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Essays on Economics of Collusion:

Cartels, Leniency and Advertising-Selling Platforms

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Summary

Overt collusion is a situation where firms communicate in an attempt to artificially increase prices and suppress competition. Such practices allow colluding firms to unlawfully increase their profits while consumers suffer due to high prices. As cartels are harmful for society, the public policy objective is to detect, punish and deter cartel activity. Cartels are illegal under European Union competition law, and the European Commission imposes severe penalties on firms that engage in collusive activities.

The standard instruments that an Antitrust Authority (AA) has on its disposal in order to fight cartels are market investigations and fines for prosecuted cartels. Along with these two instruments authorities have developed policies that facilitate prosecution and enhance cartel deterrence. Leniency Programs (LPs) offer fine reductions to cartel participants that provide information and/or evidence related to the cartel to the competition agency.

This thesis contains four papers. The first three are dealing with the design and effects of LPs, while the fourth discusses cartel enforcement in two-sided platforms, focusing on the specific but important case of media markets.

In the first paper the demand is stochastic and firms are unable to perfectly observe their rival's choices. It is well known that the equilibrium in such setting may involve periods of cooperation interrupted by periods of price war. In our work it is shown that a LP restricted to only firms that report a cartel not yet under investigation may undermine cartel stability, but does not affect the duration of price wars. *Post-investigation* leniency may have ambiguous effects on both, cartel stability and the duration of competitive periods. While they may in many instances be beneficial, when negative demand shocks are sufficiently frequent and conviction is quite likely even without the assistance of whistleblowers, LPs may produce undesirable effects.

The second paper relaxes an important and widely used assumption in the leniency-related literature, namely that every single firm possesses evidence that if delivered to the authorities, it suffices to convict the cartel with certainty (*perfect evidence*). This assumption is replaced by the assumption that cartel members possess imperfect evidence that can be divided in two parts: evidence that is commonly possessed by all cartel members (*common evidence*), as well as pieces of evidence that are exclusively possessed by every single cartel member (*specific evidence*). This assumption produces two interesting implications: first the confession by only a

subset of firms does not imply cartel conviction with certainty, and second that there may exist equilibria where both, reporting and non-reporting parties co-exist. Hence, inducing a larger number of firms to report increases the probability of cartel conviction, but at the same time requires more generous fine reductions to be offered. As in the pre-investigation period any amount of leniency offered represents also an implicit reduction of the expected fines to be paid in case of investigation and conviction, the AA can be viewed as facing a trade-off between pre- and post-investigation efficiency: a more generous amount of post-investigation leniency increases the number of informants and the probability of conviction, but for as long as the market escapes investigation it may create a more fertile ground for cartel formation. This raises the question of what is the optimal amount of post-investigation leniency, which is equivalent to asking "what is the optimal number of informants that must be induced to come forward". Our first result in this paper is that, despite the aforementioned tradeoff, it is optimal to offer sufficient incentives in order induce all firms to come forward, *i.e.* the AA must obtain all the available evidence.

The second result of this second paper relates to a common feature of LPs across jurisdictions, namely the *marker system*. The latter offers the first firm that comes in to confess the privilege to reserve its position in the informants' queue no matter how the subsequent informants' eligibility will be determined. This confession postponement, aiming to allow that firm to best prepare its reporting, may also result in a confession denial if the firm in question decides to step back and not make use of its marker. Marker systems can be of two kinds, depending on how they react in such case: some LP transfer the privilege to the second informant in line (transferable marker), while others simply cancel it. It is shown that in terms of cartel deterrence, transferable-marker systems are inferior to systems allowing for no-marker at all. Only when the marker is cancelled once not used by its first recipient can provide positive additional impact on cartel deterrence, relative to no-marker systems.

The third paper assumes that the cartel directory can determine whether any traces of evidence eventually left by any coordinating activity must be kept exclusively by the members involved in that activity, or be communicated to all cartel participants. This way, the cartel directory determines the amounts of evidence that will be common to all, or exclusively detained by each cartel participant. Given that increases in commonly-hold evidence dramatically increases the probability of cartel conviction, one intuitively expects that the cartel directory always prefers to limit common evidence to the minimum possible level, and this intuition is confirmed for many cases. However, we also show instances where cartel stability is enhanced when all the evidence becomes commonly-hold by all participants. If the design of post-investigation LP is such as to generate a "race to the Court" among all participants of a cartel under investigation, the directory knows that, once under investigation the cartel will collapse. When there is no hope of winning the post-investigation battle, cartel stability is enhanced by maximizing the "amnesty" effect, *i.e.*, the amount of leniency that will be received by at least some participants. In order to avoid "wasting amounts of leniency" the directory may decide to offer to the participant firms a lottery containing a ticket with no fine. It is shown that if the AA anticipates the cartel directory's ability to allocate the evidence, the optimal leniency may require—contrary to our previous result—an LP that results to gathering only partial evidence.

The fourth paper in this thesis examines the effectiveness of antitrust enforcement on cartel deterrence when firms operate in two-sided markers. Particular emphasis is placed on advertising-selling markets, where firms serve both, advertisers and consumers. While consumers' participation definitely creates a positive externality for advertisers, the reverse is not necessarily true: depending on whether consumers love or hate advertising, the externality advertisers create on consumers' can be positive or negative. Two types of collusion are possible for the media platforms: collude on any of the two markets and keep competing on the other (*semi-collusion*), or collude on both sides (*full collusion*). Our work compares semi-collusion on the advertising side with full collusion, from the cartel's point of view. While full collusion dominates in terms of profitability, semi-collusion may be more stable, *i.e.*, it may be sustainable in cases where full collusion is not. Hence, in media markets semi-collusion on advertising is the most likely form or collusion, unless cartel detection is unlikely and/or consumers are sufficiently ad-lovers. The high likelihood of semi-collusion raises the important question of whether, in case of cartel conviction, it is more efficient (in terms of cartel deterrence) to base fines on profits exclusively stemming from the collusive side, or on total profits. When consumers are neutral towards advertising, the increase in total profits due to semi-collusion originates from the colluding side. Since firms forego profits from the competing side in order to recoup them from the colluding market, using profits from advertising as fines basis yields a basis that is substantially enhanced.

Introduction

Collusive agreements are considered as the most serious violation of the competition law. Economic theory suggests that collusion is a situation where firms coordinate their actions, fixing prices (or restricting quantities) and unlawfully increasing their profits.[1] This increase in prices caused by unlawful coordination of firms' behavior hurts consumers by either directly extracting an amount of their surplus, or inducing some of them to turn to less desirable alternatives. In addition, because cartelists engage in counterproductive rent seeking activities, there is a loss associated with the surplus transferred from consumers to infringers. Therefore, it is broadly accepted that collusive agreements cause allocative inefficiencies.

Since cartels hurt society, public policy aims to detect and punish them with the ultimate objective to deter any such activity. Collusive agreements take various forms: price fixing, quantity restriction, sharing of market, etc. They are illegal under the European Union (EU) competition law, and the European Commission (EC) imposes severe fines on enterprises that engage in such activities.[2] However since cartels are prohibited, evidence related to collusive activities is difficult to be uncovered.[3]

The economic theory of collusion states that collusive agreements can be successful only if participants do not have any incentive to secretly undercut the collusive price, stealing other members' profits. Therefore, a collusive agreement is viable if firms are able to monitor each others' choices and credibly punish deviations from the agreement. The satisfaction of the *self-enforcing* or *incentive compatibility* constraint (ICC) is a necessary condition for a cartel to be sustainable: cartels are successful only if their members do not have an incentive to undercut the agreed price. The ICC differs from the participation constraint which requires the expected additional gain from the illegal activity to be positive. Both constraints have to be satisfied for a collusive agreement to be stable.

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[1] For example when the product is homogeneous and firms compete in prices *à* la Bertrand, colluding firms realize positive profits

[2] Between 2014 and 3/2018 32 cartel cases has been decided by the EC and the total amount of fines imposed is close to 8.5 billion €.

[3] In courts, only practices where firms coordinate actions, and therefore produce evidence, are considered illegal.

For any given frequency of investigation, simply setting sufficiently high fines could suffice to render the participation constraint impossible to satisfy and make collusion always unprofitable. However, as shown in Andreoni (1991), compliance to law is not a monotonically increasing function of the fines level: since too high fines are either impossible to impose, or are imposed with a very low probability, the expected damage to law infringers starts decreasing beyond a certain fines level. Very high fines for cartel participation may force firms out of business yielding a very concentrated structure, an outcome that is strongly undesirable for any Competition Authority. Thus, even if high fines were to be instituted, infringers know well that they would be rarely (if ever) imposed. Therefore, competition agencies apart from the budget constraints that restrict the frequency of investigations, they also face an upper bound on the level of penalties imposed to convicted violators.[4]

Taking this into account, competition authorities adopt policies, called Leniency Programs, according to which firms that report their participation in cartel arrangements and cooperate providing evidence or information that improve competition authorities' ability to prosecute and convict such infringements receive lenient treatment. Leniency Programs are described in the following section and constitute a central object of the analysis in this work.

1 Leniency Programs (LPs)

According to the International Competition Network (ICN, 2014)

"*Leniency is a generic term to describe a system of partial or total exoneration from the penalties that would otherwise be applicable to a cartel participant in return for reporting its cartel membership and supplying information or evidence related to the cartel to the competition agency providing leniency*".

Most jurisdictions have developed programs that incentivize infringers to admit the participation to a cartel and to offer quality evidence that can be used as proof of the illegal behavior. By offering immunity from fines to the first cartel member that reports and cooperates with a competition agency and a more lenient treatment in terms of fine reductions to subsequent informants, LPs induce cartel members to come forward and report the existence of a cartel.

[4] For example in US the final fine must not exceed the greatest of 100 million US$ or twice the gross pecuniary gains the violators derived from the cartel.

Wils (2016) points out some possible positive and negative aspects of leniency policies. On the positive side, there is consensus among theorists and practitioners that LPs reduce the time and cost of collecting cartel-related evidence. For given amount of resources spent on cartel enforcement, this reduction in time and cost allows the competition agency to detect and prosecute a larger number of cases. Also, allowing a deviator to also report the cartel and avoid part of the fine makes deviation more attractive, reduces trust among cartel members, and raises sharply the enforcement cost of the cartel agreement. Cartels have to develop an organizational structure that allows them to solve the coordination problem. Harrington (2008) names the effect of LPs on the value of defecting as the *deviator effect* of leniency.

On the other side, if leniency is available after the competition agency has launched an inspection, fine reductions affect the expected payoff from continuing to collude. The lower overall level of penalties renders collusion easier to sustain. Harrington (2008) names the positive effect of leniency on firms' colluding value as the *cartel amnesty effect*.

The present thesis investigates the impact of LPs on cartel deterrence. First, it examines the effect of the introduction of leniency policies when demand is uncertain and firms cannot infer the choices of their competitors. As observed in Green and Porter (1984), the presence of unexpected demand slumps and cartel-members' inability to perfectly monitor each other's actions renders the adoption of trigger strategies counterproductive: maximum punishment with permanent reversion to Nash equilibrium ruins, potentially for no reason, an otherwise profitable cartel. A solution could be a strategy of interrupting cooperation every time low sales are observed, but only for a finite number of periods (price wars).

Our first paper shows that pre-investigation leniency does not affect the duration of the competitive price wars. Post-investigation leniency may have ambiguous welfare effects, since it affects both cartel stability and the duration of price wars. LPs may produce undesirable effects when applying in situations where conviction without firms' assistance is likely and negative demand shocks are frequent.

In the second paper we deal with the effectiveness of post-investigation LP to deter unlawful collusion when firms possess imperfect evidence. We use as benchmark an "opaque" LP system that allows no information about the number of firms already willing to collaborate to be diffused. After the starting of an investigation, cartel participants find themselves in a simultaneous-game situation where they must decide whether to report without knowing the other players' action. Next, we examine whether the introduction of the marker system (see the sub-section below) assists or obstructs the performance of a LP in deterring cartel activity, compared to the opaque system. The vast majority of the literature considers that each firm possesses full evidence, so even a single firm's reporting suffices for the cartel's conviction. A consequence of this assumption is that inducing one firm to report incentivizes reporting by all firms (race to the Courthouse) and therefore, in equilibrium either none or all the firms confess. However, when each firm possesses only part of the evidence that is necessary to lead to conviction, even if a subset of firms is induced to confess, some, or all the other firms may decide to remain silent. With only a subset of firms confessing, the AA possess incomplete evidence and may be unable to obtain the cartel's conviction with certainty: in some cases the cartel members may be acquitted for lack of sufficient evidence during the judicial process. As the number of reporting firms increases, the cumulative evidence in the hands of the AA also increases, and so does the probability of conviction, but for this to happen, more generous fine reductions must be offered, rendering cartel participation in all the pre-investigation periods more attractive. The AA faces a tradeoff and one could expect optimal leniency to be such as to balance these two effects. We show, however, that whether the LP program offers a marker or does not, it is always optimal to provide sufficient incentives for all the firms to come forward.

The third paper qualifies the above result by showing that it holds when the percentage of incriminating evidence that is common within the hands of all the participants is exogenous. In this paper is assumed that the cartel directory can determine whether any traces of evidence eventually left by any coordinating activity must be kept exclusively by the members involved in that activity, or be communicated to all cartel participants. This way, the cartel directory determines the amounts of evidence that will be common to all, or exclusively detained by each cartel participant. Given that increases in commonly-hold evidence dramatically increase the probability of cartel conviction, one intuitively expects that the cartel directory always prefers to limit common evidence to the minimum possible level, and this intuition is confirmed for many cases. However, we also show instances where cartel stability is enhanced when all the evidence becomes commonly-hold by all participants. If the design of post-investigation LP is such as to generate a "race to the Court" among all participants in a cartel under investigation, the directory knows that, once under investigation the cartel will collapse. When there is no hope of winning the post-investigation battle, cartel stability is enhanced by maximizing the "amnesty" effect, *i.e.*, the amount of leniency that will be received by at least some participants. In order to avoid "wasting amounts of leniency" the directory may decide to offer to the participant firms a lottery containing a ticket with no fine. It is shown that if the AA anticipates the cartel directory's ability to allocate the evidence, the optimal leniency may require—contrary to our previous result—an LP that results to gathering only partial evidence.

1.1 Marker system

A common feature of LPs in most jurisdictions is the presence of marker system. The Organization for Economic Co-operation and Development (OECD, 2014) describes marker systems as follows:

"*Marker systems provide a mechanism for prospective leniency applicants to approach the agency with initial information about their participation in a cartel in exchange for a commitment by the agency to hold their place in line for leniency, for a finite period. This grants the marker applicant time to gather additional information through an internal investigation to complete successfully the leniency application*".

All 34 OECD members and the EU have LPs in place, and at least 30 of them appear to have some form of marker system. Some jurisdictions offer the possibility of marker to multiple applicants; the majority however does not offer such possibility. OECD members that do offer markers to subsequent applicants appear to include Canada, France, Germany, Japan, Korea, Mexico, Switzerland, Turkey, and United Kingdom. Another crucial feature of the marker system pertains to the timing during which a marker is available. There exist agencies that offer markers only before any investigation has conducted while major jurisdictions offer the possibility for potential applicants to reserve their position in the reporting line even if an investigation has launched.

The second paper of the thesis deals with the effectiveness of the marker system to enhance the LP's impact on cartel deterrence. We distinguish two cases according to whether, in case the first firm denies confession, the marker can or cannot be transferred to the next firm in the reporting line. We show that while a transferable-marker system is always inferior in terms of cartel deterrence compared to the no marker case, if the marker is permanently lost after the first firm's denial to confess, the marker system may perform better than the no marker system.

2 Cartel enforcement and LP in EU

In EU, according to 2006 Guidelines, fines are calculated in the following way: first, the basic amount of the fine may be increased or decreased due to aggravating and mitigating factors. The basic amount takes into account the undertaking's relevant turnover, the gravity and the duration of the infringement, as well as an additional amount of the value of sales to achieve deterrence. The maximum amount of the fine imposed does not exceed the 10% of annual worldwide turnover of the undertaking in the preceding business year.

The 2006 Leniency Notice, as currently applicable, defines the conditions under which fine reductions are provided: The EC grants immunity from any fine that would otherwise have been imposed to an undertaking disclosing its participation in a cartel if that undertaking is the *first to submit information and evidence* which enables EC to carry out an investigation related to the cartel, and if, at the time of the application for immunity, the Commission did not yet have sufficient evidence to adopt a decision to carry out an inspection and had not yet carried out such an inspection.[5] If no firm has received immunity on the above ground, EC grants immunity to the firm that is the first to submit information and evidence which enables the EC to find an infringement, provided that the EC did not yet have, at the time of the submission, enough evidence to make such a finding.

Firms that report their participation in a cartel and do not meet the conditions for immunity may still be eligible for a *reduction from fines*, if they provide evidence which represents *significant added value* compared to the evidence already in the EC's possession, and provided that they fulfill the same conditions of genuine, full cooperation and termination of the infringement as applicable to immunity applicants.

The first undertaking to provide such significant added value receives a reduction of 30 to 50% of the fine which would otherwise have been imposed, the second undertaking a reduction of 20 to 30%, and subsequent undertakings a reduction of up to 20%.

[5] In the period 2006-2015, in 46 out of the 54 cartel decisions with fines, immunity was granted under the EC Leniency Program to the first reporting firm, see Wils (2016).

3 Literature on LPs

The first paper that addresses the impact of LPs on enforcement towards cartels is Motta and Polo (2003). It recognizes that a LP affects cartel activity through lower fines (*pro-collusive* effect) and administrative costs. Assuming that similar lenient treatment is offered to all reporting parties and that defecting firms are not subject to conviction, this study investigates the necessity of post-investigation leniency. Motta and Polo (2003) find that, unless cartel detection is sufficiently likely, lenient treatment should be available for investigated firms.

Spagnolo (2004) focuses on LP's impact on firms' incentives to spontaneously self-report. This paper does not distinguish between the probabilities of detection and conviction; it therefore examines only the direct impact of LPs on deterrence. It shows that firms should be encouraged to unilaterally defect from the agreement and that this can be reached if the deviating firm is protected from any fines' imposition. This is called the "*protection from fines*" effect of leniency.[6]

A main result of Spagnolo (2004) is that it is optimal to reward the first informant with the sum of the fines paid by the other firms. Therefore, restricting leniency to one eligible firm reduces the possibility that such a program could be exploitable. The superiority of the *first informant rule* is also highlighted by Chen and Rey (2013). The latter shows that allowing more than one firms to benefit from fine reduction undermines the effectiveness of the LP as more informants uselessly increase the value from collusion. In addition, Harrington (2008) also supports the practice of restricting leniency only to the first in line firm as one firm's confession is enough to convict the cartel with probability 1. It shows that restricting leniency to the first informant generates the "*race to the courthouse*" by all cartel participants, thus guaranteeing maximum evidence in support of cartel conviction.

In relation to the number of eligible for leniency applicants, Sauvagnat (2014) goes a step farther by showing that, if the AA is able to offer fine reductions contingent to the number of reporting parties, costless leniency can be achieved. Promising sufficient fine reductions to the *single informant* that comes forward and reveal is enough to induce reporting by every cartel participant once an investigation is underway. In equilibrium all firms report and the AA minimizes the size of fine

[6] Spagnolo (2004) assumes that a LP can be exploited as firms can collude and confess in every period.

reductions that are finally offered. Thus, while maintaining all the advantages stemming from the use of leniency, the AA avoids the disadvantage of an *ex-ante* implicit fine reduction.

The role of private information on firms' incentive to self-report is examined in Harrington (2013). Considering that participants have private information regarding the AA's ability to prosecute and convict cartels it is shown that firms may report because of the threat of other firms' confession (*pre-emptive* effect). Sauvagnat (2015) assumes that the AA receives a binary signal, good or bad, regarding the likelihood of gathering hard evidence sufficient to lead to cartel conviction. If the cartel members are not able to infer this probability, *i.e.* to observe the signal, they may be induced to confess even if conviction is unlikely.

The fact that the colluding firms tend to keep rather than destroying cartel-related evidence is examined by Aubert et al. (2006). Assuming that the decision of keeping or not cartel-related evidence as long as the decision to deviate from the agreement are observable by the rivals, it is shown that retaining evidence may respond to the threat of deviations: even in the absence of LP if firms are able to react against opportunistic behavior, hard evidence can be used as a disciplinary device.

The impact of the marker system on the effectiveness of LPs to deter collusive agreements is investigated in Blatter *et al.* (2017). That paper considers the case where firms may possess asymmetric evidence and total evidence is cumulative, therefore reporting by one firm may not be enough to convict the cartel. Assuming a duopoly where the eligibility for leniency is restricted to the first informant, they show that if the two firms possess sufficiently symmetric evidence the introduction of the marker hurts cartel deterrence. If a marker system is in effect, only one firm confesses and the AA is not able to gather all the available evidence.

Another feature of LPs is ringleader discrimination. Some major jurisdictions do not allow firms that act as ringleaders of the cartel to benefit from fine reductions. Chen et al. (2015) show that ringleader's discrimination reduces the incentive for cartel instigation while it mitigates the destabilizing effect of LPs. Blatter *et al.* (2017) show that denying leniency to ringleaders has positive impact on deterrence when both firms possess sufficiently symmetric amount of evidence.

Some empirical findings

The efficiency of the EU LP before the 2002 revision is examined in Brenner (2009). The latter addresses the question of whether the first European LP has improved cartel deterrence and desistence. The impact of the LP on cartel stability is measured with the use of cartel's duration before and after 1996. It finds that LP resulted in a reduction on the average duration of investigations and that there exist a marginal increase in the number of prosecuted cartels following the adaption of the LP. The conclusions are in accordance with the prevailing opinion that the first EU LP did not offer sufficient incentives for cartel members to come forward and report, having only an incremental impact on cartel deterrence.

Miller (2009) investigates the effect of the US corporate LP introduced in 1993. Using data from the Department of Justice for the period between 1985 and 2005, Miller (2009) finds that the number of cartels detected increased after 1993. Also it claims that this increase was followed by a fall below the pre-leniency level, a "pattern consistent with enhanced cartel detection and deterrence capabilities".

4 A simple example with fines and leniency

In order to express the previous ideas analytically consider the following simple example: two firms producing a homogeneous good and competing in prices decide whether to collude or to compete. Both firms discount future gains using a common discount factor denoted with $\delta \in (0,1)$. After the *collusion vs. competition* decision is taken by firms, at the end of each period (*i.e.*, after sales have been completed) the Antitrust Authority (AA) investigates the market with probability $\alpha \in (0,1)$ and if firms collude, the cartel is prosecuted and the participating firms are convicted. Conviction implies that each firm pays a multiple $\mu > 1$ of the illegal gain. Following prosecution the market will be competitive for an infinite number of periods. Let the collusive profits be denoted by π.

Prior to investigation each firm chooses whether to remain loyal to the agreement or to unilaterally undercut the agreed-upon price. Assume that firms follow *trigger strategies*: any unilateral deviation from the agreement is punished with cancellation of the agreement and permanent reversion to competition; under the assumptions of Bertrand competition in homogeneous products this implies zero profits forever after. Since demand and each firm's cost are identical across periods, if a firm decides to cheat it will do it during the first period, otherwise it will remain forever loyal to the agreement until it is eventually interrupted by the AA. If both firms remain loyal and no investigation occurs at the end of the period each one of them earns π and keeps colluding for one more period. In case of successful investigation each firm earns the collusive profits but has also to pay the fine. Firms are risk-neutral, basing at each point in time their decision on whether to respect the cartel agreement or not, simply on the present value of expected collusive profits, which is:

$$V = a(\pi - \mu\pi) + (1-a)(\pi + \delta V) = \frac{\pi(1-a\mu)}{1-\delta(1-a)} \tag{1.1}$$

If one firm decides to deviate from the agreement, product homogeneity allows it to gain the entire market by undercutting the cartel price only marginally. Therefore, deviation from the agreement produces profit equal to 2π during one period, and zero profits forever after, since from the next period on, the other firm realizes the deviation and stop cooperation. The expected gain from deviation is $2\pi(1-a\mu)$.

The ICC is obtained by setting $V > 2\pi(1-a\mu)$ which yields the minimum discount factor above which collusion is sustainable:[7]

$$\delta > \frac{1}{2(1-a)} \tag{1.2}$$

At the same time for the participation constraint to be satisfied, the one period expected collusive gain must exceed the profits from competing, that is

$$\pi(1-a\mu) > 0 \Leftrightarrow \mu < \frac{1}{a}$$

Therefore, the participation constraint is not satisfied when the fine paid by convicted firms is sufficiently high. The rest of the analysis assumes that very high fines are not feasible and that firms expect positive profits when colluding. Consequently, the analysis focuses on firms' incentive to stick or to deviate from the agreement.

Observe from (1.2) that in order to improve the deterrent effect of antitrust interventions on cartels it is essential to increase the profitability from unilateral deviation.[8]

Pre-investigation leniency

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[7] Note that the absence of antitrust enforcement yields the textbook outcome according to which collusion is sustainable for $\delta > \frac{1}{2}$.

[8] By increasing the value of defection, a higher δ is required for a cartel to be sustainable.

Let the previous example of two firms considering collusion and assume that the competition agency imposes reduced fine to any firm that, while participating in a cartel agreement that has not been under investigation, it is the first one to come forward and report details about the cartel it makes part of. This lenient treatment does not affect the value of remaining loyal to the cartel agreement, but increases the value of cheating when the required fine-payment is sufficiently lower than the expected value of paying the full fine. Since a more generous fine reduction always makes the satisfaction of the ICC more difficult and rewards are excluded, offering full amnesty to the deviator maximizes the minimum δ above which collusion is sustainable. In this case the ICC is:

$$V > 2\pi \Leftrightarrow \delta > \frac{1 + \alpha\mu}{2(1 - a)} > \frac{1}{2(1 - a)} \tag{1.3}$$

Thus, offering maximum fine reductions to the firm that reports the agreement before any investigation has started, improves cartel deterrence.

In the previous simple paradigm we assume that in case of investigation conviction is certain. Relaxing this assumption, offering lenient treatment even if investigation has launched may have some positive impact on deterrence: assuming that conviction is more likely (or even certain) if firms offer cartel-related evidence, offering fine reduction after the start of an investigation may decrease the value from collusion. This occurs when the effect from an increased conviction rate offsets the impact of lower overall penalties.

Post-investigation leniency

Most jurisdictions' LPs offer also fine reductions to firms that reveal information even after the start of the investigation. Spotting a cartel does not imply conviction of its members and termination of its activity, unless the case is supported by sufficient evidence. In the most common case where initial investigation does not provide the necessary evidence, post-investigation leniency may be used in order to improve evidence collection and increase the odds in favor of cartel conviction.

Building upon the previous model, suppose that, even if a market is cartelized, the start of an investigation leads to conviction with probability $\rho \in (0,1)$. Both firms have the possibility of reporting when being under inspection and the first-to-door pays a reduced fine $\gamma\pi$ with $0 \leq \gamma < \mu$.[9] Further, assume that even if one firm reports, the cartel is condemned with probability one. Inducing reporting by at least one firm requires that unilateral reporting is more profitable than remaining silent. The firm that reports (R) while the other remains silent expects to earn the collusive profits and to pay the reduced fine: $\pi(1-\gamma)$. If this firm decides to remain silent (NR) it expects to earn the collusive gain and to keep colluding with probability $1-\rho$ and to pay the full fine with probability ρ:

$$S = (1-\rho)(\pi + \delta V') + \rho\pi(1-\mu)$$

where $V' = \frac{\pi(1-a\rho\mu)}{1-\delta(1-a\rho)}$ is the value of collusion when no firm confesses. If both firms confess the cartel is convicted with certainty and the probability of being eligible for leniency is 0.5, therefore the payoff of reporting when the other firm reports is $\pi\left(1-\frac{\gamma+\mu}{2}\right)$:

	R	NR
R	$\pi\left(1-\frac{\gamma+\mu}{2}\right), \pi\left(1-\frac{\gamma+\mu}{2}\right)$	$\pi(1-\gamma), \pi(1-\mu)$
NR	$\pi(1-\mu), \pi(1-\gamma)$	S, S

From the above unilateral reporting is achieved when leniency is sufficiently generous, that is

$$\pi(1-\gamma) \geq S \Leftrightarrow \gamma \leq \frac{\rho[\delta + \mu - (1-\alpha)\delta\mu] - \delta}{1-\delta(1-a\rho)} \tag{1.4}$$

Notice that $\frac{\rho[\delta+\mu-(1-\alpha)\delta\mu]-\delta}{1-\delta(1-a\rho)} \geq 0$ requires $\rho \geq \frac{\delta}{\delta+\mu-(1-\alpha)\delta\mu}$, therefore if ρ is low no reporting can be induced.

Unilateral reporting implies that the cartel is convicted with certainty, hence, reporting is dominant strategy for firms, as certain conviction renders each one of them willing to benefit from fine reductions: $\pi\left(1-\frac{\gamma+\mu}{2}\right) \geq \pi(1-\mu)$. Therefore, if the above condition for γ is satisfied, each firm knows that in case of investigation the cartel is convicted with probability 1 and that the probability of being eligible for leniency is 0.5:

[9] The reporting decision is taken simultaneously by both firms. If both report the name of the eligible for leniency is determined randomly.

$$V^R = (1-a)(\pi + \delta V^R) + a\pi\left(1 - \frac{\gamma + \mu}{2}\right) = \frac{\pi\left(1 - a\frac{\gamma+\mu}{2}\right)}{1 - \delta(1-a)}$$

Observe that if $V^R > V'$ the post-investigation LP may have adverse impact on deterrence. The latter holds if leniency provided is sufficiently high:

$$\gamma \leq \frac{\delta[\mu - 2 + \rho(2 - (2-\alpha)\mu)] + \mu(2\rho - 1)}{1 - \delta(1 - a\rho)}$$

However, using (1.4) the latter threshold is always lower than $\frac{\rho[\delta+\mu-(1-\alpha)\delta\mu]-\delta}{1-\delta(1-a\rho)}$, thus the leniency needed for $V^R > V'$ to hold is always greater than the minimum leniency that induces reporting in the post-investigation stage. Therefore, for a post-investigation LP to be effective, leniency offered should be sufficient but not excessively generous. Otherwise it may produce perverse effects.[10]

5 Collusion in media markets

Collusion may also prevail in markets which allow the interaction between distinct types of customers. Newspapers and more broadly advertising-selling platforms are considered as canonical two-sided markets where firms serve both advertisers and consumers. Several cases of collusion have been reported in media markets while the theoretical literature is rather scarce.

The last paper deals with the effectiveness of antitrust enforcement on cartel deterrence when firms operate in two-sided markers, with particular application in media markets. A feature of two-sided markets is that the existence and number of customers on one side affects the utility and choices of the customers on the other side, with the sign of this cross effect being usually positive. For instance, in an electronic sales platform the larger the number of suppliers the merrier the shoppers, and vice versa. Media firms (for instance, newspapers, television channels, etc.) can be viewed as such platforms serving advertisers on the one side, and readers or viewers (consumers) on the other. The particular characteristic of these platforms is that the externality that advertisers exert to consumers may not always be positive: while some consumers may like advertisements, others may exhibit aversion towards them. Therefore, if media firms collude on the advertising side while they compete on

[10] For a formal analysis of the necessity of post-investigation LP, see Chen and Rey (2013).

the other side, a decrease of advertising rates may or may not be appreciated by consumers, depending on their preferences towards advertisements.

A number of antitrust policy questions can be raised in such markets. In this study we ask first whether collusion on the advertising side (semi-collusion) can result as a rational strategic choice, instead of collusion on both sides (full collusion), or competition. While semi-collusion becomes less likely to prevail as consumers become more ad-lovers, the opposite holds for full collusion: when consumers become more ad-lovers breaking the ad-restricting cooperation and selling more advertisements not only results in higher profit in that market, but also implies an advantage of selling a superior quality product to consumers. The latter effect on the incentive to deviate is mitigated when platforms cooperate on two sides. We conclude that semi-collusion on advertising is more likely to be present than full collusion, unless detection is sufficiently unlikely and consumers are sufficiently ad-lovers. Then, considering the case of semi-collusion, we turn to the question regarding the more efficient basis for applying antitrust fines. The fines imposed on convicted firms that semi-collude over advertising could be based on either the illegal gain of the colluding side (one-sided fines), or the total illegal gain of the platform (two-sided fines). It is found that, if consumers are ad-neutral, the increase in firms' total profits originates from the colluding side and if this is the case, one-sided fines produce better deterrent results.

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Leniency Programs under Demand Uncertainty: Cartel Stability and the Duration of Price Wars

Abstract

Leniency Programs reduce sanctions against cartel members that either report spontaneously the existence of the infringement or cooperate during the investigation and facilitate prosecution. This paper investigates the impact of leniency programs on cartel stability when demand is uncertain and firms cannot perfectly observe their rival's choices. We show that pre-investigation leniency may or may not be effective in destroying the cartel, but in neither case affects the duration of price wars. Post-investigation leniency may have ambiguous welfare effects, in affecting both cartel stability and price wars duration. LPs applying in situations where leniency is not urgently needed may be not only ineffective, but also welfare reducing. Hence, in markets where negative demand shocks are sufficiently frequent, leniency policies may produce undesirable effects.

JEL Classification: K21, L12, L41

Keywords: Antitrust enforcement, Collusion, Leniency programs, Price wars

1 Introduction

Leniency Programs (LPs) aim to improve cartel deterrence by offering fine reductions to cartel members that voluntarily self-report or cooperate during investigation. The objective of this paper is to examine the effect of LPs on collusion when demand is stochastic and firms cannot perfectly observe their rival's choices. It is shown that an LP may affect both, the duration of the competitive price wars which are necessary for cartel stability, and the levels of demand uncertainty under which collusion is viable.

While it is well known that no collusive agreement can be successful without the participating firms' willingness to credibly punish deviations, there are instances where extremely hard punishments may be counterproductive. Such is the case analyzed in Green and Porter (1984, GP hereafter) where in the presence of unexpected demand slumps it is preferable for cartel-participating firms unable to perfectly monitor each other's actions to adopt strategies that are more lenient than the so-called "grim" (or "trigger") strategy.[1] In the GP context, a firm observing low sales cannot distinguish whether they are due to a deliberate price cut by a rival, or simply to sluggish demand. Punishing to infinity destroys, potentially for no reason, an otherwise profitable cartel. At the same time, not punishing at all induces cheating on the cartel agreement with certainty. An intermediate solution is a strategy that calls for interrupting cooperation every time low sales are observed (independently of their origin), but only for a limited number of periods. GP shows that a long enough period of cooperation-interruption is sufficient to induce loyalty to the cartel agreement. This way, cooperation may be disrupted for a number of periods giving its place to price wars, but stays alive in the long run.

In this paper we consider a supergame in which firms choose between competing and colluding. The demand is stochastic and a low demand state occurs with positive probability, while each firm cannot perfectly observe the price set by the rival. We show that leaving the LP available after an inspection has already been launched (post-investigation leniency) may have ambiguous effects on collusion. When negative demand shocks are frequent, post-investigation leniency may a) reduce the duration of temporary competitive price wars, and b) enlarge the set of negative-

[1] "Grim" (or "trigger") strategies correspond to a commitment to maximum punishment in case of an observed (or suspected) deviation, triggering competition for an infinite number of periods.

demand-shock probabilities for which the collusive agreement is viable. In other words, introducing leniency may in some instances enhance, instead of undermining cartel stability. Cartel stability is more likely to be enhanced when negative-demand-shocks are expected to be frequent in the future, and the probability of conviction after investigation is already high. Our conclusion is that in such cases, fine reductions not only are of little help, but also have undesirable side-effects. On the contrary, when the probability of conviction is low, leniency policies always result in a welfare improvement.

