

The predictive power of regime-switching models for stock market returns



**Interdepartmental Programme of Postgraduate Studies in
Economics (Masters in Economics)**

Thesis

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Abstract

Nowadays, nonlinearity plays an important role in predicting economic and financial variables. The objective of this paper is to investigate the out-of-sample forecasting performance of a regime-switching model for three stock markets. We examine whether the nonlinear regime-switching model improves predictions compared to linear models. We first evaluate all the models by computing forecast error criteria, we then check for statistical significance and in the end we implement a robustness test. Eventually, the regime-switching model outperforms only the random walk and has equal predictive power to the autoregressive model.

1. Introduction

The behavior of stock market returns is a subject of study that concerns a wide range of people. A large number of researchers consider stock market indices. Also, traders are interested in the stock exchange. Many have tried to estimate and predict stock markets. Some are occupied with the returns and others with the volatility or the direction of the markets. Many researchers use macroeconomic and financial variables to generate stock return predictions. Both linear and nonlinear models are applied. Nevertheless, the economic environment is volatile and many factors affect the stock markets. Hence, it is difficult to find an appropriate model to make accurate predictions.

Since the economic situation is not stable, the economic conditions can change abruptly. We want to catch the nonlinearities and that is the reason we introduce a regime-switching model. More specifically, in order to analyze and forecast stock market returns we employ a two-state Markov regime-switching model introduced by Hamilton (1989). We choose this model because it moves through time between two regimes. We focus our survey on three large and important stock markets, Canada, the United Kingdom (UK) and the United States (US).

We initially estimate the model using the full sample and we examine if it identifies the regimes consistently. Moreover, we cut the sample in two parts. The first part is used for estimation and the second is used for forecasting reasons. We proceed with one-step-ahead recursive forecasts. We also consider an AR(1) model and a random walk and we follow the same method. We evaluate the results according to the mean squared error and the mean absolute error criteria. We keep the regime-switching model as benchmark and contrast it with the two linear models. Regime-switching model performs little better than the other models. In order to evaluate if the difference of the measures is statistically significant we apply the test of Diebold and Mariano (1995). Lastly, we perform a robustness check to our out-of-sample results. We break the sample in two alternative points and we re-calculate the error criteria.

The paper is structured as follows. Section 2 outlines the literature survey. Section 3 reviews the data. In Section 4 we present the regime-switching model, the autoregressive model and the random walk. Section 5 contains the full sample estimation results of the regime-switching model. In Section 6 we conduct out-of-sample forecasts and compare the

performance of the regime-switching model with the two linear models. Finally, in Section 7 we conclude our research.

2. Literature Review

A great number of researchers have tried to forecast stock market returns with different models. Some of them use linear and others nonlinear models. We consider studies with nonlinear models and especially those with threshold and regime-switching models.

McMillan (2001) examines whether there is a nonlinear relationship between stock market returns and financial and macroeconomic variables, and whether this nonlinearity can improve the forecasting power of returns. He employs the S&P 500 monthly index from January 1970 to March 2000. For the financial and macroeconomic variables he takes monthly data of the 3-month Treasury bill (T-bill), the 12-month T-bill, unemployment, industrial production, consumer price index and money supply M1. Using model-free nonparametric methods, a nonlinear relationship is observed but only between stock market returns and interest rates. Nonparametric plots indicate some evidence of threshold effect between returns and interest rates. A STAR-type model, termed Smooth Transition Threshold Autoregressive-Exogenous (STARX), is estimated. He shows that the nonlinear models outperform the linear ones, despite the fact that the forecasting gain is marginal.

A similar research is carried out for the UK stock market by McMillan (2003). A nonlinear relationship exists between UK stock market returns and both macroeconomic and financial variables. Smooth transition (STR) threshold models are assessed. There is evidence that exponential smooth transition (ESTR) threshold model is the best predictor for the UK stock market compared to the linear model and to the logistic smooth transition (LSTR) threshold model. This result confirms the fact that investors do have different behavior between large and small returns.

Moreover, Humpe and Macmillan (2014) apply the LSTR model to the stock markets of the US and Japan. Macroeconomic variables are used to predict the stock returns. The results vary between the two stock markets and between regimes. This may happen because of the different economic situation in these two markets. Pereiro and González-Rozada (2015) propose a self-exciting threshold autoregressive model, SETAR, and compare it with linear

autoregressive (LAR) models for emerging and developed stock markets. They conclude that the SETAR model forecasts more accurately than linear models in the long-term.

McMillan and Wohar (2010) study an asymmetric ESTR model for the G7 stock markets and compare it with a symmetric ESTR and a linear model. The dividend-price ratio is used to predict stock returns. According to various forecast tests, the asymmetric ESTR model, by combining both asymmetry and nonlinearity, provides the most accurate predictions. Also, it ensures that traders receive the highest returns. Sarantis (2001) is occupied with modeling and forecasting the stock price growth rates of the G7 countries. The smooth transition autoregressive (STAR) models are considered. Linearity is rejected and cyclical behavior is detected for these markets. STAR models seem to add value in forecasting. Guidolin et al. (2009) investigate the forecasting power of a variety of linear and non-linear models for stock and bond markets with macroeconomic aggregates in the G7 countries. Nonlinearity appears to improve predictability. However, no model is found to perform best in all cases as the performance of each model is determined by the forecasting horizon, the country and the market examined.

