THE COMMON AGENCY GAME
Theory and Applications
to Political Economy

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In the political economy literature, the common agency game involves various and conflicting interest groups, seeking to influence the decisions of an incumbent government in favour of their members’ welfare. Beginning with some of its theoretical aspects, in this comprehensive survey we provide an overview of the applications of the common agency model to issues of political influence. We distinguish four areas of application; international markets, environmental policies, domestic economic policies and the political system. In addition, we describe a number of weaknesses of the common agency framework and the ways the basic model was extended for the purpose of addressing them. Finally, the dynamic version of the basic framework followed by some applications are presented, as well as some ideas for future research.

Key Words: Common Agency, Political Influence, Interest Groups, Lobbying
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Chapter 1

Introduction

Common agency is an extension of the principal-agent framework to situations in which the actions chosen by a particular individual (agent) affect not just one, but several other parties (principals) with typically conflicting preferences for the various possible actions (Bernheim and Whinston, 1986a).

Two categories of common agency exist: delegated common agency and intrinsic common agency. In the former case, several parties authorize a single common agent to make certain decisions, while in the latter, an individual possesses the inherent power to make a particular decision that affects other parties. The affected parties, then, may attempt to influence that decision.

Firstly introduced by Bernheim and Whinston (1986b) as a menu auction game, the common agency framework has been employed in the analysis of areas of economics, such as industrial organization, largely concerned with the organizational structures of firms or arrangements; public good provision, in which case a government (agent) decides about public good provision, and political economy, modelling interest groups (principals) seeking to influence the decisions of an incumbent government in favour of their members’ welfare. In this study, we overview the common agency literature concerning issues of political influence.

The two leading approaches in the political influence literature have been the political competition approach (Magee et al., 1989) and the political support approach (Hillman, 1989; Van Long and Vousden, 1991).¹ Adopting the latter one, Grossman and Helpman (1994) were the first to apply the common agency framework, in order to explain the equilibrium structure of trade protection, and since then, their model has become somewhat of theoretical consensus among researchers. This occurs for several reasons. Firstly, its framework is multisectoral. Secondly, it provides microanalytic foundations to the objective functions of the politician and the interest groups. Moreover, it is convenient to work with it.

¹See Helpman (1997).
Chapter 1. Introduction

The study is organized as follows. In Chapter 2, beginning with the presentation of the fundamental study that has spawned the common agency literature (Bernheim and Whinston, 1986), we take a look at some theoretical issues of the common agency game, concerning the existence and the characteristics of its equilibria. In Chapter 3, we provide an overview of the political influence literature based on the common agency framework, dividing four areas of application, namely international markets, environmental policies, domestic economic policies and the political system (Mallard, 2014). We, also, display a number of weaknesses of the basic model and some modified versions. In Chapter 4, we introduce a dynamic perspective of the basic common agency framework, followed by a number of applications. Finally, in Chapter 5, some concluding remarks are offered, as well as some ideas for future research.
Chapter 2

The Theory of Common Agency

In this section, we introduce some theoretical aspects of the common-agency model. The majority of the relative literature investigates the existence and the characteristics of equilibrium. In what follows, we thoroughly analyze some representative papers, and therefore we refer to some further research.

2.1 Existence of Equilibrium

2.1.1 Menu Auctions, Resource Allocation and Economic Influence

In contrast to the earlier study of auctions and competitive bidding concerning the allocation of a single, well-defined, indivisible object, Bernheim and Whinston (1986b) generalize this assumption, considering an auction, in which bidders name a "menu" of offers for every possible action available to the auctioneer. They focus on first-price menu auctions, which means that bidders pay their announced offers for the allocation chosen by the auctioneer, and that this choice maximizes the auctioneer’s payoff. It is assumed that principals have complete information, while the agent, presumably, is poorly informed. The central results of their study establish that for a refinement of the Nash Equilibrium set, in which relative preferences for the various alternatives are reflected correctly by bids, first-price menu auctions always implement efficient actions, and, also, these "truthful" equilibria are the only equilibria which possess a strong stability property.
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The Model

Bernheim and Whinston (1986b) present a game, in which an auctioneer (agent) selects an action affecting the well-being of \( M \) bidders (principals), each of whom offers a menu of payments contingent on the action chosen. The set of bidders is denoted by \( \mathcal{J} = \{ i \}_{i=1}^{M} \), and subsets by \( J \subseteq \mathcal{J} \). The possible choices for the auctioneer are given by a finite set \( S \). Bidder \( i \) receives gross monetary payoffs described by the function \( g_{i} : S \rightarrow \mathbb{R} \), while the function \( d : S \rightarrow \mathbb{R} \) indicates the disutility (in monetary terms) that the auctioneer experiences in taking each possible action. Let \( G_{J}(s) \equiv \sum_{i \in J} g_{i}(s), \forall s \in S, J \subseteq \mathcal{J} \) and
\[
S^{J} \equiv \arg\max_{s \in S} [G_{J}(s) - d(s)], \forall J \in \mathcal{J},
\]
where \( S^{J} \) contains actions that yield the highest joint payoff to the auctioneer and the members of the group \( J \). Define \( S^{\ast} \equiv S^{\mathcal{J}} ; S^{\ast} \) contains efficient actions.

In the extensive form of this game the \( M \) bidders simultaneously offer contingent payments to the auctioneer, who subsequently chooses an action that maximizes his total payoff. The strategy of each bidder consists of a function \( f_{i} : S \rightarrow \mathbb{R} \); that is he offers the auctioneer a monetary reward of \( f_{i}(s) \) for selecting action \( s \). The set of feasible strategies for each bidder is given by
\[
F_{i} \equiv \{ f_{i} \mid f_{i}(s) \geq k_{i}(s), \forall s \in S \},
\]
where the function \( k_{i}(\cdot) \) places lower bounds on the bids offered for each action. These bounds reflect the limited ability of bidders to extract payments from the auctioneer.

For a particular strategy \( f_{i} \), bidder \( i \)'s net payoff at each action \( s \) is given by the function \( n_{i}(s) \equiv g_{i}(s) - f_{i}(s) \). Moreover, \( F_{J}(s) \equiv \sum_{i \in J} f_{i}(s) \) and \( N_{J}(s) \equiv \sum_{i \in J} n_{i}(s), \forall s \in S, J \subseteq \mathcal{J} \) (thus, \( N_{J}(s) = G_{J}(s) - F_{J}(s) \)).

In a first-price menu auction the auctioneer chooses an action that maximizes his total payoff—i.e. given an element of \( \Pi_{i=1}^{M} \mathcal{F}_{i} \), the auctioneer selects an element of the set
\[
I^{\ast}({\{ f_{i} \}_{i=1}^{M}}) \equiv \arg\max_{s \in S} [F_{\mathcal{J}}(s) - d(s)].
\]

Since a menu auction \( \Gamma \) is completely specified once the action space, reward spaces, disutilities and gross payoffs have been specified, we may write \( \Gamma = [S, \{ k_{i} \}_{i=1}^{M}, \{ g_{i} \}_{i=1}^{M}, d, \{ f_{0}^{i} \}_{i=1}^{M}, s^{0} ] \) is a Nash Equilibrium for the auction \( \Gamma \) if
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$f_i^0 \in \mathcal{F}_i, \forall i, s^0 \in I^*(\{f_j^M\}_{j \neq i}),$ and—given $\{f_j^o\}_{j \neq i}$—no bidder $i$ has a feasible strategy that would yield him a net payoff greater than $n_i(s^0)$.

It is shown that $\{f_i^M\}_{i = 1}^M, s^0$ is a Nash Equilibrium of auction $\Gamma$ if and only if $\{\tilde{f}_i\}_{i = 1}^M, s^0$ is a Nash Equilibrium of auction $\tilde{\Gamma} = [S; \{\tilde{g}_i\}_{i = 1}^M, \{\tilde{d}_i\}_{i = 1}^M, d], where $\tilde{f}_i(s) = f_i(s) - k_i(s)$ and

$$\tilde{k}_i(s) \equiv 0, \quad \forall i \in \mathcal{I}, s \in S;$$
$$\tilde{g}_i(s) \equiv g_i(s) - k_i(s), \quad \forall i \in \mathcal{I}, s \in S;$$
$$\tilde{d}_i(s) \equiv d(s) - \sum_{i = 1}^M k_i(s), \quad \forall s \in S.$$

Given this fact, it shall henceforth assumed without loss of generality that $k_i(s) \equiv 0, \forall i \in \mathcal{I}, s \in S$. Thus,

$$\mathcal{F} \equiv \{f \mid f(s) \geq 0, \forall s \in S\}.$$

Results

Bernheim and Whinston (1986b) begin their analysis by completely characterizing the set of Nash Equilibria for first-price menu auctions.

**Lemma 2.1.** Consider a first-price menu auction $\Gamma$. $\{f_i^M\}_{i = 1}^M, s^0$ is a Nash Equilibrium, if-f

(i) $f_i \in \mathcal{F}, \forall i \in \mathcal{I}$
(ii) $s^0 \in I^*(\{f_i^M\}_{i = 1}^M)$
(iii) $[g_i(s^0) - d(s^0)] - [g_i(s) - d(s)] \geq [F_{-i}(s - F_{-i}(s^0))], \forall i \in \mathcal{I}, s \in S$
(iv) $\exists s_i \in I^*(\{f_i\}_{i = 1}^M), such that f_i(s_i) = 0, \forall i \in \mathcal{I}.$

Bernheim and Whinston (1986b) suggest that a large number of contingent offers will typically satisfy the four conditions of Lemma 2.1 and they want to answer the following question: *are all of these equally plausible?* They argue that they are not. They analyze a subclass of equilibria with certain appealing characteristics, that may be focal, especially in situations in which no communication occurs between bidders. In these equilibria each bidder plays a *truthful* strategy, which is clarified with the following definition:
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**Definition 2.1.** \(f_i(\cdot)\) is said to be a truthful strategy relative to \(s^0\) if and only if, \(\forall s \in S\), either

(i) \(n_i(s) = n_i(s^0)\),

or

(ii) \(n_i(s) < n_i(s^0)\), and \(f_i(s) = 0\).

\(\{f_i\}_{i=1}^M, s^0\) is said to be a Truthful Nash Equilibrium if and only if it is a Nash Equilibrium and \(\{f_i\}_{i=1}^M\) are truthful strategies relative to \(s^0\).

Note that in any truthful equilibrium each bidder offers a reward for action \(s\) that exactly reflects his net willingness-to-pay for \(s\) as opposed to \(s^0\).

In addition, it is proved that a bidder can essentially restrict himself to using truthful strategies without loss: every best-response set contains a truthful strategy.

**Theorem 2.1.** Consider a first-price menu auction \(\Gamma\) and any bidder \(i\). For any set of offers by his components, \(\{f_j\}_{j \neq i}\), bidder \(i\)’s best-response correspondence contains a truthful strategy.

Thus, the set of Truthful Nash Equilibria is an appealing refinement of the Nash set. The authors’ first task is to establish the existence of these equilibria, and to explore their properties.

In what follows, they refer to the following sets of net payoff vectors. Let

\[\Pi_\Gamma(s) \equiv \{n \in \mathbb{R}^M \mid \forall J \subseteq \mathcal{J}, \ N_J \leq [G_J(s) - d(s)] - [G_J(s^0) - d(s^0)]\},\]

where \(s^J \in S^J\), \(N_J = \sum_{i \in J} n_i\) and \(G_J(s^J) - d(s^J) \equiv \min_{s \in S} d(s)\). The Pareto efficient frontier of \(\Pi_\Gamma(s)\) is defined by

\[E_\Gamma(s) \equiv \{n \in \mathbb{R}^M \mid n \in \Pi_\Gamma(s) \text{ and } \nexists n' \in \Pi_\Gamma(s), \text{ with } n' \geq n\}.\]

Clearly the set \(E_\Gamma(s)\) is non-empty. Also for \(s', s'' \in S^*\), \(\Pi_\Gamma(s') = \Pi_\Gamma(s'') \equiv \Pi_\Gamma(S^*)\) and \(E_\Gamma(s') = E_\Gamma(s'') \equiv E_\Gamma(S^*)\).

The fundamental result concerning the set of Truthful Nash Equilibria is the following:

**Theorem 2.2.** Consider a first-price menu auction \(\Gamma\). In all Truthful Nash Equilibria the auctioneer selects \(s^0 \in S^*\), and the bidders receive payoffs in \(E_\Gamma(S^*)\). Furthermore, any net payoff vector \(n \in E_\Gamma(S^*)\) can be supported by a Truthful Nash Equilibrium.
Theorem 2.2 provides a nice characterization of the set of net payoffs that bidders can receive in a truthful equilibrium—these payoffs must lie in the set $E_\Gamma(S^*)$. In fact, for first-price menu auctions in which there are only two bidders and the auctioneer has no inherent preferences over the decision set, Theorem 2.2 leads to the following result:

**Corollary 2.1.** Consider a first-price menu auction $\Gamma$, in which there are two bidders and the auctioneer has no inherent preferences over his action set. There exist unique Truthful Nash Equilibrium bids ($s^0$ is also unique if $S^*$ is a singleton), and in a truthful equilibrium bidder $i$ receives a net payoff of $G_j(S^*) - g_j(s^1)$ (where $j \neq i$ and $s^1 \in S^1$).

One natural question to ask is whether, in a particular equilibrium, any coalition of bidders has an incentive to communicate among themselves, with the intention of arranging a stable, mutually preferable joint deviation. The authors wish to restrict attention to the set of equilibria for which no such "coalitional" deviation is possible.

For a subgroup $J$ and strategies $\{f_i\}_{i \in J}$, they define the subgroup $J$ component game relative to $\{f_i\}_{i \in \bar{J}}$ as follows:

$$\Gamma/\{f_i\}_{i \in \bar{J}} = (S, \{g_i\}_{i \in J}, \{k_i\}_{i \in J}, d - \sum_{i \in J} f_i).$$

That is, $\Gamma/\{f_i\}_{i \in \bar{J}}$ is the restriction of the game $\Gamma$ to bidders in subgroup $J$, where the strategies of the bidders in $\bar{J}$ are held fixed. Note that $\{f_i\}_{i \in J}$ causes the auctioneer to act as though he has preferences over $S$.

**Definition 2.2.**

(i) In a first-price menu auction $\Gamma$ with a single bidder ($M = 1$), $(f_1^0, s^0)$ is a Coalition-Proof Nash Equilibrium if and only if it is a Nash Equilibrium.

(ii) For a first-price menu auction $\Gamma$, where $M > 1$, $(\{f_i^0\}_{i = 1}^M, s^0)$ is self-enforcing if $(\{f_i^0\}_{i \in J}, s^0)$ is a Coalition-Proof Nash Equilibrium in the subgroup $J$ component game $\Gamma/\{f_i^0\}_{i \in \bar{J}}$, $\forall J \subset \mathcal{J}$ (where $J \neq \mathcal{J}$).

(iii) $(\{f_i^0\}_{i = 1}^M, s^0)$ is a Coalition-Proof Nash Equilibrium if it is a self-enforcing Nash Equilibrium and offers net payoffs to the $M$ bidders, that are not Pareto dominated by any other self-enforcing Nash Equilibrium.

The last striking result that (Bernheim and Whinston, 1986b) present is that all Truthful Nash Equilibria are coalition-proof, and furthermore, the set of net payoffs for the bidders that can arise in Coalition-Proof Nash Equilibria exactly
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coincides with those arising in Truthful Nash Equilibria. Thus, truthful equilibria not only possess an inherent appeal, but are also the only Nash Equilibria that are stable when (non-binding) communication is possible.

**Theorem 2.3.** Consider a first-price menu auction $\Gamma$. In all Coalition-Proof Nash Equilibria the auctioneer selects $s^0 \in S^*$, and the bidders receive payoffs in $E_\Gamma(S^*)$. Furthermore, all Truthful Nash Equilibria are coalition-proof. Thus, any payoff vector $n \in E_\Gamma(S^*)$ can be supported by a Coalition-Proof Nash Equilibrium.

2.1.2 Further Research

Few explicit models of (non-cooperative) common agency—that are tailored to highly specialized problems—had existed in the literature (Mintz and Tulkens, 1986; Bernheim and Whinston, 1985; Baron, 1985; Braverman and Stiglitz, 1982; Stiglitz, 1985), until Bernheim and Whinston (1986a) represented a first step towards developing a coherent, widely applicable, abstract framework for analyzing such instances. In their paper, they extent the standard bilateral principal-agent framework to situations in which a number of risk-neutral principals simultaneously and non-cooperatively announce incentive schemes for a common agent, whose action is not directly observable by the principals.\(^1\) Their investigation primarily concerns the following two questions: which action is implemented, and what incentives structure leads to its implementation? They find that in equilibrium, every action is implemented efficiently—the aggregate incentive scheme induces the selection of the equilibrium action at minimum cost by the agent. In addition, whenever collusive behaviour would implement the first-best action at the first-best level of cost, non-cooperative equilibrium is fully efficient. Further, when this condition fails, non-cooperative interaction does not produce a second-best outcome.

The literature concerning the analysis of the existence of equilibrium in common agency models also includes the works of Frayssé (1993), who proves the existence of an equilibrium, regardless of the number of actions, if the actions result in only two outcomes, and Reny (1999), who offers a pure strategy Nash equilibrium existence result for a large class of discontinuous games. Page Jr.