The effect of LP on cartels is first examined in Motta and Polo (2003) which stresses the importance of post-investigation leniency. Spagnolo (2004) concludes that costless cartel deterrence can be achieved when the first informant is rewarded sufficiently. Chen and Rey (2013) shows that it is optimal to provide leniency post-investigation when conviction is not very likely without self-reporting. Studying empirically the effectiveness of the early European corporate LP, Brenner (2009) concludes it has not significantly affected cartel stability, eventually because of the weakness of the provided incentives.

The paper is organized as follows: The model is described and benchmark is set in section 2. Section 3 contains the solution of the model when post-investigation leniency is introduced. In section 4 the analysis is concluded.

2 The Model with pre-investigation LP (the benchmark case)

Two firms producing homogeneous products and competing in prices play a competition *vs.* collusion game in an infinite number of periods. They both have the same discount factor $\delta \in (0,1)$ and maximize the expected sum of their future discounted profits. During a single period, firm *i*'s profit, $i = 1, 2$, is

- π^C if both firms cooperate
- $\pi^D = 2\pi^C$ if firm *i* deviates from the collusive agreement while the other charges the collusive price
- 0 if firm *i* charges the collusive price while the other deviates or if both firms compete

Due to resource limitations the Antitrust Authority (AA) investigates the industry with probability $\alpha \in (0,1)$. When a cartelized industry is investigated each firm is convicted with probability $\rho \in (0,1)$. Conviction entails a monetary fine $F = \mu\pi^j, j =$

D, C. A LP offers a cartel participating firm the option to provide cartel-related information and/or evidence, in exchange to a reduction in its fine in case the cartel is finally convicted. The reduced fine for reporting firms is $\gamma\pi^j$, with $0 \leq \gamma < \mu$. The evidence about collusion can be used by the AA only for one period and firms cannot be convicted for past violations.

At the beginning of each period each firm decides whether to collude. If at least one firm refuses to collude, competition takes place at least up to the end of the period. Assuming that a cartel agreement is in effect, each firm chooses between staying loyal to the agreement and defecting from it. Under LP, the defecting firm can also opt for denouncing the cartel, which, in that case is convicted with probability 1. In this section we restrict this option to only before any investigation is opened, leaving the case where firms may benefit from fine reductions by reporting after an investigation is launched for the next section.

During each period the demand can be either zero with probability q or positive with probability $1 - q$. Firms can observe directly neither the demand state, nor the price set by their competitor. As a consequence, a firm observing zero profits at any period cannot be sure whether this is due to its rival's defection from the agreement, or simply to low demand. Since in an infinitely repeated game that firm is bound to experience zero profits at some point, it may not find it optimal to retaliate by reverting to competition forever. Instead, after observing its profits being zero, a firm interrupts cooperation during a finite number of T periods (punishment period), during which both firms compete before reverting again to collusion.[2] During the punishment phase, firms set their price equal to marginal cost, earning zero profits for T periods.[3]

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[2] See also Tirole (1988) pp 262 on Green and Porter (1984)

[3] Marginal cost pricing is the result of assuming price competition on homogeneous products. It must be stressed that none of these assumptions is crucial for our results, as our analysis is also robust in allowing for differentiated products and/or quantity competition. What is crucial is that in all cases, by looking at its own profits, a firm cannot make a perfect inference about the other firm's behavior. For instance, if we allow for differentiated products, a firm can always cheat and jam the signal to the other firm, provided that it does not reduce its price below a level that would cause its rival to make lower profits than under a low demand state. Such constrained cheating allows the eventual cheater to obtain an only temporary, instead of permanent, interruption of cooperation. Robustness in case of Cournot competition can be shown along a similar argument.

We consider that in order to solve the coordination problem, regarding the specification of the optimal collusive price or the conditions that trigger the temporary reversion to competition, a sort of communication between firms is necessary. As explained in Green and Porter (1984) "…in view of the relative complexity of the conduct to be specified by this particular equilibrium and of the need for close coordination among its participants, it seems natural to assume here that the equilibrium arises from an explicit agreement."(Green and Porter, 1984, p.89, footnote 5). Since firms engage in a kind of communication, it is reasonable to assume that the estimation of q is common for both firms.

Each colluding firm facing the high demand state expects to earn the collusive profits and to pay the full fine in case of successful investigation. That firm keeps colluding for at least one additional period, as long as neither the collusive agreement is detected by the AA, nor a negative demand shock occurs. The observation of zero profits triggers reversion to Bertrand-Nash pricing for the T-period punishment phase, with return to cooperation afterwards. We denote this strategy as C. If the cartel is prosecuted the AA monitors the industry for an infinite number of periods, forcing firms to earn zero profits. The value of C is therefore:

$$V^C = (1-q)\pi^C(1-a\rho\mu) + \delta V^C[(1-a\rho)(1-q) + q\delta^T]$$

yielding

$$V^C = \frac{(1-q)\pi^C(1-a\rho\mu)}{1-(1-a\rho)(1-q)\delta - q\delta^{T+1}} \tag{1}$$

A defecting firm facing positive demand anticipates earning during the first period the entire monopoly profits $\pi^D = 2\pi^C$ reduced by the expected fine $a\rho\mu 2\pi^C$. Since the rival cannot distinguish between deliberate deviation and negative demand shock, competition takes again place during T periods. As always, cartel detection and punishment implies competition to infinity. Given the above, the value of secret undercutting is:

$$V^D = (1-q)2\pi^C(1-a\rho\mu) + [(1-a\rho)(1-q) + q]\delta^{T+1}V^C \tag{2}$$

No firm has incentive to undercut when

$$V^C \geq V^D \Leftrightarrow \delta^{T+1} \leq \frac{2\delta(1-a\rho)(1-q) - 1}{1 - a\rho - q(2-a\rho)} \tag{3}$$

If the RHS of (3) is positive, then there exists a lower bound on T above which the inequality is satisfied. Necessary conditions for the RHS being positive are $\delta \geq$

$\frac{1}{2(1-\alpha\rho)(1-q)}$, along with $q < \frac{1-\alpha\rho}{2-\alpha\rho}$.[4] A sufficiently high probability of negative demand shock lowers the cost of immediate defection and makes collusion impossible to sustain. Note that for $T = 0$ collusion is never sustainable.

Taking logarithms of both sides of (3) and rearranging, yields

$$T \geq T_1 \equiv \frac{\ln\left[\frac{2\delta(1-\alpha\rho)(1-q)-1}{1-\alpha\rho-q(2-\alpha\rho)}\right]}{\ln\delta} - 1 \tag{3'}$$

Since V^C decreases with T, firms benefit from choosing the shortest punishment period that secures the incentive constraint.

2.1 Pre-investigation leniency

Since reporting the cartel leads to conviction with certainty, following a firm's application for leniency, the competitive outcome prevails forever. In this case the reporting firm expects to earn $2\pi^C(1-\gamma)$ with probability $1-q$ and the competitive (zero) profits thereafter:

$$V^L = \frac{(1-q)2\pi^C(1-\gamma)}{1-q\delta^{T+1}} \tag{4}$$

The IC for cartel sustainability is $V^C \geq V^L$, which, using (4) and (1), implies

$$\delta^{T+1} \geq \frac{2(1-\gamma)[1-(1-\alpha\rho)(1-q)\delta]-(1-\alpha\rho\mu)}{q[2(1-\gamma)-(1-\alpha\rho\mu)]} \tag{5}$$

Setting $q = 0$ yields the full information result where collusion is sustainable when the AA does not offer sufficient leniency, *i.e.* $\gamma \geq 1 - \frac{1-\alpha\rho\mu}{2[1-\delta(1-\alpha\rho)]}$. Under stochastic demand, the collusive strategy is superior when the necessary punishment period is short enough, *i.e.*:

$$T \leq T_2 \equiv \frac{\ln\left[\frac{2(1-\gamma)[1-(1-\alpha\rho)(1-q)\delta]-(1-\alpha\rho\mu)}{q[2(1-\gamma)-(1-\alpha\rho\mu)]}\right]}{\ln\delta} - 1 \tag{5'}$$

When $T_1 \leq T_2$ setting $T = T_1$ induces cartel behavior with the least punishment, therefore the LP is completely ineffective. This occurs when:

$$T_1 \leq T_2 \Leftrightarrow \gamma \geq \hat{\gamma} \equiv \frac{1-2\delta(1-\alpha\rho)(1-q)+\alpha\rho\mu(1-2\delta q)}{2[1-\delta+\alpha\rho\delta(1-q)]} \tag{6}$$

Leniency destabilizes the cartel by making long punishment periods unattractive: facing a sufficiently long punishment period the prospective defector will opt for

[4] Since the positive denominator is always larger than the numerator, the latter should be also positive.

reporting the cartel rather than waiting for the restoration of cooperation. Given that the current LP in the EU absolves the informant firm for the entirety of its corresponding fine, the following lemma states the effect of fully absolvent pre-investigation leniency on cartel stability:

Lemma 1 *Assuming that the demand is zero or positive with probability q and 1-q respectively, and that each firm cannot perfectly observe the price choices of the rival, the introduction of a fully absolvent LP: i) succeeds or fails to obstruct collusion depending on whether* $q > \hat{q} \equiv \frac{2\delta(1-a\rho)-(1+a\rho\mu)}{2\delta[1-a\rho(1+\mu)]}$ *or* $q \leq \hat{q}$ *respectively, ii) has no impact on the length of the punishment period.*

Proof

Combining (3') and (5'), cartel stability requires $T_1 \leq T \leq T_2$ which is impossible when $T_1 > T_2$. When $T_1 > T_2$ the introduction of LP destabilizes the existing cartel since for $T \geq T_1$, V^C is always inferior to V^L while a reduction of T below T_1 induces defection. On the contrary, if $T_1 \leq T_2$ setting $T = T_1$ allows firms to maintain collusive behavior with the least punishment, therefore the LP is completely ineffective. The latter happens if $\gamma \geq \hat{\gamma}$. A fully absolvent LP offers $\gamma = 0$, hence it is ineffective when $\hat{\gamma} \leq 0$. Solving for q, the latter yields $q \leq \hat{q} \equiv \frac{2\delta(1-a\rho)-(1+a\rho\mu)}{2\delta[1-a\rho(1+\mu)]}$.■

When the probability of negative demand shocks is sufficiently high, firms find optimal to report the cartel because the expected loss of future earnings from the interruption of cooperation is not large enough. However, when q is low, both firms may not be willing to give up the opportunity for future cooperation, since the probability of positive future profits is high. A necessary condition for an LP with $\gamma = 0$ (full leniency) to be effective is $\hat{\gamma} > 0$, which implies

$$\mu > \hat{\mu} = \frac{2\delta(1-a\rho)(1-q)-1}{a\rho(1-2\delta q)}$$

Hence, for LP to be effective the fine in case of conviction must be large enough. Note that in case of certainty, *i.e.* when $q = 0$, the critical value of μ is $\frac{2\delta(1-a\rho)-1}{a\rho}$ which is always larger than $\hat{\mu}$. As q increases the collusive strategy becomes less attractive and a lower level of fines becomes sufficient to deter cartel activity. For any given level of fines, a higher value of q increases the value of $\hat{\gamma}$, indicating that in situations where the probability of low demand states is high, LPs providing moderate leniency are sufficient to destabilize the collusive agreement.

A numerical example[5]

Let us set the probability of detection and conviction, $a\rho = 0.12$. Letting also $\delta = 0.85$ and $\mu = 2$, an LP with full leniency ($\gamma = 0$) seems to be ineffective ($\hat{\gamma} < 0$) when the probability of negative demand shock is below 0.235. An increase of the discount factor expands the range of q for which LP is ineffective. For instance, when $\delta = 0.9$ full leniency is not sufficient for all $q < 0.3$. On the other hand, more severe enforcement results in a reduction of $\hat{q}$. For example, if $\mu = 2.5 \Leftrightarrow \hat{q} = 0.2$.

3 Allowing for post-investigation leniency

When firms can apply for leniency after an investigation has opened, an alternative collusive strategy arises for a colluding firm. This strategy, call it R, consists of maintaining cooperation as long as the cartel escapes the AA's attention, and reporting as soon as the cartel is targeted by an investigation, in order to benefit from reduction in eventual fines. Assume that the AA applies leniency according to the first informant rule, *i.e.*, the first firm that reports the collusive agreement receives a fine reduction while the other cartel participants pay the full fine, and let the reduced fine for the first informant be $\gamma^I \pi^C$, with $0 \leq \gamma^I < \mu$.[6] We assume that, if both firms decide to report, each firm has a 0.5 probability to win the reporting race and pay the reduced fine. Given the above, the value of R is:

$$V^R = (1-q)\left[(1-a)(\pi^C + \delta V^R) + a\pi^C\left(1 - \frac{\gamma^I + \mu}{2}\right)\right] + q\delta^{T+1}V^R$$

which, after rearrangement yields

$$V^R = \frac{(1-q)\pi^C\left(1 - a\dfrac{\gamma^I + \mu}{2}\right)}{1-(1-q)(1-a)\delta - q\delta^{T+1}} \tag{7}$$

[5] Using data from DoJ price-fixing cases, Bryant and Eckhart (1991) estimated the probability of cartel detection to be between 0.13 and 0.17 in a given year. Combe et al. (2008) estimated the same probability over a European sample to be around 0.13. Connor and Lande (2012) assessed that 0.2 to 0.28 of detected cartels are not convicted.

[6] The relevant theoretical literature usually concludes in favor of the first informant rule. Spagnolo (2004) claims that only the first informant should be rewarded sufficiently. Harrington (2008) also supports that restricting leniency to the first reporting firm induces the "race to the courthouse" effect when the investigation has launched. Ishibashi and Shimizu (2010) shows that providing amnesty to later applicants is of no use.

In order to be adopted, strategy R must be incentive compatible in the sense that each firm prefers to report under investigation given that the other continues colluding, which implies:[7]

$$\pi^C(1-\gamma^I) \geq \rho\pi^C(1-\mu) + (1-\rho)(\pi^C + \delta V^C) \tag{8}$$

Lemma 2 provides the condition under which firms choose to report under investigation, given that both collusive strategies, C and R, are sustainable:

Lemma 2 *For any level of forgiveness, after the opening of an investigation firms choose R instead of C when the punishment phase is long enough.*

Proof

Rearranging (8) yields $\delta^{T+1} \leq \frac{(\rho\mu-\gamma^I)[1-\delta(1-q)(1-a\rho)]-\delta(1-\rho)(1-q)(1-a\rho\mu)}{q(\rho\mu-\gamma^I)}$. ■

Furthermore, strategy R can be incentive compatible for any duration of the punishment phase, provided that the LP forgives a sufficient amount of the initial fine: setting $T = 0$ in (8) gives $\gamma^I \leq \frac{\rho\mu-\delta\{1-q+\rho[\mu-1+q-a\mu(1-q)]\}}{1-\delta+a\delta\rho(1-q)}$. Note that incentive compatibility of the alternative collusive strategy does not imply that R is more profitable than C. The profitability condition is stricter than the incentive compatibility constraint since the minimum T that renders (8) effective is always lower than the minimum T required for $V^R \geq V^C$ to hold:

$$V^R \geq V^C \Leftrightarrow T \geq T_5 \equiv \frac{\ln Z}{\ln \delta} - 1$$

where $Z = \frac{2\mu\rho-(\gamma^I+\mu)-\delta(1-q)[2(1+\rho(\mu-1))-\alpha\rho(\mu-\gamma^I)-(\gamma^I+\mu)]}{q[2\mu\rho-(\gamma^I+\mu)]}$.

Since (8) is less strict than the profitability condition above, an amount of forgiveness that suffices to satisfy (8) may in some cases not be able to also render R more profitable than C. In those cases the situation resembles to a prisoners dilemma where each firm under investigation may find privately optimal to report, even if mutually keeping secrecy and sticking to collusion might have assured to both firms higher payoffs.

For the cartel to be sustainable as long as it escapes investigation, strategy R must be preferred to both, secret undercutting and pre-investigation reporting. Concerning the first, setting $V^R \geq V^D$ yields:

$$\delta^{T+1} \leq \frac{2(1-2a\rho\mu) + a(\gamma^I+\mu) - 4\delta(1-a)(1-q)(1-a\rho\mu)}{2a\rho(1-q) + a(\gamma^I+\mu)[1-a\rho(1-q)] - 2[1-2q(1-a\rho\mu)]} \tag{9}$$

[7] See also Sauvagnat (2014)

A deviation without reporting seems to be less attractive when the consequent competitive phase is expected to be long enough. Therefore, denoting as T_3 the minimum value of T for which (9) holds, the leniency-induced strategy of colluding until investigation and reporting afterwards dominates secret undercutting when $T \geq T_3$

In order for R to also dominate the strategy of reporting before investigation, it must be that $V^R \geq V^L$. Using (7) and (4), this implies:

$$\delta^{T+1} \geq \frac{2(1-2\gamma) + a(\gamma^I + \mu) - 4\delta(1-a)(1-q)(1-\gamma)}{q[2(1-2\gamma) + a(\gamma^I + \mu)]} \tag{10}$$

Since the cost of the definite collapse of the cartel decreases with *T*, the inequality above holds for short enough punishment period. We use T_4 to denote the maximum value of *T* that makes (10) hold.

Combining the above results, the following lemma provides the sustainability condition for strategy R:

Lemma 3 *Strategy R is sustainable when q is low enough, i.e. when*

$$q \leq \bar{q} \equiv \frac{(1-a\rho)[4\delta(1-a)(1-\gamma) - a(\gamma^I + \mu) - 2(1-2\gamma)]}{4(1-a)\delta[1 - a\rho(1-\gamma+\mu)]}$$

Proof

Since sustainability implies that strategy R dominates both a) secret undercutting and b) pre-investigation reporting, it must be that $T_3 \leq T_4$. Setting the RHS of (9) to be larger than that of (10) and solving for q shows that the expression in the lemma is a necessary and sufficient condition for $T_3 \leq T_4$.■

The intuition behind lemma 3 is similar to that of lemma 1: Any collusive strategy is more likely to be sustained when the probability of negative demand shocks is low, since firms cannot disregard the expected benefit stemming from cooperation in the future.

We need to define the level of q where T_1 and T_3 are equal: setting $T_1 = T_3$ and solving for q yields:

$$q = \tilde{q} \equiv \frac{(1-a\rho)\left[\mu(2\rho-1) + \delta\big(\mu - 2 + \rho(2-(2-a)\mu)\big) - \gamma^I\big(1-\delta(1-a\rho)\big)\right]}{\delta\left[\rho\left(a\big(2+3\mu-\rho(2+(2+a)\mu) - \gamma^I(1-a\rho)\big)\right) - 4(1-\rho)\right]}$$

We are now able to summarize the effect that post-investigation leniency has on the duration of price wars:

Proposition 1 *In markets where a) the low demand state occurs with high enough probability* $(q \geq \tilde{q})$*, and b) even if no firm reports, cartel-investigation evidence alone leads to high probability of conviction* $\left(\rho \geq \hat{\rho} \equiv \frac{\mu+\gamma^{I}+2(1-2\gamma)}{\mu(2-a)+a\gamma^{I}+2(1-2\gamma)}\right)$*, offering post-investigation leniency has an adverse side-effect: it allows firms to maintain cartel stability by using shorter price-war periods in their strategy.*

Proof

Consider that both collusive strategies, C and R, are sustainable, *i.e.* $q \leq min\{\hat{q}, \bar{q}\}$ and that the leniency provided is sufficient to induce firms to report under investigation. Provided that both firms wish to adopt the shortest punishment period that induces discipline, post-investigation LP shortens (prolongs) price wars when $T_3 < (>) T_1$. Since $T_3 < T_1$ holds for $q > \tilde{q}$, when the latter holds each firm prefers to collude and reveal when the industry is investigated. In this case, the necessary punishment period is shorter than what it would have been in the absence of post-investigation leniency. On the other hand, if $\rho < \hat{\rho}$, $\tilde{q}$ is larger than $\hat{q}$ and $T_3 < T_1$ never holds. ■

When $\rho < \hat{\rho}$ $\tilde{q}$ is higher than $\bar{q}$, thus, for any $q < \bar{q}$ firms would prefer not to report the cartel even if the AA had already launched an inspection. If ρ is greater than $\hat{\rho}$, strategy R is both incentive compatible and more profitable for $q \in [\tilde{q}, \bar{q}\,]$.[8]

The above analysis shows that the impact of fine reductions on the length of competitive price wars varies as both, low demand and conviction become more or less likely. When the probability of conviction is low, the introduction of post-investigation LP always prolongs the competitive phase that is necessary for cartel stability. On the other hand, when the investigation alone can provide sufficient evidence as to render conviction highly probable without the help of any firm's post-investigation reporting, the added value of the latter is low and potentially outweighed by the cost of leniency in terms of shorter price wars. Such an outcome is possible if negative demand shocks are expected to occur frequently in the future. Moreover, the difference $T_1 - T_3$ increases with q, since the loss of cartel dissolution due to post-

[8] When $\rho \leq \hat{\rho}$ R is less profitable than C since $T_5 \geq T_3 \geq T_1$. Hence, if firms were able to choose the most profitable collusion, post-investigation leniency would have no impact on collusion for low values of ρ, since firms would choose not to report and to compete for T_1 periods even if an investigation had opened. For $\rho > \hat{\rho}$, R is more profitable than C when $q > \tilde{q}$. Both firms report after investigation and compete for T_3 periods when a demand shock occurs.

investigation reporting decreases, thus making R even more attractive than C. Note that when ρ is high enough, *i.e.* when $\rho > \frac{2\delta+(\gamma^I+\mu)(1-\delta)}{2\mu+\delta[2-\alpha\gamma^I-\mu(2-\alpha)]}$, R is more profitable than C and the effect described in proposition 1 holds for every value of $q \in (0, \bar{q}]$.[9]

The next proposition determines the effect of post-investigation leniency on the maximum level of q that allows collusion to be sustainable:

Proposition 2 *A post-investigation LP with generous fine reductions to the reporting firm may induce cartel formation in situations where the cartel would have been otherwise unstable. More specifically, fine reductions corresponding to* $\gamma^I < \tilde{\gamma}^I = \frac{\mu(2\rho-1-a\rho)-2(1-\rho)(1-2\gamma)}{1-a\rho}$ *extend the range of values of* q *for which the cartel is sustainable.*

Proof

Setting $\hat{q} < \bar{q}$ and solving for γ^I yields $\gamma^I < \tilde{\gamma}^I$. Hence, an overwhelming post-investigation leniency allows firms to sustain collusion for $q \in (\hat{q}, \bar{q}]$. Note that no collusion was sustainable for $q > \hat{q}$ when leniency was restricted only to firms reporting before an inquiry.■

Offering excessive post-investigation leniency extends collusion sustainability to values of q for which the cartel would not have been sustainable had leniency been limited to only pre-investigation reporting. An increase in the probability of conviction without reporting reduces the value of the low-demand probability threshold below which strategy C is sustainable. On the other hand, as the fine-reduction due to post-investigation leniency becomes more generous, strategy R becomes more likely to be stable. Hence, for a given probability of conviction, a more lenient treatment of the reporting parties results in a more stable collusive agreement.[10]

A numerical example

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[9] In other words, given that conviction of an investigated cartel becomes probable enough, $\tilde{q}$ becomes negative and R is both sustainable with shorter punishment periods and more profitable than C for every positive value of q below $\bar{q}$.

[10] Note that $\bar{q}$ increases while $\hat{q}$ decreases with ρ.

Consider that $\delta = 0.85$, $\mu = 2.5$, $\alpha = 0.15$ and $\gamma = 0$. Provided that post-investigation reporting is awarded with lenient treatment ($\gamma^I = 0.2$)[11] and that $\rho = \frac{2}{3}$, V^m and V^r are sustainable when q is lower than $\hat{q} = 0.253$ and $\bar{q} = 0.232$ respectively. Since $T_3 \geq T_1$ holds for $q \geq \tilde{q} = 0.327$, setting $\gamma^I = 0.2$ has clearly a positive impact on cartel deterrence, prolonging the competitive phase of the milder collusive strategy. On the other hand, if $\rho = 0.75$, the critical values of q are $\bar{q} = 0.245 > \hat{q} = 0.22 > \tilde{q} = 0.086$. Under this assumption, firms choose to collude and reveal under investigation for $0.086 \leq q \leq 0.245$ competing for $T_3 (< T_1)$ periods when they observe zero profits. For $0 \leq q < 0.086$, firms are forced to opt for the less profitable collusive strategy and to compete for longer than T_1 period. When $\rho < 0.68$, $\tilde{\gamma}^I$ is negative and excessive fine reductions described in proposition 2 are not applicable. On the contrary, if $\rho = 0.75$ the critical value of γ^I is $\tilde{\gamma}^I = 0.53$ and the effect described in proposition 2 may apply.

Note finally that the results of this paper have been developed under the assumption that q is common knowledge, or equivalently that all critical values T_h, $h = 1,2,3,4$, being common to both firms' strategies. This assumption simplifies the exposition but is by no means crucial for the results.[12] Since the collusive outcome requires communication between the two firms, the latter must agree upon a common duration of the punishment period. If their perception of q differs, the punishment must be at least as long as to induce compliance of the firm with the highest estimate of q. Since T_1 (and T_3 in the case of post-investigation LP) is increasing in q, that punishment period will also induce compliance of the firm with the lower estimate of q.[13]

4 Concluding remarks

This paper investigates the effect of LPs on firms' incentives to collude when the actions of a firm are not perfectly observable by its rival, and demand is stochastic. Assuming that the AA discovers cartels with a given probability and the fine imposed

[11] Under these parameter values, R is incentive compatible for any non-negative T. The critical value of ρ is 0.706.

[12] We are grateful to an anonymous referee for raising this issue.

[13] The upper bound to the number of punishment periods imposed by the presence of LP (T_2 for C, and T_4 for R) is decreasing in q and satisfied as long as q is lower than $\hat{q}$ and $\tilde{q}$ for C and R respectively.

in case of conviction represents a fraction of the cartel-induced profits, we show that increases in the probability of a negative demand shock affect positively the effectiveness of the LP. In markets with low probability of negative demand shocks, the LP must offer substantially higher fine reductions in order to destabilize the cartel. Under the fairly common assumption of detection and conviction rate of around 0.15 and 0.8 respectively, offering full leniency is not sufficient to destabilize the cartel if the probability of a negative demand shock is less than 0.235. On the other hand, when the LP is not effective in destroying the cartel, it will not affect either the length of the price war period.

Often the AA provides lenient treatment for reporting parties even after the opening of the inspection. In markets where the low demand state is expected to occur with high frequency and investigation alone is able to provide sufficient evidence as to make the conviction of an existing cartel highly probable, post-investigation leniency may exhibit undesirable side-effects. First, it may induce cartel-formation in circumstances where, in its absence, any attempt to coordinate pricing would have failed as being unsustainable. Second, it may shorten the punishment period during the pre-investigation life of an existing cartel. This implies longer periods of high prices and low welfare. Both side-effects are likely in cases where investigation alone can bring about powerful evidence, leading to high conviction-probability without need for self-reporting. They constitute, therefore, a strong point against post-investigation leniency with small added value. On the other hand, when investigation is not able to produce sufficient evidence for conviction, on top of being necessary for conviction, post-investigation leniency may also have an additional positive impact on welfare through increasing the duration of price wars.

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On Leniency and Markers in Antitrust

Abstract

In this paper we investigate the impact of leniency programs on firms' decision to collude. We depart from previous literature by relaxing the assumption that evidence provided by a single firm suffices to convict an existing cartel with certainty. Assuming the conviction-probability to be increasing in the number of reporting firms, we show first that efficient cartel deterrence requires incentives for all firms to report. Under a regime that secures a marker for the first in line applicant, eligibility for leniency should be extended to at least a second informant. We show that the introduction of the marker system has an ambiguous impact on cartel deterrence. In relation to the manner that the marker is secured and the cartel-related evidence is allocated, we derive the conditions under which allowing the first applicant to secure a marker enhances cartel deterrence.

JEL Classification: K21, L12, L41

Keywords: Antitrust enforcement, Collusion, Leniency programs

1 Introduction

Leniency Programs (hereafter LPs) aim to improve cartel detection and deterrence by offering fine reductions to cartel members that either voluntarily self-report before there is even an investigation, or report and generally cooperate during investigation (*pre*- and *post*-investigation leniency, respectively). Pre-investigation leniency aims at destabilizing a cartel by making deviations from its central policy more attractive. Post-investigation leniency aims to evidence acquisition that is sufficient for an already spotted cartel to be convicted. By sufficiently increasing the probability of cartel conviction, post-investigation leniency has also important impact on the pre-investigation stability of the cartel, affecting the participating firms' incentives to collude.

A common feature of many LPs is the presence of the *marker* system. The latter allows a number of reporting parties to reserve the position in queue for a finite period of time before the identities of the eligible for leniency are determined. In other words a marker removes the uncertainty for the leniency applicant about the existence of other informants and its own position in reporting line. According to Organization for Economic Co-operation and Development (OECD, 2014) the majority of jurisdictions of OECD countries seem to have some kind of marker system. Most of them (including European Commission) restrict the availability of markers to the first reporting party, while others (including Canada, France, Germany, United Kingdom etc) offer this possibility to subsequent applicants as well.

Most of the LPs-related literature assumes that even a single firm's reporting provides sufficient evidence for conviction with certainty. This implies that up to some details, all firms mainly possess similar evidence, and therefore the usefulness of any additional reporting is simply to strengthen the Antitrust Authority's (AA) ability to prove the putative infringement. However, some practitioners (see Blatter et al., 2018) observe that firms have incomplete pieces of evidence and total evidence is cumulative, with each single reporting making conviction only more likely. Also, OECD (2012) states that "authorities are likely to find themselves in situations where, while aware of the existence of a cartel as a result of a leniency application by the first applicant, they are not yet in a position to prove the infringement".

In this paper, we relax the assumption that a single firm's reporting is sufficient for conviction, assuming instead that the probability of conviction is increasing in the number of reporting firms. This corresponds to assuming that the evidence brought by subsequent informants has added value, rather than being a mere corroboration of the evidence offered by the first informant. This assumption has the interesting implication that inducing a single firm to report does not imply that all its partners have sufficient incentives to do as well. If the leniency is not sufficiently generous some firms may prefer to remain silent and avoid reporting in an attempt to restrain the conviction likelihood. We show that cartel deterrence requires the LP to provide incentives for universal reporting, *i.e.* the AA should design the LP as to obtain all the available evidence.

The rationale of rewarding subsequent leniency applicants is described in OECD (2012), where it is suggested that when the authorities "are not yet in a position to prove the infringement, the social benefits from cooperation with the second or later applicants may be large compared to the public interest of penalizing the infringers". These benefits may arise from cost savings in prosecution, increased detection rate and destabilizing effects on cartels.

A basic difference between the United States' and the European Union's LP pertains to the number of informants which are eligible for fine reductions. The US Department of Justice (DoJ) allows only the first firm that provides valuable information to receive amnesty from fines. In contrast, the European LP offers milder fine reductions to multiple informants. Parties that reveal information with significant added value can be awarded with reductions up to 50% of the fine that would have otherwise been imposed.

In this paper we compare the two systems by focusing on their impact on firms' incentive to collude. The relevant theoretical literature usually concludes in favor of the first informant rule. Spagnolo (2004) claims that only the first informant should be rewarded sufficiently. Chen and Rey (2013) notes that allowing additional firms to be eligible for leniency reduces the effectiveness of the program. Harrington (2008) also supports restricting leniency to the first reporting firm, claiming that such a policy induces a "race to the courthouse" effect once an investigation has been launched. Sauvagnat (2014) shows that leniency should be provided when only a single firm reports information; when more than one firm are willing to report, none should receive any fines reduction.

The impact of marker system on the effectiveness of LP is studied in Blatter et al. (2018). Assuming imperfect and asymmetric evidence in a duopoly and restricting the eligibility for leniency to the first informant, they show that under the marker system only one firm reports and the AA obtains only partial evidence. They show that the marker system increases the deterrence cost unless firms possess sufficiently asymmetric evidence.

Here, we show that if the first informant's position is protected by a marker, the role of post-investigation leniency in destabilizing cartels in the pre-investigation period is substantially weakened by the first informant rule. Our analysis offers support to the European practice of allowing multiple informants to benefit from lenient treatment.

We also find that the impact of the marker system on the effectiveness of a LP to deter cartel activity crucially depends on the manner that the marker is available. We show that if the marker becomes available to following applicants once its initial holder fails to comply with its information-proving obligation, the use of markers reduces the LP's power in cartel deterrence. On the contrary, if the marker is permanently lost following a denial to report by its initial recipient, the marker system may facilitate reporting by cartel members.

The rest of the paper is organized as follows: The model is described in the next section. In section 3 we analyze the benchmark case in the absence of marker system. In section 4 we introduce the marker. Section 5 concludes.

2 The model: investigation, conviction probability and fines

Consider an industry with N symmetric firms producing homogeneous goods and competing in prices for an infinite number of periods. Each firm maximizes the expected sum of future discounted profits using a common discount factor $\delta \in (\frac{1}{2}, 1)$.[1] During each period a competition *vs.* collusion game takes place. If all firms cooperate setting the collusive price, each one earns an amount of profit, π. When one firm unilaterally deviates from the agreed price, it receives $N\pi$ while the other firms get zero. The competitive gross profits are also zero. In order to lighten the analysis and without loss of generality we normalize $\pi = 1$.

[1] For $\delta < 0.5$ collusion is not sustainable even in the absence of antitrust policy.

Resource limitations allow the AA to investigate the industry with probability $a \in (0,1)$.[2] Even when the investigated firms are guilty, the start of an investigation does not necessarily imply conviction. Concentrating only on cases where the investigated firms have indeed formed a cartel, we assume that, despite the presence of the infringement, the prosecution outcome is uncertain, with the probability of conviction being non-decreasing in the amount of available evidence. We measure evidence by the change in probability of conviction that it induces. Any evidence that is a mere repetition of evidence already in the possession of the AA does not increase the probability of conviction; it is therefore considered as redundant and not taken into account.

The total amount of cartel-related evidence is decomposed in two parts: *common evidence*, denoted by z, which is evidence possessed by every participant, and *exclusive* evidence which represents pieces of evidence in the possession of a sub-group of firms. To keep matters simple, we assume that every exclusive piece is detained by only a single firm, and that the exclusive evidence pieces are distributed symmetrically among firms, each one having in its possession an amount $\Delta\rho = \frac{1-z}{N}$. We assume that AA's actions (dawn-raids etc) uncover only a portion ρ_0 of the total evidence, and that this portion contains both, common and exclusive evidence in specific portions, *i.e.*, $\rho_0 = \lambda_1 z + \lambda_2 N\Delta\rho$ with $0 \leq \lambda_h \leq 1$, $h = 1,2$; the value of ρ_0 determines the probability of conviction when no firm confesses. For simplicity, we assume that $\lambda_1 = \lambda_2$, *i.e.* the evidence unveiled by the AA's efforts consists of equal portions of common and firm-specific evidence.[3]

When a cartelized industry is investigated each convicted firm is forced to pay a fine μ, where $\mu > 1$; μ is composed of a compensation to injured parties equal to the amount of illegally obtained profits, as well as of a pure fine paid to the authorities.[4]

[2] Using data from DoJ price-fixing cases, Bryant and Eckhart (1991) estimated the probability of cartel detection to be between 0.13 and 0.17 in a given year. Combe et al. (2008) estimated the same probability over a European sample to be around 0.13.