Schaller and Van Norden (1997) focus on whether there is switching behavior in stock market returns and whether stock market returns can be predicted by economic variables. The data they use is the CRSP value-weighted monthly stock market returns for the period of January 1929 to December 1989. They construct and use excess returns. They show that switching behavior exists in the US stock market. This evidence is robust to three different specifications, switching in means, switching in variances, switching in both means and variances. They also employ a multivariate specification for the Markov switching model and find out that the past price-dividend ratio has marginal forecasting power for stock market returns after allowing for state-dependent switching. The returns react asymmetrically to the price-dividend ratio. Last but not least, they introduce one more innovation. They explore if the transition probabilities are affected by economic variables and specifically by the price dividend ratio. Again we notice an asymmetric reaction to the past price-dividend ratio.

Hess (2003) deals with competing regime-switching models for the Swiss stock market index. Unfortunately, after examination, a good estimated model does not guarantee a consistent performance out-of-sample. Consequently, further investigation needs to be done to find a model that has a good fit and forecasts accurately.

Zhu and Zhu (2013) study the excess returns of the US stock market. Fifteen economic variables are included to predict the excess stock returns. A new regime-switching combination process is introduced and it comprises uncertainty in three dimensions. This combination approach has consistent forecasting power. Also, they identify two regimes that are correlated with the business cycle. The low variance regime is related to economic growth, while the high variance regime is connected with economic decline. Excess returns are more predictable when there is recession in the economy and less predictable during economic increase.

Marcucci (2005) compares the ability of standard GARCH models with that of Markov regime-switching GARCH (MRS-GARCH) models to describe and predict the US stock market volatility at various horizons. Another study that examines volatility forecasting in the US stock market with Markov regime-switching GARCH models is this of Chang (2009). A substantial difference with Marcucci (2005) is that Chang (2009) takes into account macroeconomic variables to predict stock return dynamics. Furthermore, Hamilton and Susmel (1994) and Chen and Lin (2000) survey volatility forecasting with switching ARCH (SWARCH) models for the US and Taiwan stock market respectively. SWARCH models offer better results in forecasting than GARCH models. Liu et al. (2012) also deal with volatility of stock market returns in the US by employing a Markov-switching model.

Chen et al. (2003) investigate the dynamics of six advanced stock markets (France, Germany, United Kingdom, Japan, Switzerland and Canada). They use daily data and prove that both stock returns and volatility are asymmetrically affected by past prices of the US stock market. Particularly, by occupying a double-threshold GARCH model they show that when the news from the US is bad, the six markets are also influenced in a negative way. Contrary to this, good news from the US increases the returns of the six markets, but the impact now is clearly less than this of bad news.

Some studies examine and predict the stock market trend. Leung et al. (2000) take advantage of the existing relationship between stock returns and macroeconomic variables to predict the movements of stock markets. They compare classification models which forecast the direction of stock market indices depending on probabilities with level estimation models that predict the value of the returns. Classification models do a better job than level estimation models in predicting the sign of stock markets and bring higher profits to traders. Also, Zhang

and Zhang (2009) recommend a Markov chain model in order to predict the stock market trend in China.

Finally, we just cite some studies that forecast stock market returns using neural networks (NN). These are Qi and Maddala (1999), Qi (1999) and Pérez-Rodríguez et al. (2005).

As we have already emphasized, the economic situation is not steady. Literature above reports that stock markets are often described by cyclical behavior and asymmetry. Also, regime-switching, threshold and generally nonlinear models forecast more accurate than linear models. Nonlinearity is necessary for better results. We focus our study on a nonlinear model, specifically a regime-switching model.

3. Dataset

3.1 Data

We investigate the following three significant stock markets: the S&P/TSX Composite Index (Canada), the FTSE 100 Index (UK) and the S&P 500 Index (US). The closing prices for these three indices are obtained from *Yahoo Finance*. We have weekly data that cover the period from January 3, 2000 to November 16, 2015, implying a total of 829 observations for S&P/TSX and FTSE and 828 observations for S&P 500. We divide the sample into two parts, in-sample and out-of-sample. We take the first 2/3 of data for the in-sample estimation and the remaining 1/3 for the out-of-sample forecast evaluation. The in-sample data spans the period of January 3, 2000 to August 2, 2010 (553 for S&P/TSX and FTSE and 552 observations for S&P500). The out-of-sample starts from August 9, 2010 to November 16, 2015 (276 observations). Afterwards, we generate the returns, y_t which are the logarithmic first differences of the values of the indices, x_t . Therefore, we have $y_t = \log(x_t) - \log(x_{t-1})$.

3.2 Descriptive statistics

In this section, we depict some features of the data and their distribution. Table 1 presents the descriptive statistics for the three stock market returns.