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\(^1\)The writers provide the following example: several levels of government individually wish to raise revenues by taxing a firm, while influencing the firm’s behaviour (perhaps creating incentives for effluent abatement). They face two constraints: they are not allowed to directly observe the activities of the firm (perhaps monitoring is costly), and the profits earned by the firm must be non-negative.
and Monteiro (2003), first, establish a competitive analog to the revelation principle, which they call the implementation principle. Second, they establish a competitive taxation principle, and third, applying the concept of payoff security and the competitive taxation principle by Reny (1999), they demonstrate the existence of a Nash equilibrium for the mixed extension of the non-linear pricing game.

Monteiro and Page (2008) introduce the notion of a catalog game, in which each seller competes for a buyer of unknown type by offering her a catalog of products and prices, and their main objective is to show that Nash equilibria exist in such games. They present the efficiency tie-breaking rule, under which contracts are awarded to firms which value it most, and show that in this situation firm expected profit functions are reciprocally upper semi-continuous, and therefore a uniformly payoff secure catalog game with payoffs only upper semi-continuous in contracts has a Nash equilibrium in mixed strategies. In a subsequent paper (Monteiro and Page Jr., 2009), the same authors determine the tie-breaking rule endogenously, and show that if profit functions are continuous in contracts, then the Nash problem reduces to an implementation problem.

Finally, Carmona and Fajardo (2009) investigate the existence of equilibrium in common agency games with adverse selection. They define a menu game—which are sufficient to analyze common agency problems—as follows: first, the agent’s type follows a commonly known distribution. Afterwards, the principals simultaneously choose a menu of contracts without observing the agent’s type. In the end of the game, the agent chooses one contract (or one contract of each principal), knowing her type and the menus offered by the principals. Their main result is that every menu game, satisfying enough continuity properties, has a subgame perfect equilibrium.

2.2 Characteristics of Equilibrium

2.2.1 Conflict and Cooperation: The Structure of Equilibrium Payoffs in Common Agency

Laussel and Le Breton (1996, 2001) contribute to the theory of common agency
with complete information, by providing theoretical results to identify the structure of equilibrium payoffs focusing on the agent’s payoff. Except for the fact that the agent’s rent in Bernheim and Whinston’s model is the pure result of conflicting preferences among principals, a second reason for their focus on it is that common agency depicts many important real life situations, in which the magnitude of the agent’s rent actually matters (e.g. strategic lobbying, relationship between manufacturer and retailers, private production of public goods and auctions).

Continuing the Bernheim and Whinston’s analysis of the equilibrium payoffs, Laussel and Le Breton examine how the set of feasible actions and the utility functions of the agent and the principals affect the equilibrium payoffs of the game. To a common agency game they associate an object \( W \), which is mathematically a transferable utility (TU) cooperative game\(^2\) with the set of players being the set of principals: specifically, for each group \( S \) of principals, they calculate the highest joint payoff \( W(S) \) of the agent and principals in the group \( S \). They demonstrate that a \( n \)-dimensional vector \( u \) is a vector of equilibrium payoffs for the principals, if and only if \( u \) is a Pareto optimum of the polyhedron defined by the set of linear inequalities: \( \sum_{i \in S} u_i \leq W(N) - W(N \setminus S), \forall S. \) The meaning of the polyhedron is that the total payoff of group \( S \) can never exceed the contribution of group \( S \) to the total surplus. They show that there is a one to one relationship between the Pareto optima of this convex polyhedron, and the solutions to a system of \( n \) simultaneous equations that they call the fundamental equations.

Because of the difficulty of the characterization of all the solutions to these equations, the authors try to identify how the properties of \( W \) determine the features of the solutions. Among these features, they are primarily interested in the agent’s rent. Intuitively, the case that the agent gets a rent at equilibrium relates to the degree of conflict between the principals’ interests. The more competitive the principals are, the more the money the agent earns. Contrariwise, if the principals’ objectives converge, it is possible that the agent will not get any rent.

First, we describe the modified Bernheim and Whinston’s model by Laussel and Le Breton (2001) and the main results obtained, in order to proceed to the analysis. Therefore, two cases of games are demonstrated: the balanced and the non-balanced. Concluding, two families of applications are presented: the public common agency and the private common agency.

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\(^2\)A transferable utility (TU) cooperative game with set of players \( N = \{1, \ldots, N\} \) is a function \( W : 2^N \rightarrow \mathbb{R} \).
Bernheim and Whinston’s Results

Denote the set of principals by $N = \{1, \ldots, n\}$ and the agent will be identified by the index 0. The possible choices by the agent are given by a set $A$. Principal $i$ receives gross monetary payoffs described by the function $V_i : A \to \mathbb{R}$, while the function $V_0 : A \to \mathbb{R}$ indicates the utility (or disutility) in monetary units, that the agent experiences in taking each possible action. $2^N$ is the set of subsets of $N$ including the empty set. A common agency game is therefore completely described by an $(n+2)$ tuple $\Gamma \equiv \{A, V_0, V_1, \ldots, V_n\}$. $W_\Gamma(S) = \max_{a \in A}[\sum_{i \in S}V_i(a) + V_0(a)]$, $\forall S \in 2^N$, is the highest joint payoff for the agent and principals in group $S$ and $A_\Gamma^*(S) = \arg\max_{a \in A}[\sum_{i \in S}V_i(a) + V_0(a)]$ is the set of actions that yield this payoff.

An action is efficient if it belongs to $A_\Gamma^*(N)$. Finally, note that $W_\Gamma(\emptyset) = \max_{a \in A} V_0(a)$, where $W_\Gamma(\emptyset)$ plays the role of reservation value of the agent in the common agency game. In what follows, it is assumed that $W_\Gamma(\emptyset) = 0$.

A strategy for each principal $i$ consists of a function $T_i : A \to \mathbb{R}_+$, that is the principal offers the agent a monetary reward of $T_i(a)$ for selecting action $a$. For each action $a$, the principal gets a net payoff given by the function $U_i$ with $U_i = V_i(a) - T_i(a)$. The agent chooses an action that maximizes her total payoff, i.e. given $T \equiv (T_1, \ldots, T_n)$, the agent selects an action in the set $M(T)$ with:

$$M(T) \equiv \arg\max_{a \in A} \left[ \sum_{i \in N} T_i(a) + V_0(a) \right].$$

The common agency game is merely a game between the principals. An outcome is a Nash equilibrium if $a^* \in M(T^*)$ and $\# i \in N, T_i : A \to \mathbb{R}_+$ and $a \in M(T_i, T_{-i})$, such that $U_i(a) > U_i(a^*)$.

Now, we return to some of the results of Bernheim and Whinston (1986b), based on the new modified model.

**Definition 2.1**. $T_i$ is said to be truthful relative to $\bar{a}$ if $\forall a \in A$, either (i) $U_i(a) = U_i(\bar{a})$ or (ii) $U_i(a) < U_i(\bar{a})$ and $T_i(a) = 0$

$(T^*, a^*)$ is a truthful Nash equilibrium if and only if it is a Nash equilibrium and $T_i^*$ is a truthful strategy relative to $a^*$, $\forall i \in N$. The set of payoffs obtained by the principals in these equilibria is

$$U_\Gamma \equiv \left\{ u \in \mathbb{R}^n \mid \sum_{i \in S} u_i \leq W_\Gamma(N) - W_\Gamma(N/S), \forall S \in 2^N \right\}.$$

Also, the Pareto efficient frontier $\bar{U}_\Gamma$ of $U_\Gamma$ is
\( \bar{U}_\Gamma \equiv \{ u \in \mathbb{R}^n \mid u \in U_\Gamma \text{ and } \not\exists u' \in U_\Gamma \text{ such that } u' \geq u \} \).

The two modified theorems by Bernheim and Whinston (1986b) are the following:

**Theorem 2.2**. Consider a common agency game \( \Gamma \). In all truthful Nash equilibria the agent selects \( a^* \in A^*_\Gamma(N) \) and the principals receive payoffs in \( \bar{U}_\Gamma \). Furthermore, any vector \( u \in \bar{U}_\Gamma \) can be supported by a truthful Nash equilibrium.

**Theorem 2.3**. Consider a common agency game \( \Gamma \). In all coalition-proof Nash equilibria the agent selects \( a^* \in A^*_\Gamma(N) \) and the principals receive payoffs in \( \bar{U}_\Gamma \). Furthermore, all truthful Nash equilibria are coalition-proof. Thus, any payoff vector \( u \in \bar{U}_\Gamma \) can be supported by a coalition-proof Nash equilibrium.

### The Solution of the Fundamental Equations

Theorem 2.2* states that a vector \( u \in \mathbb{R}^n \) is a vector of equilibrium payoffs for the principals if and only if \( u \) belongs to the Pareto frontier \( \bar{U}_\Gamma \) of the polyhedron \( U_\Gamma \). But how do we know that a vector \( u \) belongs to \( \bar{U}_\Gamma \)?

Bernheim and Whinston (1986b) state that \( u \in \bar{U}_\Gamma \) if and only if \( u \in U_\Gamma \) and \( \forall i \in N, \exists S \subseteq N \) with \( i \in S \) such that

\[
\sum_{j \in S} u_j = W_\Gamma(N) - W_\Gamma(N/S).
\]

Let \( \Psi_i \equiv \{ S \subseteq N \mid i \notin S \} \). Note that from the definition of \( U_\Gamma \), \( u \in U_\Gamma \) if and only if

\[
W_\Gamma(N) - \sum_{i \in N} u_i \geq W_\Gamma(N/S) - \sum_{i \in N/S} u_i, \quad \forall S \subseteq N. \tag{2.2.1}
\]

Also, the condition above can be rewritten as: \( \forall i \in N, \exists S \subseteq N \) with \( i \in S \) such that

\[
W_\Gamma(N) - \sum_{j \in N} u_j = W_\Gamma(N/S) - \sum_{i \in N/S} u_i. \tag{2.2.2}
\]

Combining (2.2.1) and (2.2.2), it is obtained that \( u \in \bar{U}_\Gamma \) if and only if it is a solution to the following system of equations:

\[
W_\Gamma(N) - \sum_{i \in N} u_i = \max_{s \in \Psi_i} \left( W_\Gamma(S) - \sum_{i \in S} u_i \right), \quad \forall i \in N. \tag{2.2.3}
\]
(2.2.3) is a system of $n$ simultaneous equations with $n$ unknowns, which the authors call the *fundamental equations*. The left-hand side is the equilibrium payoff of the agent and the right-hand side of the $i$-th equation is the highest payoff that the agent would get if he or she disregarded the offer from principal $i$. In equilibrium, the left-hand side must be greater than or equal to any of the right-hand sides.

To each solution $u$ of (2.2.3) is associated a vector of coalitions $(S_1, \ldots, S_i, \ldots, S_n)$ with $i \notin S_i$ and

$$W_\Gamma(S_i) - \sum_{j \in S_i} u_j = \max_{S \in \Psi_i} \left( W_\Gamma(S) - \sum_{j \in S} u_j \right).$$

The following result states that any TU cooperative game $W$ arises as the TU cooperative game of a common agency game $\Gamma$.

**Proposition 2.1.** Let $W$ be an arbitrary TU cooperative game. Then there exists a common agency game $\Gamma$, such that $W = W_\Gamma$.

Subsequently, it is shown that the existence of an equilibrium with no rent is surprisingly related to the non-emptiness of the core\(^3\) of $W_\Gamma$— or equivalently to the balancedness\(^4\) of $W_\Gamma$. Bondareva (1962) and Shapley (1967) have proved that $C(W)$ is non-empty if and only if $W$ is balanced.

Let $u$ be a vector of equilibrium payoffs such that $W_\Gamma(N) - \Sigma_{i \in N} u_i = 0$. From (2.2.3), $W_\Gamma(S) - \Sigma_{i \in S} u_i \leq 0$, $\forall S \subseteq N$. Therefore, $C(W_\Gamma)$ is non-empty and $u \in C(W_\Gamma)$. Conversely, if $C(W_\Gamma)$ is non-empty and $u \in C(W_\Gamma)$, $u$ is a solution to (2.2.3).

**Theorem 2.4.** Consider a common agency game $\Gamma$. There exist equilibria, in which the agent gets no rent, if and only if the TU game $W_\Gamma$ is balanced. Further, there is a one to one relationship between the equilibria, where the agent gets no rent, and the core of $W_\Gamma$.

---

\(^3\)The core of the game $W$, denoted by $C(W)$ is the set

$$\left\{ u \in \mathbb{R}^n \mid \sum_{i \in S} u_i \leq W(S), \forall S \in 2^N, \text{ with equality for } S = N \right\}.$$  

\(^4\)Given $i \in N$, let $\Psi_i \equiv \{ S \in 2^N \mid i \in S \}$. A balanced family for the game is a vector $\delta \in \mathbb{R}_{+}^{2^n}$, such that $\Sigma_{S \in \Psi_i} \delta_S = 1$, $\forall i \in N$. The balanced family is in fact the set of coalitions $S$, such that $\delta_S > 0$ and $\delta$ restricted to this family is referred to as the vector of balancing weights. The game $W$ is said to be balanced, if for all balanced families $\delta$:

$$W(N) \geq \sum_{S \in 2^N} \delta_S W(S).$$
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The Balanced Case

Consider the case where \( W_\Gamma \) is balanced. It is known, from Theorem 2.4, that there exist equilibria in which the agent gets no rent. But this does not imply that other equilibria, in which the agent gets a rent, do not exist. The following proposition states some rather nice sufficiency results for a small number of principals and a final sufficiency result when \( W_\Gamma \) is symmetric, while the final proposition denotes a sufficient condition, which does not depend on the number of principals.

**Proposition 2.2.** (i) Consider a common agency game \( \Gamma \) with \( n = 2 \). \( \Gamma \) has the no-rent property if and only if the TU game \( W_\Gamma \) is balanced.

(ii) Consider a common agency game. If \( n = 3 \) and \( W_\Gamma \) is totally balanced, then \( \Gamma \) has the no-rent property.

(iii) Consider a common agency game. If \( n = 4 \) and \( W_\Gamma \) is exact, then \( \Gamma \) has the no-rent property.

(iv) Consider a common agency game \( \Gamma \), such that \( W_\Gamma \) is symmetric. If \( W_\Gamma \) is totally balanced, then \( \Gamma \) has the no-rent property.

Proposition 2.2(i) indicates that the no-rent property is equivalent to balancedness, if there are at most two principals. Proposition 2.2(iv) shows that when the game \( W_\Gamma \) is not symmetric, the sufficient conditions for the no-rent property to hold are highly sensitive to the number of principals. The following theorem demonstrates that if the game \( W_\Gamma \) is convex, then the no-rent property holds true regardless of the number of principals.

**Theorem 2.5.** Consider a common agency game \( \Gamma \). If \( W_\Gamma \) is convex, then \( \Gamma \) has the no-rent property.

The Non-Balanced Case

Consider now the case where \( W_\Gamma \) is not balanced, which means that in all equilibria the agent gets a rent (Theorem 2.4). There may exist multiple equilibria, thus it may matter to evaluate how small the rent can be. To answer this question, it is enough to solve the following linear program:

\[
\max_u \sum_{i \in N} u_i
\]
such that 
\[ \sum_{i \in S} u_i \leq W_\Gamma(N) - W_\Gamma(N/S), \quad \forall S \subseteq 2^N. \]

Denote by \( V_\Gamma \) the value of this maximization problem. From the duality theorem in linear programming (Rockafellar, 1970), \( V_\Gamma \) is also the value of the following minimization problem:

\[ \min_{\delta} \sum_{S \subseteq N^*} \delta_S(W_\Gamma(N) - W_\Gamma(N/S)) \]

such that
\[ \sum_{S \subseteq \Psi_i} \delta_S = 1, \quad \forall i \in N; \]
\[ \delta_S \geq 0, \quad \forall S \subseteq N. \]

The minimal rent of the agent in the common agency \( \Gamma \) is given by \( W_\Gamma(N) - V_\Gamma \).

Regarding the uniqueness of equilibrium, this will happen if there is enough orthogonality between the interest of the principals. A sufficiency result is the following.

**Proposition 2.3.** Consider a common agency game \( \Gamma \). If \( W_\Gamma \) is strongly subadditive,\(^5\) then there is a unique solution \( u \) to the fundamental equations, namely \( u_i = W_\Gamma(N) - W_\Gamma(N/\{i\}), \quad \forall i \in N \).

We can notice that not only is there a unique vector of equilibrium payoffs, but in equilibrium each principal \( i \) receives her marginal contribution, i.e. the variation in the total surplus that occurs, when the principal joint the coalition of all other principals.

**Applications**

**Public Common Agency Games.** In a public common agency game, a population \( N \) on \( n \) individuals has to select a public project \( a \) out of a feasible set \( A \). To each project \( a \) is attached a monetary cost \( C(a) \) and the monetary evaluation \( V_i(A) \) of individual \( i \). \( A = \mathbb{R}_m^+ \), i.e. a public project is a vector of quantities of \( m \) different public goods. Under this interpretation, it is reasonable to assume that both the function \( V_i \) and the function \( C \) are increasing, i.e. if \( a \geq b \) and \( a \neq b \),

\(^5\)A TU cooperative game is strongly subadditive if, \( \forall S, T \subseteq N \), such that \( S \cup T = N \), \( W(T) + W(S) - W(S \cap T) \). It is concave if the above inequality holds true for all \( S, T \subseteq N \).
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then $V_i(a) > V_i(b)$ and $C(a) > C(b)$. The common agency game, which is studied, is a game of private provision of public projects, where the selected project $a$ maximizes the profit $\Sigma_{i \in N} T_i(a) - C(a)$. It contrasts a lot with the "classical" games of private provision of public goods, where the strategies are monetary contributions instead of being monetary schedules.