[3] Assuming alternatively that $\lambda_1 \neq \lambda_2$ produces qualitatively similar results.

[4] Harrington (2014) mentions that the standard formula for cartel-related damages is $(p^c - p^n)q^c$, where p^c and q^c are the collusive price and quantity respectively and p^n is the Bertrand-Nash price. Bageri et. al (2013) shows that fines on revenues result in higher collusive prices that fines on illegal gain. In a dynamic context, Katsoulakos et al. (2015) shows that fines based on illegal profits are welfare superior to fines on revenues.

The value of μ can be neither too low, for in this case the fine cannot induce compliance, nor too high as it may curtail competition in the long run by pushing some competitors out of business.[5]

A post-investigation LP allows a cartel participating firm to provide information and/or evidence related to the existence of the cartel after the investigation's opening, in exchange for a fines reduction. As the probability that an inspected cartel is convicted increases with the amount of the evidence, it is reasonable to assume that it is also non-decreasing in the number of reporting parties. The common share of the additional evidence is provided only once by the first firm that testifies. The first informant increases the probability of conviction by $(1-\rho_0)z$, in addition to any exclusive piece it may present. When more firms confess, every subsequent informant increases the probability of conviction by $(1-\rho_0)\Delta\rho$. Hence, when $n \in [1, N]$ firms report, the probability of conviction becomes:[6]

$$\rho_n = \rho_0 + (1-\rho_0)(z + n\Delta\rho) = \frac{N(z + \rho_0 - z\rho_0) + n(1-z)(1-\rho_0)}{N}$$

A common feature among LP-related legislation in different countries is that it restricts post-investigation leniency to only a limited number m of applicants. Usually, the eligible firms are selected on a first-come-first-served basis, subject to the requirement of providing sufficient amount of evidence.[7] Even in jurisdictions where all the applicants are eligible, their treatment is asymmetric, with the "early birds" receiving substantially more generous treatment.

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[5] The US (federal) fines correspond to no more than double damages while other jurisdictions allow for up to treble damages, see Harrington (2014). A reasonable assumption for the value of μ is that $\mu \in [2,3]$. However, as Harrington (2014) points out, in practice firms found guilty by a court of law pay fines that "are probably more on the order of single rather than treble damages".

[6] Solving $\rho_n \leq 1$ yields $n \leq N$. Hence, certain conviction requires reporting by all firms. If instead we assume that one firm's reporting increases the likelihood of conviction by more than $(1-\rho_0)\Delta\rho$, *e.g.* by $(1-\rho_0)b\Delta\rho$ where $b > 1$, then confession by less than N firms suffices to raise the probability of conviction to one. Without any loss of generality we assume here that $b = 1$, *i.e.* that certain conviction requires reporting by all firms. Our results hold if we assume that $b > 1$.

[7] For instance, the US system grants leniency to a single applicant, subject to the condition that it provides substantial evidence. The EU system allows for many applicants, however, it offers them asymmetric treatment, with leniency being more generous for those that come out early and decreasing for subsequent informants.

When confessing, those eligible for leniency will receive a fine reduction proportional to their individual contribution to cartel prosecution, being required to pay only a fraction of the full fine. The reduced- to-full-fine ratio is:

$$\gamma_1(k_f) = \frac{1 - \rho_0 - k_f(1 - \rho_0)(z + \Delta\rho)}{1 - \rho_0} = 1 - k_f(z + \Delta\rho)$$

for the first informant, and

$$\gamma(k_s) = \frac{1 - \rho_0 - (1 - \rho_0)z - k_s(1 - \rho_0)\Delta\rho}{1 - \rho_0 - (1 - \rho_0)z} = \frac{N - k_s}{N}$$

for one subsequent eligible applicant. Both γ_1 and γ lie between 0 and 1, *i.e.* we rule out rewards. Both parameters k_f, k_s measure how additional information by the first and subsequent informants, respectively, is rewarded by the AA, and thus determine the generosity of the leniency offered to each eligible informant. According to the AA's specific policy, the parameters k_f, k_s, can be equal or unequal. Unequal k's reflect price discrimination, *i.e.*, that the AA awards a difference price per unit of information received by the first, or subsequent informants.[8] As we restrict leniency to non-negative fines the leniency rates are bounded from above: $\gamma_1(k_f) \geq 0$ and $\gamma(k_s) \geq 0$ imply $k_f \leq \bar{k}_f \equiv \frac{N}{1+z(N-1)}$ and $k_s \leq \bar{k}_s \equiv N$ respectively.

A firm's decision on whether to come forward and provide evidence crucially depends on that firm's perception about its position on the priority line. The accuracy of this perception depends in turn on the AA's information-diffusion policy. With respect to the latter, we examine two alternative systems.

The first, that we term "*opaque* practice" allows no firm to have information about the existence of other confessants; therefore, as the investigation proceeds no firm can be aware of its position on the priority line. Due to this information restriction, cartel participants act as if the decision on whether to come forward must be taken simultaneously by all of them. Combined with the assumption that different positions on the priority line receive asymmetric treatment, simultaneous decision implies that, when making its decision, a cartel participant is unaware of the leniency treatment it

[8] It is equally possible to replace k_s by a sequence k_i, $i = 2, \dots, m$, where m is the number of eligible-for-leniency applicants, implying that the AA accords different importance to the information provided by each different applicant. In order to keep the analysis simple, and as it turns out without loss of generality, we limit the possibility of information-price discrimination only between the first and subsequent applicants.

will finally receive. If it decides to confess thinking that $n-1$ others are going also to confess, it must assign some positive probability to being a) the first informant, b) one of the m eligible ones, or c) one of the $n-m$ that report but receive no lenient treatment. Hence, its expected fine must be a fraction $\hat{\gamma}_n\mu$ of the full fine, where:

$$\hat{\gamma}_n = \frac{\gamma_1+\gamma(m-1)+(n-m)}{n} \qquad (1)$$

with $n \in \{1, \dots, N\}$ and $m \leq n$. For the rest of the analysis we assume that the number of eligible for leniency informants can be either one or two, that is $m \in \{1,2\}$.

In the second system, often termed "*marker*" system, before providing evidence each firm has secured a position on the priority line, thus knowing exactly the kind of leniency treatment it will receive. In practice, a mixed system is often followed, where the first few applicants for leniency secure their position, while subsequent applicants only know that they will not occupy any of the already reserved positions. In this work we assume that a marker is handed only to the first-to-door applicant. The marker allows its holder some given time period in order to prepare and present the promised evidence. If at the end of that period the marker holder refuses to deliver the evidence, the marker may or may not become available for another potential applicant. As the transferability of the marker has important implications for the LP's efficiency, we examine both cases.

The timing of the game is as follows. At the beginning of every period each firm decides whether to collude or not; if at least one firm refuses to collude, competition takes place at least up to the end of the period. If a cartel agreement is reached, in all subsequent periods each firm chooses between staying loyal or defecting from it. A deviation from the collusive price implies that the market will be competitive ever after (trigger strategies). At the end of each period, after firms have set their prices and made the current period profit, the AA randomly decides with probability a whether to investigate the industry. Collusion evidence can be used for only one period, *i.e.* firms cannot be convicted for past violations. In case of investigation, each cartel participant chooses between reporting or not. Focusing on the deterring impact of leniency policies, we make the simplifying assumption that, regardless of whether it leads to conviction, an investigation implies the definite dissolution of the collusive agreement.[9] Finally, the cartel is convicted with probability ρ_n, depending on the amount of evidence collected.

3 The opaque practice

Once an investigation has started each firm faces a multi-person prisoners' dilemma, and as the AA follows a full-secrecy policy, an investigation node corresponds to a subgame where the reporting decision is taken simultaneously by all firms.[10] The superscript "0" indicates hereafter equilibrium values under AA's opaque policy.

In order to find the equilibrium of the investigation-subgame, we must determine each firm's best reply function. Consider a node where the AA decides to investigate. If a firm thinks that $n-1$ others, $n \in [2, N]$, are about to report, it expects to pay the full fine with probability ρ_{n-1}, and no fine in case of unsuccessful prosecution. Reporting, on the other hand, reduces that period's profit by a percentage $\rho_n \hat{\gamma}_n$ and is the best reply to $n-1$ other firms choosing to report when:[11]

$$1 - \rho_n \hat{\gamma}_n \mu \geq \rho_{n-1}(1-\mu) + 1 - \rho_{n-1} \tag{2}$$

which simplifies to $\varphi(n) = \rho_{n-1} - \rho_n \hat{\gamma}_n \geq 0$ or equivalently

$$\varphi(n) \equiv \frac{N\rho_n\left[k_f(1+(N-1)z)+k_s\right] - Nn(1-z)(1-\rho_0)}{N^2 n}$$

where $N\rho_n = n(1-z)(1-\rho_0) + N(z + \rho_0 - z\rho_0)$.

Note that $\frac{\partial \varphi(n)}{\partial n} = -\frac{\left[k_f(1+(N-1)z)+k_s\right][\rho_0+(1-\rho_0)z]}{Nn^2} < 0$. The following lemma links the number of reporting parties with the incentive to denounce the agreement, given the presence of at least one applicant:

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9 We assume that, no matter whether the investigation leads or not to conviction, the AA monitors the investigated market for an infinite number of periods, forcing firms to compete forever after. Assuming instead that convicted firms keep colluding produces qualitatively similar results.

10 Simultaneous reporting is meant to represent that before deciding whether to confess, a firm must "guess" how many others are about to report. "Guessing" correctly is equivalent to assuming that during the reporting process each potential whistleblower is notified about the number of the other reporting parties and taking this into account decides its action. At the end of the reporting phase, the names of those eligible for leniency are determined randomly.

11 Observe that allowing an investigated cartel to continue its collusive activity affects both sides of (2) positively.

Lemma 1 *In case of investigation, for every $n \in [2, N]$ the incentive to report is monotonically decreasing in the number of other informants.*

Since the incentive to report is negatively affected by the number of reporting parties, if a firm decides to report as part of a group with $n-1$ other informants, it is also willing to report when thinking that there are fewer informants in the group. For any given n, setting $\varphi(n) = 0$, determines a relation between k_f and k_s that allows at most n firms to come forward. Solving this relation for k_f, obtains

$$k_{n,k_s}^0(k_s; n) \equiv \frac{1}{1+(N-1)z}\left[\frac{Nn(1-z)(1-\rho_0)}{N\rho_n} - k_s\right] \qquad (3)$$

For any reward per unit of information offered to the second informant, for the LP to provide sufficient incentive for n informants to come forward, the per-unit-of-information reward offered to the first informant must be no less than k_{n,k_s}^0 as defined above. Due to the negative sign of the coefficient of k_s, expression (3) defines a trade-off between rewards to the second and first informant that makes the pair (k_f, k_s) sufficient to induce n firms "racing to the court," even if they know that $n-2$ of them will receive no reward for the information they will provide.

We define the value of $k_{n,k_s}^0$when $k_s = 0$ $(m = 1)$ as k_n^0:

$$k_n^0 = \frac{Nn(1-z)(1-\rho_0)}{[1+(N-1)z]N\rho_n}$$

Corollary 1 *When $k_{n+1,k_s}^0 > k_f \geq k_{n,k_s}^0$ the equilibrium of the post-investigation subgame involves $n \in [2, N]$ informants.*

Note that for every $n \in [2, N-1]$, the pair (k_{n,k_s}^0, k_s) determines multiple equilibria of the investigation subgame. These equilibria are qualitatively similar: they contain the same number of informants, differing only with respect to the identity of the reporting firms. More serious is the potential existence of another equilibrium where no firm comes forward, investigated right below.

Lemma 1 establishes that a firm's incentive to confess is reduced with the number of other firms that this firm thinks have also decided to confess. This rule applies to firms that think that at least another cartel member is going to report. However, the situation of the unique informant is different and not described by lemma 1: when thinking that no other firm has decided to confess, a firm may decide not to come forward even if it would have done so under the assumption that some another firm has already decided to confess. Because the first informant offers all the common evidence the rewards are larger, but also the consequence in terms of increasing the conviction probability graver. When the latter creates a sufficiently strong disincentive, universal non-reporting is an equilibrium, along with the equilibria mentioned earlier. Compared to them, the non-reporting equilibrium is Pareto dominant, and for this reason it is very important for the AA to design its policy as to eradicate it.

Lemma 2 *Universal non-reporting is an equilibrium of the investigation subgame when* $k_f < k_1^0$*, where*

$$k_1^0 \equiv \frac{N(1-\rho_0)}{1+(N-1)(z+\rho_0-z\rho_0)} \tag{4}$$

Proof

If a firm reports assuming that no other does so, it expects to pay the reduced fine with probability ρ_1, and to receive nothing thereafter. If it chooses to remain silent as everybody else, it expects to pay the full fine with probability ρ_0 and no fine otherwise. Reporting is the best reply to all other firms remaining silent when:

$$1-\rho_1\gamma_1\left(k_f\right)\mu \geq 1-\rho_0+\rho_0(1-\mu)$$

Solving the above for k_f yields the value k_1^0 stated in (4).■

Subject to the constraint that the resulting fine is nonnegative, the value of k_1^0 in (4) represents the minimal implicit price per piece of information at which the AA must purchase the first informant's evidence—both, common and exclusive— in order to induce at least a single firm to come forward when thinking that all the others will remain silent. Substituting k_1^0 into the definition of $\gamma_1$yields:

$$\gamma_1(k_1^0) \equiv \frac{N\rho_0}{1+(N-1)(z+\rho_0-z\rho_0)}$$

Note that $\gamma_1(k_1^0)$ always decreases with z, thus, when the unique informant possesses a large amount of the common share of evidence, it requires more generous fine reductions in order to come forward.

If the LP provides sufficient incentives for a single informant to come forward, the race to the court is not guaranteed: other firms may not follow suit, and to the extent that they possess pieces of evidence not available to the first informant, conviction is not 100% certain. As we will see later, this may also have serious implications for the effectiveness of the LP in deterring cartel formation.

Lemma 3 *For* $k_f = max\{k_1^0, k_{n,k_s}^0\}$ *at least* n *firms reveal under investigation.*

Proof

When $k_1^0 \geq k_n^0 \Leftrightarrow \rho_0 \leq \frac{[1+(N-1)z]z}{(1-z)[n-1-(N-1)z]}$ (for positive k_s the latter threshold increases rendering $k_1^0 \geq k_{n,k_s}^0$easier to hold) offering $k_f = k_n^0$ to the first informant implies that either n or no firms confess. Note that the payoff when n informants exist is $1-\rho_n\mu\hat{\gamma}\left(k_{n,k_s}^0, k_s\left(k_{n,k_s}^0\right)\right)$ for each reporting firm whereas if all firms remain silent each one earns $1-\rho_0\mu$.
As

$$\rho_n\hat{\gamma}\left(k_{n,k_s}^0, k_s\left(k_{n,k_s}^0\right)\right) = \frac{(n-1)(1-z)(1-\rho_0)+N(z+\rho_0-z\rho_0)}{N} \geq \rho_0$$

holds for every $\rho_0 \leq 1$ and assuming that firms coordinate on the most profitable equilibrium, reporting by n parties requires $k_f = k_1^0$ to be offered. When $k_1^0 < k_n^0$, offering $k_f = k_n^0$ is enough for the same outcome (n informants) to be achieved.

Solving (3) for k_s and setting $k_f = k_1^0$ yields that $k_s(k_1^0) > 0$ holds for $\rho_0 > \frac{[1+(N-1)z]z}{(1-z)[n-1-(N-1)z]}$. Therefore, offering k_1^0 to the first and $k_s(k_1^0)$ to the second informant is not possible when $k_1^0 > k_n^0$. In the latter case k_1^0 should be offered to only one informant.■

Only by imposing fines $\gamma_1 \leq \gamma_1(k_1^0)$ to the first informant the AA can be certain that at least one firm will come forward. Had a single firm's reporting been able to raise the conviction probability to 1—as is commonly assumed in the literature—a fine $\gamma_1 \leq \gamma_1(k_1^0)$ would have been sufficient to induce universal reporting.[12] However, if the probability of conviction increases monotonically with the number of informants, offering sufficient incentive for the first informant to come forward may not always induce a universal reporting.

Note that if it is feasible to offer leniency rates contingent on the number of informants, setting $k_f = k_{n,k_s}^0$ would be sufficient to induce reporting by $n \geq 2$ firms even if $k_1^0 \geq k_{n,k_s}^0$: offering $k_f = k_1^0$ or $k_f = k_{n,k_s}^0$ if the number of eligible informants is either one or $n \geq 2$ respectively would be enough to induce reporting by $n \geq 2$ informants as every firm would be motivated to confess as the unique

[12] That is $\rho_1 = 1$ which under the assumption of firms' symmetry with respect to the evidence they possess implies $z = 1$. In the latter case $k_1^0 = 1-\rho_0 \geq k_{n,k_s}^0 = \frac{-k_s}{N}$.

informant.[13] For the rest of the analysis assume that leniency rates are not possible to be contingent on the number of informants, therefore, the condition described in lemma 3 must hold.

Both k_1^0 and k_{N,k_s}^0 depend on the values of (N, z, ρ_0), whereas k_{N,k_s}^0 depends also on the value of k_s promised to the second informant if $m = 2$. For every value of k_s, there exists a constellation (N, z, ρ_0) such that $k_1^0 \equiv k_{N,k_s}^0$. Solving the latter for z obtains

$$z_a \equiv \frac{\sqrt{[k_s+N(1-k_s)]^2+4N^3\rho_0(N-1)}-(k_s+2N\rho_0)(N-1)-N}{2(N-1)N(1-\rho_0)} \tag{5}$$

It can be shown that $z > z_a$ is equivalent to $k_1^0 > k_{N,k_s}^0$, hence when $z > z_a$ offering $k_f = k_{N,k_s}^0$ to the first informant is not enough to induce universal reporting. In such case offering $k_f = k_1^0$ for the first in line and no leniency for any subsequent informant induces reporting by every cartel participant. This implies that when the evidence brought-in by the first comer increases significantly the conviction rate, inducing the unique informant to come forward suffices to trigger a race to report by all firms. Otherwise, even if some firms report, others may find it preferable to hold back.

Corollary 2 *Inducing the unique informant to report triggers reporting by every firm when $z \geq z_a$. Otherwise the same outcome requires $k_f = k_{N,k_s}^0$ and k_s to be offered.*

Regarding the equilibrium selection, we consider some random mechanism (perhaps focal points) determining which firm belongs to each group. We assume that in case of multiple equilibria, equilibrium selection takes place at the beginning of the investigation. Thus, at the beginning of the game where firms must adopt an open loop strategy, all firms know that there will be equilibrium with n firms reporting and $N - n$ remaining silent, but no firm knows which equilibrium will be selected and therefore to which group it will belong in the occurrence of an investigation. Instead, if a typical investigation-subgame has multiple equilibria involving both reporting, and non-reporting firms, each firm assigns a probability for being in the reporting group.

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[13] In case where $z = 1$, promising $k_f = 0$ if $n \geq 2$ and $k_f = \frac{N(1-\rho_0)}{1+(N-1)}$ if $n = 1$ would be enough to induce universal reporting with no fine reductions in equilibrium, as in Sauvagnat (2014). For $0 \leq z < 1$ some leniency is necessary, even if leniency rates are contingent on the number of informants.

Equilibrium of the entire game

The previous analysis implies that depending of the characteristics of the LP (generosity and number of eligible firms) the equilibrium of the subgame starting from a node where the AA decides investigation, none, many, or all the firms involved in the cartel may report information and/or evidence.

When firms decide whether to cheat or to remain loyal to the agreement, they compare the value of collusion to the gain from unilateral deviation. We assume that the firm that unilaterally defects is absolved from any fine imposition, therefore the value of cheating is N. The purpose of the AA is, with the use of LP, to minimize the cartel value and consequently to increase the minimum δ above which the collusive agreement is sustainable.

Now we turn to the case where the AA promises $k_f = k^0_{n,k_s} > k^0_1$ to the first eligible firm and k_s to one subsequent firm in order to induce post-investigation reporting by n parties. Initially, each participant expects to earn the collusive profits and, in case of successful -with probability ρ_n- investigation, to pay the full fine with probability $\frac{N-2}{N}$ or a reduced fine with probability $\frac{2}{N}$. Therefore, the value of collusion where n firms report under investigation is:

$$\hat{V}^0_n = \frac{1 - a\rho_n \hat{\gamma}\left(k^0_{n,k_s}\right)\mu}{1-\delta(1-a)}$$

with $\hat{\gamma}\left(k^0_{n,k_s}\right) = \frac{\gamma_1\left(k^0_{n,k_s}\right)+\gamma(k_s)+(N-2)}{N} = \frac{\psi}{\rho_n N^2}$

and $\psi = N^2(z+\rho_0 - z\rho_0) + n(N-1)(1-z)(1-\rho_0)$

The number of informants affects the value of the cartel through the conviction rate as well as through the level of the expected fine. The following proposition determines the optimal number of firms induced to report under investigation:

Proposition 1 *Offering the minimum amount of leniency that induces every firm to report when an investigation is underway, always dominates in terms of ex-ante deterrence any asymmetric case where $n \leq N-1$ firms report.*

Proof

Substituting for $\gamma_1\left(k^0_{n,k_s}\right)$ and $\gamma(k_s)$ into $\hat{V}^0_n$ yields

$$\hat{V}^0_n = \frac{N^2[1 - a\mu(z+\rho_0 - z\rho_0)] - a\mu n(1-z)(1-\rho_0)(N-1)}{N^2[1-\delta(1-a)]}$$

Note that $\frac{\partial \hat{V}_n^0}{\partial n} = -\frac{a\mu(N-1)\mu(1-z)(1-\rho_0)}{N^2[1-\delta(1-a)]} < 0$, *i.e.* the value of the cartel is decreasing in the number of reporting firms. Hence, it is always optimal to set $n = N$, *i.e.* to induce reporting by every cartel participant.■

Observe that the cartel value is higher as the overall level of fine decreases:

$$\frac{\partial \hat{V}_n^0}{\partial \hat{\gamma}} = -\frac{a\mu\rho_n}{1-\delta(1-a)} < 0$$

while $\frac{\partial \hat{\gamma}\left(k_{n,k_s}^0\right)}{\partial n} = -\frac{(1-z)(1-\rho_0)(z+\rho_0-z\rho_0)}{{\rho_n}^2 N^2} < 0$, *i.e.* more lenient treatment is required for more informants to be attracted. Thus, more informants have a positive impact on cartel sustainability as the value of the cartel increases when fines are lower, while the latter have to be low in order to attract more informants. In the contrary, as

$$\frac{\partial \hat{V}_n^0}{\partial \rho_n} = -\frac{a\mu[n(N-1)(1-z)(1-\rho_0)+N^2(z+\rho_0-z\rho_0)]}{N[1-\delta(1-a)][n(1-z)(1-\rho_0)+N(z+\rho_0-z\rho_0)]} < 0$$

the effect of the likelihood of conviction on the value of cartel is negative, as expected. At the same time the probability of conviction increases with the number of informants: $\frac{\partial \rho_n}{\partial n} = \frac{(1-z)(1-\rho_0)}{N}$. Therefore, n has a parallel impact on cartel stability through ρ_n: more informants increase the likelihood of conviction which in turn decreases the value of collusion, $\frac{\partial \hat{V}_n^0}{\partial \rho_n}\frac{\partial \rho_n}{\partial n} < 0$.

Proposition 1 states that $\frac{\partial \hat{V}_n^0}{\partial \hat{\gamma}}\frac{\partial \hat{\gamma}\left(k_{n,k_s}^0\right)}{\partial n} + \frac{\partial \hat{V}_n^0}{\partial \rho_n}\frac{\partial \rho_n}{\partial n} < 0$ always holds: the benefit that the increased likelihood of conviction has on deterrence outweighs the adverse impact that the reduced level of overall fines generates. Hence, offering substantial post-investigation leniency improves both the deterrence and the prosecution, as the set of the created cartels is minimized and every investigated cartel is condemned.

Finally observe that $\hat{V}_n^0$ is independent of k_s. As $k_{N,k_s}^0 = \frac{N(1-z)(1-\rho_0) - k_s}{1+z(N-1)}$ increases when k_s lowers and $\gamma_1\left(k_{N,k_s}^0\right) + \gamma\left(k_s\left(k_{N,k_s}^0\right)\right) = 1 + \rho_0 + z(1-\rho_0)$, *i.e.* the expected fine is unaffected by k_s, we can hereafter assume that also when $z < z_a$ the number of eligible informants is one ($m = 1$), that is $k_s = 0$.

Assumption *When no marker is available only the first informant is eligible for leniency. The only eligible for leniency firm receives* $k_f = k_N^0 = \frac{N(1-z)(1-\rho_0)}{1+z(N-1)}$ *if* $k_N^0 > k_1^0$ *and* $k_f = k_1^0$ *otherwise.*

Let us now define strategy C of the entire game as the usual trigger strategy with the additional feature of dictating to remain silent in case of investigation. A firm that plays C expects with probability $1-a$ to keep receiving the collusive profits, and with probability $a\rho_0$ to pay the fine μ, and keep receiving the competitive profit for an infinite number of periods. The value of C is therefore:

$$V^C = (1-a)(1+\delta V^C) + \alpha[(1-\rho_0) + \rho_0(1-\mu)]$$

Solving for V^C yields:

$$V^C = \frac{1-a\rho_0\mu}{1-\delta(1-a)}$$

As mentioned before, when $k_1^0 \geq k_N^0$ promising $k_f = k_N^0$ to one firm, is not sufficient to induce any reporting: as $V^C > \hat{V}_N^0$ holds for every $n \in [1, N]$, firms select to coordinate on the most profitable C which entails that no one confesses under investigation. Hence, offering $k_f = k_1^0 > k_N^0$ to the first informant induces reporting by every participant. In this case the value of the cartel becomes:

$$\tilde{V}_N^0 = \frac{1-a\mu\frac{\gamma_1(k_1^0)+(N-1)}{N}}{1-\delta(1-a)}$$

For the rest of the analysis consider that the number of eligible for leniency firms under the *no marker* regime is $m=1$, *i.e.* that only the first in line reporting firm receives fine reduction. The value of collusion becomes:[14]

$$V_N^0 = \begin{cases} \hat{V}_N^0 = \dfrac{N(1-a\mu) + a\mu(1-z)\big(1-\rho_0\big)}{N[1-\delta(1-a)]} & if \;\; z < z_a \\ \tilde{V}_N^0 \;\; if \;\; z \geq z_a \end{cases} \tag{6}$$

where from (5) $z \geq z_a \equiv \frac{\sqrt{1+4N\rho_0(N-1)}-2\rho_0(N-1)-1}{2(N-1)(1-\rho_0)}$.

4 Marker

Now consider that the first-to-door applicant can secure its position and the AA announces that the privileged first position is no longer available. Besides this piece of information, potential subsequent informants remain unaware of the total number of informants that may have shown up already as well as of their precise position on

[14] Observe again that if offering leniency rate contingent on the number of informants was possible, promising k_1^0 to the unique informant and k_n^0 if $n \geq 2$ would be enough to induce universal reporting with $k_f = k_N^0$, regardless of the level of z. In such case the cartel value would be always equal to $\hat{V}_N^0$.

the informants' queue. Usually the marker is secured only for a specific time period considered necessary for its holder to organize and present the promised evidence. If at the expiry date the holder has failed to deliver the evidence, its position ceases to be secured, and we assume that this is common knowledge. We also assume that if a firm has denied confession as marker holder, it shows no interest in confessing as subsequent applicant, and this is common knowledge as well.[15] We distinguish two types of marker system according to the way the AA may treat a confession denial by the marker holder: the marker is either transferred to the next firm in the priority line, or permanently lost. As all firms know whether the marker has been already attributed (although they may not know the identity of the holder) both marker systems are unable to bring in more than one informant unless they offer leniency to at least one more applicant from the lot.

4.1 Scrolling *marker*

First, we analyze the case where the marker is transferred to the next applicant when a previous holder chooses to remain silent. If the marker holder indeed confesses, the subsequent $N-1$ firms take their reporting decision simultaneously, knowing that the first-informant position is not available. If the first marker holder decides to remain silent the marker is transferred to the second in line and in case of the second marker holder's reporting the $N-2$ subsequent firms take the reporting decision simultaneously, etc.

Since further reporting cannot be induced without making sure that some leniency is also offered to at least one applicant on top of the marker holder, we assume that $m=2$. If a subsequent firm thinks that $n-1$ others are going to report, it deduces that by remaining silent it pays the full fine with probability ρ_{n-1}, whereas, by reporting it takes a leniency-winning stake with probability $\frac{1}{n-1}$ at the price of increasing the probability of conviction by $\Delta\rho$. Hence it will report if:

$$\rho_{n-1} \geq \rho_n \frac{\gamma(k_s) + (n-2)}{n-1}$$

which yields

[15] Using very mild restrictions on the fines structure it can be shown that confessing as marker holder dominates confessing as subsequent informant. We state it as assumption in order to avoid burdening the analysis.

$$k_s \geq k_n \equiv \frac{N(n-1)(1-z)}{[n(1-z)+N(z+\theta)]} \tag{7}$$

where $\theta = \frac{\rho_0}{1-\rho_0}$. Observe first that k_n is decreasing in z, since a larger common evidence reduces the impact of additional reporting on the conviction rate, and therefore the price of the reporting lottery. Second, note that since any $k_s \geq k_n$ is able to bring forward n informants, the rule of offering only fine reductions and no positive rewards, *i.e.* $\gamma(k_s) \geq 0$, requires that $k_s \leq N$. As $k_n \leq N \Leftrightarrow (1-z) + N(z+\theta) \geq 0$ always holds, just offering $k_s = k_n$ to one subsequent applicant suffices to attract any exogenously determined number n of informants (given that the marker recipient confesses).

The following lemma defines the necessary treatment for the marker recipient to report:

Lemma 4 *Assuming a regime that offers a transferrable marker to the first-to-door applicant, the latter requires at least $k_f = k_1^0$ in order to come forward, where k_1^0 is defined in (4).*

Proof

See the Appendix.■

When the LP offers sufficient incentive for n firms to report when the cartel is under investigation, when making the decision of whether to join the cartel and respect the agreement, each firm anticipates that in case of investigation it will be the marker holder with probability $\frac{1}{N}$, and a subsequent leniency recipient with probability $\frac{m-1}{N} = \frac{1}{N}$ for the case of $m = 2$ assumed here. The value of collusion where n firms report under investigation is:

$$V_n^1 = \frac{1 - a\rho_n\mu\hat{\gamma}(k_1^0, k_n)}{1-\delta(1-a)} \tag{8}$$

where $\hat{\gamma}(k_1^0, k_n) = \frac{\gamma_1(k_1^0)+\gamma(k_n)+(N-2)}{N}$

The next proposition states that eligibility for leniency should be extended in order to allow the AA to obtain maximum evidence:

Proposition 2 *Consider that the AA offers a transferable marker to the first in line applicant:*

i. Maximum efficiency in cartel-formation deterrence requires a LP design offering incentives that induce reporting by all firms, i.e. the AA should obtain *all the available evidence. Necessary for the latter is to offer sufficient leniency to at least one subsequent applicant.*

ii. It is always superior in terms of cartel deterrence to maintain the uncertainty among cartel participants, regarding their position in reporting line.

Proof

See the Appendix.■

Proposition 2 shows that the DoJ's leniency regime, where i) only one informant receives leniency, ii) its position is reserved (marker), and iii) in case of marker holder's withdrawal the marker can be transferred to another applicant, may not attain maximum efficiency in deterring cartel formation. While the amount of leniency offered may be sufficient in order to attract the (important) first informant, as leniency is restricted to one firm, no other party has incentive to increase the likelihood of conviction by reporting, therefore the number of eligible and the number of actual informants coincide. According to proposition 2, leniency should be offered to at least one more applicant, and according to proposition 1 this additional leniency should be offered without marker.

The above provides an argument that supports the European system's practice to extend the eligibility to subsequent applicants. As mentioned before, the DoJ's LP restricts the eligibility to the first informant which implies that when firms possess imperfect evidence only a single firm's testimony is obtained. On the other side, the European system manages to extract evidence from multiple firms, a fact that seems to improve not only the cartel detection and the collection of fines but cartel deterrence as well.

4.2 Non-scrolling *marker*

Let us now analyze the case where the marker is withdrawn once the marker holder denies confession. In this case if the marker recipient confesses all subsequent firms take the reporting decision simultaneously, as in the transferred marker case. If the holder remains silent the marker is lost and the $N-1$ firms take their decision as in the no marker regime.

Consider that the AA provides incentive to the $n-1$ subsequent parties to come forward, following a confession by the first in line, *i.e.* $k_s = k_n$ is offered to one additional applicant. Consequently, if the marker holder confesses the total number of informants is n.

Let us keep the assumption from the previous section that without marker (*opaque* system) leniency is offered only to the first-to-door applicant. Incentive for universal reporting (race to the court) is provided if the value of k_f is given by (3) for $k_s = 0$:

$$k_n^0 = \frac{Nn(1-z)}{[1+(n-1)z][n(1-z)+N(z+\theta)]}$$

Lemma 5 *In a regime that offers a non-transferable marker to the first-to-door applicant; inducing universal reporting is always optimal.*

Proof

Reporting by the marker holder implies a conviction rate equal to ρ_n, while if remaining silent conviction takes place with probability $\rho_{n-\nu}$, with $\nu \in [0, n]$. Therefore the marker holder has sufficient incentive to confess when:

$$1 - \rho_n \gamma_1(k_f)\mu \geq (1-\rho_{n-\nu}) + \rho_{n-\nu}(1-\mu)$$

which is equivalent to $\frac{[N-k(1+(N-1)z)][n(1-z)(1-\rho_0)+N(z+\rho_0-z\rho_0)]}{N^2} \leq z + \rho_0 - z\rho_0 + \frac{(1-z)(1-\rho_0)(n-\nu)}{N}$ or $k_f \geq k_{1\nu}^n$, where

$$k_{1\nu}^n = \frac{\nu N(1-z)}{[1+z(N-1)][n(1-z)+N(z+\theta)]}$$

Substituting $k_f = k_{1\nu}^n$ into $\gamma_1(k_f)$, we obtain the corresponding value of the cartel:

$$V_{n\nu}^1 = \frac{1 - a\rho_n\mu\frac{\gamma_1(k_{1\nu}^n) + \gamma(k_n) + (N-2)}{N}}{1-\delta(1-a)}$$

or $V_{n\nu}^1 = \frac{N^2[1-a\mu(z+\rho_0-z\rho_0)]-a\mu(1-z)(1-\rho_0)[n(N-1)+1-\nu]}{N^2[1-\delta(1-a)]}$.

