UNIVERSITY OF MACEDONIA

DEPARTMENT OF ECONOMICS

EXAMINING THE CONCEPT OF MONEY NEUTRALITY IN TWO ADVANCED ECONOMIES

BY ANTONIOS GKIATAS

SUPERVISOR: PROFESSOR STYLIANOS FOUNTAS

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Abstract

The neutrality of money is an economic theory that states that changes in the money supply only affect nominal variables and not real variables in the long run. The term «neutrality of money» was first introduced by acclaimed economist and Nobel Prize winner Friedrich Hayek and was a result of conversation and debate that dates back to the 18th century. Since then it has become a staple idea for classical economists and it has influenced monetary policies all around the world.

The purpose of this paper is to examine the basic idea of money neutrality in the long run and the short run in two major economies (USA and Canada). In order to do so we have used the Johansen co-integration test, Error correction models, VAR models and four time series (Real Gross Domestic Product, M1 money supply, M2 money supply and the Consumer's Price index)for each of these two economies. Our results confirm the neutrality proposition in the long run but not in the short run.

CONTENTS

1. Introduction

1.1 Government policies and Money Neutrality

There are two types of policies a government can use to impact the local economy: fiscal and monetary policy. Fiscal policy is the use of government revenue collection (taxes) and expenditure (spending) to influence the economy. A change in fiscal policies can influence economic variables such as the aggregate demand, savings and investments, income distributions and others.

There are three types of fiscal policy:

- Neutral, which means that the government's deficit spending remains roughly the same through a period of time. This usually happens when the economy is in neither a recession nor a boom.
- Expansionary, which involves increasing government spending that at times increases the deficit between spending and revenue. This type of policy normally occurs at times of recession.
- Contractionary, which means that the government's deficit spending is even lower than usual. This normally happens at times of high inflation rates and potential asset bubbles.

Monetary policy is the process by which the monetary authority of a country, typically the central bank or currency board, controls either the cost of very short-term borrowing or the monetary base, often targeting an inflation rate or interest rate to ensure price stability and general trust in the currency. Monetary policy is exercised through actions such as modifying the interest rate, buying or selling government bonds, and changing the amount of money banks are required to keep in the vault (bank reserves).

There are two types of monetary policy:

- Expansionary, which increases the money supply in order to lower unemployment, boost private-sector borrowing and consumer spending, and stimulate economic growth.
- Contractionary, which slows the rate of growth in the money supply or outright decreases the money supply in order to control inflation. When necessary, contractionary monetary policy can decrease economic growth,

increase unemployment and depress consumer spending and borrowing to achieve a controllable rate of inflation.

The impact of monetary policy in free market economies has been a subject of strenuous debate over the decades. There have been a lot of different views about the use of money supply as a tool and its effect on business cycles. One of the most well known is the money neutrality theory.

Neutrality of money is the idea that a change in money supply affects only nominal variables in the economy such as prices, wages, and exchange rates, with no effect on real variables, like employment, real GDP, and real consumption. It is an idea credited to the classical economists and the classical school of thinking and is related to the classical dichotomy (real variables can be examined without having knowledge of their nominal counterparts).

The neutrality of money idea is basically based on the Quantity Theory of Money(from now on QTM). The QTM started in the 16^{th} century as the import of gold and silver(therefore more coins) from the New World to Europe resulted to a rise in inflation. This led economists of that time period to the assumption that more money doesn't necessarily mean more output. QTM states that there is a direct relationship between money supply and prices of goods and services sold. Therefore, if we double the amount of money an economy has, the prices will also double, causing inflation. In its simplest form, this theory is expressed by the following equation:

$$\mathbf{MV = PT}$$

This is known as the Fisher Equation. It took its name from the acclaimed economist Irving Fisher. M denotes money supply, V denotes velocity of money, P is the average price level and T denotes the volume of transactions of goods and services(it generally denotes an economy's output, hence why GDP is being used regularly). In its most basic form, the theory states that V and T are constant in the short term. Therefore, an increase in M will lead to an equal increase in P. These assumptions have been heavily criticized, especially the assumption that velocity is constant.

The theory also assumes that the quantity of money, which is exogenous, is the main influence of economic activity in a society. A change in money supply results in changes in price levels and/or a change in supply of goods and services. Also, T is determined by labor ,technology ,capital ,natural resources, organization and knowledge. According to the QTM, the economy is in equilibrium and at full employment.

In essence, the theory states that the value of money depends on the amount of money available in the economy(law of supply and demand). If we increase the amount of money, its value will decrease and therefore our purchasing power will decrease. To sum things up, it will cost more money to buy the same goods and services.

The concept of neutrality can be found going back to the 18th century in some writings by Scottish philosopher and economist David Hume. It became a subject of debate and disagreement for centuries, but it wasn't until the early 20th century that it came at the forefront of economic thinking. It was Austrian economist and philosopher Friedrich Hayek who introduced the term and helped enhance the debate. Since then there have been many arguments regarding its validity. The original classical view stated that neutral money exist both in the short term and in the long term. The two main criticisms came from two of the most influential economists of the 20th century, John Maynard Keynes and Milton Freedman. Keynes completely rejected the idea, stating that monetary policy tools can balance economic cycles and therefore impact the real economic variables. On the other hand, Milton Freedman believed that neutral money exist in the long run, and strongly disagreed with the constant changing of monetary policies, going as far as saying they did more harm than good.

The monetarist school of thinking states that growth in money supply that surpasses the growth of output leads to inflation, as there is too much money behind too little production of goods. Thus, they believed that money supply should be kept under a limit in order to control inflation.

In this paper we will examine the concept of neutrality in two advanced economies (USA and Canada) using eight simple econometric models. We will use econometric tools such as unit root tests, co-integration procedures, Vector Error Correction models and VAR models. Before the econometric research there will be a short overview of these two economies through the years as well as a detailed literature review.

1.2 An extended overview of the economies

1.2.1 United States of America

The **United States of America** (**USA**) is a federal republic composed of 50 states. At 3.8 million square miles (9.8 million km^2), the United States is one of the world's largest countries. It has a population of over 325 million people, making it the third-most populous country in the world. The capital is Washington, D.C. and the largest city by population is New York City.

The United States is one of the founding members of institutions such as the the United Nations, World Bank, International Monetary Fund, Organization of American States (OAS), and others. It is a highly developed country with an economy that is based on the free market-private enterprise combination, with minimal input from the government in areas in areas such as health care, transportation, and retirement. The United States boasts a highly diversified and innovative industrial sector, an abundance of natural resources and high productivity. It is the world's second largest importer and the third largest exporter of goods. Though its population is only 4.3% of the world total, the U.S. holds 33.4% of the total wealth in the world. The United States ranks among the highest nations in several measures of socioeconomic performance, including average wage, human development, per capita GDP, and productivity per person. It also possesses the most advanced and powerful military force in the world, making up a third of global military spending, and is a leading political, cultural, and scientific force internationally.

The U.S. dollar is the currency most used in international transactions and is the world's foremost reserve currency. Several countries use it as their official currency, and in many others, it is the *de facto* currency. Its largest trading partners are China, Canada, Mexico, Japan, Germany, South Korea, United Kingdom, France, India, and Taiwan. The US also has one of the world's largest stock exchange markets, the NYSE(New York Stock Exchange). Foreign investments made in the U.S. total over $4 trillion, while American investments in foreign countries total to almost $6 trillion. Debt held by the public, a measure of national debt, was approximately 77% of GDP in 2017, ranked the 40rd highest out of 207 countries. Income inequality ranked 41st highest among 156 countries in 2017.

1.2.2 Canada

Canada is a country located in the northern part of North America. It covers 9.98 million square kilometres (3.85 million square miles), making it the world's second-largest country. Canada's southern border with the United States is the world's longest bi-national land border. It has a population of 37 million people and its capital city is Ottawa. Canada’s population is highly urbanized, with 82 percent of the almost 37 million people concentrated in large and medium-sized cities. Its three largest metropolitan areas are Toronto, Montreal, and Vancouver. It is considered a sparsely populated country in large due to the extreme climate conditions.

The country has a unique economic system that combines the private and public enterprises and the highest economic freedom in the world with a public to private property ratio of 40:60. Canada is one of the most prosperous nations in the world with its GDP ranking 10th nominally and 17th by PPP. It is one of the major exporters of agricultural products and also has a very extended service industry. Canada is a developed country that ranks among the highest in international measurements of government transparency, civil liberties, quality of life, economic freedom, and education. Income inequality ranked 117th highest among 156 countries in 2017. It is also a part of several major international institutions and groups such as the the United Nations, the North Atlantic Treaty Organization, the G7(formerly G8), the G20, the North American Free Trade Agreement and the Asia-Pacific Economic Cooperation forum.

Canada has the fourth highest total estimated value of natural resources, valued at US$33.2 trillion in 2016. Its debt/GDP ratio stands at 89,60% as of 2017. It has the world's third largest proven petroleum reserves and is the fourth largest exporter of petroleum. It is considered one of the premier energy superpowers due to the abundance of natural resources. Its currency is the Canadian dollar, which has an exchange rate of 0,774US dollars as of September 2018. The main export partners include the US, the Eurozone, China, Japan, Mexico, India and South Korea. Canada mainly imports goods from the US, the Eurozone, China, Mexico, Japan, the UK, and South Korea.

Canada closely resembles the U.S. in its market-oriented economic system, patterns of production, and high living standards. Leading sectors include automotive and other manufactured goods, forest products, minerals, and petroleum. Is also has one of the world’s longest coastlines, which fueled the growth of the commercial fishing and seafood industry(one of the largest in the world). The Canadian economy is unusual among developed economies due to the importance of the primary sector, and especially the oil and logging industries, to its growth and viability.

2. Literature Review

There have been many empirical studies and therefore an extended literature on the subject of money neutrality and the general impact of money in an economy. The results have been varied and therefore there isn't a dominating point of view regarding this subject. One of the first studies on money neutrality was done by **Fisher and Seater(1993)**. Using ARIMA framework in testing Long run money neutrality and Long run money super-neutrality[1]* in the US, they found little support for the proposition.

Boschen and Otrok(1994) re-examined the Fisher and Seater(1993) evidence by leaving the Great Depression period(1930-1939) out, therefore using the same ARIMA framework for two sub-periods(1869-1929 and 1940-1992), thinking that endogenous changes in money supply were made during that time period. They conclude that LRMN[2]* eventually holds for the two sub-periods.

Based on the Fisher and Seater's(1993) ARIMA framework, **Frederick Wallace(1999)** examined the concept of neutrality for the Mexican economy. Using both M1 and M2 analysis, as well as a dummy variable for a period in which banks were nationalized, he came to the conclusion that a change in money supply does not affect real output in the Mexican economy. Based again on the Fisher and Seater(1993) study, **Serletis and Krause(1996)** examined the hypothesis of money neutrality for 10 developed countries using long, low frequency data. They concluded that Long run neutrality stands for 8 of them.

Another important study by **Bae and Ratti(2000)** studied the cases of Argentina and Brazil. Testing long run neutrality and super-neutrality using time series for real output and money M2 and using data for a time period of 1884 to 1996 and 1914 to 1995 respectively, they concluded that neutrality stands but super-neutrality was not confirmed. **Bae et al(2005)** using a fractionally integrated autoregressive moving average model (ARFIMA) and based again on the Fisher and Seater(1993) framework, examined the effect of money to real output for six countries and confirmed the neutrality proposition for five of them. They further concluded that in a low inflation economy, a monetary shock could bring positive effects toward output even if long run money neutrality does hold in those countries.

Puah et al(2008b) extended the research for 10 member countries of the South East Asian Central Banks (SEACEN) using annual monetary data from 1950 to 2001. The

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*the proposition that permanent, exogenous changes to the growth rate of the money supply ultimately lead to equal changes in the nominal interest rate and leave the level of real variables unchanged

*Long Run Money Neutrality

results have been mixed, as LRMN stands for only 5 of them. For three of those countries(Indonesia, Taiwan and Thailand)a positive connection between a monetary expansion and output growth seems to exist.

Moreover, some studies have indicated that different measures of monetary aggregates can provide different results on LRMN tests. **Tan and Baharumshah (1999)** examined the effect of three different monetary variables(M1,M2 and M3) to real output and prices in the Malaysian economy. They studied those effects using monthly time series from 1975 to 1995. The econometric analysis consisted of a Johansen multivariate co-integration analysis, Vector Error Correction Modeling and Granger causality. The results showed that in a small economy with high sustained growth rates M3 performed better than M1 and M2 as it has the strongest causal effect on real output. Furthermore, **Leong and McAleer (2000)** studied the proposition of long run money neutrality in Australia, using quarterly data from 1975 to 1995. Again, different types of money supply provided different results to the analysis. The results indicated that the long run neutrality of money proposition could not be rejected when M1 was used as the measure of money supply. However, when M3 was used, money was non neutral. The same conclusion was made by **Wallace(2005)** for the case of Guatemala. Based on the Fisher and Seater tests, he found that LRMN holds when M1 is the money supply measure, but does not hold when M2 takes its place.

Puah and Jayaraman (2007) investigated the causal relationship between capital stock prices and other microeconomic variables(including money supply M2) using quarterly data from 1997 to 2004 in Fiji. They utilized ADF and PP unit root tests as well as Johansen Cointegration, Error correction model and Granger causality tests for the empirical analysis. The study revealed that stock prices are cointegrated with real economic activities and they adjust rather fast from short run deviation to long-run equilibrium. They also found that real GDP, M2, and exchange rate Granger cause stock price in the short-run.

Tang et al. (2013) investigated the LRMN proposition in Singapure for the 1980 to 2009 period using Divisia[3]* money and simple sum monetary aggregates. They based their work on the Fisher and Seater(1993) neutrality tests and concluded that neutrality does not hold when both the simple-sum money and Divisia money are employed. In other words, monetary tools have long lasting impact on real economic activity and therefore an expansionary monetary policy can be used to stimulate economic growth.

*Barnett (1980) introduced a weighted monetary aggregate as an alternative to simple-sum money. Each monetary component asset is weighted differently in the Divisia monetary aggregate according to its monetary services(Tang et al. 2013)

There have been numerous pieces on the subject of long run money neutrality with mixed results. The conclusions are diverse and depend on the countries that are being examined, the choice of broad or narrow monetary aggregates and even the time periods for each study. This topic remains very interesting for economic researchers and the debate continues as more studies on this subject are being made.

