INVESTIGATING THE DYNAMICS OF COMMODITY MARKETS

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Abstract

Many economic time-series present significant changes in their behavior in the wake of "extreme" events such as a financial crises or war. When such changes arise in the (time-series) data, a powerful statistical tool can be employed, i.e. that of Markov switching models. This dissertation employs the Markov regime switching framework to capture the time-varying dynamics in 5 commodity indexes over the period 1995:10-2015:09. These indices are the following: (a) the Commodity Agricultural Raw Materials Index; (b) the Commodity Metals Price Index; (c) the Commodity Fuel (energy) Index; (d) the Commodity Food Price Index; and (e) Gold. Also, the return of S&P 500 is used as an additional regressor. The empirical results show that the S&P 500 and the food index have a positive and statistically significant effect on all commodity returns.

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1. INTRODUCTION

Commodities markets, both historically and in modern times, have significant economic impact on nations and people. The impact of commodity markets throughout history is still not fully known, but it has been suggested that rice futures may have been traded in China as long ago as 6,000 years. Shortages on critical commodities have sparked wars throughout history (such as in World War II, when Japan ventured into foreign lands to secure oil and rubber), while oversupply can have a devastating impact on a region by devaluing the prices of core commodities. Energy commodities such as crude oil are closely watched by countries, corporations and consumers alike. The average Western consumer can become significantly impacted by high crude oil prices. Alternatively, oil-producing countries in the Middle East (that are largely dependent on petrodollars as their source of income) can become adversely affected by low crude oil prices. Unusual disruptions caused by weather or natural disasters can be an impetus for price volatility, but can also cause regional food shortages.

Commodity markets have gained significant investor interest in recent years. According to the Investment Company Institute, total net assets of commodity exchange traded funds grew from $1bn in 2004 to more than $100bn in 2010. Commodity markets, particularly those for precious metals, have also been proposed as a vehicle for hedging investors' exposure to inflation risk. This has featured prominently recently due to central bank implementation of quantitative easing policies combined with increased uncertainty about future inflation rates. Increases in commodity prices, notably crude oil, have also been linked to economic recessions and deterioration in growth prospects. Ten of eleven postwar recessions were preceded by sharp increases in the price of crude petroleum.

The examination of the commodity price dynamics is quite important for countries relying heavily on exports of primary commodities, and hence facing the possibility of a trade shock in case of drop in commodity prices. Further, the need to understand the factors influencing the behavior of commodity prices has taken on a new urgency in recent years, as non-oil commodity prices have fallen sharply and persistently in real terms since the early 1980s (Borensztein and Reinhart 1994).

The objective of this dissertation is, by using commodity spot price indexes over the period 1995-2015, to examine the dynamics of commodity prices. Section two reports the literature review about the behavior and determinants of commodity prices, section three describes the

data set in our methodology while section four presents the empirical results. Finally section five concludes.

2. LITERATURE REVIEW

2.1 THE BEHAVIOR OF COMMODITY PRICES

2.1.1 The Behavior of Commodity Prices

Figure 2-1 presents the behavior of four commodity indices (foods, raw materials, metals and a non-energy index) and oil. There are several interesting points that can be made with regard to the behavior of commodity prices over the period 1960-2008.

- Decline of the 1980's: Commodity prices fell persistently from the eighties until the end of 1990's.
- Upsurge of 2000's: After 2000 commodity prices, predominantly the price of metals and oil, moved upward.

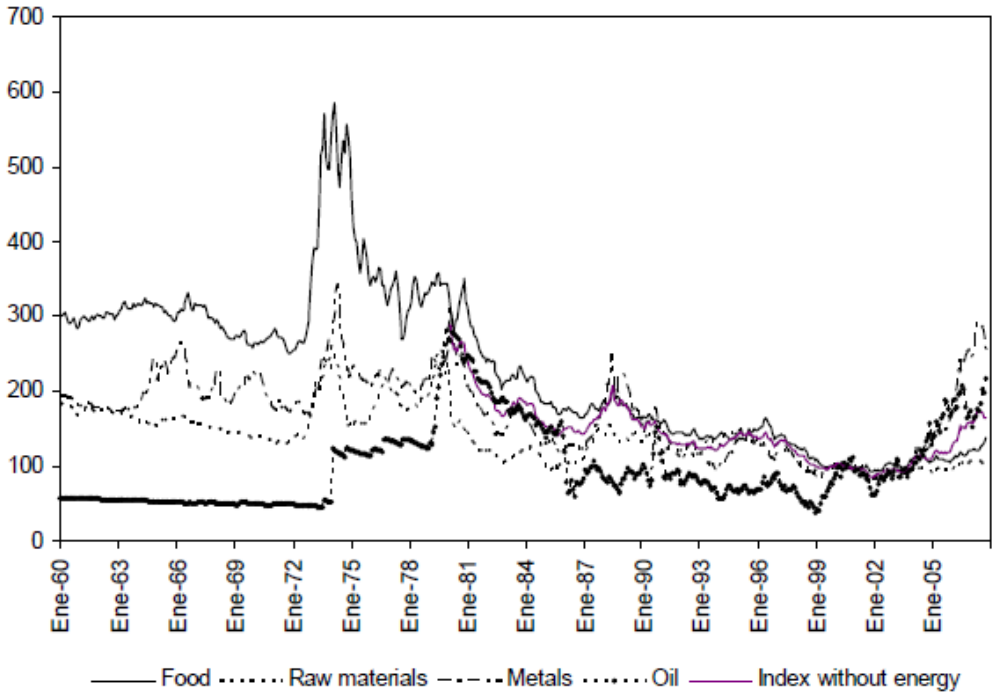

Figure 2-1: The Behavior of Real Commodity Prices

Source: IMF

The sharp fall in commodity prices after the 1980's can be attributed to demand-driven factors. Indeed, during the early 1980's the industrial production in many industrial countries was weak, as several of these countries experienced prolonged and deep recessions, and at the

same time the dollar (i.e. the currency at which commodity prices are denominated) appreciated in real terms by nearly 50 percent (Borensztein and Reinhart, 1994).

However, after 1984, despite the depreciation of the dollar and the robust growth record of several major industrial countries, commodity prices did not seem to pick up, as the "demand-driven" model would suggest. A number of reasons have been put forward to account for the persistent weakness in commodity prices after 1984 (Borensztein and Reinhart, 1994).

The most important, seems to be the response of developing countries to the debt crisis of the 1980s and the economic developments in the transition economies of Eastern Europe and the former Soviet Union (Borensztein and Reinhart, 1994). Specifically, since the mid-1980s, the supplies of commodities by developing markets increased, as these economies expanded their commodity exports in an attempt to service their increasing debt obligations (Aizenman and Borensztein (1988); Gilbert (1989)). Further, many countries of the former Soviet Union sharply increased their exports of various metals.

In general, the (relative) price of commodities presents a downward tendency. Indeed, Bleaney and Greenaway (1993) examined the ratio of primary commodity prices to the price of manufactured goods over the period 1900- 1991 and they concluded that there was a statistically significant long-run downward trend in the price of commodities.

There could be several explanations for this downward trend in the price of commodities (see Cuddington (1992)). To begin with, foods are regarded as less than normal goods, in the sense that the demand for them exhibits low income elasticity; hence an increase in income will increase by less the demand for such goods. Then, different rates of technological progress between commodities and manufactured goods favor the production of the latter. Finally, there are different degrees of competition in commodity and manufactured good markets, the former being more competitive, and hence the tendency for lower prices. Indeed, Bloch and Sapsford (1997), using a sample spanning the period 1948-1986 to estimate the coefficients of a structural model for the price of 24 commodities relative to manufactured product prices, found that a net trend in the terms of trade of -1.5% per year could be explained by wages and mark-up pricing in the manufactured goods.

Fig. 2-2 to 2-8 presents the downtrend in the price of some commodities relative to the Unit Value Index of Exports (UVIE) a proxy index for the prices of manufactured goods from 20 industrial countries. The group of commodities with a downward trend consists of coffee, maize, rubber, wheat, cotton, gold and oil. Figure 2.9 presents the UVIE and the industrial

production index for developed countries. The vertical grey lines of the graphs are used to show the periods of recession.

