as a result of the weak relationship between the US traded assets and the foreign exchanges.

Hedging can significantly reduce the risk of a diversified portfolio, such as an equity index, when the underlying assets are negotiated in markets presenting strong linkages and sharing common channels. As financial interdependence enhances and contagion incites volatility spillovers, the need for hedging increases. Investors should therefore be aware of the information flows as well as the continuously changing cross relationships between

their assets, and consider them in their investment decisions.

Appendix

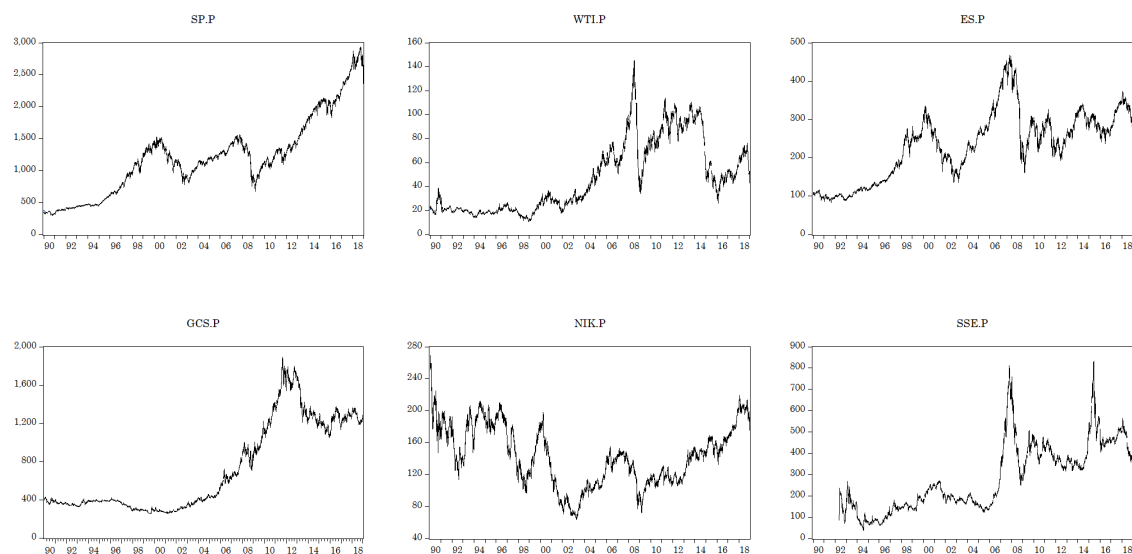


Figure 2.35: Plot of all the series used

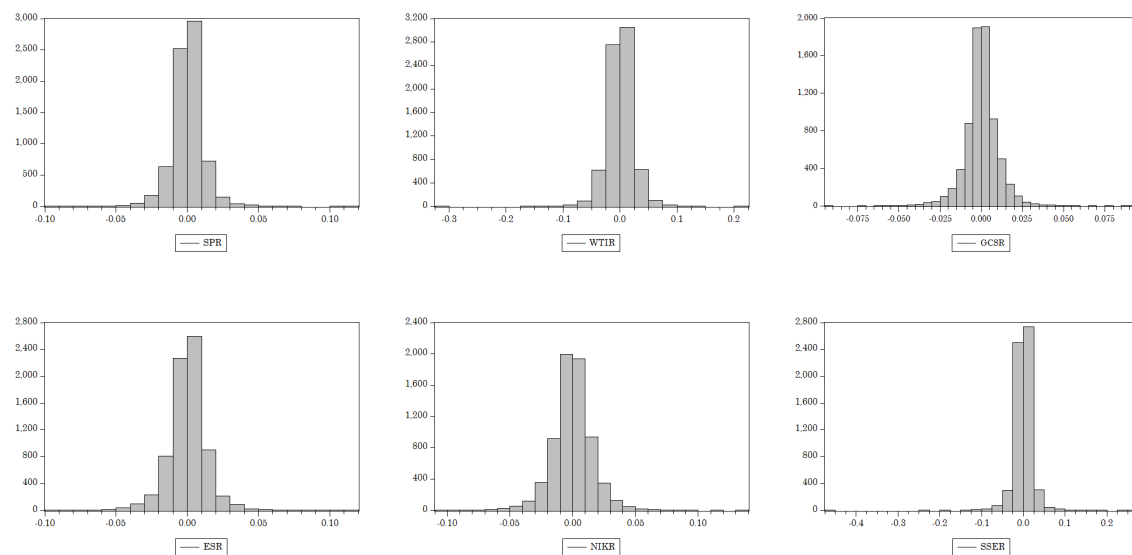


Figure 2.36: Histograms of all the returns for all series

Table 2.11: Unit Root Test

Null Hypothesis: SPR has a unit root			Null Hypothesis: WTIR has a unit root		
Exogenous: Constant			Exogenous: Constant		
Lag Length: 33 (Automatic - based on AIC, maxlag=35)			Lag Length: 14 (Automatic - based on AIC, maxlag=35)		
	t-statistic	Prob.		t-statistic	Prob.
Augmented DF statistic	-15.39056	0.0000	Augmented DF statistic	-20.75847	0.0000
Test critical values	1% level	-3.431072	Test critical values	1% level	-3.431072
	5% level	-2.861743		5% level	-2.861743
	10% level	-2.566920		10% level	-2.566920