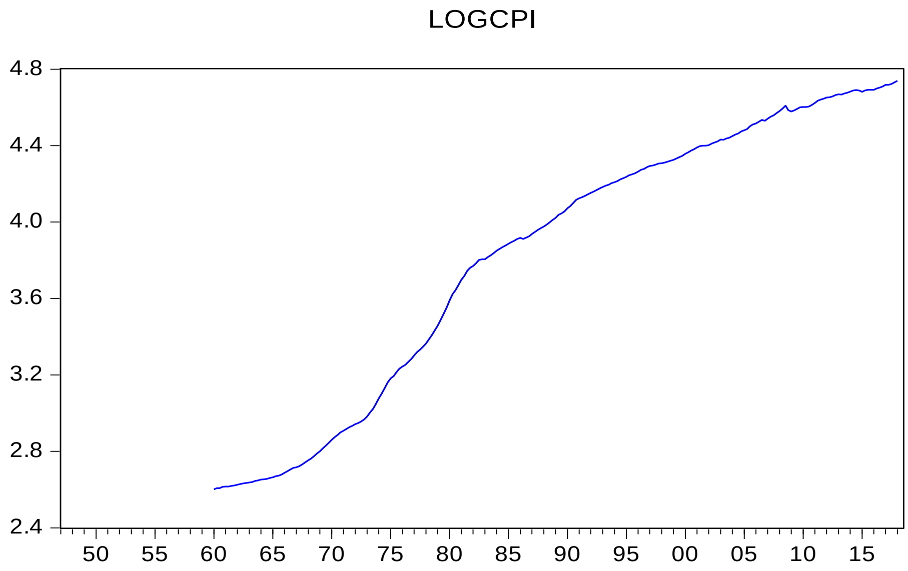
3.DATA AND METHODOLOGY

3.1 DATA

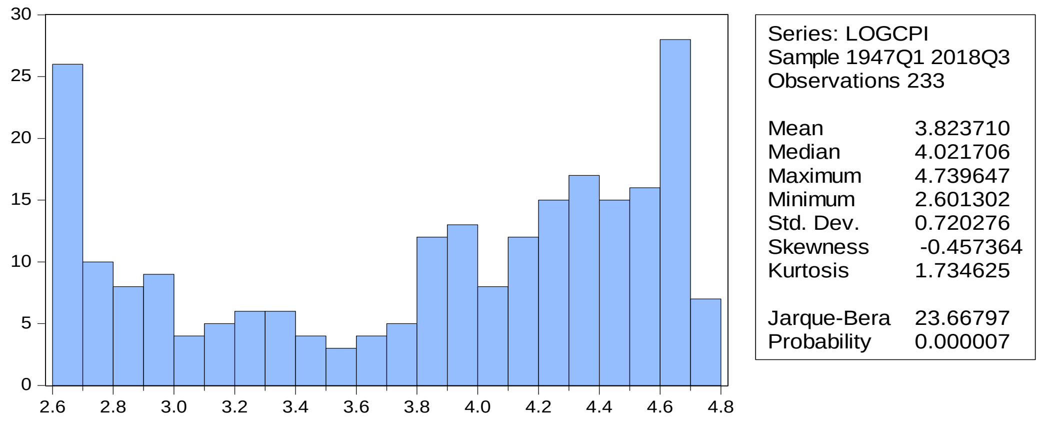
For this paper i used quarterly time series for real GDP, M1[4]* money supply, M2 money supply and CPI(Consumer's Price Index) for USA and Canada. All these time series were used in logarithmic form to prevent heteroskedasticity. For the co-integration analysis we used data in logarithmic form at the levels. For the Vector Autoregressive Models(VARs) we used first differences of the logarithmic data. The main source for the data was the https://fred.stlouisfed.org/ website which is the website of the Eighth District of the Federal Reserve System in the US. A more accurate depiction of every time series used as well as their graphic form can be seen below:

For USA:

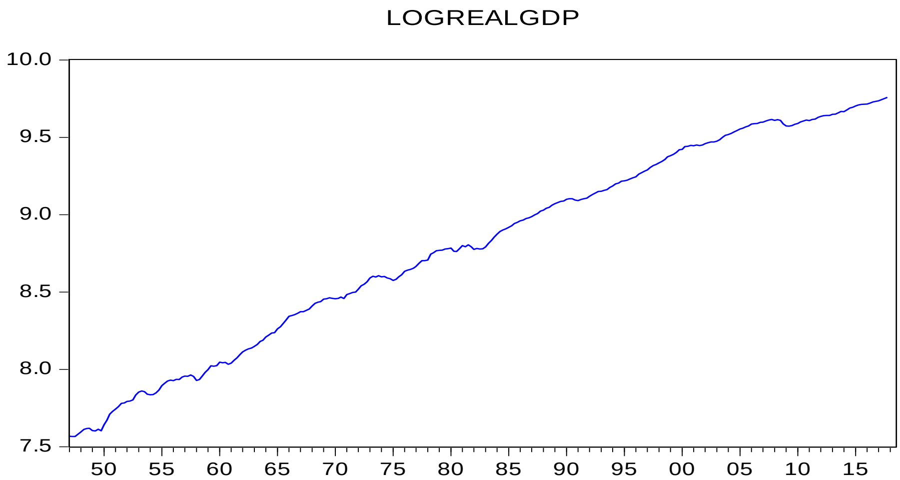
- Consumer's Price Index in 2010 American dollars, quarterly data in logarithmic form, 1960:1 to 2018:1.

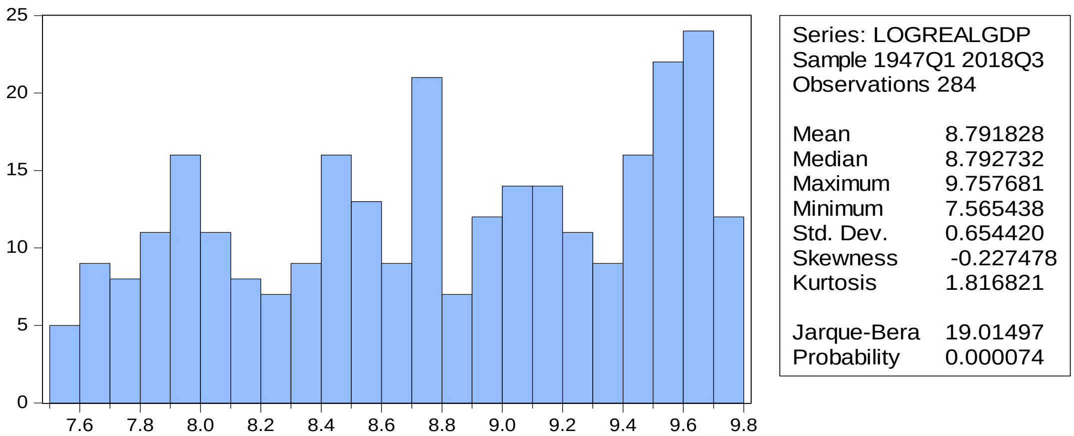


*includes physical currency and coin, demand deposits, travelers checks, other checkable deposits and negotiable order of withdrawal (NOW) accounts. M2 is M1 plus savings deposits, money market securities, mutual funds and other time deposits.(source https://www.investopedia.com/terms/m/m2.asp)

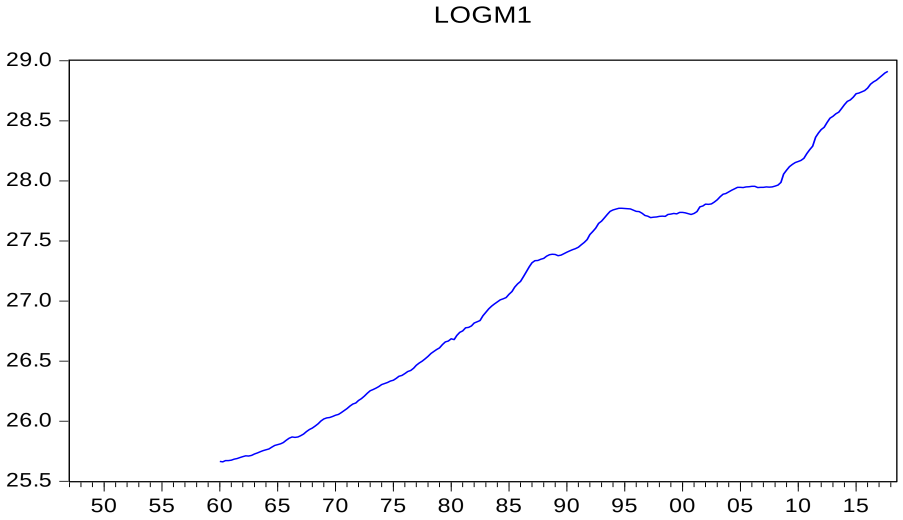


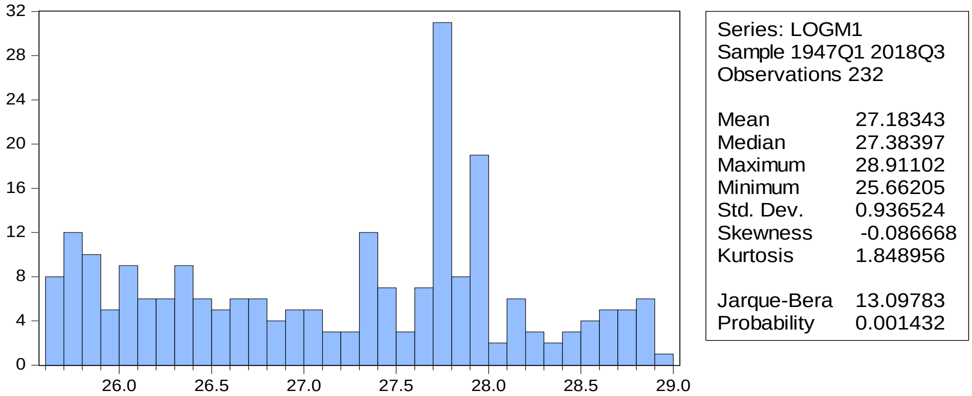
- Real GDP in 2009 dollars, quarterly seasonally adjusted data in logarithmic form, 1947:1 to 2017:4.





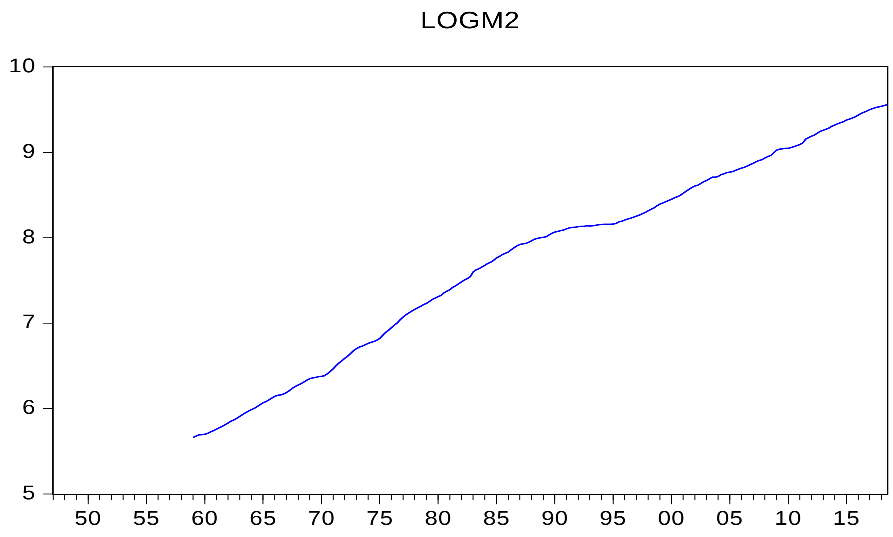
- M1 money supply, quarterly seasonally adjusted data in logarithmic form, 1960:1 to 2017:4.

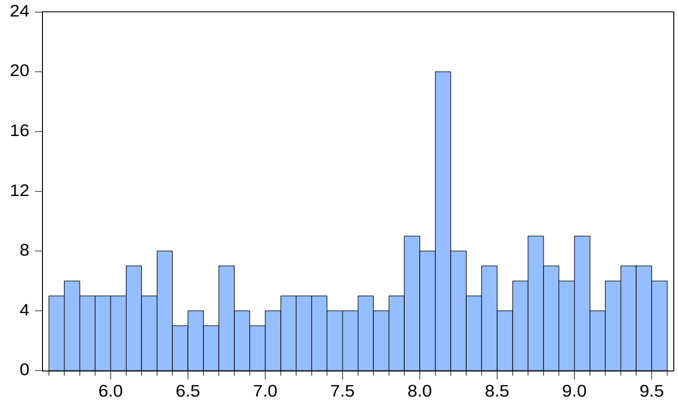




- M2 money supply, monthly(turned quarterly[5]*) seasonally adjusted data in logarithmic form, 1959:1 to 2018:3.

*The monthly data for M2 were turned into quarterly data by summing them for every three quarters and then dividing them by 3. (Q1+Q2+Q3)/3 etc.



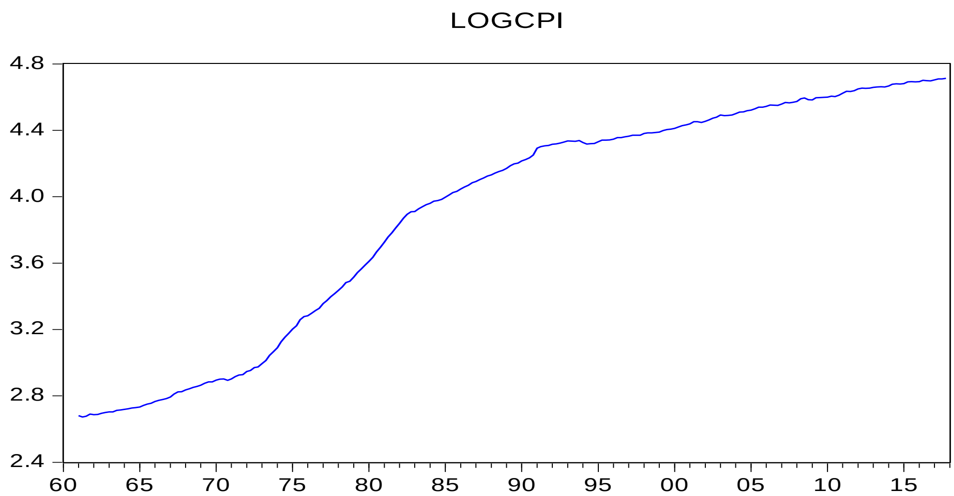


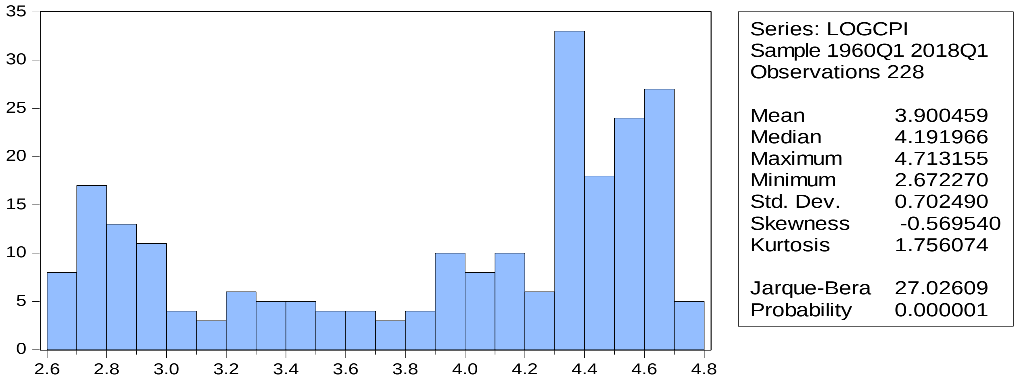
Series: LOGM2
Sample 1947Q1 2018Q3
Observations 239

Mean	7.748854
Median	7.998974
Maximum	9.557237
Minimum	5.662382
Std. Dev.	1.143028
Skewness	-0.256916
Kurtosis	1.882831
Jarque-Bera	15.05789
Probability	0.000537