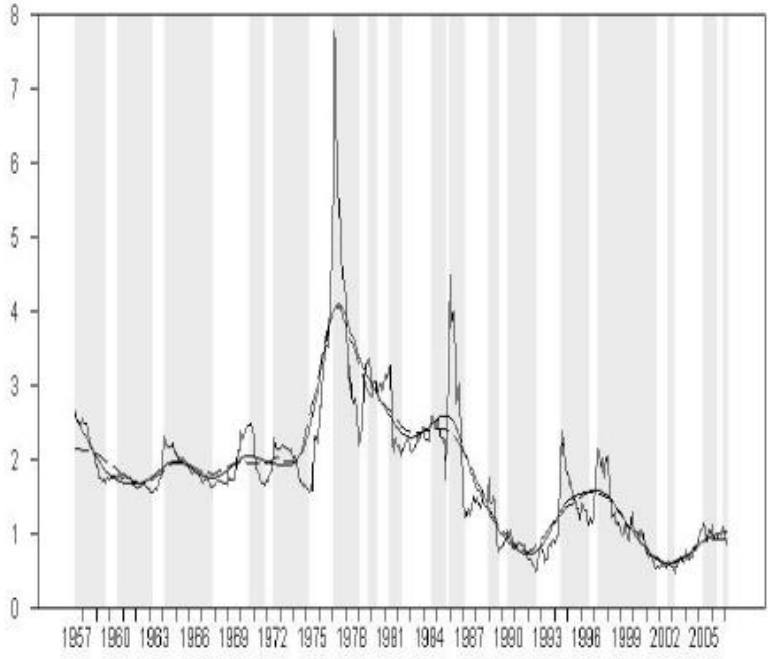

Figure 2-2: The Relative Price of Coffee

Source: Arango *et al.,* 2008

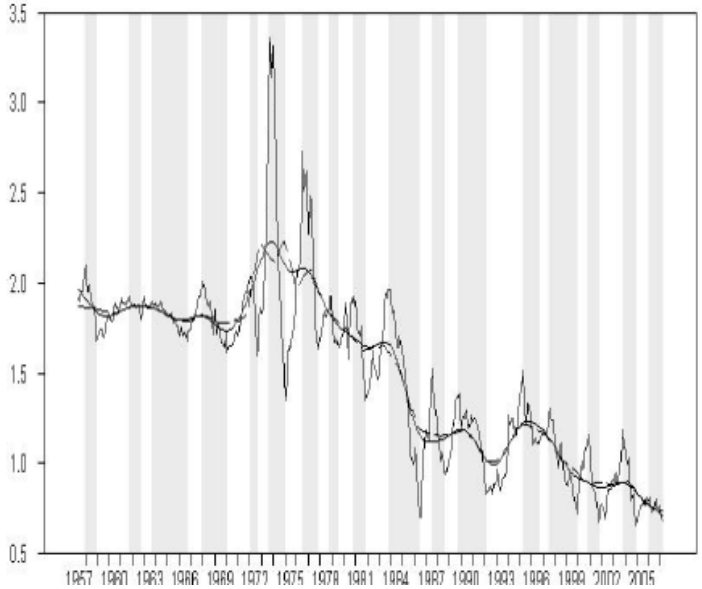

Figure 2-3: The Relative Price of Cotton

Source: Arango *et al.* 2008

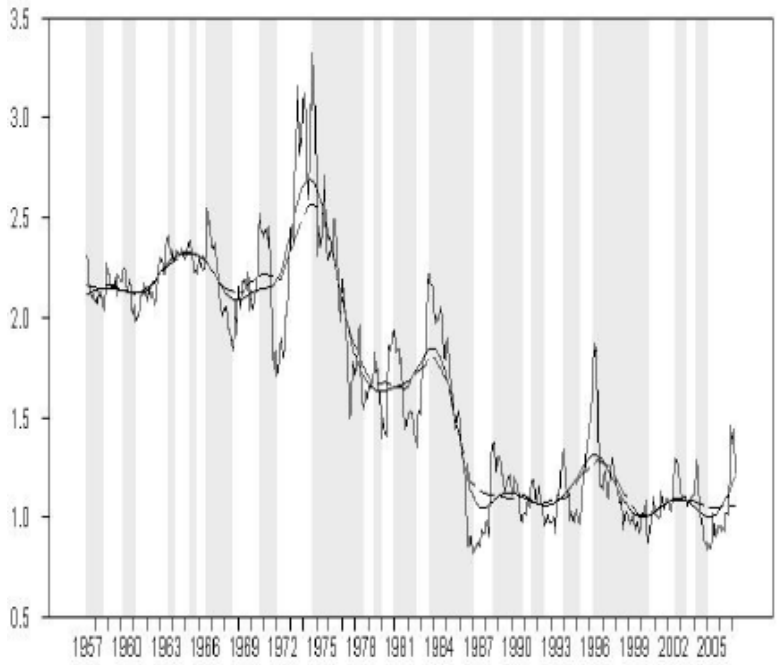

Figure 2-4: The Relative Price of Maize

Source: Arango *et al.* 2008

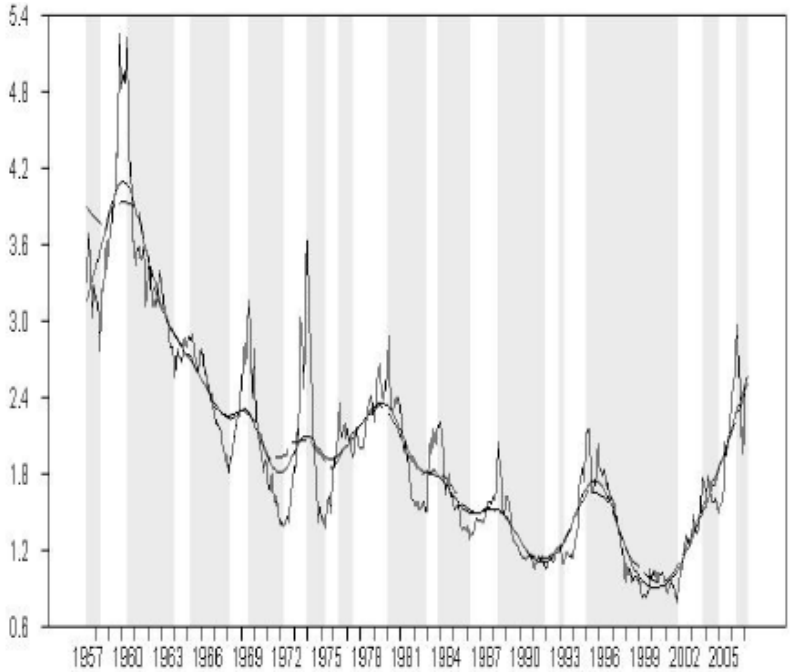

Figure 2-5: The Relative Price of Rubber

Source: Arango *et al.,* 2008

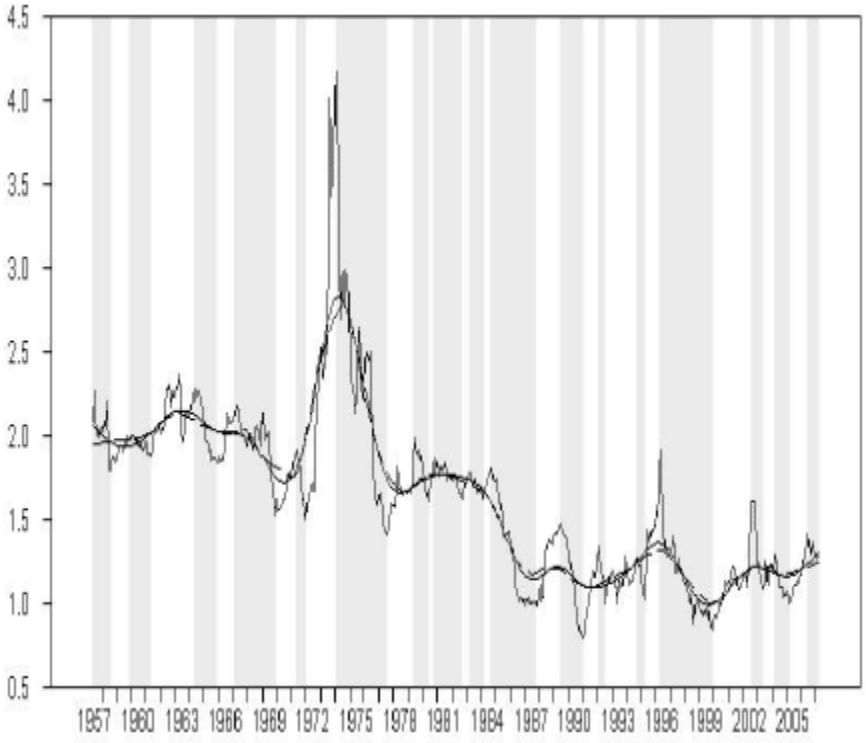

Figure 2-6: The Relative Price of Wheat

Source: Arango *et al.,* 2008

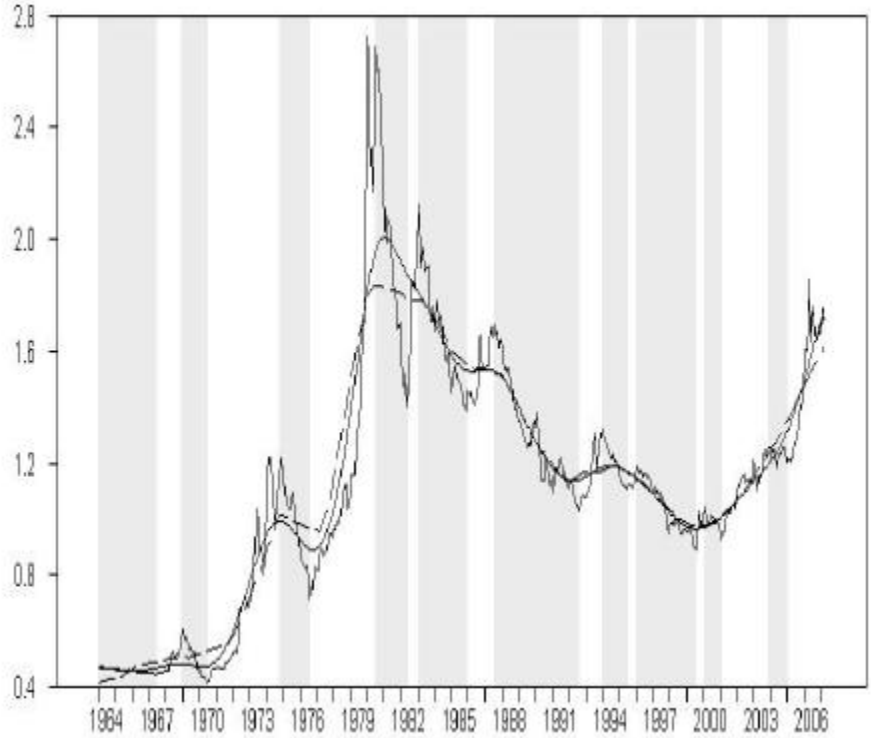

Figure 2-7: The Relative Price of Gold

Source: Arango *et al.,* 2008

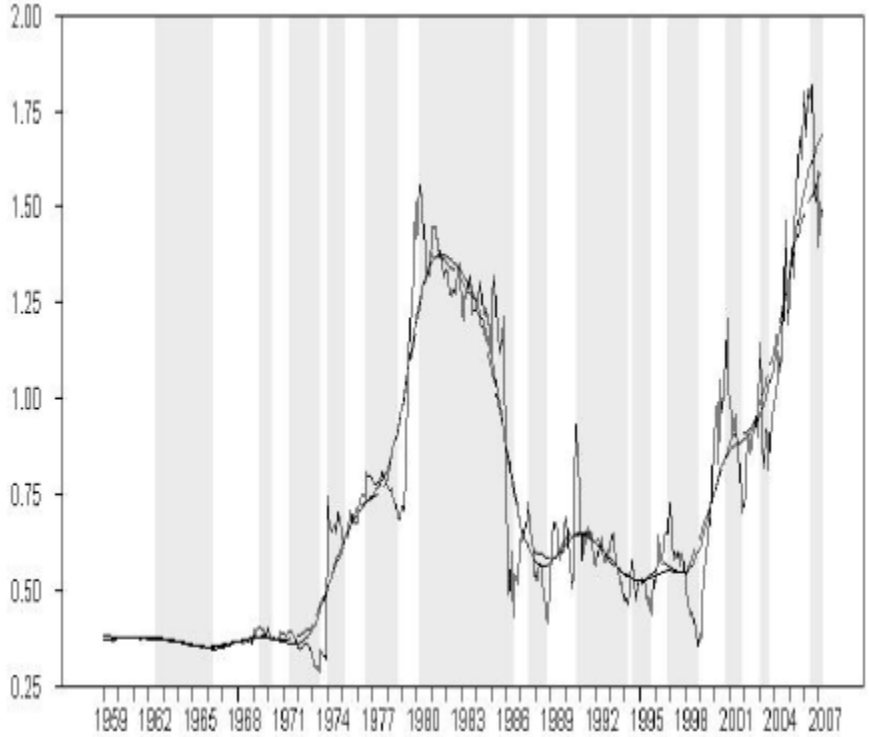

Figure 2-8: The Relative Price of Oil

Source: Arango *et al.,* 2008

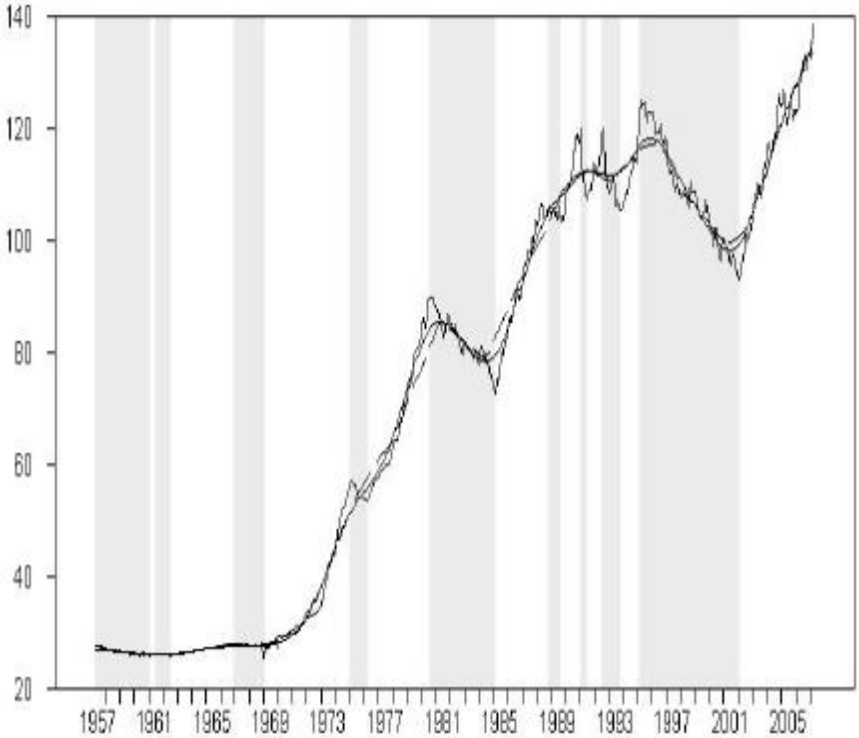

Figure 2-9: Manufactured Goods Prices

Source: Arango *et al.,* 2008

2.1.2 The Co-movements of Commodity Prices

Several studies (Harri et al., 2009, Reboredo, 2012,) have focused on the co-movements among commodities and measures of economic activity. Pindyck and Rotemberg (1990) studied the co-movements of seven commodities (wheat, cotton, copper, gold, crude oil, lumber and cocoa) over the period 1960- 1985. From their empirical findings, the authors could not attribute the co-movements among commodity prices to current and expected future values of macroeconomic activity. Cashin et al. (1999) studied the (excess) co-movements of commodity prices by measuring the extent to which their cycles are coordinated. From their findings the authors rejected the idea of cycle coordination on the part of commodity prices.

2.2 THE DETERMINANTS OF COMMODITY PRICES

2.2.1 Demand Factors in the Determination of Commodity Price