From Theorem 2.2*, it is concluded that in all equilibria the public project is Pareto efficient. It is expected that the total amount of monetary contributions will exactly match the cost of the project, when the interests of the principals are not too much conflicted.

**Definition 2.3.** A common agency game $\Gamma \equiv \{A, V_0, V_1, \ldots, V_n\}$ is comonotonic, if $[V_i(a) - V_i(b)] [V_j(a) - V_j(b)] \geq 0, \forall i, j \in N, \forall a, b \in A$.

**Proposition 2.4.** Consider a common agency game $\Gamma$. If $\Gamma$ is comonotonic, then $W_\Gamma$ is convex.

The property of comonotonicity is a very demanding property on the profile of valuation functions $(V_i)_{i \in N}$ since, up to the treatment of indifferences, it requires that the principals order the public projects alike.

The "congruence" condition below is weaker than comonotonicity. It relies on the definition of fictitious common agency games $\Gamma_\mu \equiv \{A, V_0, V_1, \ldots, \mu V_i, \ldots, V_n\}$, in which principal $i$'s gross monetary payoff associated to any $a \in A$ is $\mu V_i(a), \mu \in (0, 1]$, i.e. in which $\mu$ measures the size of principal $i$.

**Definition 2.4.** A common agency game $\Gamma \equiv \{A, V_0, V_1, \ldots, V_n\}$ has the congruence property, if $\forall i \in N, i \notin S, \forall \mu \in (0, 1], \forall S, T \in 2^N$, with $S \subset T$, $v_i^T \geq v_i^S$.

Whenever the game $\Gamma$ has the congruence property, it can be proved that $W_\Gamma$ is convex.

**Private Common Agency Games.** In a private common agency game, the action chosen by the agent is a list on $n$ actions and principal $i$ only cares about the $i$-th component in the list. Moreover, it is assumed that the set of each principal’’s feasible actions, is a sublattice of $\mathbb{R}^K$ and that the utility functions of the principals are increasing. This means that the agent supplies to each principal a $K$-dimensional vector of goods or services, and for each principal more is better. In most of the applications Laussel and Le Breton consider, the agent is a firm whose cost function is derived from its technology, and the principals are customers of this firm.
Formally, a private common agency game is a common agency game \( \Gamma = \{ A, V_0, V_1, \ldots, V_n \} \), such that \( A = \Pi_{i \in N} A_i \) and \( V_i(a) \equiv V_i(a_i), \ \forall \ i \in N, \ \forall a = (a_1, a_2, \ldots, a_n) \in A \). An action is an \( n \)-tuple of actions and principal \( i \) is concerned only by the \( i \)-th coordinate of the action. Otherwise, there are no restrictions on \( V_0 \).

Hereafter, the authors focus on the case where \( A_i \subset \mathbb{R}^K, \ \forall \ i \in N \). Let \( a_i \geq b_i, \ \forall i \in N, \) if \( a_i^k \geq b_i^k, \ \forall k = 1, \ldots, K, \) and let \( a_i \succ b_i, \) if \( a_i \geq b_i \) and there exists \( k \) such that \( a_i^k \geq b_i^k \). The "meet" \( a_i \land b_i \) and the "join" \( a_i \lor b_i \) of \( a_i \) and \( b_i \) are defined as follows:

\[
\begin{align*}
a_i \land b_i & \equiv (\min(a_i^1, b_i^1), \ldots, \min(a_i^K, b_i^K)) \\
a_i \lor b_i & \equiv (\max(a_i^1, b_i^1), \ldots, \max(a_i^K, b_i^K)).
\end{align*}
\]

Assume that \( A_i \) is a sublattice of \( \mathbb{R}^K \), i.e. if \( a_i, b_i \in A_i \), then \( a_i \land b_i \in A_i \) and \( a_i \lor b_i \in A_i \). Also, assume that \( A_i \) has a lowest element \( a_i \). Let \( a \geq b, \) if \( a_i \geq b_i, \ \forall i \in N \) and \( a \succ b, \) if \( a \geq b \) and \( \exists \ i \) such that \( a_i \succ b_i \). \( A \) is a sublattice of \( \mathbb{R}^{nK} \), with a lowest element \( a \equiv (a_1, \ldots, a_n) \) for the partial order \( \geq \).

Assume that \( \forall i \in N, \ V_i \) is increasing with respect to \( \succeq \), i.e. \( a_i \succ b_i \) implies \( V_i(a_i) \geq V_i(b_i) \) and that \( V_0 \) is decreasing with respect to \( \geq \), i.e. \( a \succ b \) implies \( V_0(a) \geq V_0(b) \). According to the normalization defined before, this implies: \( V_0(a) = V_0(b) = 0 \). They refer to the cos function as being the function \( C \equiv -V_0 \), where \( C \) is increasing and \( C(a) = 0 \).

A private common agency game refers to situations in which an agent provides to each principal \( i \) a quantity \( a_i^k \) of good or service \( k, \ \forall k = 1, \ldots, K \). Each principal concerns only about the quantities she obtains. The production of these quantities is costly for the agent. In the common agency game, each principal announces to the agent a monetary payment contingent on the quantities that she will receive.

**Definition 2.5.** The function \( C \) is supermodular if \( C(a) + C(b) \leq C(a \land b) + C(a \lor b) \), \( \forall a, b \in A \). The function \( C \) is strictly supermodular if the inequality is strict, whenever \( a \) and \( b \) cannot be compared with respect to \( \geq \).

**Definition 2.6.** The function \( C \) is submodular if \( C(a) + C(b) \geq C(a \land b) + C(a \lor b) \), \( \forall a, b \in A \). The function \( C \) is strictly submodular if the inequality is strict, whenever \( a \) and \( b \) cannot be compared with respect to \( \geq \).

When \( C \) is submodular or supermodular, there are costs that are complements or substitutes, respectively.
Proposition 2.5. Consider a private common agency game $\Gamma$. If $C$ is submodular and $V_i$ is supermodular, $\forall i \in N$, then $\Gamma$ has the no-rent property.

The proposition above states that there are no equilibria in which the agent makes rent in the case of complementary costs, while the following result—combined with the Theorem 2.4—implies that if $C$ is strictly supermodular, then the agent will always get a rent.

Proposition 2.6. Consider a private common agency game $\Gamma$. If $C$ is strictly supermodular, then the TU cooperative game $W_\Gamma$ is strictly subadditive.

When $C$ is supermodular, necessary and sufficient conditions on $C$ for $W_\Gamma$ to be concave are not known. Thus, we can sometimes rely on the following "disagreement property", which straightforwardly implies the concavity of $W_\Gamma$ and therefore the existence of a unique solution of (2.2.3).

Definition 2.7. A common agency game $\Gamma \equiv \{A, V_0, V_1, \ldots, V_n\}$ has the disagreement property, if $\forall i \in N, i \notin S$, $\forall \mu \in (0, 1]$, all $S, T \in 2^N$ with $S \subset T$, $v_T^i \leq v_S^i$.

2.2.2 Further Research

Proceeding the study of Laussel and Le Breton (1996, 2001), Billette de Villemeur and Versaevel (2003) attempt to bridge the gap between the private and public common agency, by widening the applicability of the common agency theory to real-world cases that fall in between the two classes of games. Examples include situations in which principals are interested in reselling received quantities on a final market for substitutable or complementary goods. They prove that Proposition 2.6—a result in the class of private common agency models—remains valid in situations in which each principal’s gross monetary payoff depends not only on the quantities $a_i$ she receives, but also directly on the quantities $a_{-i}$ received by the other principals.

Martimort (1996) uses the theory of multiprincipals—which describes how different incentive mechanisms compete with each others—in order to open the black box of the government organization. First, he defines a regulatory charter as an allocation of different regulatory rights among several regulators. Second, he argues that the limits in the exercise of the regulatory authority correspond in fact to an optimal organizational response to the threat of regulators’ non-benevolence. He presents a simple model of the interaction between two
regulatory agencies (principals) controlling non-cooperatively the same regulated firm (agent). He explores the possible costs on this non-cooperative provision of incentives, when regulators are benevolent. Finally, he shows how non-benevolent agencies can be better controlled when powers are separated. Separation is good, since it reduces the regulators’ discretion and the scope for their non-benevolent behavior.

As we saw earlier, in a public common agency game under complete information, Bernheim and Whinston (1986b) argued that we should focus on the class of truthful Nash equilibria of such games, because, first, they yield an efficient outcome, second, truthful equilibria are generally coalition-proof, and third, truthful Nash equilibria have been found attractive in applied research, since the distribution of truthful payoffs is easily characterized by means of simple inequalities. Despite these attractive properties, the use of truthful contribution schedules raises two sets of issues. First, can we find other equilibrium schedules and still maintain efficiency? What is the impact of alternative choices of extensions for the characterization of the principals’ payoffs? Second, given that the choice of extensions might have a significant impact on the distribution of equilibrium payoffs, can we find any rationale for relying on some particular extensions and, if so, what are the consequences?

Martimort and Stole (2009) address these questions. They show that the set of equilibrium payoffs of the delegated common agency game under complete information, that can be achieved when the principals’ and the agent’s choices are obtained at a differentiability point, is entirely described by considering only equilibria with truthful schedules. Asymmetric information can sometimes provide a rationale to select within that set of truthful payoffs. They conclude with the relaxation of the differentiability requirement; they find that inefficient equilibria may be sustained, and asymmetric information does not provide any rationale for focusing on differentiable schedules or restrict the equilibrium set.
Chapter 3

Common Agency and Political Influence

It is common knowledge that in representative democracies, governments form their policy taking into consideration not only the concerns of the general electorate, but also the pressures exerted by special interests. Groups of individuals with common interests usually attempt to advance them by participating in the political process, in order to influence policy outcomes. Policy outcomes, then, are a result determined by two factors: interest groups’ desire for policies favoring their members and politicians’ desire for re-election.

These organized interests employ a variety of methods (e.g. campaign contributions, transmission of information, endorsements, grassroots campaigns, media campaigns and lobbying), with the purpose of influencing politicians’ decisions in favor of their members’ welfare. As (Tullock, 1972) suggested, these favors are likely to be quite specific, such as receiving an import quota or settling an antitrust action, because contributors prefer payoffs that are beneficial primarily for themselves. The literature normally relates to these activities undertaken by the interest groups as lobbying.

In this chapter, we present applications of the common agency model concerning political influence, following the division made by Mallard (2014). He distinguishes four areas, in which models such as these have been used: the analysis of the political determination of policies regarding international markets, the environment, the domestic economy and the examination of the nature of the political system itself. Beginning with issues concerning foreign markets and the pathbreaking Protection for Sale (Grossman and Helpman, 1994), we take a retrospective look at each of the above areas of study.

1Interest groups may provide legislators with information about the possible effects of a potential policy or about other legislators’ attitude toward it. Likewise, they may attempt to educate the general public, with the intention to shape the public opinion in a way that will be beneficial for their cause (see Grossman and Helpman (2001)).
3.1 Foreign Market Issues

3.1.1 Protection for Sale

Grossman and Helpman (1994) develop a model in which interest groups (principals) seek to influence an incumbent government’s (agent) choice of trade policy by making political contributions. These political contributions offered by interest groups are valued by politicians for their potential use in the coming election. This ability to contribute gives interest groups their privileged position in the eyes of the government.

The lobbying process goes as follows. Each organized interest group sets a contribution schedule, which corresponds every policy vector that might be chosen by the government to a campaign contribution level. The government then sets a policy vector and collects from each lobby the contribution related to its policy choice. An equilibrium is a set of contribution schedules, such that each lobby’s schedule maximizes the total utility of the lobby’s members, considering as given the schedules set by the other lobby groups.

The Model

Consider a small economy, which is populated by individuals with identical preferences but different factor endowments. Each individual maximizes utility given by

\[ u = x_0 + \sum_{i=1}^{n} u_i(x_i), \]

where \( x_0 \) is consumption of good 0 and \( x_i \) is consumption of good \( i, i = 1, 2, \ldots, n \). The sub-utility functions \( u_i(\cdot) \) are differentiable, increasing and strictly concave. Good 0 serves as numeraire, with a world and domestic price equal to 1. \( p_i^* \) is the exogenous world price of good \( i \), while \( p_i \) represents its domestic price. With these preferences, an individual spending an amount \( E \), consumes \( x_i = d_i(p_i) \) of good \( i, i = 1, 2, \ldots, n \) (where the demand function \( d_i(\cdot) \) is the inverse of \( u_i'(x_i) \)) and \( x_0 = E - \sum_i p_i d_i(p_i) \) of the numeraire good. Indirect utility takes the form

\[ V(p, E) = E + s(p), \]
where \( \mathbf{p} = (p_1, p_2, \ldots, p_n) \) is the vector of domestic prices of the non-numeraire goods and 
\[
s(p) \equiv \sum_i u_i [d_i(p_i)] - \sum_i p_i d_i(p_i)
\]
is the consumer surplus derived from these goods.

Good 0 is manufactured from labor alone with constant returns to scale and an input–output coefficient equal to 1. Assuming that the aggregate supply of labor is sufficient to ensure a positive supply of this good, the wage rate equals 1 in a competitive equilibrium. Production of each non-numeraire good requires labor and a sector-specific input, and the technologies for these goods exhibit constant returns to scale. With the wage rate fixed at 1, the aggregate reward to the specific factor used in producing good \( i \) depends only on the domestic price of that good \( (p_i) \). This reward is denoted by \( \pi_i(p_i) \).

The government is restricted to implement only trade taxes and subsidies. The result of these policies is a wedge between domestic and world prices. A domestic price in excess of the world price implies an import tariff for a good that is imported and an export subsidy for one that is exported. In contrast, domestic prices below world prices correspond to import subsidies and export taxes. The net revenue from all taxes and subsidies, expressed on a per capita basis, is given by

\[
r(p) = \sum_i (p_i - p_i^*) \left[ d_i(p_i) - \frac{1}{N} y_i(p_i) \right],
\]

where \( N \) measures the total (voting) population and \( y_i(p_i) = \pi'(p_i) \) is domestic output of good \( i \). Assuming that the government redistributes revenue uniformly to all of the country’s voters, then \( r(p) \) gives the net government transfer to each individual.

In Grossman and Helpman’s model, the various owners of the specific factor in industry \( i \), with their common interest in protection (or export subsidies) for their sector overcome free-rider problems discussed by Olson (1965) and may choose to join forces for political activity.\(^2\) They simply assume that in some exogenous set of sectors, denoted \( L \), the specific-factor owners have been able to organize themselves into lobby groups, while in the remaining sectors (if any), the individual owners of the specific factors remain unorganized.

The lobby representing an organized sector \( i \) makes its political contribution contingent on the trade-policy vector implemented by the government.

\(^2\)In his seminal work, Olson (1965) argues that collective action will not take place, if individuals are rational egoists and the group is large. The reason is that interest groups trade in collective, or public, goods, which are characterized by non-excludability. Thus, a rational, self-interested individual should prefer to free-ride; to enjoy the benefits of the collective goods without contributing to the costs.
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The contribution schedule offered by lobby $i$ is denoted by $C_i(p)$. The lobby customizes this schedule to maximize the total welfare (income plus consumer surplus less contributions) of its members, $V_i = W_i - C_i$, where $W_i$ is their gross-of-contribution total welfare. We note that

$$W_i(p) \equiv \ell_i + \pi_i(p_i) + \alpha_i N[r(p) + s(p)], \quad (3.1.2)$$

where $\ell_i$ is the total labor supply (and also the labor income) of owners of the specific input used in industry $i$, and $\alpha_i$ is the fraction of the voting population that owns some of this factor.

The government cares about the total level of political contributions, because they can be used to finance campaign spending, and about aggregate well-being, because a government that has delivered a high standard of living is more likely to be re-elected. A linear form for the government’s objective function is chosen, namely,

$$G = \sum_{i \in L} C_i(p) + aW(p), \quad a \geq 0, \quad (3.1.3)$$

where $W$ represents aggregate, gross-of-contributions welfare. Aggregate gross welfare equals aggregate income plus trade tax revenues plus total consumer surplus; that is

$$W(p) = \ell + \sum_{i=1}^{n} \pi_i(p_i) + N[r(p) + s(p)]. \quad (3.1.4)$$

An equilibrium of a two-stage non-cooperative game, in which the lobbies simultaneously choose their political contribution schedules in the first stage and the government sets policy in the second, is a set of contributions functions $\{C^*_i(p)\}$, one for each organized lobby group, such that each one maximizes the joint welfare of the group’s members given the schedules set by the other groups and the anticipated political optimization by the government, and a domestic price vector $p^*$, that maximizes the government’s objective taking the contributions schedules as given.

---

3The government’s welfare function could alternatively written as $\tilde{G} = a_1 \sum_i C_i + a_2(W_i - \sum_i C_i)$, where $a_1$ is the weight the government attaches to campaign contributions, and $a_2$ is the weight it attaches to net aggregate welfare. Maximizing $\tilde{G}$ is equivalent to maximizing $G$ in (3.1.3), with $a = a_2 / (a_1 - a_2)$, provided that a dollar spent in the campaign is more valuable by politicians than a dollar in the hands of the public ($a_1 > a_2$).
The Structure of Protection

The interaction between the various lobbies and the government in this economy has the structure of a menu-auction problem. Grossman and Helpman (1994) allow the government’s choice set (of domestic price vectors) to be continuous. They do so, because although Bernheim and Whinston (1986b) limited their analysis about the structure of equilibrium for a class of such problems to situation where players bid for a finite set of objects, their main results apply also when, as here, the auctioneer can choose from a continuum of possible actions.