Descriptive statistics			
	S&P/TSX	FTSE	S&P500
Mean	0,0006	-6.84E-05	0,0004
Median	0,0031	0,0019	0,0017
Maximum	0,1282	0,1258	0,1136
Minimum	-0,1754	-0,2363	-0,2008
Std. Dev.	0,0249	0,0250	0,0254
Skewness	-0,9443	-1,0831	-0,8182
Kurtosis	9,9712	14,8240	9,8838
Jarque-Bera	1799,7	4985,2	1725,2
Probability	0,0000	0,0000	0,0000

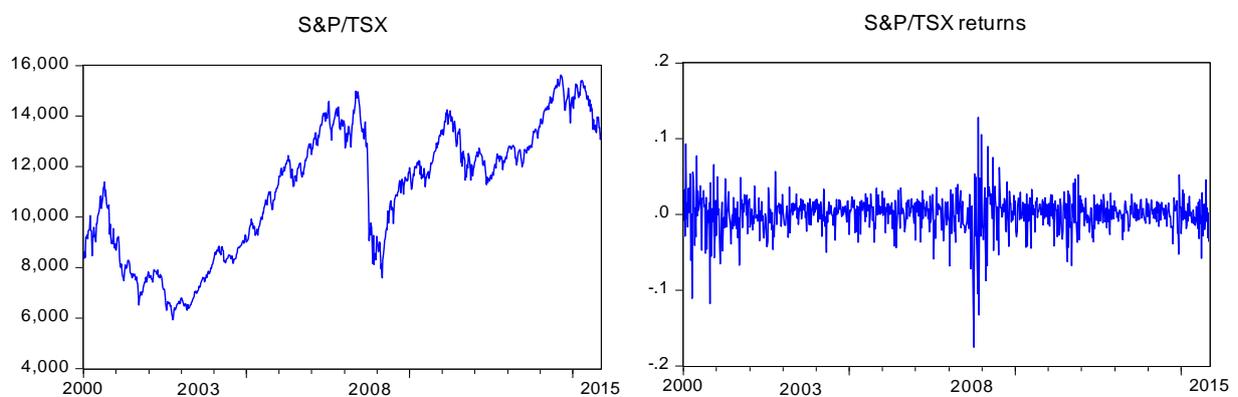
Table 1

We observe that S&P/TSX and S&P500 have on average positive returns and FTSE has on average negative returns. The standard deviation is almost the same for all the indices and is relatively small, nearly to 2,5%. Hence, the markets do not seem to have many ups-and-downs. The skewness for all the variables is smaller than zero. So, the information is not symmetrically distributed and we have left skewed distributions. The kurtosis is greater than three for all stock markets and we have leptokurtic distributions. Furthermore, the Jarque-Bera statistic is too big and the probability is zero for all stock markets. Therefore, we reject the null hypothesis that the series follow a normal distribution.

3.3 Graphs

The graphs of the stock markets give us some information about their behavior. In Figure 1, we can see the graphs of both levels and returns.