Null Hypothesis: GCSR has a unit root			Null Hypothesis: RESID01SP_WTI has a unit root		
Exogenous: Constant			Exogenous: Constant		
Lag Length: 11 (Automatic - based on AIC, maxlag=35)			Lag Length: 34 (Automatic - based on AIC, maxlag=35)		
	t-statistic	Prob.		t-statistic	Prob.
Augmented DF statistic	-26.18611	0.0000	Augmented DF statistic	0.028010	0.9600
Test critical values	1% level	-3.431072	Test critical values	1% level	-3.431072
	5% level	-2.861743		5% level	-2.861743
	10% level	-2.566920		10% level	-2.566920

Null Hypothesis: RESID02SP_GCS has a unit root		
Exogenous: Constant		
Lag Length: 17 (Automatic - based on AIC, maxlag=35)		
	t-statistic	Prob.
Augmented DF statistic	-1.098422	0.7187
Test critical values	1% level	-3.431072
	5% level	-2.861743
	10% level	-2.566920

Table 2.12: BEKK estimation coefficients for ES-WTI

Constants	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.000425	0.000114	3.723635	0.0002
C(2)	0.000371	0.000197	1.885377	0.0594
Equation Estimated				
$GARCH=M+A1A1(RESD(-1)^2)+B1B1GARCH(-1)$				
Variance Equation Coefficients				
M(1,1)	1.74e-06	1.05e-07	11.72341	0.0000
M(1,2)	-4.02e-08	1.03e-07	2.326808	0.6839
M(2,2)	3.50e-06	4.43e-07	6.758067	0.0000
A1(1,1)	0.253339	0.005409	46.83565	0.0000
A1(2,2)	0.228789	0.005782	39.56932	0.0000
B1(1,1)	0.962500	0.001689	569.9340	0.0000
B1(2,2)	0.970708	0.001631	595.2067	0.0000
Log likelihood	40634.47	Schwarz IC		-11.11111
Avg. log likelihood	2.780516	Hannan-Quinn IC		-11.11668
Akaike IC	-11.11960			

Table 2.13: BEKK estimation coefficients for ES-GCS

Constants	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.00448	0.000114	3.932495	0.0001
C(2)	3.49e-05	9.00e-05	0.387914	0.6981
Equation Estimated				
$GARCH=M+A1A1(RESD(-1)^2)+B1B1GARCH(-1)$				
Variance Equation Coefficients				
M(1,1)	2.26e-06	2.03e-07	11.16724	0.0000
M(1,2)	2.03e-07	5.38e-08	3.773616	0.0200
M(2,2)	2.24e-07	3.33e-08	6.729897	0.0000
A1(1,1)	0.292377	0.007363	39.71072	0.0000
A1(2,2)	0.184253	0.002501	73.67390	0.0000
B1(1,1)	0.950447	0.002364	402.0610	0.0000
B1(2,2)	0.982634	0.000423	2321.726	0.0000
Log likelihood	46501.73	Schwarz IC		-12.71704
Avg. log likelihood	3.181999	Hannan-Quinn IC		-12.72261
Akaike IC	-12.72553			

Table 2.14: BEKK estimation coefficients for NIK-WTI

Constants	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.000323	0.000151	2.137805	0.0325
C(2)	0.000366	0.000209	1.747111	0.0806
Equation Estimated				
$GARCH=M+A1A1(RESD(-1)^2)+B1B1GARCH(-1)$				
Variance Equation Coefficients				
M(1,1)	5.40e-06	4.92e-07	10.96500	0.0000
M(1,2)	2.05e-07	2.12e-07	0.966465	0.3338
M(2,2)	3.56e-06	5.41e-07	6.580694	0.0000
A1(1,1)	0.295467	0.007924	37.28639	0.0000
A1(2,2)	0.227875	0.006109	37.29959	0.0000
B1(1,1)	0.945044	0.003073	307.4850	0.0000
B1(2,2)	0.970932	0.001755	553.2529	0.0000
Log likelihood	36701.31	Schwarz IC		-10.62499
Avg. log likelihood	2.659130	Hannan-Quinn IC		-10.63083
Akaike IC	-10.63391			