Let \( P \) denote the set of domestic price vectors from which the government may choose. \( P \) is bounded, so that each domestic price \( p_i \) must lie between some minimum \( \bar{p}_i \) and some maximum \( \tilde{p}_i \), i.e. \( p_i \in [\bar{p}_i, \tilde{p}_i] \). Lemma 2.1 implies that an equilibrium to the trade-policy game can be characterized as follows:

**Proposition 3.1.** (B-W) \((\{C^o_i\}_{i \in L}, p^o)\) is a subgame-perfect Nash equilibrium of the trade-policy game if-f:

(i) \( C^o_i \) is feasible \( \forall i \in L \);

(ii) \( p^o \) maximizes \( \sum_{i \in L} C^o_i(p) + aW(p) \) on \( P \);

(iii) \( p^o \) maximizes \( W_j(p) - C^o_j(p) + \sum_{i \in L} C^o_i(p) + aW(p) \) on \( P \), \( \forall j \in L \);

(iv) \( \forall j \in L, \exists p^j \in P \) that maximizes \( \sum_{i \in L} C^o_i(p) + aW(p) \) on \( P \), such that \( C^o_j(p^j) = 0 \).

Condition (i) restricts each lobby’s contribution schedule to be among those that are feasible (i.e. contributions must be non-negative and no greater than the aggregate income available to the lobby’s members). Condition (ii) demonstrates that, given the contribution schedules offered by the lobbies, the government sets trade policy to maximize its own welfare. Condition (iii) states that, for every lobby \( j \), the equilibrium price vector must maximize the joint welfare of that lobby and the government, given the contribution schedules offered by the other lobbies. Finally, condition (iv) requires that for every \( i \) there must exist a policy that elicits a contribution of zero from lobby \( i \), which the government finds equally attractive as the equilibrium policy \( p^o \).
Assume now that the lobbies set political-contribution functions that are differentiable, at least around the equilibrium point $p^o$. The fact that $p^o$ maximizes $V_j + G$ implies that a first-order condition is satisfied at $p^o$, namely,

$$\nabla W^o_j(p^o) - \nabla C^o_j(p^o) + \sum_{i \in L} \nabla C^o_i(p^o) + a \nabla W(p^o) = 0, \quad \forall j \in L. \quad (3.1.5)$$

However, the government’s maximization of $G$ requires the first-order condition

$$\sum_{i \in L} \nabla C^o_i(p^o) + a \nabla W(p^o) = 0. \quad (3.1.6)$$

Taken together (3.1.5) and (3.1.6) imply

$$\nabla C^o_i(p^o) = \nabla W^o_i(p^o), \quad \forall i \in L. \quad (3.1.7)$$

Equation (3.1.7) establishes that the contribution schedules all are locally truthful around $p^o$; that is, each lobby sets its contribution schedule so that the marginal change in the contribution for a small change in policy matches the effect of the policy change on the lobby’s gross welfare.

Grossman and Helpman extend the notion of "truthfulness" to define a truthful contribution schedule. This is a contribution schedule that everywhere reflects the true preferences of the lobby. It pays to the government, for any policy $p$, the excess (if any) of lobby $j$’s gross welfare at $p$ relative to some base level of welfare. Formally, a truthful contribution takes the form

$$C^T_j(p, B_j) = \max \left[ 0, W_j(p) - B_j \right] \quad (3.1.8)$$

for some $B_j$.\(^4\)

Truthful Nash equilibria (TNE) have an interest property. It is proved that the equilibrium price vector of any TNE satisfies

$$p^o = \arg \max_{p \in \mathcal{P}} \left[ \sum_{j \in L} W_j(p) + aW(p) \right]. \quad (3.1.9)$$

Equation (3.1.9) says that in equilibrium, truthful contribution schedules induce the government to behave as if it were maximizing a social-welfare function.

\(^4\)We saw in Chapter 2 that players incur essentially no cost from playing truthful strategies, because the set of best responses to any strategies played by one’s opponents includes a strategy that is truthful. Also, all equilibria supported by truthful strategies, and only these equilibria, are stable to non-binding communication among the players. For these reasons, truthful Nash equilibria may be focal among the set of Nash equilibria.
that weights different members of society differently, with individuals represented by a lobby group receiving a weight $1 + a$, and those not so represented receiving the smaller weight of $a$.

Return now to the characterization of equilibrium trade policies that can be supported by differentiable contribution schedules. Sum (3.1.7) over $i$ and substitute the result into (3.1.6) to derive

$$\sum_{i \in L} \nabla W_i(p^o) + a \nabla W(p^o) = 0. \quad (3.1.10)$$

Equation (3.1.10) characterizes the equilibrium domestic prices supported by differentiable contribution functions. Notice that this is just the first-order condition that is necessary for the maximization in (3.1.9).

In what follows, the authors calculate how marginal policy changes affect the welfare of the various groups in society. Looking first at the members of some lobby $i$, they find from (3.1.1) and (3.1.2) that

$$\frac{\partial W_i}{\partial p_j} = (\delta_{ij} - \alpha_i)y_j(p_j) + \alpha_i(p_j - p_j^*)m_j'(p_j), \quad (3.1.11)$$

where $m_j(p_j) \equiv N d_j(p_j) - y_j(p_j)$ denotes the net import demand function and $\delta_{ij}$ is an indicator variable that equals 1 if $i = j$ and 0 otherwise. Equation (3.1.11) states that lobby $i$ gains both from an increase in the domestic price of good $i$ above its free-trade level and from a decrease in the price of any other good (because $m_j' < 0$). The specific-factor owners benefit more from an increase in the price of their industry’s output the larger is the free-trade supply of the good. The benefit to lobby $i$ that results from a decline in the price of another good $j$ falls as the share of the members of lobby $i$ in the total population shrinks, and it vanishes completely in the limit when $\alpha_i = 0$.

To determine how a policy change has an impact on the gross welfare of the entire group of individuals who are actively trying to influence policy, they sum

\[ \sum_{j \in L} W_j(p) + a W(p) = \sum_{j \in L} W_j(p) + a \left[ \sum_{j \in L} W_j(p) + W_{-j}(p) \right] \]

\[ = (1 + a) \sum_{j \in L} W_j(p) + a W_{-j}(p). \]
the expressions in (3.1.11) for all \( i \in L \), to derive

\[
\sum_{i \in L} \frac{\partial W_i}{\partial p_j} = (I_j - \alpha_L)y_j(p_j) + \alpha_L(p_j - p_j^*)m_j'(p_j),
\]

(3.1.12)

where \( I_j \equiv \Sigma_{i \in L} \delta_{ij} \) is an indicator variable that equals 1 if industry \( j \) is organized and 0 otherwise, while \( \alpha_L \equiv \Sigma_{i \in L} \alpha_i \) denotes the fraction of the total population of voters who are represented by a lobby. Equation (3.1.12) reveals that, starting from free-trade prices, lobby members as a whole benefit from a small increase in the domestic price of any good that is produced by an organized industry and, provided \( \alpha_L > 0 \), from a small decline in the price of any good that is produced by an unorganized industry.

Finally, to compute the effect of a marginal price change on aggregate welfare, they use the definition of \( W \) in (3.1.4). They find

\[
\frac{\partial W}{\partial p_j} = (p_j - p_j^*)m_j'(p_j),
\]

(3.1.13)

which reveals that marginal deadweight loss grows as the economy deviates further and further from free trade. Substituting (3.1.12) and (3.1.13) into (3.1.10), they solve for the domestic prices in political equilibrium, assuming that these prices lie in the interior of \( \mathcal{P} \), and they express the result in terms of the equilibrium ad valorem trade taxes and subsidies, which are defined by \( t_{oi} \equiv (p_{oi} - p_i^*)/p_i^* \).

**Proposition 3.2.** (Equilibrium Policies) If the lobbies use contribution schedules that are differentiable around the equilibrium point, and if the equilibrium lies in the interior of \( \mathcal{P} \), then the government chooses trade taxes and subsidies that satisfy

\[
\frac{t_{oi}}{1 + t_{oi}} = \frac{I_i - \alpha_L}{a + \alpha_L} \left( \frac{z_{oi}}{e_{oi}} \right), \quad \text{for } i = 1, 2, \ldots, n,
\]

where \( z_{oi} = y_i(p_{oi})/m_i(p_{oi}) \) is the equilibrium ratio of domestic output to imports (negative for exports) and \( e_{oi} = -m_i'(p_{oi})/m_i(p_{oi}) \) is the elasticity of import demand or of export supply (the former defined to be positive, the latter negative.

Proposition 3.2 implies that all else equal, industries that have high import demand or export supply elasticities (in absolute value) will have smaller ad valorem deviations from free trade. This is true for two reasons. First, the government may incur a political cost from creating deadweight loss (if \( a > 0 \)). To the extent that this is so, all else equal, it will prefer to raise contributions from sectors where the cost is small. Second, even if \( a = 0 \), if \( \alpha_L > 0 \), the
members of lobbies as a group will share in any deadweight loss that results from trade policy. The owners of specific inputs in industries other than \( i \) will bid more to avoid protection in sector \( i \) the greater is the social cost of that protection.

We can also note that all sectors that are represented by lobbies are protected by import tariffs or export subsidies in the political equilibrium. In contrast, imports subsidies and export taxes are applied to all sectors that have no organized representation. In other words, the organized interest groups collectively manage to raise the domestic prices of goods from which they derive profit income and to lower the prices of goods that they only consume.

The smaller is the weight that the government places on a dollar of aggregate welfare compared with a dollar of campaign financing, the larger in absolute value are all trade taxes and subsidies. An interior solution remains possible, however, even if the government cares only about contributions \((a = 0)\). This is because the interest groups themselves do not want the distortions to grow too large. As the share of voters who are members of one interest group or another increases, equilibrium rates of protection for the organized industries decline. At the extreme, when all voters belong to an interest group \((\alpha_L = 1)\) and all sectors are represented \((I_i = 1, \forall i)\) then free trade prevails in all markets, because the various interest groups neutralize one another. On the other hand, if interest-group members constitute a negligible fraction of the voting population \((\alpha_L = 0)\), then no trade taxes or subsidies will be applied to goods not represented by a lobby (for which \( I_i = 0 \)).

**Political Contributions**

Grossman and Helpman focus on truthful Nash equilibria—the equilibria that arise when lobbies announce truthful contribution schedules. With this restriction, the competition between the lobbies involves only a choice of the scalars \( \{B_i\} \). Given these "anchors" for the contribution functions, the truthfulness requirement dictates the shapes of the schedules.

From the definition (3.1.8), we see that whenever the lobby makes a positive contribution to the government in equilibrium, the net welfare to lobby \( i \) will be \( B_i (W_i - C_i = B_i) \). The lobby therefore wishes to make \( B_i \) as large as possible and the contributions as small as possible, but without going so far as to induce the government to deviate from \( p^o \) to some alternative policy that might damage its interests.
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Thus, each lobby must worry about what policy would be selected if it were to raise its $B_i$ to a level where the government would choose to neglect its interests entirely. Define $p^{-i}$ as the policy that would emerge from political maximization by the government, if the contributions offered by lobby $i$ were zero; that is

$$p^{-i} = \arg\max_{p \in \mathcal{P}} \sum_{j \in L, j \neq i} C^T_j (p, B^o_j) + aW(p), \quad \text{for } i \in L. \quad (3.1.14)$$

Lobby $i$ will raise its $B_i$ to the point where the government is just indifferent between choosing the policy $p^{-i}$ and choosing the equilibrium policy $p^o$. This indifference is expressed by the following equation:

$$\sum_{j \in L, j \neq i} C^T_j (p^{-i}, B^o_j) + aW(p^{-i}) = \sum_{j \in L} C^T_j (p^o, B^o_j) + aW(p^o), \quad \forall i \in L. \quad (3.1.15)$$

These two sets of equations allow us to solve for the net welfare levels of the various lobbies in a truthful Nash equilibrium with positive contributions by all lobbies. As a consistency check, it must be made sure that at $B^o_i$ lobby $i$ would make no contributions were the policy $p^{-i}$ to be chosen by the government. This requires $W_i(p^{-i}) \leq B^o_i$, $\forall i \in L$.

The authors examine three cases, to see how the equilibrium contributions are determined in different situations.

**A Single Organized Lobby.** In this case, there is only one politically active lobby group, which represents the interests of the specific-factor owners in some industry $i$. The equilibrium policy vector here provides protection for sector $i$, i.e. $p^o_i > p^*_i$, and so long as $\alpha_i > 0$, it calls for import subsidies and export taxes on all other goods, i.e. $p^o_j < p^*_j$, for $j \neq i$. It is known that the government would opt for free trade in the absence of any contributions from the one and only interest group; thus (3.1.14) gives $p^{-i} = p^*$. From (3.1.15), the equilibrium campaign contribution of lobby $i$ is $C^T_i (p^o, B^o_i) = aW(p^*) - aW(p^o)$.$^6$

It is obvious that the lobby contributes an amount that is proportional to the excess burden that the equilibrium trade policies impose on society. In this political equilibrium, the politicians derive exactly the same utility as they would have achieved by allowing free trade in a world without influence payments.

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$^6$Since there is only one politically active lobby group, $\sum_{j \in L, j \neq i} C^T_j (p^{-i}, B^o_j) = 0$, for $i \in L$. 
In other words, a lobby that faces no opposition from competing interests captures all of the surplus from its political relationship with the government.

**All Voters Represented as Special Interests.** Now, all of the voters are represented in the political process by one lobby or another. The political competition in this case results in free trade, i.e. \( p^o = p^* \). Nonetheless, each lobby must make a positive campaign contribution in order to induce the government to choose this outcome rather than one that would be still worse from its perspective. For example, if there are only two non-numeraire goods and two lobbies, using (3.1.15) we have

\[
C_i^T(p^o, B^o_i) = \left[ C_j^T(p^{-i}, B^o_j) + aW(p^{-i}) \right] \\
- \left[ C_j^T(p^o, B^o_j) + aW(p^o) \right], \quad \text{for } i = 1, 2; j \neq i.
\]

(3.1.16)

By the definition of \( p^{-i} \) and the fact that \( p^{-i} \neq p^* = p^o \), the right-hand side of (3.1.16) is positive for \( i = 1, 2 \). Thus both lobbies must actively contribute to the incumbent government in order to support the free-trade outcome. When all voters are active in the process of buying influence, the rivalry among competing interests is most intense, and the government captures all of the surplus from the political relationships.

To answer which of the two lobbies makes the larger contribution, the authors transform (3.1.16) as

\[
C_i^T(p^o, B^o_i) = \left[ W_j(p^{-i} + aW(p^{-i})) \right] \\
- \left[ W_j(p^*) + aW(p^*) \right], \quad \text{for } i = 1, 2; j \neq i.
\]

(3.1.17)

(3.1.17) implies that each lobby \( i \) must contribute to the politicians an amount equal to the difference between what its rival and the government could jointly achieve were lobby \( i \) not itself active in the political process, and what the two actually attain in the full political equilibrium. Thus, each lobby pays according to the political strength of its rival.

**Represented Special Interests are Highly Concentrated.** In the final example, the ownership of the specific factors is so highly concentrated that interest-group members account for a negligible fraction of the total voting population. In this case, the political equilibrium has positive protection for all organized sectors. But since \( \alpha_i = 0, \forall i \), the members of each interest group receive only a
negligible share of government transfer payments and derive only an insignificant share of the surplus from consuming non-numeraire products. Thus, no lobby is willing to contribute toward trade intervention in any sector other than its own. The common agency problem here is the same as for a set of separate principal-agent arrangements between each industry lobby and the government. As in the first case, each lobby $i$ must compensate the government for the political cost of providing protection—it pays $a$ times the deadweight loss imposed by the industry policy $p^o_i$. But with no political rivalry between the special interests, each industry group captures all of the surplus from its own political relationship with the government.

3.1.2 Further Research

The work of Grossman and Helpman (1994) has been the trigger for the theoretical and empirical Protection for Sale literature. Rama and Tabellini (1998) use a similar approach to analyze the joint determination of product and labor market distortions in a small open economy. In contrast to the many sectors of Grossman and Helpman’s model, they study a small open economy with only two sectors: manufacturing and agriculture. In the former sector there are two organized groups—capital owners and union members—, while factors of production in the latter are not organized in political lobbies. The government can only affect the manufacturing sector with two policy instruments: a tariff and a minimum wage. Thus, unlike Grossman and Helpman, Rama and Tabellini focus on how different factors in the same sector lobby the government for distortionary policies that, on the one hand, create rents (the tariff) and, on the other, determine the distribution of these rents (the minimum wage). They find that despite the conflict of interest on labor market policies, the two policy instruments are complements, and that the equilibrium level of labor and product market distortions is not modified by social pacts between capital and labor.

An extension of the Grossman–Helpman model is that of (Fung et al., 2009). In their paper, they investigate the formation of strategic export subsidy and strategic import tariff under Cournot and Bertrand competition. They identify the winners and losers from lobbying and they derive optimal trade policies and politically-determined trade policies. Moreover, they highlight the possibility that if the government is not a benevolent dictator and it concerns only about contributions, then welfare may be improved by lobbying.
Haaparanta (1996) studies international competition for foreign investment (FDI), modelling national governments as principals and a multinational corporation as the single agent. He shows that in a world where countries or regions use subsidies to compete for investments: (i) the allocation of investment may be affected by competition, even if all countries subsidize investment, (ii) a country in which "competitiveness" in terms of production costs deteriorates may increase its share of FDI and (iii) a country may lose FDI even if it pays larger subsidies than other countries. Konishi et al. (1999) construct a political economy model in which a domestic firm and a foreign firm influence the domestic government’s trade policy through their campaign contributions. After the domestic government chooses its trade policy, the foreign firm may avoid trade restrictions by choosing FDI instead of exporting as the means of serving the domestic market. Thus, the foreign firm can use FDI as a threat to influence trade policy. They characterize equilibrium contributions of the two firms and show that the domestic government prefers a voluntary export restraint (VER) to a tariff.

