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Thesis Title: Competition Issues Related to Extended Warranties

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Contents

Abstract

The aim of this study is to investigate extended warranties, prices and profits in a model with two competing retailers selling two substitute goods and one manufacturer. Our model comprises four cases that differ from one another depending on the identity of the extended warranty (EW) provider. i) Case N, where neither the retailers, nor the manufacturer provide extended-warranties (EWs), the latter being supplied by an open market that is fully competitive. ii) Case R, in which the EW providers are the retailers. iii) Case M, in which the manufacturer is the EW provider and finally case RM, in which the retailers sell the manufacturer's EWs. We find that in case R the product price is lower compared to that in any of the other cases. Also the chain profits are higher in that case. The full price is higher in cases R, M and RM than the full price in the competitive EW market (case N). Another interesting result is that as product differentiation decreases, extended warranty price decreases too.

1. Introduction

Extended warranties (EWs) give rise to a highly profitable market. An Extended warranty is usually offered with a durable consumer product and represents insurance, protecting consumers against product failure, after the base warranty (BW) has expired. The terms of the EW may be the same as the BW or differ in the terms of cost sharing such as return costs, partial product exclusions and additional services. In most the purchase of an EW is optional and it can take place either at the moment of the basic good's purchase, or later on, eventually before, but in some cases even after the BW's expiration. The provider of the EW can be either the product's manufacturer, the retailer, or even a third party.

Customers buy EWs because of the "peace of mind" they provide, whereas EW providers sell them because of the profits they generate. Selling EWs may increase dramatically the profits of the agent that provides them, and for this reason are usually promoted aggressively.. Berner. R. (2004). EWs have profit margins around 50% - 60% which sometimes represents up to 18 times the margins of basic product's sales. Another interesting feature of EWs is that they can generate valuable information about product quality, problematic product parts, and other information which can help the designers and the manufacturers to improve the product.

2. Literature Review

According to Richard and Fox, (1985), manufacturers offer basic warranties (BWs) that contain very similar terms, and as a consequence BWs cannot be used as a marketing tool. With extended warranties (EW) manufacturers, retailers or third parties tend to differentiate themselves from one another. Even in the same industry, the offering of EW may differ in terms, duration, pricing, selling practices (selling the EW at the point of product sale or after the BW expires), promotion and advertising.

Padmanabhan and Rao RC (1993) examine the extended warranties in the automobile industry. They assume that consumers are heterogeneous in risk-preferences. Their study shows that the BW covers the least risk-averse customers and the EW covers the more risk-averse customers. Risk-averse consumers are more likely to buy EWs if the BW is short (less than three years in the automobile industry). Also the product price has a positive effect in the EW demand while BW length has a negative effect.

In Padmanabhan (1995), a firm can make price discrimination by offering a self-selecting menu of base warranty and extended warranties. Low usage customers will choose low warranty coverage because the probability of product failure is lower in their case. High usage customers are willing to pay more for insurance because they know that the high tense usage of the product increases the probability of product failure. As a result the firm sets the prices in such a way that the product warranty bundle is a better choice for the low usage customers compared with no purchase or the choice of the product and the extended warranty and the product + extended warranty is a better choice for the high usage types compared with no purchase or the choice of product alone.

In NA and Padmanabhan (1998), it mentions that if consumers differ in their valuations of a working unit, then the monopolist manufacturer may offer partial warranties (and low quality) to low valuation consumers, and full warranties (and high quality) to high valuation consumers. In case that a third party enters the warranties market then he can interact only with the low valuation customers (low quality product) because the high valuation customers have a full manufacturer warranty coverage. In case that the manufacturer decides to drop the low valuation customers, then the third party entrance is deterred. As a result competition increases the price paid by high valuation customers, manufacturer's profits and total surplus also decreases. If the manufacturer sells to both high and low valuation consumers, the independent insurer enters the market in equilibrium, and sells extended warranties to the under-insured low valuation consumers. Because the markets for the product and for warranty insurance are intimately linked and entry of a third party into the insurance market affects consumer behavior in the product market, manufacturer profits may actually increase.

Hollis A (1999). Should it be legally permissible for manufacturers to set restrictions in the extended warranties market? In other words, what are the outcomes when retailers and third parties are restricted to sell only the manufacturer's extended warranties? If the product market is characterized by monopoly or oligopoly, then the monopolist may exclude the light users. In that case, the monopolist should be prevented from excluding the third parties from the extended warranty market. On the other hand, even if the product market is itself characterized by competition, then tying in the secondary market for extended warranties may lead to a decrease in welfare.

3. Manufacturer vs. Retailer

As mentioned above, the EWs can be offered by the manufacturer, the retailer or by a third party, like insurance companies.

Yiwen *et all*. (2005), assumes we have one manufacturer who sells two substitutes to two competing retailers. The manufacturer's BW and a retailer's EW are bundled with the product. It considers two scenarios. First no retailer offering EWs and second both retailers offering EW. The results show that the manufacturer has no incentive to offer its corresponding base warranty when the retailers offer EWs; while in the absence of the EWs, the manufacturer will provide its corresponding base warranty.

Kunpeng Li *et all*. (2012), assumes we have three models.

- Centralized model (Model C), where the manufacturer sells his product and EWs directly to the customers
- Retailer model (Model R) where the manufacturer sells his product to his customers through a retailer. The retailer decides the final product price and offers his EW to the customers.
- Morel manufacturer (Model M), where the manufacturer sells his product to the customers through the retailer. The retailer sets the final product's price maximizing his profits given the manufacturers wholesale price. Finally in model M the manufacturer sells to the customers his/her EWs.

They compare model R (retailer offers his EWs) and model M (manufacturer sells his EWs to the customers) with model C and they find that model R generates higher system profits than model M. This is something we expect because in model R, the retailer can influence directly the product's and the EW's demand through the final product's price and the EW's price. In model M, the manufacturer can choose the EW price and influence indirectly only the final product's price through his decision of wholesale price. In this case he cannot directly influence the product demand nor manipulate the final product price and the EW price, resulting lower system profits. Furthermore both the retailer and the manufacturer are better off in model R in comparison with model M. Model R, therefore, makes a Pareto improvement over Model M.

The wholesale price will be higher in model R in contrast with model M because in model M, the manufacturer wants to increase the demand of EWs. Despite the higher wholesale price, the retail price may be lower in model R because the retailer wants to increase the product demand. By increasing the product demand he/she increases the demand for EWs.

4. Quality Signaling

When customers cannot observe the product quality, warranty length and coverage often operate as quality signals. Long warranty and high coverage mean that the product is probably of a high quality whereas short warranty and low coverage mean the opposite, because of repair and replacement costs. If a manufacturer produces a low quality product with high failure probability, then he can offer nothing but a short and low coverage warranty, because he cannot afford the costs of repairing a product with long warranty. On the other hand, a high quality manufacturer will offer a long warranty, because he knows that the probability of failure of his high quality is low. This way he uses his warranty as a good quality product to his customers.

