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An empirical analysis of the Fisher effect in the G7.

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Abstract

This paper tests the existence of Fisher Effect in G7 countries. The Fisher effect postulates that real interest rate is constant and that nominal interest rate and expected inflation rate move one to one together. We employ Engle and Granger 2- step and Johansen's method cointegration procedure and we find support for long-run and positive relationship between nominal interest rate and inflation rate in all G7 except for Japan. From the error correction model derived we examine the dynamic relationship between the two variables and we find that the full Fisher Effect applies only for Canada and USA. Moreover, our analysis is extended to Granger-Causality test and Impulse Response Functions to investigate the short-run relation between the nominal interest rate and inflation rate.

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1. Introduction

The long-run Fisher hypothesis, formalized by Irving Fisher (1930) states that a permanent shock (change) in the rate of inflation will cause an equal change in the nominal interest so that the real interest rate is not affected by monetary shocks in the long run. So, for holding this long run relation, the ex-ante real interest rate which according to Fisher's model is the difference between the nominal interest rate and the expected inflation rate must be mean-reverting. If the nominal interest rate and the inflation rate are each integrated of order one, denoted I(1), then the two variables should cointegrated with a slope coefficient of unity so that the real interest rate is covariance stationary. The hypothesis has been studied extensively and plays a major role in application of monetary policy. If the Fisher effect holds in long-run, then the monetary policy will have no influence on the real interest rate, since in this case, any change in the expected inflation will be an offset by a change in the nominal rate of interest, leaving the ex-ante rate unchanged.

Many empirical analyses have been done various models have been proposed and tested using data from many countries (developed and developing) for the validity of the Fisher effect and the existence of the Fisher effect has been subject to debate by many economists and researchers. From a macroeconomic view, the Fisher effect is the basis of neutrality monetary models (changes in the money supply affect nominal variables but not real variables) and it attributes to explaining the movement and behavior economic fundamentals (i.e exchange rate). Actually, inflation affects consumers and economy at a large extent and because of the difference between nominal and interest rate, which affects all intertemporal savings and investment decisions in the economy, the understanding of the Fisher link between inflation, nominal and real interest real contribute to gaining knowledge about how each economy runs as a whole and how different economies interact.

A variety of empirical techniques have been used to test the Fisher hypothesis. However, nowadays cointegration and error correction methodologies have been more often used to investigate whether the nominal interest rate and some measure of expected inflation cointegrate with a unitary coefficient which is evidence that the strong form Fisher hypothesis describes an empirically valid long-run equilibrium relation.

Our analysis is organized as follows: Section 2 refers to literature review relate to Fisher effect. Section 3 gives us information about methodology that we follow and the data. In Section 4 we investigate the integrating properties of the variables using unit root and stationarity tests. In Section 5 we use cointegration tests to examine the long-run relationship and also using the VECM to examine the dynamic relationship between the variables. In section 6 test the short- run relation of variables using Granger-causality test and IRF. And in end section 7 closes with a brief summary and conclusions.

2. Literature review

In spite of the relatively large number of empirical analyses have been done for the Fisher effect, its validity is yet under investigation. The results are mixed, with different studies using different tests, different countries and different samples ending up in different conclusions.

In theoretical framework, clearly, the Fisher relations holds in models (i.e. Sidrauski, 1967) in which the real interest rate is determined by a relation such as the modified golden rule and therefore does not depend on monetary variables. On the other hand, it is violated in models in which the Tobin (1965) and Mundell (1963) effect applies - in such models, an increase in the inflation rate results in a decrease of the real rate and consequently in a less than one-to-one increase of the nominal interest rate due to the impact of inflation on wealth and subsequently savings. Also, Darby (1975) and Feldstein (1976) point out that the effects of tax would result in a more than one-to-one adjustment to expected inflation, while Shome, Smith and Pinkerton (1988) suggest a premium needs to be incorporated in nominal interest rates to account for covariance risk.