As $\frac{\partial V_{n\nu}^1}{\partial n} = -\frac{a\mu(N-1)(1-z)(1-\rho_0)}{N^2[1-\delta(1-a)]} < 0$, it is optimal to provide $k_f = k_{1\nu}^N = \frac{\nu(1-z)(1-\rho_0)}{1+z(N-1)}$ for the marker holder and $k_s = k_N$ to one subsequent applicant.■

Assume now that regardless of the marker recipient's reporting decision $k_N = (N-1)(1-z)(1-\rho_0)$ is offered to one of the subsequent reporting firms. Therefore, following the marker holder's denial to confess, one of the $N-1$ others receive $k_f = k_N$. Note that when the marker holder fails to confess and $k_N \geq k_1^0$ holds, offering $k_f = k_N$ to one informant induces reporting by n firms provided that

$k_N \geq k_n^0$. If $k_N < k_1^0$ no firm reports for $k_f = k_N$. Defining the level of z below which reporting is possible in case of marker holder's denial to report, $k_N \geq k_1^0$ holds when $z \leq z_b$ with

$$z_b \equiv \frac{N - 2 + \sqrt{N}\sqrt{N - 4(1-\rho_0)} - 2(N-1)\rho_0}{2(N-1)(1-\rho_0)}$$

Note further that if $k_N < k_{N-1}^0$, less than $N-1$ subsequent firms report once the marker holder failed to confess. $k_N < k_{N-1}^0$ holds for $z < z_c$ where

$$z_c \equiv \frac{\sqrt{N}\sqrt{N^3 - 2(2-\rho_0)N^2 + N[8-(8-\rho_0)\rho_0] - 4(1-\rho_0)} - 2(1-\rho_0) - N(N-2+\rho_0)}{2(N-1)(1-\rho_0)}$$

The following proposition compares the deterrent impact of the *marker* and the *no marker* systems once the marker is lost after its holder's decision to withhold the evidence and given that k_N is provided for the subsequent firms regardless of the marker holder's reporting decision:

Proposition 3 *Consider a non-transferable marker system:*

i. *When $z_c < z_a < z_b \Leftrightarrow \rho_0 > \frac{1}{N(N-2)^2}$ for $z_c < z < z_a$, the opaque and marker systems produce similar results in terms of ex-ante deterrence. For $z_a < z < z_b$ the marker system enhances ex-ante deterrence compared to the opaque. For $z > z_b$ and for $z < z_c$ the opaque system is superior in terms of ex-ante deterrence compared to the marker system.*

ii. *When $z_b < z_a \Leftrightarrow \rho_0 < \frac{1}{N(N-2)^2}$ the opaque and marker system produce similar results in terms of ex-ante deterrence if further $z_c < z < z_b$. Otherwise the opaque system produce better deterrence compared to the marker system.*

Proof

See the Appendix.■

Name $k_N^{1\prime} = \frac{(1-z)(1-\rho_0)}{1+z(N-1)}$ $\left(k_N^{1\prime\prime} = \frac{N(1-\rho_0)}{1+z(N-1)}\right)$ the minimum k_f that makes the marker holder to come forward when every other (no) firm confesses following the marker holder's denies to confess, that is when $z_c < z < z_b$ $(z > z_b)$. It can be easily verified that $k_N^{1\prime} < k_N^{1\prime\prime}$ and that $k_N > k_N^{1\prime}$. The expected fine for any subsequent applicant is equal to the marker holder's actual fine:

$$\gamma_1(k_N^{1\prime}) = \frac{\gamma(k_N) + (N-2)}{N-1} = \frac{N - 1 + z + \rho_0 - z\rho_0}{N} = \hat{\gamma}(k_N^{1\prime}, k_N) > \gamma(k_N)$$

Therefore, the actual reduced fine that one eligible subsequent applicant pays is always lower than that of the marker holder, when $z_c < z < z_b$. The certainty that the marker creates for the latter renders this applicant less demanding in terms of the leniency requested.

The previous analysis implies that the marker holder has strong incentive to come forward when all the others are going to do the same regardless of what the first in line decides. If further $z > z_a$, which implies that under the *no marker* regime the first–to-door requires $k_1^0 (> k_N^0)$ to come forward, the marker acts as a mechanism that induces firms to compromise with a lower level of leniency, that is $\hat{\gamma}(k_N^{1\prime}, k_N)$. This results in a value of collusion which is lower under the *marker* system. When $z > z_b$ the marker holder recognizes the gravity of its reporting, as remaining silent implies universal non-reporting: confession by this firm needs a more generous treatment to be offered, a fact that reduces the overall lever of expected fines raising the value of collusion, and finally stabilizing the collusive agreement.

In the graph below (figure 1) the black, dashed and dotted lines represent the value of the cartel under the *opaque*, the non-scrolling and the scrolling *marker* system respectively, for the following values of the parameters: $a = .15$, $\mu = 2$, $\delta = .9$, $N = n = 4$ and $\rho_0 = .7$. The common share of evidence z is on the horizontal axis. For $z_c = .026 < z < z_a = .38$ the value of collusion under the *no marker* and the non-scrolling *marker* systems coincide. For $.38 < z < z_b = .64$ the value of the cartel under the non-scrolling *marker* is lower, while for $z > .64$ the *no marker* system is superior. Comparing the two *marker* systems, the non-scrolling results in lower deterrence when $z > .64$. For every z the scrolling *marker* produces worse deterrent results compared to the no marker regime.

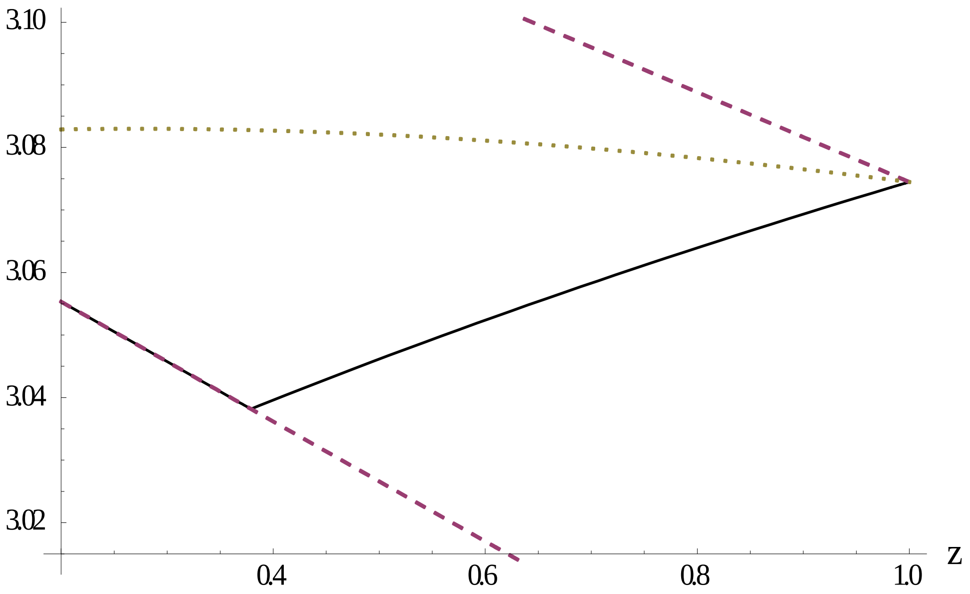

Figure 1: cartel value

Furthermore, we consider useful to discuss the robustness of proposition 3 with respect to the assumption that k_N is offered to one subsequent applicant, regardless of the marker holder's reporting decision. If instead we suppose that k_{N-1}^0 or k_1^0, depending on the level of z, is offered to one informant, all $N-1$ subsequent firms are induced to confess following the marker recipient's denial to come forward. When $z > (<) z_d \equiv \frac{\sqrt{1+\rho_0[2+4N(N-2)+\rho_0]}-(2N-3)\rho_0-1}{2(N-1)(1-\rho_0)}$, $k_1^0 > (<) k_{N-1}^0$ holds and promising $k_f = k_1^0$ $\left(k_f = k_{N-1}^0\right)$ to one subsequent applicant in case of the holder's denial to confess, is enough to trigger universal reporting if additionally $k_f = k_N^{1\prime}$ and $k_s = k_N$ are offered to the marker recipient and to one subsequent informant respectively. The marker holder recognizes that remaining silent (confessing) implies a conviction likelihood equal to ρ_{N-1} $(\rho_N = 1)$. Thus, $k_f = k_N^{1\prime}$ is enough to induce reporting by the marker holder and $k_s = k_N$ secures that all subsequent firms also come forward. In both cases the resulting collusive value is $\hat{V}_N^0$:[16]

Proposition 4 *If* $k_f = k_1^0$ $\left(k_f = k_{N-1}^0\right)$ *is offered to one subsequent applicant when* $z > (<) z_d$*, the non-transferable marker regime produces at least equal deterrent outcome compared to the opaque system: if* $z > z_a$ *the non-transferable*

[16] In equilibrium $k_f = k_N^{1\prime}$ and $k_s = k_N$ are offered to the marker holder and to one additional applicant respectively. Offering k_1^0 or k_{N-1}^0 is just a credible threat that induces the marker recipient to compromise with the least possible reward.

marker system has better deterrent results while if $z \leq z_a$ both systems are equivalent in terms of deterrence.

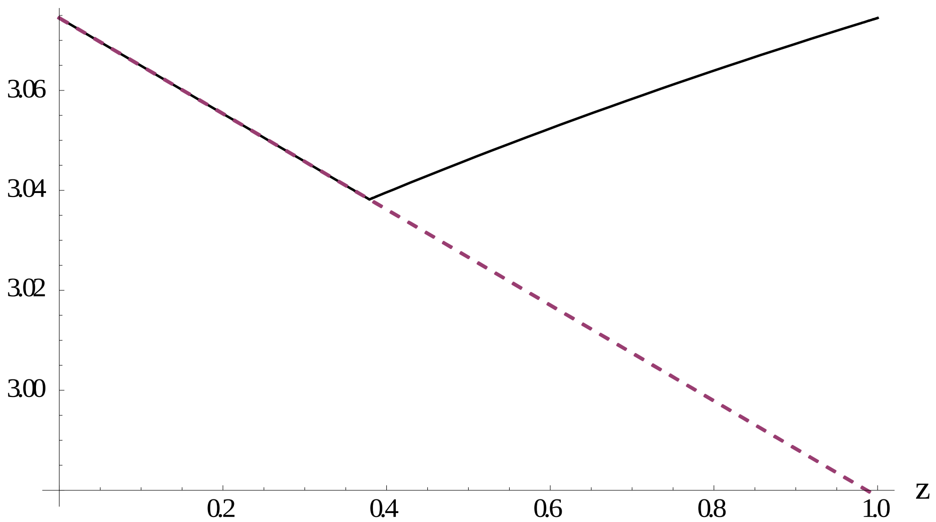

Figure 2: cartel value

Figure 2 depicts the collusive values of the *opaque* (black line) and the non-scrolling (dashed line) *marker* systems under the parameter values of figure 1. In this case if $z > z_a = .38$ the non-transferrable marker induces the marker holder to compromise with the minimum possible reward resulting in a lower cartel value and consequently in better deterrent outcome.

Thus, persuading the marker recipient that remaining silent implies that all the others are going to confess is the key in order to induce confession with the lowest possible reward. In such case the non-transferrable marker regime minimizes the value of the cartel and as a result enhances deterrence compared to the *opaque* system.

5 Concluding remarks

As OECD (2012) states "it is often the case that co-operation from the second applicant is of particular value because its testimony and other evidence it presents can be used to corroborate the evidence submitted by the first applicant. Co-operation of subsequent applicants may contribute to proving additional facts either in terms of duration, product or geographic scope or the composition of the cartel". In this paper we show that it is always optimal in terms of both deterrence and detection to induce every firm to reveal the evidence when being under inspection. This occurs because the impact of reduced overall fines is always lower than the effect that the higher likelihood of conviction has on the profitability of collusion and consequently on cartel deterrence.

It has been highlighted by leniency applicants' representatives and practitioners that transparency and certainty are crucial parameters that should be taken into consideration for the implementation of the LP. ICN (2014) mentions that "a leniency applicant needs to be able to foresee with a high degree of certainty how it will be treated if it reports anticompetitive conduct and what the consequences will be if it does not come forward".

The adoption of a marker system succeeds to eliminate the uncertainty, at least for the first-to-come applicant, reserving for the latter a position in line that secures its eligibility for a given lenient treatment. Here we show that offering information to applicants about the availability of leniency affects the effectiveness of the LP, depending on the way that the marker is secured: if the latter is repeatedly obtainable, regardless of the reporting decision by the marker holder, the marker system requires higher overall level of fine reductions. This increases the profitability of collusion and consequently it hurts deterrence. Otherwise, a marker that is not transferable in case of failed-reporting by the marker holder could induce universal reporting with the first applicant to be less demanding in terms of leniency. In such case the marker acts as a mechanism that induces confession in instances where in the absence of it reporting would require more generous fine reductions to be offered.

Admitting that a marker is provided to the first applicant, as applied in major jurisdictions like the DoJ and EC, we show that it is always preferable to extend the eligibility for leniency to -at least- a second applicant in order to achieve universal reporting. This occurs because the disincentive to collude that emanates from the secured conviction surpasses the cost of lower overall fines needed for that. The latter may suggest an argument that justifies the EC's (among others) practice to offer temperate fine reductions to subsequent informants.

Indeed, our assumptions do not allow evaluating other aspects of the marker system. The latter offers a possibility to infringers to come forward in an early stage, likely before the opening of an investigation, and to reserve a position in queue, providing the necessary time to gather sufficient information. This aspect is out of the present paper's scope and requires additional model specifications in order to be examined.

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Appendix

Proof of lemma 4

The incentive to report for a marker holder always increases with the number of other reporting parties: $1 - \rho_n\gamma_1\mu - (1 - \rho_{n-1}\mu) = (\rho_{n-1} - \rho_n\gamma_1)\mu$

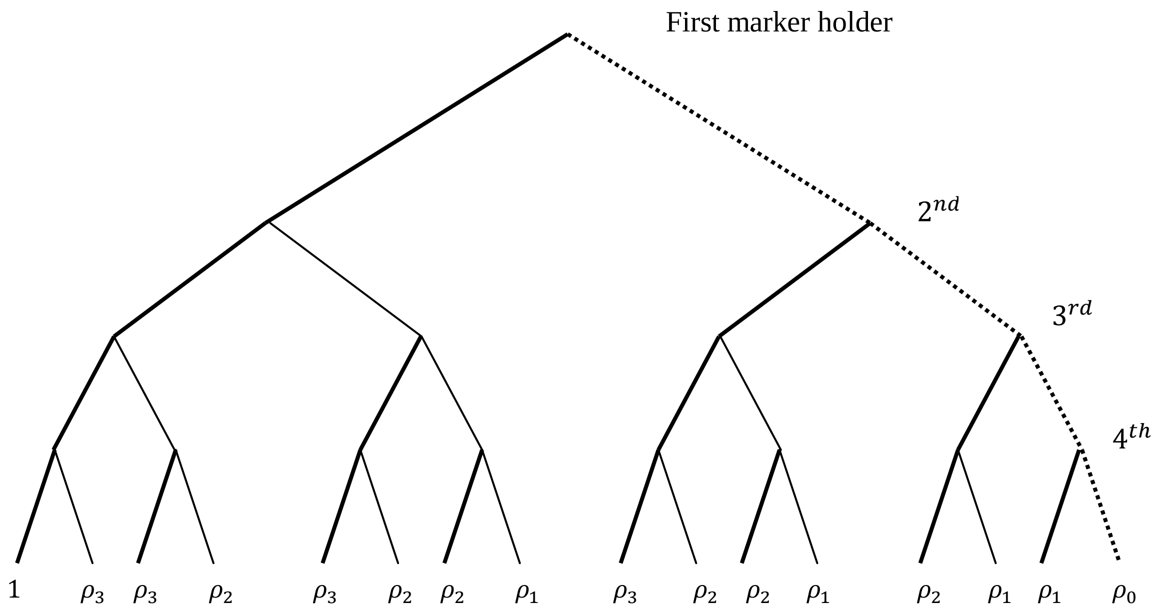
$$\frac{\partial(\rho_{n-1} - \rho_n\gamma_1)}{\partial n} = \frac{k_f(1-z)(1-\rho_0)[1+(N-1)z]}{N^2} > 0$$

Assume that $N-1$ firms have previously claimed and subsequently denied the marker, and now the N^{th}firm contemplates whether receiving it or not. As in case of confession that firm will be the first and unique informant, it will confess iff $k_f \geq k_1^0$ is satisfied. If $k_f < k_1^0$, the $(N-1)^{th}$ firm will face the same dilemma knowing that, on the one hand none of the previous firms has confessed, and on the other hand that in case it denies confession and the marker goes to the N^{th} firm, the latter will find optimal not to confess. Thus, the $(N-1)^{th}$ firm also considers itself in the position of the first and unique informant, and since $k_f < k_1^0$ has been assumed not to hold, the $(N-1)^{th}$ marker recipient will not confess either. Backwards induction yields that if $k_f < k_1^0$ does not hold no firm will honor the marker: every previous marker recipient knows that its reporting implies a conviction rate at least equal to ρ_1 while remaining silent entails a conviction rate equal to ρ_0. Hence, if the last in line is not to report as the unique informant, no other has any incentive to come forward as a marker holder.

If $k_f \geq k_1^0$ is offered to this applicant, the $(N-1)^{th}$ recipient recognizes that reporting entails conviction with probability ρ_1 or ρ_2. On the other side, if remaining silent the N^{th}marker recipient confesses and conviction is the investigation outcome with probability ρ_1. The previous implies that the $(N-1)^{th}$ marker recipient confesses if at least $k_f = k_1^0$ is offered. Under the same rationale, the first in line knows that its reporting implies that the cartel will be convicted with probability ρ_n while remaining silent results in conviction with probability, at least, equal to ρ_{n-1}. Consequently the first marker holder has no incentive to remain silent, even if the action of its reporting entails a probability of conviction equal to ρ_n. Therefore, $k_f = k_1^0$ is necessary for at least one informant to exist and sufficient to persuade any marker holder to confess.

Consider the following $N = 4$ paradigm where $k_s = k_4 = 3(1-z)(1-\rho_0)$ is offered to the first subsequent applicant. If $k_f < k_1^0 = \frac{4(1-\rho_0)}{1+3(z+\rho_0-z\rho_0)}$ the forth marker receiver denies confessing as the unique informant. The third recognizes that reporting entails conviction likelihood equal to ρ_2 (the forth confesses as subsequent if $k_s \geq k_2$, which is the case) while remaining silent implies that conviction occurs with probability ρ_0 and thus denies to confess. The second receiver knows that remaining silent implies that conviction occurs with probability ρ_0 while reporting implies conviction with probability ρ_3. Similarly the first in line knows that remaining silent implies that conviction occurs with probability ρ_0 while its reporting convicts the cartel with probability 1. Consequently no one reports if $k_f < k_1^0$.

Consider now that $k_f \geq k_1^0 = \frac{4(1-\rho_0)}{1+3(z+\rho_0-z\rho_0)}$ is offered to the marker holder. In this case the last receiver confesses as the unique informant. At the same time every firm confesses given that one other firm confesses ($k_s = k_4$). The third receiver knows that reporting implies conviction probability equal to ρ_2 while remaining silent implies that conviction occurs with probability ρ_1(the last confesses). The second receiver knows that remaining silent implies that conviction takes place with probability ρ_2 while reporting convicts the cartel with probability ρ_3. Similarly the first in line knows that remaining silent implies that conviction occurs with probability ρ_3 while reporting implies certain conviction. Therefore, for $k_f = k_1^0 = \frac{4(1-\rho_0)}{1+3(z+\rho_0-z\rho_0)}$ and $k_s \geq k_2$, the first marker holder confesses and all three subsequent firms do the same.



Proof of proposition 2

Substituting for $\gamma_1(k_n^1)$ and $\gamma(k_n)$ into V_n^1 and taking the derivative with respect to n yields

$$\frac{\partial V_n^1}{\partial n} = \frac{-a\mu(1-z)(1-\rho_0)\left[N-2+\left(1-\rho_0\right)z(N-2)(N-1)+\rho_0[2+N(N-2)]\right]}{N^2[1-\delta(1-a)][1+(N-1)(z+\rho_0-z\rho_0)]}$$

The numerator's expression in brackets is positive if $z > -\frac{N-2+\rho_0[2+N(N-2)]}{(N-2)(N-1)(1-\rho_0)}$ which always holds. Therefore, $\frac{\partial V_n^1}{\partial n} < 0$ always holds. As the value of the cartel reduces with the number of reporting parties, it is always optimal to set $N = n$, *i.e.* to induce reporting by all firms.

Using (4), (6), (7) and (8) $\hat{V}_N^0 < V_N^1$ holds for $\rho_0 \leq 1$. At the same time $\tilde{V}_N^0 < V_N^1$ always holds as apart from k_1^0 which is offered to the first informant in the marker case (the same if offered only to the first informant in the no marker case) some additional leniency is required for one subsequent applicant for additional informants to exist.

Proof of proposition 3

First, $z_a < (>) z_b$ holds for $\rho_0 > (<) \frac{1}{N(N-2)^2}$. Also the $N-1$ subsequent firms confess even if the marker holder remains silent only when $k_N \geq max\{k_{N-1}^0, k_1^0\} \Leftrightarrow z_c < z < z_b$. If $z < z_c$ and marker holder denies to confess, at most $N-2$ reporting firms exist. Consider that $\rho_0 > \frac{1}{N(N-2)^2}$. For $z_c < z < z_b$ the marker holder knows that remaining silent implies that $N-1$ others are going to report and its incentive to confess is:

$$1-\gamma_1\left(k_f\right)\mu \geq (1-\rho_{N-1}) + \rho_{N-1}(1-\mu)$$

which yields

$$k_f \geq k_N^{1\prime} = \frac{(1-z)(1-\rho_0)}{1+z(N-1)}$$

For $z_c < z < z_b$ the value of the cartel becomes

$$\frac{1-a\mu\hat{\gamma}(k_N^{1\prime}, k_N)}{1-\delta(1-a)} = \hat{V}_N^0$$

where $\hat{\gamma}(k_N^{1\prime}, k_N) = \frac{\gamma_1(k_N^{1\prime}) + \gamma(k_N) + (N-2)}{N}$, $\gamma(k_N) = \frac{1 + (z + \rho_0 - z\rho_0)(N-1)}{N} < 1$ and $\gamma_1(k_N^{1\prime}) = \frac{N - 1 + (z + \rho_0 - z\rho_0)}{N} > \gamma(k_N)$. Therefore, under the non-scrolling *marker* the collusive value is equal to the value of the cartel under the *no marker* regime for $z_c < z < z_b$. For $z_a < z < z_b$ the collusive value under the *no marker* system is $\tilde{V}_N^0 > \hat{V}_N^0$.

If $z < z_c$ the marker holder knows that remaining silent implies that less than $N - 1$ subsequent are going to come forward and therefore the holder's confession requires $k_f > k_N^{1\prime}$. Consequently the collusive value in this case is always greater than $\hat{V}_N^0$.

If $z > z_b$ the unique marker holder recognizes that remaining silent implies that firms will coordinate on the most profitable equilibrium where no investigated firm confesses and that the conviction probability will be ρ_0 (see lemma 3). The marker holder confesses only if $1 - \gamma_1(k_f)\mu \geq 1 - \rho_0 + \rho_0(1 - \mu)$ or if $\rho_0 \geq \gamma_1(k_f)$ which yields

$$k_f \geq k_N^{1\prime\prime} = \frac{N(1 - \rho_0)}{1 + z(N-1)}$$

The value of the cartel is

$$\tilde{V}_N^1 = \frac{1 - a\mu\hat{\gamma}(k_N^{1\prime\prime}, k_N)}{1 - \delta(1 - a)}$$

where $\hat{\gamma}(k_N^{1\prime\prime}, k_N) = \frac{\gamma_1(k_N^{1\prime\prime}) + \gamma(k_N) + (N-2)}{N} = \frac{N[N - (1-\rho_0)(2-z)] + (1-z)(1-\rho_0)}{N^2}$, $\gamma_1(k_N^{1\prime\prime}) = \rho_0 < \gamma(k_N)$. Notice that the expected fine is lower under the marker regime, that is

$$\hat{\gamma}(k_1^0) - \hat{\gamma}(k_N^{1\prime\prime}, k_N) = \frac{(N-1)(1-\rho_0)(1-\rho_0)[1 + (N-1)(1-\rho_0)z + \rho_0(2N-1)]}{N^2[1 + (N-1)(1-\rho_0)z + \rho_0(N-1)]} > 0$$

which implies $\tilde{V}_N^1 > \tilde{V}_N^0$.

On the Allocation of Evidence among Cartelists under a Leniency Program

Abstract

The impact of leniency programs on cartelists' decision to allocate the incriminating evidence is investigated. Firms are allowed to possess either exclusive or common pieces of cartel-related evidence. The cartel organization is able to allocate the incriminating evidence in an attempt to enhance the sustainability of the illicit agreement. Assuming that the Antitrust Authority (AA) provides incentives that induce confession, reporting is either partial or universal. It is shown that in the former case the cartel organization selects to split and equally share the evidence (each firm possesses only exclusive pieces) whereas in the latter case every firm may possess perfect evidence. Unless the conviction of an investigated cartel is unlikely, when the AA optimally anticipates the cartel's ability to allocate the evidence, only partial information is obtained.

JEL Classification: K21, L12, L41

Keywords: Antitrust enforcement, Collusion, Leniency programs

1 Introduction

Leniency Programs (LPs) aim to enhance cartel deterrence by offering fine reductions to infringers that provide cartel-related evidence to the Antitrust Authority (AA) and cooperate during investigation. Here, we examine the impact that a LP has on cartel organization's decision to allocate and distribute the incriminating evidence among cartel members.

Cartel coordination requires communication among cartel members regarding issues such as setting the price level, attenuating differences that could threaten the cartel's stability, etc. This interaction leaves traces, *e.g.* pieces of hard evidence. Taking into account self-reporting schemes, Agisilaou (2012) investigates the incentives of cartelists to keep rather than destroying hard evidence.[17] Aubert et. al. (2006) show that if firms are able to react after a rival's observable deviation from the agreement, hard evidence may act as a threat against such opportunistic behavior, resulting in increased cartel sustainability.

In the present paper we examine the impact of leniency policies on firms' incentives to allocate the incriminating evidence. In contrast to the majority of the related literature, we assume that even if a firm decides to come forward and to cooperate with the AA bringing forward the incriminating evidence it possesses, the illicit agreement is not necessarily convicted with certainty. This implies that offering incentives for a single firm to report may not necessarily trigger a race to the court by all cartel members, as is usually predicted by the literature. Even knowing that some rival is ready to provide information, in some cases the remaining firms may withhold the evidence they possess in an attempt to prevent an increase of the conviction probability.[18]

While evidence concerning bilateral communications between specific cartel members may be considered to stay within the hands of those involved, who keeps the evidence from communications related to applying central cartel decisions is a more complicated issue, since many, or all cartel members may be involved. As we show, the distribution of such evidence among cartel members may have significant consequences on the cartel's stability; it is therefore natural to assume that, to the

[17] It is shown that – even in the absence of LP – not destroying hard evidence may be stability-enhancing.

[18] See also Blatter et. al. (2018) for a study related to the effectiveness of LPs, when cartel-related evidence is cumulative.

extent that it is possible, the cartel's organization has an interest in controlling it. To make matters simple, we assume the presence of a cartel directory with a unique objective, to enhance cartel stability, and ultimately cartel value. The agency is able to monitor the quantity of evidence that reaches each member's office. Of course, the ideal solution would have been for the agency to erase all the evidence. This however is in most cases practically unfeasible, and once full evidence eradication is ruled out, erasing evidence reduces the probability of conviction, but eventually increases the expected fines, since it attracts additional punishment in case of conviction. Instead of (or along with) erasing evidence, the cartel directory may in some cases affect cartel stability by controlling how much evidence remains in the hands of each individual firm. When post-investigation leniency is available, this is equivalent to controlling each firm's individual incentive to report, thus having important impact on both, the cartel stability in the absence of investigation, as well as the probability of cartel's conviction in case it is spotted and investigated.

As mentioned in Motta (2004), "institutional arrangements to sustain collusion might differ: from a well organized cartel-like structure where a central office takes the main decisions, to situations where firms just find some form of communication to sustain the agreement". Although one might consider the idea of division and allocation of the evidence to be somehow artificial, the fact that such a system could be in effect should not be excluded. The Organization for Economic Co-operation and Development (OECD, 2012) describes cartels as "sophisticated and capable of learning" and insists that they "would seek to strategically exploit any feature of a leniency program". In that spirit it also conjectures that the availability of leniency programs may motivate the cartel organization to adopt specific ways of evidence sharing, in order to achieve maximum fine reductions for its members.

The present work aims at formalizing the above conjecture by showing both, how evidence allocation can affect cartel stability, and how it can in turn be affected by the design of leniency programs. Considering the existence of many potential agreements differing among them in the way the traces left by it are distributed among its members, the cartel's directory aims to allocate the evidence in such a way that the most profitable among stable agreements is selected. Our work demonstrates that while in most instances the evidence must be split among firms, in some cases the cartel directory will agree to let each individual firm possess a copy of all the available evidence.

The next section describes the basic assumptions of the model. Section 3 determines how firms would select to allocate the incriminating evidence. Section 4 concludes.

2 Model

Consider $N = 2$ symmetric firms producing homogeneous good and competing in prices for an infinite number of periods. Each firm maximizes the expected sum of future discounted profits using a common discount factor $\delta \in (\frac{1}{2}, 1)$. In each period firms choose between competing and colluding:

- If both firms cooperate setting the collusive price, each one earns half the monopoly profit, normalized to be equal to 1;
- When one firm unilaterally deviates from the agreed price, it receives the entire monopoly profit, normalized to 2, while the other gets zero;
- The competitive profits are zero.

The AA investigates the industry with probability $a \in (0,1)$. Even if the AA has spotted a case where firms have indeed formed a cartel, it is not certain that prosecution will end up in conviction, due to imperfections and noise in the available evidence. It is, however, natural to assume that as the total amount of evidence in the hands of the AA increases, the probability of conviction being the prosecution outcome increases. Hence, we measure an amount of evidence by the conviction probability it induces when in the hands of the AA and/or the Court, and the term "piece of evidence" corresponds to increments of that probability. Pieces of evidence that repeat already known information are valueless in that they do not increase the probability of conviction, and since we measure evidence by the latter, they are considered as zero evidence.

The total amount of cartel-related evidence consists of evidence possessed by both participants, denoted by $z \in [0,1]$, and exclusive pieces of evidence distributed symmetrically between firms, each one possessing an amount $\Delta\rho = \frac{1-z}{2}$. If the AA succeeds to obtain the total amount of evidence, an investigated cartel is convicted with certainty. AA's initial investigation (down-raids, etc.) uncovers only a portion ρ_0 of the total evidence, assuring from the outset a ρ_0 probability of successful prosecution. This uncovered evidence is composed by common and firm-specific pieces in portions λ_1, λ_2, respectively, hence the probability of conviction when no firm confesses is $\rho_0 = \lambda_1 z + \lambda_2 2\Delta\rho = z(\lambda_1 - \lambda_2) + \lambda_2$, with $0 \leq \lambda_h \leq 1$, $h = 1,2$. For simplicity, we assume that $\lambda_1 = \lambda_2$, *i.e.* the evidence unveiled by the AA's efforts consists of equal portions of common and firm-specific evidence. This implies a neutral effect of z on ρ_0.[19]

Conviction for participating in a cartel entails a fine $\mu \geq 1$.[20] A LP allows a firm to reduce that fine in exchange for providing cartel-related evidence, and the fine reduction is proportional to the amount of evidence provided. Since repeated information is redundant, the commonly possessed evidence not in the hands of the AA is considered as offered only once, by the first firm in line to testify. This increases the probability of conviction, relative to ρ_0, by amount $(1 - \rho_0)z$, in addition to any other piece of exclusive evidence the first informant may present. When both firms confess, the second firm's exclusive evidence renders the conviction more likely by $(1 - \rho_0)\Delta\rho$ and the conviction of the investigated cartel occurs with certainty.

As in Buccirossi and Spagnolo (2006), we assume that the existence of hard evidence requires explicit agreement between participants, *i.e.* evidence can only be deliberately produced by the parties involved.[21] Consider that an amount of evidence is the inevitable by-product of the cartel's proper functioning. If the two firms agree to both retain in common a percentage z of that evidence while splitting the remaining evidence evenly, each one retains $z + \Delta\rho = \frac{1+z}{2}$. When no evidence is allowed to be simultaneously in the hands of both firms, firms possess only exclusive pieces of evidence, that is $z = 0$, and a single firm possesses evidence equivalent to a conviction likelihood of $\frac{1}{2}$.

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[19] Observe that z affects ρ_0 positively when $\lambda_1 > \lambda_2$. Results remain qualitatively unaffected if $\lambda_1 > \lambda_2$.

[20] According to Harrington (2014) the US federal fines correspond to no more than double damages. Other jurisdictions allow up to treble damages. Bageri et. al (2013) and Katsoulakos et al. (2015) underline the superiority of fines based on illegal profits compared to fines on revenues.

[21] Consider evidence as signed contracts or illegal agreements, audio and/or visual records of meetings, etc. Buccirossi and Spagnolo (2006) examine possible negative effects of leniency policies on the feasibility of illegal exchange. They treat hard evidence as a "hostage" that restricts the opportunistic behavior which undermines the trust among wrongdoers.

While the level of the common share z does not affect the value of ρ_0, it becomes important when it comes to the reporting decision. If one firm confesses conviction occurs with probability ρ_1:

$$\rho_1(z) = \rho_0 + (1 - \rho_0)(z + \Delta\rho) = \frac{z(1 - \rho_0) + 1 + \rho_0}{2} \tag{1}$$

Observe that ρ_1 is increasing in z. In fact $z = 0$ implies $\rho_1 = \frac{1+\rho_0}{2}$ while $z = 1$ implies $\rho_1 = 1$, meaning that a single firm's reporting is sufficient to lead to cartel conviction with certainty.

When confessing, the first-to-door informant receives a fine reduction proportional to its contribution to cartel prosecution. The reduced to full fine rate is given by the following expression:

$$\gamma_1(k) = \frac{1 - \rho_0 - k(1 - \rho_0)(z + \Delta\rho)}{1 - \rho_0} = 1 - k(z + \Delta\rho) \tag{2}$$

The parameter k measures how each additional percentage-increase in the probability of conviction is valued by the AA, and thus determines the generosity of the leniency offered to the eligible informant. Fine reductions are limited as to never become net rewards, *i.e.*, γ_1 lies between 0 and 1, which implies the parameter k is bounded from above by a value $\hat{k}$ such that $\gamma_1(\hat{k}) = 0$ where

$$\hat{k} \equiv \frac{2}{1 + z} \geq 1 \tag{3}$$

Solving the above for z yields

$$z < \hat{z} \equiv \frac{2 - k}{k} \tag{3'}$$

Let the full fine μ be determined by law and assume that the AA has determined the value of k, call it k^*, subject to the constraint that $k \leq \hat{k}$. By assumption, firms' reporting choices do not affect k^*, therefore firms perceive the latter as exogenous. As $\hat{k}$ decreases with z while the determined leniency level (k^*) is independent of z, circumstances under which k^* leads to negative fines may exist (for sufficiently high z). In such cases, as fines are restricted to non-negative level, the reduced fine takes the determined level ($\gamma_1(k^*)$) as long as it is non-negative or otherwise it equals zero:

$$\gamma_1(k) = max\{\gamma_1(k^*), 0\}$$

or equivalently $k = min\{k^*, \hat{k}\}$.