Finally, Di Gioacchino et al. (2008) examine the role played by the distribution of domestic wealth in determining a country’s level of access to international lending and Mayer and Mourmouras (2005) model the relationship between an aid-providing international financial institution (IFI) (e.g. International Monetary Fund (IMF)) and an aid-receiving government, whose economic policy choices are influenced by a domestic interest group. The main result of the former study is that a country’s ability to enter international credit markets does not depend on its aggregate wealth but rather on its distribution. The more unequal the distribution of wealth the more political pressure the dominant domestic political actors can exert to impose their interests. A result from the latter study is that the less a government’s political support depends on the general public’s wellbeing, the less unconditional assistance is provided by the IFI and the more distorting policies are adopted by the receiving country’s government. Also, conditional aid programs raise welfare of the general public in the receiving country, as well as of the world as a whole.\footnote{In conditional aid the IFI makes assistance contingent on less distorting economic policies, and unconditional aid is provided without such conditions.}
3.2 Environmental Policies

3.2.1 The Political Economy of Pollution Taxes in a Small Open Economy

In his paper, Fredriksson (1997) develops a positive theory that predicts pollution tax policy outcomes in a small open economy. He employs a common agency model with complete information, in which an environmental and an industry lobby group each offer a menu of prospective campaign contributions in return for pollution tax policy choices made by the government. Similarly to the Grossman–Helpman model, re-election is the only concern of the government, and the probability of electoral success depends on aggregate campaign contributions and on aggregate social welfare. Moreover, the government is not assumed to face an explicit challenger in an upcoming election; instead, the model incorporates implicit political competition.

Fredriksson finds that the political equilibrium tax rate on pollution differs from the Pigouvian rate. This can be explained by lobby group membership, the government’s weight on aggregate social welfare relative to lobbying activities and the tax elasticity of pollution. Moreover, he shows that the equilibrium pollution tax rate may decrease in the pollution abatement subsidy, because of effects on the political forces determining the tax rate. The abatement subsidy reduces the industry’s marginal cost of pollution, and hence output increases, which may lead to an increase in pollution. Thus, lobbying activity may be stimulated by the industry lobby group because a higher total pollution level makes a low pollution tax rate more important. Simultaneously, the lower pollution per unit of output may reduce the lobbying effort of the environmental lobby group.

The Model

The Economy. A small open economy has two sectors: one sector produces the non-polluting numeraire good \( z \), the other sector produces the polluting good \( x \). The economy is populated by \( N \) heterogeneous citizens of three different types: workers, industrialists (factor owners) and environmentalists. \( N \) is normalized to one. All types of citizens have labor income, whereas only environmentalists have environmental concerns, and only industrialists have some
factor income from production of good $x$. Environmentalists derive disutility from the pollution associated with the local production of good $x$, and share identical separable preferences.

Environmentalists’ utility is given by

$$U^E = c_z + u(c_x) - \theta X,$$  \hfill (3.2.1)

where $c_z$ is consumption of the numeraire good $z$ with world and domestic price equal to one, $c_x$ is consumption of good $x$ with world and domestic price equal to $p^*$, and $u(c_x)$ is a strictly concave and differentiable sub-utility function. Production of good $x$ is given by $X$, and $\theta \geq 0$ is an exogenously given damage coefficient of pollution from each unit of production of good $x$. The sum of disutilities suffered by environmentalists equal total damages. All industrialists and workers share identical additively separable preferences. Industrialists’ and workers’ utility is given by

$$U^I = U^W = c_z + u(c_x).$$ \hfill (3.2.2)

It is assumed that free trade prevails in both markets. The only government’s environmental policy instrument is a pollution tax $t \in T$, $T \in \mathbb{R}$, levied on pollution associated with production of good $x$. The producer’s price, after the pollution tax is set, is then given by

$$p = p^* - t\theta.$$ \hfill (3.2.3)

Total revenue from pollution taxes equals

$$\tau(t) = t\theta X(p),$$ \hfill (3.2.4)

where $\tau(t)$ is assumed to be distributed uniformly to all individuals. The total endowment of labor equals $l$. Each individual has one unit of labor. The non-polluting numeraire good $z$ is produced by labor alone with a constant returns to scale technology, and an input-output coefficient equal to one by choice of unit. It is also assumed that the supply of labor is sufficiently large for the supply of good $z$ to be positive. A competitive equilibrium then implies a wage rate equal to one.

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8It is assumed that environmentalists do not realize that their consumption of good $x$ implies there is more production of good $x$, and therefore more damages suffered by them; each individual’s impact is negligible.
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Good $x$ is produced by two inputs with a constant returns to scale technology: labor and an immobile sector-specific input, available in inelastic supply, indivisible and non-tradeable (e.g. sector-specific capital). Supply is a function of the producer’s price, $X(p)$, where it is assumed that $X_p > 0$ and $X_{pp} = 0$. With a wage rate equal to one and the world market price fixed, the specific factor reward, denoted by $\Pi$, depends on the producer’s price $p$, i.e. $\Pi = \Pi(p)$. By Hotelling’s lemma, the supply curve for good $x$ is given by

$$X(p) = \Pi_p(p).$$

(3.2.5)

Individuals with similar interests in the $x$ sector are assumed to organize themselves into lobby groups, offering campaign contributions to the government. Individuals who derive disutility from pollution join the environmental lobby group, and individuals with income from the specific factor join the industry lobby group. Workers are assumed not to be organized. The existence of lobby groups is, then, exogenous and the membership is fixed. Let $i$ denote the type of lobby group, $E$ for environmental and $I$ for industry lobby group. $\alpha^i$ is the fraction of the population with membership in lobby group $i$. The environmental and industry lobby group make campaign contributions denoted by $\Lambda^E(t)$ and $\Lambda^I(t)$, respectively, which depend on the pollution tax rate selected by the government. An environmentalist solves the following problem:

$$\max_{c_z, c_x} U^E = c_z + u(c_x) - \theta X$$

subject to

$$l + \tau = c_z + p^* c_x + \Lambda^E/\alpha^E,$$

(3.2.6)

whereas an industrialist solves

$$\max_{c_z, c_x} U^I = c_z + u(c_x)$$

subject to

$$l + \tau + \Pi/\alpha^I = c_z + p^* c_x + \Lambda^I/\alpha^I,$$

(3.2.6a)

and a worker solves

$$\max_{c_z, c_x} U^W = c_z + u(c_x)$$

subject to

$$l + \tau = c_z + p^* c_x.$$
An individual spending $Y$ consumes $c_x = d(p^*)$ of good $x$, where the demand curve $d(p^*)$ for good $x$ is the inverse of $u_c(c_x)$ and consumption of the numeraire good $z$ equals $c_z = Y - p^*d(p^*)$. The indirect utility function for an environmentalist is expressed as $V^E(p^*, t, Y) = Y + u[d(p^*)] - p^*d(p^*) - \theta X$, and for an industrialist and workers who share the same indirect utility function, $V^I(p^*, t, Y) = Y + u[d(p^*)] - p^*d(p^*)$, where $u[d(p^*)] - p^*d(p^*)$ is the consumer surplus derived from consumption of good $x$.

**The Lobby Groups’ and the Government’s Utility Functions.** The utility of the environmental and industry lobby groups in the absence of campaign contributions are, respectively,

$$\Omega^E(t) \equiv \alpha^E[-\theta X(p) + \tau(t) + l]$$  

(3.2.7)

and

$$\Omega^I(t) \equiv \Pi(p) + \alpha^I[\tau(t) + l],$$  

(3.2.8)

where $\alpha^I[\tau(t) + l], i \in E, I$ is the share of total pollution tax revenue and labor income allocated to lobby group $i$, and $-\alpha^E\theta X(p)$ is the aggregate disutility from pollution of the environmental lobby group. There is no explicit competition between politicians. The incumbent government maximizes the probability of re-election facing an implicit challenger by the maximization of a weighted sum of aggregate campaign contributions and aggregate social welfare. The gross aggregate social welfare attained at the pollution tax rate $t$, ignoring contributions, is given by

$$\Omega^A(t) \equiv \tau(t) + l + \Pi(p) - \alpha^E\theta X(p).$$  

(3.2.9)

Since only interest groups make political contributions, the government’s utility function becomes

$$v^G = \sum_{i \in L} \Lambda^i(t) + a\Omega^A(t),$$  

(3.2.10)

where $L$ is the set of lobby groups, and $a \geq 0$ is the exogenously given weight that the government places on aggregate social welfare relative to campaign contributions. The government is implicitly assumed to take both the political value of funds and the indirect cost associated with the loss of welfare into consideration in the determination of the parameter $a$. 
The Game. In the first stage of the two-stage extensive form game between the incumbent government and the two lobby groups, each lobby group $i$ simultaneously offers the government a continent campaign contribution schedule $\Lambda^i(t)$. Hence, a lobby group’s strategy consists of a continuous function $\Lambda^i : T \rightarrow \mathbb{R}_+$; it offers the government a monetary contribution $\Lambda^i(t)$ for selecting policy $t$ from the government’s one-dimensional choice set $T$.

In the second stage, the government selects a tax policy and receives from each lobby the contribution associated with the policy selected. Lobby group $i$ receives gross monetary payoffs described by the continuous function $\Omega^i : T \rightarrow \mathbb{R}_+$. The government’s strategy is a policy scalar that maximizes its total utility, given the strategies of the lobby groups. The government selects $t^0 = \arg\max_{t \in T} [\sum_{i \in L} \Lambda^i(t) + a\Omega^A(t)]$, where $t^0$ denotes the equilibrium pollution tax rate.

For this menu auction, a set of contribution schedules $\{\Lambda^0_i\}_{i \in L}$, and a policy $t^0$ is a Subgame Perfect Nash Equilibrium, if each contribution schedule is feasible; the policy $t^0$ maximizes the government’s welfare $\sum_{i \in L} \Lambda^i(t) + a\Omega^A(t)$, taking the contribution schedules as given; and given the schedule of lobby group $j$, $\{\Lambda^0_j\}_{j \neq i}$, and the government’s anticipated decision rule, no lobby group $i$ has a feasible strategy that yields a net payoff greater than the equilibrium net payoff.

The Political Equilibrium

Following Proposition 3.1 in Grossman and Helpman (1994) and assuming that the equilibrium pollution tax lies in the interior of $T$, we have

**Proposition 3.3.** $(\Lambda^0_i)_{i \in L}, t^0)$ is a Subgame Perfect Nash Equilibrium if-f

(i) $\Lambda^0_i$ is feasible $\forall i \in L$;

(ii) $t^0$ maximizes $\sum_{i \in L} \Lambda^0_i(t) + a\Omega^A(t)$ on $T$;

(iii) $t^0$ maximizes $\Omega^j(t) - \Lambda^0_j(t) + \sum_{i \in L} \Lambda^0_i(t) + a\Omega^A(t)$ on $T$, $\forall j \in L$;

(iv) $\forall j \in L$, $\exists t^{-j} \in T$, that maximizes $\sum_{i \in L} \Lambda^0_i(t) + a\Omega^A(t)$ on $T$, such that $\Lambda^0(t^{-j}) = 0$.

Condition (i) requires that each lobby group’s contribution schedule is feasible, i.e. contributions are non-negative and less than or equal to the total income of the lobby group’s members. Condition (ii) stipulates that the government sets the pollution tax policy to maximize its own welfare, given the offered
contribution schedules. Condition (iii) states that the equilibrium pollution tax maximizes the joint welfare of each lobby group \( j \) and the government, given the other lobby group’s equilibrium contribution schedule. Finally, condition (iv) establishes the following: before deciding on which campaign contribution schedule to offer, lobby group \( j \) first decides which level of net welfare to use as a base level for the design of its schedule. This “anchor” level will at a minimum be raised until its least favorable policy implies a contribution equal to zero.

**The Equilibrium Characterization.** Condition (iii) in Proposition 3.3 implies that the following first order condition holds for lobby group \( j \) at \( t^0 \):

\[
Ω_j^t(t^0) - Λ_j^θ(t) + ∑_{i ∈ L} Λ_i^θ(t^0) + aΩ_A^t(t^0) = 0, \quad ∀ j ∈ L.
\]  

(3.2.11)

The government’s maximization of \( v^G \) in condition (ii) yields

\[
∑_{i ∈ L} Λ_i^θ(t^0) + aΩ_A^t(t^0) = 0,
\]  

(3.2.12)

which together with (3.1.9) implies

\[
Ω_j^t(t^0) = Λ_j^θ(t^0), \quad ∀ i ∈ L.
\]  

(3.2.13)

This implies that the contribution schedules are *locally truthful* around \( t^0 \); i.e. around the equilibrium point, each lobby \( i \) formulates its contribution schedule so that the marginal change in the contribution for a small change in tax policy equals the impact on the lobby group’s gross welfare of the tax change. Substituting (3.2.13) into (3.2.12), yields

\[
∑_{i ∈ L} Ω_j^t(t^0) + aΩ_A^t(t^0) = 0.
\]  

(3.2.14)

This is the equilibrium characterization assuming differentiable contribution functions.

**The Endogenous Pollution Tax.** Using (3.2.3)–(3.2.5), (3.2.7) and (3.2.8), the partial derivatives of the lobby groups utility functions with respect to the tax in the absence of contributions are

\[
Ω_i^E(t) = α^E[X_pθ^2 + θ(X - tX_pθ)],
\]  

(3.2.15)
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and

\[ \Omega_t^I(t) = -X\theta + \alpha^I(X - tXp\theta). \]  (3.2.16)

Equation (3.2.15) states that the environmental lobby group benefits from an increase in the tax levied on pollution from production of good \( x \), because pollution falls. Equation (3.2.16) states that the industry lobby group benefits from a decrease in the tax levied on pollution from production of good \( x \), adjusted for the extent to which the changes in tax revenues are channelled back to its members. The partial derivative of the government’s aggregate welfare function with respect to the pollution tax is obtained using (3.2.3)–(3.2.5) and (3.2.9):

\[ \Omega_t^A(t) = Xp\theta^2(\alpha^E - t). \]  (3.2.17)

From (3.2.17) the social welfare maximizing pollution tax selected by the government in the absence of lobby groups can be calculated.

**Proposition 3.4.** In the absence of lobby groups, the government selects an equilibrium pollution tax rate equal to \( t^0 = \alpha^E \).

In the absence of lobby groups, the tax rate selected equals the proportion of the population that derives disutility from pollution. At \( t^0 = \alpha^E \) the tax payment of the polluter sector \( x \) exactly equals the marginal disutility to society. The political equilibrium is the following:

**Proposition 3.5.** If the lobby groups employ campaign contribution schedules that are differentiable around the equilibrium point, and if the equilibrium lies in the interior of \( T \), the government selects a pollution tax that satisfies

\[ t^0 = \frac{\alpha^E(1 + \alpha)\delta^0}{(\alpha^E + \alpha^I + a)(1 + \delta^0) - (1 + a)}, \]  (3.2.18)

where \( \delta^0 = -[dX(p^0)\theta/dt][t/X(p^0)\theta] \) is the tax elasticity of total pollution.

The implications of Proposition 3.5 are the following: the environmental and industry lobby groups unambiguously influence the equilibrium tax rate positively and negatively, respectively. In the political equilibrium the tax rate depends only on the tax elasticity of pollution, lobby group membership and the government’s weight on aggregate social welfare. First, treating \( \delta^0 \) as a parameter, the tax rate is decreasing in (independent of) \( \delta^0 \), if \( (\alpha^E + \alpha^I) < (=) 1 \). Second, if all individuals are organized in lobby groups \( (\alpha^E + \alpha^I = 1) \), the pollution tax rate equals \( \alpha^E \). In this case, the government maximizes the aggregate welfare of
all lobby group members by selecting the efficient tax rate and, thus, maximizing deadweight loss. In this way, the government maximizes total campaign contributions. Third, the smaller the government’s weight on social welfare, the larger the relative political influence of the interest groups.

The discussion of Proposition 3.5 assumed a constant tax elasticity of pollution. This assumption is now relaxed. In order for the equilibrium characterized in (3.2.14) to be a utility maximum, the second-order condition of the corresponding expression is required to be negative. Assume that this requirement holds. This implies that the following condition holds. \textit{Assumption A1:} 
\[ 1 - 2(\alpha^E + \alpha^I) - a < 0. \]

\textbf{Proposition 3.6.} \textit{In equilibrium, (i) the equilibrium pollution tax is decreasing in the world market price, if some individuals are workers, and (ii) total pollution is increasing in the world market price.}

The rate of change of the tax rate depends on the damage coefficient, lobby group membership and the government’s weight on aggregate social welfare. The tax rate is decreasing in the world market price, because output increases in this price. It becomes more important for the industry lobby group to bid for a lower tax rate as output increases.

\textbf{Proposition 3.7.} \textit{In equilibrium, the pollution tax rate is increasing (decreasing) in the environmental lobby group membership, if the sum of: (i) the change in total tax revenue and (ii) the government’s valuation of the environmental lobby group’s marginal disutility from pollution is negative (positive).}

An increase in the membership of the environmental lobby group has three effects. First, the total disutility from pollution to the environmental lobby group increases. Second, aggregate social welfare is more heavily affected by pollution. Third, the environmental lobby group’s share of total pollution tax revenue increases.