In empirical framework, Carmichael and Stebbing (1983), propose another relationship between inflation, nominal and real interest rates to that of Fisher. They have claimed that when the Fisher hypothesis is applied to financial assets the Fisher hypothesis may be inverted, namely, 'that the nominal rate of interest on these assets may be approximately constant over the longer term, with the real rate of interest moving inversely one-for-one with the rate of inflation'. This so called Inverted Fisher effect or Fisher paradox. They have used quarterly data from USA for the period 1953-1978 and for Australia for two periods 1965-1981 and 1963-1981 and found evidence of the Inverted Fisher Effect. However Moazzami (1991), using the same dataset on three-month Treasury bills and consumer prices for USA and Australia (such as Carmichael and Stebbing) cannot find the same long run inverted Fisher relationship. Also Choudhry (1997), employing a longer time period from 1955-1994, using data from France, Belgium and Germany, found some support for a partial adjustment, nonetheless, little support for a Fisher inversion.

The long run neutrality proposition has been investigated in a number of studies. King and Watson (1997), have contributed to the literature on testing the long-run neutrality by applying tests based on coefficient restrictions in bivariate vector autoregressive models (VAR models).They used US quarterly data from 1949-1 to 1990-4. They observed that meaningful neutrality tests can only be constructed if both nominal and real variables satisfy certain non-stationarity conditions. Especially, they found that the Fisher effect tests are possible if the inflation and interest rates series are integrated of order one and do not cointegrate. In this study they found that the Fisher hypothesis is not confirmed. Likewise, Koustas and Serletis (1999), using quarterly data for 11 OECD countries that begin between 1957 and 1972 and ends in 1995 and paying explicit attention to the integration and cointegration properties, since meaningful Fisher effect tests critically depend on the property of stationarity in the first differences of the variable and no-cointegration, they did not find evidence of the Fisher effect. Also applying the same framework of King and Watson (1997), Engsted (1995) have studied 13 OECD countries from 1962 to 1993 rejected long-run neutrality of inflation with respect to real interest rates. And Rapach (2003), studying 14 OECD countries for a sample that begins between 1949 and 1965 and ends between 1994 and 1996 found no evidence of the Fisher effect.

The emergence of the literature on unit roots and cointegration tests provided an important impulse to the empirical testing of the Fisher hypothesis. However the results from cointegration tests of Fisher hypothesis are mixed.

Mishkin (1992), in an attempt to explain why there was strong evidence of a Fisher effect for some periods and not for others, argued that a Fisher effect would only appear in samples where inflation and interest rates displayed stochastic trends. The reasoning behind this was that when the two series exhibit trends, they would trend together, resulting in a strong evidence correlation between them. This involved determining the univariate statistical properties of the respective time series, namely, inflation and interest rates. Using US monthly data from 1953-1 to 1990-12 and applying Dickey Fuller and Phillips tests for existence of unit roots in series, he observed that both the levels of inflation and interest rates contained a unit root. After, he carried out cointegration tests for a common trend in inflation and interest rates revealed the existence of a long-run Fisher effect, but the absence of a short-run relationship. As predicted, a Fisher effect was displayed for the post-war period until October 1979 in which evidence was strongest of stochastic trends in inflation and interests rates. However, there was no evidence of a trend and consequently a Fisher effect for the pre-war period and October 1979 to September 1982.

Likewise, Bonham (1991), Pelaez (1995) and Wallace and Warner (1993), covering a similar time period, confirmed Mishkin's findings that inflation contained a unit root. Wallace and Warner (1993), using US quarterly data from 1948-1 to 1990-4 they found that inflation and interest rates are I(1) in the majority of cases and applying the Johansen and Juselius (1990) cointegration test provided support for long and short Fisher relationship. Bonham (1991), using US monthly data from 1955-1 to 1990-3 and applying the Dickey Fuller test he found that the inflation and interest rates are I(1) but rejected the cointegration relation between inflation and nominal interest rate. Pelaez (1995) using US quarterly data from 1959-1 to 1993-4 and applying both the

Engle Granger two-step procedure and Johansen's vector autoregressive error correction mechanism he did not find evidence of a Fisher relationship.