5. Differentiation

While BWs are used as product quality signals, EWs are used as instruments for product differentiation. The BW is usually tied-in with the base product and is sold as an inseparable part of it, its price being included in the price of the product. They exist to protect the customers from unexpected product failures and most of the times BWs are similar between similar products. EWs are mostly sold as separate products and a customer can choose among a variety of EWs which differ in price, duration and terms. This way the EW can be customizable and meet customer's needs. The providers of EWs can choose the price, the length and the terms of their EWs and differentiate themselves from one another, even if they sell the same product.

6. Customer Foresight

Customer foresight can affect the product price as well as the EW price. Completely rational consumers take into account the EW price and terms before choosing the retailer from whom they buy their product. Consumers with low or no foresight (we can call them myopic or naive), don't consider the EW's price before purchasing the product, simply choosing the retailer who offers the lower product price (we assume that all retailers offer EWs of the same duration). The degree of customer foresight is very important for the retailers and for the competition between them. When consumers are rational, the product price and the EW price may be higher than in the case of low customer foresight. When the customer foresight is low, retailers compete stiffly preferring to lower the product price, eventually even under cost, in order to attract more consumers, since by increasing its product sales, a retailer can gain profits through increased sales of EW's.

7. Two-sided markets

"*A two-sided market is a meeting place for two sets of agents who interact through an intermediary or platform. Getting both sides on board is fundamental for the development and success of each platform and to our understanding of the pricing strategies followed by market participants. In considering the platform pricing strategies, the economics literature has demonstrated that the characteristic that* *distinguishes two-sided markets is that the pricing structure (the relative prices charged to each side), matters .The fact that two-sided markets are based on externalities, such as group or network externalities, and that the structure of prices matters, as much as their level, has important consequences for regulation. First, we should not expect to find a direct relationship between the price charged on one side and the incremental cost of serving that side. The prices that maximize consumer welfare will in fact often depart from the cost-reflective ones. Second, any change imposed to the price on one side will also change the price on the other side of the platform, thus creating a waterbed effect.*" (Financial Times)

In our case we can consider that we have one agent (the retailer) who sells two complementary products, the basic product and the EW. The demand of one product can affect the demand of the other product.

In the case of myopic or otherwise naïve customers, the demand of the basic product can affect in a positive way the demand of EWs. This seems to be the most common case in reality because in most cases customers are attracted more by the basic product's price. This is happening for two reasons. Firstly, there is not much advertising for the EWs and secondly, usually the EWs are offered to customers when they have already purchased the basic product.

In case of customer foresight (the customers are rational), we have a two way relationship in the products demands. The demand of the basic product can affect the demand of EWs and vice versa. This happens because, before the purchase decision, customers are not attracted only by the product's price, but from the EW's price too.

Fillol *et all.* (2014), studies the case of airports as platforms. The airports can gain revenues from aeronautical and non-aeronautical services. The revenues from the aeronautical services are gained by the landing fees they charge the airline companies which in turn rolls this charges to the end customers' tickets. The revenues from the non-aeronautical services are gained from fees on the revenues of the airport's shops.

The above bears significant resemblance to the EW situation, if we consider the airport as the retailer, landing fees as the basic product price, and the fees on the revenues of the airport's shops as the EW's price. Airports also set prices considering the degree of customer foresight. With naive customers, the airport must lower significantly the landing fees so it can attract more travelers to the airport shops. This will increase the revenues from the non-aeronautical services. If we consider rational customers (high customer foresight), then the airport can charge higher landing fees because now customers take into account the surplus that they can gain from shopping so they are willing to pay more for the ticket.

Again if we relate their conclusions in our case, the retailer can lower the price of his product even under the cost, if he has to deal with naïve customers. This will increase the demand of the basic product and probably will boost up the demand of EW. Selling more EWs the retailer can rise significantly his revenues.

On the other hand, if the retailer deals with rational customers, then he knows that customers will take into account the retailer's EWs' prices. In this case he can attract customers even with a higher product price, if he sets a competitive EW price.

8. The Model

Consider a manufacturer providing the basic product to two retailers who sell it to consumers, competing against each other in prices. Assume the product is bundled with a BW provided by the manufacturer, and consider the following four cases according to the identity of the EW provider. In the first case (case N), neither the retailers, nor the manufacturer provide EWs, the latter being supplied by a fully competitive EWs market. In the second case (case R) EWs are provided by the retailers. In the third case (case M), it is the manufacturer that provides the EWs. Finally in case four, (case RM), the retailers are selling the manufacturer's EWs. We also assume consumers to be heterogeneous and myopic (no foresight). Let x and P_1 , P_2 , represent wholesale and retail prices, respectively. After a consumer has decided the retailer that will buy his product from, the EW provider offers him the possibility to buy EW. We assume that all EW have standard length and terms, therefore the consumer's decision depends only upon the EW's price. We also assume that both the EW and product costs are zero.

Notations

x	Wholesale price
q_1, q_2	Product quantity
P_1 , P_2	Product Price
w_1 , w_2	EW price
P_f	Full price (the sum of product price and EW price)
Π_1 , Π_2	Retailers' profits
Π_m	Manufacturer's profits
Π_{Ch}	Chain profits
w_x	EW wholesale price
q_w	EW quantity

The superscript of each notification will declare the case of each notation. For example P_1^M is the product retail price in Case M.

8.1 Product Demand Functions

We have two demand functions for the products. One for each retailer. The Product demand functions are the same in all cases.

$$\begin{aligned} p_1 &= 1 - bq_1 - \theta bq_2 \\ p_2 &= 1 - bq_2 - \theta bq_1 \end{aligned} \tag{1}$$

Where q_1, is the demand of retailer 1 and q_2, is the demand of retailer 2. p_1, p_2 are the prices of retailer 1 and retailer 2.

θ is the product price sensitivity of the consumers.

8.2 EW Demand Functions

In our model we have demand functions only in cases R, M and RM. In case N none of our retailer or the manufacturer are providing EWs.

In case R we have EW demand functions of the below form.

$$\begin{aligned} q_{w1}^R &= q_1(p_1^r, p_2^r) - \beta w \\ q_{w2}^R &= q_2(p_1^r, p_2^r) - \beta w \end{aligned} \tag{2}$$

Where: q_1^w, q_2^w, are the demands of the EWs of retailer 1 and retailer 2.

β is the EW price sensitivity of the consumers.

The demand for EWs of each retailer depends on the demand of the product. It cannot exceed the demand of the product. In other words it will be equal with the product demand, if all customers that purchase the product, purchase the EW too. Otherwise the demand of EWs will be lesser than the demand of the final product. If the EW price is very low (near zero), we can see that the demand of EWs will be almost equal with the demand of product. This means that if the EWs are offered almost for free, then all the customers would take them.

In case M the EW demand function will be:

$$q_w^M = (q_1 + q_2) - 2\beta w \tag{3}$$

In case RM the EW demand function is the same as the one in case R.