We construct an infinitely-repeated game played by the AA, the cartel directory and the two firms. Let the parameters μ, and α be determined from the outset by the

AA and also let ρ_0 (the efficiency of the AA's own investigation) be exogenously given and common knowledge to all players. At each period, a three- or four- stage game is played according to whether the market is investigated by the AA. The stages of the stage-game are as follows:

1. The AA sets the policy parameters, *i.e.* it determines k;
2. Each firm decides whether to collude or not.[22] If firms collude the cartel organization determines z;
3. Each firm chooses between staying loyal or undercutting the agreed price.[23] In case of subsequent cartel conviction, the firm that has unilaterally deviated from the agreement is absolved from the imposition of any fine. Therefore, the value of cheating is equal to 2, which corresponds to the total cartel profit without fines payment;
4. The AA spots and investigates the cartelized industry with probability a. With probability $1 - a$, the cartel is not investigated, the stage-game ends and a new one starts all over in the next period. In case of investigation, the presence of post-investigation LP requires each participant firm to individually make a new decision, namely whether reporting or not (reporting decision). Depending on the number of firms that finally decide to report, the cartel is convicted with probability ρ_n, $n = 0{,}1{,}2$. We assume for simplicity that, once a cartel is spotted, its life ends even if it escapes conviction. This is justified by the fact that the AA keeps monitoring the sector tightly.[24] Hence, when arriving at a reporting-decision node, the game ends.

Since at most one firm is eligible for fine reduction, if both firms report "nature" determines which will be the lucky one to benefit from leniency.[25] When $n = 2$ each reporting firm expects to receive lenient treatment with probability $\frac{1}{2}$. We define the expected fine when more than one firm confesses as $\hat{\gamma}_2\mu$ where

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[22] If at least one firm refuses to collude, competition takes place at least up to the end of the period.

[23] A deviation from the collusive price implies that the market will be competitive ever after.

[24] Assuming instead that firms keep colluding even after a successful investigation does not affect the quality of the results.

[25] For instance, one firm will get to the AA sooner than the other. If the AA does not diffuse information relative to firms that have decided to collaborate, it is well possible that both firms decide to confess, each hoping to be the first comer.

$$\hat{\gamma}_2 = \frac{\gamma_1(k) + 1}{2} \tag{4}$$

Finally, suppose that cartel evidence can be used for only one period, *i.e.* firms cannot be convicted for past violations.

3 Optimal evidence allocation

Assume a cartel has survived up to some period at which the AA decides to investigate the case. If a post-investigation LP is in place, firms must decide whether to report or not. Assuming that both firms make this decision simultaneously, the following lemma describes the equilibrium of the investigation sub-game:[26]

Lemma 1

- *For* $z \leq \bar{z}(k) \equiv \frac{(2+k)(1-\rho_0)-2k}{k(1-\rho_0)}$ *no investigated firm confesses.*
- *For* $\bar{z}(k) < z \leq \bar{\bar{z}}(k) \equiv \frac{2(1-\rho_0)-k}{2(1-\rho_0)+k} \leq 1$ *one firm confesses*
- *Both firms confess when* $z > max\{\bar{z}, \bar{\bar{z}}\}$

Proof

See the Appendix.■

Solving $z \leq \bar{z}(k)$ and $z \leq \bar{\bar{z}}(k)$ for k yields $k \leq \bar{k}(z) \equiv \frac{2(1-\rho_0)}{1+z(1-\rho_0)+\rho_0}$ and $k \leq \bar{\bar{k}}(z) \equiv \frac{2(1-z)(1-\rho_0)}{1+z}$ respectively. Since both $\bar{k}(z)$ and $\bar{\bar{k}}(z)$ are monotonically decreasing in z, they take their maximum value when $z = 0$. We define

$$k_{01} \equiv \bar{k}(0) = \frac{2(1-\rho_0)}{1+\rho_0} \tag{5}$$

and

$$k_{02} \equiv \bar{\bar{k}}(0) = 2(1-\rho_0) \tag{6}$$

For $z = 1$, $\bar{k}(z)$ reaches its minimum value equal to $(1-\rho_0)$, which represents the minimum level of k that is necessary for at least one firm to report.

When $k > k_{01}$ there is no value of $z \geq 0$ that can induce universal non reporting in the equilibrium of the investigation subgame: even if the cartel keeps each firm with

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[26] Simultaneous reporting is equivalent to assuming that firms take the reporting decision in a random order and in case of universal reporting, at the end of the reporting phase nature determines the name of the eligible for leniency one. In other words, someone will win the race to the Court, but no player is able to predict whether the winner will be itself or the other party.

the minimum possible percentage of evidence $\left(\frac{1}{2}\right)$ there will be at least one whistleblower. Similarly, when $k > k_{02}$, there is no value of $z > 0$ that in case of investigation can prevent both firms from racing to the Court.

We determine the collusive values according to the number of reporting parties in the equilibrium of the investigation stage. Recall that at any period, for all the parameter values that assure cartel stability, if no investigation takes place each firm expects to collude for at least one more period (and potentially many more), while if an investigation takes place the cooperation stops from the next period on, whether the cartel is convicted or not. Thus, when $z \leq \bar{z}(k)$, each firm expects a) in case of non-investigation to keep colluding for one more period earning the collusive profit, b) in case of unsuccessful investigation-prosecution to interrupt the collusion but pay no fines, and c) in case of successful investigation-prosecution, to stop colluding and pay the full fine; its profit in this case is equal to 1 (one half of the normalized monopoly profit), reduced by μ. The value of collusion where no firm confesses in the equilibrium of the investigation subgame is:

$$V_0 = (1-a)(1+\delta V_0) + a(1-\rho_0) + \alpha\rho_0(1-\mu) = \frac{1-a\rho_0\mu}{1-\delta(1-a)} \qquad (7)$$

When $z > max\{\bar{z}, \bar{\bar{z}}\}$, *i.e.* when both firms are induced to confess, each firm expects to earn the collusive gain, and, to pay either the full or the reduced fine with equal probability, since only one firm is eligible for leniency. In case of no investigation each firm continues colluding for, at least, one more period. The value of collusion becomes:

$$V_2 = (1-a)(1+\delta V_2) + \alpha(1-\hat{\gamma}_2\mu)$$

which after some rearrangement yields:

$$V_2(z;k) = \frac{1-a\hat{\gamma}_2\mu}{1-\delta(1-a)} = \frac{4-a\mu[4-k(1+z)]}{4[1-\delta(1-a)]} \qquad (8)$$

Consider now the case $\bar{z}(k) < z < \bar{\bar{z}}(k)$, where there is only one reporting firm in the equilibrium of the investigation subgame. When a firm decides whether to remain loyal to the cartel, it anticipates that with probability a there will be investigation in which case one firm will decide to report, but, due to the existence of two symmetric equilibria, it cannot predict which firm will be the reporting one. Unless there is a binding agreement on some coordinating device, in case of conviction each firm expects with probability to receive either the reduced or the full fine.[27] Its expected fine is $\hat{\gamma}_2\mu$, and the value of its cartel participation is:

$$V_1 = \frac{1 - a\rho_1\hat{\gamma}_2\mu}{1 - \delta(1 - a)}$$

which after replacing ρ_1 and $\hat{\gamma}_2$ from (1) and (4) yields:

$$V_1(z;k) = \frac{8 - a\mu[4 - k(1 + z)][1 + z(1 - \rho_0) + \rho_0]}{8[1 - \delta(1 - a)]} \tag{9}$$

Cartel stability requires that at stage 3 no firm prefers to deviate from the agreement. Recall that, by assumption, a firm that unilaterally deviates from the agreement and reports before the launch of the audit is absolved from any fine imposition (full leniency for pre-investigation informants). Hence, the cartel directory's decision on z, as well as any preceding decision of the AA concerning post-investigation leniency (in our case, the value of k) do not affect the value of deviating from the agreement. Cartel stability therefore requires that V_0, $V_1(z)$, or $V_2(z)$, according to the case, be no less than 2, the normalized monopoly profit from this market.

At stage 2 firms cooperatively decide the level of z that maximizes the cartel value, thus enhancing the sustainability of the agreement. From simple inspection of (8) and (9) we see that $V_1(z) \geq V_2(z)$. The following lemma compares the cartel value under no reporting to $V_1(z)$:

Lemma 2 *For* $k \leq k_{01}$, $V_0 \geq V_1(z)\big(\geq V_2(z)\big)$ *always holds .*[28]

Proof

See the Appendix.■

Lemma 2 states that when universal non-reporting is feasible in the post-investigation stage, it is also the most profitable outcome. Since V_0 does not depend on z, for pairs (k, ρ_0) with $k \leq k_{01}$ the cartel directory is indifferent among evidence distributions $z \in [0, \bar{z}(k)\big)$. In other words, when no reporting can be obtained as

[27] Note that this case differs from that where both firms report in equilibrium in that here, conviction is not certain, since one firm's portion of exclusive evidence will not be presented to the Court.

[28] Note that $\hat{k} \geq k_{01}$ holds when $z \leq \frac{2\rho_0}{1-\rho_0}$. For $z > \frac{2\rho_0}{1-\rho_0}$, $k = k_{01}$ implies net rewards to the eligible informant and thus cannot be offered.

equilibrium of the investigation subgame the cartel is equally satisfied with any value of z provided that it that does not upset the no reporting equilibrium.[29]

We turn now to determine the optimal choice of z given that $k \geq k_{01}(\rho_0)$. The latter implies that $\bar{z} < 0$ and therefore for every $z \in [0,1]$ reporting by at least one firm cannot be avoided. The cartel directory must now choose between either a value of z that satisfies $\bar{z}(k) < 0 < z \leq \bar{\bar{z}}(k)$ (one informant) or a value of z such that $z > max\{\bar{z}, \bar{\bar{z}}\}$ (universal reporting), by comparing the maximized values of $V_1(z)$, $V_2(z)$ over the relevant range of z.[30]

Lemma 3 *When both firms confess in the investigation stage the cartel becomes more profitable as z increases. On the contrary, if only one investigated firm reports the cartel's profitability is decreasing in z.*

Proof

See the Appendix.■

While a formal proof is contained in the Appendix, the intuition of lemma 3 can be developed in relatively simple terms. With both firms confessing, conviction becomes certain and all that the directory aims is to ensure that the AA pays through fine-reductions for an as-large-as-possible portion of the evidence. For instance, opting for $z = 1$ when the race to the Courts is unavoidable implies that the winner of the leniency ticket will receive a fine reduction in proportion to the entire available evidence, whereas by setting $z = 0$ the reduction is based on only half of the evidence. The other half is still brought in, but receives no fines-reduction, since the second firm to arrive is not eligible for leniency. Obviously, with $z = 1$ the equilibrium outcome of the post-investigation subgame is universal reporting, since even a single firm's confession implies conviction with certainty. On the other hand, $\bar{z}(k) < 0$ implies that even setting $z = 0$ does not avoid confession, and if also $\bar{\bar{z}}(k) > 0$, only one firm will decide to come forward, which implies that there is some positive probability to avoid conviction. What the proof of Lemma 3 shows is that when $\bar{z}(k) < 0$, for every $z \in [0, \bar{\bar{z}})$ the cartel's main preoccupation must be to

[29] This lies on the assumption that the AA's investigation uncovers equal portions of each type of evidence. If instead $\lambda_1 > \lambda_2$ is assumed, *i.e.* that down-raids uncover more of the common evidence, then V_0 decreases with z.

[30] While we know that $V_1(z) \geq V_2(z)$ holds for all z, this is of no much use here since V_1, V_2 may reach their maximum at different values of z.

let as little evidence as possible in the hands of AA. Hence, the best choice among all the values of z inducing single reporting is the smallest possible one, implying that when $\bar{z}(k) < 0$, the optimal choice of the cartel directory is to set $z = 0$.

When no reporting by any firm is impossible, lemma 3 limits the cartel's optimal choice of z between two values, 0 or 1. Setting $z = 1$ in (8) we obtain the maximum value of the collusion where both investigated firms confess:

$$V_2^* = V_2(1) = \frac{2 - a\mu(2-k)}{2[1-\delta(1-a)]} \tag{10}$$

From (1) the conviction probability when only one firm confesses and $z = 0$ is

$$\ddot{\rho}_1 \equiv \rho_1(0) = \frac{1+\rho_0}{2}$$

and since in the decision not to undercut the cartel price each firm assigns a probability $\frac{1}{2}$ to be the reporting party, using (2) the expected fine when $z = 0$ is $\hat{\gamma}_1\mu$ with

$$\hat{\gamma}_1 = \frac{1}{2}\left(\frac{2-k}{2} + 1\right) = \frac{4-k}{4}$$

and the collusive value becomes $V_1^* = (1-a)(1+\delta V_1^*) + \alpha(1-\ddot{\rho}_1) + \alpha\ddot{\rho}_1(1-\hat{\gamma}_1\mu)$ which yields the maximized cartel value when only one firm confesses in the investigation subgame:

$$V_1^* = V_1(0) = \frac{8 - a\mu(4-k)(1+\rho_0)}{8[1-\delta(1-a)]} \tag{11}$$

Using (10) and (11) we obtain a value of k, such that $k > (<) k_{03}$ implies that $V_1^* < (>) V_2^*$:

$$k_{03} \equiv \frac{4(1-\rho_0)}{3-\rho_0} \tag{12}$$

The following proposition describes the consequences of every choice of k on the equilibrium of the subgame, assuming that the cartel directory will always respond with the value-maximizing choice of z, call it z^*:[31]

Proposition 1 *When the AA's initial investigation is relatively efficient, yielding an initial probability of conviction* $\rho_0 \geq \frac{1}{3}$ *, then i) for* $k^* < k_{01}$, $z^* \in [0, \bar{z}]$ *and no firm confesses; ii) for* $k_{01} < k^* < k_{03}$ $z^* = 0$ *and one firm confesses; iii) for* $k^* > k_{03}$ $z^* = 1$ *and both firms confess.*

[31] Note that when $k = k^*$ the latter is offered as long as $k^* \leq \hat{k}$. If $k^* > \hat{k}$ the eligible firm pays no fine, that is $k = \hat{k}$.

When $\rho_0 < \frac{1}{3}$ *i) for* $k_{01} < k^* < k_{02}$ $z^* = 0$ *and one firm confesses and; ii) for* $k^* > k_{02}$ $z^* \in \left[\frac{2-k}{k}, 1\right]$ *and both firms confess.*

Proof

See the Appendix.■

When the leniency provided is not sufficiently generous, the cartel-directory will spread the incriminating evidence as much as possible in order to induce as few firms as possible to collaborate in case of investigation. On the contrary, generous leniency schemes induce the cartel directory to let in the hands of each firm as much information as possible. This somewhat paradoxical result is easily explained if one thinks that universal reporting is in this case unavoidable, hence cartel stability is enhanced by maximizing the "amnesty effect".[32] By letting as much evidence as possible in the hands of each firm, the directory increases the winning prize of the "race to the Courts" lottery and through this the expected value of staying loyal to the cartel agreement before investigation.

Given the above, proposition 2 determines the optimal choice of k assuming that the AA's objective is to minimize cartel stability.

Proposition 2 *When the AA's own investigation is sufficiently efficient, i.e., when* $\rho_0 \geq \frac{1}{3}$, *optimal cartel deterrence requires offering a LP with* $k^* = k_{01}$, *otherwise, maximum cartel deterrence is obtained by offering* $k^* = k_{02}$.

Proof

See the Appendix.■

The last proposition states that the choice of the AA on the offered leniency rate and consequently the number of reporting firms and the subsequent amount of gathered evidence depends on how likely is the conviction in the absence of firms' confession. The profitability of the cartel is always non-increasing in the probability of conviction. When the latter is low $\left(\rho_0 < \frac{1}{3}\right)$ maximizing the profitability of the cartel demands the cartel to restrict the amount of evidence that each member possesses in an attempt to minimize the likelihood of successful prosecution. In this case the AA optimally selects to offer sufficient incentives that prevent any firm to remain silent in case of investigation. Offering $k^* = k_{02}$ entails that firms possess

[32] According to Harrington (2008) the "cartel amnesty effect" of LP describes how fine reductions affect the expected payoff from cartel continuation.

$z \geq \frac{\rho_0}{1-\rho_0}$ and that the eligible for leniency firm pays no fine. Maximum fine's reduction induces both firms to come forward despite that the likelihood of failed investigation is large.

When the likelihood of failed investigation narrows $\left(\rho_0 > \frac{1}{3}\right)$ the incentive of each investigated firm to remain silent weakens and the cartel finds it profitable to extend the common share of evidence in order to benefit from maximum fine reductions. In such case optimal cartel deterrence requires the AA to offer moderate reporting incentives that rule out universal reporting inducing firms to partially report despite that cartel conviction is already likely.

With respect to the optimal choice of the leniency rate, Charistos and Constantatos (2016) considers the value of z as exogenous and concludes that the AA should always provide sufficient incentives in order to assure 100% conviction, without paying attention to the presence of the amnesty effect. By endogenizing the value of z, the present analysis conditions this result on the efficiency of the AA's independent investigation. When the latter is high, yielding *ex-ante* a sufficiently high probability of conviction, insisting on making that probability 100% may reduce the cartel-deterring impact of the LP. By properly adjusting the value of z, the cartel directory may increase the AA's bill for universal reporting, and through this, reduce the expected fine. On the other hand, when the conviction probability is low without the assistance of whistleblowers, the AA's priority must be to guarantee 100% conviction.

4 Concluding remarks

One question this paper attempts to answer is how colluding firms would select to allocate the incriminating evidence between the participants of the conspiracy. First, we show that if the provided incentives to confess are sufficiently weak, universal non-reporting is a target that can be credibly achieved, and since it is also the most profitable outcome, the cartel directory will opt for it. The goal of inducing no firm to collaborate with the authority is served by allowing little evidence to be shared by both firms: the common evidence must be below a given threshold.

Then, assuming that the AA provides sufficient incentives in order to eliminate universal non-reporting, the selected level of shared evidence affects both the likelihood of cartel conviction and the level of expected penalty. If further the leniency provided is not enough to rule out partial non-reporting the cartel organization selects to split and equally share the evidence. If firms are not able to avoid universal reporting (and consequently certain conviction) the cartel value is maximized when firms possess a large amount of common evidence which allows them to benefit from maximum fine reductions.

When the AA anticipates the cartel's ability to manipulate the amount of evidence that each member holds and its own efforts lead to sufficiently likely conviction, reporting is either partial or universal depending on the generosity of the LP. As conviction is already likely the AA's choice that maximizes cartel deterrence (minimizes the profitability of the cartel) involves moderate fine reductions that induces both reporting and non-reporting members to co-exist in occurrence of investigation's opening. In this case each member possesses only exclusive pieces and consequently the AA is not able to gather all the available evidence.

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Appendix

Proof of lemma 1

Non-reporting (NR) by all firms entails payment of the full fine μ with probability ρ_0 while reporting (R) by a single firm implies that the confessant pays the reduced fine $\gamma_1\mu$ with probability ρ_1 (recall that no fine is paid if the cartel is not convicted).

	R	NR
R	$1-\hat{\gamma}_2\mu, 1-\hat{\gamma}_2\mu$	$1-\rho_1\gamma_1\mu\ , 1-\rho_1\mu$
NR	$1-\rho_1\mu, 1-\rho_1\gamma_1\mu$	$1-\rho_0\mu, 1-\rho_0\mu$

When thinking that the other firm is not going to report, a firm decides to remain silent if $1-\rho_0\mu \geq 1-\rho_1\gamma_1\mu$. The incentive to remain silent when no firm reports can be written as $\varphi_1 = 1-\rho_0\mu-(1-\rho_1\gamma_1\mu)$ which after replacing ρ_1 and γ_1 by their equivalent from (1) and (2) yields:

$$\varphi_1(z) = \frac{\mu(1+z)[(2-kz)(1-\rho_0)-k(1+\rho_0)]}{4}$$

which is positive when $z \leq \bar{z}(k)$, the latter being given by the lemma. Due to symmetry, when $z \leq \bar{z}(k)$, no firm has incentive to report thinking that the other will not do so, hence, universal non-reporting is an equilibrium.

If a firm believes that the other is going to report, by remaining silent it expects to pay the full fine with probability ρ_1, whereas by reporting it expects to pay $\hat{\gamma}_2\mu$ with probability $\rho_2 = 1$. The incentive to remain silent when the other reports is therefore: $\varphi_2 = 1-\rho_1\mu-(1-\hat{\gamma}_2\mu\ )$ which, after replacing ρ_1 and $\hat{\gamma}_2$ by their equivalent, reduces to

$$\varphi_2(z) = \frac{\mu[2(1-\rho_0)(1-z) - k(1+z)]}{4}$$

The latter is positive when $z \leq \bar{\bar{z}}(k)$, where $\bar{\bar{z}}$ is given in lemma 1. Note that $\bar{\bar{z}}(k)$ may be either higher or lower than $\bar{z}(k)$, which implies two different cases to be considered.

I. Case 1 ($\bar{z}(k) < \bar{\bar{z}}(k)$):

(i) For every $z \in [\bar{z}, \bar{\bar{z}}]$, $\varphi_1(z) < 0$ while $\varphi_2(z) > 0$, therefore, the strategy pair {report, do not report} is a Nash equilibrium. Due to symmetry, the strategy pair {do not report, report} represents a Nash equilibrium, as well. These equilibria are symmetric differing only with respect to the identity of the reporting firm.

(ii) For every $z \in [\bar{\bar{z}}, 1]$, both $\varphi_1(z)$ and $\varphi_2(z)$ are negative implying that to report becomes dominant strategy.

(iii) For every $z \in [0, \bar{z}]$, $\varphi_1(z)$ and $\varphi_2(z)$ are positive, therefore do not report becomes dominant strategy for both players.

II. Case 2 ($\bar{\bar{z}}(k) < \bar{z}(k)$):

For every $z \in [\bar{\bar{z}}, \bar{z}]$, $\varphi_1(z) > 0$ while $\varphi_2(z) < 0$, hence there exist two equilibria: either none or both firms confess. Note that the payoff from remaining silent when the other does the same is $1 - \rho_0\mu$. The payoff from reporting when the other reports is $1 - \hat{\gamma}_2\mu$. Assume further that when multiple equilibria exist, firms coordinate on the equilibrium that is Pareto optimal. The no-reporting (reporting) equilibrium is more profitable when $\rho_0 < (>)\hat{\gamma}_2 \Leftrightarrow z < (>)\frac{4(1-\rho_0)-k}{k}$. If $\bar{z}(k) > \frac{4(1-\rho_0)-k}{k}$ and $\frac{4(1-\rho_0)-k}{k} < 1$ both firms confess for $z \in \left(\frac{4(1-\rho_0)-k}{k}, min\{\bar{z}, 1\}\right)$. Note that $\frac{4(1-\rho_0)-k}{k} < 1$ holds when $k > 2(1-\rho_0)$ and $\bar{z}(k) > \frac{4(1-\rho_0)-k}{k}$ holds for $k < \frac{(1-\rho_0)(2\rho_0-1)}{\rho_0}$. Since $2(1-\rho_0) < \frac{(1-\rho_0)(2\rho_0-1)}{\rho_0}$ never holds, $\bar{z}(k) \leq \frac{4(1-\rho_0)-k}{k}$ and $\frac{4(1-\rho_0)-k}{k} < 1$ never hold simultaneously, thus for every $z < \bar{z}(k)$ universal non reporting is the Pareto optimal equilibrium and therefore no investigated firm confesses.

Proof of lemma 2

First, we compare the cartel values with no informants to the corresponding ones when one firm decide to come forward and provide information: Using (7) and (9)

$V_0 > V_1(z)$ holds for $z < z' = \frac{(1-\rho_0)(4+k)}{k(1-\rho_0)}$. Recall from lemma 1 that non-reporting by both firms is the outcome of the investigation subgame if $z < \bar{z}(k) \equiv \frac{(2+k)(1-\rho_0)-2k}{k(1-\rho_0)}$. Obviously, for every (k, ρ_0), $\bar{z}(k) < z'$, therefore when the pair (k, ρ_0) is such that the combination (*do not report, do not report)* is equilibrium of the subgame, it is also the strategy that maximizes the value of the cartel.[33]

Proof of lemma 3

Using (8) it is shown that:

$$\frac{\partial V_2(z)}{\partial z} = \frac{ak\mu}{4[1-\delta(1-a)]} \geq 0$$

Setting $z = 1$ in $V_2(z)$ we obtain the maximum value of the collusion where both firms confess under investigation, that is $V_2(1) = \frac{2-a\mu(2-k)}{2[1-\delta(1-a)]}$.

The derivative of $V_1(z)$ with respect to z yields $\frac{\partial V_1(z)}{\partial z} = \frac{-a\mu[(2-kz)(1-\rho_0)-k]}{4[1-\delta(1-a)]}$ which is negative (positive) when $z < (>) \frac{2(1-\rho_0)-k}{k(1-\rho_0)}$. Observe that single-firm reporting is sustainable if further $z \leq \bar{\bar{z}}(k) \equiv \frac{2(1-\rho_0)-k}{2(1-\rho_0)+k}$ (see lemma 1) and that $\bar{\bar{z}}(k) < \frac{2(1-\rho_0)-k}{k(1-\rho_0)}$ holds for every $k > 0$. Therefore we conclude that when single reporting is sustainable $\frac{\partial V_1(z)}{\partial z} < 0$ holds.

Proof of Proposition 1

By setting the RHS of (12) greater or equal to k_{01} and solving for ρ_0 we obtain that $k_{01} \leq (>) k_{03}$ iff $\rho_0 \geq (<) \frac{1}{3}$. For what follows, recall also that, from (5) and (6), $k_{01} < k_{02}$.

(i) First consider that $\rho_0 \geq \frac{1}{3}$ which implies $k_{01} < k_{03} < k_{02}$, $k_{01} \leq \hat{k}$.

Comparing (3) with (5) and (12) yields also that $\rho_0 \geq \frac{1}{3}$ is equivalent to both $k_{01} \leq \hat{k}$ and $k_{03} \leq \hat{k}$. Note that for every $k \in [k_{01}, k_{03}]$ the constraint of nonnegative fines is never binding and $V_1^* \geq V_2^*$ holds, while single reporting is sustainable. Consequently, the directory sets $z = 0$ and one firm confesses under investigation.

[33] In other words, when private incentives lead both firms to no reporting, this actions-combination is also collectively optimal.

For $k_{03} < k < min\{\hat{k}, k_{02}\}$, the constraint of nonnegative fines is never binding as well but now $V_1^* < V_2^*$: the directory sets $z = 1$ and both firms confess when investigated.

(ii) Now consider the case where $\rho_0 < \frac{1}{3}$ which implies that $k_{03} < k_{01} < k_{02}$ holds. We examine the two cases $k_{01} < k < k_{02}$ and $k > k_{02}$ separately:

- $k_{01} < k < k_{02}$

For $k_{01} < k < k_{02}$ universal non-reporting is not sustainable. There exist two possibilities with respect to the number of informants: if the directory chooses z such that $z < \bar{\bar{z}}(k)$ only one firm confesses and according to lemma 3 the cartel value is decreasing in z for the entire $[0, \bar{\bar{z}}(k)]$ Thus, if the directory is to set $z < \bar{\bar{z}}(k)$, it will set $z = 0$ and the cartel value will be V_1^*, as given by (11).

If the cartel decides a value of z such that $z > \bar{\bar{z}}(k)$ both firms confess and the value of the cartel is given by (8), which is increasing in z, provided that fines are nonnegative. Thus, if $\hat{z} = \frac{2-k}{k} \geq 1$, which holds *iff* $k \leq 1$, the cartel sets $z = 1$. If, however, $\hat{z} < 1$, since fines cannot be negative (rewards) we assume that the leniency winner receives no fines. Below it is shown that the cartel directory is indifferent among any value of $z \in [\hat{z}, 1]$.

For $\bar{\bar{z}}(k) < z < \hat{z} = \frac{2-k}{k}$ (from 3') both firms confess and $k < \hat{k}$. As $\frac{\partial V_2(z)}{\partial z} > 0$, in the interval $[\bar{\bar{z}}(k), \hat{z}]$ the maximum value of the cartel is obtained when $z = \frac{2-k}{k}$ and is equal to:

$$\breve{V}_2 = V_2(\hat{z}) = \frac{2 - a\mu}{2[1 - \delta(1 - a)]}$$

Observe that the previous is equivalent to offering $k = \hat{k}$: Substituting $k = \hat{k}$ in (8) we obtain the collusive value which is again $\breve{V}_2$. The comparison of V_1^* and $\breve{V}_2$ yields that $V_1^* > \breve{V}_2$ holds for $k > \frac{4\rho_0}{1+\rho_0}$. As $\frac{4\rho_0}{1+\rho_0} < 1$ while $k_{01} > 1$ for $\rho_0 < \frac{1}{3}$, when $k_{01} < k < k_{02}$ and $\rho_0 < \frac{1}{3}$ firms set $z = 0$ and one firm confesses.

- $k > k_{02}$

Offering $k > k_{02}$ implies that both firms confess for every z. When $\rho_0 < \frac{1}{3}$ offering $k > k_{02}$ is possible if $\hat{k} \geq k$ and the latter holds when $z < \hat{z}$. As it was shown above $\breve{V}_2$ is the maximized value of $V_2(z)$ (zero fines for any $z \geq \hat{z}$). Therefore in this case firms set $z \geq \hat{z}$ and both firms confess.

Proof of Proposition 2

Consider that $\rho_0 \geq \frac{1}{3}$ $(k_{01} < k_{03} < k_{02})$. From proposition 1 when $k_{01} < k \leq k_{03}$ then $V_1^* \geq V_2^*$, hence the directory sets $z = 0$ and only one firm reports. Therefore, for $\rho_0 \geq \frac{1}{3}$ the cartel sets $z = 0$ and one investigated firm confesses when $k_{03} \geq k \geq k_{01}$. Substituting $k = k_{01}$ from (5) into (11) we get the respective cartel value:

$$\tilde{V}_1^*(k_{01}) = \frac{4 - a\mu(1 + 3\rho_0)}{4[1 - \delta(1 - a)]}$$

For $k > k_{03}$, $V_1^* < V_2^*$ holds, therefore the directory sets $z = 1$, and both firms compete which one will supply all the evidence to the AA. In this case, using (12) and (10) the cartel value becomes:

$$\tilde{V}_2^*(k_{03}) = \frac{3 - \rho_0 - a\mu(1 + \rho_0)}{[1 - \delta(1 - a)](3 - \rho_0)}$$

Simple calculations show that that $\tilde{V}_1^* \leq \tilde{V}_2^*$ holds for every $\rho_0 \in \left[\frac{1}{3}, 1\right]$. Thus, setting $k = k_{01}$ is optimal.

Consider now that $\rho_0 < \frac{1}{3}$. From the proof of proposition 1, $V_1^* > \breve{V}_2$ holds for holds for $0 < \rho_0 < \frac{1}{3}$. Thus, setting $k = k_{02} + \varepsilon$ is optimal.

Collusion and Antitrust Enforcement in Advertising-Selling Platforms

Abstract

This paper underlines the impact of indirect network externalities on the effectiveness of antitrust enforcement to deter collusion between advertising-selling platforms. Since two-sided collusion is less likely to be sustained as consumers (*e.g.* readers/viewers) become more ad-avoiders while the opposite is true for one-sided collusion, firms may be induced to semi-collude (collude on advertising while competing for consumers) instead of colluding on both sides. When firms semi-collude on advertising and consumers are neutral towards advertisements, the imposition of fines based on the illegal gain of the colluding side (one-sided fines) enhances cartel deterrence compared to fines based on the total illegal profits (two-sided fines).

JEL Classification: K21, L12, L41

Keywords: Collusion, Media markets, Two-sided markets

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1 Introduction

Advertising-financed media are two-sided platforms selling services to both, advertisers and consumers (*e.g.* readers, viewers, etc.).[1] In a typical two-sided platform, the behavior of buyers in one side affects the utility of buyers in the other. Usually, the presence of buyers in one market has positive impact on the demand of buyers on the other market: for instance cardholders and merchants are mutually happy by the presence of each other in the market of payment cards. However, the presence of advertisers distinguishes media from other two-sided markets. While the presence of consumers has an unambiguously positive effect on the advertisers' willingness-to-pay, the reverse is not always true: consumers may exhibit either appreciation or aversion towards advertisements. For example, buyers of fashion magazines may be delighted by the presence of stylish advertisements, whereas TV viewers may be annoyed by advertising-related program interruptions. This difference between advertisement-selling media-platforms and other two-sided platforms may have important consequences on firms' behavior.

The media-platforms market has had its fair share of anti-trust cases, with several cases of collusion being reported and investigated. In some of these cases platforms have been accused for colluding only on the advertising side, while maintaining competition vis-à-vis final consumers.[2] Such s*emi-collusion* looks *a priori* as an inferior strategy in two respects. First, full collusion always obtains no lesser profits compared to any form of partial collusion, and second, multimarket contact is known to enforce cartel stability.

This paper investigates the sustainability conditions of different degrees of collusion and examines the appropriate type of law-enforcement fines against semi-collusion. Since the Anti-Trust Authority (AA) is unable to monitor every sector at every moment, the investigation of any given sector at any point in time occurs with probability less than one. Considering that a down raid at a platform's headquarters may reveal collusion-related evidence not only in the market under investigation but also in the other market, full collusion exposes the cartel at a higher risk of conviction. Hence, while the *ex post* profit is higher with full collusion, semi-

[1] See Rysman (2009) for a survey of the two-sided markets related literature.

[2] In 2018, the South African Competition Commission found that printed-media companies were engaged in price fixing against advertising agencies. See Dewenter et al. (2011) and Lefouli and Pinho (2018) for additional examples of collusion in media markets.

collusion may be superior in terms of *expected* profit. When is the importance of reducing the exposure to investigation risk sufficiently strong as to make semi-collusion superior to full collusion? The answer depends on consumers' attitude towards advertising.

Semi-collusion on the advertising side (SC henceforth) means restricting the quantity of advertisements in order to increase their price, and this can be perceived by final consumers as an either increase or decrease in the quality of the product they purchase.[3] If consumers dislike advertising, semi-collusion acts as a coordinated increase in the quality of all platforms' final product (reading copies, broadcasting hours, etc.). This increases the profitability of the consumers' market under any type of firm conduct—competition or cooperation—however its impact under no cooperation is more important. Thus, by colluding only on the advertising side firms may already obtain the larger part of cooperation associated benefit. Taking into account the aforementioned increase in exposure to the risk of being convicted, the additional profit provided from colluding also on the consumers' side may be simply not worth. Conversely, when consumers are ad-lovers, reducing quantity at the advertising side reduces product quality and exposes firms to low profits in case of non-cooperation on the consumers' side.

The above arguments relate also consumers' attitude to cartel stability. We show that *ceteris paribus* as consumers become more ad-haters, full-collusion becomes more difficult to sustain while the opposite is true for SC. Intuitively, when consumers become more ad-averse the fruit of cooperating on the advertising side is mostly reaped as profit from the final-product side, thus making deviation on the final-product market more tempting. An agreement that limits cooperation in only the advertising market becomes easier to sustain, since any deviator will find itself immediately punished by selling a final product of lower quality (lots of ads) and under competitive conditions. On the contrary, when consumers are ad-lovers a platform that breaks cooperation and sells more advertisements not only makes higher profit in that market, but also competes on the consumer side with the advantage of selling a superior quality product. Since a large part of the cooperation benefit is

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[3] If consumers dislike advertising, a rational cartel directory may restrict advertising at a level below the one that maximizes profit from the advertising market, taking into account the beneficial impact o from such restriction on the profit stemming from the consumers' market.

obtained through profits at the advertising market, if cooperation survives that stage it is easy to be maintained at the final-product stage, as well.