Crowder and Hoffman (1996), examine the long-run dynamic relationship between short-term nominal interest rate and inflation using quarterly data for the period 1952- 1991 they observed that the 3-month US Treasury Bill rate and the inflation rate are cointegrated and applying a bivariate vector error correction model (VECM) they found that the VECM suggest a long-run Granger-cause ordering from the inflation rate to the nominal interest rate. This implies that the inflation rate contains information about the future path of the interest rate.

Weidmann (1997), suggest a threshold cointegration (TC) model to test for the Fisher effect. Using monthly data of the German 12-month Treasury bills and the consumer index price (CPI) from 1967-1 to 1996-6, he shows that the stochastic process governing the bivariate system of inflation and interest rates depends on the level of variables and can be designed as a TC model. The TC model explains the downward bias of the coefficient estimates, the sample and country sensitivity and cannot reject the long run Fisher relationship. However, the TC model is mainly applied in industrialized countries where there is price stability and inflation and interest rates seldom occur negative and persistently high values.

Westerlund (2008), tested the Fisher hypothesis using quarterly data from 1980 to 2004 in a cointegrated panel of 20 OECD countries and could not reject the Fisher effect. Panels generally add power to contegration tests due to the added cross sectional dimension, however, they also impose restrictions, particularly on the cross-sectional dependencies, that may not hold in the data.

Wong, Ka-fu and Haijun Wu (2003), using monthly data from G7 countries and eight Asian countries, test the hypothesis at short-run and long-run horizons applying OLS and Instrumental Variables regressions. They found more support for the hypothesis at long run horizons, as well as a positive relationship between long-horizon nominal stock returns and expected inflation but not between long-horizon nominal stock returns and contemporaneous inflation. Also, Berument Hakan, and Mohamed Mehdi (2002), test whether the Fisher hypothesis holds for a monthly sample of 12 developed (1957M4-1998M5) and 14 developing (1957M5-1998M5) countries using an instrumental variable technique. They found that the strong version of the Fisher hypothesis could not be rejected for 9 out of 12 developed countries and for 7 out of 14 developing countries.

Authors	Countries	Sample	Methodology	Finding
Carmichael and Stebbing (1983)	USA	1953Q1-1978Q4	Cointegration	Evidence of IFH
	Australia	1965Q3-1981Q4	Cointegration	Evidence of IFH
		1963Q1-1981Q4	Cointegration	Evidence of IFH
Moazzami (1991)	USA	1953Q1-1978Q4	Coint.-VECM	No Evidence of IFH
	Australia	1965Q3-1981Q4	Coint.-VECM	No Evidence of IFH
		1963Q1-1981Q4	Coint.-VECM	No Evidence of IFH
Choudhry (1997)	Belgium	1958Q1-1994Q4	Cointegration	No Evidence of IFH
	France	1958Q1-1994Q4	Cointegration	No Evidence of IFH
	Germany	1955Q1-1994Q4	Cointegration	No Evidence of IFH
King and Watson (1997)	USA	1949Q1-1990Q4	VAR	No Evidence of FH
Koustas and Serletis (1999)	11OECD countries	1957-1972 and ends in 1995	Engle-Granger 2-stepCoint. VAR(Kin-Wat)	No Evidence of FH
Engsted (1995)	13OECD countries	1962Q1-1993Q1	VAR(King-Watson)	No Evidence of FH
Rapach (2003)	14OECD countries	1949-1965 ends 1994-1996	VAR(King-Watson)	No Evidence of FH
Mishkin (1992)	USA	1953M1-1990M12	Cointegration test	Evidence of long run FH
Bonham (1991)	USA	1955M1-1990M3	Engle-Granger 2-st	No Evidence of FH
Pelaez (1995)	USA	1959Q1-1993Q4	Engle-Granger 2-st. Johansen Coint.	No Evidence of FH
Wallace and Warner (1993)	USA	1948Q1-1990Q4	Johansen-Jesulius Coint.	Evidence of long and short run FH
Crowder and Hoffman (1996)	USA	1952Q1-1994Q$	Johansen, IRF VECM	Evidence of long run FH
Weidmann (1997)	Germany	1967M1-1996M6	Threshold Coint.	Evidence of long run FH
Westerlund (2008)	20 OECD	1980Q1-2004Q4	Cointegration Panels	Evidence of long run FH
Wong, Ka-fu and HaijunWu (2003)	G7 and 8 Asian Countries	1958Q1-1999Q4	OLS, Instrumental Variables	Evidence of long run FH
Berument Hakan, and Mohamed Mehdi J,(2002)	12 developed countries	1957M4-1998M5	Instrumental Variables	Evidence of long run FH in 9 countries
	14 developing countries	1957M5-1998M5	Instrumental Variables	Evidence of long run FH in 7 countries