8.3 Profit Functions

8.3.1 Retailers' Profit Functions

In **case N** and in **case M** we have the Retailer same profit functions, because as mentioned above, in case one none of the retailers or the manufacturer are selling EWs and in model M, manufacturer is the one selling EWs. Retailers in that case are generating profits only by selling the basic product.

The Retailers' profit functions in cases N and M are:

$$\Pi_1 = (p_1 - x)q_1(p_1, p_2) \qquad \textbf{(4)}$$

$$\Pi_2 = (p_2 - x)q_2(p_1, p_2)$$

In case R, retailers are gaining profits from both product and the EWs sales.

$$\Pi_1^R = (p_1 - x)q_1(p_1, p_2) + [q_1(p_1, p_2) - \beta w_1]w_1$$

$$\Pi_2^R = (p_2 - x)q_2(p_1, p_2) + [q_2(p_1, p_2) - \beta w_2]w_2 \qquad \textbf{(5)}$$

In case RM, retailers are gaining profits from both product and the EWs sales.

In this model, the manufacturer sells his product and his EWs to the final customers through the two retailers. In other words the retailers sell exclusively the official manufacturer's EWs to the customers. They buy the product and the EWs from the manufacturer at their wholesale prices and then they set retail prices for both of them, given the wholesale prices.

$$\Pi_1^{RM} = (p_1 - x)q_1(p_1, p_2) + (w_1 - w_x)[q_1(p_1, p_2) - \beta w_1]$$

$$\Pi_2^{RM} = (p_2 - x)q_2(p_1, p_2) + (\mathrm{w}_2 - w_x)[q_2(p_1, p_2) - \beta w_2] \tag{6}$$

8.3.2 Manufacturer's Profit Functions.

In **cases N and R** manufacturer has the same profit function because they are gaining profits only by selling the product to the retailers in a wholesale price x.

$$\Pi = (q_1 + q_2)x \tag{7}$$

In **case M** the manufacturer's profits are:

$$\Pi^M = [q_1(p_1, p_2) + q_2(p_1, p_2)]x + [q_1(p_1, p_2) + q_2(p_1, p_2) - 2\beta p^w]p^w \tag{8}$$

In **case RM:**

$$\Pi^{RM} = [q_1(p_1, p_2) + q_2(p_1, p_2)]x + [q_1(p_1, p_2) + q_2(p_1, p_2) - 2\beta w]\mathrm{w}_x \tag{9}$$

8.4 Optimal Solutions for All Cases

We can see now the optimal prices, quantities and profits of all cases.

Table 1

	Case N	Case R
x	$\frac{1}{2}$	$\frac{1}{2}$
P_1 , P_2	$\frac{2\theta - 3}{2(\theta - 2)}$	$\frac{1 + b\beta(1+\theta)(-3+2\theta)}{1 + 2b\beta(-2+\theta)(1+\theta)}$
q_1 , q_2	$\frac{-1}{2b(-2+\theta)(1+\theta)}$	$\frac{b\beta(-1+\theta^2)}{(b - b\theta^2)(1 + 2b\beta(-2+\theta)(1+\theta))}$
w_1, w_2	0	$\frac{-1}{2(1 + 2b\beta(-2+\theta)(1+\theta))}$

P_f	$\frac{1}{2}$	$\frac{1+2b\beta(1+\theta)(2\theta-3)}{2+4b\beta(\theta-2)(\theta+1)}$
Π_1 , Π_2	$\frac{1-\theta}{4b(-2+\theta)^2(1+\theta)}$	$\frac{\beta[-1-4b\beta(-1+\theta^2)]}{4[1+2b\beta(-2+\theta)(1+\theta)]^2}$
Π_m	$\frac{-1}{2b(-2+\theta)(1+\theta)}$	$\frac{b\beta(-1+\theta^2)}{(b-b\theta^2)(1+2b\beta(-2+\theta)(1+\theta))}$
Π_{Ch}	$\frac{3-2\theta}{2b(-2+\theta)^2(1+\theta)}$	$\frac{\beta(-3+4b\beta(3+\theta-2\theta^2))}{2(1+2b\beta(-2+\theta)(1+\theta))^2}$
w_x	-	-

Table 2

	Case M	Case RM
x	$\frac{1+2b\beta(-2+\theta)(1+\theta)}{1+4b\beta(-2+\theta)(1+\theta)}$	$\frac{1}{2}$
P_1 , P_2	$\frac{1+2b\beta(1+\theta)(-3+2\theta)}{1+4b\beta(-2+\theta)(1+\theta)}$	$\frac{1+b\beta(1+\theta)(-3+2\theta)}{1+2b\beta(-2+\theta)(1+\theta)}$
q_1 , q_2	$\frac{2b\beta(-1+\theta)(1+\theta)}{(b-b\theta^2)(1+4b\beta(-2+\theta)(1+\theta))}$	$\frac{b\beta(-1+\theta^2)}{(b-b\theta^2)(1+2b\beta(-2+\theta)(1+\theta))}$
w_1 , w_2	$\frac{1}{-1+4b\beta(2+\theta-\theta^2)}$	$\frac{-1}{2(1+2b\beta(-2+\theta)(1+\theta))}$
P_f	$\frac{2b\beta(1+\theta)(2\theta-3)}{1+4b\beta(\theta-2)(\theta+1}$	$\frac{2b\beta(1+\theta)(2\theta-3)}{1+4b\beta(\theta-2)(\theta+1}$
Π_1 , Π_2	$\frac{4b\beta^2(1-\theta^2)}{(1+4b\beta(-2+\theta)(1+\theta))^2}$	$\frac{\beta(-1-4b\beta(-1+\theta^2))}{4(1+2b\beta(-2+\theta)(1+\theta))^2}$
Π_m	$\frac{\beta(-1+8b\beta(2+\theta-\theta^2))}{(1+4b\beta(-2+\theta)(1+\theta))^2}$	$\frac{b\beta(-1+\theta^2)}{(b-b\theta^2)(1+2b\beta(-2+\theta)(1+\theta))}$
Π_{Ch}	$\frac{\beta(-1+8b\beta(3+\theta-2\theta^2))}{(1+4b\beta(-2+\theta)(1+\theta))^2}$	$\frac{\beta(-3+4b\beta(3+\theta-2\theta^2))}{2(1+2b\beta(-2+\theta)(1+\theta))^2}$

9. Comparisons

We will set b=β=1 to the optimal prices of tables 1 and 2, and we will compare the optimal solutions among cases N, R and M. We will not compare with model RM because as mention above, the solutions of model RM are the same with the ones of model R.

By setting b=β=1 we can see the optimal solutions related to product differentiation. It is necessary to find a variety of θ that satisfies the prices and profits of the three cases so none of them are negative.