The traditional “demand-approach” focuses on demand factors as the underlying determinants for the behavior of commodity prices; two important such variables, which were studied in the literature, were the level of industrial production for industrial countries and the real exchange rate of the U.S. dollar. The real exchange rate of the U.S. dollar is taken into consideration since the prices of commodities are denominated in dollars, and hence they must be deflated by a dollar-denominated price index, whereas the relevant measure for the non-U.S. industrial countries is the price of commodities relative to output prices in those countries.

Within this framework, Borensztein and Reinhart (1994), relying on quarterly data for the period 1970:Q1-1992:Q3, estimated a model where the dependent variable was the IMF non-oil all-commodity index (deflated by the US GNP deflator) and as explanatory variable the industrial production index (for industrial countries, seasonally adjusted) and the US real exchange rate (relative to other industrial countries). The results showed that the estimated elasticity for industrial production and the real exchange rate were 1.99 and -1.52, respectively. Also, the authors took broader view of "world" demand that extends beyond the industrial countries and includes output developments in Eastern Europe and the Former Soviet Union.

Jacks and Stuermer (2015) looked at the long-run determinants of commodity prices by analyzing annual data on prices and production levels for 14 agricultural, metal, and soft commodities over a 162-year period from 1850 to 2012. The authors identified differences in the type of shock driving prices of the various types of commodities, and they related these differences to commodity types which presumably reflect differences in long-run elasticity of supply and demand. The empirical evidence led the researchers to the conclusion that demand shocks strongly dominated the supply shocks in the determination of commodity prices.