3. Research Methodology

3.1 The model

The Fisher hypothesis state in the long- run, inflation and nominal interest rates move one-for-one, meaning that real interest rates remain unchanged in the long run:

$$i_t = r_t + \pi_t^e \quad (1)$$

where i_t is the nominal interest rate at time t
r_t is the real interest rate at time t
and π_t^e is the expected inflation at time t.
As usual we assume rational expectations so that expected inflation is equal to actual inflation ($\pi_t = \pi_t^e + e_t$).
So, the Fisher regression model can be written as:

$$i_t = r_t + \pi_t - e_t \quad (2)$$

where e_t is the residuals of the regression
When inflation and nominal interest rates behave each as an I(1) process in our samples, then they should be cointegrated with a slope coefficient a_1.

$$i_t = a_0 + a_1\pi_t + e_t \quad (3)$$

where a_1 is the coefficient of inflation.

If a_1 is statistically equal to one then the strong form of the Fisher effect or the full Fisher effect is implied and there is a one to one relation between the nominal interest rate and inflation.
If a_1 is positive and less than one then the weak form of the Fisher effect or the partial Fisher effect is implied.
If a_1 is more one then it is implied that the nominal interest rate is taxed and this state that there is a more than one to one relation between the inflation and nominal interest rate. (Darby 1975)

3.2 Econometric Methodology

Our analysis is testing for cointegration between the nominal interest rate and the inflation rate. Cointegration states that there is a long-run relationship to which a system converges over time. Conversely, if the variables are not cointegrated, they trend to drift apart.

The basic problem of testing the equation (3) using standard assumptions and OLS models is that variables may be not stationary and that can be lead to no valid results. However, if the two variables inflation and nominal interest rate are non-stationary and are integrated of order one then the Fisher hypothesis can be tested using cointegration methods.

Firstly, we apply unit root or stationary tests to see in what order are integrated the variables and if the variables have unit root we can continue in testing cointegration.