In conclusion, both profitability and stability considerations point *ceteris paribus* towards semi-collusive (full-collusive) agreements in markets where consumers are ad-averse (ad-lovers). Since SC consists, both theoretically and practically a conceivable risk for competition, the next question is, how can fines be designed in order to most effectively deter its formation.

Fines on convicted cartel participants are (roughly) based on illegally earned profits, defined as the difference between profits earned and profits that would have been earned had competition prevailed. The issue from applying this principle in cases of SC is whether the fines base must be limited on illegal profits stemming from the side where cooperation was effective (one-sided fines), or be extended on profits from both markets (two-sided fines). Our analysis shows that, with ad-neutral consumers, SC increases profits relative to competition on that market but *reduces profits* on the other. Hence, one-sided fines result in higher actual fine and produce better deterrence compared to two-sided fines.

Related literature

Related to our work are two strands of literature. The first, mostly empirical, relates to consumer attitude towards advertising in media markets. The second deals with issues of collusion among platforms and more generally among firms serving related markets.

The evidence on consumers' attitude towards advertising is not unanimous. Sonnac (2000) shows the effect of advertising to be country-specific and depending on the type of media. Wilbur (2008) concludes that an increase in advertisement time decreases the median audience size. On the contrary, Kaizer and Song (2009) using data from the German magazine market concludes that consumers either appreciate or are indifferent towards advertisements. Argentesi and Filistrucchi (2007) and van Cayselee and Vanormelingen (2009) find that advertising has no effect on the sales of Italian and Belgian daily newspapers respectively. From the literature it can be inferred that magazines' readers are on average ad-lovers, while TV viewers are ad-avoiders and newspapers' readers are rather indifferent towards advertisements. Theoretical work, such as Gabszewicz *et al.* (2002, 2012) assumes that advertising has no effect on readers' demand for newspapers; the work of Kind *et al.* (2016)

assumes that TV viewers' suffer a nuisance cost when the broadcast is interrupted by advertisements.

The impact of indirect network externalities on firms' incentive to collude is studied in Ruhmer (2011). It shows that two-sided collusion is less likely to be sustained when network externalities become stronger and more asymmetric. Evans and Schmalensee (2007) claim that one-sided collusion cannot be profitable since the supra-competitive profits will be competed away on the non-colluding side. However, Rhumer (2011) shows that one-sided collusion is generally profitable. Regarding the sustainability conditions, Rhumer (2011) finds that when network externalities are symmetric two-sided collusion is never harder to be sustained compared to one-sided collusion while the answer is ambiguous when network externalities are asymmetric.

Focusing on the market of newspapers and assuming that consumers appreciate advertising; Dewenter *et al.* (2011) compares competition in both markets, semi-collusion in advertising quantities and full collusion. They show that semi-collusion on advertising reduces copy prices and advertising quantity compared to competition, while both readers' surplus and firms' profits are higher under semi-collusion.

Lefouili and Pinho (2018) explore the sustainability of collusion and how collusion affects prices, using an infinitely repeated version of Armstrong (2006). Allowing firms to imperfectly collude in a two-sided duopoly where both platforms set prices on both sides simultaneously they confirm that two-sided collusion is less likely to be sustainable when the level of cross-group externalities increases. The prices in both sides are higher under two sided-collusion compared to competition. When firms semi-collude and the cross-group externalities exerted on the colluding side are positive the collusive price on one side is lower than the competitive one.

The present work is also related to the literature of cartel enforcement in multimarket contact when demand relationships are present. Choi and Gerlach (2013) analyze the impact of antitrust interventions when two firms interact in demand-related markets. They show that whether products are substitutes or complements, crucially affects whether prosecution in one market enhances or hurts cartel stability in the other market. Therefore, the nature of the demand linkage affects optimal cartel formation *i.e.* whether firms prefer sequential or simultaneous cartels.

The paper is organized as follows: In section 2 we describe the basic assumptions of the model and section 3 analyze and compare the stage game outcomes. Section 4 examines the impact that different forms of collusion and fines basis has on firms' incentive to collude. Section 5 concludes.

2 Model

Consider two platforms (firms) $i = 1,2$ serving two groups of customers, advertisers and consumers, denoted by A and R respectively. Both platforms interact for an infinite number of periods. In each period, firms decide whether to compete on both sides, to collude on both sides, or to collude only on side A (hereafter, full- and semi-collusion, respectively). They maximize the expected sum of future discounted profits under a common discount factor $\delta \in (0,1)$.

At the beginning of each period a two stage game begins where at the first stage firms simultaneously set the level of advertising, *e.g.*, the amount of advertising contained per hour of broadcast, or in each printed copy sold; at the second stage, the platforms simultaneously decide the price of their product (*e.g.* price of printed copies). After all the decisions have been made, the two markets clear simultaneously. Before making their decisions, the two platforms may decide to collude on either both markets, or just the advertising one. If at least one firm refuses to collude, competition takes place in both markets at least up to the end of the period, where the issue will be decided anew. Whether one- or two-sided, a collusive agreement is illegal and cannot be based on any sort of binding contract. Hence, even if collusion is initially agreed, when making their pricing decision, firms may defect from the agreement, unless some future punishment is imposed to the defector. We assume that firms follow trigger strategies which implies that a deviation from the collusive agreement breaks the cooperation apart forever after.

In each period, after both firms have completed their sales, the Antitrust Authority (AA henceforth) may audit the market. We assume that if the AA investigates a cartelized market, the cartel is always uncovered and the participating firms are convicted. However, resource limitations allow the AA to investigate a cartelized market with probability $a \in (0,1)$.[4] We assume that if firms have colluded on both markets, the AA's investigation in one market unveils the existence of collusion in the

[4] Bryant and Eckhart (1991) and Combe et al. (2008) estimated the probability of cartel detection to be between .13 and .17 and around .13 respectively in a given year.

other market as well.[5] For example, if firms collude on both sides A and R, putting under anti-trust scrutiny market A uncovers also all the evidence related to the existence of collusion on side R.[6] Therefore, the probability of global conviction emanating from investigation on a single side is $a(1-a)$. Since there is also a probability a^2 that both sides are simultaneously investigated, bringing the probability of cartel prosecution in case of two-sided collusion becomes $a^f = a^2 + 2a(1-a) = a(2-a)$.

Conviction entails a monetary fine which is a *fraction of* the illegally earned profits. In case of one-sided collusion, the fine can be based on profit from either solely the collusive side, or the firm's total activity on both markets. We term fines as one- or two-sided according to their corresponding *base*.[7] In order to have deterring effects, a fine must be at least equal to the profit earned, but at the same time, excessive fines may be socially harmful, as they may force unable-to-pay firms out of business. A reasonable assumption is that the fine imposed to convicted infringers is finite and close to twice the gain from the illegal activity. This assumption will be relaxed when the value of the fines multiplier becomes qualitatively important for the results.[8] Finally, we assume that evidence related to the collusion can be used by the AA only for one period and therefore firms cannot be convicted for past violations.

[5] In 2014, the Hungarian Competition Authority (GVH) fined four publishers for coordinating both retail and advertising prices (case number: Vj/23/2011). In a prior stage, GVH has established that publishers shared information related to the prices of advertisements.

[6] We assume that at each period, the set of markets that will be subject to investigation is decided prior to starting any investigation. This implies that there is a probability that both the A and R markets be investigated at the same period. Since the probability of investigating any single market is a, if firms collude on both markets, the AA spots both cartels with probability $a^f = a^2 + 2a(1-a) = a(2-a)$, since inspecting one side reveals the cartel on the other side, as well. Alternatively assuming that the detection of full collusion occurs with any probability $a^f \in (a, a(2-a)]$ does not affect the quality of the results.

[7] If firms collude only on side A, one may argue that there are no illegal profits from side R. This, however, is true only if the two sides are completely independent. To the extent that collusion on A may also affect profits on R, the choice of fines-base is a matter of policy. Central in our analysis is to determine whether using one- or two-sided fines is more effective in deterring the formation of cartel agreements.

[8] The US (federal) fines correspond to no more than the greater between double damages and twice the colluding firms' gain; while other jurisdictions allow for up to treble damages, see Harrington (2014).

2.1 Demand and cost specification

Following Dewenter *et al.* (2011) we assume the preferences of the representative consumer are given by the following utility function:[9]

$$U_R = q_i(1 + \gamma s_i - p_i) + q_j(1 + \gamma s_j - p_j) - \frac{1}{2}(q_i^2 + q_j^2 + 2\theta q_i q_j)$$

where $j \neq i$, q_i is the number of copies (or the TV program quantity) sold by firm i, and s_i is the quantity of advertising contained in each copy or unit of program content. The parameter $\theta \in (0,1)$ measures "brand-name" differentiation on the consumers' side: for values of θ close to 1 the final product of the two platforms is considered as almost homogeneous, whereas values of θ close to zero imply two almost independent markets. The parameter γ represents consumers' attitude towards advertising: for $\gamma < 0$ consumers are ad-averse, whereas for $\gamma > 0$ they appreciate advertisements.[10]

The maximization of U_R yields the representative consumer's inverse demand for broadcast or printed copies as given by the following expression:

$$p_i = 1 + \gamma s_i - q_i - \theta q_j$$

Rearranging yields the following demand function:

$$q_i(\gamma) = \frac{1 - \theta - (p_i - \theta p_j) + \gamma(s_i - \theta s_j)}{1 - \theta^2} \tag{1}$$

Regarding the advertising side, a usual assumption in the related literature is that the utility that advertisers derive from reaching a consumer is independent of the number of consumers reached. As Anderson and Jullien (2016) note, this implies that the value per unit of demand is constant. Each advertiser's per consumer utility is analogously specified:

$$U_A = s_i(\kappa - r_i) + s_j(\kappa - r_j) - \frac{1}{2}(s_i^2 + s_j^2 + 2\beta s_i s_j)$$

where κ denotes the relative size of the advertisers' market and r_i is interpreted as the price that an advertiser has to pay in order to access a consumer. The maximization of

[9] The utility function has been introduced in Singh and Vives (1984). Anderson and Jullien (2016) provide an insight regarding the use of the representative consumer approach in media economics.

[10] While θ represents a "quasi-horizontal" differentiation parameter, the parameter γ introduces an element of vertical differentiation, defining as higher quality a printed copy that contains either more, or less advertising, depending on consumers' attitude.

the above provides the advertisers' marginal willingness-to-pay for platform i's advertising per-consumer:

$$r_i(s_i, s_j) = \kappa - s_i - \beta s_j \tag{2}$$

where $\beta \in (0,1)$ denotes the degree of differentiation in the advertisers' side. The expression above represents the inverse demand function for advertising. In order to avoid gratuitous complexity we assume that $\beta = \theta, \gamma \in (-1,1)$ and $\kappa = 1$.[11]

For simplicity we ignore any costs, thus the profits of firm i are equal to

$$\pi_i = q_i(r_i s_i + p_i) \tag{3}$$

where the first and second part represent profits from side A and from side R respectively. Note that the assumption of quantity competition on the advertising side seems reasonable if we admit that TV channels or printed media are constrained with respect to the available for advertisements time or space.

3 Stage game equilibrium and comparison

In order to obtain the equilibrium we solve the stage game backwards for each possible case, namely both-sides competition, semi-collusion, and full-collusion. Before doing this, it is useful to derive some more general expressions describing second-stage equilibrium values when there is no collusion on side R.

Assuming that the two firms compete on side R, let the superscript $h = N, SC$, denote the two possible cases, both sides competition and semi-collusion, respectively. Maximizing π_i *w.r.t.* p_i and solving for p_i yields the prices as functions of s_i:[12]

$$p_i(s_i, s_j) = \frac{2[1 - s_i(r_i(s_i, s_j) - \gamma)] - \beta[1 + s_j(r_j(s_i, s_j) + \gamma) + \beta(1 + \gamma s_i)]}{4 - \beta^2} \tag{4}$$

Since platforms are symmetric, and so are the advertisers, second stage and overall game equilibria are symmetric, so we can write $p_1 = p_2 = p^h$, and $s_1 = s_2 = s^h$. Using symmetry and (4) we obtain the price expression when firms compete in prices:

$$p^h(s^h; \beta, \gamma) = \frac{1 - \beta - s^h[1 - s^h(1 + \beta) - (1 - \beta)\gamma]}{2 - \beta} \tag{4'}$$

[11] The effects of the two parameters, θ and β, are qualitatively symmetric; therefore this assumption does not obscure any meaningful comparisons.

[12] A more detailed derivation of the equilibrium is provided in the Appendix A.

$h = N, SC$. The above expression holds for the cases of semi-collusion and both sides competition, and describes the second-stage equilibrium prices for given (symmetric) levels of advertising already determined at the first stage. Expression (4′) can be substituted backwards into the first-stage profit function only in case of semi-collusion, but can also be helpful in simplifying of both sides competition. Using (1) we can determine the (symmetric) second-stage equilibrium quantity for any given (symmetric) levels of advertising as $q^h = \frac{1+\gamma s^h - p^h}{1+\beta}$, $h = N, SC$, which, after substituting p^h from (4') becomes

$$q^h(s^h; \beta, \gamma) = \frac{1 + s^h[1 - s^h(1+\beta) + \gamma]}{(2-\beta)(1+\beta)} \tag{5}$$

Define profit from the advertising market as $\pi_A^h = q^h r^h s^h$, and total profit as $\pi^h = \pi_A^h + \pi_R^h = q^h(r^h s^h + p^h)$, $h = N, SC$ from (3). Using symmetry and expressions (2), (4′), and (5) we obtain an expression for a single firm's equilibrium overall profit in case of competition on the R market:

$$\pi^h(s^h; \beta, \gamma) = \frac{(1-\beta)[1 + s^h(1 - s^h(1+\beta) + \gamma)]^2}{(2-\beta)^2(1+\beta)} \tag{6}$$

while its equilibrium profit from the advertising side is

$$\pi_A^h(s^h; \beta, \gamma) = \frac{s^h[1 - s^h(1+\beta)][1 + s^h(1 - s^h(1+\beta) + \gamma)]}{(2-\beta)(1+\beta)}, h = N, SC.$$

The expression for total welfare is

$$w^h = w_A^h + w_R^h + 2\pi^h = (1 - r^h)q^h s^h + (1 + \gamma s^h - p^h)q^h + 2\pi^h$$

where

$$w_A^h(s^h; \beta, \gamma) = (1 - r^h)q^h s^h = \frac{{s^h}^2[1 + s^h(1 - s^h(1+\beta) + \gamma)]}{2-\beta}$$

and

$$w_R^h(s^h; \beta, \gamma) = (1 + \gamma s^h - p^h)q^h = \frac{[1 + s^h(1 - s^h(1+\beta) + \gamma)]^2}{(2-\beta)^2(1+\beta)} = \frac{\pi^h(s^h; \beta, \gamma)}{1-\beta}$$

represent the welfare of advertisers and consumers respectively. After substitution of (4'), (5) and (6) into the above, total welfare becomes:

$$w^h = \frac{[1 + s^h(1 - s^h(1+\beta) + \gamma)]\left[(3 - 2\beta)\left(1 + s^h(1+\gamma)\right) - {s^h}^2(1-\beta^2)\right]}{(2-\beta)^2(1+\beta)} \tag{7}$$

which holds for $h = N, SC$, *i.e.* when there is no collusion on the R side.

3.1 Competition on both sides

Consider now that firms compete on side A as well as on side R. Given $p_i(s_i, s_j)$ from (4) and using (3) the maximization of π_i *w.r.t.* s_i yields

$$s_i(s_j) = \frac{(1+\gamma)(2-\beta^2) - s_j\beta[2-\beta(1+\beta)]}{2(2-\beta^2)}$$

Solving the 2×2 system of equations gives the equilibrium level of advertising, when firms compete in both sides:

$$s^N(\gamma) = \frac{(1+\gamma)(2-\beta^2)}{(1+\beta)[4-\beta(2+\beta)]} \tag{8}$$

Observe that $0 \leq s^N \leq 1$ always holds. Substituting s^N into (2) yields

$$r^N(\gamma) = \frac{2(1-\beta) - \gamma(2-\beta^2)}{4-\beta(2+\beta)}$$

Note that $r^N > 0$ requires a further assumption to be made:

$$\gamma < \gamma_1 \equiv \frac{2(1-\beta)}{2-\beta^2}$$

Substituting s^N from (8) into (4′) gives $p^N = p^N(s^N; \gamma)$. A sufficient condition for $p^N \geq 0$ is that $\beta < .85$. If $\beta \geq .85$, there exist two critical values of γ, $\gamma_3 < \gamma_2 \leq 0$ such that, for $\gamma_3 < \gamma < \gamma_2 \leq 0$, $p^N < 0$ (see the Appendix A). For simplicity, we assume $\beta < .85$.[13]

The substitution of s^N in (6) yields the total competitive profits

$$\pi^N(\gamma) = \frac{(1-\beta)\psi^2}{(2-\beta)^2(1+\beta)^3[4-\beta(2+\beta)]^4} \tag{9}$$

where $\psi(\gamma) = 2(1-\beta)(2-\beta^2)\gamma(2+\gamma) + \beta^5 + 5\beta^4 + 2\beta^3 - 22\beta^2 - 4\beta + 20$ while the profits from the advertising side are

$$\pi_A^N(\gamma) = \frac{(2-\beta^2)(1+\gamma)[2(1-\gamma) - \beta(2-\beta\gamma)]\psi}{(2-\beta)(1+\beta)^3[4-\beta(2+\beta)]^4}$$

The welfare of the advertisers and consumers is given by $w_A^N = w_A^N(s^N; \beta, \gamma) = \frac{\psi[(2-\beta^2)(1+\gamma)]^2}{(2-\beta)(1+\beta)^3[4-\beta(2+\beta)]^4}$ and $w_R^N = w_R^N(s^N; \beta, \gamma) = \frac{\pi^N(\gamma)}{1-\beta}$ respectively. Finally, setting $s^h = s^N$ in (7) yields the total welfare in case of competition.

3.2 Semi-collusion on advertising

[13]The analysis holds even assuming $\beta \geq .85$, but in that case becomes more complicated since the non-negativity of the final product's price requires additional assumptions on the value of γ: for $\beta \geq .85$ our results hold as well, unless $\gamma_3 < \gamma < \gamma_2 \leq 0$.

Now we turn to the case where firms collude on advertising while competing for consumers. We distinguish two cases according to the base used on calculating the fines imposed upon convicted firms. In the first, fines are calculated as a percentage of a firm's total profits (two-sided fines- TSF). In the second, fines are based solely on the profits stemming from the colluding side, ignoring the effect that semi-collusion may have on the profitability of the other side (one-sided fines- OSF). As we show, each fine basis has distinct effects on both, stage profitability and the viability of the collusive agreement.

3.2.1 Two-sided fines (SC2)

Suppose that following a successful investigation, the AA imposes sanctions on prosecuted firms based on the sum of their illicit profits obtained from both sides. Since the case at hand involves collusion in only one market, at the beginning of each period, each cartelist expects the cartel to remain undetected with probability $1-a$, which also represents the probability that at the end of the period the cartelist will earn the collusive gain and have the opportunity to keep colluding for at least one more period. At the same time each cartel member expects to be investigated and condemned with probability a, in which case it will be forced to pay a fine equal to $f^{SC2}=2(\pi^{SC2}-\pi^{N})$. Since we assume that convicted firms are monitored and therefore forced to compete for ever after, the value of remaining loyal to the cartel is:

$$V^{SC2}=(1-a)(\pi^{SC2}+\delta V^{SC2})+a\left(\pi^{SC2}-f^{SC2}+\frac{\delta}{1-\delta}\pi^{N}(\gamma)\right)$$

where each firm's gain in the semi-collusion case is denoted by $\pi^{SC2}=\pi_A^{SC2}+\pi_R^{SC2}$ with π_A^{SC2} and π_R^{SC2} representing respectively the profits from sides A and R. Rearranging the above yields the following:

$$V^{SC2}(\gamma)=\frac{(1-2\alpha)\pi^{SC2}(\gamma)+\alpha\left(\frac{2-\delta}{1-\delta}\right)\pi^{N}(\gamma)}{1-\delta(1-a)} \tag{10}$$

Since firms still compete for consumers, the optimal price as function of the advertising quantity is again given by (4). As firms maximize the value of the cartel, which, in case of SC2, is equivalent to maximizing joint profits, the level of advertising in the SC2 case is obtained by using $s=s_i=s_j$. Maximizing joint profits with respect to s yields:

$$s^{SC2}(\gamma) = \frac{1+\gamma}{2(1+\beta)} \leq 1 \tag{11}$$

Note that the same value of s^{SC2} could have been obtained by maximizing (6) with respect to s^h. Therefore, s^{SC2} is the level of the advertising that maximizes the stage profits when firms compete on side R.

Substituting s^{SC2} into (4) gives

$$p^{SC2}(\gamma) = \frac{(3-2\beta)\gamma^2 + 2\gamma(1-\beta) + 3 - 4\beta^2}{4(1+\beta)(2-\beta)}$$

In the Appendix A it is shown that a sufficient condition for $p^{SC2} \geq 0$ is that $\beta <$ .864, which is fulfilled since we already assume $\beta < .85$.[14]

The total maximized profits of each firm are

$$\pi^{SC2}(\gamma) = \pi_A^{SC2}(\gamma) + \pi_R^{SC2}(\gamma) = \frac{(1-\beta)[4(1+\beta) + (1+\gamma)^2]^2}{16(1+\beta)^3(2-\beta)^2} \tag{12}$$

where the profits from the colluding side are given by

$$\pi_A^{SC2}(\gamma) = \pi_A^{SC2}(s^{SC2}; \beta, \gamma) = \frac{(1-\gamma^2)[4(1+\beta) + (1+\gamma)^2]}{16(1+\beta)^3(2-\beta)}$$

Advertisers' and consumers' welfare is respectively provided by the following expressions:

$$w_A^{SC2}(\gamma) = w_A^{SC2}(s^{SC2}; \beta, \gamma) = \frac{(1+\gamma)^2[4(1+\beta) + (1+\gamma)^2]}{16(1+\beta)^3(2-\beta)} \tag{13}$$

$$w_R^{SC2}(\gamma) = w_R^{SC2}(s^{SC2}; \beta, \gamma) = \frac{\pi^{SC2}(\gamma)}{1-\beta}$$

Total welfare in the SC2 case is obtained by adding to the sum of the two expressions in (13) the total profit of the two firms, *i.e.*, $w^{SC2}(\gamma) = w_A^{SC2} + w_R^{SC2} + 2\pi^{SC2}$. The expression is omitted since it is too complex to provide any intuition.

Observe that all the equilibrium values in SC2 are independent of competition policy parameters: neither the fine level, nor the probability of cartel detection affect the coordinated quantity of advertising. Hence, equilibrium profits and welfare are also invariant to the enforcement parameters.

3.2.2 One-sided fines (SC1)

[14] If $\beta \geq .864$ the price that consumers pay in case of SC2 is negative if $\gamma_5 < \gamma < \gamma_4$ where $\gamma_5 < 0$ and $\gamma_4 \geq 0$.

Instead of using total profits as fines base, an alternative approach is to base fines on the difference between collusive and competitive profits stemming only from the collusive side. In this case the fine is

$$f^{SC1} = \mu(\pi_A^{SC1} - \pi_A^N)$$

with $\mu \geq 1$ to represent the fine multiplier and π_A^{SC1} the collusive profits from the colluding side A under SC1.

When both firms collude each firm expects to earn the collusive gain and keep colluding for one more period, with probability $1 - a$. Otherwise, firms are detected and forced to pay f^{SC1}:

$$V^{SC1} = (1 - a)(\pi^{SC1} + \delta V^{SC1}) + a\left(\pi^{SC1} - f^{SC1} + \frac{\delta}{1 - \delta}\pi^N\right)$$

where π^{SC1} denote each firm's total collusive gain under one-sided fines. Rearranging yields

$$V^{SC1}(\gamma, \beta, a) = \frac{\pi^{SC1} - af^{SC1} + \alpha\dfrac{\delta}{1 - \delta}\pi^N}{1 - \delta(1 - a)} \tag{14}$$

Adopting this alternative fine structure allows us to make the following observations:

Proposition 1 For $\gamma \neq 0$, $\pi^{SC1} = \pi^{SC1}(\gamma, \beta, a, \mu)$. Also $\pi^{SC1}(\gamma, \beta, a, \mu) \leq \pi^{SC2}(\gamma, \beta)$ and $\pi^{SC1}(0, \beta) = \pi^{SC2}(0, \beta)$

Proof

See the Appendix A.■

From (10) and (14) it is obvious that contrary to TSF, OSF do affect the price-quantity decisions of the semi-colluding firms. This explains the fact that net profits from semi-collusion are lower under OSF. That the per-period equilibrium profits are lower under SC1 compared to SC2 does not imply directly that OSF are more effective to deter semi-collusion on advertising. The analysis regarding the sustainability of semi-collusion on advertising is postponed for the next section.

3.3 Full collusion (FC)

Consider that the two platforms decide to collude on both sides A and R. In such instance, if the cartel is investigated in one market, its illegal activity on the other market will also be revealed. The cartel remains undetected with probability $(1 -$

$a^f) = (1-a)^2$, or gets convicted with probability $a^f = 2a(1-a) + a^2 = a(2-a)$:[15]

$$V^{FC} = \left(1 - a^f\right)(\pi^{FC} + \delta V^{FC}) + a^f \left[\pi^{FC} - 2(\pi^{FC} - \pi^N) + \frac{\delta}{1-\delta}\pi^N\right]$$

We denote with $\pi^{FC} = \pi_A^{FC} + \pi_R^{FC}$ each firm's gain when they fix both advertising level and consumers' prices, with π_A^{FC} and π_R^{FC} to represent the profits from side A and R respectively. Rearranging the above gives the value of the collusion when firms collude on both sides:

$$V^{FC}(\gamma) = \frac{(1 - 2a^f)\pi^{FC}(\gamma) + a^f \left(\frac{2-\delta}{1-\delta}\right)\pi^N}{1 - \delta(1 - a^f)} \tag{15}$$

The equilibrium level of advertising when firms fully collude is equal to the level of advertising when firms semi-collude on side A (the detailed derivation of all the expressions below is provided in the Appendix A):

$$s^{FC}(\gamma) = s^{SC2}(\gamma) = \frac{1+\gamma}{2(1+\beta)}$$

The price that consumers pay when firms fully collude is:

$$p^{FC}(\gamma) = \frac{1}{2} + \frac{(\gamma+1)(3\gamma-1)}{8(1+\beta)}$$

Inserting s^{FC} and p^{FC} into q_i into (1) yields:

$$q^{FC}(\gamma) = \frac{4(1+\beta) + (\gamma+1)^2}{8(1+\beta)^2}$$

Total profits of each firm, consumers' and total welfare are given by the following expressions:

$$\pi^{FC}(\gamma) = \frac{[4(1+\beta) + (\gamma+1)^2]^2}{64(1+\beta)^3} \tag{16}$$

$$w_R^{FC}(\gamma) = \pi^{FC}(\gamma)$$

$$w^{FC}(\gamma) = \pi^{FC}(\gamma)\left[5 - \frac{8(1+\beta)}{4(1+\beta) + (\gamma+1)^2}\right]$$

3.4 Comparison

In this sub-section we compare the outcomes of competition and different types of collusion. We present how the enforcement's choice affects firms' profitability and

[15] In any case colluding firms receive the collusive gain, they pay twice the difference between the collusive and the competitive gain and then they compete forever.

welfare. Two-sided competition implies higher advertising level compared to SC2 or full collusion, namely $s^N - s^{SC2} = \frac{(2-\beta)\beta(1+\gamma)}{2(1+\beta)[4-\beta(2+\beta)]} \geq 0$. Recall that (6) is maximized at $s^h = s^{SC2}$. Therefore π^{SC2} is the maximum level of profits when firms compete on side R, regardless of whether firms compete or collude on side A. The following lemma compares the profitability and welfare of SC2, two-sided competition and full collusion. The expressions involved are given by (9), (12), (13) and (16), and a proof is contained in the Appendix A:

Lemma 1 For $\gamma \in (-1,1)$ and $\beta \in (0,1)$

i. $0 \leq \pi^N \leq \pi^{SC2} \leq \pi^{FC}$

ii. $0 \leq \frac{\partial \pi^{SC2}}{\partial \gamma} \leq \frac{\partial \pi^{FC}}{\partial \gamma}$

iii. $w^{FC} < w^{SC2} < w^N$ *and* $w_R^{FC} < w_R^N < w_R^{SC2}$

The first part of the lemma is as expected: collusion in both markets yields higher profits than collusion in only the A side. If $\gamma > 0$, the profit increase relative to both-sides competition originates from the A side. Since advertising increases the quality of the final product, the advertising market is complementary in terms of sales to the R market: higher sales on the A market imply a demand increase in the R market. When firms semi-collude the increase in total profits originates from the colluding side A if A is complementary to R. For sufficiently negative γ, the increase in total profits emanates from the competing side R: ad-averse consumers appreciate the decrease in the amount of advertising, a fact that increases the demand in side R.

The second part shows that in all cases profits increase with γ, *i.e.* as the two sides become more complementary (or less sales-rival). This effect is more pronounced in the case of FC since full coordination allows the platforms to better exploit increases in profit opportunities. The third part shows welfare rankings. While the three cases rank in terms of total welfare as expected—more competition, higher welfare—the ranking in terms of consumers' welfare contains a surprise: SC2 is welfare superior to full competition.[16] Despite that consumers and firms are better off when firms semi-collude, total welfare is lower under semi-collusion for every $\gamma \in (-1,1)$: the

[16] Note that Dewenter *et al.* (2011) find that semi-collusion may lead to higher welfare compared to two-sided competition when the size of the advertising market is large. Under our assumption ($\kappa = 1$) semi-collusion always reduces welfare.

decrease in advertisers' welfare offsets the increase in total profits and consumers' welfare.

4 Cartel Stability

In this section we examine the stability of full- and semi-collusion. Our aim is first to determine whether and when semi-collusion is a rational decision, and second to analyze the impact of an alternative fines basis on deterring cartel activity. Before proceeding, note that deviation-cum-reporting has the advantages and disadvantages of a simple deviation, plus the advantage of shielding the deviator from an eventual investigation by the AA and the corresponding punishment. Hence, defection is always rationally accompanied by reporting.

4.1 SC2 vs. FC

We start with the SC2 case where the platforms decide the level of advertising collusively and consumers' prices competitively, and in case of cartel conviction fines are imposed on the total profit of each participating firm. Note that the satisfaction of the participation constraint for SC2 requires the expected gain from colluding to exceed the competitive profits we have that $\pi^{SC2} - 2\alpha(\pi^{SC2} - \pi^N) \geq \pi^N$ which implies that $(\pi^{SC2} - \pi^N)(1 - 2\alpha) \geq 0$. The latter holds for all $a \leq .5$. Values of $a > .5$ deter semi-collusion, yet such values are generally considered as unrealistic.[17]

If a firm decides to defect from the agreed level s^{SC2} it expects to earn the one-period defecting profits, denoted by $\hat{\pi}^{SC2}$, and to continue competing forever after. Its value is therefore:

$$\hat{V}^{SC2} = \hat{\pi}^{SC2} + \frac{\delta}{1-\delta}\pi^N \tag{17}$$

Requiring the above to be no higher than the firm-value from remaining loyal as given in (10) yields the minimum value of the discount factor δ, call it δ^{SC2}, above which SC2 is sustainable:

$$\delta^{SC2}(\gamma) \equiv \frac{\hat{\pi}^{SC2} - \pi^{SC2} + 2a(\pi^{SC2} - \pi^N)}{(1-a)(\hat{\pi}^{SC2} - \pi^N)} \tag{18}$$

---

[17] Recall that for simplicity we have set the fine multiplier $\mu = 2$. In the general case where $\mu \geq 1$, the participation constraint holds if $1 - \alpha\mu > 0$ or $a \leq \frac{1}{\mu}$.

where π^{SC2} and π^{N} are given in (12) and (9) respectively. Note that $\delta^{SC2}(\gamma) < 1$ holds if $a < a^{SC2}(\gamma) = \frac{\pi^{SC2}-\pi^{N}}{\hat{\pi}^{SC2}+2\pi^{SC2}-3\pi^{N}}$, which implies that a sufficiently high probability of detection induces full compliance.[18]

In order to determine $\hat{\pi}^{SC2}$ we need first to obtain the optimal level of advertising in case of deviation. A deviator i maximizes π_i under the constraints that its rival sets $s_j = s^{SC2}$ and that consumer prices are given by (4). The maximization yields (see the Appendix A):

$$\hat{s}^{SC2}(\gamma) = \frac{(1+\gamma)\left[4+\beta\left(2-\beta(1+\beta)\right)\right]}{4(1+\beta)(2-\beta^2)} \tag{19}$$

Substitution of $s_i = \hat{s}^{SC2}$ and $s_j = s^{SC2}$ into (4) and (2) gives $\hat{p}^{SC2}$, $\hat{r}^{SC2}$and the price of the loyal competitor $\hat{p}_j^{SC2}$. Inserting the previous into (1) yields $\hat{q}^{SC2}$ and the deviation profits are equal to the following:

$$\hat{\pi}^{SC2}(\gamma) = \hat{q}^{SC2}(\hat{r}^{SC2}\hat{s}^{SC2} + \hat{p}^{SC2}) = \frac{\left(\psi_1\gamma^2 + 2\psi_1\gamma + \tilde{\psi}_1\right)^2}{256(1+\beta)^3(8-6\beta^2+\beta^4)^2(1-\beta)} \tag{20}$$

$\psi_1 = \beta^5 + \beta^4 + 4\beta^3 - 12\beta^2 - 8\beta + 16$

$\tilde{\psi}_1 = 17\beta^5 + 33\beta^4 - 44\beta^3 - 108\beta^2 + 24\beta + 80$

The following lemma describes how the sustainability of SC2 is affected by the consumers' taste for advertisements:

Lemma 2 *For every* $a \leq 1/2, \frac{\partial \delta^{SC2}}{\partial \gamma} \geq 0.$

Proof

See the Appendix A.■

The lemma above implies that when SC2 satisfies the participation constraint, SC2 is less likely to be sustainable as consumers become more ad-lovers. This happens because the advertising restriction resulting from collusion on side A leaves consumers less satisfied with the final product of the loyal firms, thus making

[18] It can be verified that the maximum a that allows collusion to be feasible ($\delta^{SC2} = 1$) is $a^{SC2}(-1) = .25$. For $.25 < a < .5$ while the participation constraint is satisfied semi-collusion on A is never viable: the incentive constraint is stricter than the participation constraint.

deviation more lucrative on side R as well.[19] Lemma 2 also entails that $\frac{\partial a^{SC2}}{\partial \gamma} \leq 0$: as consumers are more ad-lovers, the critical a below which $\delta^{SC2} \leq 1$ holds lowers.