2.2.2 Macroeconomic Factors in the Determination of Commodity Prices

Monetary policy and interest rates could be another determinant of commodity prices (see Frankel (2005, 2006)). Specifically, higher (short-term) interest rates lead to an expansion of commodities supply, and in conjunction with reduced demand for commodities, the commodity prices will drop. The literature has documented that other demand determinants of commodity prices are fiscal balances (IMF, 2006), and the impact of former Soviet Union (Borensztein and Reinhart, 1994).

Gargano and Timmermann (2012), using a range of commodity spot price indexes over the period 1947-2010, investigated the predictability of commodity returns at a monthly, quarterly and annual horizons. The authors used predictive variables capturing the state of the economy, such as bond spreads (the return difference between long-term corporate and government bonds), growth in money supply, growth rate in industrial production, and the unemployment rate. The results of the study revealed that out-of-sample commodity-return predictability varies considerably across different horizons, as there was modest evidence of out-of-sample predictability on monthly movements in metals and raw industrials commodity spot price indexes. The important predictors on a monthly basis were the T-bill rate, the default return spread, and money supply growth. At a longer time horizon, the authors found that out-of-sample commodity price predictability strengthens considerably. Variables such as the T-bill rate, investment-capital ratio, money supply growth and also the rate of inflation were found to have predictive power over changes in raw industrials and metals spot prices. Then, at an annual horizon, the evidence is even stronger with a host of similar predictor variables apparently capable of predicting movements in commodity prices. The only variable that consistently predicted commodity spot price movements, at all time horizons (monthly, quarterly and annual horizons), was the growth in the narrow money supply, M1.

2.2.3 Supply Factors in the Determination of Commodity Prices

Borensztein and Reinhart (1994) augmented the traditional demand-approach in the determination of real commodity prices by taking into consideration the supply-side of the market. To proxy the supply-side factors, the authors used the volume of primary commodities imported (primary commodity imports excluding oil for 14 industrial countries including the United States) by industrial countries as a price determinant of the price equation. The 14 industrial countries that comprise the supply index are Austria, Belgium, Canada, Denmark, France, Germany, Italy, Japan, Netherlands, Norway, Sweden, Switzerland, United Kingdom, and the United States. The coefficient on this supply variable was found to have the expected sign, i.e. a negative sign indicating that an expansion in supply, other things being equal, reduces commodity prices, and to be statistically significant. The supply coefficient at -0.9 suggests that an increase in commodity supply translates to an almost proportional decline in its price.

Thomas et al. (2010) examined the supply side of the commodities markets as a key price determinant. The authors employing as explanatory variables in their model the VIX to proxy for the risk aversion, the implied volatility of the S&P 500 index options and geopolitical risk, came to the conclusions that, on the one hand, speculation does not drive commodity prices, and on the other hand, the long-run dynamics in the real oil price can be traced down to a combination of supply-side factors, like oil production from non-OPEC and demand-side factor, like demand from OECD countries and emerging markets.

Brooks and Prokopczuk (2011) concluded that commodities have very different stochastic properties, and therefore it is suboptimal to consider them as a single, unified asset class. The sample consisted of daily spot price data, over the 25-year period 1985-2010, for the commodities traded in the US: namely crude oil (CL), gasoline (HU), gold (GC), silver (SI), soybeans (S) and wheat (W). The first two are energy commodities, the second two are metals, and the last two are agricultural commodities.

Specifically, the authors examined a number of issues for six major commodity markets. First, they investigated whether volatility does indeed behave stochastically. Second, they examined the volatility of volatility, its persistence, and whether changes in prices and volatility are correlated. They additionally allowed for jumps in both prices and volatility. They also investigated the linkages between commodity markets considered. They determined whether volatilities are correlated across markets and whether the prices or the volatilities of different commodities jump at the same time.

The findings were the following. First, within the stochastic volatility framework, the models allowing for jumps provide a considerably better fit to the data than those which do not, although there is little to choose between the models allowing for jumps in returns only and those allowing for jumps in both returns and volatility. Second, alternate signs were observed in the relationships between returns and volatilities for different commodities – negative for crude oil and equities, close to zero for gasoline and wheat, and positive for gold, silver, and soybeans. The authors attributed these differences to variations in the relative balances of speculators and hedgers across the markets.

2.3 STRUCTURAL BREAKS IN COMMODITY PRICES

Pala (2013), using monthly data over the period 1990:01-2011:08 and Granger Causality, investigated the relationship between crude oil price index and food price index. The Food and Agricultural Organization (FAO) Price Index consists of five food commodities (meat, dairy, cereals, oils and fats and sugar) and the Brent crude oil price index were used as derivative for oil price. The author made use of the Zivot and Andrews (1992) test to test for unit-roots allowing for one endogenously determined structural break. Further, the author used the Clemente-Monates-Reyes test (Clemente et al, 1998) to test for unit-root allowing for two structural breaks. This test allows for a break in the slope, and a break in both the intercept and the slope of the time series.

The results of the study indicated evidence for breaks after August and November 2008. Specifically, the break date for crude oil was 2008:08 and for food 2008:07. The Clemente-Monates-Reyes test indicated 2008:08 and 2008:11 as the optimal break dates for the intercept and the slope of the food price index and the oil price index. The author also found a clear long-run relationship between these series for the full and sub sample. Co-integration regression coefficient is negative at the 1990:01-2008:08 time period, but adversely positive at the 2008:11-2011:08 time period.

Mariscal and Powel (2014) applied the Impulse Indicator Saturation (IIS) and the Step-Indicator Saturation (SIS) method to endogenously identify breaks in long-term commodity price series. These statistical techniques provide a general procedure for detecting an unknown number of structural breaks, occurring at unknown times, with unknown duration and magnitude, anywhere in the sample. The index used by the authors to analyze commodity prices was the Grilli and Yang (1988) Commodity Price Index (GYCPI), a weighted average of 24 commodities barring oil, with weights having been calculated using the average export

share of the period 1977-1979. Further, the authors, taking into consideration the manufacturing unit value index (MUV), constructed an Index of Relative Commodity Prices by taking logarithms of the GYCPI and MUV.

The results showed that the price of commodities relative to manufactured goods did decline, but the decline was not stable, having a number of breaks. Specifically, the authors found two major negative breaks, one in 1921 and the other in 1985. Having estimated relevant breaks, the authors estimated a set of error correction models for commodity prices.

3. DATA AND METHODS OF ANALYSIS

3.1 DATA

The total sample consists of monthly observations on commodity prices spanning the period 1995:10 to 2015: 09. The rate of return for each data series is specified as the ratio of the price in period t to price in period t-1 and subtracting 1 from this ratio. The time series of the sample involves the following commodity price indices, which have been derived from the Index Mundi website[1].

- **Agriculture**: The Commodity Agricultural Raw Materials Index includes Timber, Cotton, Wool, Rubber, and Hides Price Indices.
- **Metals**: The Commodity Metals Price Index includes Copper, Aluminum, Iron Ore, Tin, Nickel, Zinc, Lead, and Uranium Price Indices.
- **Energy**: The Commodity Fuel (energy) Index includes Crude oil (petroleum), Natural Gas, and Coal Price Indices.
- **Food:** Commodity Food Price Index includes Cereal, Vegetable Oils, Meat, Seafood, Sugar, Bananas, and Oranges Price Indices.
- **Gold:** Gold (UK), 99.5% fine, US$ per troy ounce.

Figure 1 (Appendix A), shows that all series had an upward trend until 2008, when a sudden drop in all series was observed, and then all series picked up their upward trend.

Table 3-1 shows the descriptive summary statistics of the six commodity markets considered. Several points are worth noting. The energy index has the highest mean monthly return of 0.66%, while the lowest average monthly return of 0.047% is observed for the

[1] http://www.indexmundi.com/commodities/?commodity=food-price-index&months=240

agricultural index. The energy index has the highest standard deviation of 7.02%, and the agricultural index the lowest of 3.02%. The volatility of the equity market (as this is captured by the monthly return on the S&P 500 Index) is about the same with that of metals, i.e. 4.4%.

Table 3-1: Descriptive statistics of commodity indexes

	AGRICUL	ENERGRY	FOOD	GOLD	METALS	SP500
Mean	0.000470	0.006609	0.001328	0.005234	0.003213	0.005993
Med	0.000992	0.014810	0.001404	0.000871	0.002825	0.010674
Max	0.083423	0.179388	0.092391	0.173680	0.152050	0.107723
Min	-0.1619	-0.2361	-0.1580	-0.1173	-0.1988	-0.1694
Std Dev.	0.030273	0.070266	0.030374	0.038277	0.046314	0.044059
Skew	-0.7092	-0.5925	-0.3894	0.533848	-0.0256	-0.6537
Kurt.	6.196485	3.731032	5.800335	4.628987	4.492262	3.999264
Jarq	121.7840	19.30530	84.13316	37.77767	22.20182	26.96756
Prob	0.000000	0.000064	0.000000	0.000000	0.000015	0.000001
Obs	239	239	239	239	239	239

3.2 METHODS OF ANALYSIS

Markov regime-switching (MRS) models constitute a generalisation of the simple dummy variables approach, allowing regimes (called states) to occur several periods over time. According to this model, random variable Y_t takes on values in each period t depending on the state $S_t = m$, where m is a possible state. $m = 1,2, \dots, M$.