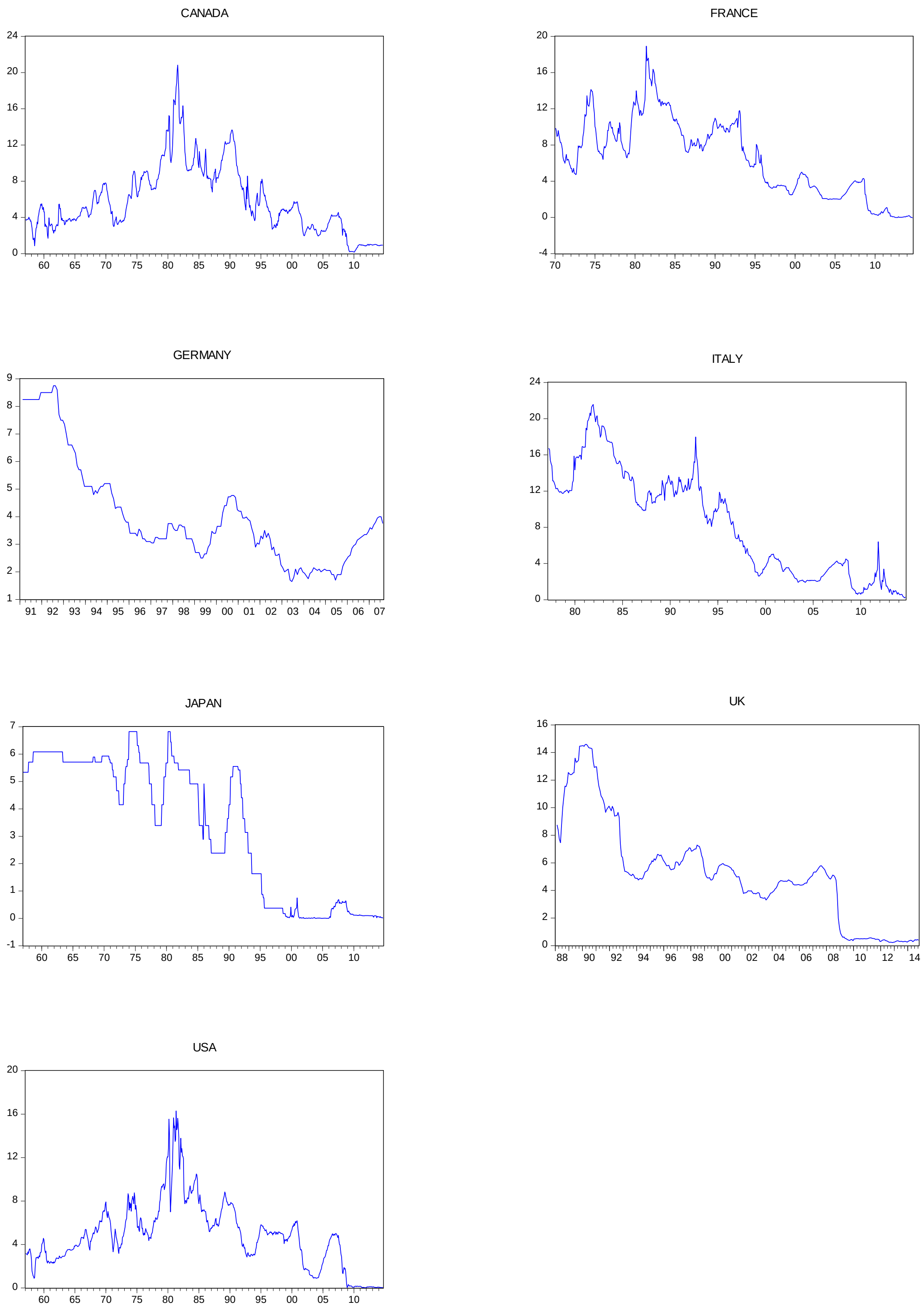
The two most common procedures for testing cointegrating systems between in variables and time series are the Engle-Granger Two- step and Johansen Technique based on VARs. The Engle-Granger 2-step or Engle-Granger's is based on testing the stationarity of the residuals in equation (3). If i_t and π_t are integrated of order 1 I(1)and the residuals e_t are integrated of order zero I(0), then there is a long- run relationship and long-run Fisher hypothesis holds. When $e_t \sim$ I(0) and a_1 is not significantly different from one then the real interest rate is mean- reverting and the full Fisher effect holds. But, if $e_t \sim$ I(0) and a_1 is significantly less than one then the real interest rate is non stationary and that means that the weak form of Fisher effect is implied.

The Johansen cointegration test is a procedure for testing cointegration between in I(1) time series. This test permits more than one cointegrating relationship so is more generally applicable from Engle-Granger test which is based on testing the stationarity of the residuals from a single estimated cointegrating relationship. There are two types of Johansen test either with trace statistics or with max eigenvalue statistics and the inferences may be a little bit different.