θ should be $0 \le \theta \le \frac{\sqrt{3}}{2}$. If for example θ is greater than $\frac{\sqrt{3}}{2}$ then retailers' optimal profits in case R will be negative

Proposition 1

The final product retail price will be lower in case R and higher in case N, while product retail price in case N will be in between them i.e., $\forall \theta \in [0, \frac{\sqrt{3}}{2}]$, $p^R < p^M < p^N$

Proof

Replacing product retail price for each case, from tables 1 and 2 and setting b=β=1, we obtain that,

$$\frac{\theta(-1+2\theta)-2}{2(-1+\theta)\theta-3} < 1+\frac{2(1+\theta)}{4(-1+\theta)\theta-7} < \frac{2\theta-3}{2(-2+\theta)} \quad \forall \theta \in [0, \frac{\sqrt{3}}{2}]$$

Concerning, $p^R < p^M$

we note that $\frac{\theta(-1+2\theta)-2}{2(-1+\theta)\theta-3} \le 1+\frac{2(1+\theta)}{4(-1+\theta)\theta-7} \Leftrightarrow \frac{1}{2}\left(1-\sqrt{7}\right) < \theta < \frac{1}{2}\left(1+\sqrt{7}\right)$

Since, $[0, \frac{\sqrt{3}}{2}] \subseteq [\frac{1}{2}\left(1-\sqrt{7}\right), \frac{1}{2}\left(1+\sqrt{7}\right)]$, Then, for $\forall\theta\in[0,\frac{\sqrt{3}}{2}]$, $p^R<p^M$.

In case R, retailers can lower the basic products' retail price to increase the demand for EWs. This way they may increase their total profits.

Replacing product retail price for cases M and N, from tables 1 and 2 and setting b=β=1, we obtain that,

$$1+\frac{2(1+\theta)}{4(-1+\theta)\theta-7}<\frac{2\theta-3}{2(-2+\theta)}\Leftrightarrow\frac{1}{2}\left(1-\sqrt{7}\right)<\theta<\frac{1}{2}\left(1+\sqrt{7}\right)$$

Since, $[0,\frac{\sqrt{3}}{2}]\subseteq[\frac{1}{2}\left(1-\sqrt{7}\right),\frac{1}{2}\left(1+\sqrt{7}\right)]$, then for $\forall\theta\in[0,\frac{\sqrt{3}}{2}]$, $p^M<p^N$

Proposition 2

The EW price will be higher in case R than in case M, i.e., $w^R>w^M$

Proof

We find w^R,w^M, by setting b=β=1 to the EW prices of table 1 and 2.

We obtain that $\frac{1}{6-4(-1+\theta)\theta}>\frac{1}{-1+4\left(2+\theta-\theta^2\right)}\Leftrightarrow\frac{1}{2}\left(1-\sqrt{7}\right)<\theta<\frac{1}{2}\left(1+\sqrt{7}\right)$,

i.e as shown in proposition one $\forall\theta\in[0,\frac{\sqrt{3}}{2}]$, $w^R>w^M$

Proposition 3

The wholesale price will be lower in case M than in cases R and N, which is equal to 0.5, i.e., $x^M<x^R=x^N=0.5$

Proof

Replacing wholesale prices from tables 1 and 2 and setting as before b=β=1,

$$\frac{2(-1+\theta)\theta-3}{4(-1+\theta)\theta-7}=\frac{3}{7}<0.5$$

Proposition 4

Retailers' profits will be higher in case R than in case N. In case N manufacturers and retailers are gaining profits only from the basic products' sales, i.e., $\Pi_{1,2}^{R}>\Pi_{1,2}^{N}$

Proof

We find $\Pi_{1,2}^{R},\Pi_{1,2}^{N}$, by setting b=β=1 to the retailers' profits in table 1 and we obtain that, $\dfrac{4-4\theta^{2}}{\left(7-4(-1+\theta)\theta\right)^{2}}>\dfrac{1-\theta}{4(-2+\theta)^{2}(1+\theta)} \quad \Leftrightarrow \dfrac{4}{49}>\dfrac{1}{16}$

Proposition 5

Manufacturer's profits will be higher in case R than in case M which in turn are greater than manufacturer's profits in case N, i.e., $\Pi_{m}^{R}>\Pi_{m}^{M}>\Pi_{m}^{N}$.

Proof

We find $\Pi_{m}^{R},\Pi_{m}^{M},\Pi_{m}^{N}$, by setting b=β=1 to the manufacturers' profits in table 1 and 2 and we obtain that, $\dfrac{1}{3+2\theta-2\theta^{2}}>\dfrac{15-8(-1+\theta)\theta}{\left(7-4(-1+\theta)\theta\right)^{2}}>\dfrac{-1}{2(-2+\theta)(1+\theta)}\Leftrightarrow \quad \dfrac{15}{49}>\dfrac{15}{49}>\dfrac{1}{4}$

Proposition 6

Chain profits will be higher in case R and M than in case N, i.e., $\Pi_{Ch}^{R}>\Pi_{Ch}^{N}$, $\Pi_{Ch}^{M}>\Pi_{Ch}^{N}$

Proof

Replacing channel profits from tables 1 and 2, we find $\Pi_{Ch}^{R}, \Pi_{Ch}^{N}$ and we obtain that,

$$\frac{4\left(3+\theta-2\theta^2\right)-3}{2\left(1+2\left(-2+\theta\right)\left(1+\theta\right)\right)^2} > \frac{3-2\theta}{2\left(-2+\theta\right)^2\left(1+\theta\right)} \Leftrightarrow \frac{1}{2} > \frac{3}{8}$$

Doing the same as before for Π_{Ch}^{M}, we obtain that,

$$\frac{23+8\left(1-2\theta\right)\theta}{\left(7-4\left(-1+\theta\right)\theta\right)^2} > \frac{3-2\theta}{2\left(-2+\theta\right)^2\left(1+\theta\right)} \Rightarrow \frac{23}{49} > \frac{3}{8}$$

Proposition 7

Basic products' quantities will be higher in case R than in case M, which in turn are higher than quantities in case N, i.e., $q_{1,2}^{R} > q_{1,2}^{M} > q_{1,2}^{N}$

Proof

Replacing channel profits from tables 1 and 2, we find $\Pi_{Ch}^{R}, \Pi_{Ch}^{N}$ and we obtain that,

$$\frac{9+4\theta-8\theta^2}{2\left(3-2\left(-1+\theta\right)\theta\right)^2} > \frac{23+8\left(1-2\theta\right)\theta}{\left(7-4\left(-1+\theta\right)\theta\right)^2} > \frac{3-2\theta}{2\left(-2+\theta\right)^2\left(1+\theta\right)} \Rightarrow \quad \frac{1}{2} > \frac{23}{49} > \frac{3}{8}$$

Figure 1: Basic Product retail price, full price and wholesale price related to product differentiation.