Graphs of levels and returns



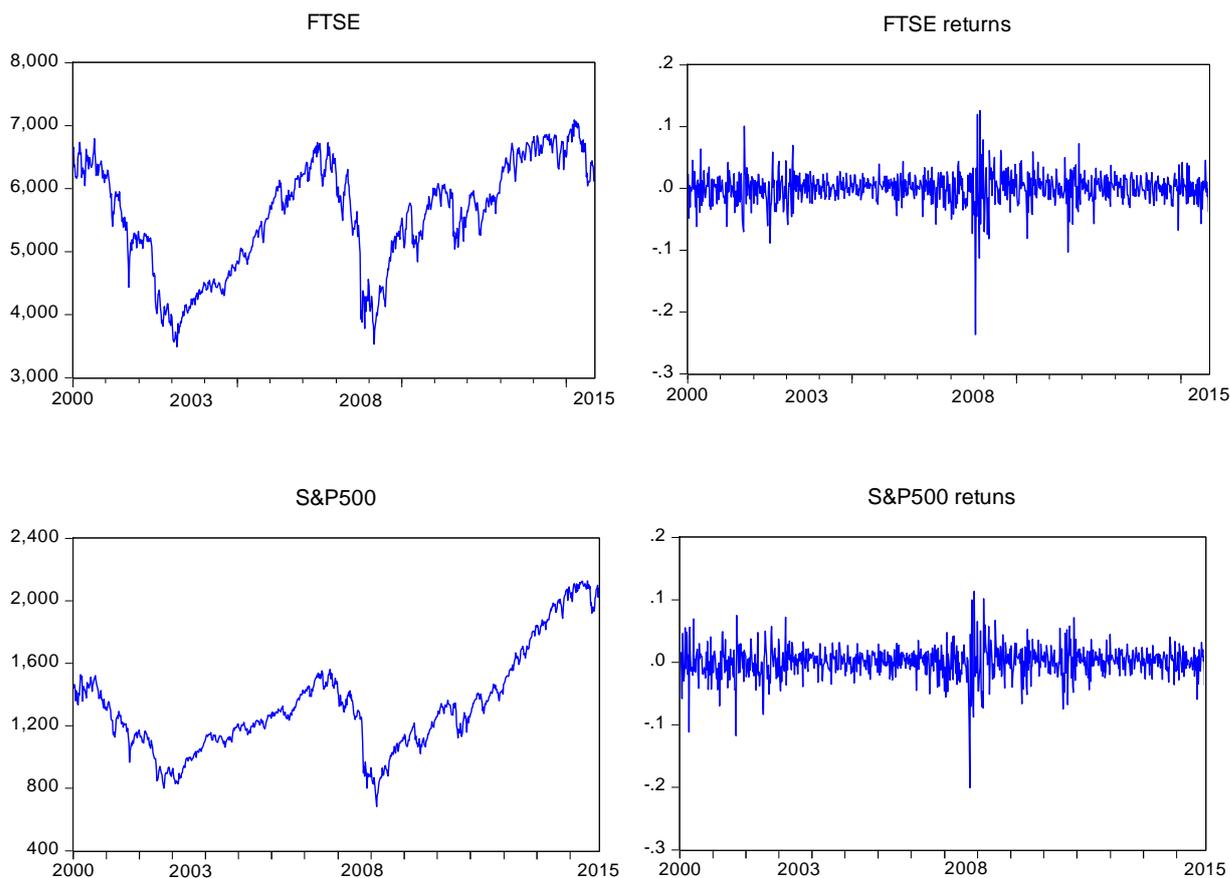


Figure 1

No index seems to have trend. Moreover, their movement is in general smooth. Only in 2008 we observe steep recession at the stock markets. This happens probably due to the global financial crisis.

3.4 Unit root test

We now test the stationarity of the variables. The tests we use for this reason are the Augmented Dickey-Fuller (ADF) test and the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test. We test both the natural logarithms of the levels (logs) and the returns. The results are given in Table 2:

	Unit root tests			
	ADF t-statistic		KPSS LM-stat	
	Logs	Returns	Logs	Returns
S&P/TSX	-1,657	-9,084***	2,379***	0,058
FTSE	-1,824	-8,217***	0,969***	0,122
S&P500	-0,637	-17,850***	1,404***	0,253

Note: *** denotes rejection of null hypothesis at 1% significance level.

Table 2

For the ADF test, we include the intercept and we take into account the Akaike information criterion (AIC) to select the lag length. The null hypothesis for the ADF test is that the variable we examine has a unit root. First, we focus on the logarithms of the series. Since the t-statistic is greater than the critical values for all variables, we cannot reject the null hypothesis. Thus, the variables have a unit root and they are not stationary. Now we take the returns. We see that we reject the null hypothesis at 1% level of significance as the t-statistic is smaller than the critical values for all variables and we have stationarity. For the KPSS test, we include only an intercept, we choose the default (Bartlett kernel) spectral estimation method and the Newey-West Bandwidth. The null hypothesis for the KPSS test differs from that of the ADF test and the null hypothesis now is that the variable is stationary. The LM-stat for the logarithms is greater than the critical values and we reject the null hypothesis at 1% significance level, so the variables have a unit root. As for the returns, the LM-stat is smaller than the critical values and we have stationarity. So, both tests reach the same inference. All our series are I(1).

4. The models

4.1 Regime-switching model

We want to estimate a model in order to predict the stock market returns. The model we apply is a first-order Markov regime-switching model with two states. This model was firstly introduced by Hamilton (1989). The model is nonlinear and moves through time from one regime to the other abruptly based on a probability law. The probability of being in a regime depends on the previous state and that is the reason our model is of order one. In addition, this probability is constant and is described by a Markov chain. The model allows for different specification in the conditional mean of each regime. Also, it allows for different variances and so we have regime heteroskedasticity. The model we use has the following form:

$$y_t = \begin{cases} c_0 + c_1 y_{t-1} + u_t, u_t \sim iidN(0, \sigma^2) \\ c'_0 + c'_1 y_{t-1} + u_t, u_t \sim iidN(0, \sigma'^2) \end{cases},$$

where y_t is the stock market return at time t , y_{t-1} is the stock market return at time $t-1$, c_0, c_1 are the coefficients in the first regime and c'_0, c'_1 are the coefficients in the second regime. So, we have an AR(1) model that changes states.

4.2 Autoregressive model and random walk

We introduce two more models in order to evaluate the regime-switching model. The models we consider in our investigation are the autoregressive model of order 1, AR(1) model:

$$y_t = c_0 + c_1 y_{t-1} + v_t$$

and the random walk without drift:

$$y_t = y_{t-1} + \varepsilon_t.$$

Both are linear contrary to the regime-switching model that is nonlinear.

5. Full sample estimation results

5.1 Estimation of the model

We estimate the regime-switching model for our three stock market indices taking into account the whole sample. In Table 3 we see the results of the regression.

Full sample estimation			
	S&P/TSX	FTSE	S&P500
c_0	-0,0057 (0,0031)*	-0,0086 (0,0042)**	-0,0050 (0,0032)
c_0'	0,0024 (0,0007)***	0,0017 (0,0008)**	0,0024 (0,0008)***
c_1	-0,1853 (0,0722)**	-0,1342 (0,0879)	-0,0494 (0,0893)
c_1'	-0,0334 (0,0455)	-0,0661 (0,0444)	-0,1299 (0,0452)***
$\log(\sigma)$	-3,1913 (0,0659)***	-3,1121 (0,0813)***	-3,1800 (0,0789)***
$\log(\sigma')$	-4,1793 (0,0465)***	-4,0792 (0,0448)***	-4,1085 (0,0444)***

Note: Sigma is the standard deviation. c_0 , c_1 and $\log(\sigma)$ represent regime 1. c_0' , c_1' and $\log(\sigma')$ represent regime 2. The numbers in parentheses are the standard errors. ***, ** and * denote rejection of the null hypothesis at 1%, 5% and 10% significance level respectively. The null hypothesis claims that a coefficient is zero and its rejection means that the coefficients are statistically significant.