\textbf{Proposition 3.8.} \textit{In equilibrium, the pollution tax is increasing (decreasing) in the industry lobby group membership, if total revenue is increasing (decreasing) in the tax rate.}

Whether total pollution tax revenue is increasing (decreasing) in the tax rate depends on whether the direct effect of a change in the pollution \( X \) tax is larger (smaller) than the indirect effect on output through the change in the producer’s price \( \theta X_p \). The industry lobby group becomes less concerned with the tax rate, when its membership increases, since a larger fraction of the tax revenue is redistributed back to its members.
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Proposition 3.9. In equilibrium, the pollution tax rate is increasing (decreasing) in the government’s weight on aggregate social welfare, if $t < (>) \alpha_E$.

When the government’s weight on aggregate welfare increases, the deviation of the pollution tax rate from the optimal level is reduced.

3.2.2 Further Research

One of the many studies concerning the characteristics of endogenous environmental policy in a common agency model of politics, is that of Aidt (1998). He distinguishes between lobby groups that are functionally specialized, i.e. they act as advocates for only one aspect of environmental policy, and lobby groups that have multiple goals, e.g. trade unions and employers’ associations. They assume that firms in each industry of a small open economy use an input (raw materials, clean water etc.) that has an external effect on the well-being of consumers (smoke, toxic waste water etc.), and the government can use only production taxes-cum-subsidies and input taxes-cum-subsidies as policy instruments, which can be used to affect the use of the externality generating input and to motivate the firm to use a less polluting production technology, respectively. A result of Aidt’s study is that regardless of the type of the lobby groups, only the tax-cum-subsidy on the externality generating activity includes an environmental adjustment.

Yu (2005) develops a model with two opposing interest groups—an environmentalist group and an industrialist group—compete directly (lobbying the government) and indirectly (persuading the public) for political influence in a three-stage game. In the first stage the "indirect" competition takes place, in which interest groups attempt to persuade the public to indirectly influence the government’s policy. The "direct" competition follows in the second stage, in which interest groups exert pressure to the government to directly influence its policy. In the final stage, the government, which cares both about political contributions and public support, chooses a politically optimal environmental policy. The three main results of Yu’s paper are the following: (i) the direct and indirect competition are complements, i.e. the group that has more political influence in direct competition, also exerts relatively more effort in indirect competition, (ii) an increase in the effectiveness of public persuasion will induce substitution between direct and indirect competition, and (iii) the degree of public environmental awareness itself will determine not only how strict the
environmental policy will be, but also it will affect the pattern of direct and indirect competition by both interest groups.

Finally, Fredriksson (1999), Damania et al. (2004) and Lai (2006) investigate the effect of trade liberalization on the government’s choice of environmental policy. In the model of the first paper, rival environmental and industry lobby groups seek to influence the government’s pollution tax policy in a tariff-protected sector. He demonstrates that the political pressures on the pollution tax from the two lobbies fall as the tariff decreases, and that total pollution may increases with more liberal trade because the equilibrium pollution tax falls. In the second study it is shown that trade liberalization increases (decreases) the pollution tax, if the level of corruption is sufficiently high (low). Lastly, Yu considers a country that imposes a minimum standard on an imported polluting good, and finds that trade liberalization tends to tighten the minimum standard, reduces imports of the polluting good, and decreases environmental disutilities.

3.3 Domestic Economic Policies

3.3.1 Common Agency with Rational Expectations

Consider an economy which is subject to stochastic shocks, and the policy process takes place after the shocks are realized. The principals must act before the shocks are realized, given their expectations about the policy variable. When the shocks are realized and the policy process occurs, the \textit{ex ante} actions are fixed, so the principals and the agent may be tempted to take unanticipated or surprise actions. However, the principals form their expectations \textit{ex ante}, rationally considering this \textit{ex post} equilibrium. Therefore, equilibrium is an interrelated two-stage process. At an even earlier stage, where the constitutional rules of the common agency are being designed, the interrelated two-stage nature of the process must be taken into account.

Dixit and Jensen (2003) extend the common agency model to show how the principals must change their incentive schedules in equilibrium to take into consideration the rational expectations constraint.\footnote{The authors present an example of monetary policy, in which private sector wage and price settlements are made \textit{ex ante}, given expectations of inflation, and governments \textit{ex post}}
The Model

Consider a common agency with \( n > 1 \) principals and one agent. The agent chooses a policy variable \( x \), after the realization of two types of stochastic shocks, represented by vectors \( \theta \) and \( \epsilon \). Therefore, the agent’s policy is a function \( x(\theta, \epsilon) \). The principals form rational expectations \( x^e \) of the policy after \( \theta \) is realized but before \( \epsilon \) is realized; therefore the expectations are a function \( x^e(\theta) \). Let \( \phi(\theta, \epsilon) \) denote the joint probability density function of the shocks, \( f_\theta \) the marginal density function of \( \theta \), and \( g(\epsilon|\theta) \) the conditional density function of \( \epsilon \). Unconditional expectations are denoted by \( E_{\epsilon}(\cdot|\theta) \), and expectations with respect to the distribution of \( \theta \) by \( E_\theta(\cdot) \). Now the condition for rationality of expectations can be written as

\[
x^e(\theta) = E_{\epsilon}[x(\theta, \epsilon)|\theta] \equiv \int x(\theta, \epsilon)g(\epsilon|\theta)d\epsilon, \quad \forall \theta.
\]

(3.3.1)

The objective function of principal \( i \) takes the form

\[
U_i(x, x^e; \theta, \epsilon).
\]

(3.3.2)

This should be considered as an "indirect utility function", where the \( \text{ex ante} \) actions taken by the principal in the light of the expectations \( x^e \) are optimized out, recognizing that the \( \text{ex post} \) payoffs from such actions will also depend on the realization of the shocks and on the actual policy chosen \( \text{ex post} \) by the agent.

The authors first characterize the complete contingent commitment policy that each principal \( i \) would most prefer. For this optimization they construct the Lagrangian

\[
L = E\left\{U_i[x(\theta, \epsilon), x^e(\theta); \theta, \epsilon]\right\} + E_\theta\left(\lambda_i(\theta)\{x^e(\theta) - E_{\epsilon}[x(\theta, \epsilon)|\theta]\}\right),
\]

where \( \lambda_i(\theta) \) is the multiplier on (3.3.1). The first-order conditions are \( \partial U_i/\partial x = \lambda_i(\theta) \) and \( \partial E_{\epsilon}(U_i|\theta)\partial x^e = -\lambda_i(\theta) \), respectively, where \( U_i \) is evaluated at \( x = x(\theta, \epsilon) \) and \( x^e = x^e(\theta) \). These conditions are combined to eliminate the multiplier so as to characterize optimal commitment policy from the perspective of desire to create surprise inflation to get higher output. However, the private sector’s rational expectations result to an equilibrium in which the average output is not higher and inflation is higher.
principal $i$:
\[
\frac{\partial U_i}{\partial x} = -E\left(\frac{\partial U_i}{\partial x^e}\right|_\theta).
\] (3.3.3)
This reflects that $E(\partial U_i/\partial x + \partial U_i/\partial x^e|\theta) = 0$, i.e. in the preferred equilibrium, the expected gain of a fully expected marginal increase in $x$ is zero for any realization of $\theta$.

The policy chosen in each realization is the Nash equilibrium of a game, in which each principal offers an incentive contract to the common agent. The contracts are committed to before expectations are formed but delivered after the shocks are realized. The contract offered by principal $i$ is written as a policy-and-state-contingent transfer schedule of the form $k_i(\theta, \epsilon) + T_i(x, \theta, \epsilon)$. The principals choose these schedules non-cooperatively, so we seek a Nash equilibrium in schedules.

The agent’s objective function is assumed to be exogenous and takes the following form:
\[
U^{Agent} = A \sum_{i=1}^{n} a_i U_i + H(x) + B \sum_{i=1}^{n} p_i [k_i(\theta, \epsilon) + T_i(x, \theta, \epsilon)],
\] (3.3.4)
with $A, B \geq 0$ and $a_i, p_i > 0$. The first term of (3.3.4) reflects a weighted average of the principals’ objective functions, the second term is the agent’s direct concern for the policy instrument, and the third term is a weighted average of the explicit incentives. $a_i$ and $p_i$ are the exogenous weights that capture the relative importance of the principals in the influence process.

When deciding on $T_i(x, \theta, \epsilon)$, each principal knows that the agent chooses $x$ to maximize (3.3.4). The first-order condition for this is
\[
A \sum_{i=1}^{n} a_i \frac{\partial U_i}{\partial x} + H'(x) + B \sum_{i=1}^{n} p_i \frac{\partial T_i}{\partial x} = 0.
\] (3.3.5)
Principal $i$ maximizes $E\{U_i - c_i[k_i(\theta, \epsilon) + T_i(x, \theta, \epsilon)]\}$, where $c_i > 0$ is a parameter that captures the weight of the “currency” of principal $i$’s transfers to the agent relative to the principal’s utility.

Each principal’s maximization is subject to several constraints. First, each principal recognizes that the common agent will choose $x$ according to (3.3.5). Moreover, the principals recognize that expectations will be formed rationally as defined by (3.3.1). Because it is mathematically complex to substitute out for
the expectations using this equation, the authors use the simpler and mathematically equivalent approach of regarding each principal as if he chooses the expectations also, subject to the constraint (3.3.1). Finally, each principal recognizes the agent’s participation constraint, \( E(U_{\text{Agent}}) \geq u_0 \), where \( u_0 \) is some outside opportunity utility.

Consider, now, the decision of, say, principal 1. The Lagrangian is

\[
\mathcal{L}_1 = E\left\{ U_1[x, x^e(\theta); \theta, \epsilon] - c_1[k_1(\theta, \epsilon) + T_1(x, \theta, \epsilon)] \right\} \\
+ E_{\theta}\left\{ \lambda_1(\theta)\left\{ x^e(\theta) - E_{\epsilon}[x(\theta, \epsilon)|\theta, \epsilon] \right\} \right\} \\
+ \mu_1 E\left\{ \sum_{i=1}^{n} a_i U_i[x, x^e(\theta); \theta, \epsilon] + H(x) + B \sum_{i=1}^{n} p_i[k_i(\theta, \epsilon) + T_i(x, \theta, \epsilon)] - u_0 \right\},
\]

where \( \lambda(\theta) \) and \( \mu_1 \) are the multipliers on the rational expectations constraint and participation constraint, respectively. The principal’s choice variables are the functions \( k_1(\theta, \epsilon) \) and \( T_1(x, \theta, \epsilon) \), and the "as if" choice of \( x^e(\theta) \), above, taking as given the other principals’ choice functions. From (3.3.5), \( k_1 \) plays no role for the determination of \( x \). Furthermore, the Lagrangian depends only on the expectation \( E(k_1) \), not the full function \( k_1(\theta, \epsilon) \). Therefore, \( E(k_1) \) can be regarded as principal 1’s choice variable. The first-order condition for this is

\[-c_1 + \mu_1 p_1 B = 0, \quad \text{or}, \quad \mu_1 = \frac{c_1}{B p_1}.\]

Using this, the Lagrangian can be rewritten as

\[
\mathcal{L}_1 = E(U_1) + E_{\theta}\left\{ \lambda_1\left[x^e - E_{\epsilon}(x|\theta)\right] \right\} \\
+ \frac{c_1}{B p_1} E\left[ A \sum_{i=1}^{n} a_i U_i + H + B \sum_{i=2}^{n} p_i(k_i + T_i) - u_0 \right].
\]

The schedule has an effect via its effect on \( x \). Any marginal change in \( T_1(\cdot, \theta, \epsilon) \) will cause a marginal change in \( x(\theta, \epsilon) \), which will alter principal 1’s Lagrangian.
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by

$$dL_1 = \left\{ \frac{\partial U_1}{\partial x} - \lambda_1 + \frac{c_1}{Bp_1} \left[ A \sum_{i=1}^{n} a_i \frac{\partial U_i}{\partial x} + H'(x) + B \sum_{i=2}^{n} p_i \frac{\partial T_i}{\partial x} \right] \right\} \phi(\theta, \epsilon) dx(\theta, \epsilon)$$

$$= \left( \frac{\partial U_1}{\partial x} - \lambda_1 - c_1 \frac{\partial T_1}{\partial x} \right) \phi(\theta, \epsilon) dx(\theta, \epsilon),$$

(3.3.6)

The optimal choice of $T_1$ must therefore satisfy

$$\frac{\partial T_1}{\partial x} = \frac{1}{c_1} \left[ \frac{\partial U_1}{\partial x} - \lambda_1(\theta) \right]$$

(3.3.7)

A simple function satisfying this requirement is

$$T_1(x, \theta, \epsilon) = \frac{1}{c_1} \left\{ U_1[x, x^e(\theta); \theta, \epsilon] - \lambda_1(\theta) x \right\}.$$  (3.3.8)

Next, the first-order condition for the choice of $x^e$ is

$$E_\epsilon \left( \frac{\partial U_1}{\partial x^e} \theta \right) + \lambda_1(\theta) + \frac{c_1}{Bp_1} E_\epsilon \left( A \sum_{i=1}^{n} a_i \frac{\partial U_i}{\partial x^e} \theta \right) = 0$$

(3.3.9)

and provides the solution for $\lambda_1(\theta)$. Similar conditions apply for all principals.

In (3.3.8) if $c_1 = 1$ and $\lambda_1(\theta) = 0$, then (3.3.7) would say that the equilibrium transfer schedule must be what Grossman and Helpman (1994) called _locally truthful_. Likewise, the particular solution (3.3.8) would simplify to just $T_1(x, \theta, \epsilon) = U_1[x, x^e(\theta); \theta, \epsilon]$. The constant term in the incentive schedules have been separated as $k_1(\theta, \epsilon)$. Thus, each principal’s schedule (for each realization of the shocks) differs from its own utility function only by a constant. Put differently, the schedules become the familiar schedules of the common agency equilibrium that Grossman and Helpman originally called _globally truthful_. Dixit and Jensen’s equilibrium is a generalization of Grossman and Helpman’s in two senses. First, the factors $c_i$ allow the transfers to figure differently in the principals’ and the agent’s objective functions. Second, the multiplier $\lambda_1(\theta)$ captures the added effect of the rational expectation constraint, which is important in many macroeconomics and tax policy applications.
Properties of the Equilibrium Policy. After some algebra, Dixit and Jensen (2003) conclude that
\[
\sum_{i=1}^{n} \left( A a_i + B \frac{p_i}{c_i} \right) \left[ \frac{\partial U_i}{\partial x} + E_e \left( \frac{\partial U_i}{\partial x^e} \bigg| \theta \right) \right] = H'(x) - (n-1) \sum_{i=1}^{n} A a_i E_e \left( \frac{\partial U_i}{\partial x^e} \bigg| \theta \right).
\]
(3.3.10)

This equation implicitly defines the actual policy \( x \) that will emerge in the equilibrium. The left hand side is a weighted sum of the principals’ marginal utilities from an equal increase in actual and expected \( x \), and the second-order condition for that optimization is that the left hand side should be decreasing, that is positive for smaller \( x \) and negative for larger \( x \). In the special case where the agent cares only about the transfers he receives, i.e. the case of \( A = 0 \) and \( H(x) = 0 \), the left hand side of (3.3.10) must indeed equal zero. Thus, if the common agent has only "selfish" objectives, he implements a policy which in an average of the principals’ preferred policies. However, when \( A > 0 \) and \( H(x) \neq 0 \), this will no longer be the case.

3.3.2 Further Research

Chortareas and Miller (2004) and Campoy and Negrete (2008) extend the basic principal-agent model in central banking to allow for the presence of a second potential principal (a disaffected interest group). In the former study, it is shown that when an interest group possesses an output bias, central banker inflation contracts do not deliver the optimal policy outcomes because the contract offered by the interest group dominates the contract offered by the government. When the interest group possesses an inflation bias, then the outcome depends on the willingness of the two principals to incur contract costs. In the latter study, is is proved that inflation bias is eliminated when both principals disagree over the objective and the contracts designed by the government and the interest group link incentives, respectively, to inflation and output.

Testa (2005) examines the determination of price liberalization in a common agency model in which the citizens, organized in lobby groups, influence the government through direct threats, such as social unrest and strikes. She demonstrates that as far as all groups have an instrument to pressurize the legislator, social surplus is maximized. Furthermore, transfers to the legislators
are not necessary from all lobbies for the social surplus maximizing policy to be implemented.

3.4 The Political System

3.4.1 Committees and Special Interests

It is commonly accepted that committees can be viewed as an efficient mechanism for the aggregation of decentralized and imperfect information. However, in many cases one single or several opposing interest groups may want to influence committee members and distort their decisions. Felgenhauer and Grüner (2008) examine the ways in which special interests may affect the decisions of committees in such cases, and how excessive distortions of a committee can be prevented by the institutional arrangement. Their model clarify the role of several institutional dimensions such as committee members’ abilities, rules for the ex post release of information on individual and collective voting behavior of committee members, and the size of the committee.

They argue that the decision quality is higher in a closed committee, because incentives to give in to external pressures do not decline with the probability of being pivotal. Moreover, they show that under open voting, a larger number of lower quality decisions may be made. Finally, under closed voting, a lower quality of experts within a committee may improve the quality of the committee’s decision under interest group influence.