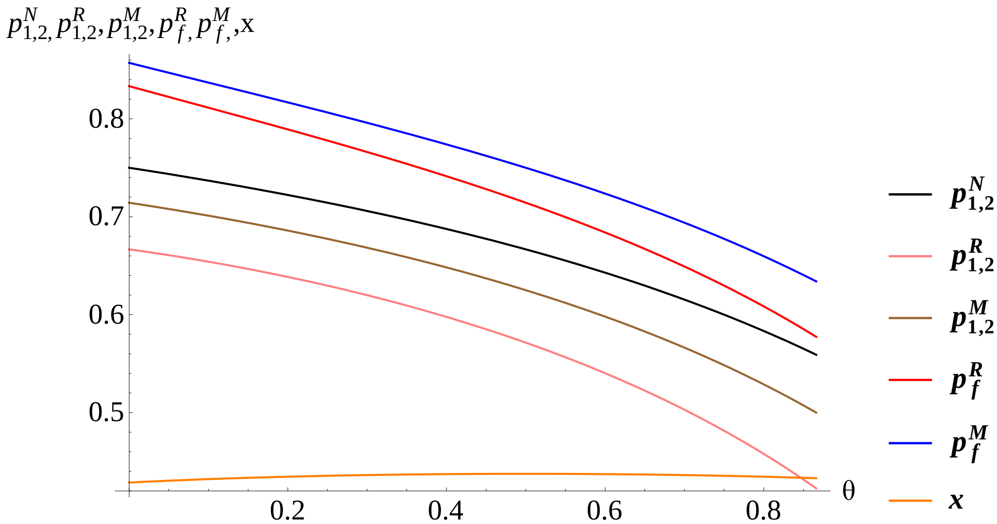


Figure 2: EW price related to product differentiation.

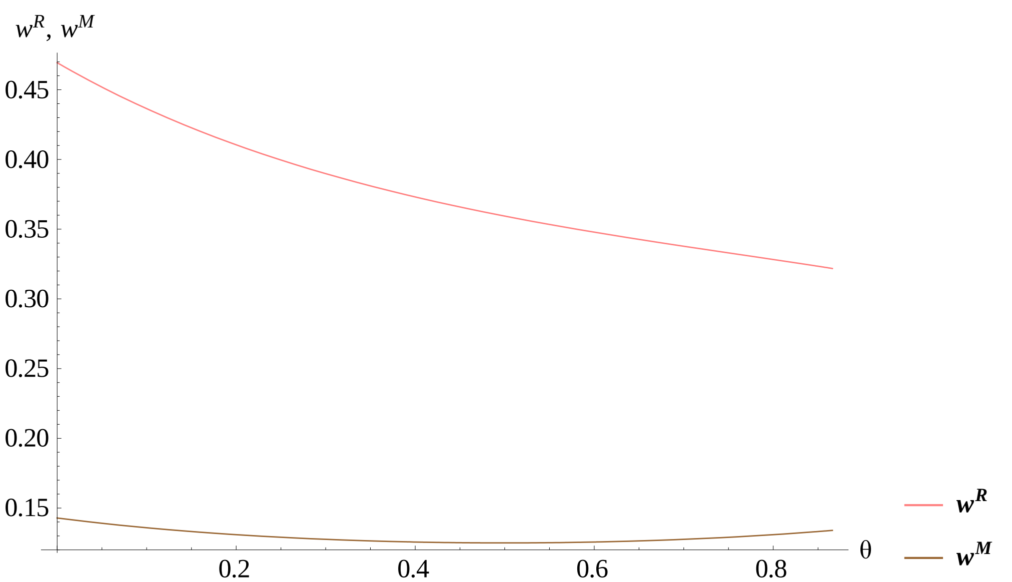
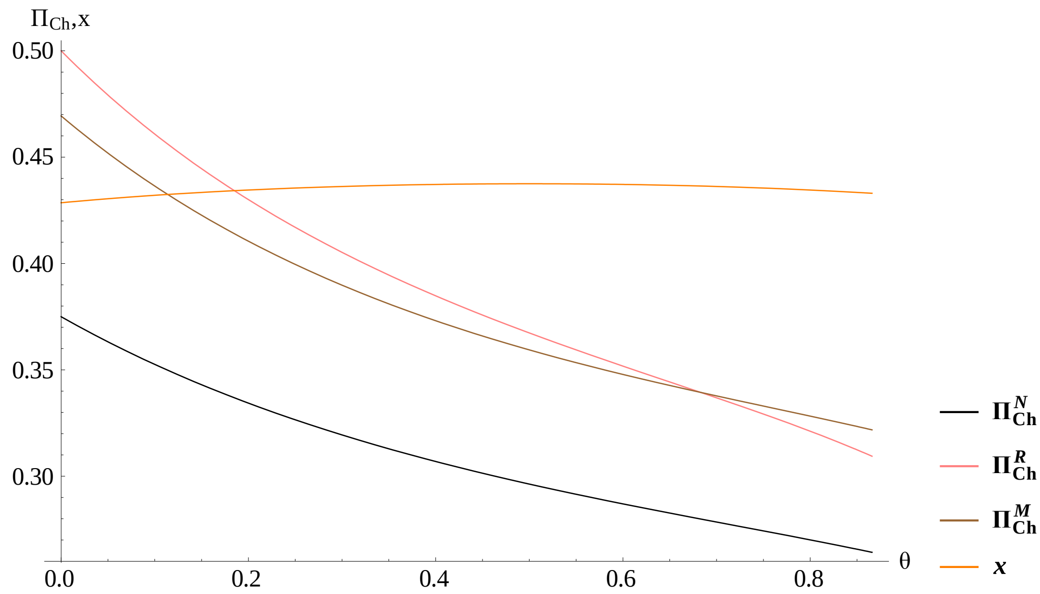


Figure 3: Chain profits related to product differentiation.



Now if $b = \theta = 1$, then Then β should be: $0 \le \beta < \frac{1}{8}$ or $\beta \ge \frac{1}{2}$ for product retail price to be non-negative at the same time in both R and M cases.

Figure 4: Basic products' retail price related to customer sensitivity in EW price.