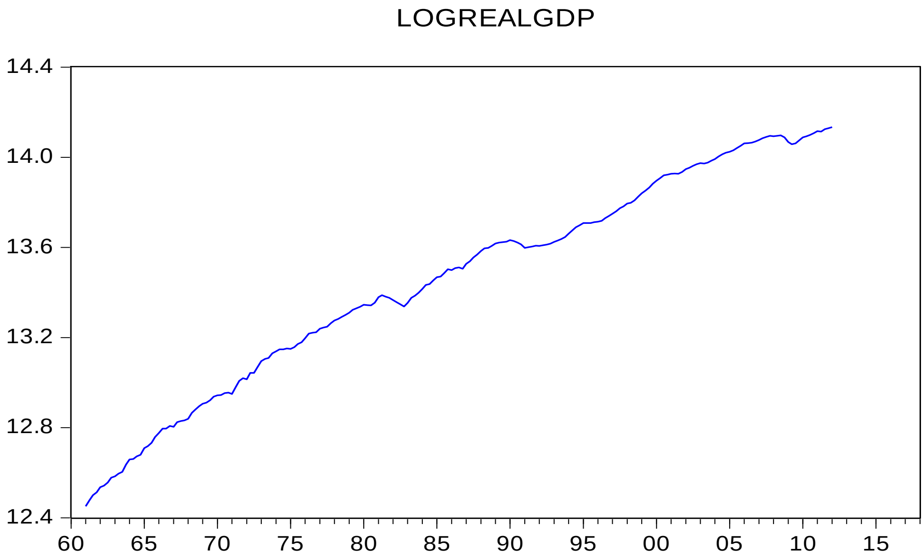
<u>For Canada:</u>

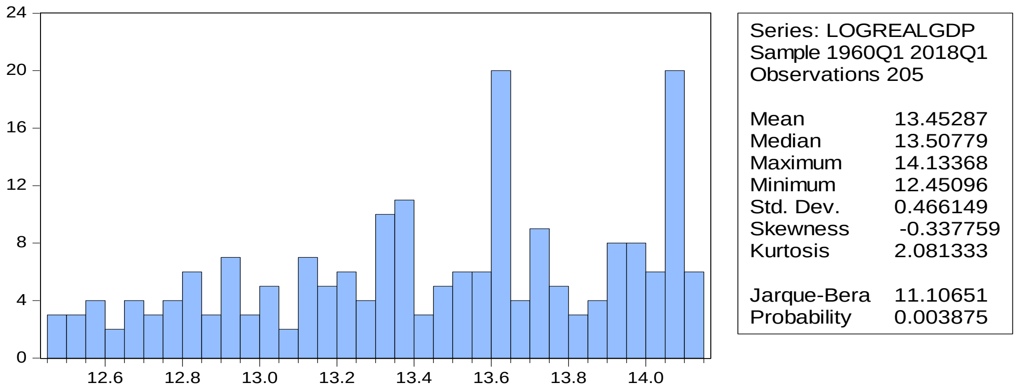
- Consumer's Price Index(CPI), 2010=1, quarterly data in logarithmic form, 1961:1 to 2017:4



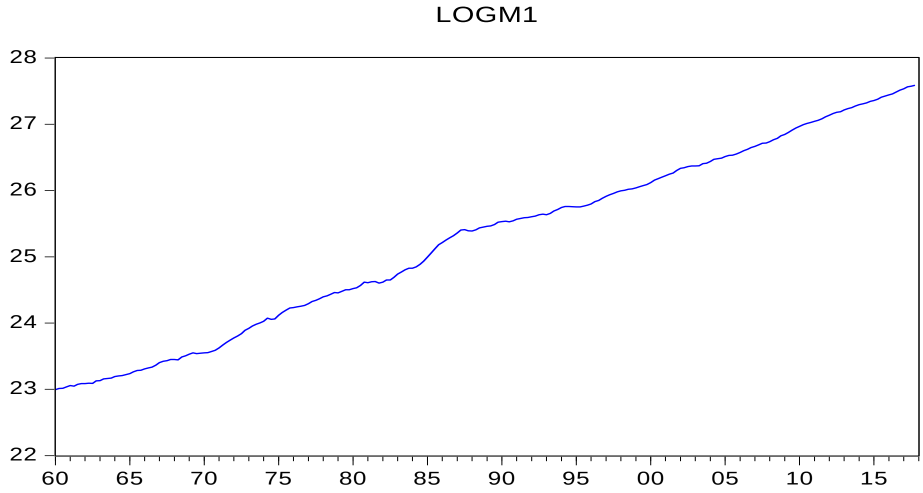


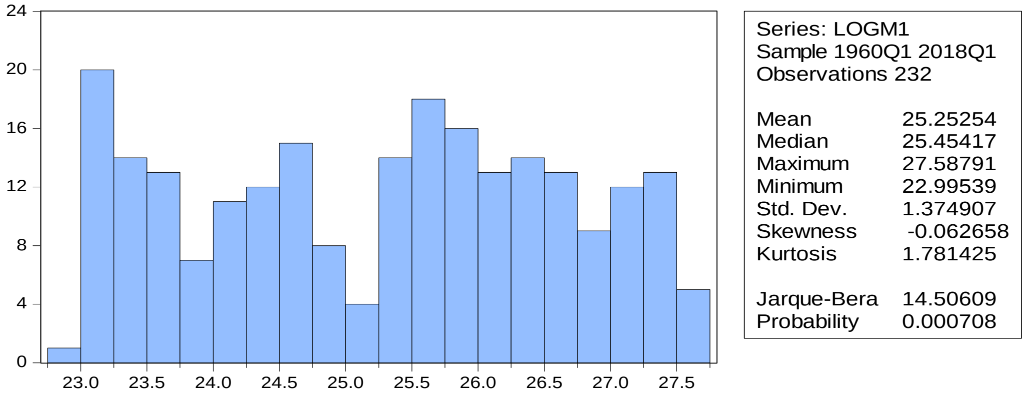
- Real GDP in 2002 Canadian dollars, quarterly seasonally adjusted data in logarithmic form, 1961:1 to 2012:1.



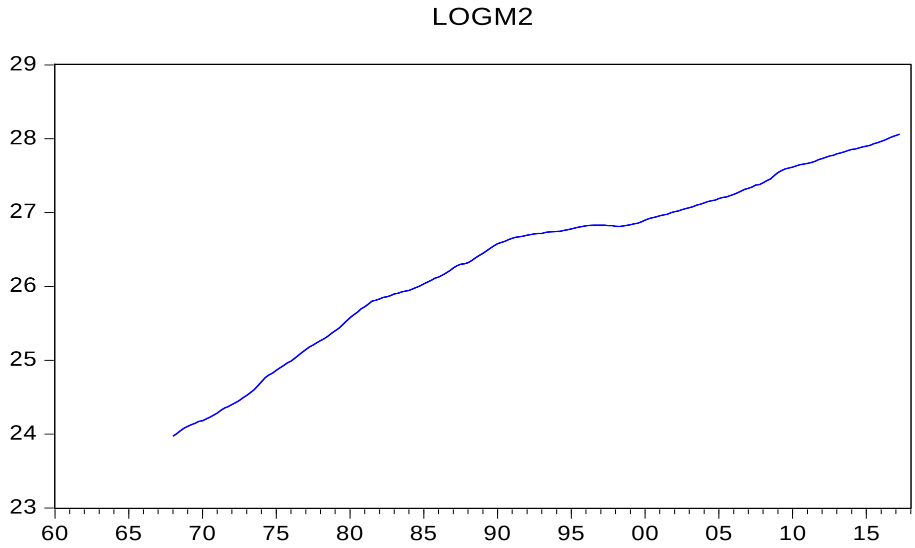


- M1 money supply in national currency, quarterly seasonally adjusted data in logarithmic form, 1960:1 to 2017:4.

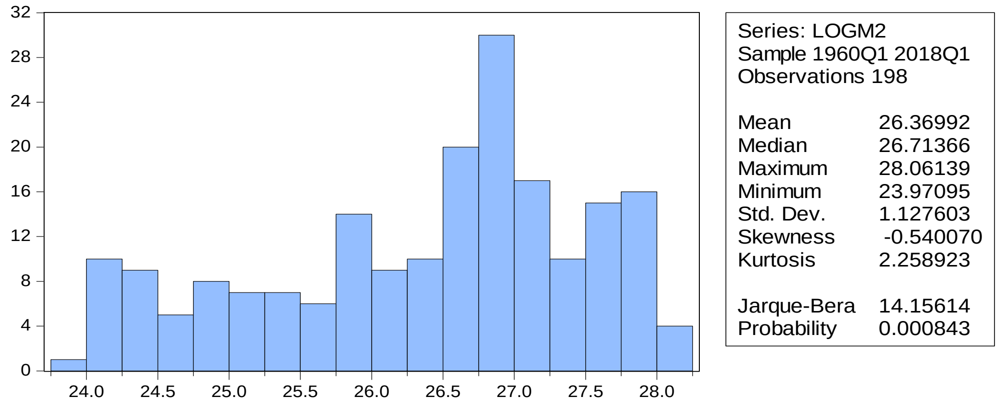




- M2 money supply in Canadian dollars, monthly(turned quarterly[6]*) seasonally adjusted data in logarithmic form, 1968:1 to 2017:2.



*The monthly data for M2 were turned into quarterly data by summing them for every three quarters and then dividing them by 3. (Q1+Q2+Q3)/3 etc.



3.2 METHODOLOGY

3.2.1 The concept of Stationarity

The concept of stationarity is an important part of a time series analysis. It can strongly influence its behaviour and properties – e.g. persistence of shocks will be infinite for non-stationary series(Brooks, 2008). A time series is said to be stationary when its mean and variance are constant over time and the value of the covariance depends solely on the distance/gap between the two distinct time periods, meaning the current period and its past lag, regardless the time at which the covariance is calculated (Gujarati 1995). A time series is said to be strictly stationary if the joint distribution of any set of n observations $X(t_1)$, $X(t_2)$….,$X(t_n)$ is the same as the joint distribution of $X(t_{1+k})$, $X(t_{2+k})$…..$X(t_{n+k})$ for all n and k(Maddala & Lahiri,2009).

If the variables in the regression are not stationary, then it can be proved that the standard assumptions for asymptotic analysis will not be valid. In other words, the "t-ratios" will not follow a t-distribution, so we cannot undertake hypothesis tests about the regression parameters and coefficients(Brooks, 2008). It is also important to underline the trap of spurious regressions. If two variables are trending overtime, a regression of one on the other could have a high R^2 even if the two are unrelated (Brooks, 2008).

There are two models that are frequently used to characterize non stationarity: The random walk model with drift:

$$y_t = \mu + y_{t-1} + u_t \quad (1)$$

and the deterministic trend process:

$$y_t=a+\beta t+u_t \qquad (2)$$

where u_t is iid in both cases(Brooks,2008).

Let's assume that model (1) takes the following form:

$$y_t = \mu + \phi y_{t-1} + u_t$$

We have three cases:

1. $\phi<1 \Rightarrow \phi^T=0$ as $T \to \infty$

So the shocks to the system gradually die away

2. $\phi=1 \Rightarrow \phi^T=1 \ \forall\ T$

So shocks persist in the system and never die away.

3. $\phi>1$.

Now shocks become more influential as the time goes on, because if $\phi>1$, $\phi^3>\phi^2>\phi$ etc. (Brooks,2008).

In case 1, the series is stationary. In cases 2 and 3, the series is non-stationary(stochastic non-stationarity). Case 3 is very rare in economics and finance, so we mostly perform stationarity tests with the first two cases as conflicting hypotheses.

This kind of series are important and meaningful when we carry out research, because a stationary variable has the tendency to return to its mean and fluctuations around the mean value will have a certain width, meaning that any shocks caused in the process, will be absorbed sooner or later. On the contrary, non-stationary time series tend to fluctuate randomly forever, after a shock has occurred. This is due to their time varying mean and variance occasionally (Gujarati 1995).

Let's go back to our 2 characterizations of non-stationarity:

$$y_t = \mu + y_{t-1} + u_t \quad (1)$$

$$y_t=a+\beta t+u_t \qquad (2)$$

The two require different treatments to achieve stationarity. The second case, known as deterministic non-stationarity requires detrending. The first case is known as stochastic non-stationarity. If we let:

$$\Delta y_t = y_t - y_{t-1}$$

$$\text{and } L y_t = y_{t-1}$$

$$\text{so } (1-L) y_t = y_t - L y_t$$

We take (1) and subtract y_{t-1} from both sides:

$$y_t - y_{t-1} = \mu + u_t$$

$$\Delta y_t = \mu + u_t$$

We say that we induced stationarity by "differencing once"(Brooks,2008).

Although trend-stationary and difference-stationary series are both 'trending' over time,we need to use the correct methodology for each case. «*If first differences of a trend-stationary series were taken, it would 'remove' the non-stationarity, but at the expense of introducing an MA(1) [7]* structure into the errors. Conversely if one tried to de-trend a series which has stochastic trend, then the non-stationarity would not be removed»*.(Brooks,2008,p323).

In regards to the stochastic model(random walk with drift), we can give the following definition: A time series y_t is said to be integrated of order 1 or I(1) if Δy_t is a stationary time series. A stationary time series is said to be I(0). A random walk is a special case of an I(1) series, because if y_t is a random walk, Δy_t is a random series or white noise[8]*. A time series is said to be integrated of order 2 or I(2) if Δy_t is I(1), and so on. If $y_t \sim I(1)$ and $u_t \sim I(0)$, then their sum $Z = y_t + u_t \sim I(1)$(Maddala & Lahiri,2009).

3.2.2 Unit Root tests

3.2.2.1 The Dickey-Fuller and Philips-Perron tests

The most well known work on testing for a unit root in time series was done by Dickey and Fuller (Fuller, 1976; Dickey and Fuller, 1979). The basic objective of the test is to examine the null hypothesis that $\phi = 1$ in

$$y_t = \phi y_{t-1} + u_t \quad (3)$$

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*Moving average model of first degree: $y_t = \mu + u_t + \theta u_{t-1}$

*White noise is a process where $E(e_t)=0$, $V(e_t)=\sigma^2$ and $cov(e_t, e_{t+\kappa})=0$ $\forall$ t and $\kappa \neq 0$

against the one-sided alternative $\phi < 1$. Thus the hypotheses of interest are H0: series contains a unit root versus H1: series is stationary(Brooks,2008).

In practice we use the following regression:

$$\Delta y_t = \psi y_{t-1} + u_t \quad (4)$$

so that a test of $\phi = 1$ is the same as a test of $\psi = 0$ (since $\phi - 1 = \psi$). Dickey-Fuller (DF) tests are also known as τ -tests, and can be conducted allowing for an intercept, or an intercept and deterministic trend, or neither, in the test regression. The model for the unit root test in each case is:

$$y_t = \phi y_{t-1} + \mu + \lambda t + u_t \quad (5)$$

The tests can also be written, by subtracting y_{t-1} from each side of the equation, as

$$\Delta y_t = \psi y_{t-1} + \mu + \lambda t + u_t \quad (6)$$

since $\phi - 1 = \psi$(Brooks,2008).So if $\psi=0$, the series contains a unit root. If $\psi<0$, the series is stationary.

The test statistics for the original DF tests are:

Test statistic=$\frac{\hat{\psi}}{\widehat{SE(\psi)}}$

«The test statistics do not follow the usual t-distribution under the null hypothesis, since the null is one of non-stationarity, but rather they follow a non-standard distribution»(Brooks, 2008,p328).

The tests are valid only if u_t is white noise. We assume that u_t is not autocorrelated, but would be so if there was autocorrelation in the dependent variable (y_t) which has not been modelled. The solution is to 'augment' the test using p lags of the dependent variable. The new model is:

$$\Delta y_t = \psi y_{t-1} + \sum_{i=1}^{p} a_i \, \Delta y_{t-1} + u_t \quad (7)$$

The test is known as an augmented Dickey--Fuller (ADF) test. We still test the ψ parameter with the same critical values from the previous DF test(Brooks, 2008).

Phillips and Perron have developed a more advanced way of testing unit root non-stationarity. *«The tests are similar to ADF tests, but they incorporate an automatic correction to the DF procedure to allow for autocorrelated residuals»(Brooks,2008, p 330).*In other words, the test takes into account the possibility that the residuals are not White Noise. More often than not, the PP test gives the same conclusions as the ADF test and has a lot of the same limitations.