Table 2.15: BEKK estimation coefficients for NIK-GCS

Constants	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.000286	0.00015	1.909935	0.0561
C(2)	0.0000351	0.0000934	0.3762	0.7068
Equation Estimated				
$GARCH=M+A1A1(RESD(-1)^2)+B1B1GARCH(-1)$				
Variance Equation Coefficients				
M(1,1)	0.00000583	0.000000512	11.38872	0.0000
M(1,2)	0.000000401	8.73e-08	4.597355	0.0000
M(2,2)	0.000000217	3.49e-08	6.236599	0.0000
A1(1,1)	0.318348	0.00861	36.97461	0.0000
A1(2,2)	0.183607	0.00258	71.16196	0.0000
B1(1,1)	0.937162	0.003395	276.0775	0.0000
B1(2,2)	0.982842	0.000454	2163.883	0.0000
Log likelihood	42272.62	Schwarz IC		-12.23963
Avg. log likelihood	3.06279	Hannan-Quinn IC		-12.24547
Akaike IC	-12.24855			

Table 2.16: BEKK estimation coefficients for SSE-WTI

Constants	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.000364	0.000158	2.303784	0.0212
C(2)	0.000501	0.000220	2.281967	0.0225
Equation Estimated				
$GARCH=M+A1A1(RESD(-1)^2)+B1B1GARCH(-1)$				
Variance Equation Coefficients				
M(1,1)	5.73E-07	1.05E-07	5.437387	0.0000
M(1,2)	1.30E-07	1.36E-07	0.957427	0.3384
M(2,2)	2.83E-06	4.59E-07	6.169694	0.0000
A1(1,1)	0.219645	0.002379	92.30814	0.0000
A1(2,2)	0.211777	0.006290	33.66872	0.0000
B1(1,1)	0.977473	0.000360	2715.159	0.0000
B1(2,2)	0.975227	0.001578	617.8846	0.0000
Log likelihood	32498.52	Schwarz IC		-10.20727
Avg. log likelihood	2.554915	Hannan-Quinn IC		-10.21352
Akaike IC	-10.21683			

Table 2.17: BEKK estimation coefficients for SSE-GCS

Constants	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.000391	0.000158	2.472658	0.0134
C(2)	7.86e-05	9.67e-05	0.813307	0.4160
Equation Estimated				
$GARCH=M+A1A1(RESD(-1)^2)+B1B1GARCH(-1)$				
Variance Equation Coefficients				
M(1,1)	6.06Ee-07	1.13e-07	5.384389	0.0000
M(1,2)	-1.30e-08	5.33e-08	-0.243670	0.8075
M(2,2)	3.07e-07	4.57e-08	6.709757	0.0000
A1(1,1)	0.228987	0.002373	96.51656	0.0000
A1(2,2)	0.194776	0.002929	66.50367	0.0000
B1(1,1)	0.975771	0.000370	2639.996	0.0000
B1(2,2)	0.980332	0.000622	1575.382	0.0000
Log likelihood	37605.97	Schwarz IC		-11.81338
Avg. log likelihood	2.956444	Hannan-Quinn IC		-11.81964
Akaike IC	-11.82295			

Table 2.18: Descriptive Statistics of the Hedging Effectiveness on S&P500

S&P500	WTI Hedge		GCS Hedge	
	HE Dynamic	HE Static	HE Dynamic	HE Static
Mean	0.04587	-0.003843	0.013883	0.000593
Median	0.001928	0.003060	-0.001355	0.004025
Maximum	0.636564	0.323490	0.547442	0.158938
Minimum	-0.262628	-1.033019	-1.220525	-0.259102
Std. Dev.	0.121170	0.094470	0.090116	0.034187
Skewness	1.618164	-2.001435	-0.670190	-0.730970
Kurtosis	5.523380	18.43272	24.74419	5.626608

Table 2.19: Descriptive Statistics of the Hedging Effectiveness on other indices

ES	HE_WTI	HE_GCS	NIK	HE_WTI	HE_GCS
Mean	4.84E-05	2.02E-05	Mean	3.98E-06	1.31E-05
Median	1.89E-05	5.72E-06	Median	-1.54E-07	4.62E-06
Maximum	0.002577	0.000617	Maximum	0.000691	0.000462
Minimum	-0.000291	-0.000167	Minimum	-0.000167	-0.000335
Std. Dev.	0.000187	5.27E-05	Std. Dev.	3.77E-05	3.66E-05
Skewness	8.370903	4.893732	Skewness	6.587771	2.656271
Kurtosis	92.44944	38.45736	Kurtosis	73.66869	30.35801

SSE	HE_WTI	HE_GCS
Mean	-0.010981	0.002959
Median	-0.019111	-0.002702
Maximum	0.447761	0.175562
Minimum	-0.385510	-0.145196
Std. Dev.	0.066953	0.027723
Skewness	1.009614	1.534687
Kurtosis	8.412786	8.948987

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