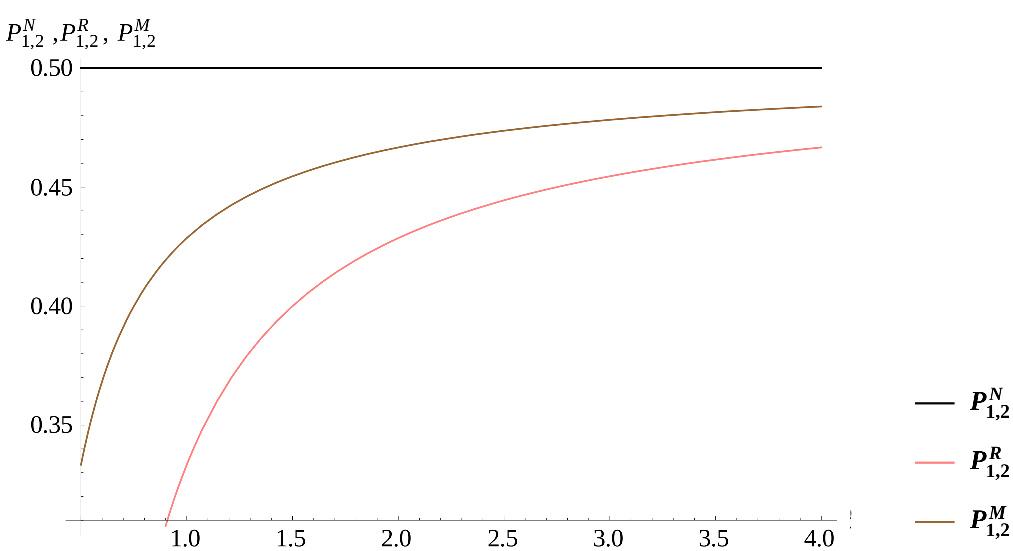
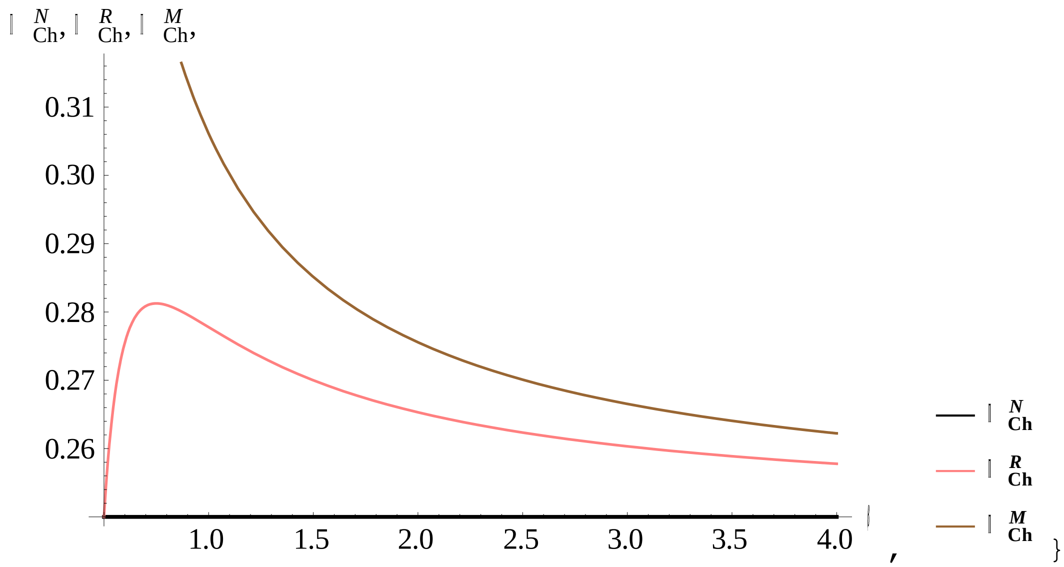


Figure 5: Chain Profits related to customer sensitivity in EW price.



10. Numerical Example

We can put some values to the coefficients b, β and θ to make a numerical example.

We can set: $b = 1$, $\beta = 1$, $\theta = 0,3$

By replacing these values to the table with the optimal solutions, we can have the prices and the profits of our example.

	NOEW	REW
x^*	0.5	0.5
P_1^* , P_2^*	0.706	0.620
q_1^*, q_2^*	0.226	0.292
w_1^*, w_2^*		0.146
Π_1^*, Π_2^*	0.047	0.056
Π_m^*	0.226	0.292
Π_{Ch}^*	0.319	0.405

	MEW	**RMEW**
x^*	0.436	0.5
P_1^*, P_2^*	0.668	0.620
q_1^*, q_2^*	0.255	0.292
w_1^*, w_2^*	0.128	0.146
Π_1^*, Π_2^*	0.059	0.056
Π_m^*	0.271	0.292
Π_{Ch}^*	0.390	0.405
w_x^*		0

10.1 Results compassions

Wholesale price x^*

As we can see the wholesale price is the same and equal to 0.5, except the RMEW, in which it is 0.436, which means that it is 12.80% lower than the other models.

Product retail price

Let's put the retail prices in descending order.

N	0,706
M	0,668
R	0,62

The retail price of the product is higher in the NOEW model than the other models. Also the product retail prices are equal between RMED and REW. The higher price in the NOEW model can be explained from the ability of the retailer to make profits only from the basic product. On the other models, the retailers can lower the price of the basic product with the incentive to increase the demand of the basic product. Such a strategic plan can increase the demand of the EWs and may lead to higher total profits.

Now let's see a panel in which we represent the basic product retail prices of each model in the diagonal and in the other sells we can see the percent change of each price to the other models.

	N	R	M
N	**0,706**	8,60%	3,80%
R	-8,60%	**0,62**	-4,80%
M	-3,80%	4,80%	**0,668**

For example the basic product retail price of the REW model, is 8,6% lower than the product price in NOEW in which it happens to be 0,706.

Now we will do the same for the product quantities, EW prices, retailers' profits, manufacturer's profits and chain profits.

Product quantities in decreasing order.

R	0,292
M	0,255
N	0,226

Percentage differences of product quantities in all four models.

	N	R	M
N	**0,226**	-6,60%	-2,90%
R	6,60%	**0,292**	3,70%
M	2,90%	-3,70%	**0,255**

EW prices in decreasing order.

R	0,146
M	0,128
N	-

Percentage differences of of EW prices in the four models.

	N	REW	MEW
N	-	-	-
R	-	**0,146**	1,8%
M	-	-1,80%	**0,128**

Retailers' profits in decreasing order.

M	0,059
R	0,056
N	0,047

Percentage differences of retailers' profits in all four models.

	NW	R	M
N	**0,047**	-0,90%	-1,20%
R	0,90%	**0,056**	-0,30%
M	1,20%	0,30%	**0,059**

Manufacturer's profits in decreasing order.

R	0,292
M	0,271
N	0,226

Percentage differences of manufacturer's profits in all four models.

	N	R	M
N	**0,226**	-6,60%	-4,50%
R	6,60%	**0,292**	2,10%
M	4,50%	-2,10%	**0,271**

Chain profits in decreasing order.

R	0,405
M	0,39
N	0,319

Percentage differences of chain profits in all four models.