Turning to the case of FC, recall that the necessary condition for participation in the fully collusive agreement requires $\pi^{FC} - 2a^f(\pi^{FC} - \pi^N) \geq \pi^N$, or $(\pi^{FC} - \pi^N)(1 - 2a^f) \geq 0$. The latter holds for all $a^f < .5$ or equivalently $a \leq .29$. Examining the stability condition for the case of full collusion requires deriving the respective incentive compatibility constraint, which again requires determining the profits from deviation. As firms choose the level of advertising and consumers' prices sequentially, deviation can occur either on side A or on side R but not on both.[20]

Consider first that a firm chooses to unilaterally defect on the agreed advertising level. This implies that consumers' prices are indeed the competitive ones, $p_i(s_i, s_j)$, given by (4). Given that the loyal partner sets $s_j = s^{FC}(\gamma) = \frac{1+\gamma}{2(1+\beta)}$, the deviator maximizes (3) *w.r.t.* s_i. The optimal defecting level of advertising is the same as in the SC2 case, provided by (19) while the respective defecting profits are given by (20).

Suppose now that firm i decides to offer the agreed advertising quantity at the first stage, and then to deviate from the agreement, charging a lower price to consumers. In this case, $\hat{s}^{FC}(\gamma) = s^{FC}$, $\hat{r}^{FC}(\gamma) = r^{FC}$ and $p_j = p^{FC}$ hold. The maximization of the stage profits provides the level of prices that the defecting firm selects to set:

$$\hat{p}^{FC}(\gamma) = \frac{\gamma^2(6-\beta) + 2\gamma(2-\beta) + \beta(3-4\beta) + 6}{16(1+\beta)}$$

Since firm j is expected to comply with the collusive price p^{FC}, using (1) the defecting level of q_i is:

$$\hat{q}^{FC}(\gamma) = \frac{(2-\beta)[4(1+\beta) + (\gamma+1)^2]}{16(1+\beta)^2(1-\beta)}$$

[19] Note that $s^N - s^{SC2}$ increases with γ while the rate of collusive to defecting profits $\left(\frac{\pi^{SC2}}{\hat{\pi}^{SC2}}\right)$ is decreasing in γ.

[20] Two stages imply that the outcome of the first stage is observed before the decision of the second stage is taken. Consequently, it is impossible to defect on both sides: a defection on the A-side must immediately end the cooperation. In other words, either there is cooperation on A and defection on R, or there is defection on A and competition on R.

Then, the defecting profits, in case of collusion in both sides with deviation to occur on the prices for consumers, are:

$$\hat{\pi}^{FC}(\gamma) = \hat{q}^{FC}(\hat{r}^{FC}\hat{s}^{FC} + \hat{p}^{FC}) = \frac{(2-\beta)^2[4(1+\beta)+(1+\gamma)^2]^2}{256(1+\beta)^3(1-\beta)} \qquad (21)$$

The following lemma comes from the direct comparison between (20) and (21):

Lemma 3 *Deviation from full collusion is more profitable on consumers' prices, i.e.* $\hat{\pi}^{FC} \geq \hat{\pi}^{SC2}$ *always holds for* $\gamma \in (-1,1)$.

The above is quite natural since deviation on the first stage and competing in the second is less profitable than earning the jointly maximized profits in the first stage and then unilaterally undercutting the agreed consumers' price at the second stage. Taking this into account, the binding incentive constraint is $V^{FC} \geq \hat{\pi}^{FC} + \frac{\delta}{1-\delta}\pi^N$, which,using (15) and (21) becomes:

$$\delta \geq \delta^{FC}(\gamma) \equiv \frac{\hat{\pi}^{FC} - \pi^{FC} + 2a^f(\pi^{FC} - \pi^N)}{(1-a^f)(\hat{\pi}^{FC} - \pi^N)} \qquad (22)$$

Note that $\delta^{FC}(\gamma) < 1$ holds if $a^f < \frac{\pi^{FC} - \pi^N}{\hat{\pi}^{FC} + 2\pi^{FC} - 3\pi^N}$ or equivalently if $a < a^{FC}(\gamma)$ where

$$a^{FC}(\gamma) = 1 - \frac{\hat{\pi}^{FC} + \pi^{FC} - 2\pi^N}{\sqrt{(\hat{\pi}^{FC} + 2\pi^{FC} - 3\pi^N)(\hat{\pi}^{FC} + \pi^{FC} - 2\pi^N)}}.$$

Lemma 4 *For every* $a^f \leq .5$, $\frac{\partial \delta^{FC}}{\partial \gamma} \leq 0$.

Proof

See the Appendix A.■

The lemma above implies that when the participation constraint is satisfied, two-sided collusion becomes more profitable and more likely to be sustainable as consumers' appreciation for advertisements increases.[21] Notice also that lemma 4 implies that the maximum level of a below which $\delta^{FC}(\gamma) < 1$ holds increases with γ, that is $\frac{\partial a^{FC}}{\partial \gamma} \geq 0$.

While the difficulty-to-sustain semi-collusion on advertising increases with γ, the opposite is true for full-collusion. As consumers' appreciation for advertisements increases, an ad-restrictive agreement lowers quality of the R-side product, which increases the importance of price-fixing in terms of profits. This also explains the

[21] It can be also verified numerically that the maximum a that renders $\delta^{FC}(\gamma_1) = 1$ is $a^{FC}(\gamma_1) \cong .155$. For $a > .155$ (or equivalently $a^f > .285$) two-sided collusion is never sustainable.

greater sensitivity of total profits *w.r.t.* γ when firms fully collude compared to when they semi-collude (see lemma 1). Due to the fact that fully-collusive profits respond sharply to increases of γ, the impact of collusive profits on the level that γ affects the sustainability of collusion is greater when firms collude on both sides. Hence, as profits always increase with the taste parameter, when full collusion is the case, sustaining cooperation is easier given that consumers become more ad-lovers.

Note that Lefouili and Pinho (2018) find that as the level of cross-group externalities increases, the gain from a deviation increases more than the severity of the punishment following a deviation, thus implying that collusion is less likely to occur when network externalities are stronger. In our setting for all γ, increases in γ increase the sustainability of full collusion. Our assumption that platforms select advertising quantities and consumers' prices sequentially mitigates further the impact that a more ad-loving attitude of consumers has on platforms' incentive to deviate: optimal deviation happens on consumers' prices a fact that renders defection even less tempting as consumers become more ad-lovers.

The following result concerns relative sustainability of semi- and full collusion. Recall from lemmata 2 and 4 that δ^{SC} and δ^{FC} are both monotonic and respectively increasing and decreasing functions of γ. Assuming that the two functions intersect for some $\gamma \in (-1, \gamma_1)$, define γ^* such as $\delta^{FC}(\gamma^*) = \delta^{SC}(\gamma^*)$. Also define $a^*(\gamma)$ the level of a where $\delta^{FC} = \delta^{SC}$ holds.

Table 1 shows that when consumers are indifferent towards advertisements, that is $\gamma = 0$, $\delta^{FC}(0) > \delta^{SC}(0)$ holds for all $a \geq .05$. Observe that for $a < a^*(0) < .05$, $\delta^{FC} = \delta^{SC}$ occurs for $\gamma = \gamma^* < 0$. Therefore, when $a < a^*(0)$, $\gamma^* < 0$ and $\delta^{FC} < \delta^{SC}$ holds for every $\gamma > 0$. Furthermore, $\delta^{FC}(\gamma_1) = \delta^{SC}(\gamma_1)$ holds for $a = a^*(\gamma_1)$ where $a^*(\gamma_1) < .08$. This implies that for $a > .08$ $\delta^{FC} > \delta^{SC}$is the case for all $\gamma < \gamma_1$.

β	.01	.25	.5	.75	.8
$a^*(0)$	.045	.046	.044	.03	.023
$a^*(\gamma_1)$	.077	.07	.061	.041	.033

Table1

Summarizing the above along with lemmata 2 to 4 we can conclude that whether full- collusion is more or less likely to be sustainable than semi-collusion depends on consumers' taste for advertisements:

Proposition 2 If $a > a^*(\gamma_1)$, then $\gamma^* > \gamma_1$, therefore for every $\gamma \in (-1, \gamma_1)$ $\delta^{FC} > \delta^{SC}$; if $a < a^*(\gamma_1)$, then $\gamma^* \in (-1, \gamma_1)$, hence $\delta^{FC} \geq (\leq) \delta^{SC}$ according to whether $\gamma \leq (\geq) \gamma^*$.

The result above implies that relative sustainability of semi- and -full collusion depends on consumers' attitude towards advertising. In markets where consumers are ad-averse, SC2 is more likely to be sustainable, whereas in markets populated by ad-lovers full-collusion may be easier to survive. Turning to the role of the probability of detection, recall that compared to semi-collusion, full collusion offers higher advantages, but is also more likely to be detected. For low values of a the higher profitability of full collusion dominates the risk of being prosecuted, rendering full collusion easier to be viable. If instead the probability of detection is *a priori* high, semi-collusion offers a so substantial reduction of the effective probability of detection that it pays-off to forego the additional profitability of full collusion.

The following graph (figure 1) depicts two pairs of $(\delta^{FC}(\gamma), \delta^{SC}(\gamma))$ lines, each pair traced for a different value of a. The thick and thin solid lines depict δ^{FC} and δ^{SC}, respectively as functions of γ, for $a = .05$; the dotted and dashed ones depict the same functions for $a = .1$. In all cases $\beta = .5$, which implies (see Table 1) that $a^*(0) = .044$ and $a^*(\gamma_1) = .061$. Since $a = .05$ lies between these two critical values, δ^{FC} and δ^{SC} intersect at some value of $\gamma = \gamma^*$ within the $(0, \gamma_1)$ interval. For values of $\gamma < \gamma^*$ there are values of the discount rate for which semi-collusion is stable while full collusion is not, and the opposite holds true for $\gamma > \gamma^*$. On the other hand, when $a = .1 > a^*(\gamma_1) = .061$, which implies that for every $\gamma \in (-1, \gamma_1)$, $\delta^{FC} > \delta^{SC}$, *i.e.* full collusion is less stable independently of γ.

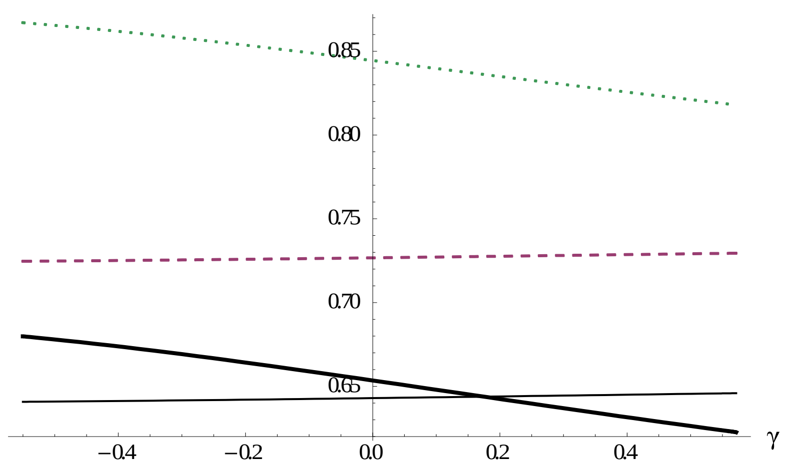

Figure 1: SC2 vs. FC

The question is when both forms of collusion are sustainable, which is the one that will be chosen? The following lemma deals with the issue.

Lemma 5 *For every* $\delta \leq (\geq)\tilde{\delta},\ V^{SC}(\gamma) \leq (\geq) V^{FC}(\gamma)$, *with*

$$\tilde{\delta}(\gamma, a) \equiv \frac{(1-2a^{f})\pi^{FC} - (1-2a)\pi^{SC2} + 2(a^{f}-a)\pi^{N}}{(1-a)\left[(1-2a^{f})\pi^{FC} - \left(1-a(3-2a)\right)\pi^{SC2} + a\pi^{N}\right]}$$

and $\frac{\partial\tilde{\delta}(\gamma,a)}{\partial\gamma} < 0$.

Proof

See the Appendix A.■

Restricting our attention only to cases where both SC2 and FC are sustainable, that is $\delta \in [max\{\delta^{FC}, \delta^{SC}\}, 1]$ when $a \in (0, min\{a^{FC}, a^{SC}\})$, we want to identify when firms prefer semi- to full collusion. Since $\delta^{FC}(0) < 1$, implies $\tilde{\delta}(0, a) > 1$, the negativity of $\partial\tilde{\delta}/\partial\gamma$ guarantees that for $\gamma < 0$, $\tilde{\delta} > 1$, hence if both forms of collusion are feasible, FC is always superior to SC2 in terms of firm value.[22]

However, since $\frac{\partial\tilde{\delta}}{\partial\gamma} < 0$, there may exist $\gamma \in (0, \gamma_1]$ where $\tilde{\delta}(\gamma) < 1$ and while firms have the possibility to choose they select to semi-collude when $\delta \in (\tilde{\delta}, 1]$. Notice that $\tilde{\delta} < 1$ may hold only when a is sufficiently large, namely above a critical value $\tilde{a}(\gamma, \beta)$ such that

$$\tilde{a} \equiv \frac{\pi^{N} - 5\pi^{SC2} + 4\pi^{FC} - \sqrt{(\pi^{N})^2 - 2\pi^{N}\pi^{SC2} + 9(\pi^{SC2})^2 - 16\pi^{FC}\pi^{SC2} + 8(\pi^{FC})^2}}{K}$$

where $K = \frac{\beta^2\left[4(1+\beta)+(1+\gamma)^2\right]^2}{16(1+\beta)^3(2-\beta)^2}$.

The following proposition summarizes the previous analysis:

Proposition 3 *Consider that* $a \in (0, min\{a^{FC}, a^{SC}\})$ *and that both SC2 and full collusion are sustainable, that is* $\delta \in [max\{\delta^{FC}, \delta^{SC}\}, 1]$. *When* $a < \tilde{a}$, *then for every* $\delta \in [max\{\delta^{FC}, \delta^{SC}\}, 1]$, $V^{SC} < V^{FC}$ *while for* $a \geq \tilde{a}$, $V^{SC} > (<) V^{FC}$ *if* $\delta \in (\tilde{\delta}, 1]$ $\left(\delta \in [0, \tilde{\delta})\right)$.

While the *per-period* gains are always higher under full collusion, SC2 may be more profitable in the long run, due to its lower expected fine. Greater *per period*

[22] Recall from footnote 21 that the maximum a that renders $\delta^{FC}(\gamma_1) = 1$ is $a^{FC} \cong .15$. Since $\tilde{\delta}(0, .15) \geq 1.048$, $\tilde{\delta}(0, a) > 1$ is the case for all $a < .15$.

gains affect any form of collusion in two ways: a) they increase the collusive value, and b) they increase the fine each firm will pay in case of cartel detection. When γ is positive, as γ increases, approaching its highest admissible value, the difference in *per-period* collusive gains between FC and SC2 is becoming larger, thus increasing the relative strength of both in FC compared to SC2. If in addition the probability of detection is also high, the negative effect of *per period* gains described in b) dominates the direct positive effect in a). In a market where consumers are sufficiently intense ad-lovers and the AA is sufficiently vigilant, semi-collusion is a superior form of long-run cooperation, despite its lower *per period* profits.

Table B.2 in the Appendix B contains some numerical simulations regarding the critical discount factors. From this table we can infer that when γ is close to its maximum value the necessary condition $\tilde{a} < min\{a^{FC}, a^{SC}\}$ holds when the parameter of differentiation is low. Otherwise, $\tilde{a}$ is greater than $min\{a^{FC}, a^{SC}\}$ and consequently there exist no values of a that satisfy both $\tilde{\delta}(\gamma, a) < 1$ and $max\{\delta^{FC}, \delta^{SC}\} < 1$. In the latter case firms always select to collude on both sides when this is feasible.

4.2 Semi-collusion on side A: one-sided fines (SC1) vs. SC2

In this subsection we compare the effectiveness of the benchmark TSF and OSF to deter semi-collusion on side A. Considering SC1, when each firm decides to unilaterally deviate, it expects to earn the respective gain $\hat{\pi}^{SC1}$ interrupting cooperation forever. Note that with $\hat{\pi}^{SC1}$ we denote the sum of profits of the deviating firm from side A and the respective profits from the other side:

$$\hat{V}^{SC1} = \hat{\pi}^{SC1} + \frac{\delta}{1-\delta}\pi^N \tag{23}$$

In order to render the analysis tractable, assume further that consumers are indifferent towards advertisements, that is $\gamma = 0$. A discussion regarding this assumption follows at the end of the section. Using (14) and (23) we solve $\hat{V}^{SC1} \leq V^{SC1}$ and we obtain the critical discount factor above which SC1 can be sustained:

$$\delta^{SC1} \equiv \frac{\hat{\pi}^{SC1} - \pi^{SC1} + a\mu(\pi_A^{SC1} - \pi_A^N)}{(1-a)(\hat{\pi}^{SC1} - \pi^N)} \tag{24}$$

The maximization of V^{SC1} yields that the level of advertising under SC1 is (for a detailed derivation see the Appendix A):

$$s^{SC1}(0) = s^{SC2}(0) = \frac{1}{2(1+\beta)}$$

given that $a \leq \bar{\alpha} \equiv \frac{5-\beta(1+4\beta)}{(2-\beta)(3+2\beta)\mu}$. Therefore, for $a \leq \bar{\alpha}$ the equilibrium values of SC1 are equal to those of SC2 for $\gamma = 0$ (see proposition 1): $\pi^{SC1}(0) = \pi^{SC2}(0)$, $\hat{\pi}^{SC1}(0) = \hat{\pi}^{SC2}(0)$, $\pi_A^{SC1}(0) = \pi_A^{SC2}(0)$. The following lemma states that for $a = \bar{\alpha}$ is enough to fully prevent collusion under OSF:

Lemma 6 *For $a \geq \bar{\alpha}$, $\delta^{SC1} \geq 1$ always holds.*

The above implies that there exists no incentive for the AA to unnecessarily spend resources by increasing a above $\bar{\alpha}$. For the rest of the analysis we assume $a \leq \bar{\alpha}$.

We proceed to the comparison of the effectiveness of one-sided to the effectiveness of two-sided fines to deter one-sided collusion. Using (18) and (24), $\delta^{SC2}(0) < \delta^{SC1}$, *i.e.* SC1 is less likely to be sustainable than SC2 if

$$2(\pi^{SC2}(0) - \pi^N) < \mu(\pi_A^{SC2}(0) - \pi_A^N)$$

which holds for $\mu > \tilde{\mu}$ where $\tilde{\mu} \equiv \frac{2(\pi^{SC2}(0)-\pi^N)}{\pi_A^{SC2}(0)-\pi_A^N}$.

The following proposition compares the deterrent effect of two-sided fines and one-sided fines:

Proposition 4 *Consider that a) firms collude on side A and compete on side R and b) consumers are indifferent towards advertising. Then, $\delta^{SC2}(0) < \delta^{SC1}$ holds for every $\mu > \tilde{\mu}(\beta)$, where $\tilde{\mu}(\beta) < 2$.*

An analytical proof of Proposition 4 is included in the Appendix A. Figure 2 depicts both critical discount factors, δ^{SC2} in the solid line and δ^{SC1} in the dashed line, while the fine multiplier μ is on the horizontal axis. Figure 2 illustrates that, unless μ is sufficiently lower than two, OSF improves cartel deterrence compared to TSF.

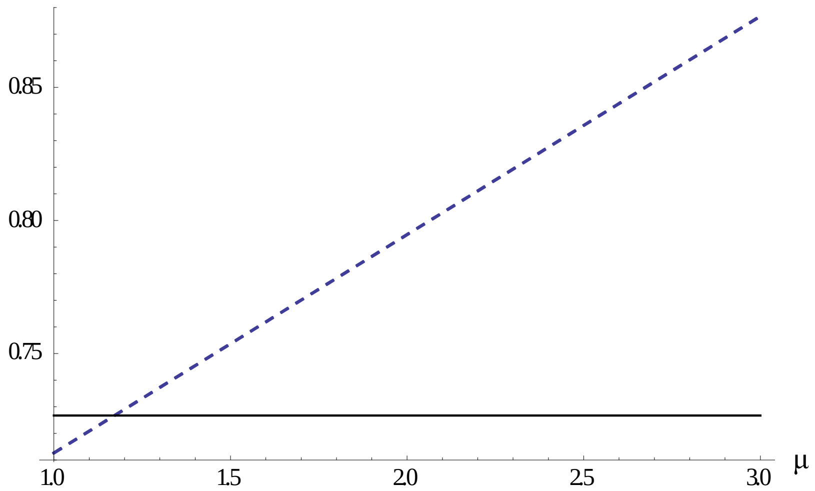

Figure 2: OSF vs. TSF ($\gamma = 0$)

The result above implies that if firms collude on side A and compete on side R and consumers are indifferent towards advertising, any fine multiplier equal to two (as it is often in practice) is more efficient cartel-deterrent when applied on only profits from the collusive side than on profits from both sides. When firms semi-collude on advertising, if consumers are ad-neutral the increase in total profits (compared to competition) originates solely from the colluding side. Since the expected fine is *ceteris paribus* higher when μ is applied only on the colluding side, using as base only the illegal gain from the colluding market and ignoring the effect from the competing side in order to punish cartel participants is, besides being legally reasonable, it is also more effective.

The above conclusion is affected by consumers' attitude towards advertising. If instead of being ad-neutral consumers are ad-lovers, the above conclusion holds *a fortiori*. Since the quantity restriction on the side A reduces consumers' willingness-to-pay on side R, semi-collusion increases profits relative to competition on A but *reduces profits* on R. Hence, choosing the A-side's illegal profits as base results in higher monetary fine.

Things get blurred when consumers exhibit ad-aversion. In this case the A-side quantity restriction enhances product quality and willingness-to-pay on the R-side, leading to higher profits from that side. Applying OSF in cases of ad-aversion allows therefore firms to shield part of their profit due to semi-collusion. Figure 3 below compares $\delta^{SC1}(\gamma)$ (dashed line) to $\delta^{SC2}(\gamma)$ (solid line). For positive values of γ both, the nominal value of the fine and the expectation to pay it are higher under δ^{SC1}. When consumers' ad-aversion is only moderate, δ^{SC1} still yields higher expected fines and is therefore more efficient. As ad-aversion increases, the reduction in the nominal value of the fine under OSF becomes very important, making TSF the more efficient than OSF. Note that in cases of pronounced ad-aversion, semi-collusion *reduces* profits on the A-side, pumping all its additional profitability from the R-side. In this case, OSF result in no (positive) fines, therefore they are not applicable.

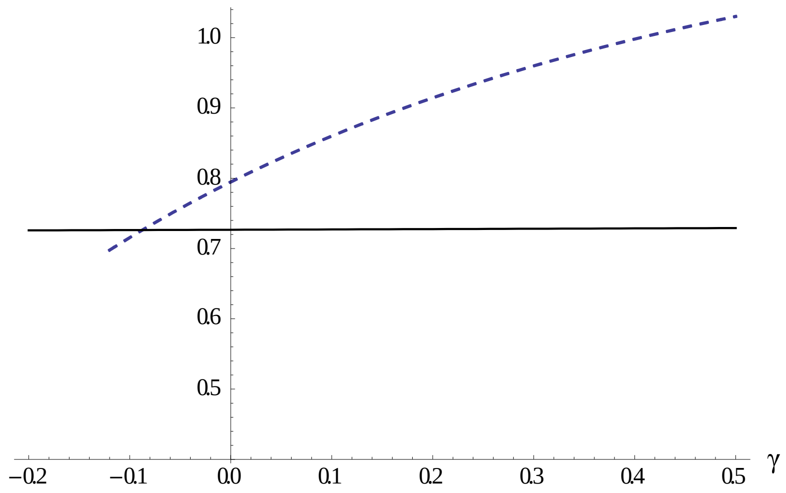

Figure 3: OSF vs. TSF

While in case of sufficiently strong ad-aversion TSF are more effective in deterring semi-collusion, the question is whether such deterrence is desirable, since semi-collusion increases consumers' welfare. In fact, semi-collusion acts as an agreement among firms to reduce a negative externality on the A-side, embodying higher quality into their final product. The problem is that consumers (and firms) gain in this case additional surplus at the expense of advertisers, therefore depends on the AA's ethical judgment as to whether advertisers' surplus is equally important to that of consumers. If the surplus of both categories of buyers is considered as equally important, semi-collusion reduces total buyer welfare and the AA must intervene. The same conclusion is reached when a social planner wants to protect total surplus, including profits, since, semi-collusion reduces total welfare.

5 Concluding remarks

This work analyzed some potential forms of collusion among advertising-selling platforms, such as press and broadcasting. Typically, such platforms sell advertising spots to interested agents, and newspapers, magazines, or hours of broadcasting to consumers. We analyzed the stability and profitability of different forms of collusion as well as their welfare effects. Our analysis is primarily positive in nature, trying to determine when semi- or full-collusion represents the more attractive form of cooperation for cartel participants. We find that the answer depends on consumers' attitude towards advertising. Another major point of this study pertains to the design of an efficient cartel-deterring policy, by determining when it is more efficient to use full or one-side profits as base for calculating the fines to be imposed on convicted cartel participants.

Due to their "platform" character, collusion in media markets can take three forms: a) collusion on both markets (full collusion), b) semi-collusion on the advertising market, and c) semi-collusion on the readers' market. Against cases b) and c) the AA may threaten infringers with fines that are based on their illegal profit from either the collusive side, or their operation on both sides. Our aim was to first examine the welfare impact of each type of collusion, and second to analyze the impact the choice of fines base may have on the stability of each type of collusive agreement. Due to time limitations, the scope of this work is limited on cases a) and b) above, leaving case c) for future research.

Bringing cartel-stability issues into the picture, we show that contrary to full-collusion, one-sided collusion becomes harder to sustain as consumers become less ad-avoiders. Due to its greater exposure to detection, full collusion may in some cases not be viable, even if it is more profitable in terms of expected profit. Thus, firms may be forced to compromise with a less profitable agreement. We show this to always be the case unless antitrust enforcement is sufficiently weak and consumers are sufficiently ad-lovers. Thus, our work rationalizes the decision of colluding only over advertising while competing for consumers, and determines the circumstances making this form of collusion preferable.

Finally, we examined whether applying one- or two-sided fines is a more effective deterrent of semi-collusion on the advertising side. Due to the complexity of the issue we limited the formal analysis to the case where consumers are indifferent about the amount of advertising contained in the product they purchase. Since the increase in total profits (compared to competition) originates solely from the colluding side, the expected fine is *ceteris paribus* higher when it is applied only on the colluding side. Therefore we show that, compared to two-sided fines, one-sided fines are *ceteris paribus* more effective in reducing the sustainability of semi-collusion.

While we believe that the points answered in this work constitute significant advancement in the literature of anti-trust in two-sided markets, many important questions have been left unanswered, requiring further research. As already mentioned, the issue of semi-collusion on the consumers' side has been neglected in this analysis. When can this type of collusion be more profitable and/or more sustainable than either full collusion, or semi-collusion on the advertising side? And, when it represents a real threat, what is the most effective choice of fines base against it? We are currently working on the issue, hoping to be able to present some analytical results in the near future.

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Appendix A

Two-sided competition

Substitution of (1) and (2) into (3) yields

$$\pi_i = \frac{\left[p_i + s_i r_i\left(s_i, s_j\right)\right]\left[1 - p_i + \gamma s_i - \beta\left(1 - p_j + s_j\gamma\right)\right]}{1-\beta^2}$$

Taking the derivative of the above *w.r.t.* $p_i$yields

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1 - 2p_i - s_i\left[r_i\left(s_i, s_j\right) - \gamma\right] - \beta\left(1 - p_j + s_j\gamma\right)}{1-\beta^2}$$

and setting $\frac{\partial \pi_i}{\partial p_i} = 0$ gives

$$p_i = \frac{1 - s_i\left[r_i\left(s_i, s_j\right) - \gamma\right] - \beta\left(1 - p_j - s_j\gamma\right)}{2}$$

for $i = 1,2$. Solving the 2×2 system gives $p_i\left(s_i, s_j\right)$ (eq. (4) in the text).

Using (4) and (3) we obtain

$$\pi_i\left(s_i, s_j\right) = \frac{{\xi_0}^2}{(1-\beta^2)(4-\beta^2)^2}$$

where $\xi_0 = 2(1 + \gamma s_i) - r_i s_i(2 - \beta^2) - \beta\left[1 + s_j\left(r_j + \gamma\right) + \beta(1 + \gamma s_i)\right]$

The derivative *w.r.t.* s_i yields

$$\frac{\partial \pi_i\left(s_i, s_j\right)}{\partial s_i} = -\frac{2\xi_1\xi_2}{(1-\beta^2)(4-\beta^2)^2}$$

where $\xi_1 = 2s_i(2 - \beta^2) + s_j\beta[2 - \beta(1 + \beta)] - (1 + \gamma)(2 - \beta^2)$ and

$$\xi_2 = 2 - s_i^2(2-\beta^2) - s_i\left[s_j\beta\left(2-\beta(1+\beta)\right) - (1+\gamma)(2-\beta^2)\right] - \beta\left[1+\beta+s_j\left(1-s_j+\gamma\right)\right]$$

Setting $\frac{\partial\pi_i(s_i,s_j)}{\partial s_i} = 0$ gives $s_i(s_j) = \frac{(1+\gamma)(2-\beta^2)-s_j\beta[2-\beta(1+\beta)]}{2(2-\beta^2)}$. Solving the 2×2 system of equations gives $s^N(\gamma)$.[23] Since $s^N \le 1 \Leftrightarrow \gamma \le \frac{2(1-\beta^2)+\beta(2-\beta^2)}{2-\beta^2}$ and $2(1-\beta^2)+\beta(2-\beta^2) \ge 2-\beta^2 \Leftrightarrow 2 \ge \beta(1+\beta)$ always holds, then $0 \le s^N \le 1$ is always the case.

Substituting s^N for s_i, s_j into (4) yields the equilibrium price:

$$p^N = \frac{(2-\beta^2)\gamma[(6(1-\beta)+\beta^3)\gamma + (4(1-\beta)+\beta^3)] - (1-\beta)^2[\beta^4 - 6(1+\beta)(2-\beta^2)]}{(2-\beta)(1+\beta)[4-\beta(2+\beta)]^2}$$

Then, substituting s^N into (2) and s^N, p^N into (1) yields $r^N = \frac{2(1-\beta)-\gamma(2-\beta^2)}{4-\beta(2+\beta)}$ and the competitive quantity $q^N = \frac{1+\gamma s^N - p^N}{1+\beta}$ respectively.

The denominator of p^N is positive and the numerator is a trionyme in γ with positive coefficient of the squared term, that is $(2-\beta^2)[6(1-\beta)+\beta^3] > 0$. The discriminant of the trionyme is

$$d_1 = (2-\beta^2)\left[(2-\beta^2)(4(1-\beta)+\beta^3)^2 - 4(6(1-\beta)+\beta^3)\left(\beta^4 - 6(1+\beta)(2-\beta^2)\right)\right]$$

which simplifies to:

$$d_1 = (2-\beta^2)[4-\beta(2+\beta)]^2[4\beta^5 - \beta^4 - 24\beta^3 + 22\beta^2 - 16(1-\beta)]$$

where $d_1 > 0$ if $\beta \ge 85$. Consequently, when $\beta < .85$, $p^N > 0$ holds. When $\beta \ge 85$ the trionyme has two roots:

$$\gamma_{2,3} = \frac{-(2-\beta^2)[4(1-\beta)+\beta^3] \pm \sqrt{d_1}}{2(2-\beta^2)[6(1-\beta)+\beta^3]} \le 0$$

and is positive except for values of γ between its roots, *i.e.*, $\gamma_3 < \gamma < \gamma_2$.

Semi-collusion on A under two-sided fines

Provided that $s = s_i = s_j$, maximizing

$$V(s;\gamma,a) = \frac{(1-2\alpha)\pi(s;\gamma,\beta) + \alpha\left(\frac{2-\delta}{1-\delta}\right)\pi^N(\gamma)}{1-\delta(1-a)}$$

is equivalent to maximizing $\pi(s;\gamma,\beta)$ where from (6)

$$\pi(s;\gamma,\beta) = \frac{(1-\beta)[1+s(1-s(1+\beta)+\gamma)]^2}{(1+\beta)(2-\beta)^2}$$

[23] The root $s_i(s_j)$ comes from setting $\xi_1 = 0$. The two roots coming from $\xi_2 = 0$ yield zero profits and therefore are ignored.

Since $a < \frac{1}{2}$ and neither the denominator nor the second part of the numerator in the expression of $V(s;\gamma,a)$ contains s, $V(s;\gamma,a)$ is a linear monotonic transformation of $\pi(s;\gamma,\beta)$, hence maximizing $V(s;\gamma,a)$ *w.r.t.* s is equivalent to maximizing the instantaneous profit $\pi(s;\gamma,\beta)$. Using (6), the derivative of $\pi(s;\gamma,\beta)$ *w.r.t.* s is

$$\frac{\partial\pi(s;\gamma,\beta)}{\partial s} = \frac{2(1-\beta)[1+s(1-s(1+\beta)+\gamma)][1-2s(1+\beta)+\gamma]}{(1+\beta)(2-\beta)^2} \qquad (A.1)$$

and

$$\frac{\partial^2\pi(s;\gamma,\beta)}{\partial s^2} = \frac{2(1-\beta)\left[1+2\beta-6s(1+\beta)\big(s(1+\beta)-(1+\gamma)\big)+\gamma(2+\gamma)\right]}{(1+\beta)(2-\beta)^2}$$

The numerator of (A.1) has three roots: One that sets the second term in square brackets equal to zero, and is $s = s^{SC2}(\gamma) = \frac{1+\gamma}{2(1+\beta)}$, and two roots of the first term in square brackets, which are $s = \frac{1+\gamma\pm\sqrt{5+4\beta+\gamma(2+\gamma)}}{2(1+\beta)}$. The second and third roots yield zero profits and therefore are ignored. Evaluating the second derivative at $s = s^{SC2}$, yields $\frac{\partial^2\pi(s;\gamma,\beta)}{\partial s^2}\Big|_{s=s^{SC2}} = \frac{-(1-\beta)[5+4\beta+\gamma(2+\gamma)]}{(1+\beta)(2-\beta)^2} < 0$.

Substituting s^{SC2} into (4) and (2) gives the respective equilibrium values of p_i and r_i:

$$p^{SC2}(\gamma) = \frac{(3-2\beta)\gamma^2+2\gamma(1-\beta)+3-4\beta^2}{4(1+\beta)(2-\beta)}$$

$$r^{SC2}(\gamma) = \frac{1-\gamma}{2}$$

Observe that the denominator of p^{SC2} is positive and the numerator is a trionyme in γ with positive coefficient of the squared term. The discriminant of the trionyme is $d_2 = -8\beta^3+13\beta^2-4(2-\beta)$ which can be shown to be positive only for $\beta >$ .864: Since $d_2'(\beta) = -24\beta^2+26\beta+4 > 0$, $d_2(\beta)$ is monotonically increasing in $\beta \in [0,1]$ and for $\beta \cong .864$ $d_2 = 0$. Hence, for $\beta < .864$, $p^{SC} > 0$ always holds while for $\beta > .864$ $p^{SC} < 0$ holds for $\gamma_5 < \gamma < \gamma_4$ where $\gamma_4 = \frac{\beta-1+\sqrt{d_2}}{3-2\beta}$ and $\gamma_5 = -\frac{1-\beta+\sqrt{d_2}}{3-2\beta} < 0$.