Table 3

Looking at the standard deviation, we notice that the high volatility regime for all three stock indices is the first one, while the low volatility regime is the second one. According to the coefficients c_0 and c_0' , we see that for all three stock markets high volatility is associated with negative returns while low volatility is associated with positive returns.

5.2 Transition probabilities

As we have already mentioned, the probabilities that the process moves from one regime to the other are constant and characterized by a Markov chain. These probabilities are called constant Markov transition probabilities and we can see them in Table 4:

Markov transition probabilities		
	1→2	2→1
S&P/TSX	0,068	0,024
FTSE	0,119	0,028
S&P500	0,057	0,020

Table 4

We notice that the transition probabilities from regime 1 to regime 2 and vice versa are really small and as a consequence, the probabilities of staying at the same regime are large. Therefore, when the model is at one state, the probability of changing state is very low.

Also, looking at the expected regime durations in Table 5 we can confirm that indeed the model remains for long periods at each regime and does not have the tendency to change regime frequently.

Expected durations		
	Regime 1	Regime 2
S&P/TSX	14,807	41,882
FTSE	8,377	35,774
S&P500	17,410	50,965

Table 5

Furthermore, all three indices spend more time at the low volatility state. Low volatility indicates less risk.

5.3 Regime classification measure

According to Ang and Bekaert (2002), we can calculate a measure in order to assess the quality of regime classification. This measure is called Regime Classification Measure (RCM) and the formula for a model with two regimes is the following:

$$RCM = 400 \times \frac{1}{T} \sum_{t=1}^T p_t (1 - p_t),$$

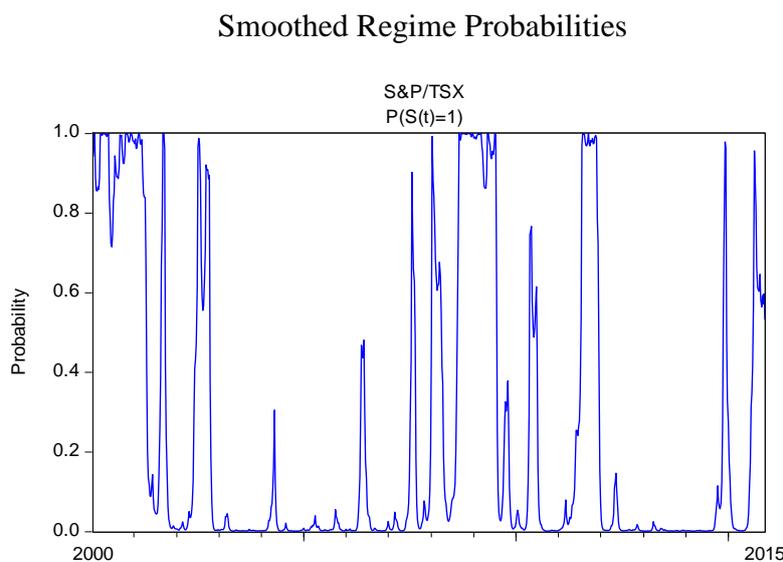
where p_t are the smoothed regime probabilities and T is their total number. When the regime-switching model cannot successfully separate the regimes, then we have weak regime inference. If p_t is close to 1 or 0, the regime-switching model is ideal and it classifies regimes abruptly. The fixed term in the form is used to keep the RCM statistic between 0 and 100. Low RCM implies good regime classification. So, when RCM is 0 we have perfect regime classification. On the other hand, a value of 100 denotes that we cannot observe any information about the regimes. Now, in our analysis we find the following values for the RCM statistic (Table 6):

Regime classification measure	
	RCM
S&P/TSX	19,640
FTSE	21,332
S&P500	19,982

Table 6

The RCM statistic is relatively low for all the indices. Therefore, we can conclude that the regime classification for the model in all three cases is good enough, but not ideal. The regime-switching model of S&P/TSX produces the sharpest regime classification, followed by S&P500 and FTSE.

Figure 2 shows the smoothed probabilities of the high volatility regime for the three stock market returns. The smoothed regime probabilities indicate in which regime the market is throughout the period under scrutiny.