	N	R	M
N	**0,319**	-8,60%	-7,10%
R	8,60%	**0,405**	1,50%
M	7,10%	-1,50%	**0,39**

11. Proofs of Models' Optimal Solutions

11.1 Proof of Case N Optimal Solutions

From equations **(1)** we can find the reaction functions**:**

$$q_1 = \frac{1-\theta+\theta p_2 - p_1}{b(1-\theta^2)} \quad q_2 = \frac{1-\theta+\theta p_1 - p_2}{b(1-\theta^2)} \qquad \textbf{(10)}$$

We replace equations **(10)** into profit functions **(4)**:

$$\Pi_1 = (p_1 - x)\frac{1-\theta+\theta p_2 - p_1}{b(1-\theta^2)} \qquad \Pi_2 = (p_2 - x)\frac{1-\theta+\theta p_1 - p_2}{b(1-\theta^2)} \qquad \textbf{(11)}$$

The two retailers generates profits only from product sales and they maximize their profits at the price where $\frac{\partial \Pi_1}{\partial p} = 0$, $\frac{\partial \Pi_2}{\partial p} = 0$.

$$\frac{\partial \Pi_1}{\partial p_1} = 0 \Rightarrow p_1 = \frac{1}{2}[1 + x + (-1 + p_2)\theta]$$

$$\frac{\partial \Pi_2}{\partial p_2} = 0 \Rightarrow p_2 = \frac{1}{2}[1 + x + (-1 + p_1)\theta] \qquad \textbf{(12)}$$

From equations **(11), (12)**:

$$p_1 = \frac{-1-x+\theta}{-2+\theta}, \quad p_2 = \frac{-1-x+\theta}{-2+\theta} \tag{13}$$

From equations **(10)**, **(13)**:

$$q_1 = \frac{-1+x}{b(-2+\theta)(1+\theta)} = q_2 \tag{14}$$

From manufacturer's profit function in equation **(7)** and from equation **(14)**:

$$\Pi_m = \frac{2(-1+x)x}{b(-2+\theta)(1+\theta)} \tag{15}$$

The manufacturer maximize his profits at the price that, $\frac{\partial \Pi_m}{\partial x} = 0$.

$$\frac{\partial \Pi_m}{\partial x} = 0 \Rightarrow x^{N*} = \frac{1}{2} \tag{16}$$

By replacing **(16)** into equations **(13)**, **(14)**:

$$p_1^{N*} = \frac{-\frac{3}{2}+\theta}{-2+\theta} = p_2^{N*} \tag{17}$$

$$q_1^{N*} = -\frac{1}{2b(-2+\theta)(1+\theta)} = q_2^{N*} \tag{18}$$

From equations **(11)**, **(13)**, **(14)**:

$$\Pi_1^{N*} = -\frac{-1+\theta}{4b(-2+\theta)^2(1+\theta)} = \Pi_2^{N*}$$

From equations **(15)**, **(16)** we can calculate the manufacturer's profits.

$$\Pi_m^{N*} = -\frac{1}{2b(-2+\theta)(1+\theta)}$$

The chain profits are the sum of retailers' and manufacturer's profits

$$\Pi_{Ch}^{N*} = \Pi_1^{N*} + \Pi_2^{N*} + \Pi_m^{N*}$$

$$\Pi_{Ch}^{N*} = \frac{3-2\theta}{2b(-2+\theta)^2(1+\theta)}$$

11.2 Proof of Case R Optimal Solutions

Replacing the reaction functions in equations **(10),** into the profit functions **(5)**, we have:

$$\Pi_1 = (p_1 - x)\frac{1-\theta+\theta p_2 - p_1}{b(1-\theta^2)} + [\frac{1-\theta+\theta p_2 - p_1}{b(1-\theta^2)} - \beta p_1^w]p_1^w$$

$$\Pi_2 = (p_2 - x)\frac{1-\theta+\theta p_1 - p_2}{b(1-\theta^2)} + [\frac{1-\theta+\theta p_1 - p_2}{b(1-\theta^2)} - \beta p_2^w]p_2^w \qquad \textbf{(19)}$$

We want to see now, which product and EW prices will the retailers set to maximize their profits.

$$\frac{\partial \Pi_1}{\partial p_1} = 0 \Rightarrow p_1 = \frac{1}{2}[1 - w_1 + x + \theta(p_2 - 1)]$$

$$\frac{\partial \Pi_1}{\partial w_1} = 0 \Rightarrow w_1 = \frac{\theta p_2 - p_1 - \theta + 1}{2b\beta(1-\theta^2)}$$

$$\frac{\partial \Pi_2}{\partial p_2} = 0 \Rightarrow p_2 = \frac{1}{2}[1 - w_2 + x + \theta(p_1 - 1)]$$

$$\frac{\partial \Pi_2}{\partial w_2} = 0 \Rightarrow w_2 = \frac{\theta p_1 - p_2 - \theta + 1}{2b\beta(1-\theta^2)} \qquad \textbf{(20)}$$

By solving the system of equations **(12)**, we can find the final product end EW prices.

$$p_1 = \frac{1-2b\beta(1+x-\theta)(1+\theta)}{1+2b\beta(\theta-2)(\theta+1)} = p_2 \quad \textbf{(21)}$$

By replacing **(13)**, into the reaction functions **(10)**, we will find the optimal product sales of retailers in relation with the wholesale price x.

$$q_1 = -\frac{2b\left(-1+x\right)\beta\left(-1+\theta^2\right)}{\left(b-b\theta^2\right)\left(1+2b\beta\left(-2+\theta\right)\left(1+\theta\right)\right)} = q_2 \quad \textbf{(22)}$$

We see that the two retailers have symmetric prices and sell the same quantities, as $p_1 = p_2$ and $q_1 = q_2$

Manufacturer has sold a total quantity of $q_1 + q_2$ to the retailer, at the wholesale price x. From his profit function **(7)** and the equation **(22)**:

$$\Pi_m = -\frac{4b\left(-1+x\right)x\beta\left(-1+\theta^2\right)}{\left(b-b\theta^2\right)\left(1+2b\beta\left(-2+\theta\right)\left(1+\theta\right)\right)} \quad \textbf{(23)}$$

The manufacturer will maximize his profits, when: $\frac{\partial \Pi^m}{\partial x} = 0$.

It will happen, when the wholesale price is: $x^R = \frac{1}{2}$. **(24)**

From equations (20), (21), (22), (23), (24) we can find the optimal retailers' product prices, EW prices and product quantities, manufacturer's and retailers' profits.

$$P_1^{R*} = \frac{1+b\beta\left(1+\theta\right)\left(-3+2\theta\right)}{1+2b\beta\left(-2+\theta\right)\left(1+\theta\right)} = P_2^{R*}$$

$$q_1^{R*} = \frac{b\beta\left(-1+\theta^2\right)}{\left(b-b\theta^2\right)\left(1+2b\beta\left(-2+\theta\right)\left(1+\theta\right)\right)} = q_2^{R*}$$

$$w_1^{R*} = -\frac{1}{2\left(1+2b\beta\left(-2+\theta\right)\left(1+\theta\right)\right)} = w_2^{R*}$$

$$\Pi_1^{R*} = -\frac{\beta[1+4b\beta(-1+\theta^2)]}{4[1+2b\beta(-2+\theta)(1+\theta)]^2} = \Pi_2^{R*}$$

$$\Pi_m^{R*} = \frac{b\beta\left(-1+\theta^2\right)}{\left(b-b\theta^2\right)\left(1+2b\beta\left(-2+\theta\right)\left(1+\theta\right)\right)}$$