For the sake of simplicity we consider a simple model with just two states ($M = 2$) that is, the discrete, but unobserved, random variable S_t takes on two values $S_t = 1$ or S_t=2.We assume that the random variable of interest Y_{jt} (for $j = 1,2, \dots 6$) follows a process that depends on the random variable S_t, according to the following model (see Lindgren (1978) and Baum, et. al. (1980)).

$$Y_{j,t} = \mu_{S_t} + U_t \qquad 3.1$$

$$U_t \sim N(0, \sigma_{S_t}^2) \qquad 3.2$$

$$\mu_{S_t} = \mu_1 Z_{1t} + \mu_2 Z_{2t} \qquad 3.3$$

$$\sigma_{S_t}^2 = \sigma_1^2 Z_{1t} + \sigma_2^2 Z_{2t} \qquad 3.4$$

$$\mu_s = c_{0,s} + \sum_{k=1}^{K} c_{k,s} Y_{k,t-1} \quad (s = 1,2) \qquad 3.5$$

$$k = 1,2, \dots, K = 6.$$

Where μ_{S_t} the process' conditional mean, which depends on the value of the state variable S_t, and U_t is the disturbance term for the time period $t = 1,2,..,T$. In the above model we can directly observe the values of y_t obtained at time $t = T$, but we can only make an inference about the value of S_t, where S_t is a random variable. We denote the set of these values as $\Omega_t = \{y_T, y_{T-1}, \dots, y_0\}$. Also, $Z_{1t} = 1$ when we are in the state 1 and zero otherwise, and $Z_{2t} = 1$ when we are in the second regime and zero otherwise. Note that the model depicted by Eq. (3.1)-(3.4) suggest that there is regime heteroscedasticity, that is, the variance of the series depends on the regime the series is on.

If we are in regime 1 (i.e. when $S_t = 1$), which is supposed to describe the observed data for $t = 1,2,..,t_0$ the model becomes,

$$Y_{j,t} = \mu_1 + U_{1t}$$

$$\mu_1 = c_{0,1} + \sum_{k=1}^{6} c_{k,1} Y_{k,t-1}$$

$$U_{1t} \sim N(0, \sigma_1^2)$$

Then at date t_0 a significant change in the average level of the series is supposed to take place and the data generating process, which is supposed to describe the observed data for $t = t_0 + 1, t_0 + 2, .., T$(i.e. when $S_t = 2$), has as follows

$$Y_{j,t} = \mu_2 + U_{2t}$$

$$\mu_2 = c_{0,2} + \sum_{k=1}^{6} c_{k,2} Y_{k,t-1}$$

$$U_{2t} \sim N(0, \sigma_2^2)$$

If the states are known a priori, then the problem boils down to dummy-variable regression approach, but in practice the prevailing regime is not directly observable. Further, the model suggests that the states occur randomly according to the value of the unobserved state variable Z_t. So when $Z_t = 0$, we will be in state $S_t = 1$, while $Z_t = 1$ we will be in state $S_t = 2$.

The random variable S_t evolves over time according to the following conditional probabilities stipulated by a Markov process

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \qquad 3.6$$

Where P is known as the transition matrix and $p_{ij}(t) = P(S_t = j|S_{t-1} = i) = p_{ij}$ denotes the (time-invariant) probability of that the variable $Y_t$will move from regime i in the previous period $(t-1)$ to the new regime j in the current period. So for example p_{12} is the probability that the random variable Y_t will move from state $S_t = 1$ to state $S_t = 2$. It is easy to show that[2]

$$P(S_t = m) = \sum_{j=1}^{2} P(S_{t-1} = j) P(S_t = m | S_{t-1} = j) \qquad 3.7$$

In a Markov process the future state of a variable depends solely on its present state, while it is independent of all its past states[3]. For example, the probability that we move from state $S_t = 1$ in time $t-1$ to state $S_t = 2$ in time t (p_{12}) depends on the state the variable was in period $t-1$.

One important question when dealing with Markov switching models is the following: given that we are actually in regime j (for example a regime of high volatility) how long on average this regime lasts? The expected duration of regime j is given by

$$E(D) = \frac{1}{1 - p_{jj}} \qquad 3.8$$

[2] For example, the probability of transition from state 1 into state 2 is

$$p_{12} = P(S_{t-1} = 1)P(S_t = 2|S_{t-1} = 1) + P(S_{t-1} = 2)P(S_t = 2|S_{t-1} = 2)$$

[3]The fact that a Markov chain is a random process where all information about the future is included in the present state can be seen from the following expression

$$\Pr(s_{t+1} = i|s_0 = j_0, s_1 = j_1, \dots, s_t = j) = \Pr(s_{t+1} = i|s_t = j)$$

4. EMPIRICAL RESULTS

4.1 UNIT ROOT TESTS

The conducted unit-root tests assume no trend in the data (as this is evident from Fig. 2 in Appendix A), but with a drift parameter (i.e. a constant) in the econometric specification. This specification, known as random walk with drift, has an increasing time-varying mean and variance, and hence it is non-stationary[4].

Table 4-1 presents the results of the unit root tests for the returns on a number of time-series variables. The table reports the estimated t-statistic of the ADF test. As the table shows, all variables are stationary.

Table 4-1: Unit root test

Variable (Return of Index)	ADF t −Statistic	Order of integration
Agriculture	-11.83^*	$I(0)$
Energy	-11.51^*	$I(0)$
Food	-9.73^*	$I(0)$
Gold	-13.63^*	$I(0)$
Metals	-10.98^*	$I(0)$
S&P 500	-14.21^*	$I(0)$

Note: For all variables except the "gold" variable, the model that is tested is that of random walk with drift. *Indicates statistically significant estimate at 5%. Estimation period: 1995M10-2015M9. The estimated model for the time-series variable Y_t, within the augmented Dickey–Fuller (ADF) test framework, is $\Delta Y_t = \delta + \varphi Y_{t-1} + \sum_{i=1}^{p} a_i \Delta Y_{t-i} + \varepsilon_t$, where ε_t is a pure white-noise error. The number of lagged difference terms (ΔY_{t-i}) to include is often determined empirically, the idea being to include enough terms so that the error term is serially uncorrelated.

[4]Specifically, the mean of the process increases by a constant δ in each time period compared with the previous time period, while the variance of the process increased by a constant σ^2, which is the variance of the disturbance term.

4.2 MARKOV SWITCHING RESULTS

4.2.1 Parameter Estimates

Tables 4-2, 4-3 and 4-4 present the results from the estimation of the two-state Markov model (3.1). The model has identified two distinct regimes, each with a different mean and standard deviation. Further, the assumption of regime heteroscedasticity has been employed, that is, each regime is assumed to a have a different volatility. Regimes are classified into low volatility and high volatility regime based on their standard deviations.

For each commodity index two estimations are reported; the first (initial) includes all explanatory variables and the second (final) includes only explanatory variables that are statistically significant either in the low or in the high volatility regime. To end up with the second model we remove (from the initial version of the model) the explanatory variable(s) which is (are) statistically insignificant (i.e. having the highest p-value) under both regimes. For example, consider the case of the Agricultural Index returns, the most statistically insignificant regressor is the first lag of energy (-0.0447), and hence the model is re-estimated without this explanatory variable.