The Model

Agents. A homogeneous population delegates a binary decision to a committee which consists of three members, \( i = 1, 2, 3 \). Each of these members obtains a private signal about the true state of the world \( s \), which can be either 1 or 0. Each of the two states is realized with probability 1/2. The signal of individual \( i \) is denoted by \( s_i \in \{0, 1\} \). Each signal is correct with probability \( p \). A policy \( x \), which is 1 or 0, is chosen according to majority voting. The individual votes are denoted by \( x_i \in \{0, 1\} \). Besides this, there are one or two interest groups (principals) who try to influence the committee decision.
Preferences. Each committee member $i$ derives the utility

$$u_i = y + t_i,$$  

(3.4.1)

where $t_i$ is the total payment made to agent $i$ and

$$y = \begin{cases} 
1, & \text{if } x = s \\
0, & \text{if } x \neq s. 
\end{cases}$$  

(3.4.2)

The public’s payoff is a multiple of $y$. In the absence of transfers, each committee member, therefore, wants to choose according to the general public’s interest.

It is assumed that there are two interest groups $j = 0, 1$ with utility functions

$$u_j = \theta_j x - \sum_{i=1}^{3} t_{ij},$$  

(3.4.3)

and $\theta_1 = -\theta_0 > 1$. $t_j = (t_{1j}, t_{2j}, t_{3j})$ denotes lobby $j$’s vector of transfers and $t_{ij}$ denotes the transfer from group $j$ to voter $i$. Interest group 0 prefers policy $x = 0$ and group 1 prefers policy $x = 1$ and both groups have an identical valuation for the respective preferred policy.

Timing. First, the interest groups choose a transfer scheme. These transfers will be conditioned either (3.4.1) upon the observable individual votes, $x_i$, or (3.4.2) upon the chosen policy $x$. For simplicity, it is assumed that the transfer schemes are known to all three committee members.

Next, nature draws a state $s$. Each of the two states is realized with probability $1/2$. Each committee member privately observes the signal $s_i$, which is correct with probability $p$. Finally, committee members choose a policy according to majority voting and transfers are paid in the pre-specified way. Society and committee members observe ex post whether the decision was appropriate. However, it is assumed that this is not verifiable. Hence, committee members cannot be directly rewarded for correct decisions.

Strategies and Equilibrium Concept. A voter’s strategy maps transfers and the private signal into the set of possible votes. Interest groups’ strategies are required to be a Nash equilibrium given the Bayesian strategies of voters. Each player has four pure strategies: truthful voting, always voting for one of the two alternatives and inverse voting. Players who vote truthfully are called sincere voters. The authors focus on pure strategies equilibria.
Common Agency

**A Modified First Price Sealed Bid Auction.** In what follows, Felgenhauer and Grünner (2008) show that a mixed strategies equilibrium of the corresponding (discontinuous payoff) game exists. Such an equilibrium has the property that the interest groups’ efforts do not always cancel out. It may rather be the case that the difference between transfers induces some agents to vote insincerely. Conditions are derived, which guarantee that secret voting is strictly preferred to open voting from a social point of view.

Both interest groups make simultaneous announcements about their transfers and based on these offers, which become common knowledge among the committee members (but not to the opposing interest group), each member casts a vote. Hence, the game is similar to a first price sealed bid auction, with the difference that the difference between the offers has to be above a certain threshold in order to ensure that the vote changes.

![Figure 3.1: Regions I and II](image)

**Secret Voting Scheme.** Under secret voting, the transfers can only be made contingent on the policy chosen by the committee as a whole, that is, the offers $t_{ij}$ are made contingent on $x$.

Each member $i$’s equilibrium voting decision depends on the transfers he obtains from both lobbies. If a lobby $j$ wants to purchase voter $i$ regardless of his signal, then $j$ has to outbid its rival at least by a certain threshold, denoted by
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\[ \Delta_i. \] This threshold always depends on the other two voters’ equilibrium strategies. It may also depend on the voter’s received signal \( s_i \). Then, committee member \( i \)'s optimal vote \( x_i \) is

\[
x_i = \begin{cases} 
  s_i, & \text{if } t_{ij} \neq s_i - t_{i-j} \leq \Delta_i \\
  j, & \text{otherwise.}
\end{cases}
\]

(3.4.4)

that is, \( i \) votes sincerely if the difference in the transfers \( t_{ij} \neq s_i - t_{i-j} \) (the first term being the transfers from the group, which does not coincide with \( i \)'s signal \( s_i \)) is smaller than the threshold.

The following proposition refers to Figure 3.1.

**Proposition 3.10.** Consider a committee under closed voting.

(i) For all \((\theta, p)\) combinations in region II in Figure 3.1, there exists only one equilibrium. In this equilibrium, the transfers satisfy \( t_{ij} = 0, \forall i, j \). This equilibrium maximizes the social surplus.

(ii) For all \((\theta, p)\) combinations in region I in Figure 3.1, there is no equilibrium guaranteeing that the socially best policy is chosen.

(iii) For all \((\theta, p)\) combinations in region I in Figure 3.1, an equilibrium not guaranteeing the socially best policy exists.

**Open Voting Scheme.** Under open voting, the transfers can be made contingent on the vector of votes.

**Proposition 3.11.** Consider a committee under open voting with general contracts. Open voting always leads to a policy bias.

**Comparison of Closed and Open Voting.** According to Propositions 3.10 and 3.11, there are \((\theta, p)\) combinations in which closed voting is better for society than open voting. In those cases, open voting generates a policy bias while closed voting always yields the appropriate decision. For another set of \((\theta, p)\) combinations, both institutional setups yield a policy bias. Without any additional information on the type of the equilibrium, a welfare ranking in those cases cannot be provided.

**Proposition 3.12.** Either \( \theta \) and \( p \) are such that

(i) a closed voting mechanism guarantees that the best policy is chosen, whereas under open voting the best policy is generally not chosen, or
(ii) both voting mechanisms do not guarantee the social optimum.

**Enlarging the Committee: The Condorcet Jury Theorem**

One of the fundamental insights in voting theory is that more individuals are involved, the better results are yielded by the aggregation of decentralized information. The present framework may yield the opposite result.

**Proposition 3.13.** Consider the common agency setup with open voting. There are parameters $p$ and $\theta$ such that in equilibrium, a single voter is more likely to make a correct decision than in an equilibrium in an open committee of three.

### 3.4.2 Further Research

In a related paper, Felgenhauer (2010) asks whether transparency, which means that the lobbies have access to sensitive information, could be socially beneficial in the presence of external influence, and shows that in the presence of opposing groups, it may be socially optimal.

Tommasi and Weinschelbaum (2007) and Bordignon et al. (2008) both assess whether government functions should be centralized or decentralized if lobbying behavior is taken into account. In the former study, it is demonstrated that the optimal institutional arrangement depends on the relative importance of an externality problem in the provision of public goods, and a collective action problem among principals (citizens) in controlling political agents. Decentralization is preferable when the externality effect is less important than the differences of the coordination effect. According to the latter study, whether a government function should be (de-)centralized depends on how the interests of the regional lobbies are positioned with respect to the function to be (de-)centralized. When regional lobbies have conflicting interests, then lobbying is less damaging for social welfare under centralization than under decentralization. However, when they have aligned interests, lobbying is less damaging for social welfare under decentralization.

Ujhelyi (2009) studies whether regulatory caps on contributions to political campaigns are desirable. He concludes that when the cost of entering the political competition is zero, a symmetric cap on campaign contributions always implements the efficient policy, but when lobbying costs exist, a cap on contributions no longer guarantees efficiency, and may reduce social welfare relative
to the status quo. On the other hand, Prat (2002) shows that under reasonable parametric assumptions, a ban on campaign contributions improves voter welfare in a utilitarian sense.

Helpman and Persson (2001) examine the interaction between lobbying and legislative bargaining in a US-style congressional system and a European-style parliamentary system. They find that the concentration of agenda-setting powers in the coalition supporting the executive in parliamentary systems and the effective veto powers of these coalition members, produce greater legislative cohesion in parliamentary systems, which affects the strategic interaction between lobbies as well as between lawmakers.

Finally, Ruta (2008) extends the common agency with rational expectations mentioned above (Dixit and Jensen, 2003) to the case in which the agent can commit to follow its policy. The results of his study imply that the desirability of politicization of monetary policy in a currency union depends critically on whether the monetary authority can commit to follow its policies.

### 3.5 Weaknesses

Despite the indubitable fruitfulness of the common agency approach, a number of weaknesses remain. For instance, Mitra (1999), Siqueira (2001) and Le Breton and Salanie (2003) do not take for granted that principals are able to overcome the Olsonian collective action problem to form active interest groups.

Mitra (1999) endogenizes the formation of lobbies within the Grossman and Helpman (1994) framework for trade policy determination. To do this, he adds a first stage in which agents with common interests decide whether or not to bear the costs of getting organized. He shows that when everyone in the population owns a specific factor, free trade is an equilibrium outcome either when the government is highly responsive to political contributions or when decision-making is highly social welfare oriented. A high affinity on the part of the government for political contributions leads to the formation of a large number of opposing lobbies which cancel each other out.

Siqueira (2001) looks at two possible scenarios. In the first, principals, whether organized or not, are assumed to move simultaneously and independently. In the second, it is assumed that the smallest group of principals has a first-mover advantage. The main results of his study is that in the first case, partial cooperation is generally self-defeating from an organizing principal’s perspective.
despite a strengthening of agent incentives and effort. Therefore, there is no incentive for principals to organize. However, in the second case, it is shown that not only is the ability to move first individually beneficial to cooperating principals, but the outcome in terms of agent incentives and effort is constrained Pareto efficient, better than even the standard, third-best common agency outcome.

Le Breton and Salanie (2003) consider a model in which the interest groups are not a priori organized or unorganized and the type of politician is not common knowledge. They show—as suggested by Olson (1965)—that the supporters of a decision are more likely to be successful if they are less numerous. Moreover, a group is more likely to be successful if it is more heterogeneous; it is better to have small supporters allied with large ones rather than a population of identical supporters.

As Mallard (2014) claims, except for the formation of interest groups, there are also weaknesses that relate to the assumptions usually made about the nature of lobbying, the political system and behavioral demands, and that relate to supporting empirical analysis.

Ansolabehere et al. (2003) argues that campaign contributions should be viewed primarily as a type of consumption good, rather than as a market for buying political benefits. Chiesa and Denicolò (2009) study all the subgame perfect Nash equilibria—not only truthful ones—of a class of common agency games with perfect information, providing a complete characterization of equilibrium payoffs. They show that the equilibrium, that is Pareto-dominant for the principals, is not truthful when there are more than two of them.

In general, in the literature it is assumed that voters are influenced by campaign spending in an ad hoc way. Notable exceptions to this are the papers by Prat (2002) and Grossman and Helpman (1996). The first of these develops a microfounded model of campaign advertising with multiple lobbies by combining a signaling model of non-informative advertising with common agency. The second one studies the competition between two political parties, which have fixed positions on some issues but vary their positions on others in order to attract votes and campaign contributions, for seats in legislature. It is shown that each party is induced to behave as if it were maximizing a weighted sum of aggregate welfares of informed voters and members of special interest groups.

It has been demonstrated in the experimental literature that the subgame perfect equilibrium concept, upon which the common agency model is based,
does not accurately predict actual behavior. This happens because the concept is too complicated for players to implement, and players are concerned about fairness for other players in the game as well as for themselves. Kirchsteiger and Prat (2001) ask whether the truthful equilibrium is the only reason-
able equilibrium. In their model, principals behave in a simpler way; instead of making positive offers on all, or most, possible alternatives, as the truthful equilibrium requires, each principal makes only one strictly positive offer, on the alternative that she hopes to get. They call such strategy and the corresponding equilibrium (if it exists) natural. From an experiment they run, to ascertain which class of equilibrium better predicts behavior, they obtain mixed results. On the one hand, the natural equilibrium is a much better predictor of the outcome of the game; it is chosen in 65% of the matches. On the other hand, most subjects do not use natural contributions schedules, as 81% of the chosen schedules include positive offers on more than one alternative. However, the truthful equilibrium is a poor predictor of the subjects’ strategies, too, because truthful contributions are observed in only 17% of the cases.

In two recent papers, Chesnokova (2014) allows for the marginal contribution of a lobby to be decreasing in the total amount of money collected by the government, and Boultzis (2015) assumes that the agent’s objective need not be increasing in contributions, as well as the agent has the ability to reject contributions from principals, either partially or in whole. In the former paper, it is shown that in contrast to the Grossman-Helpman model, as the number of lobbies increases, the political influence of each lobby reduces. In the latter, the notions of quasi truthful strategies and quasi truthful equilibria are defined, and it is shown that these equilibria are efficient.

Finally, many empirical studies estimate the protection equation that emerges from the Protection for Sale model and have found the parameter estimates to support the propositions of its theoretical applications (Goldberg and Maggi, 1999; Gawande and Usree, 2000; Mitra et al., 2002; Eicher and Osang, 2002; McCalman, 2004). A surprising finding of Goldberg and Maggi (1999) is that the weight of welfare in the government’s objective function is many times larger than the weight of contributions, whereas Gawande and Usree (2000) find that these weights are almost equal. Overall, this empirical literature tends to support the original framework, although Imai et al. (2009) raise a number of doubts over the precise methodologies employed.
In this section, we introduce an extensive form of the common-agency model: the dynamic common agency. In this case, the agent contracts with the principals repeatedly over time. Primarily focusing on the theoretical framework, we also take a look at a number of applications of the above model to the political economy and other areas of economics.

4.1 A General Model

The static model of common agency, analyzed earlier, has been widely used in many areas of economics, including political economy. However, political choices are rarely made only once, and their future implications are usually more important than their temporary consequences. Thus, if a commitment between the politician and the lobbyists cannot exist, a need of a dynamic perspective arises.

Bergemann and Välimäki (2003) extend the static common agency game, considering a general model of dynamic common agency with symmetric information. They introduce the notion of marginal contribution equilibria in the static common agency game, and afterwards they study the existence of such an equilibrium in the dynamic framework. The first model they analyze (agenda game) is a two-period game in which in the first period, the agent chooses the available actions, and in the second stage, the common agency game is played, with the set of actions as determined in the first period. Then, they proceed to a general model of dynamic common agency, considering deterministic as well as stochastic transitions between states.
The Model

Payoffs. The principals are indexed by \( i \in \mathcal{J} = \{1, \ldots, I\} \). Time is discrete and indexed by \( t = 0, 1, \ldots, T \), where \( T \) can be finite or infinite. In period 0, the agent selects an action \( \alpha_0 \) from a finite set of available actions \( \mathcal{A}_0 \), and principal \( i \) offers a reward schedule \( r_i(a_0) \in \mathbb{R}^+ \). The stage may change over periods and the payoff relevant state of the world in period \( t \) is denoted by \( \theta_t \in \Theta \). For simplicity, it is assumed that \( \Theta = \{\theta_1, \ldots, \theta_K\} \) for some \( K < \infty \). Only the agent makes directly payoff relevant choices in any of the periods, thus it is assumed that the transition function \( q(\theta_{t+1} | \alpha_t, \theta_t) \) is a Markovian in the sense that the distribution of the payoff relevant state in period \( t+1 \) depends only on the current action \( \alpha_t \) and the current state \( \theta_t \).

Setting \( h_1 = (\theta_0, \alpha_0, r_0, \theta_1) \), the histories for \( t > 1 \) in the game are given by

\[
h_t = (h_{t-1}, \alpha_{t-1}, r_{t-1}, \theta_t),
\]

where \( r_t \) is the profile of reward schedules in period \( t \) and \( a_t \in \mathcal{A}(\theta_t) \), the set of available actions in state \( \theta_t \). It is assumed that \( \bigcup_{\theta \in \Theta} \mathcal{A}(\theta) \) is finite and the principals can commit in every period to the reward schedules.

Taking the action \( a_t \) in period \( t \), the agent bears a cost of \( c(a_t, \theta_t) \). The benefit to principal \( i \) is \( v_i(a_t, \theta_t) \), which may again depend on \( \theta_t \). After a history \( h_t \), the aggregate reward paid by a subset of principals \( S \subset \mathcal{J} \) for an action \( a_t \) is

\[
r_S(a_t, h_t) \triangleq \sum_{i \in S} r_i(a_t, h_t),
\]

and the aggregate benefits for the principals are

\[
v_S(a_t, \theta_t) \triangleq \sum_{i \in S} v_i(a_t, \theta_t).
\]

For \( S = \mathcal{J} \), the aggregate rewards and benefits are denoted by \( r(a_t, h_t) \triangleq r_{\mathcal{J}}(a_t, h_t) \) and \( v(a_t, \theta_t) \triangleq v_{\mathcal{J}}(a_t, \theta_t) \) respectively. It is assumed that \( v_i(a_t, \theta_t) \geq 0 \) and \( c(a_t, \theta_t) \geq 0 \), \( \forall a_t, \theta_t \), and also that there is an action \( a_t \in \mathcal{A}(\theta_t) \), such that \( c(a_t, \theta_t) = 0 \), \( \forall \theta_t \).

All players maximize expected discounted value and their common discount factor for future periods is \( \delta \in (0, 1) \).

Social values. Assuming that utility between the agent and the principals is transferable, Pareto efficiency coincides with surplus maximization. The value of the socially efficient program is denoted by
and the value of efficient program with a subset $S$ of principals and the agents is denoted by $W_S(\theta_t)$. These values are obtained from the following dynamic programming equation:

$$W_S(\theta_t) = \max_{a_t \in A(\theta_t)} \mathbb{E}\{v_S(a_t, \theta_t) - c(a_t, \theta_t) + \delta W_S(\theta_{t+1})\}.$$ 

The efficient action for the set $S$ in state $\theta_t$ is denoted by $a^*_S \triangleq a^*_S(\theta_t)$ and for the entire set $\mathcal{J}$, it is $a^* \triangleq a^*_\mathcal{J}(\theta_t)$. The value of a set of principals $\mathcal{J} \setminus S$ is similarly denoted by $W_{\setminus S}(\theta_t)$.