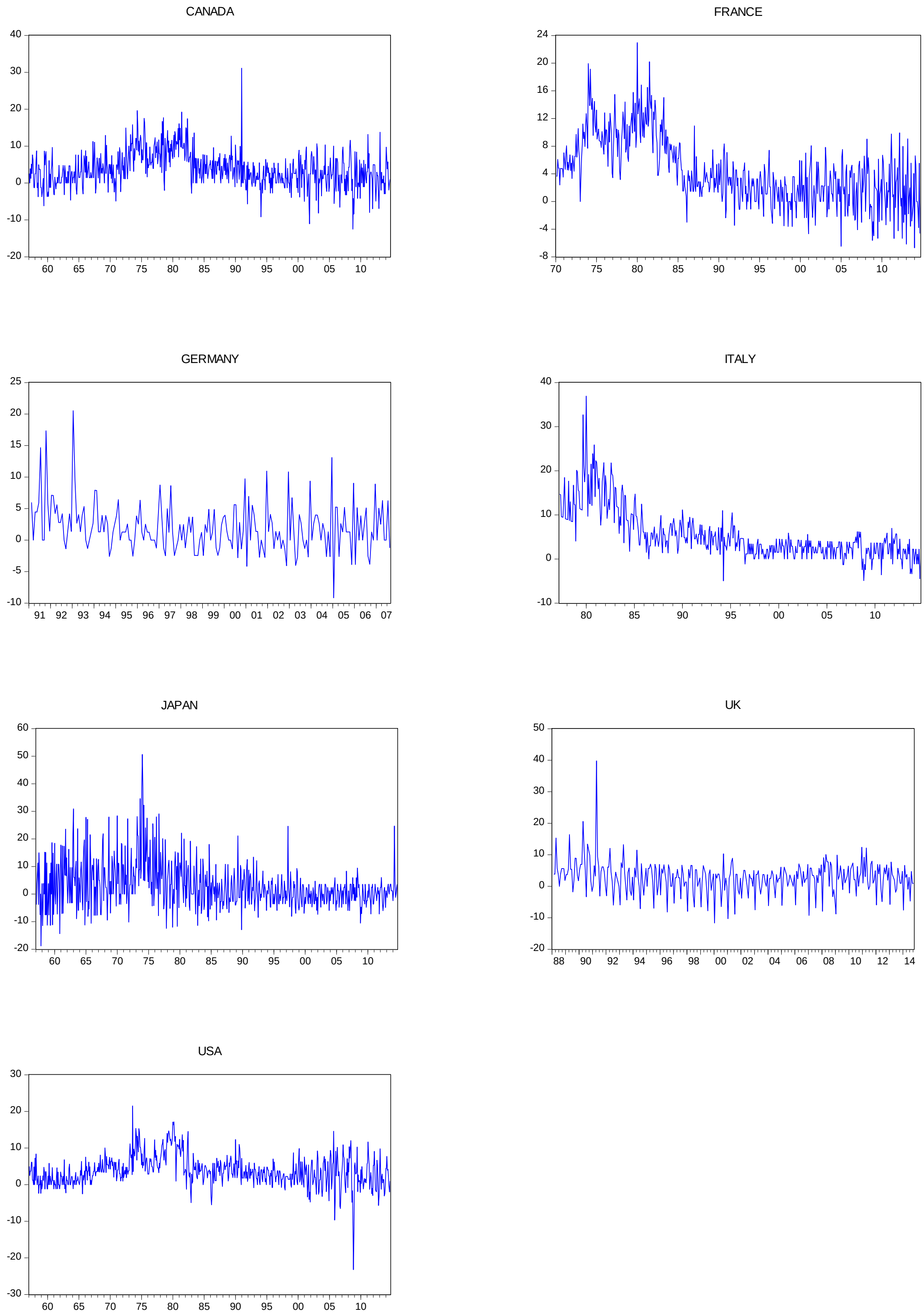
The presence of a cointegration between variables suggests a long- term relationship among the variables under consideration and consequently a VECM can be applied to examine the dynamic relationship between the nominal interest rate and inflation rate.

In the next step, using the VECM we apply Granger Causality test and Impulse Response Functions between the two time series to test the interactions between these variables.