The most important criticism for these unit root tests is that their power is low if the process is stationary but with a root close to the non-stationary boundary. So, for example, consider the following process with coefficient 0.95:

$$y_t = 0{,}95\ y_{t-1} + u_t$$

In this case the null hypothesis of a unit root should be rejected. There have been many instances were these tests have shown difficulty at deciding, for example, whether $\phi = 1$ or $\phi = 0.95$, especially with small sample sizes(Brooks, 2008).

3.2.3 Co-integration

An important aspect of time series analysis and the association between different time series is the concept of Co-integration. Let's assume that we have two time series y_t and x_t and both are integrated of order one or I(1). According to the Co-integration Theorem created by Granger in 1981 and developed further by Engle and Granger in 1987 there is a possibility of finding a linear combination in the form of $y_t=\beta\ x_t+ e_t$ that is I(0) (Brooks,2008).

In most cases, if two variables that are I(1) are regressed on each other, then the combination will also be I(1). Furthermore, if variables with differing orders of integration are combined, the combination will have an order of integration equal to the largest. If $X_{i,t} \sim I(d_i)$ for i = 1, 2, 3,..., k so that there are k variables each integrated of order d , and letting

$Z_t=\sum_{i=1}^{k} a_i X_{i,t}$ (8)

Then $z_t \sim I(\max d_i)$. z_t in this context is simply a linear combination of the k variables X_i . Rearranging (8)

$X_{1,t}=\sum_{i=2}^{k} \beta_i X_{i,t}+z_t'$

where $\beta_i =-\alpha_i/\alpha_1$,$z_t' = z_t/\alpha_1$, i = 2,..., k (Brooks,2008).

Economically speaking, two time series are co-integrated if despite their short term differences and fluctuations, there is a long run relationship between them. The linear combination cancels out the stochastic trends in the two series. If there is no co-integration, then there is no long run association between the variables and the estimation results will be spurious (Maddala, 1988).

In other words, there are many time series that are not stationary but move together over time. If variables are co-integrated, it means that a linear combination of them will be stationary. There may be up to r linearly independent co-integrating relationships(where r≤k-1, where k is the number of the time series or variables) also known as co-integrating vectors.

3.2.3.1 The Johansen technique for co-integration

According to Maddala (1988), it is better to apply the Johansen co-integration if we have more than 2 variables in each model. The Engle-Granger can test for one co-integrating equation while the Johansen test is superior as it can identify more than one co-integrating equations.

To use Johansen method, we have to turn the VAR[9]* of the form:

$Y_t = \beta_1 \; y_{t-1} + \beta_2 \; y_{t-2} + \cdots + \beta_k \; y_{t-k} + u_t$

gX1 gXg gX1 gXg gX1 gXg gX1 gX1

into a VECM[10]*:

$\Delta Y_t = \Pi Y_{t-k} + \Gamma_1 \Delta Y_{t-1} + \Gamma_2 \Delta Y_{t-2} + ... + \Gamma_{k-1} \Delta Y_{t-(k-1)} + u_t$

Where $\Pi = (\sum_{j=a}^{k} \beta_i) - I_g$ and $\Gamma_i = \sum_{j=1}^{i} \beta_j - I_g$

Π is a long run coefficient matrix since $\Delta Y_{t-l} = 0$(Brooks,2008).

Brooks (2008) argues that the essence of this test lies in the examination of the Π matrix. The rank of the matrix is equal to the number of its characteristic roots(eigenvalues) that are non-zero. If the variables are not co-integrated, the rank of Π will be zero.

There are two test statistics for co-integration under the Johansen approach, the trace and the max statistics:

$\lambda_{trace}(r) = -T\sum_{i=r+1}^{g} ln(1 - \hat{\lambda}_i)$ and

$\lambda_{max}(r, r + 1) = -T \ln(1 - \hat{\lambda}_{r+1})$

where «*r is the number of co-integrating vectors under the null hypothesis and $\hat{\lambda}_i$ is the estimated value for the ith ordered eigenvalue from the Π matrix. Intuitively, the larger is $\hat{\lambda}_i$, the more large and negative will be ln(1 − $\hat{\lambda}_i$)*

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*Vector Autoregressive model, explained later in this chapter.
*Vector Error Correction Model, explained later in this chapter.

and hence the larger will be the test statistic. Each eigenvalue will have associated with it a different co-integrating vector, which will be eigenvectors. A significantly non-zero eigenvalue indicates a significant co-integrating vector»(Brooks,2008,p351).

λ_{trace} is a joint test where the null is that the number of co-integrating vectors is less than or equal to r against an alternative that there are more than r. λ_{max} tests each eigenvalue separately, with the null hypothesis that the number of co-integrating vectors is r against an alternative of r + 1(Brooks,2008). The idea of the test is that we keep rejecting the null hypothesis for both the eigenvalue and maximum eigenvalue statistic, until we fail to do so.

$H0 : r = 0$ versus $H1 : 0 < r \leq g$

$H0 : r = 1$ versus $H1 : 1 < r \leq g$

$H0 : r = 2$ versus $H1 : 2 < r \leq g$

…………………

$H0 : r = g - 1$ versus $H1 : r = g$.

3.2.4 VECM(Vector Error Correction Model)

Initially, when series were found to be non-stationary, the normal response was to take the first differences and then use them on the modeling process. The problem with this method is that it takes away the long run dynamics of each series.

For example: $\Delta Y_t=\alpha \, \Delta X_t + u_t$ with Y_t and X_t both I(1).

In the long run the difference terms are zero, meaning that the variables are no longer changing. So $\Delta Y_t=0$ and $\Delta X_t=0$. Assuming that Y_t and X_t are co-integrated(connected in the long run), such an approach would limit our understanding of dynamics for the variables of this particular model. The best way to deal with this problem is to use both first difference and level terms:

$\Delta Y_t= \beta_1 \, \Delta X_t + \beta_2(Y_{t-1} - \gamma X_{t-1}) + u_t$

This model is known as an Error Correction Model(ECM) and the $Y_{t-1} - \gamma X_{t-1}$ is known as the error correction term. Assuming that Y_t and X_t are co-integrated, the error correction term will be I(0) which makes the regression non-spurious(we know that ΔY_t and ΔX_t are I(0)). The coefficient γ is the co-integrating coefficient. The error term, which appears in the equation lagged one period, expresses the speed of adjustment (correction) of the previous error term (gap between the variables) to the current period. In other words, it expresses the speed of adjustment of the variables from the short run fluctuations to the long run equilibrium.

3.2.5 VAR(Vector Autoregressive Model)

Vector Autoregressive Models(VARs) were popularized by Sims(1980). «*A VAR is a systems regression model (i.e. there is more than one dependent variable) that can be considered a kind of hybrid between the univariate time series models and the simultaneous equations models*»(Brooks,2008, p290). In a VAR model, current values of dependent variables are regressed against *k* lags of themselves and the rest of the variables as well as the error terms. The simplest case is a bivariate VAR:

$$Y_{1t} = \beta_{10} + \beta_{11}\, y_{1t-1} + \cdots + \beta_{1k}\, y_{1t-k} + \alpha_{11}\, y_{2t-1} + \cdots + \alpha_{1k}\, y_{2t-k} + u_{1t}$$

$$Y_{2t} = \beta_{20} + \beta_{21}\, y_{2t-1} + \cdots + \beta_{2k}\, y_{2t-k} + \alpha_{21}\, y_{1t-1} + \cdots + \alpha_{2k}\, y_{1t-k} + u_{2t}$$

where u_{it} is a white noise term with $E(u_{it}) = 0$, $(i = 1, 2)$, $E(u_{1t}u_{2t}) = 0$(Brooks,2008).

The goal of a VAR model is to trace the relationship between the dependant and the exogenous variables for each regression.

3.2.6 Causality tests, Variance Decomposition and the Impulse Response Function

«*It is likely that, when a VAR includes many lags of variables, it will be difficult to see which sets of variables have significant effects on each dependent variable and which do not. In order to address this issue, tests are usually conducted that restrict all of the lags of a particular variable to zero*»(Brooks,2008,p297).

The most important test for causality in a VAR context is the Granger Causality test. Causality tests seek to answer simple questions of the type, "Do changes in y_1 cause changes in y_2?" If y_1 causes y_2, lags of y_1 should be significant in the equation for y_2. If this is the case and not vice versa, it would be said that y_1 "Granger-causes" y_2 or that there exists unidirectional causality from y_1 to y_2(Brooks,2008).

«Block F-tests and an examination of causality in a VAR will suggest which of the variables in the model have statistically significant impacts on the future values of each of the variables in the system. But F-test results will not, by construction, be able to explain the sign of the relationship or how long these effects require to take place. That is, F-test results will not reveal whether changes in the value of a given variable have a positive or negative effect on other variables in the system. Such information will, however, be given by an examination of the VAR's impulse responses and variance decompositions»(Brooks,2008, p298-299).

The impulse response function shows the response of each concerned variable in the linear system to a shock from system variables and the variance decomposition function shows the portion of the variance in the forecast error for each variable due to innovations to all variables in the system (Enders, 1995). The shocks are applied to the error term.

For example, let's use the simplest form of a bivariate VAR:

$Y_{1t} = \beta_{10} + \beta_{11} y_{1t-1} + \alpha_{11} y_{2t-1} + u_{1t}$

$Y_{2t} = \beta_{20} + \beta_{21} y_{2t-1} + \alpha_{21} y_{1t-1} + u_{2t}$

A change in u_{1t} will cause an immediate change in Y_{1t}. That change will cause a change in Y_{2t} in the next period and then a change in Y_{1t} in the period after that etc. With the impulse response function we can examine how long a shock lasts and how it affects the other variables in the system.

Variance Decompositions display the proportion of changes in the dependent variable caused by shocks in their own time series versus shocks to the other variables in the system. A shock to the ith variable will directly affect that variable of course, but it will also be dynamically spread on all the other variables in the system. «*Variance decompositions determine how much of the s-step-ahead forecast error variance of a given variable is explained by innovations to each explanatory variable for s = 1, 2,….»(*Brooks,2008, p300).

It is important to stress that the ordering of the variables plays a critical role in variance decomposition and impulse response function analysis. In most cases, economic theory gives a platform for the right ordering. In any case, the ordering of the variables is crucial to the results of the analysis due to the existence of correlation in the residuals.

4. RESULTS AND ANALYSIS

4.1 Unit root tests

The results of the ADF and the PP tests are presented below. With these tests we test the null hypothesis that a unit root exists against the alternative of no unit roots in the time series. A detailed depiction of the tests will be presented in the appendix section.

For the USA:

ADF panel

variables	p-value	t-statistic
Logcpi - levels	0.9504	-0.924341
Logcpi - first differences	0.0290	-3.086351
Logrealgdp - levels	0.8660	-1.376018
Logrealgdp – first differences	0	-11.59300
Logm1 - levels	0.8561	-1.408903
Logm1 – first differences	0	-7.096265
Logm2 - levels	0.9522	-0.908636
Logm2 – first differences	0	-7.027094

PP panel

variables	p-value	t-statistic
Logcpi - levels	0.9970	0.084582
Logcpi - first differences	0	-5.292410
Logrealgdp - levels	0.9339	-1.051417
Logrealgdp – first differences	0	-11.19043
Logm1 - levels	0.8235	-1.510589
Logm1 – first differences	0	-6.894081
Logm2 - levels	0.9576	-0.858826
Logm2 – first differences	0	-7.020144

The tests confirm that the four time series used in this model for the USA are integrated of order one, or I(1), which means that we can proceed with the co-integration analysis.

For Canada:

<u>ADF panel</u>

variables	p-value	t-statistic
Logcpi - levels	0.9423	-0.989172
Logcpi - first differences	0.1113	-2.523430
Logrealgdp - levels	0.3631	-2.429668
Logrealgdp – first differences	0	-10.43192
Logm1 - levels	0.4338	-2.296245
Logm1 – first differences	0.0001	-5.355784
Logm2 - levels	0.4075	-2.344730
Logm2 – first differences	0.1106	-2.527458

<u>PP panel</u>

variables	p-value	t-statistic
Logcpi - levels	0.9975	0.140730
Logcpi - first differences	0	-8.854798
Logrealgdp - levels	0.2202	-2.744162
Logrealgdp – first differences	0	-10.52494
Logm1 - levels	0.5112	-2.156412
Logm1 – first differences	0	-9.884216
Logm2 - levels	0.5836	-2.025111
Logm2 – first differences	0.0001	-5.312538

In the ADF test, we can observe that logcpi and logm2 seem to be integrated of order two(p-value>5% in first differences), or I(2). On the other hand, the PP procedure corrects these results, and both of these time series seem to be I(1). Since the PP procedure is the stronger one(taking into consideration the possibility of autocorrelated residuals) we will accept the PP results. In other words, the tests confirm that the four time series used in this model for Canada are integrated of order one, or I(1), which means that we can proceed with the co-integration analysis.

4.2 Co-integration, VECMs and VAR models.

4.2.1 USA Models.

We start our co-integrating analysis with two models, one for real GDP and M1, and one for real GDP and M2, both in logarithmic form.

- <u>Real GDP and M1 model</u>

Table 1: Johansen co-integration Real GDP-M1

Sample (adjusted): 1961Q2 2017Q4
Included observations: 227 after adjustments
Trend assumption: Linear deterministic trend
Series: LOGREALGDP LOGM1
Lags interval (in first differences): 1 to 4

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None	0.055441	13.18995	15.49471	0.1080
At most 1	0.001068	0.242595	3.841466	0.6223

Trace test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None	0.055441	12.94735	14.26460	0.0799
At most 1	0.001068	0.242595	3.841466	0.6223

Max-eigenvalue test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

For the trace test, we begin testing the null hypothesis of no co-integrating equation against the alternative of one or more co-integrating equations. We can see that the trace statistic is smaller than the 0.05 critical value(13,18995<15,49471), so we cannot reject the null hypothesis at 5%. We also cannot reject the null hypothesis at 10%, as p-value>0,10. According to the trace test, there is no co-integrating relationship between real GDP and money supply M1.

For the Maximum Eigenvalue test, we can observe that the max-Eigen statistic is smaller than the 0.05 critical value(12,94735<14,26460). As a result, we cannot reject the null hypothesis of no co-integrating equation at 5%. At 10%, we can reject the null hypothesis as p-value=0,0799<0,10. Since we focus our analysis on the 5%

level of significance, we conclude that the Maximum Eigenvalue test shows zero co-integrating equations for the two time series. In other words, the model confirms the idea of money neutrality in the long run, given that M1 doesn't influence the real output of the economy.