Table 4-2: Estimation of the Markov Model for the Agriculture, Food and Energy Indexes

Regime	Estimate	AGRICULTURE		FOOD		ENERGY	
		Initial	Final	Initial	Final	Initial	Final
High volatility regime	C	-0.0104	-0.0097	0.0015	0.0014	0.0025	-0.0206*
	Agricult.(-1)	-0.2157		0.1489*	0.0948	-0.0053	
	Energy(-1)	-0.0447		0.0286	-0.0033	0.1911*	0.7057*
	Food(-1)	-0.1945	-0.2105	0.2772*	0.3570*	0.2189	
	Gold(-1)	0.4852*	0.4455*	-0.0071		0.1894	
	Metals(-1)	0.7724*	0.5251*	-0.0355		-0.0078	
	SP500(-1)	0.5325*	0.4952*	0.0386	0.0343	0.2055*	0.5293*
Low volatility regime	C	0.0010	0.0018	-0.0110	-0.0408*	0.0068*	0.0247*
	Agricult.(-1)	0.0890		-0.5964*	0.2400*	0.1404*	
	Energy(-1)	-0.0027		-0.1654	-0.9969*	0.3092*	-0.1774**
	Food(-1)	0.1960*	0.2151*	1.2712*	2.0617*	0.5286*	
	Gold(-1)	0.0188	0.0279	0.1375		0.3106*	
	Metals(-1)	0.0149	0.0145	0.2460		0.0576	
	SP500(-1)	-0.0175	-0.0256	0.2773**	1.0366*	0.2758*	-0.0334

Table 4-3: Estimation of the Markov Model for the Metals and Gold Indexes

Regime	Estimate	METALS		GOLD	
		Initial	Final	Initial	Final
High volatility regime	C	0.0006	0.0092	0.0042*	0.0042*
	Agriculture (-1)	0.1435		-0.0588	-0.0588
	Energy (-1)	-0.0800		0.0201	0.0201
	Food (-1)	-0.1324	0.3523**	0.1855*	0.1855*
	Gold (-1)	0.1465**		0.2137*	0.2137*
	Metals (-1)	0.5388*	0.0539	0.0895	0.0895
	SP500 (-1)	0.1865*	0.5997*	-0.0086	-0.0086
Low volatility regime	C	-0.0034	-0.0020	-0.0201*	-0.0201*
	Agriculture (-1)	-0.2881		0.9614*	0.9614*
	Energy (-1)	0.2302**		-0.3138*	-0.3138*
	Food (-1)	0.6115*	-0.0587	-0.8274*	-0.8274*
	Gold (-1)	-0.1505		-0.4912*	-0.4912*
	Metals (-1)	-0.3483**	0.2947*	-0.1195*	-0.1195*
	SP500 (-1)	0.5318*	0.1594*	0.2627*	0.2627*

Notes: *, ** indicates statistically significant t-statistic at 5% and 10%, respectively level of significance. Sample: 1995M10-2015M9. The initial model includes as explanatory variables all lagged variables. Standard errors are not reported for brevity.

Table 4-4: Summary Results

Regime	Estimate	INDEX				
		Agriculture	Energy	Food	Gold	Metals
High volatility regime	Agriculture (-1)			+		
	Energy (-1)		+			
	Food (-1)			+	+	+
	Gold (-1)	+			+	
	Metals (-1)	+				
	SP500 (-1)	+	+			+
Low volatility regime	Agriculture (-1)		+	+	+	
	Energy (-1)		+	+	-	
	Food (-1)	+	+	+	+	
	Gold (-1)		+		-	
	Metals (-1)				-	+
	SP500 (-1)		+	+	+	+

So, based on the results reported in the above three tables, we can see how indexes affect each other in each regime. First, the returns of the agricultural index, under the high volatility regime, are positively affected by gold, metal and S&P 500 while under the low volatility

regime the index returns are influenced only by the returns of the food index. Second, the returns of the energy index, under the low volatility regime, are affected positively by all indexes taken into consideration except for metals while under the high volatility regime the index returns are influenced only by S&P 500 and their own lag. Third, the food returns are affected positively by agriculture returns and their own lag under the high volatility regime while under the low volatility regime are influenced by agriculture, energy, S&P 500 and its own lag. Fourth, the gold returns, under the high volatility regime, are affected positively by food returns and its own lag, while under the low volatility regime are influenced positively by food, agriculture and S&P 500 and negatively by metals, energy and their own lag. Finally, the metals returns under the high volatility regime are affected positively by S&P500 and food while under the low volatility regime are influenced only by S&P500 and their own lag.

4.2.2 Volatility, Transition Probabilities, Expected Durations and Long-term probabilities under the two Regimes

Table 4-5 shows the standard deviations in each regime, the transition probabilities and the expected durations. The transition probabilities indicate the probability of staying in the same regime. The expected durations indicate how long each index will stay in each regime (the duration of staying is counted in months because data are monthly). Finally the long term probabilities indicate the probability of being in each regime after a long time.

Table 4-5: Volatilities, Transition Probabilities, Expected Durations and Long-Term Probabilities

Commodity	STAND. DEVIATION		TRANSITION PROB.		EXP.DURATION		LONG-TERM PROBABILITY	
	High volatility regime	Low volatility regime	From high to high volatility regime	From low to low volatility regime	High volatility regime	Low volatility regime	High volatility regime	Low volatility regime
Agriculture	0.0312	0.0214	0.7862	0.9475	4.67	19.04	0.1971	0.8029
Food	0.0240	0.0043	0.9769	0.4137	43.32	1.70	0.9618	0.0382
Energy	0.0668	0.0024	0.8954	0.08	9.56	1.08	0.8979	0.1021
Gold	0.0351	0.0017	0.9190	0.067	12.35	1.07	0.9201	0.0799
Metals	0.0558	0.0319	0.9794	0.9944	48.77	180.63	0.2137	0.7863

Long-term unconditional probabilities π_j, are computed by a simple method. Given the vector of probabilities $\pi = \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix}$, the fact that $\sum_{j=1}^{2} \pi_j = 1$, and the transition matrix $P = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix}$ we can write the following

$$[P^T - I_2]\pi = 0_2$$

Where I_2 indicate a 2×2 identity matrix and 0_2 is a 2×1 column-vector of zeros. From the above system we have

$$-(1 - p_{11})\pi_1 - (p_{22} - 1)\pi_2 = 0$$
$$(1 - p_{11})\pi_1 + (p_{22} - 1)\pi_2 = 0$$

Since these equations are multiples of each other we can make use of one of them and combine it with the fact that $\pi_1 + \pi_2 = 1$ in order to get

$$(1 - p_{11})\pi_1 + (p_{22} - 1)\pi_2 = 0$$
$$\pi_1 + \pi_2 = 1$$

Hence,

$$\pi_1 = \frac{1 - p_{22}}{2 - (p_{11} + p_{22})}$$
$$\pi_2 = \frac{1 - p_{11}}{2 - (p_{11} + p_{22})}$$

Where π_1 denotes the long term probability for the high volatility regime and p_{11} denotes the transition probability from the high volatility regime to the high volatility regime whereas π_2 denotes the long term probability for the low volatility regime and p_{22} denotes the transition probability from the low volatility regime to the low volatility regime.

From the table 4-5 we can identify the characteristics of the commodity index returns in each regime.

First, **in the case of the agriculture index**, the standard deviation of its return is $\boldsymbol{\sigma = 3.12\%}$ under the high volatility regime and $\boldsymbol{\sigma = 2.14\%}$ under the low volatility regime. The transition probabilities indicate the probability that the agriculture index will remain in the high volatility regime in period **t** given that it was in the same regime in the previous month is 78%, while the probability of remaining in the low volatility regime given that in the previous month it was in the same regime is about 95%. In general these estimations indicate that the regimes are highly persistent, with the low volatility regime being more persistent than the high volatility regime. The expected duration of the series under high volatility regime is 4.6 months

and 19 months under the low volatility regime. The long-term probabilities of being in the high and in the low volatility regime are:

$$\pi_1 = \frac{1 - p_{22}}{2 - p_{11} - p_{22}} = \frac{1 - 0.9475}{2 - 0.7862 - 0.9475} = 0.1971$$

$$\pi_2 = 1 - \pi_1 = 1 - 0.1971 = 0.8029$$

Figure 3-a (Appendix B) shows the probability that the return of the agricultural index is in the high or low volatility regime at each point in time. As seen from the figure, the probability that the agricultural index is in the first or second regime varies frequently, but for most time the probability is quite high that it is in the low volatility regime. The probability of being in the low volatility regime was very low mainly over the period 2002-2006 and 2012-2014, while for the remaining years the probability of being in the low volatility regime was quite high.