Graph 1. Nominal interest rate time-series



Graph 2. Inflation rate time-series



3.3 The data

We retrieved monthly data, for the consumer price index (CPI) and the short-terms interest rates from the International Monetary Fund's online data base for the G7 countries: United States, Japan, Germany, United Kingdom, France, Italy and Canada. The data have been obtained from the beginning of the base. The sample that we used for each country is illustrated in tables 1 to 6. Inflation rates were calculated from the first differences of the natural logarithm of the CPI, multiplied by 1200 to get annualized rates in percent. We picked for the short-term interest rate the 3-month Treasury bill for all countries. The graphs 1 and 2 show the graphs of nominal interest rate and inflation time series in G7 countries.

4. Unit root tests

The stationarity or non stationarity of a series can strongly influence its behavior properties (for instance, persistence of shocks will be infinite for nonstationary series). If the variables in the regression model are not stationary, then it can be proved that the standard assumptions for asymptotic analysis will not be valid. In other words, the usual 't- ratios' will not follow a t-distribution, so we cannot validly undertake hypothesis tests about the regression parameters. The possible endogeneity of regressors is a problem not well handled by OLS. Classical regression properties hold only for cases where variables are stationary (integrated of order zero). But by contrast most economic variables are integrated of order one, error correction mechanisms or long run relationships may exist and therefore certain combinations of I(1) variables are likely to be I(0) and hence amenable to OLS estimation. Where this is so, the variables are said to be cointegrated and OLS estimates of such cointegrated variables may be supercosistent in the sense of collapsing to their true values more quickly than if the variables had been stationary. The first step is to determine the degree of integration of the individual series under investigation, thus the empirical analysis begins by examining this with univariate tests. Where a cointegrating relationship cannot be found, no long run relationship among the variables can be demonstrated and we have the case of spurious regression.

If a non-stationary series, y_t must be differenced *d* times before it becomes stationary, then it is said to be integrated of order *d.* We write $y_t \sim$ I(d).And if $y_t \sim$ I(d) then $\Delta^d y_t \sim$ I(0).So, a series or variable is determined to be

I(1) if one unit root is found in levels and stationary is found the first differences.

The majority of economic and financial variables usually contain a single unit root.

Our analysis will apply the ADF and Ng-Perron unit root test and KPSS stationary test.

4.1 Autoregressive Unit root tests

The early and pioneering work on testing for a unit root in time series was done by Dickey and Fuller (Dickey and Fuller 1979,Fuller 1976).we use the AR (1) model $y_t = \varphi y_{t-1} + u_t$ where u_t is a white noise process $u_t \sim (0, \sigma^2)$ and test the null hypothesis that φ=1 against the one-sided alternative φ<1.so we have

H_0: φ=1 (series contains a unit root)

vs. H_1: φ<1 (series is stationary)

We usually use the empirical regression: $\Delta y_t = \psi y_{t-1} + u_t$ (4)

So that a test of φ=1 is equivalent to a test of ψ=0 (since φ-1=ψ) and a test of φ<1 is equivalent to a test of ψ<0.

The test statistics are defined as:

$$\text{test statistic}=\frac{\hat{\psi}}{SE(\hat{\psi})}$$

Where $\hat{\psi}$ is the least square estimate and is the usual standard error estimate. The test is a one-sided left tail test.

The test statistic does not follow the usual t-distribution under the null, since the null is one of non-stationarity, but rather follows a non-standard distribution. And critical values are derived from Monte Carlo experiments.