Since there is no co-integration, we proceed with the VAR modeling of the variables in order to examine short run dynamics. We use the first differences of the two time series in logarithmic form. The initial task in estimating the VAR model is to determine the optimum order of lag length. This is important since under-parameterization would tend to bias the results and over-parameterization would diminish the power of tests. We used 2 lags based on the results given by most of the lag length criteria (the SC was the only exception), as we can see below:

Table 2: Lag length criteria for VAR model, Real GDP and M1
VAR Lag Order Selection Criteria
Endogenous variables: R_LGREALGDP R_LGM1
Exogenous variables: C
Sample: 1947Q1 2018Q3
Included observations: 223

Lag	LogL	LR	FPE	AIC	SC	HQ
0	1415.169	NA	1.07e-08	-12.67416	-12.64361	-12.66183
1	1489.766	147.1862	5.70e-09	-13.30732	-13.21565*	-13.27031
2	1496.665	13.48838*	5.55e-09*	-13.33332*	-13.18053	-13.27164*
3	1500.039	6.536486	5.58e-09	-13.32770	-13.11380	-13.24135
4	1501.022	1.885978	5.74e-09	-13.30064	-13.02562	-13.18962
5	1505.891	9.257439	5.69e-09	-13.30844	-12.97230	-13.17274
6	1510.309	8.322610	5.67e-09	-13.31219	-12.91494	-13.15183
7	1511.666	2.531088	5.81e-09	-13.28849	-12.83012	-13.10345
8	1512.626	1.772400	5.97e-09	-13.26122	-12.74174	-13.05151

* indicates lag order selected by the criterion

We then proceed with the VAR modeling:

Table 3: VAR model, Real GDP and M1
Vector Autoregression Estimates

Sample (adjusted): 1960Q4 2017Q4

Included observations: 229 after adjustments

Standard errors in ( ) & t-statistics in [ ]

	R_LGREALGDP	R_LGM1
R_LGREALGDP(-1)	0.286506	-0.144103
	(0.06587)	(0.08306)

	[4.34955]	[-1.73487]
R_LGREALGDP(-2)	0.177647	-0.056935
	(0.06591)	(0.08311)
	[2.69540]	[-0.68506]
R_LGM1(-1)	0.010778	0.531311
	(0.05242)	(0.06611)
	[0.20560]	[8.03724]
R_LGM1(-2)	0.035205	0.154372
	(0.05184)	(0.06538)
	[0.67905]	[2.36133]
C	0.003379	0.005973
	(0.00101)	(0.00128)
	[3.33915]	[4.68045]

As we can see in table 3 the real GDP in time t is affected by itself in times t-1 and t-2(as t-stat>1,96 in these cases, so we can reject the null hypothesis that the coefficient is zero) but it is not affected by M1 in time t-1(t-stat=0,20560<1,96) and time t-2(t-stat=0,67905<1,96) at the 5% level of significance. This means that a change in the value of M1 in times t-1 and t-2 will not affect Real GDP in time t. In order to further examine this result we use the Granger causality test:

Table 4: Granger Causality/Block Exogeneity Test, Real GDP and M1.

Null hypothesis	Probability
R_LGM1 does not Granger cause R_LGREALGDP	0,5579
R_LGREALGDP does not Granger cause R_LGM1	0,0880

The Granger causality test confirms the previous results, as for the null hypothesis that R_LGM1 does not Granger cause R_LGREALGDP the p-value is larger than 0,05(in this case 0,5579). So the null hypothesis is not rejected at 5% level of significance. In the same way, we cannot reject the hypothesis that R_LGREALGDP does not Granger cause M1, as p-value=0,0880>0,05.

It is obvious that this model confirms the money neutrality proposition in the long run and the short run, meaning that M1 money supply does not affect the Real output of the USA economy.

- <u>Real GDP and M2 model</u>

Table 5:Johansen co-integration Real GDP-M2

Sample (adjusted): 1960Q4 2017Q4
Included observations: 229 after adjustments
Trend assumption: Linear deterministic trend
Series: LOGREALGDP LOGM2
Lags interval (in first differences): 1 to 6

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None	0.039384	12.44733	15.49471	0.1367
At most 1	0.014074	3.245901	3.841466	0.0716

Trace test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None	0.039384	9.201430	14.26460	0.2698
At most 1	0.014074	3.245901	3.841466	0.0716

Max-eigenvalue test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

For the trace test, we begin testing the null hypothesis of no co-integrating equation against the alternative of one or more co-integrating equations. We can see that the trace statistic is smaller than the 0.05 critical value(12,44733<15,49471), so we cannot reject the null hypothesis at 5%. We also cannot reject the null hypothesis at 10%, as p-value>0,10. According to the trace test, there is no co-integrating relationship between real GDP and money supply M2.

For the Maximum Eigenvalue test, we can observe that the max-Eigen statistic is smaller than the 0.05 critical value(9,201430<14,26460). As a result, we cannot reject the null hypothesis of no co-integrating equation at 5%. We also cannot reject the null hypothesis at 10% levels of significance. Since we focus our analysis on the 5% level of significance, we conclude that the Maximum Eigenvalue test shows zero co-integrating equations for the two time series. In other words, the model confirms the idea of money neutrality in the long run, given that M2 doesn’t influence the real output of the economy.

Since there is no co-integration, we proceed with the VAR modeling of the variables in order to examine short run dynamics. We use the first differences of the two time series in logarithmic form. We used 3 lags based on the results given by AIC(Akaike Information Criterion) and LR criteria.

Table 6: **Lag length criteria for VAR model, Real GDP and M2**

VAR Lag Order Selection Criteria
Endogenous variables: R_LGREALGDP R_LGM2
Exogenous variables: C
Sample: 1947Q1 2018Q3
Included observations: 227

Lag	LogL	LR	FPE	AIC	SC	HQ
0	1540.206	NA	4.46e-09	-13.55248	-13.52230	-13.54030
1	1621.254	159.9526	2.26e-09	-14.23131	-14.14078*	-14.19478*
2	1625.609	8.518732	2.25e-09	-14.23444	-14.08356	-14.17356
3	1630.994	10.43757	2.23e-09*	-14.24664*	-14.03541	-14.16141
4	1632.333	2.572845	2.28e-09	-14.22320	-13.95162	-14.11361
5	1637.869	10.53440*	2.25e-09	-14.23673	-13.90480	-14.10279
6	1638.377	0.959003	2.32e-09	-14.20597	-13.81368	-14.04768
7	1639.268	1.662632	2.39e-09	-14.17857	-13.72593	-13.99592
8	1640.882	2.987068	2.44e-09	-14.15755	-13.64456	-13.95055

* indicates lag order selected by the criterion

We then proceed with the VAR modeling:

Table 7: VAR model, Real GDP and M2

Vector Autoregression Estimates
Sample (adjusted): 1960Q1 2017Q4
Included observations: 232 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	R_LGREALGDP	R_LGM2
R_LGREALGDP(-1)	0.249676	-0.143910
	(0.06657)	(0.05344)
	[3.75064]	[-2.69309]
R_LGREALGDP(-2)	0.177812	0.017677
	(0.06752)	(0.05420)
	[2.63363]	[0.32616]
R_LGREALGDP(-3)	0.018196	0.062086
	(0.06474)	(0.05196)
	[0.28109]	[1.19476]
R_LGM2(-1)	0.137637	0.633080
	(0.08079)	(0.06485)
	[1.70361]	[9.76165]
R_LGM2(-2)	0.116594	-0.075916

	(0.09654)	(0.07750)
	[1.20772]	[-0.97961]
R_LGM2(-3)	-0.072219	0.176403
	(0.08192)	(0.06576)
	[-0.88162]	[2.68267]
C	0.001148	0.004901
	(0.00140)	(0.00112)
	[0.82095]	[4.36547]

As we can see in table 8 the real GDP in time t is affected by itself in times t-1 and t-2(as t-stat>1,96 in these cases, so we can reject the null hypothesis that the coefficient is zero at 5%) but it is not affected by M2 in time t-1(t-stat=1,70361<1,96), in time t-2(t-stat=1,20772<1,96) and time t-3(t-stat=-0,88162<1,96). This means that a change in the value of M2 in times t-1, t-2 and t-3 will not affect Real GDP in time t at 5%. We can also see that M2 in t-1 does affect Real GDP in time t at 10% level of significance(t-stat=1,70361>1,65). In order to further examine these results we use the Granger causality test:

Table 8: Granger Causality/Block Exogeneity Test, Real GDP and M2.

Null hypothesis	Probability
R_LGM2 does not Granger cause R_LGREALGDP	0.0161
R_LGREALGDP does not Granger cause R_LGM2	0.0502

For the null hypothesis that R_LGM2 does not Granger cause R_LGREALGDP the p-value is smaller than 0,05(in this case 0,0161). So the null hypothesis is rejected at 5% level of significance. In the same way, we narrowly accept the hypothesis that R_LGREALGDP does not Granger cause M2, as p-value=0,0502>0,05. So in this case, M2 does Granger Cause Real GDP in the short run.

This model confirms the money neutrality proposition in the long run, but not in the short run since the Granger causality test rejects this hypothesis.

Obviously, since the two models above have only two variables, the results may suffer from omitted variable bias. In order to strengthen the conclusions, we proceed with the creation of two new models, adding the price variable (CPI=Consumer's Price Index) to the analysis.

- Real GDP, M1 and CPI model

Table 9: Johansen co-integration, Real GDP-M1-CPI

Sample (adjusted): 1961Q2 2017Q4
Included observations: 227 after adjustments
Trend assumption: Linear deterministic trend
Series: LOGREALGDP LOGM1 LOGCPI
Lags interval (in first differences): 1 to 4

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None	0.097209	28.84310	29.79707	0.0641
At most 1	0.022882	5.629221	15.49471	0.7388
At most 2	0.001649	0.374565	3.841466	0.5405

Trace test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.097209	23.21388	21.13162	0.0251
At most 1	0.022882	5.254657	14.26460	0.7094
At most 2	0.001649	0.374565	3.841466	0.5405

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

For the trace test, we begin testing the null hypothesis of no co-integrating equation against the alternative of one or more co-integrating equations. We can see that the trace statistic is smaller than the 0.05 critical value(28,84310<29,79707), so we cannot reject the null hypothesis at 5%.On the other hand, we can reject the null hypothesis at 10%, as p-value=0,0641<0,10. According to the trace test, there is no co-integrating relationship between real GDP, money supply M1 and the CPI at the 5% level of significance.

For the Maximum Eigenvalue test, we can observe that the max-Eigen statistic is greater than the 0.05 critical value(23,21388>21,13162). As a result, we can reject the null hypothesis of no co-integrating equation at 5% and 10% level of significance. For the null hypothesis that there is at most one co-integrating equation against the alternative that there are more than one co-integrating equations, we cannot reject the null hypothesis since the max Eigen statistic is smaller than the 0,05 critical value(5,254657<14,26460). Since we focus our analysis on the 5% level of significance, we conclude that the Maximum Eigenvalue test shows one co-integrating equation for the three time series.

Since the Maximum Eigenvalue test gives one co-integrating equation between the three variables, we proceed with the Vector Error Correction modeling:

Table 10:VECM, Real GDP-M1-CPI

Vector Error Correction Estimates
Sample (adjusted): 1961Q1 2017Q4
Included observations: 228 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1		
LOGREALGDP(-1)	1.000000		
LOGM1(-1)	-0.141455		
	(0.16147)		
	[-0.87603]		
LOGCPI(-1)	-0.363610		
	(0.20354)		
	[-1.78640]		
C	-3.781252		
Error Correction:	**D(LOGREALGDP)**	**D(LOGM1)**	**D(LOGCPI)**
CointEq1	-0.017720	-0.001661	0.001824
	(0.00394)	(0.00537)	(0.00257)
	[-4.49921]	[-0.30939]	[0.71057]
D(LOGREALGDP(-1))	0.214537	-0.146292	0.082304
	(0.06521)	(0.08889)	(0.04250)
	[3.28995]	[-1.64582]	[1.93658]
D(LOGREALGDP(-2))	0.138578	-0.072519	-0.002464
	(0.06555)	(0.08935)	(0.04272)
	[2.11404]	[-0.81160]	[-0.05767]
D(LOGREALGDP(-3))	-0.019952	-0.027236	0.059446
	(0.06275)	(0.08554)	(0.04090)
	[-0.31795]	[-0.31841]	[1.45348]
D(LOGM1(-1))	0.009797	0.516732	0.037315
	(0.05177)	(0.07056)	(0.03374)
	[0.18925]	[7.32318]	[1.10603]
D(LOGM1(-2))	0.087640	0.110448	0.003891
	(0.05857)	(0.07984)	(0.03817)
	[1.49629]	[1.38339]	[0.10194]
D(LOGM1(-3))	-0.104558	0.076190	0.009146
	(0.05150)	(0.07020)	(0.03357)
	[-2.03021]	[1.08532]	[0.27249]

D(LOGCPI(-1))	-0.147880	-0.012935	0.578985
	(0.10300)	(0.14040)	(0.06713)
	[-1.43568]	[-0.09213]	[8.62464]
D(LOGCPI(-2))	-0.070577	0.076858	-0.018870
	(0.12051)	(0.16427)	(0.07854)
	[-0.58564]	[0.46788]	[-0.24025]
D(LOGCPI(-3))	-0.088850	-0.054327	0.337097
	(0.10531)	(0.14355)	(0.06863)
	[-0.84371]	[-0.37847]	[4.91151]
C	0.008013	0.006027	-0.000789
	(0.00135)	(0.00184)	(0.00088)
	[5.92415]	[3.26874]	[-0.89513]

We can see that the REALGDP has a negative adjustment coefficient(-0.017720) and is also statistically significant(t-statistic=-4,49921).A significant negative coefficient shows that there is a tendency from short term fluctuations to long term equilibrium condition. For M1, the adjustment coefficient has the right sign but is not statistically significant(t-stat=-0.30939). For CPI, the adjustment coefficient is both non-negative and statistically insignificant(t-stat=0,71057).

In order to check the significance of the M1 coefficient in the model we use the Likelihood Ratio test(LR test). We perform the test by imposing a restriction on the previous Error Correction Model that the M1 coefficient is equal to zero. The results can be seen below:

Table 11: Likelihood ratio test on the M1 cointegrating coefficient

Cointegration Restrictions:

B(1,2)=0

Convergence achieved after 5 iterations.

Not all cointegrating vectors are identified

LR test for binding restrictions (rank = 1):

Chi-square(1)	0.526669
Probability	0.468011

The p-value in this case is much bigger than 0,05(0,468011). In this case we cannot reject the null hypothesis that the M1 coefficient is zero. In other words, this model confirms the long run neutrality proposition even with the existence of a co-integrating relationship.

For the examination of short run dynamics,we use the Granger causality test:

Table 12: Granger Causality, Real GDP-M1-CPI model

Null hypothesis	Probability
R_LGM1 does not Granger cause R_LGREALGDP	0.2092
R_LGCPI does not Granger cause R_LGREALGDP	0.0005
R_LGREALGDP does not Granger cause R_LGM1	0.1596
R_LGCPI does not Granger cause R_LGM1	0.9677
R_LGREALGDP does not Granger cause R_LGCPI	0.0468
R_LGM1 does not Granger cause R_LGCPI	0.3329

For the null hypothesis that R_LGM1 does not Granger cause R_LGREALGDP the p-value is larger than 0,05(in this case 0,2092). So the null hypothesis is accepted and therefore M1 does not Granger cause real GDP in the short run. We can also see that R_LGM1 does not Granger cause R_LGCPI, which means that there is no short run causality between M1 and CPI.

- <u>Real GDP, M2 and CPI model</u>

Table 13: Johansen co-integration Real GDP-M1-CPI

Sample (adjusted): 1961Q2 2017Q4
Included observations: 227 after adjustments
Trend assumption: Linear deterministic trend
Series: LOGREALGDP LOGM2 LOGCPI
Lags interval (in first differences): 1 to 4

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.091091	30.40892	29.79707	0.0425
At most 1	0.034883	8.728102	15.49471	0.3911
At most 2	0.002939	0.668150	3.841466	0.4137

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.091091	21.68082	21.13162	0.0418
At most 1	0.034883	8.059952	14.26460	0.3727
At most 2	0.002939	0.668150	3.841466	0.4137

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

For the trace test, we begin testing the null hypothesis of no co-integrating equation against the alternative of one or more co-integrating equations. We can see that the trace statistic is greater than the 0.05 critical value(30,40892>29,79707), so we reject the null hypothesis at 5%.For the null hypothesis that there is one co-integrating equation, we can see that the trace statistic is smaller than the 0,05 critical value(8,728102<15,49471). According to the trace test, there is no co-integrating relationship between real GDP, money supply M2 and the CPI at the 5% level of significance.