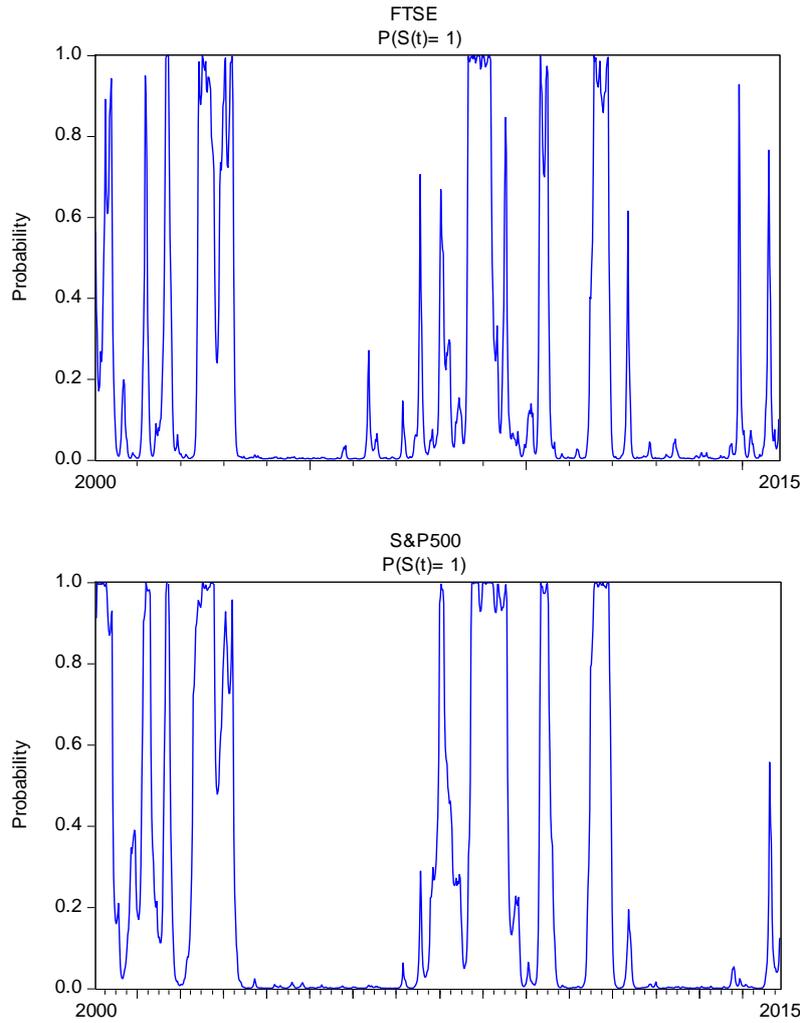


Figure 2

We verify that all three stock markets stay longer at the low volatility regime. All the analysis above is based on the full sample estimation. In the next section, we focus on the main goal of our study that is the out-of-sample forecastability of the model.

6. Out-of-sample forecast evaluation

We want to investigate whether the regime-switching model can provide more accurate forecasts than the simple AR(1) model and the random walk model. As we have already stated, we divide the sample in two parts and we keep the last 276 observations for the out-of-sample evaluation. Since we use the returns, the total number of observations is now 828 for S&P/TSX and FTSE and 827 for S&P500. We follow the recursive method and we perform one-step-ahead forecasts out-of-sample.

6.1 Recursive method

We assume that our sample has T observations and $T=R+P$. The first R observations are used to estimate the models and the last P are used for forecast purposes. Also, we generate one-step-ahead forecasts. The procedure is the following. In the beginning we estimate the models for the first $T-P$ observations and we find the forecast for the $T-P+1$ observation. Afterwards, we estimate again the models for the $T-P+1$ observations and the forecast for the $T-P+2$ observation is derived. Each time we increase the in-sample observations by one, we estimate the models for these observations and we get the next one-step-ahead forecast. When we estimate the models for the $T-1$ observations we finally take the last forecast. In the end we have P forecasts.

6.2 Error criteria

In order to evaluate the predictive accuracy of the models we employ two criteria, the Mean Squared Error (MSE) and the Mean Absolute Error (MAE). The smaller their value is, the larger the forecasting strength of the model. We calculate these two measures with the following formulas:

$$MSE = \frac{1}{P} \sum_{t=1}^P (f_t - y_t)^2 \quad \text{and} \quad MAE = \frac{1}{P} \sum_{t=1}^P |f_t - y_t|,$$

where f_t is the forecasting value at time t , y_t is the actual value at time t and p is the sample size. Additionally, we know that the error (e_t) is the difference between the true value and the prediction and so $e_t = f_t - y_t$. Hence, MSE is the average of the square of the errors and MAE is the mean of the absolute errors. We can also call them forecast error criteria.

To compare the models we take the ratios of the values of the measures. We use the regime-switching model as benchmark. Particularly, we put the value of the MSE of the regime-switching model as the numerator and the value of the MSE of the AR(1) model as denominator. After, we divide the value of the MSE of the regime-switching model by the value of the MSE of the random walk. We do the same for the MAE criterion. In all cases, the regime-switching model is on the numerator. So, if the ratio is less than unity, the regime-switching model outperforms the model at the denominator, otherwise the other model appears to be more reliable. We see the results in Table 7:

Forecasting performance of the models				
		S&P/TSX	FTSE	S&P500
MSE	RS/AR	0,983	0,985	0,983
	RS/RW	0,433	0,448	0,444
MAE	RS/AR	0,991	0,994	0,990
	RS/RW	0,661	0,684	0,655

Note: RS indicates the regime-switching model. AR indicates the AR(1) model. RW indicates the random walk model.

Table 7

According to both criteria, there is evidence that RS has stronger predictive power than the other two models for all three stock market indices. However, the difference between models is not big and we cannot say with confidence that RS outperforms AR. Therefore, we need to examine whether the differences are statistically significant. In order to continue our examination and to further explore the forecasting accuracy of the models we perform the Diebold and Mariano (1995) test.