The total chain profits are $\Pi_{Ch}^{R*} = \Pi_1^{R*} + \Pi_2^{R*} + \Pi_m^{R*}$

$$\Pi_{Ch}^{R*} = \frac{\beta\left(-3+4b\beta\left(3+\theta-2\theta^2\right)\right)}{2\left(1+2b\beta\left(-2+\theta\right)\left(1+\theta\right)\right)^2}$$

11.3 Proof of Case M Optimal Solutions

By replacing the retailer quantities of equation **(14)** into the manufacturer's profit function **(8)**:

$$\Pi_m = [2\frac{-1+x}{b\left(-2+\theta\right)\left(1+\theta\right)}]x + [2\frac{-1+x}{b\left(-2+\theta\right)\left(1+\theta\right)} - 2\beta w^m]w^m$$

Manufacture will maximize his profits at the point that:

$$\frac{\partial \Pi_m}{\partial x} = 0,\ \frac{\partial \Pi_m}{\partial w_m} = 0$$

$$\frac{\partial \Pi_m}{\partial x} = 0 \Rightarrow x = \frac{1-w_m}{2}$$

$$\frac{\partial \Pi_m}{\partial w_m} = 0 \Rightarrow w_m = \frac{1-x}{2b\beta\left(2+\theta-\theta^2\right)}$$

From the above EW the wholesale equation in relation with the wholesale price, we can find the manufacturer's optimal EW price and wholesale price.

$$x^{M*} = \frac{1+2b\beta(-2+\theta)(1+\theta)}{1+4b\beta(-2+\theta)(1+\theta)} \quad \textbf{(25)}$$

$$w_m^{M*} = \frac{1}{-1+4b\beta\left(2+\theta-\theta^2\right)} \quad \textbf{(26)}$$

Now by replacing **(25), (26)** into equations **(13)**, **(14)** we can find the optimal quantities and prices in case M.

$$q_1^{M*} = \frac{2b\beta(-1+\theta)(1+\theta)}{\left(b-b\theta^2\right)\left(1+4b\beta(-2+\theta)(1+\theta)\right)} = q_2^{M*}$$

$$p_1^{M*} = \frac{1+2b\beta(1+\theta)(-3+2\theta)}{1+4b\beta(-2+\theta)(1+\theta)} = p_2^{M*}$$

Now by replacing the optimal manufacturer's wholesale price and EW price and the retailers' optimal final product price to their profit functions, we can find their final profits, as well as the total chain profits.

$$\Pi_1^{M*} = \left[\frac{2b\beta(-1+\theta)(1+\theta)}{\left(b-b\theta^2\right)\left(1+4b\beta(-2+\theta)(1+\theta)\right)} - \frac{1+2b\beta(-2+\theta)(1+\theta)}{1+4b\beta(-2+\theta)(1+\theta)}\right]\frac{2b\beta(-1+\theta)(1+\theta)}{\left(b-b\theta^2\right)\left(1+4b\beta(-2+\theta)(1+\theta)\right)} = \Pi_2^{M*}$$

$$\Pi_1^{M*} = \frac{4b\beta^2\left(1-\theta^2\right)}{\left(1+4b\beta(-2+\theta)(1+\theta)\right)^2} = \Pi_2^{M*}$$

Manufacturer's profits will be:

$$\Pi_m^{M*} = (q_1^{M*}+q_2^{M*})x^* + [(q_1^{M*}+q_2^{M*}) - 2\beta \mathrm{w}_m^{M*}]\mathrm{w}_m^{M*}$$

$$\Pi_m^{M*} = \frac{\beta\left(-1+8b\beta\left(2+\theta-\theta^2\right)\right)}{\left(1+4b\beta(-2+\theta)(1+\theta)\right)^2}$$

Finally, the **chain profits** in this case will be the sum of the manufacturer's and the two retailers' total profits.

$$\left.\begin{array}{l}\Pi_{Ch}^{M*} = \Pi_{1}^{\mathrm{M}*} + \Pi_{2}^{\mathrm{M}*} + \Pi_{m}^{\mathrm{M}*} \\ \Pi_{1}^{\mathrm{M}*} = \Pi_{2}^{\mathrm{M}*}\end{array}\right\} \Pi_{Ch}^{M*} = \Pi_{1}^{\mathrm{M}*} + \Pi_{m}^{\mathrm{M}*}$$

$$\Pi_{Ch}^{M*} = \frac{\beta\left(-1+8b\beta\left(3+\theta-2\theta^{2}\right)\right)}{\left(1+4b\beta\left(-2+\theta\right)\left(1+\theta\right)\right)^{2}}$$

11.4 Case MR Optimal Solutions

If we use the same mythology as in case R but using the cases' RM retailers' and manufacturer's profit function, then we will have the same optimal solutions as the ones in model R.

12. Conclusions

We examined the product retail and wholesale price, the profits, the product quantities, the EW prices as well as the EWs wholesale prices, when the provider of the EWs is the manufacturer, the retailer or when the retailer provides the official manufacturer's EWs. We also examined a case in which there are no EWs or in other words, EWs are provided by a free EW market.

From the results we can see that the outcomes between the case in which the retailer is the provider of EWs are exactly the same as the ones when the retailer is the official provider of manufacturer's EWs. This is reasonable, because the EW wholesale price has a negative effect in the product wholesale price. The manufacturer will give the EW free of charge to the retailers in order maximize his own profits.

The wholesale price of the product is the same in all models except from the model in which manufacturer is the EW provider as a result to be expected since the manufacturer can earn profits not only from selling the basic product, but also from selling EWs. By reducing the product's wholesale price, he may increase the demand for EWs and finally managing to end up with more profits.

In cases R and M, as β increases, the product retail price increases too, tending to be equal with the product retail price in model N. This means that when customers start to become more sensitive in the EW price, the demand of EWs decreases. With a low demand for EWs retailers in model R and the manufacturer in model M start to loose profits. To abate the loss, they increase the product price and decrease the EW price.

Chain profits will be higher in model R than in case M or N. In this case, retailers can affect both the product retail price and the EW retail price managing to gain more profits. The profits are decreasing as θ increases, leading to Bertrand Paradox when θ tends to 1.

The full price will be higher when the EW provider is the manufacturer than the retailer but both of them are higher than the retail price in the full competitive EW market (Case N).

When a retailer provides EWs, he can sell his product in a lower price than his product cost, which in our case is the wholesale price.

An interesting result is that as product differentiation decreases, extended warranty price decreases too. This might happen because as product differentiation decreases, the product retail price decreases. With a product of a lower price customers are willing to pay less for an extended warranty which means that they became more sensitive in EW price when the retail price is lower.

In our numerical example we can see the above outcomes. Chain profits are 8,6% higher in REW model than the profits in NOEW model. This explains the aggressive marketing of retailers to promote EWs. The chain profits are 1.5% lower in model MEW in contrast with the chain profits in model REW, which explains that retailers have the ability to make more profits, controlling both EW and product retail price.

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