For the Maximum Eigenvalue test, we can observe that the max-Eigen statistic is greater than the 0.05 critical value(21,68082>21,13162). As a result, we can reject the null hypothesis of no co-integrating equation at 5% and 10% level of significance. For the null hypothesis that there is at most one co-integrating equation against the alternative that there are more than one co-integrating equations, we cannot reject the null hypothesis since the max Eigen statistic is smaller than the 0,05 critical value(8,059952<14,26460). Since we focus our analysis on the 5% level of significance, we conclude that the Maximum Eigenvalue test shows one co-integrating equation for the three time series.

Since both tests give one co-integrating equation between the three variables, we proceed with the Vector Error Correction modeling:

Table 14:VECM, Real GDP-M2-CPI

Vector Error Correction Estimates
Sample (adjusted): 1961Q1 2017Q4
Included observations: 228 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1

LOGREALGDP(-1)	1.000000		
LOGM2(-1)	-0.008754 (0.20339) [-0.04304]		
LOGCPI(-1)	-0.522026 (0.31303) [-1.66763]		
C	-6.953122		

Error Correction:	D(LOGREALGDP)	D(LOGM2)	D(LOGCPI)
CointEq1	-0.015143 (0.00370) [-4.09197]	-0.003609 (0.00316) [-1.14316]	0.002249 (0.00244) [0.92209]
D(LOGREALGDP(-1))	0.182975 (0.06565) [2.78714]	-0.115183 (0.05601) [-2.05663]	0.073389 (0.04327) [1.69603]
D(LOGREALGDP(-2))	0.141268 (0.06545) [2.15843]	-0.000576 (0.05583) [-0.01031]	-0.008307 (0.04314) [-0.19255]
D(LOGREALGDP(-3))	-0.010972 (0.06227) [-0.17620]	0.066173 (0.05312) [1.24570]	0.053587 (0.04104) [1.30564]
D(LOGM2(-1))	0.156221 (0.08252) [1.89304]	0.580419 (0.07040) [8.24452]	0.039070 (0.05439) [0.71829]
D(LOGM2(-2))	0.061535 (0.09502) [0.64763]	-0.004644 (0.08106) [-0.05729]	0.018132 (0.06263) [0.28952]
D(LOGM2(-3))	-0.046280 (0.08059) [-0.57425]	0.130493 (0.06875) [1.89799]	0.006544 (0.05312) [0.12320]
D(LOGCPI(-1))	-0.123889 (0.10354) [-1.19651]	-0.164402 (0.08833) [-1.86120]	0.573042 (0.06825) [8.39657]
D(LOGCPI(-2))	-0.101964 (0.11921) [-0.85532]	0.288246 (0.10170) [2.83432]	-0.006799 (0.07858) [-0.08653]
D(LOGCPI(-3))	-0.124417 (0.10825) [-1.14936]	-0.037328 (0.09235) [-0.40422]	0.321297 (0.07135) [4.50315]
C	0.005614 (0.00154) [3.65213]	0.004449 (0.00131) [3.39264]	-0.000897 (0.00101) [-0.88514]

We can see that the REALGDP has a negative adjustment coefficient(-0.015143) and is also statistically significant(t-statistic=-4,09197).A significant negative coefficient shows that there is a tendency from short term fluctuations to long term equilibrium condition. For M2, the adjustment coefficient has the right sign but is not statistically significant(t-stat=-1,14316). For CPI, the adjustment coefficient is both non-negative and statistically insignificant(t-stat=0,92209).

In order to check the significance of the M2 coefficient in the model we use the Likelihood Ratio test(LR test). We perform the test by imposing a restriction on the previous Error Correction Model that the M2 coefficient is equal to zero. The results can be seen below:

Table 15: Likelihood ratio test on the M2 cointegrating coefficient.

Cointegration Restrictions:
B(1,2)=0
Convergence achieved after 3 iterations.
Not all cointegrating vectors are identified
LR test for binding restrictions (rank = 1):

Chi-square(1)	0.000790
Probability	0.977583

The p-value in this case is much bigger than 0,05(0,977583). In this case we cannot reject the null hypothesis that the M2 coefficient is zero. In other words, this model confirms the long run neutrality proposition even with the existence of a co-integrating relationship.

For the examination of short run dynamics,we use the Granger causality test:

Table 16: Granger Causality test, Real GDP-M2-CPI model

Null hypothesis	Probability
R_LGM2 does not Granger cause R_LGREALGDP	0.0334
R_LGCPI does not Granger cause R_LGREALGDP	0.0001
R_LGREALGDP does not Granger cause R_LGM2	0.1689
R_LGCPI does not Granger cause R_LGM2	0.0226
R_LGREALGDP does not Granger cause R_LGCPI	0.1085
R_LGM2 does not Granger cause R_LGCPI	0.5730

For the null hypothesis that R_LGM2 does not Granger cause R_LGREALGDP the p-value is lesser than 0,05(in this case 0,0334). So the null hypothesis is rejected and therefore M2 does Granger cause real GDP in the short run.

4.2.2 Canada models

We start our co-integrating analysis with two models, one for real GDP and M1, and one for real GDP and M2, both in logarithmic form.

- <u>Real GDP and M1 model</u>

Table 1: Johansen co-integration, Real GDP-M1

Sample (adjusted): 1962Q4 2012Q1
Included observations: 198 after adjustments
Trend assumption: Linear deterministic trend
Series: LOGREALGDP LOGM1
Lags interval (in first differences): 1 to 6

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None	0.062793	13.81527	15.49471	0.0881
At most 1	0.004911	0.974738	3.841466	0.3235

Trace test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None	0.062793	12.84054	14.26460	0.0829
At most 1	0.004911	0.974738	3.841466	0.3235

Max-eigenvalue test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

For the trace test, we begin testing the null hypothesis of no co-integrating equation against the alternative of one or more co-integrating equations. We can see that the trace statistic is smaller than the 0.05 critical value(13,81527<15,49471), so we cannot reject the null hypothesis at 5%. On the other hand, we can reject the null hypothesis at 10%, as p-value=0.0881<0,10. According to the trace test, there is no co-integrating relationship between real GDP and money supply M1 at the 5% level of significance.

For the Maximum Eigenvalue test, we can observe that the max-Eigen statistic is smaller than the 0.05 critical value(12,84054<14,26460). As a result, we cannot reject the null hypothesis of no co-integrating equation at 5%. At 10%, we can reject the null hypothesis as p-value=0,0829<0,10. Since we focus our analysis on the 5% level of significance, we conclude that the Maximum Eigenvalue test shows zero co-integrating equations for the two time series. In other words, the model confirms the idea of money neutrality in the long run, given that M1 doesn't influence the real output of the economy.

Since there is no co-integration, we proceed with the VAR modeling of the variables in order to examine short run dynamics. We use the first differences of the two time series in logarithmic form. The initial task in estimating the VAR model is to determine the optimum order of lag length. This is important since under-parameterization would tend to bias the results and over-parameterization would diminish the power of tests. We used 5 lags based on the results given by most of the lag length criteria (the SC and HQ are the exceptions), as we can see below:

Table 2:Lag length criteria for VAR model, Real GDP-M1

VAR Lag Order Selection Criteria
Endogenous variables: R_LGREALGDP
R_LGM1
Exogenous variables: C
Sample: 1960Q1 2018Q1
Included observations: 196

Lag	LogL	LR	FPE	AIC	SC	HQ
0	1198.518	NA	1.71e-08	-12.20937	-12.17592	-12.19582
1	1235.057	71.96066	1.23e-08	-12.54140	-12.44105*	-12.50078*
2	1238.767	7.229519	1.23e-08	-12.53844	-12.37119	-12.47073
3	1245.830	13.62120	1.19e-08	-12.56969	-12.33554	-12.47490
4	1252.934	13.55709	1.15e-08	-12.60137	-12.30032	-12.47949
5	1259.184	11.79695*	1.13e-08*	-12.62432*	-12.25637	-12.47536
6	1261.952	5.169271	1.14e-08	-12.61175	-12.17690	-12.43571
7	1263.417	2.706106	1.17e-08	-12.58589	-12.08414	-12.38276
8	1265.942	4.612231	1.19e-08	-12.57084	-12.00219	-12.34062

* indicates lag order selected by the criterion

We then proceed with the VAR modeling:

Table 3:VAR Model, Real GDP-M1

Vector Autoregression Estimates
Sample (adjusted): 1962Q3 2012Q1
Included observations: 199 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	R_LGREALGDP	R_LGM1
R_LGREALGDP(-1)	0.301900	-0.069914
	(0.07268)	(0.12067)
	[4.15394]	[-0.57938]
R_LGREALGDP(-2)	0.033802	-0.168754
	(0.07539)	(0.12517)
	[0.44838]	[-1.34822]
R_LGREALGDP(-3)	0.113876	-0.196164
	(0.07494)	(0.12442)
	[1.51964]	[-1.57661]
R_LGREALGDP(-4)	0.094466	0.051912
	(0.07491)	(0.12438)
	[1.26108]	[0.41738]
R_LGREALGDP(-5)	-0.045457	0.306766
	(0.07129)	(0.11837)
	[-0.63763]	[2.59160]
R_LGM1(-1)	0.057194	0.442114
	(0.04257)	(0.07068)
	[1.34357]	[6.25523]
R_LGM1(-2)	0.107632	-0.053365
	(0.04560)	(0.07571)
	[2.36027]	[-0.70482]
R_LGM1(-3)	-0.058554	0.299274
	(0.04442)	(0.07375)
	[-1.31821]	[4.05788]
R_LGM1(-4)	0.018920	-0.267064
	(0.04563)	(0.07577)
	[0.41460]	[-3.52477]
R_LGM1(-5)	-0.034420	0.150805
	(0.04240)	(0.07041)
	[-0.81169]	[2.14191]
C	0.002145	0.009337
	(0.00150)	(0.00249)
	[1.43095]	[3.75138]

As we can see in table 3 the real GDP in time t is affected by itself in time t-1 (as t-stat=4,15394>1,96 in this case, so we can reject the null hypothesis that the coefficient is zero) and is also affected by M1 in time t-2(t-stat=2,36027>1,96) at the 5% level of significance. This means that a change in the value of M1 in time t-2 will affect Real GDP in time t. In order to further examine this result we use the Granger causality test:

Table 4: Granger Causality/Block Exogeneity Test, Real GDP and M1.

Null hypothesis	Probability
R_LGM1 does not Granger cause R_LGREALGDP	0,0306
R_LGREALGDP does not Granger cause R_LGM1	0,0298

For the null hypothesis that R_LGM1 does not Granger cause R_LGREALGDP the p-value is smaller than 0,05(in this case 0,0306). So the null hypothesis is rejected at 5% level of significance. In the same way, we reject the hypothesis that R_LGREALGDP does not Granger cause R_LGM1, as p-value=0,0298<0,05. So in this case, M1 does Granger Cause Real GDP in the short run.

It is obvious that this model confirms the money neutrality proposition in the long run but not in the short run, and thus confirming the monetarist idea of the neutrality concept.

- <u>Real GDP and M2 model</u>

Table 5:Johansen co-integration Real GDP-M2

Sample (adjusted): 1969Q4 2012Q1
Included observations: 170 after adjustments
Trend assumption: Linear deterministic trend
Series: LOGREALGDP LOGM2
Lags interval (in first differences): 1 to 6

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.072832	15.54189	15.49471	0.0492
At most 1	0.015678	2.686362	3.841466	0.1012

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**

None	0.072832	12.85552	14.26460	0.0825
At most 1	0.015678	2.686362	3.841466	0.1012

Max-eigenvalue test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

For the trace test, we begin testing the null hypothesis of no co-integrating equation against the alternative of one or more co-integrating equations. We can see that the trace statistic is narrowly greater than the 0.05 critical value(15,54189<15,49471), so we c reject the null hypothesis at 5%. For the null hypothesis that there is at most one co-integrating equation against the alternative that there are more than one co-integrating equations, we cannot reject the null hypothesis because the trace statistic is smaller than the 0,05 critical value(2,686362<3,841466). According to the trace test, there is one co-integrating relationship between real GDP and money supply M2.

For the Maximum Eigenvalue test, we can observe that the max-Eigen statistic is smaller than the 0.05 critical value(12,85552<14,26460). As a result, we cannot reject the null hypothesis of no co-integrating equation at 5%. On the other hand, we can reject the null hypothesis at 10% levels of significance. Since we focus our analysis on the 5% level of significance, we conclude that the Maximum Eigenvalue test shows zero co-integrating equations for the two time series.

Since the Trace test and the Maximum Eigenvalue test give different results, we proceed with the Vector Error Correction Model analysis. Our focus is on the two error-correction coefficients. If neither of the two is negative and statistically significant, we will accept the Maximum Eigenvalue results that there is no-cointegrating relationship between the variables(the Maximum Eigenvalue test is considered the more efficient one). If there is one or more negative and statistically significant coefficients, we will accept the Trace test results that there is a cointegrating relationship between the variables.

Table 6:VECM, Real GDP and M2

Vector Error Correction Estimates
Sample (adjusted): 1969Q3 2012Q1
Included observations: 171 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
LOGREALGDP(-1)	1.000000
LOGM2(-1)	-0.609713 (0.06711) [-9.08473]

C	2.405801	

Error Correction:	D(LOGREALGDP)	D(LOGM2)
CointEq1	0.010347	0.004873
	(0.00296)	(0.00279)
	[3.49720]	[1.74573]
D(LOGREALGDP(-1))	0.330240	-0.035509
	(0.07824)	(0.07381)
	[4.22087]	[-0.48107]
D(LOGREALGDP(-2))	-0.079550	0.040512
	(0.08251)	(0.07784)
	[-0.96418]	[0.52048]
D(LOGREALGDP(-3))	0.114725	0.097751
	(0.08258)	(0.07791)
	[1.38919]	[1.25466]
D(LOGREALGDP(-4))	-0.052471	0.020285
	(0.08326)	(0.07855)
	[-0.63018]	[0.25823]
D(LOGREALGDP(-5))	-0.169619	0.109650
	(0.07767)	(0.07327)
	[-2.18387]	[1.49645]
D(LOGM2(-1))	0.031794	0.622837
	(0.08122)	(0.07662)
	[0.39147]	[8.12893]
D(LOGM2(-2))	-0.064586	-0.022296
	(0.09523)	(0.08984)
	[-0.67820]	[-0.24817]
D(LOGM2(-3))	-0.010851	0.272094
	(0.09305)	(0.08779)
	[-0.11661]	[3.09944]
D(LOGM2(-4))	-0.013351	-0.263730
	(0.09448)	(0.08913)
	[-0.14132]	[-2.95898]
D(LOGM2(-5))	-0.128736	0.192795
	(0.08055)	(0.07600)
	[-1.59813]	[2.53692]
C	0.010161	0.002395
	(0.00199)	(0.00188)
	[5.10146]	[1.27458]

We can see in the CointEq1 row that none of the two coefficients is negative and statistically significant. The D(LOGREALGDP) coefficient is statistically significant but positive, and the D(LOGM2) coefficient is statistically insignificant and positive. We therefore accept the Maximum Eigenvalue test result that there is no co-integrating relationship between the variables and we proceed with the VAR modeling for the examination of short run dynamics. We begin with the Lag Length Criteria panel:

Table 7: Lag length criteria for VAR model, Real GDP-M2

VAR Lag Order Selection Criteria
Endogenous variables: R_LGREALGDP
R_LGM2
Exogenous variables: C
Sample: 1960Q1 2018Q1
Included observations: 168