Inserting s^{SC2} and p^{SC2} into (1) (or equivalently $s^h = s^{SC2}$ in (5)) yields:

$$q^{SC2}(\gamma) = \frac{4(1+\beta)+(1+\gamma)^2}{4(1+\beta)^2(2-\beta)}$$

The total profits of each firm are by (6) for $s = s^h$ which after substitution of s^{SC2} yields (12).

Name $\gamma_1^s(\gamma_2^s)$ the level of γ above (below) which $\pi_A^{SC2} > \pi_A^N$ ($\pi_R^{SC2} > \pi_R^N$) holds. For every $\gamma < \gamma_1^s$ the increase on total profits due to SC2 comes solely from the non-colluding side. The following table depicts that both γ_1^s and γ_2^s are negative and close to zero.

β	.01	.25	.4	.5	.6	.75	.8
γ_1^s	-.003	-.07	-.108	-.135	-.164	-.214	-.234
γ_2^s	-.001	-.02	-.04	-.06	-.089	-.15	-.177

Table A.1

Optimal deviation from s^{SC2}

Substituting $s^{SC2} = \frac{1+\gamma}{2(1+\beta)}$ and $\hat{s}$ in (4) yields the price of the defecting and the loyal firm respectively:

$$\hat{p}^{SC2} = \frac{2\left[1 - \hat{s}\left(\hat{r}^{SC2} - \gamma\right)\right] - \beta\left[1 + s^{SC2}\left(1 - s^{SC2} - \beta\hat{s} + \gamma\right) + \beta(1 + \gamma\hat{s})\right]}{4 - \beta^2}$$

and

$$\hat{p}_j^D = \frac{2\left[1 - s^{SC2}\left(1 - s^{SC2} - \beta\hat{s} - \gamma\right)\right] - \beta\left[1 + \hat{s}\left(\hat{r}^{SC2} + \gamma\right) + \beta\left(1 + \gamma s^{SC2}\right)\right]}{4 - \beta^2}$$

Then, the maximization of $\hat{\pi}^{SC2}(\gamma) = \hat{q}^{SC2}(\hat{r}^{SC2}\hat{s} + \hat{p}^{SC2})$

with $\hat{q}^{SC2} = \frac{1-\beta-\left(\hat{p}^{SC2}-\beta\hat{p}_j^D\right)+\gamma\left(\hat{s}-\beta s^{SC2}\right)}{1-\beta^2}$ and $\hat{r}^{SC2} = 1 - \hat{s} - \beta s^{SC2}$ *w.r.t.* $\hat{s}$ gives

$$\hat{s} = \hat{s}^{SC2} = \frac{(1+\gamma)\left[4 + \beta\left(2 - \beta(1+\beta)\right)\right]}{4(1+\beta)(2-\beta^2)}$$

as the unique solution that yields positive profits.

Proof of proposition 1: semi-collusion on A under one-sided fines

Using (4'), (5) and (6) and provided that $s = s_1 = s_2$ and $\pi_A(s;\beta,\gamma) = p(s)q(s)s$ maximizing

$$V_1(s;\gamma,a) = \frac{\pi(s;\beta,\gamma) - a\mu\left(\pi_A(s;\beta,\gamma) - \pi_A^N(\gamma)\right) + \alpha\frac{\delta}{1-\delta}\pi^N(\gamma)}{1 - \delta(1-a)}$$

w.r.t. s yields:

$$\frac{\partial V_1(s;\gamma,a)}{\partial s}=\frac{\frac{\partial \pi(s;\beta,\gamma)}{\partial s}-a\mu\frac{\partial \pi_A(s;\beta,\gamma)}{\partial s}}{1-\delta(1-a)}=0\Leftrightarrow\frac{\partial \pi(s;\beta,\gamma)}{\partial s}-a\mu\frac{\partial \pi_A(s;\beta,\gamma)}{\partial s}=0$$

Hence, we expect that under SC1 the enforcement parameter a does affect the stage equilibrium outcome. Given (4) we get

$$\pi(s;\beta,\gamma)=\frac{(1-\beta)[1+s(1-s(1+\beta)+\gamma)]^2}{(1+\beta)(2-\beta)^2}$$

and

$$\pi_A(s;\beta,\gamma)=\frac{s[1-s(1+\beta)][1+s(1-s(1+\beta)+\gamma)]}{(1+\beta)(2-\beta)}$$

Denote $v_i(s;\beta,\gamma)=\pi(s;\beta,\gamma)-a\mu\big(\pi_A(s;\beta,\gamma)-\pi_A^N(\gamma)\big)$

The derivative of $v_i(s;\beta,\gamma)$ *w.r.t.* s gives

$$\frac{\partial v_i}{\partial s}=\frac{\partial \pi(s;\beta,\gamma)}{\partial s}-a\mu v(s;\beta,\gamma)$$

where $v(s;\beta,\gamma)=\frac{4s^3(1+\beta)^2-3s^2(1+\beta)(2+\gamma)-2s(\beta-\gamma)+1}{(1+\beta)(2-\beta)}$

and $\frac{\partial \pi(s;\beta,\gamma)}{\partial s}$ is given by (A.1) . Recall that $\frac{\partial \pi(s;\beta,\gamma)}{\partial s}=0$ yields $s=s^{SC2}(\gamma)=\frac{1+\gamma}{2(1+\beta)}$ while $v(s^{SC2};\beta,\gamma)=\frac{-\gamma\left[4(1+\beta)+(1+\gamma)^2\right]}{4(1+\beta)^2(2-\beta)}<0$ for $\gamma>0$. Hence, if s^{SC1} denotes the level of advertising under SC1, unless $\gamma=0$ when $s^{SC1}=s^{SC2}$, s^{SC1} is function of a. Since (6) is maximized for $s=s^{SC2}$, when $s^{SC1}\neq s^{SC2}$, that is when $\gamma\neq 0$, $\pi^{SC2}>\pi^{SC1}$ holds, where $\pi^{SC1}(\beta,\gamma,a,\mu)$ denotes the equilibrium profits under one-sided fines. If $\gamma=0$, $s^{SC1}=s^{SC2}$ and $\pi^{SC1}=\pi^{SC2}$.

Full collusion

Firms maximize their collusive value first *w.r.t* $s=s_i=s_j$ and then *w.r.t.* $p=p_i=p_j$. Maximizing $V(p,s;\gamma,a)=\frac{[1-2\alpha(2-a)]\pi(p,s;\gamma)+\left(\frac{2-\delta}{1-\delta}\right)\alpha(2-a)\pi^N}{1-\delta(1-a)^2}$ *w.r.t.* p is equivalent to maximizing the joint profits: $\frac{\partial V(p,s;\gamma,a)}{\partial p}=\frac{[1-2\alpha(2-a)]\frac{\partial \pi(p,s;\gamma)}{\partial p}}{1-\delta(1-a)^2}=0\Leftrightarrow\frac{\partial \pi(p,s;\gamma)}{\partial p}=0$. Hence, under full collusion the parameter a does not affect the stage equilibrium outcome. Given (4) we get

$$\pi(p,s;\gamma)=\frac{(1-p+s\gamma)\big[p+s\big(1-s(1+\beta)\big)\big]}{1+\beta}$$

and

$$\frac{\partial\pi(p,s;\gamma)}{\partial p}=\frac{1-2p-s[1-s(1+\beta)-\gamma]}{1+\beta}$$

Setting $\frac{\partial\pi(p,s;\gamma)}{\partial p}=0$ gives $p=p^F=\frac{1-s[1-s(1+\beta)-\gamma]}{2}$. Substituting $p=p^F$ gives

$$\pi(s;\gamma)=\frac{\left[1+s\left(1-s(1+\beta)\right)+\gamma\right]^2}{4(1+\beta)}$$

Taking the derivative of $\pi(s;\gamma)$ *w.r.t.* s and equating to zero yields the collusive level of advertising in case of full collusion:

$$\frac{\partial\pi(s;\gamma)}{\partial s}=\frac{\left[1+s\left(1-s(1+\beta)\right)+\gamma\right][1-2s(1+\beta)+\gamma]}{2(1+\beta)}$$

Setting $\frac{\partial\pi(s;\gamma)}{\partial s}=0$ yields $s_i=s^{FC}(\gamma)=\frac{\gamma+1}{2(1+\beta)}$ which yields maximum profits as $\left.\frac{\partial^2\pi(s;\gamma)}{\partial s^2}\right|_{s=s^{FC}}=-\frac{5+4\beta+\gamma(2+\gamma)}{4(1+\beta)}<0$. Substituting s^{FC} into p^F and r_i yields the respective values:

$$p^{FC}(\gamma)=\frac{1}{2}+\frac{(\gamma+1)(3\gamma-1)}{8(1+\beta)}$$

$$r^{FC}(\gamma)=\frac{1-\gamma}{2}$$

Inserting s^{FC} and p^{FC} into q_i yields:

$$q^{FC}(\gamma)=\frac{4(1+\beta)+(\gamma+1)^2}{8(1+\beta)^2}$$

Total profits of each firm are

$$\pi^{FC}(\gamma)=q^{FC}(p^{FC}+r^{FC}s^{FC})=\frac{[4(1+\beta)+(\gamma+1)^2]^2}{64(1+\beta)^3}$$

The welfare of the advertisers is given by

$$w_A^{FC}(\gamma)=(1-r^{FC})q^{FC}s^{FC}=\frac{(\gamma+1)^2[4(1+\beta)+(\gamma+1)^2]}{32(1+\beta)^3}$$

Consumers' welfare is provided by the following expression:

$$w_R^{FC}(\gamma)=(1+\gamma s^{FC}-p^{FC})q^{FC}=\frac{[4(1+\beta)+(\gamma+1)^2]^2}{64(1+\beta)^3}$$

Finally, the sum of consumers' welfare, advertisers' welfare and the profits of the two firms yields the total welfare in case of full collusion:

$$w^{FC}(\gamma)=\frac{[4(1+\beta)+(\gamma+1)^2][12(1+\beta)+5(\gamma+1)^2]}{64(1+\beta)^3}$$

Optimal deviation from full collusion

Since deviating from s^{FC} results in similar outcome to SC2 we proceed to the case of deviation from the agreed consumers' prices. Substituting $\hat{s}^{FC}(\gamma) = s^{FC}$, $\hat{r}^{FC}(\gamma) = r^{FC}$ and $p_j = p^{FC}$ in (1) yields:

$$\hat{q}^{FC} = \frac{4[2+\gamma(2+\gamma)] - 8p_i(1+\beta) - \beta[4\beta - (1-\gamma)(3+\gamma)]}{8(1-\beta)(1+\beta)^2}$$

Then substituting $r_i = r^{FC}$, $s_i = s^{FC}$ and $q_i = \hat{q}^{FC}$ into (3) yields

$$\hat{\pi}^{FC} = \frac{[1+4p_i(1+\beta) - \gamma^2][4(2+\gamma(2+\gamma) - 2p_i(1+\beta) - \beta^2) - \beta(1-\gamma)(3+\gamma)]}{32(1-\beta)(1+\beta)^2}$$

The derivative of the above *w.r.t.* p_i gives

$$\frac{\partial\hat{\pi}^{FC}}{\partial p_i} = \frac{\beta(1-\gamma)(3+\gamma) + 2[3+\gamma(2+3\gamma)] - 16p_i(1+\beta) - 4\beta^2}{8(1-\beta)(1+\beta)^2}$$

while $\frac{\partial^2\hat{\pi}^{FC}}{\partial {p_i}^2} = \frac{-2}{1-\beta^2} < 0$. Setting $\frac{\partial\hat{\pi}^{FC}}{\partial p_i} = 0$ yields $p_i = \hat{p}^{FC}(\gamma)$.

Proof of lemma 1

First, using (7) the total welfare in case of competition is

$$w^N = \frac{\psi\zeta_1}{(2-\beta)^2(1+\beta)^3[4-\beta(2+\beta)]^4}$$

$\zeta_1 = \gamma(2+\gamma)(\beta^3 + 2\beta^2 - 12\beta + 10)(2-\beta^2) + 68 - \beta(2\beta^5 + 8\beta^4 - 13\beta^3 - 54\beta^2 + 66\beta + 56)$. The welfare of consumers in case of two-sided competition is given by

$$w_R^N = w_R^N(s^N;\beta,\gamma) = \frac{\psi^2}{(2-\beta)^2(1+\beta)^3[4-\beta(2+\beta)]^4}$$

Recall that the welfare of consumers in case of SC2 is

$$w_R^{SC2}(\gamma) = w_R^{SC2}(s^{SC2};\beta,\gamma) = \frac{[4(1+\beta) + (1+\gamma)^2]^2}{16(1+\beta)^3(2-\beta)^2}$$

while the total welfare is given by the following:

$$w^{SC2}(\gamma) = \frac{[4(1+\beta) + (1+\gamma)^2]}{16(1+\beta)^3(2-\beta)^2}$$

Since in case of competition in side R, consumers' welfare is $w_R^h(s^h;\beta,\gamma) = \frac{\pi^h(s^h;\beta,\gamma)}{1-\beta}$ and $\pi^{SC2} > \pi^N$, consumers are better off under SC2 than under two-sided competition.

When firms fully collude, total profits, consumers' and total welfare are given by (15).

$$\pi^{FC} - \pi^{SC2} = \frac{\beta^2[4(1+\beta) + (\gamma+1)^2]}{64(1+\beta)^3(2-\beta)^2} \geq 0$$

$$w^N - w^{SC2} = \frac{\beta(1+\gamma)^2\zeta_2}{16(1+\beta)^3[4-\beta(2+\beta)]^4} \geq 0$$

$\zeta_2 = \gamma(2+\gamma)[4-\beta(4+\beta)][8-(4-\beta)\beta(1+\beta)](4-3\beta) + 640 +$
$\beta(4\beta^6 - 9\beta^5 - 129\beta^4 + 16\beta^3 + 732\beta^2 - 448\beta - 800) > 0.$

$$w^{SC2} - w^{FC} = \frac{\beta[4(1+\beta) + (1+\gamma)^2][\gamma(2+\gamma)(8-5\beta) + 12(2-\beta^2) - \beta]}{16(1+\beta)^3[4-\beta(2+\beta)]^4} \geq 0$$

$$w_R^N - w_R^{FC} = -\frac{(2-\beta)^2[4(1+\beta) + (1+\gamma)^2]^2[4-\beta(2+\beta)]^4 - 64\psi}{64(2-\beta)^2(1+\beta)^3[4-\beta(2+\beta)]^4} \geq 0$$

$$\frac{\partial\pi^{SC2}}{\partial\gamma} = \frac{(\gamma+1)(1-\beta)[4(1+\beta) + (\gamma+1)^2]}{4(1+\beta)^3(2-\beta)^2}$$

$$\frac{\partial\pi^{FC}}{\partial\gamma} = \frac{(2-\beta)^2}{4(1-\beta)}\frac{\partial\pi^{SC2}}{\partial\gamma}$$

$$\frac{\partial\pi^{FC}}{\partial\gamma} - \frac{\partial\pi^{SC2}}{\partial\gamma} = \frac{\partial\pi^{SC2}}{\partial\gamma}\left[\frac{(2-\beta)^2}{4(1-\beta)} - 1\right] \geq 0$$

Proof of lemma 2

Recall from (18) that $\delta^{SC2} = \frac{\hat{\pi}^{SC2} - \pi^{SC2} + 2a(\pi^{SC} - \pi^N)}{(1-a)(\hat{\pi}^{SC2} - \pi^N)}$. Taking the derivative of δ^{SC2} *w.r.t.* γ yields

$$\frac{\partial\delta^{SC}}{\partial\gamma} \propto \left[\frac{\partial\hat{\pi}^{SC2}}{\partial\gamma} - \frac{\partial\pi^{SC2}}{\partial\gamma} + 2a\left(\frac{\partial\pi^{SC}}{\partial\gamma} - \frac{\partial\pi^N}{\partial\gamma}\right)\right](1-a)(\hat{\pi}^{SC2} - \pi^N)$$
$$- (1-a)\left(\frac{\partial\hat{\pi}^{SC2}}{\partial\gamma} - \frac{\partial\pi^N}{\partial\gamma}\right)[\hat{\pi}^{SC2} - \pi^{SC2} + 2a(\pi^{SC2} - \pi^N)]$$

where $\pi^N = b_1\frac{\partial\pi^N}{\partial\gamma}$, $\pi^{SC2} = b_2\frac{\partial\pi^{SC2}}{\partial\gamma}$ and $\hat{\pi}^{SC2} = b_3\frac{\partial\hat{\pi}^{SC2}}{\partial\gamma}$ with

$$b_1 = \frac{1}{8}\left[2(1+\gamma) + \frac{(1+\beta)\left(4-\beta(2+\beta)\right)^2}{(1-\beta)(2-\beta^2)(1+\gamma)}\right]$$

$$b_2 = \frac{1}{4}\left[1+\gamma + \frac{4(1+\beta)}{1+\gamma}\right]$$

$$b_3 = \frac{1}{4}\left[1+\gamma + \frac{16(1+\beta)(1-\beta)(2+\beta)(2-\beta^2)}{(1+\gamma)(16-8\beta-12\beta^2+4\beta^3+\beta^4+\beta^5)}\right]$$

We can write the previous expression as

$$(1-a)(1-2a)\left[\frac{\partial\pi^{SC2}}{\partial\gamma}(b_2-b_3)\frac{\partial\hat{\pi}^{SC2}}{\partial\gamma}+\frac{\partial\pi^N}{\partial\gamma}\left(\frac{\partial\pi^{SC2}}{\partial\gamma}(b_1-b_2)-\frac{\partial\hat{\pi}^{SC2}}{\partial\gamma}(b_1-b_3)\right)\right]$$

After substitutions we conclude that:

$$\frac{\partial\delta^{SC2}}{\partial\gamma}\propto\frac{(1-a)(1-2a)(2-\beta)^2\beta^6(1+\gamma)^5}{256(1+\beta)^4(2+\beta)^2(2-\beta^2)^2[4-\beta(2+\beta)]^4}\varphi(\beta)\qquad(\mathrm{A.}2)$$

where $\varphi(\beta)=32-8\beta-36\beta^2+4\beta^3+9\beta^4+\beta^5$. The RHS of (A.2) has the sign of $\varphi(\beta)$, since the latter is multiplied by a positive expression. Note first that at $\beta=1$, $\varphi(1)=2>0$. Since $\beta=1$ is an upper bound on β, if it also holds that $\varphi'(\beta)<0$ then for every $\beta\in[0,1]$, $\varphi(\beta)>0$. With respect of the sign of the derivative, note that:

$$\varphi'(\beta)=-8-72\beta+12\beta^2+36\beta^3+5\beta^4$$

$$\varphi''(\beta)=-72+24\beta+108\beta^2+20\beta^3$$

$$\varphi'''(\beta)=24+216\beta+60\beta^2>0$$

Since $\varphi'''(\beta)>0$, the second derivative is always increasing, taking values between $\varphi''(0)=-72<0$ and $\varphi''(1)=80>0$. It follows that the first derivative is decreasing around 0 and increasing around 1. Since $\varphi'(0)=-80<0$ and $\varphi'(1)=-27<0$, $\forall\beta\in[0,1]$ the derivative $\varphi'(\beta)<0$, which guarantees that $\varphi(\beta)$ is positive for all the admissible values of β. This implies that for $<\frac{1}{2}$, the RHS of (A.2) is positive and therefore δ^{SC2} is increasing in γ.

Proof of lemma 4

Recall from (22) that $\delta^{FC}=\frac{\hat{\pi}^{FC}-\pi^{FC}+2a^f(\pi^{FC}-\pi^N)}{(1-a^f)(\hat{\pi}^{FC}-\pi^N)}$. Taking the derivative of δ^{FC} *w.r.t.* γ yields:

$$\frac{\partial\delta^{FC}}{\partial\gamma}\propto\left[\frac{\partial\hat{\pi}^{FC}}{\partial\gamma}-\frac{\partial\pi^{FC}}{\partial\gamma}+2a^f\left(\frac{\partial\pi^{FC}}{\partial\gamma}-\frac{\partial\pi^N}{\partial\gamma}\right)\right](1-a^f)(\hat{\pi}^{FC}-\pi^N)$$
$$-(1-a^f)\left(\frac{\partial\hat{\pi}^{FC}}{\partial\gamma}-\frac{\partial\pi^N}{\partial\gamma}\right)[\hat{\pi}^{FC}-\pi^{FC}+2a^f(\pi^{FC}-\pi^N)]$$

with $\pi^N=b_1\frac{\partial\pi^N}{\partial\gamma}$, $\pi^{FC}=b_2\frac{\partial\pi^{FC}}{\partial\gamma}$ and $\hat{\pi}^{FC}=b_2\frac{\partial\hat{\pi}^{FC}}{\partial\gamma}$ and

$$b_2=\frac{1}{4}\left[1+\gamma+\frac{4(1+\beta)}{1+\gamma}\right]$$

Rearranging the numerator of $\frac{\partial \delta^{FC}}{\partial \gamma}$ (the denominator is always positive) yields the expression below:

$$\frac{\partial \delta^{FC}}{\partial \gamma} \propto -(1-a^f)(1-2a^f)\frac{\partial \pi^N}{\partial \gamma}(b_1-b_2)\left(\frac{\hat{\pi}^{FC}}{b_2}-\frac{\pi^{FC}}{b_2}\right)$$

Since $\hat{\pi}^{FC} > \pi^{FC}$, $\frac{\partial \pi^N}{\partial \gamma} > 0$ and $b_1 - b_2 = \frac{(2-\beta)^2\beta^2(1+\beta)}{8(1-\beta)(2-\beta^2)(1+\gamma)} \geq 0$, the RHS of the above has the opposite sign of the expression $(1-2a^f) = [1-2a(2-a)] = 2a^2 - 4a + 1$, a trionyme which is negative (positive) between (outside) its two roots $a = \frac{2\pm\sqrt{2}}{2}$. The larger root clearly violates the assumption $a < 1$, but the smaller root is $\frac{2-\sqrt{2}}{2} \cong .29$. Hence, for $a < .29$ or equivalently $a^f \leq .5$ the trionyme is positive and $\frac{\partial \delta^{FC}}{\partial \gamma} < 0$.

Proof of lemma 5

The first part of the lemma, namely that $V^{SC}(\gamma) > V^{FC}(\gamma) \Leftrightarrow \delta > \tilde{\delta}(\gamma, a)$, is obtained using (10) and (15). For the derivative note that:

$$\begin{aligned}\frac{\partial \tilde{\delta}}{\partial \gamma} \propto \left[\frac{\partial \pi^{FC}}{\partial \gamma}(1-2a^f) - \frac{\partial \pi^{SC2}}{\partial \gamma}(1-2a) + 2a(1-a)\frac{\partial \pi^N}{\partial \gamma}\right](1-a)\big[(1-2a^f)\pi^{FC} \\ - \big(1-a(3-2a)\big)\pi^{SC2} + a\pi^N\big] \\ - (1-a)\left[(1-2a^f)\frac{\partial \pi^{FC}}{\partial \gamma} - \big(1-a(3-2a)\big)\frac{\partial \pi^{SC2}}{\partial \gamma} \right. \\ \left. + a\frac{\partial \pi^N}{\partial \gamma}\right][(1-2a^f)\pi^{FC} - \pi^{SC2}(1-2a) + 2\alpha(1-a)\pi^N]\end{aligned}$$

with $\pi^N = b_1\frac{\partial \pi^N}{\partial \gamma}$, $\pi^{FC} = b_2\frac{\partial \pi^{FC}}{\partial \gamma}$ and $\pi^{SC2} = b_2\frac{\partial \pi^{SC2}}{\partial \gamma}$ and b_1, b_2, are defined in the proof of lemma 2.

Rearranging the above expression yields:

$$\frac{\partial \tilde{\delta}}{\partial \gamma} \propto a(1-a)(1-2a)(1-2a^f)\frac{\partial \pi^N}{\partial \gamma}(b_1-b_2)\left(\frac{\partial \pi^{SC2}}{\partial \gamma}-\frac{\partial \pi^{FC}}{\partial \gamma}\right)$$

Since $\frac{\partial \pi^{SC2}}{\partial \gamma} - \frac{\partial \pi^{FC}}{\partial \gamma} < 0$ (lemma 1) and $b_1 - b_2 = \frac{\beta^2(2-\beta)^2(1+\beta)}{8(1-\beta)(1+\gamma)(2-\beta^2)} \geq 0$, then $\frac{\partial \tilde{\delta}}{\partial \gamma} < 0$.

Semi-collusion on A under one-sided fines when consumers are indifferent towards advertisements

Setting $\gamma = 0$ in (6) yields $\pi(s;\beta,0) = \frac{(1-\beta)[1+s(1-s(1+\beta))]^2}{(2-\beta)^2(1+\beta)}$ and $\pi_A(s;\beta,0) = \frac{s[1-s(1+\beta)][1+s(1-s(1+\beta))]}{(2-\beta)(1+\beta)}$. Then we define $v_i(s;\beta,0) = \pi(s;\beta,0) - a\mu(\pi_A(s;\beta,0) - \pi_A^N)$ and taking its derivative for s yields that $\frac{\partial v_i}{\partial s} = \frac{\partial \pi(s;\beta,0)}{\partial s} - a\mu\frac{\partial \pi_A(s;\beta,0)}{\partial s}$ equals the expression below:

$$\frac{[1-2s(1+\beta)]\left[2(1-\beta)\left(1+s\big(1-s(1+\beta)\big)\right) - a\mu(2-\beta)\left(1+2s\big(1-s(1+\beta)\big)\right)\right]}{(2-\beta)^2(1+\beta)}$$

Setting $\frac{\partial v_i}{\partial s} = 0$ gives $s = s^{SC2}(0) = \frac{1}{2(1+\beta)}$. Also

$$\left.\frac{\partial^2 v_i}{\partial s^2}\right|_{s^{SC2}(0)} = \frac{a\mu(2-\beta)(3+2\beta)+\beta(1+4\beta)-5}{(2-\beta)^2(1+\beta)} < 0$$

for $a \le \bar{\alpha} \equiv \frac{5-\beta(1+4\beta)}{(2-\beta)(3+2\beta)\mu}$.[24]

The table below shows that for different levels of the differentiation parameter β, $\bar{\bar{\alpha}} \le \bar{\alpha}$ is always the case where $\bar{\bar{\alpha}}$ denotes the level of a below which $\delta^{SC1} < 1$ holds when $s = s^{SC1}$:

β	.1	.25	.5	.75	.8
$\bar{\alpha}$	.4	.37	.29	.177	.15
$\bar{\bar{\alpha}}$	.22	.2	.175	.12	.105

Table A.2

Proof of proposition 4

Using (18) and (24), $\delta^{SC2}(0) < \delta^{SC1}$ yields $> \tilde{\mu} \equiv \frac{2(\pi^{SC2}(0)-\pi^N)}{\pi_A^{SC2}(0)-\pi_A^N}$. If $\tilde{\mu}$ is lower than the fine multiplier under two-sided fines, then for equal fine multipliers ($\mu = 2$) one-sided fines deter semi-collusion more frequently: $\tilde{\mu} < 2 \Leftrightarrow \pi^{SC2}(0) - \pi^N < \pi_A^{SC2}(0) - \pi_A^N$. Therefore, if the difference in profits from the colluding side exceeds

[24] Note that $\frac{\partial v_i}{\partial s} = 0$ yields two more values of s associated to maxima of v_i when $a > \bar{\alpha}$: $s_{2,3}^{SC1} = \frac{1-\beta-a\mu(2-\beta)\pm\sqrt{[1-\beta-a\mu(2-\beta)][5-\beta-4\beta^2-a\mu(3+2\beta)(2-\beta)]}}{2(1+\beta)[1-\beta-a\mu(2-\beta)]}$ with $\left.\frac{\partial^2 v_i}{\partial s^2}\right|_{s_{2,3}^{SC1}} = \frac{2[5-\beta(1+4\beta)-a\mu(2-\beta)(3+2\beta)]}{(2-\beta)^2(1+\beta)} < 0$ for $a > \bar{\alpha}$. Since $a = \bar{\alpha}$ is enough to fully prevent semi-collusion on A and for the sake of simplicity the case where $a > \bar{\alpha}$ is ignored.

the difference on total profits, then $\delta^{SC1} > \delta^{SC2}$ holds at least for $\mu = 2$. Taking into account that $\pi^{SC2} - \pi^N > 0$ and $\pi^{SC2} - \pi^N = \pi_A^{SC2} - \pi_A^N + \pi_R^{SC2} - \pi_R^N$, if $\pi_R^{SC2} - \pi_R^N < 0$ then $\pi^{SC2} - \pi^N < \pi_A^{SC2} - \pi_A^N$ always holds. Since

$$\pi_R^N - \pi_R^{SC2} = \frac{\beta^2[32(1+\beta) - 4\beta^2(3+17\beta) + \beta^4 + \beta^5(20+4\beta)]}{16(1+\beta)^3[4-\beta(2+\beta)]^4} > 0$$

it follows that $\pi^{SC2} - \pi^N < \pi_A^{SC2} - \pi_A^N$ and therefore $\delta^{SC1} > \delta^{SC2}$ holds for $\mu \geq \tilde{\mu}$ where $\tilde{\mu} < 2$.

Appendix B

Table B.1: Semi-collusion (SC1 *vs.* SC2) ($\mu = 2$)

β	δ^{SC1}	$\delta^{SC2}(0)$
.25	.62	.608
.5	.675	.643
.75	.77	.705

$\alpha = .05$

β	δ^{SC1}	$\delta^{SC2}(0)$
.25	.73	.694
.5	.8	.726
.75	.93	.785

$\alpha = .1$

β	δ^{SC1}	$\delta^{SC2}(0)$
.25	.85	.79
.5	.93	.82
.75	>1	.875

$\alpha = .15$

β	δ^{SC1}	$\delta^{SC2}(0)$
.25	.98	.898
.5	>1	.926

$\alpha = .2$

Table B.2: FC *vs.* SC2

β	δ^{FC}	δ^{SC2}	$\tilde{\delta}$
.1	.6	.59	1.03
.25	.615	.608	1.037
.5	.653	.643	1.042

$\alpha = .05 \ \ \gamma = 0$

β	δ^{FC}	δ^{SC2}	$\tilde{\delta}$
.25	.65	.608	1.048
.5	.678	.64	1.049
.75	.754	.7	1.05

$\alpha = .05 \ \ \gamma = -.5$

β	δ^{FC}	δ^{SC2}	$\tilde{\delta}$
.1	.533	.59	1.001
.25	.563	.609	1.016
.5	.622	.645	1.032
.75	.726	.71	1.042

$\alpha = .05 \ \gamma = \gamma_1$

β	δ^{FC}	δ^{SC2}	$\tilde{\delta}$
.1	.79	.677	1.06
.25	.815	.694	1.072
.5	.84	.726	1.086
.75	.92	.785	1.096

$\alpha = .1 \ \gamma = 0$

β	δ^{FC}	δ^{SC2}	$\tilde{\delta}$
.25	.84	.693	1.1
.5	.86	.724	1.104
.75	.93	.779	1.106

$\alpha = .1 \ \gamma = -.5$

β	δ^{FC}	δ^{SC2}	$\tilde{\delta}$
.1	.741	.677	.976
.25	.767	.695	1.016
.5	.818	.73	1.06
.75	.907	.79	1.08

$\alpha = .1 \ \gamma = \gamma_1$

β	δ^{FC}	δ^{SC2}	$\tilde{\delta}$
.1	1.032	.775	1.071
.25	1.043	.79	1.097
.5	1.07	.82	1.126
.75	1.129	.875	1.146

$\alpha = .15 \ \gamma = 0$

β	δ^{FC}	δ^{SC2}	$\tilde{\delta}$
.25	1.068	.79	1.155
.5	1.087	.818	1.163
.75	1.14	.87	1.168

$\alpha = .15 \ \gamma = -.5$

β	δ^{FC}	δ^{SC2}	$\tilde{\delta}$
.1	1.01	.775	.997
.25	1.026	.79	1.043
.5	1.056	.821	1.092
.75	1.12	.88	1.127

$\alpha = .15\ \ \gamma = \gamma_1$

Table B.3: OSF vs. TSF

β	s^{SC1}	π^{SC1}	$\hat{s}^{SC1}$	$\hat{\pi}^{SC1}$	δ^{SC1}	δ^{SC2}
.5	.293	.19	.397	.195	.651	.642
.75	.247	.113	.405	.124	.758	.703

$\alpha = .05\ \ \gamma = -.1$

β	s^{SC1}	π^{SC1}	$\hat{s}^{SC1}$	$\hat{\pi}^{SC1}$	δ^{SC1}	δ^{SC2}
.1	.505	.3684	.525	.3686	.7	.59
.25	.445	.302	.501	.304	.664	.608
.5	.372	.214	.483	.22	.694	.643
.75	.324	.125	.491	.138	.786	.706

$\alpha = .05\ \ \gamma = .1$

β	s^{SC1}	π^{SC1}	$\hat{s}^{SC1}$	$\hat{\pi}^{SC1}$	δ^{SC1}	δ^{SC2}
.1	.581	.416	.568	.4161	.8	.59
.25	.513	.337	.57	.34	.71	.608
.5	.431	.235	.548	.242	.717	.644
.75	.381	.136	.556	.151	.8	.709

$\alpha = .05\ \ \gamma = .25$

β	s^{SC1}	π^{SC1}	$\hat{s}^{SC1}$	$\hat{\pi}^{SC1}$	δ^{SC1}	δ^{SC2}
.25	.581	.38	.636	.381	.746	.609
.5	.49	.26	.612	.268	.734	.645

$\alpha = .05\ \ \gamma = .4$

β	s^{SC1}	π^{SC1}	$\hat{s}^{SC1}$	$\hat{\pi}^{SC1}$	δ^{SC1}	δ^{SC2}
.5	.557	.294	.686	.304	.75	.645
.75	.42	.144	.598	.16	.807	.71

$\alpha = .05\ \ \gamma = \gamma_1$

β	s^{SC1}	π^{SC1}	$\hat{s}^{SC1}$	$\hat{\pi}^{SC1}$	δ^{SC1}	δ^{SC2}
.1	.512	.368	.525	.368	.908	.677
.25	.452	.302	.5	.303	.81	.69
.5	.382	.213	.481	.218	.83	.72
.75	.346	.125	.487	.134	.955	.786

$\alpha = .1\ \ \gamma = .1$

β	s^{SC1}	π^{SC1}	$\hat{s}^{SC1}$	$\hat{\pi}^{SC1}$	δ^{SC1}	δ^{SC2}
.25	.531	.336	.567	.337	.912	.694
.5	.454	.234	.543	.238	.88	.727
.75	.435	.134	.546	.140	.985	.788

$\alpha = .1\ \ \gamma = .25$

β	s^{SC1}	π^{SC1}	$\hat{s}^{SC1}$	$\hat{\pi}^{SC1}$	δ^{SC1}	δ^{SC2}
.25	.61	.377	.633	.378	.99	.694
.5	.525	.258	.606	.262	.922	.728
.6	.507	.215	.603	.219	.938	.747

$\alpha = .1\ \ \gamma = .4$

β	s^{SC1}	π^{SC1}	$\hat{s}^{SC1}$	$\hat{\pi}^{SC1}$	δ^{SC1}	δ^{SC2}
.5	.607	.29	.677	.293	.955	.73
.75	.49	.14	.585	.145	1	.79

$\alpha = .1\ \ \gamma = \gamma_1$