The marginal contribution of principal $i$ and, respectively, of a subset of principals $S \subset \mathcal{J}$ is defined by

$$M_i(\theta_t) \triangleq W(\theta_t) - W_{\setminus i}(\theta_t), \quad (4.1.1)$$
$$M_S(\theta_t) \triangleq W(\theta_t) - W_{\setminus S}(\theta_t). \quad (4.1.2)$$

It is emphasized that for all social values $W_S(\theta_t)$, the agent is always implicitly included in the set $S$ of principals, whereas for all marginal contributions $M_S(\theta_t)$, the agent is always excluded from set $S$.

After discussing the main results of Bernheim and Whinston (1986b), the authors provide an additional characterization of the set of Nash equilibrium payoffs for the static game.

**Definition 4.1.** *(Marginal contribution equilibrium)* A marginal contribution equilibrium of the common agency game is a truthful Nash equilibrium with $n_i = M - i$, $\forall i$.

In other words, all principals receive their marginal contribution to the social welfare as their equilibrium net payoff in a marginal contribution equilibrium. The characterization of the games with marginal contribution equilibria and their truthful equilibrium payoffs follows.
Chapter 4. Dynamic Common Agency

**Theorem 4.1. (Existence and uniqueness)**

(i) A marginal contribution equilibrium exists if and only if

\[ \forall S \subseteq J, \sum_{i \in S} M_i \leq M_S. \]  

(4.1.3)

(ii) The truthful Nash equilibrium payoff set is a singleton if and only if the game has a marginal contribution equilibrium.

(iii) If \( M_S \) is superadditive,

\[ \forall S, T, S \cap T = \emptyset, \ M_S + M_T \leq M_{S \cup T}, \]  

(4.1.4)

then the truthful equilibrium is unique.

(4.1.3) is referred to as weak superadditivity of the marginal contributions, and it requires that the total marginal contributions of each principal \( i \in S \) to \( J \) is less than the marginal contributions of the entire set \( S \) to \( J \). Superadditivity condition (4.1.4) is a sufficient condition for (4.1.3) and it agrees with it if \( |J| = 2 \).

**Agenda Setting**

Bergemann and Välimäki (2003) modify the static common agency game by endogenizing the set of action available to the agent. More precisely, the agent is allowed to set the agenda in the initial period by selecting a subset of the exogenously given set of feasible actions. In this period, the principals are capable of influencing the selection of the subset by the agent. In the subsequent period, the principals bid on the actions in the selected subset. This game, referred to as agenda setting game, extends over two periods and is the simplest dynamic common agency model.

It proceeds as follows. In period 0, each principal bids on the subset \( A \) chosen by the agent from the set of feasible actions \( \mathcal{A} \). If the agent selects the subset \( A \) for the second stage, she bears no cost and she receives a reward \( r_i(A) \) from principal \( i \). However, the eventual choice of the agent in period 1 is restricted to the subset \( A \).

The agent seeks high equilibrium payoffs. Thus, in the first stage, she selects actions that increase competition between principals in the second stage. In general, the agent does not have to include the socially optimal action in the
subset of actions that maximizes the equilibrium payoff to the agent in the final stage.

The set of actions available to the agent in the first period is the set of all subsets of \( \mathcal{X} \), or \( 2^\mathcal{X} \). Assuming that a marginal contribution equilibrium in period 1 exists for all subsets \( A \), the payoffs in period 1 are given directly by their marginal contribution relative to the set \( A \) and by the residual rent for the agent. Let

\[
W_S(A) \doteq \max_{a \in A} \{ v_s(a) - c(a) \},
\]

and

\[
M_S(A) \doteq W(A) - W_{-S}(A).
\]

By assumption, every set \( A \) induces a marginal contribution equilibrium in the second stage with payoffs given by \( M_i(A) \) for principal \( i \) and by \( W(A) - \sum_{i \notin S} M_i(A) \) for the agent. The social value the set \( S \) of principals can achieve jointly with the agent is denoted by

\[
\hat{W}_S \doteq \max_{A \in 2^{\mathcal{X}}} \left\{ W(A) - \sum_{i \notin S} M_i(A) \right\}.
\]

The recursive contribution of a set \( S \) of principals is defined as

\[
\hat{M}_S \doteq \hat{W} - \hat{W}_{-S}.
\]

The agenda game can now by analyzed as a static common agency game. The first result is that all truthful equilibria of the agenda game are efficient. All subsets \( A \), which include the efficient action \( a^* \) permit the realization of the efficient surplus tomorrow, thus the equilibrium choice of \( A \) is not unique; it includes all subsets which include \( a^* \). Denote by \( \mathcal{X}^* \) the set of all such subsets:

\[
\mathcal{X}^* = \{ A \in 2^\mathcal{X} \mid a^* \in A \}.
\]

**Theorem 4.2. (Agenda game)**

(i) Every truthful equilibrium of the agenda game is efficient: \( A \in \mathcal{X}^* \), \( a = a^* \).

(ii) The following three statements are equivalent:

(a) the agenda game has a marginal contribution equilibrium;
(b) \( \forall \subseteq J : \sum_{i \in S} \hat{M}_i \leq \hat{M}_S ; \)

(c) \( \forall A \subseteq \mathcal{A}, S \subseteq J : \)

\[ W(\mathcal{A}) - \sum_{i \in S} M_i(\mathcal{A}) \geq W(A) - \sum_{i \in S} M_i(A). \]

In the truthful equilibrium of the agenda game, agenda and action choice will be efficient. But the option to increase rent extraction tomorrow allows the agent to extract more surplus from the principals than she could in the static game. The next corollary identifies the principals who will see a decrease in their equilibrium payoff due to the possibility of agenda setting by the agent.

**Corollary 4.1.** *(Equilibrium payoffs in agenda game)* The equilibrium transfers \( \{r_i(\mathcal{A})\}_{i \in \mathcal{A}} \) have the properties:

(i) \( \forall S, \Sigma_{i \in S} (M_i - r_i(\mathcal{A})) \leq \hat{M}_S ; \)

(ii) \( \forall i, \exists S, \text{such that } i \in S \text{ and } \Sigma_{j \in S} (M_j - r_j(\mathcal{A})) = \hat{M}_S. \)

**Dynamic Common Agency**

**Truthful Equilibrium.** In the dynamic game, a reward strategy for principal \( i \) is a sequence of reward mappings

\[ r_i : A_t \times H_t \to \mathbb{R}_+, \]

which allocates a non-negative reward, possibly contingent on the entire past history of the game, to every action \( a_t \in \mathcal{A}_t \). An agent’s strategy is a sequence of actions over time

\[ a_i : A_t \times_{i=1}^{t} \mathbb{R}_+^{\left| \mathcal{A}_i \right|} \times H_t \to \mathcal{A}_t, \]

which depends on the profile of reward schedules in period \( t \) and history until period \( t \). The strategies that depend on \( h_t \) only through \( \theta_T \) are called Markov strategies.

The expected discounted payoff with a history \( h_t \) for a given sequence of reward policies \( r \) and action profiles \( a \) is denoted by \( V_0(h_t) \) for the agent and \( V_i(h_t) \) for principal \( i \). When \( a \) and \( r \) are Markov policies, then the values are given by \( V_0(\theta_t) \) and \( V_i(\theta_t) \) if the state is \( \theta_t \) in period \( t \). \( \mathbb{E}V_i(a_t, \theta_t) \) represents the
expectation of the continuation value in period \( t + 1 \) if in period \( t \) the action was \( a_t \) and the state was \( \theta_t \). Henceforth, for simplicity, the expectations operator \( E[\cdot] \) is omitted, while the transition from \( \theta_t \) to \( \theta_{t+1} \) may be stochastic.

**Definition 4.2.** (Markov perfect equilibrium) The strategies \( \{ r_i(a_t, \theta_t) \}_{i \in J} \) and \( a(\cdot, \theta_t) \) form a Markov perfect equilibrium (MPE) if

(i) \( \forall \theta_t, r'(\cdot), a(r'(\cdot), \theta_t) \) is a solution to

\[
\max_{a_t \in A_t} \left\{ r'(a_t, \theta_t) - c(a_t, \theta_t) + \delta V_0(a_t, \theta_t) \right\},
\]

(ii) \( \forall i, \theta_t \), there is no other reward function \( r'_i(a_t, \theta_t) \), such that

\[
v_i(a', \theta_t) - r'_i(a', \theta_t) + \delta V_i(a', \theta_t) > v_i(a_t, \theta_t) - r_i(a_t, \theta_t) + \delta V_i(a_t, \theta_t),
\]

where \( a \) and \( a' \) are best responses to \( (r_i(\cdot), r_{-i}(\cdot)) \) and \( (r'_i(\cdot), r_{-i}(\cdot)) \), respectively.

Truthful strategies reflect accurately each principal’s net willingness to pay, as in the static game, with the difference that the allocation relative to which truthfulness is defined is now an action \( a_t \) and a state \( \theta_t \). The intertemporal net benefit \( n_i(a_t, \theta_t) \) of an allocation \( a_t \) in the state \( \theta_t \) is the flow benefit \( v_i(a_t, \theta_t) - r_i(a_t, \theta_t) \) and the continuation benefit \( \delta V_i(a_t, \theta_t) \):

\[
n_i(a_t, \theta_t) \triangleq v_i(a_t, \theta_t) - r_i(a_t, \theta_t) + \delta V_i(a_t, \theta_t).
\]

**Definition 4.3.** (Truthful (Markov) Strategy)

(i) A reward function \( r_i(a_t, \theta_t) \) is truthful relative to \( (a, \theta_t) \) if \( \forall a_t \in A_t(\theta_t) \), either:

(a) \( n_i(a_t, \theta_t) = n_i(a, \theta_t) \), or

(b) \( n_i(a_t, \theta_t) < n_i(a, \theta_t) \) and \( r_i(a_t, \theta_t) = 0 \).

(ii) The strategies \( \{ r_i(\cdot) \}_{i=1}^I \) and \( a(\cdot, \theta_t) \) are an MPE in truthful strategies if they are an MPE and \( \{ r_i(\cdot) \}_{i=1}^I \) are truthful strategies relative to \( a(\cdot, \theta_t) \).

**Theorem 4.3.** A Markov perfect equilibrium in truthful strategies exists.

**Characterization.** The marginal contribution of principal \( i \) is

\[
M_i(\theta_t) \triangleq W(\theta_t) - W_{-i}(\theta_t).
\]
Chapter 4. Dynamic Common Agency

\( W(\theta_t | a_t) \) is defined to be the social value of the program which starts with an arbitrary and not necessarily efficient action \( a_t \), but thereafter chooses an intertemporally optimal action profile. Similarly, let \( M_i(\theta_t | a_t) \triangleq W(\theta_t | a_t) - W_{-i}(\theta_t | a_t) \).

The maximal value the agent and a subset \( J \setminus S \) of principals can achieve along the equilibrium path is obtained by selecting \( a_t \) so as to solve

\[
\max_{a_t} \left\{ W(\theta_t | a_t) - v_S(a_t, \theta_t) - \sum_{i \in S} \delta V_i(a_t, \theta_t) \right\}.
\]

The net value \( n_S(\theta_t) \) of the set \( S \) of principals in truthful equilibrium must then satisfy the following inequality in every period:

\[
n_S(\theta_t) \leq W_i(\theta_t) - \max_{a_t \in \mathcal{A}} \left\{ W(\theta_t | a_t) - v_S(a_t, \theta_t) - \sum_{i \in S} \delta V_i(a_t, \theta_t) \right\}.
\]

The following result is obtained:

**Theorem 4.4. (Efficiency)**

1. All MPE in truthful strategies are efficient.
2. \( \forall S \subseteq J \),
   \[
   \sum_{i \in S} V_i(\theta_t) \leq M_S(\theta_t).
   \]

**Marginal Contribution Equilibrium.** A marginal contribution equilibrium is a Markov perfect equilibrium in truthful strategies in which the payoff of each principal coincides with his marginal contribution, or \( \forall i, \theta_t, \ V_i(\theta_t) = M_i(\theta_t) \).

**Theorem 4.5. (Marginal Contribution Equilibrium)** The marginal contribution equilibrium exists if-f

\[
\sum_{i \in S} (m_i(\theta_t) - M_i(\theta_t | a_t)) \leq W_i(\theta_t) - W(\theta_t | a_t), \ \forall a_t, \theta_t, S. \quad (4.1.5)
\]

(4.1.5) can be directly interpreted as the trade-off between rent extraction and efficiency gains. A reformulation of this inequality provides a link between the static and dynamic conditions for the existence of a marginal contribution equilibrium. For any state \( \theta_t \), define

\[
\hat{M}_S(\theta_t) \triangleq W(\theta_t) - \max_{a_t \in \mathcal{A}} \left\{ W(\theta_t | a_t) - v_S(a_t, \theta_t) - \sum_{i \in S} \delta M_i(a_t, \theta_t) \right\}
\]
as the recursive contribution of a subset $S$ of principals. The main difference between the marginal contribution, $M_S(\theta_t)$, and the recursive contribution, $\hat{M}_S(\theta_t)$, is that the former attributes the entire future marginal contribution of coalition $S$ to its members, while the latter attributes only the sum of individual marginal contributions. These two notions are equivalent if and only if the marginal contributions are additive. Likewise, if the marginal contributions are (strictly) superadditive, or $\Sigma_{i \in S} M_i(\theta_t) < M_S(\theta_t)$, it is shown that $\hat{M}_S(\theta_t) < M_S(\theta_t)$.

**Corollary 4.2.** A marginal contribution equilibrium exists if and only if for all $\theta_t, S$

$$\sum_{i \in S} \hat{M}_i(\theta_t) \leq \hat{M}_S(\theta_t).$$

**Corollary 4.3.** (Uniqueness) A MPE in truthful strategies is unique if and only if it is a marginal contribution equilibrium.

### 4.2 Further Research and Applications

Aidt and Dutta (2004) extend the Dynamic Common Agency game presented above by allowing the principals to have only one incentive instrument available; the decision to reappoint the agent or not. They say that a sequence of performance standards displays strategic consensus if the agent prefers to meet both standards at all times, both principals support her reappointment, and she is reappointed with certainty. Their main result is that all equilibrium paths display strategic consensus.

Mourmouras and Mayer (2004) consider a dynamic version of their earlier common agency model (Mayer and Mourmouras, 2002), in which an International Financial Institution is willing to provide assistance to a reforming government which competes with interest groups opposing reform. Their model, in which every period is similar to the initial period, is extended by Paloni and Zanardi (2006), who specify a fully dynamic common agency model, analyzing the implementation of policy reforms supported by the IFI. They show that conditionality may increase the social welfare of the recipient country.

In his study, Ishihara (2015) examines the conditions under which the interest groups can commit to their contribution agreement. He considers an infinitely repeated game in which a politician makes the decisions and the lobbyists voluntarily offer financial contributions contingent to the observed decision.
in each period. He finds that in the initial period of the optimal punishment for a deviating principal, the agent chooses a decision that harms the principal, with the addition of a sanction fine transferred from the principal to the agent. After the initial period, the decision chosen by the agent may be jointly beneficial for the players.

Finally, Boyce (2010) models conservationists and harvesters attempting to influence the regulator’s harvest quota allocations and Filson (2005) applies the dynamic common agency model to a movie distribution concept modelling two distributors and one exhibitor.
Chapter 5

Conclusion and Ideas for Future Research

This study has highlighted the contribution of the common agency game not only to the specific area of Political Economy, but also to many other economic issues. Beginning with some theoretical aspects, in Chapter 2, we identified the origin of the basic model and how it was established in the game theory literature. In Chapter 3, which involves our principal concerns of this study, we demonstrated the fruitfulness of the model in capturing the political economy setting, in which various interest groups seek to influence a politician’s policy choice in order to favor their members. We also refer to a number of weaknesses of the ‘Protection for Sale’ model and the ways that some of them have been addressed by the subsequent literature. In the final part of our study, Chapter 4, a dynamic version of the theoretical (static) common agency model and some applications were introduced.

Despite the broad political economy literature based on the common agency approach, there are still many aspects that have not been covered yet. For instance, regarding the formation of the interest groups, its optimal size has not been specified. Moreover, the existing literature has not identified the reason that professional lobbyists are employed, and whether should they be common or exclusive.

Within the common agency framework it is assumed that the interest groups influence the decisions of a government offering financial contributions. However, another way of influence could be the transmission of information. For instance, an interest group could be capable of gathering information about an issue and then provide it to the government, or alternatively, could take advantage of the unawareness of the politician and induce her to choose its favorable policy. Bennedsen and Feldmann (2006) has modelled interest groups providing both money and information to an incumbent government, but their model
was not based on the common agency framework. Another possible extension could be the introduction of alternative and probably more realistic means of influence, other than financial contributions and transmission of information (e.g. offer for a future career to a politician, election of an interest group member as a policymaker).

The introduction of political threats to an electoral competition possibly answers why, in fact, there is so little money in politics compared to the value of the favors campaign contributions buy (Tullock, 1972). Chamon and Kaplan (2013) and Motz (2015) predict that an interest group will contribute only to one of the two candidates competing in a race. However, the model could be extended to situations in which an interest group has a strong motive for seeing the challenger elected. This may lead to split contributions, in which one interest group contributes to both candidates.

Finally, an extension of the model of Chesnokova (2014) considering a model of electoral competition with the marginal contribution of a lobby to be decreasing in the total amount of money collected by the government would be very interesting.
References


Ishihara, A. (2015), ‘Relational political contribution under common agency’, *Available at SSRN 2039488*.


REFERENCES