Lag	LogL	LR	FPE	AIC	SC	HQ
0	1062.480	NA	1.13e-08	-12.62477	-12.58758	-12.60967
1	1172.652	216.4089	3.19e-09	-13.88872	-13.77715*	-13.84344*
2	1178.253	10.86744	3.13e-09	-13.90777	-13.72182	-13.83230
3	1185.382	13.66458	3.01e-09	-13.94502	-13.68469	-13.83937
4	1188.299	5.521871	3.05e-09	-13.93213	-13.59742	-13.79629
5	1197.853	17.85654*	2.86e-09	-13.99825	-13.58916	-13.83222
6	1202.549	8.664407	2.83e-09*	-14.00653*	-13.52306	-13.81031
7	1204.039	2.714510	2.92e-09	-13.97665	-13.41880	-13.75025
8	1205.489	2.607112	3.01e-09	-13.94630	-13.31407	-13.68971

* indicates lag order selected by the criterion

We accept the conclusions of the FPE and AIC that the optimum number of lag length is six. We then proceed with the VAR modeling:

Table 8:VAR modeling, Real GDP-M2

Vector Autoregression Estimates
Sample (adjusted): 1969Q4 2012Q1
Included observations: 170 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	R_LGREALGDP	R_LGM2
R_LGREALGDP(-1)	0.419701	0.010323
	(0.07831)	(0.07239)
	[5.35957]	[0.14260]
R_LGREALGDP(-2)	-0.035056	0.062992
	(0.08369)	(0.07737)
	[-0.41887]	[0.81419]
R_LGREALGDP(-3)	0.130827	0.112689
	(0.08429)	(0.07792)
	[1.55213]	[1.44621]
R_LGREALGDP(-4)	-0.021998	0.044960
	(0.08547)	(0.07901)
	[-0.25738]	[0.56904]
R_LGREALGDP(-5)	-0.202741	0.115946
	(0.08514)	(0.07871)
	[-2.38130]	[1.47315]
R_LGREALGDP(-6)	0.181267	0.094558
	(0.07938)	(0.07338)
	[2.28355]	[1.28857]

R_LGM2(-1)	0.066733 (0.08563) [0.77934]	0.601018 (0.07916) [7.59262]
R_LGM2(-2)	-0.070424 (0.09989) [-0.70503]	-0.001892 (0.09234) [-0.02048]
R_LGM2(-3)	0.020808 (0.09760) [0.21319]	0.259013 (0.09023) [2.87069]
R_LGM2(-4)	-0.014826 (0.09797) [-0.15134]	-0.233526 (0.09057) [-2.57848]
R_LGM2(-5)	-0.023729 (0.09902) [-0.23965]	0.168933 (0.09153) [1.84557]
R_LGM2(-6)	-0.036834 (0.08307) [-0.44340]	0.079885 (0.07679) [1.04025]
C	0.004989 (0.00151) [3.30221]	-0.000637 (0.00140) [-0.45576]

As we can see in table 9, the real GDP in time t is affected by itself in time t-1 (as t-stat=5,35957>1,96 in this case, so we can reject the null hypothesis that the coefficient is zero), t-5(as t-stat=-2,38130>1,96 in absolute value) and t-6(as t-stat=-2,28355>1,96). On the other hand, it is not affected by M2 at any time. This means that a change in the value of M2 at any time up to six lags will not affect Real GDP in time t. In order to further examine this result we use the Granger causality test:

Table 9: Granger Causality/Block Exogeneity Test, Real GDP and M2

Null hypothesis	Probability
R_LGM2 does not Granger cause R_LGREALGDP	0.8580
R_LGREALGDP does not Granger cause R_LGM2	0.0085

The Granger causality test confirms the previous results, as for the null hypothesis that R_LGM2 does not Granger cause R_LGREALGDP the p-value is larger than 0,05(in this case 0,8580). So the null hypothesis is not rejected at 5% level of significance. On the other hand, we reject the hypothesis that R_LGREALGDP does not Granger cause M2, as p-value=0,0085<0,05.

It is obvious that this model confirms the money neutrality proposition in the long run and the short run, meaning that M2 money supply does not affect the Real output of the Canadian economy.

- Real GDP, M1 and CPI model

Table 10:Johansen co-integration, Real GDP-M1-CPI

Sample (adjusted): 1962Q4 2012Q1
Included observations: 198 after adjustments
Trend assumption: Linear deterministic trend
Series: LOGREALGDP LOGM1 LOGCPI
Lags interval (in first differences): 1 to 6

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None	0.066469	22.43105	29.79707	0.2751
At most 1	0.036473	8.812488	15.49471	0.3830
At most 2	0.007326	1.455837	3.841466	0.2276

Trace test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None	0.066469	13.61856	21.13162	0.3971
At most 1	0.036473	7.356651	14.26460	0.4478
At most 2	0.007326	1.455837	3.841466	0.2276

Max-eigenvalue test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

For the trace test, we begin testing the null hypothesis of no co-integrating equation against the alternative of one or more co-integrating equations. We can see that the trace statistic is smaller than the 0.05 critical value(22,43105<29,79707), so we cannot reject the null hypothesis at 5%. We also cannot reject the null hypothesis at 10%, as p-value=0,2751>0,10. According to the trace test, there is no co-integrating relationship between real GDP, money supply M1 and CPI.

For the Maximum Eigenvalue test, we can observe that the max-Eigen statistic is smaller than the 0.05 critical value(13,61856<21,13162). As a result, we cannot reject the null hypothesis of no co-integrating equation at 5%. We also can't reject the null hypothesis at the 10% level of significance since p-value=0,3971>0,10. Since we focus our analysis on the 5% level of significance, we conclude that the Maximum Eigenvalue test shows zero co-integrating equations for the three time series. In other words, the model confirms the idea of money neutrality in the long run.

Since there is no co-integration, we proceed with the VAR modeling of the three variables. We begin with the Lag Length criteria panel.

Table 11: Lag Length Criteria for VAR modeling, Real GDP-M1-CPI

VAR Lag Order Selection Criteria
Endogenous variables: R_LGREALGDP
R_LGM1 R_LGCPI
Exogenous variables: C
Sample: 1960Q1 2018Q1
Included observations: 196

Lag	LogL	LR	FPE	AIC	SC	HQ
0	1839.838	NA	1.45e-12	-18.74325	-18.69307	-18.72293
1	1933.277	183.0643	6.14e-13	-19.60487	-19.40417*	-19.52362*
2	1942.858	18.47730	6.11e-13	-19.61080	-19.25957	-19.46860
3	1959.904	32.35243	5.63e-13	-19.69290	-19.19115	-19.48977
4	1977.633	33.10567	5.15e-13	-19.78197	-19.12969	-19.51789
5	1991.363	25.21911*	4.91e-13*	-19.83024*	-19.02743	-19.50522
6	1998.546	12.97326	5.00e-13	-19.81170	-18.85837	-19.42574
7	2001.008	4.370568	5.35e-13	-19.74498	-18.64112	-19.29808
8	2008.809	13.61205	5.43e-13	-19.73274	-18.47836	-19.22491

* indicates lag order selected by the criterion

We choose five lags and we proceed with the VAR modeling of the variables. We use first differences of the variables in logarithmic form:

Table 12: VAR model, Real GDP-M1-CPI

Vector Autoregression Estimates
Sample (adjusted): 1962Q3 2012Q1
Included observations: 199 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	R_LGREALGDP	R_LGM1	R_LGCPI
R_LGREALGDP(-1)	0.310395	-0.078808	0.062697
	(0.07410)	(0.12444)	(0.05854)
	[4.18868]	[-0.63329]	[1.07098]
R_LGREALGDP(-2)	0.000810	-0.166202	-0.044946
	(0.07755)	(0.13023)	(0.06126)
	[0.01045]	[-1.27623]	[-0.73365]
R_LGREALGDP(-3)	0.133385	-0.202114	0.084543
	(0.07678)	(0.12893)	(0.06066)
	[1.73727]	[-1.56758]	[1.39383]
R_LGREALGDP(-4)	0.055294	0.077864	-0.001797

	(0.07741)	(0.13000)	(0.06116)
	[0.71428]	[0.59896]	[-0.02939]
R_LGREALGDP(-5)	-0.009713	0.324969	0.144728
	(0.07381)	(0.12394)	(0.05831)
	[-0.13161]	[2.62193]	[2.48217]
R_LGM1(-1)	0.049322	0.441023	0.043403
	(0.04269)	(0.07169)	(0.03372)
	[1.15538]	[6.15204]	[1.28699]
R_LGM1(-2)	0.108739	-0.043949	-0.004295
	(0.04573)	(0.07680)	(0.03613)
	[2.37760]	[-0.57224]	[-0.11887]
R_LGM1(-3)	-0.050734	0.304641	-0.017858
	(0.04450)	(0.07473)	(0.03516)
	[-1.14006]	[4.07652]	[-0.50798]
R_LGM1(-4)	0.024973	-0.267244	0.002114
	(0.04578)	(0.07689)	(0.03617)
	[0.54546]	[-3.47588]	[0.05846]
R_LGM1(-5)	-0.034011	0.146938	0.067276
	(0.04268)	(0.07167)	(0.03371)
	[-0.79693]	[2.05027]	[1.99543]
R_LGCPI(-1)	0.033056	-0.097521	0.341574
	(0.09199)	(0.15447)	(0.07267)
	[0.35935]	[-0.63131]	[4.70035]
R_LGCPI(-2)	-0.114708	-0.087136	0.043344
	(0.09432)	(0.15839)	(0.07451)
	[-1.21616]	[-0.55013]	[0.58171]
R_LGCPI(-3)	0.030683	-0.044789	0.155671
	(0.09229)	(0.15499)	(0.07291)
	[0.33246]	[-0.28899]	[2.13509]
R_LGCPI(-4)	-0.167918	0.151052	0.261820
	(0.09293)	(0.15606)	(0.07342)
	[-1.80693]	[0.96793]	[3.56630]
R_LGCPI(-5)	0.132138	0.055419	0.061599
	(0.09080)	(0.15249)	(0.07173)
	[1.45521]	[0.36344]	[0.85871]
C	0.002920	0.009106	-0.002451
	(0.00168)	(0.00282)	(0.00133)
	[1.74068]	[3.23218]	[-1.84941]

We can see that R_LGM1 in time t-2 affects the Real GDP value in time t since t-stat=2,37760>1,96. We proceed with the Granger causality test in order to further examine the short run relationship.

Table 13: Granger Causality/Block Exogeneity Test, Real GDP-M1-CPI

Null hypothesis	Probability
R_LGM1 does not Granger cause R_LGREALGDP	0.0412

R_LGCPI does not Granger cause R_LGREALGDP	0.2812
R_LGREALGDP does not Granger cause R_LGM1	0.0232
R_LGCPI does not Granger cause R_LGM1	0.8465
R_LGREALGDP does not Granger cause R_LGCPI	0.0324
R_LGM1 does not Granger cause R_LGCPI	0.2405

For the null hypothesis that R_LGM1 does not Granger cause R_LGREALGDP the p-value is smaller than 0,05(in this case 0,0412). So the null hypothesis is rejected and therefore M1 does Granger cause real GDP in the short run. We can also see that R_LGM1 does not Granger cause R_LGCPI, which means that there is no short run causality between M1 and CPI.

This model confirms the money neutrality idea in the long run, but not in the short run.

- Real GDP, M2 and CPI model

Table 14:Johansen co-integration, Real GDP-M2-CPI

Sample (adjusted): 1969Q4 2012Q1
Included observations: 170 after adjustments
Trend assumption: Linear deterministic trend
Series: LOGREALGDP LOGM2 LOGCPI
Lags interval (in first differences): 1 to 6

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None	0.072020	21.00518	29.79707	0.3573
At most 1	0.035009	8.298474	15.49471	0.4340
At most 2	0.013092	2.240264	3.841466	0.1345

Trace test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**

None	0.072020	12.70670	21.13162	0.4796
At most 1	0.035009	6.058210	14.26460	0.6058
At most 2	0.013092	2.240264	3.841466	0.1345

Max-eigenvalue test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

For the trace test, we begin testing the null hypothesis of no co-integrating equation against the alternative of one or more co-integrating equations. We can see that the trace statistic is smaller than the 0.05 critical value(21,00518<29,79707), so we cannot reject the null hypothesis at 5%. We also cannot reject the null hypothesis at 10%, as p-value=0,3573>0,10. According to the trace test, there is no co-integrating relationship between real GDP, money supply M2 and CPI.

For the Maximum Eigenvalue test, we can observe that the max-Eigen statistic is smaller than the 0.05 critical value(12,70670<21,13162). As a result, we cannot reject the null hypothesis of no co-integrating equation at 5%. We also cannot reject the null hypothesis at the 10% level of significance since p-value=0,4796>0,10. Since we focus our analysis on the 5% level of significance, we conclude that the Maximum Eigenvalue test shows zero co-integrating equations for the three time series. In other words, the model confirms the idea of money neutrality in the long run.

Since there is no co-integration, we proceed with the VAR modeling of the three variables. We begin with the Lag Length criteria panel.

Table 15: Lag Length Criteria for VAR modeling, Real GDP-M2-CPI.

VAR Lag Order Selection Criteria
Endogenous variables: R_LGREALGDP R_LGM2 R_LGCPI
Exogenous variables: C
Sample: 1960Q1 2018Q1
Included observations: 168

Lag	LogL	LR	FPE	AIC	SC	HQ
0	1630.094	NA	7.77e-13	-19.37016	-19.31438	-19.34752
1	1778.560	289.8636	1.48e-13	-21.03048	-20.80734*	-20.93992*
2	1786.541	15.29647	1.49e-13	-21.01835	-20.62785	-20.85986
3	1801.805	28.71149	1.39e-13	-21.09292	-20.53507	-20.86652
4	1815.526	25.31804	1.31e-13	-21.14912	-20.42392	-20.85480
5	1833.503	32.53023*	1.18e-13*	-21.25599*	-20.36343	-20.89375
6	1839.328	10.33147	1.23e-13	-21.21819	-20.15827	-20.78802
7	1843.850	7.860478	1.30e-13	-21.16489	-19.93761	-20.66680
8	1847.145	5.608228	1.39e-13	-21.09696	-19.70233	-20.53095

* indicates lag order selected by the criterion

We choose five lags and we proceed with the VAR modeling of the variables. We use first differences of the variables in logarithmic form:

Table 16: VAR model, Real GDP-M2-CPI

Vector Autoregression Estimates
Sample (adjusted): 1969Q3 2012Q1
Included observations: 171 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	R_LGREALGDP	R_LGM2	R_LGCPI
R_LGREALGDP(-1)	0.458515	0.031586	0.102579
	(0.07937)	(0.07437)	(0.06799)
	[5.77689]	[0.42473]	[1.50878]
R_LGREALGDP(-2)	-0.101983	0.010392	-0.064685
	(0.08620)	(0.08076)	(0.07384)
	[-1.18313]	[0.12867]	[-0.87606]
R_LGREALGDP(-3)	0.217784	0.165146	0.144604
	(0.08493)	(0.07957)	(0.07275)
	[2.56438]	[2.07542]	[1.98775]
R_LGREALGDP(-4)	-0.113030	-0.011306	-0.050573
	(0.08786)	(0.08233)	(0.07526)
	[-1.28641]	[-0.13733]	[-0.67194]
R_LGREALGDP(-5)	-0.058051	0.165556	0.174440
	(0.08041)	(0.07534)	(0.06888)
	[-0.72195]	[2.19749]	[2.53264]
R_LGM2(-1)	0.112981	0.663807	0.079504
	(0.08294)	(0.07771)	(0.07104)
	[1.36226]	[8.54241]	[1.11911]
R_LGM2(-2)	-0.140573	-0.068754	0.068130
	(0.09967)	(0.09339)	(0.08538)
	[-1.41034]	[-0.73621]	[0.79797]
R_LGM2(-3)	0.074210	0.311006	-0.000364
	(0.09740)	(0.09126)	(0.08343)
	[0.76195]	[3.40810]	[-0.00436]
R_LGM2(-4)	-0.053824	-0.279980	0.036054
	(0.09770)	(0.09155)	(0.08369)
	[-0.55088]	[-3.05839]	[0.43078]
R_LGM2(-5)	-0.028896	0.214628	0.081672
	(0.08470)	(0.07936)	(0.07255)
	[-0.34115]	[2.70443]	[1.12566]
R_LGCPI(-1)	0.103143	0.085867	0.261603
	(0.09269)	(0.08684)	(0.07939)
	[1.11283]	[0.98877]	[3.29502]
R_LGCPI(-2)	-0.111018	-0.152375	-0.041431
	(0.09454)	(0.08858)	(0.08098)
	[-1.17427]	[-1.72018]	[-0.51160]

R_LGCPI(-3)	0.119502	0.164270	0.093690
	(0.09312)	(0.08725)	(0.07977)
	[1.28326]	[1.88270]	[1.17452]
R_LGCPI(-4)	-0.294005	-0.070958	0.223371
	(0.09466)	(0.08869)	(0.08108)
	[-3.10606]	[-0.80009]	[2.75491]
R_LGCPI(-5)	0.192565	0.037867	0.067654
	(0.09021)	(0.08452)	(0.07727)
	[2.13459]	[0.44800]	[0.87550]
C	0.004976	-4.56E-05	-0.003804
	(0.00148)	(0.00138)	(0.00126)
	[3.37305]	[-0.03296]	[-3.01028]

We can easily observe that R_LGM1 does not affect the Real GDP value in time t at none of the five past times(t-stat<1,96 in all cases). We proceed with the Granger causality test in order to further examine the short run relationship.