6.3 Diebold and Mariano Test

Taking into account Diebold and Mariano (1995), we consider the time series $\{y_t\}_{t=1}^P$ and two forecasts for this series, $\{f_{it}\}_{t=1}^P$ and $\{f_{jt}\}_{t=1}^P$. The forecast errors are $\{e_{it}\}_{t=1}^P$ and $\{e_{jt}\}_{t=1}^P$. The loss differential is $d_t = [g(e_{it}) - g(e_{jt})]$, where $g(\cdot)$ is the loss function and i, j are two models we compare. In our analysis we have two loss functions, the one refers to the squared errors and the other to the absolute errors. Diebold and Mariano (1995) recommend a test with null hypothesis $H_0: E[g(e_{it})] = E[g(e_{jt})]$ or $E[d_t] = 0$. The null hypothesis says that two models have equal predictive accuracy and thus the difference of their forecast errors is not statistically significant. The alternative hypothesis is $H_1: E[d_t] \neq 0$. In this case there is statistical significance and one model is more accurate than the other. The test refers to non-nested models. According to Diebold and Mariano (1995) we take that $\sqrt{P}(\bar{d} - \mu) \xrightarrow{d} N(0, V)$, where \bar{d} is the sample mean loss differential and $\bar{d} = \frac{1}{P} \sum_{t=1}^P [g(e_{it}) - g(e_{jt})]$. For large samples, the sample mean loss differential \bar{d} approaches the normal distribution with mean μ and variance V/P . The large-sample statistic that follows

the standard normal distribution and tests the null hypothesis is $DM = \frac{\bar{d}}{\sqrt{\hat{V}}} \sqrt{P}$, where \hat{V} is a consistent estimator of the variance V . The results of the test appear in Table 8.

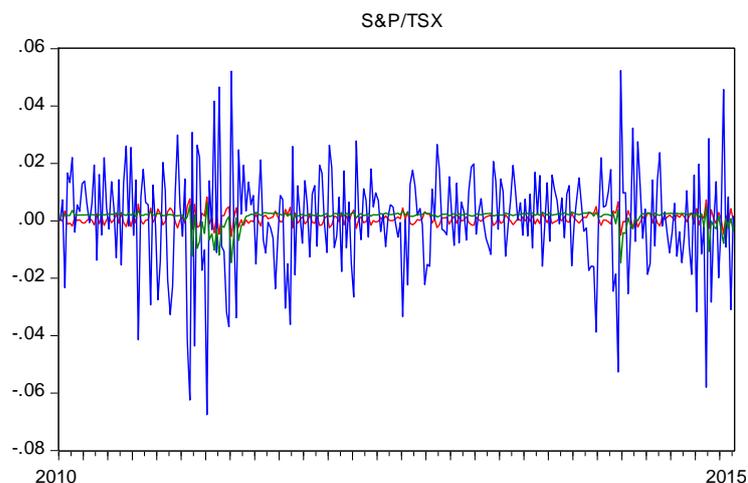
	DM-test statistic					
	S&P/TSX		FTSE		S&P500	
	DM-test	p-value	DM-test	p-value	DM-test	p-value
RSvsAR (MSE)	-0,7378	0,230303	-0,6318	0,263770	-0,8103	0,208875
RSvsRW (MSE)	-5,6033	1,05E-08	-5,4505	2,51E-08	-6,1011	5,27E-10
RSvsAR (MAE)	-0,7620	0,223037	-0,6182	0,268238	-1,0774	0,140641
RSvsRW (MAE)	-7,3750	8,22E-14	-6,5506	2,86E-11	-8,2917	1,11E-16

Note: RS indicates the regime-switching model. AR indicates the AR(1) model. RW indicates the random walk model.

Table 8

We take a 5% significance level. Since the DM-statistic approaches the standard normal distribution, we can reject the null hypothesis when $|DM| > 1,96$ and we cannot reject it when $|DM| < 1,96$. Looking at Table 8, we see that when we compare RS with AR, we cannot reject the null hypothesis of equal predictive accuracy. Nevertheless, when we compare RS with RW, we notice that we reject the null hypothesis. We can verify our results looking at the p-values. In the first case the p-values are too big and exceed by far the significance level 0,05. In the second case, the p-values are almost zero and certainly less than 0,05. Figure 3 shows the out-of-sample forecasted returns of our models for the three stock markets. It is obvious that RS and AR predict returns pretty close to each other contrary to RW.