Second, **in the case of the food index**, the standard deviation of its return is $\boldsymbol{\sigma} = \mathbf{2.40}\%$ under the high volatility regime and $\boldsymbol{\sigma} = \mathbf{0.43}\%$ under the low volatility regime. The transition probabilities are 97% for staying in the high volatility regime, while the probability of remaining in the low volatility regime is about 41%. These estimations indicate that the high volatility regime is highly persistent while the low volatility regime is choppy. The expected duration of the series under the high volatility regime is 43 months and 1.7 months under the low volatility regime. The long-term probabilities of being in the high and in the low volatility regime are:

$$\pi_1 = \frac{1 - p_{22}}{2 - p_{11} - p_{22}} = \frac{1 - 0.4137}{2 - 0.9769 - 0.4137} = 0.9618$$

$$\pi_2 = 1 - \pi_1 = 1 - 0.9618 = 0.0382$$

Third, **in the case of the energy index**, the standard deviation of its return is $\boldsymbol{\sigma} = \mathbf{6.68}\%$ under the high volatility regime and $\boldsymbol{\sigma} = \mathbf{0.24}\%$ under the low volatility regime. The transition probabilities are 89% for staying in the high volatility regime, while the probability of remaining in the low volatility regime is about 8%. These estimations indicate that the high volatility regime is highly persistent while the low volatility regime is very floating. The expected duration of the series under the high volatility regime is 9 months and 1 month under the low volatility regime. The long-term probabilities of being in the high and in the low volatility regime are:

$$\pi_1 = \frac{1 - p_{22}}{2 - p_{11} - p_{22}} = \frac{1 - 0.08}{2 - 0.8954 - 0.08} = 0.8979$$

$$\pi_2 = 1 - \pi_1 = 1 - 0.8979 = 0.1021$$

Fourth, **in the case of the gold index**, the standard deviation of its return is $\boldsymbol{\sigma} = \mathbf{3.51}\%$ under the high volatility regime and $\boldsymbol{\sigma} = \mathbf{0.17}\%$ under the low volatility regime. The transition probabilities are 91.9% for staying in the high volatility regime, while the probability of remaining in the low volatility regime is about 6.7%. These estimations indicate that the high volatility regime is highly persistent while the low volatility regime is very floating. The expected duration of the series under the high volatility regime is 12 months and 1 month under the low volatility regime. The long-term probabilities of being in the high and in the low volatility regime are:

$$\pi_1 = \frac{1 - p_{22}}{2 - p_{11} - p_{22}} = \frac{1 - 0.067}{2 - 0.9190 - 0.067} = 0.9201$$

$$\pi_2 = 1 - \pi_1 = 1 - 0.9201 = 0.0799$$

Finally, **in the case of the metals index**, the standard deviation of its return is $\boldsymbol{\sigma} = \mathbf{5.58}\%$ under the high volatility regime and $\boldsymbol{\sigma} = \mathbf{3.19}\%$ under the low volatility regime. The transition probabilities are 97% for staying in the high volatility regime, while the probability of remaining in the low volatility regime is about 99%. These estimations indicate that both regimes are highly persistent with the low volatility regime to be more persistent than the high volatility regime. The expected duration of the series under the high volatility regime is 48 months and 180 month under the low volatility regime. The long-term probabilities of being in the high and in the low volatility regime are:

$$\pi_1 = \frac{1 - p_{22}}{2 - p_{11} - p_{22}} = \frac{1 - 0.0319}{2 - 0.0558 - 0.0319} = 0.2137$$

$$\pi_2 = 1 - \pi_1 = 1 - 0.2137 = 0.7863$$

Figure 3-e (Appendix B) shows the probability that the return of the metals index is under the high or the low volatility regime at each point in time. As seen from the figure, the probability of being in the high volatility regime was very high mainly over the period 2005-2011, while for the remaining years the probability of being in the low volatility regime was also very high.

5. CONCLUSION

The purpose of the Markov Regime-Switching model is to identify, without ex-ante knowledge, the periods of high and low volatility and the characteristics of the commodity index return in each regime. This is a potentially useful approach to model time series with time-varying dynamics. If the dates of the regimes switches are known, we can simply use dummy variables.

Six commodity indexes were used in order to estimate an MRS model that splits the data on the returns of commodity indexes into two distinct regimes, the low and the high volatility regime. The S&P 500 and the food index were found to affect positively and statistically significant all indices, mainly under the low volatility regime where these two indices affect four out of five index returns taken under consideration. Another important result is that the gold returns, under the low volatility regime, are affected either positively or negatively but in any case statistically significant by all indexes. Of all six indexes under consideration five (i.e. the Agriculture index, the Food index, the Gold index, the Metals index and the Energy index) the food index, the energy index and the gold index were found to be more persistent in high volatility regime while the agriculture index and the metals index were more persistent in the low volatility regime. The long term probabilities come to certify this conclusion. Finally the expected durations in conjunction with the smoothed probabilities graphs shows us exactly the periods of the sample that returns were either in the high volatility regime or in the low volatility regime. This is more obvious for the agriculture index and the metals index because of the high constancy of their both regimes.