Table 17: Granger Causality/Block Exogeneity Test, Real GDP-M2-CPI

Null hypothesis	Probability
R_LGM2 does not Granger cause R_LGREALGDP	0.6248
R_LGCPI does not Granger cause R_LGREALGDP	0.0306
R_LGREALGDP does not Granger cause R_LGM2	0.0146
R_LGCPI does not Granger cause R_LGM2	0.3534
R_LGREALGDP does not Granger cause R_LGCPI	0.0233
R_LGM2 does not Granger cause R_LGCPI	0.0030

For the null hypothesis that R_LGM2 does not Granger cause R_LGREALGDP the p-value is greater than 0,05(in this case 0,6248). So the null hypothesis is not rejected and therefore M2 does not Granger cause real GDP in the short run.

This model confirms the neutrality of money concept in the long and the short run.

5.CONCLUSIONS

In this paper we examined the concept of money neutrality for the economies of the United States of America and Canada using four different models for each country. The goal of this paper is to add another point of view in the long lasting debate regarding the effect of monetary policies on the real economy. In the case of the USA, the simple straightforward models consisting of the Real GDP and money supply variables confirm the long run money neutrality. In the short run, the RGDP-M1 model gives no causality between the M1 and the RGDP variables, in contrast to the RGDP-M2 that gives strong short run causality between the variables(M2 does Granger Cause RGDP). The addition of the CPI variable creates models that have in both cases one cointegrating relationship thus are connected in the long run. But since in this case the t-stats are not applicable for tests, we used the Likelihood Ratio test to check the statistical significance of the M1 and M2 cointegrating coefficients. In both cases, the coefficients were shown to be non significant. In the short run, the Granger causality tests shows that the M1 money supply does not Granger cause Real GDP, but the M2 does. In the case of Canada, the long run money neutrality is confirmed by all the models. It is important to observe that the addition of the CPI variable in the Canadian models gives stronger results of no-cointegration and therefore adding more power to the long run neutrality hypothesis. In the short run the results are varied, as the RGDP-M1 and the RGDP-M1-CPI models give causality relationships between money supply and the RGDP variables, while the M2 models don't.

In essence, this paper confirms the long run neutrality of money. In other words, a monetary policy that changes the amount of money that exists in an economy does not change the real variables and therefore does not impact the real economy. This result can be added to the varied results of the existing studies about this subject, results that change from country to country and at times even from the change of monetary variables used in the model(M1,M2,M3, Divisia money etc).

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7. APPENDIX

1.UNIT ROOT TESTS – USA VARIABLES

- CPI

Null Hypothesis: LOGCPI has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 3 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.924341	0.9504
Test critical values:	1% level	-3.998635	
	5% level	-3.429570	
	10% level	-3.138293	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGCPI) has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.086351	0.0290
Test critical values:	1% level	-3.458845	
	5% level	-2.873974	
	10% level	-2.573472	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: LOGCPI has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 11 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		0.084582	0.9970
Test critical values:	1% level	-3.998104	
	5% level	-3.429313	
	10% level	-3.138142	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGCPI) has a unit root
Exogenous: Constant
Bandwidth: 5 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-5.292410	0.0000
Test critical values:	1% level	-3.458594	
	5% level	-2.873863	
	10% level	-2.573413	

*MacKinnon (1996) one-sided p-values.

- M1

Null Hypothesis: LOGM1 has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.408903	0.8561
Test critical values:	1% level	-3.998457	
	5% level	-3.429484	
	10% level	-3.138243	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGM1) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.096265	0.0000
Test critical values:	1% level	-3.458719	
	5% level	-2.873918	
	10% level	-2.573443	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: LOGM1 has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 10 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-1.510589	0.8235
Test critical values:	1% level	-3.998280	
	5% level	-3.429398	
	10% level	-3.138192	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGM1) has a unit root
Exogenous: Constant
Bandwidth: 2 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-6.894081	0.0000
Test critical values:	1% level	-3.458719	
	5% level	-2.873918	
	10% level	-2.573443	

*MacKinnon (1996) one-sided p-values.

- M2

Null Hypothesis: LOGM2 has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.908636	0.9522
Test critical values:	1% level	-3.997250	
	5% level	-3.428900	
	10% level	-3.137898	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGM2) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.027094	0.0000
Test critical values:	1% level	-3.457865	
	5% level	-2.873543	
	10% level	-2.573242	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: LOGM2 has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 9 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-0.858826	0.9576
Test critical values:	1% level	-3.997083	
	5% level	-3.428819	
	10% level	-3.137851	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGM2) has a unit root

Exogenous: Constant
Bandwidth: 1 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-7.020144	0.0000
Test critical values:	1% level	-3.457865	
	5% level	-2.873543	
	10% level	-2.573242	

*MacKinnon (1996) one-sided p-values.

- Real GDP

Null Hypothesis: LOGREALGDP has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.376018	0.8660
Test critical values:	1% level	-3.990935	
	5% level	-3.425841	
	10% level	-3.136094	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGREALGDP) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-11.59300	0.0000
Test critical values:	1% level	-3.990935	
	5% level	-3.425841	
	10% level	-3.136094	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: LOGREALGDP has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 4 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-1.051417	0.9339
Test critical values:	1% level	-3.990817	
	5% level	-3.425784	
	10% level	-3.136061	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGREALGDP) has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 12 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-11.19043	0.0000
Test critical values:	1% level	-3.990935	
	5% level	-3.425841	
	10% level	-3.136094	

*MacKinnon (1996) one-sided p-values.

2.UNIT ROOT TESTS – CANADA VARIABLES

- CPI

Null Hypothesis: LOGCPI has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 4 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.989172	0.9423
Test critical values:	1% level	-3.999740	
	5% level	-3.430104	
	10% level	-3.138608	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGCPI) has a unit root
Exogenous: Constant
Lag Length: 3 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.523430	0.1113
Test critical values:	1% level	-3.459627	
	5% level	-2.874317	
	10% level	-2.573656	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: LOGCPI has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 11 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*

Phillips-Perron test statistic		0.140730	0.9975
Test critical values:	1% level	-3.998997	
	5% level	-3.429745	
	10% level	-3.138397	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGCPI) has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 10 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-8.854798	0.0000
Test critical values:	1% level	-3.999180	
	5% level	-3.429834	
	10% level	-3.138449	

*MacKinnon (1996) one-sided p-values.

- M1

Null Hypothesis: LOGM1 has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 5 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.296245	0.4338
Test critical values:	1% level	-3.999180	
	5% level	-3.429834	
	10% level	-3.138449	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGM1) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 4 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.355784	0.0001
Test critical values:	1% level	-3.999180	
	5% level	-3.429834	
	10% level	-3.138449	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: LOGM1 has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 7 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*

Phillips-Perron test statistic		-2.156412	0.5112
Test critical values:	1% level	-3.998280	
	5% level	-3.429398	
	10% level	-3.138192	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGM1) has a unit root
Exogenous: Constant
Bandwidth: 1 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-9.884216	0.0000
Test critical values:	1% level	-3.458719	
	5% level	-2.873918	
	10% level	-2.573443	

*MacKinnon (1996) one-sided p-values.

- M2

Null Hypothesis: LOGM2 has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 5 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.344730	0.4075
Test critical values:	1% level	-4.006566	
	5% level	-3.433401	
	10% level	-3.140550	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGM2) has a unit root
Exogenous: Constant
Lag Length: 4 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.527458	0.1106
Test critical values:	1% level	-3.464460	
	5% level	-2.876435	
	10% level	-2.574788	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: LOGM2 has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 10 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-2.025111	0.5836
Test critical values:	1% level	-4.005318	
	5% level	-3.432799	
	10% level	-3.140195	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGM2) has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 0 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-5.312538	0.0001
Test critical values:	1% level	-4.005562	
	5% level	-3.432917	
	10% level	-3.140265	

*MacKinnon (1996) one-sided p-values.

- Real GDP

Null Hypothesis: LOGREALGDP has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.429668	0.3631
Test critical values:	1% level	-4.003902	
	5% level	-3.432115	
	10% level	-3.139793	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGREALGDP) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-10.43192	0.0000
Test critical values:	1% level	-4.003902	
	5% level	-3.432115	
	10% level	-3.139793	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: LOGREALGDP has a unit root

Exogenous: Constant, Linear Trend
Bandwidth: 6 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-2.744162	0.2202
Test critical values:	1% level	-4.003675	
	5% level	-3.432005	
	10% level	-3.139728	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(LOGREALGDP) has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 5 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-10.52494	0.0000
Test critical values:	1% level	-4.003902	
	5% level	-3.432115	
	10% level	-3.139793	

*MacKinnon (1996) one-sided p-values.

3.GRANGER CAUSALITY TESTS – USA MODELS

REAL GDP-M1

VAR Granger Causality/Block Exogeneity Wald Tests
Sample: 1947Q1 2018Q3
Included observations: 229

Dependent variable: R_LGREALGDP

Excluded	Chi-sq	df	Prob.
R_LGM1	1.167208	2	0.5579
All	1.167208	2	0.5579

Dependent variable: R_LGM1

Excluded	Chi-sq	df	Prob.
R_LGREALGDP	4.860250	2	0.0880
All	4.860250	2	0.0880

REAL GDP-M2

VAR Granger Causality/Block Exogeneity Wald Tests
Sample: 1947Q1 2018Q3
Included observations: 232

Dependent variable: R_LGREALGDP

Excluded	Chi-sq	df	Prob.
R_LGM2	10.30927	3	0.0161
All	10.30927	3	0.0161

Dependent variable: R_LGM2

Excluded	Chi-sq	df	Prob.
R_LGREALGDP	7.804919	3	0.0502
All	7.804919	3	0.0502

REAL GDP-M1-CPI

VEC Granger Causality/Block Exogeneity Wald Tests
Sample: 1947Q1 2018Q3
Included observations: 228

Dependent variable: D(LOGREALGDP)

Excluded	Chi-sq	df	Prob.
D(LOGM1)	4.534884	3	0.2092
D(LOGCPI)	17.65141	3	0.0005
All	24.14281	6	0.0005

Dependent variable: D(LOGM1)

Excluded	Chi-sq	df	Prob.
D(LOGREALGDP)	5.172386	3	0.1596
D(LOGCPI)	0.258272	3	0.9677
All	6.080218	6	0.4143

Dependent variable: D(LOGCPI)

Excluded	Chi-sq	df	Prob.
D(LOGREALGDP)	7.963237	3	0.0468
D(LOGM1)	3.408149	3	0.3329
All	10.33313	6	0.1113

REAL GDP-M2-CPI

VEC Granger Causality/Block Exogeneity Wald Tests
Sample: 1947Q1 2018Q3
Included observations: 228

Dependent variable: D(LOGREALGDP)

Excluded	Chi-sq	df	Prob.
D(LOGM2)	8.711401	3	0.0334
D(LOGCPI)	21.81840	3	0.0001
All	30.21640	6	0.0000

Dependent variable: D(LOGM2)

Excluded	Chi-sq	df	Prob.
D(LOGREALGDP)	5.040043	3	0.1689
D(LOGCPI)	9.567865	3	0.0226
All	18.36559	6	0.0054

Dependent variable: D(LOGCPI)

Excluded	Chi-sq	df	Prob.
D(LOGREALGDP)	6.063772	3	0.1085
D(LOGM2)	1.997098	3	0.5730
All	8.983423	6	0.1745

4.GRANGER CAUSALITY TESTS –CANADA MODELS

REAL GDP-M1

VAR Granger Causality/Block Exogeneity Wald Tests
Sample: 1960Q1 2018Q1
Included observations: 199

Dependent variable: R_LGREALGDP

Excluded	Chi-sq	df	Prob.
R_LGM1	12.32117	5	0.0306
All	12.32117	5	0.0306

Dependent variable: R_LGM1

Excluded	Chi-sq	df	Prob.
R_LGREALGDP	12.39522	5	0.0298
All	12.39522	5	0.0298

REAL GDP-M2

VAR Granger Causality/Block Exogeneity Wald Tests
Sample: 1960Q1 2018Q1
Included observations: 170

Dependent variable: R_LGREALGDP

Excluded	Chi-sq	df	Prob.
R_LGM2	2.592220	6	0.8580
All	2.592220	6	0.8580

Dependent variable: R_LGM2

Excluded	Chi-sq	df	Prob.
R_LGREALGDP	17.20899	6	0.0085
All	17.20899	6	0.0085

REAL GDP-M1-CPI

VAR Granger Causality/Block Exogeneity Wald Tests
Sample: 1960Q1 2018Q1
Included observations: 199

Dependent variable: R_LGREALGDP

Excluded	Chi-sq	df	Prob.
R_LGM1	11.57152	5	0.0412
R_LGCPI	6.266086	5	0.2812
All	18.67023	10	0.0447

Dependent variable: R_LGM1

Excluded	Chi-sq	df	Prob.
R_LGREALGDP	13.01553	5	0.0232
R_LGCPI	2.019153	5	0.8465
All	14.21784	10	0.1633

Dependent variable: R_LGCPI

Excluded	Chi-sq	df	Prob.
R_LGREALGDP	12.17873	5	0.0324
R_LGM1	6.742670	5	0.2405
All	22.04663	10	0.0149

REAL GDP-M2-CPI

VAR Granger Causality/Block Exogeneity Wald Tests
Sample: 1960Q1 2018Q1
Included observations: 171

Dependent variable: R_LGREALGDP

Excluded	Chi-sq	df	Prob.
R_LGM2	3.490760	5	0.6248
R_LGCPI	12.32631	5	0.0306
All	14.87190	10	0.1368

Dependent variable: R_LGM2

Excluded	Chi-sq	df	Prob.
R_LGREALGDP	14.16800	5	0.0146
R_LGCPI	5.541316	5	0.3534
All	19.02654	10	0.0399

Dependent variable: R_LGCPI

Excluded	Chi-sq	df	Prob.
R_LGREALGDP	13.00812	5	0.0233
R_LGM2	17.98047	5	0.0030
All	36.13611	10	0.0001