Forecasted returns of the models



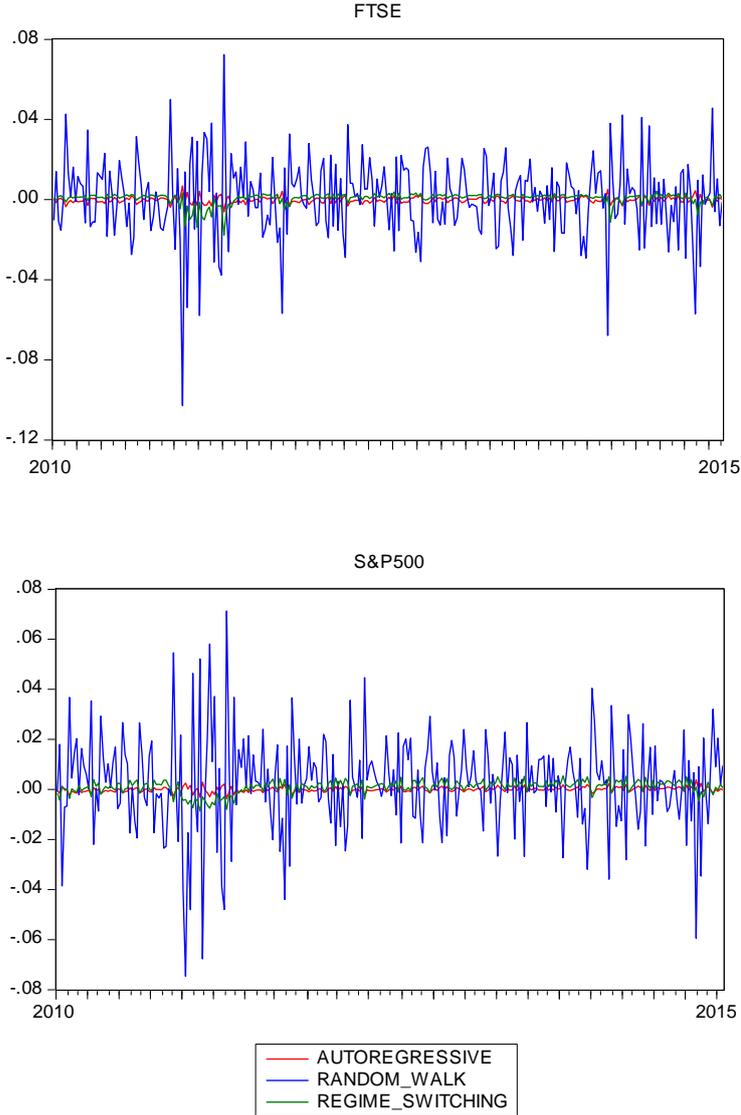


Figure 3

6.4 Robustness Test

The forecasting analysis conducted above is based on 276 out-of-sample observations. The sample was divided arbitrarily. Now we proceed with a robustness check regarding the way we split our sample. Particularly, we increase and decrease the out-of-sample observations by 70 and we investigate the forecasting performance of our models in these two cases. We want to see if the results will remain qualitatively the same. We only examine the error criteria. We first have 206 observations and afterwards 346 out-of-sample observations. We follow again the recursive way with one-step-ahead forecasts. The results are reported in Tables 9 and 10 respectively.

Forecasting performance of the models
(206 out-of-sample observations)

		S&P/TSX	FTSE	S&P500
MSE	RS/AR	0,9934	0,9899	0,9820
	RS/RW	0,4296	0,4403	0,4130
MAE	RS/AR	0,9962	0,9908	0,9835
	RS/RW	0,6674	0,6843	0,6327

Note: RS indicates the regime-switching model. AR indicates the AR(1) model. RW indicates the random walk model.

Table 9

Forecasting performance of the models
(346 out-of-sample observations)

		S&P/TSX	FTSE	S&P500
MSE	RS/AR	1,0004	1,0033	1,0039
	RS/RW	0,4336	0,4703	0,4634
MAE	RS/AR	0,9985	0,9982	1,0006
	RS/RW	0,6689	0,6979	0,6723

Note: RS indicates the regime-switching model. AR indicates the AR(1) model. RW indicates the random walk model.

Table 10

We observe that all values in Table 9 are qualitatively the same as these in Table 7, namely less than unity and thus RS seems to be better than the other models for all stock market returns. We can say the same for Table 10 except for the values in bold which are slightly greater than unity. According to these values, RS is not better than AR for S&P/TSX and FTSE in terms of the MSE criterion and for S&P500 in terms of both criteria.

However, we notice that when we compare RS and AR, all the values are close to unity and this also happens in Table 7 where we regard the 276 out-of-sample observations. The fact that the values are close to unity probably means that the two models do not differ significantly. Moreover, RS and AR are found to have equal forecasting power according to the Diebold and Mariano test for the 276 out-of-sample observations (Table 8). So despite the fact that not all values stay qualitatively the same, the results are robust to the break-point of the sample.

7. Conclusion

A large number of people worldwide are interested in the stock exchange. Volatility and returns of stock markets concern stockbrokers and researchers. We analyze weekly stock market returns of Canada, the United Kingdom and the United States for almost fifteen years. We present a first-order Markov regime-switching specification with two states and we initially examine its in-sample properties. Thereafter, we cut our sample into the in-sample and the out-of-sample part. We keep the first part for estimation purposes and the second one for the forecasting evaluation. We attempt to evaluate the forecasting power of the regime-switching model relative to that of an AR(1) model and a random walk model. The mean squared error (MSE) and the mean absolute error (MAE) criteria are used to assess the results generated by the one-step-ahead recursive method.

The regime-switching model seems to be slightly better than the other models in all cases. To examine if this outcome is accurate, we apply the Diebold and Mariano (1995) test. The regime-switching model outperforms the random walk, but has equal predictive ability to the AR(1) model. Finally, there is strong evidence that even if we change the break-point of the sample, the results stay the same.

In summary, although our regime-switching model has a relatively good fit in-sample, the out-of-sample predictions are not very accurate. Consequently, we cannot consider this specification as a trustworthy model to derive forecasts. Generally speaking, it is hard to insert a consistent model which performs efficiently out-of-sample. The literature supports nonlinear models. Thus, further research should be done and more parameters have to be considered for a more effective model to be generated.

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Further Reading

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