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APPENDIX A

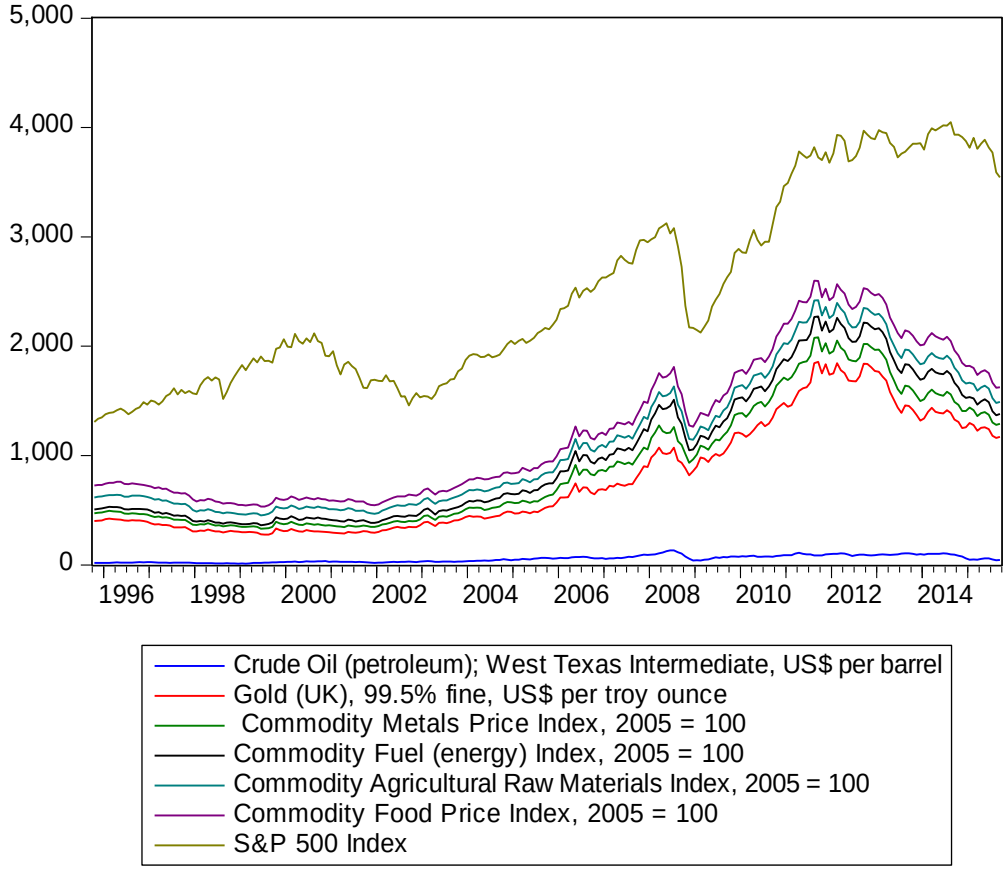


Figure 1: The Levels of the Indexes

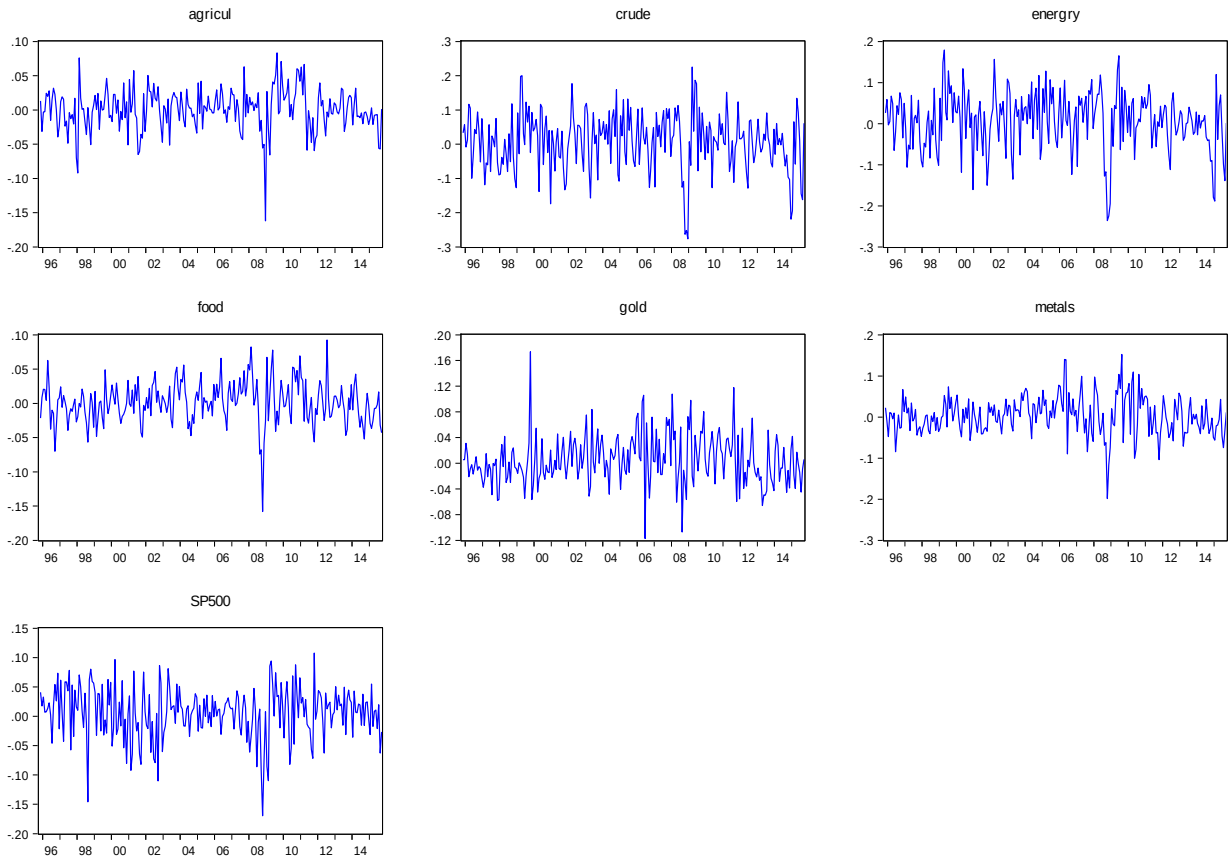
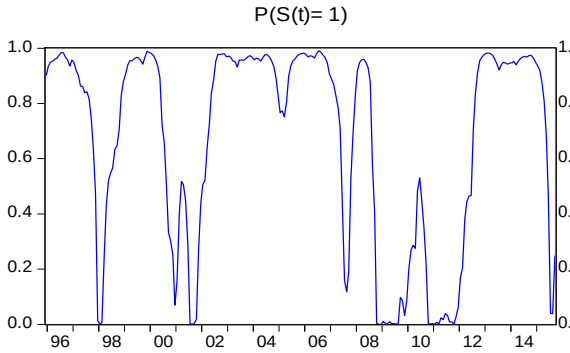


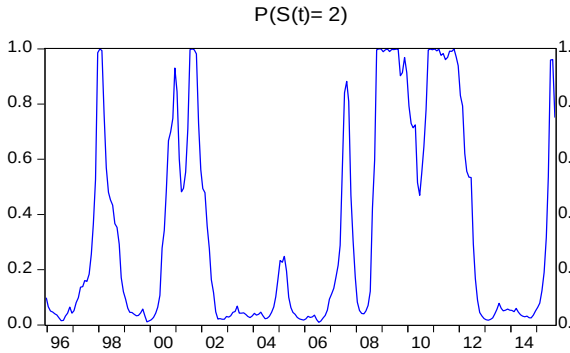
Figure 2: Commodity Monthly Returns

APPENDIX B

(a)

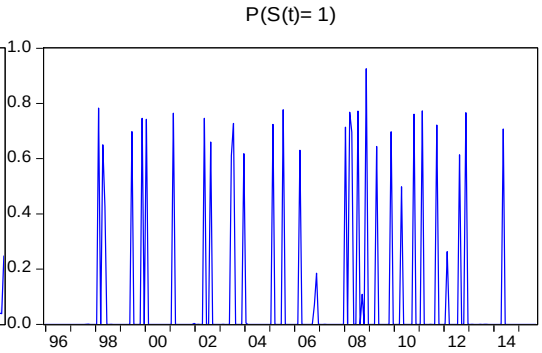
Agriculture Smoothed Regime Probabilities

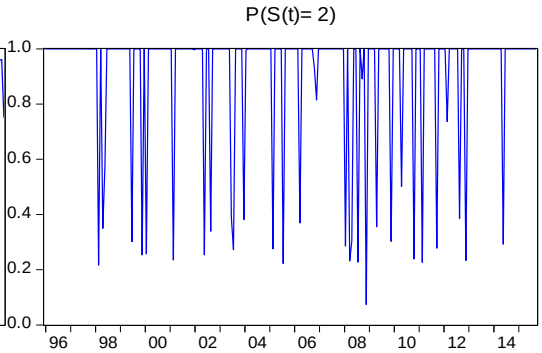




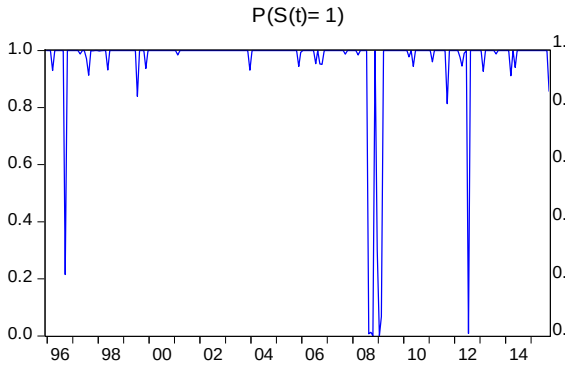
(b)

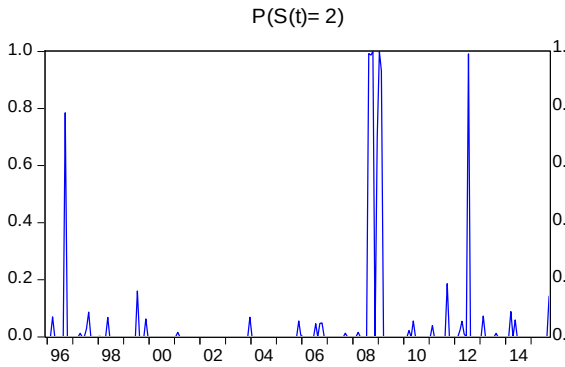
Smoothed Regime Probabilities - Energy





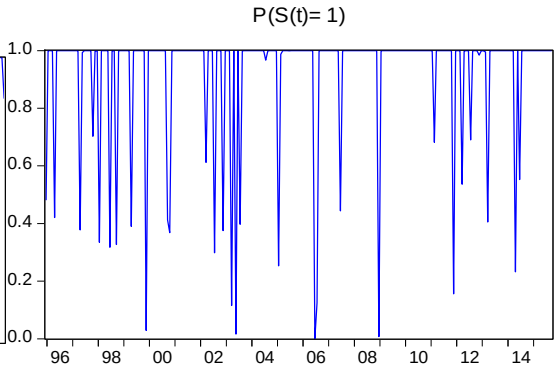
Smoothed Regime Probabilities-Food

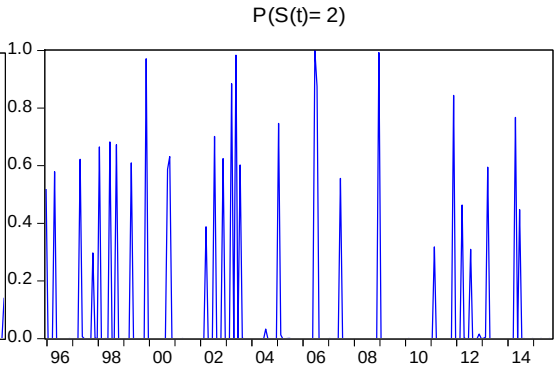




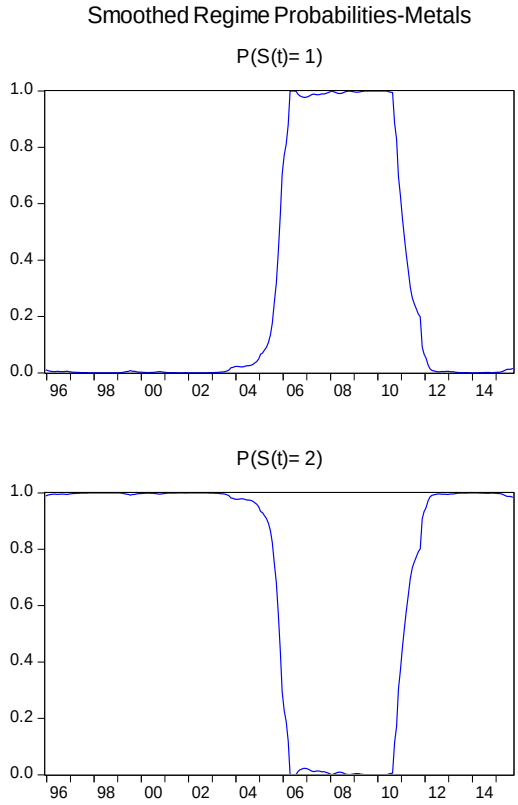
(c)

Smoothed Regime Probabilities-Gold





(d)



(e)

Figure 3: Smoothed Probabilities of the Indexes