4.2 The Augmented Dickey-Fuller (ADF) Unit Root Test

The unit root tests described above are only valid if the time series y_t is well characterized by an AR(1) model with white noise errors. However, many financial time series have a more complicate dynamic structure than is captured by a simple AR(1) model. In particular, u_t will be autocorrelated if there was autocorrelation in the dependent variable of the regression (Δy_t) which we have not modelled. The solution is to 'augment' the test using p lags of the dependent variable. The ADF test is based on estimating the test regression:

$$y_t = \beta' D_t + \varphi y_{t-1} + \sum_{i=1}^{p} a_i \Delta y_{t-i} + u_t \quad (5)$$

where D_t is a vector of deterministic terms (intercept, trend), the p lagged difference terms, Δy_{t-i}, are used to approximate the ARMA structure of the errors and the value of p is set so that the error u_t is serially uncorrelated. The error term is also assumed to be homoscedastic. The specification of the deterministic terms depends on the assumed behavior of y_t under the alternative hypothesis of trend stationarity.
The alternative (empirical) of the ADF test regression is:

$$\Delta y_t = \beta' D_t + \psi y_{t-1} + \sum_{i=1}^{p} a_i \Delta y_{t-i} + u_t \quad (6)$$

The ADF test statistics are defined as: $ADF_t = \frac{\hat{\psi}}{SE(\hat{\psi})}$

A problem now arises in determining the optimal number of lags of the dependent variable. For this problem, we usually use some information criteria like Akaike Information Criteria, Schwarz Information Criteria, Hannan-Quinn Information Criteria etc.

4.3 Ng-Perron Unit Root Test

Other known unit roots test are the Phillips-Perron and (PP) unit root test (1988), the Elliot, Rothenberg, and Stock unit root test (1996) and the Ng-Perron (2001) unit root test.

Phillips-Perron have developed a more theory of nonstationarity. The tests are similar to ADF tests but they differ in how they deal with serial correlation and heteroskedasity in the residuals. And the tests usually give the same conclusions as the ADF tests.

In general, the ADF and PP tests have very low power against I(0) alternatives that are close to being I(1). That is, unit root tests cannot distinguish highly persistent stationary processes from non-stationary processes very well. Also, the power of unit root tests diminish as deterministic terms are added to the test regressions. That is, tests that include a constant and trend in the test regression have less power than tests that only include a constant in the test regression. For maximum power against very persistent alternatives the recent tests proposed by Elliot, Rothenberg, and Stock (1996) and the Ng-Perron(2001) unit root test can be used.

Elliot, Rothenberg, and Stock (1996) suggest a more powerful unit root test: the generalized least squares version of the ADF test. They found that powers of ADF tests are lower than those of the limiting power functions when deterministic components (mean or trend) are included in the data process.

Ng-Perron (2001) unit root test building on some of their own work in 1996 and work by Elliot, Rothenberg, and Stock (1996), new methods to deal with both of these problems. Their tests, in contrast to many of the other 'new' unit root tests that have been suggested over the years, seems to have caught on as a good alternative to the traditional ADF and PP tests. The family of NP tests (which includes among others, ADF and and PP test statistics) have the following featured. First, the time series are de-meaned or detrended by applying a GLS estimator. This step arises to improve the power of the tests when there is a large AR root and reduces size distortions when there is a large negative MA root ion the differenced series. The second feature of this test is a modified lag selection (or transaction selection) criteria. It turns out that the standard lag selection procedures used in specifying the ADF regression (or for calculating the long run variance for the PP statistic) tend to inderfit, i.e., choose too a small a lag length, when there is a large negative MA root. This creates additional size distortion in unit root tests. The NP modified lag selection criteria accounts for this tendency.

They use the GLS detrending procedure of ERS to create efficient versions of the modified PP tests of Perron and Ng (1996). These efficient modified PP tests do not exhibit the severe size distortions of the PP tests for errors with large negative MA or AR roots, and they can have substantially higher power than the PP tests especially when ψ is close to unity.

Using the GLS detrended data $\tilde{y}_t$, the efficient modified PP tests are defined as:

$$MZ_\alpha = (\mathrm{T}^{-1}\,\tilde{\mathrm{y}}_t^2 - \mathrm{f}_0)/2\kappa$$

$$MSB = \left(\frac{\kappa}{f_0}\right)^{1/2}$$

$$MZ_t = MZ_\alpha \times MSB$$

$$MPT = \begin{cases} \left(\bar{c}^2\kappa - \bar{c}T^{-1}\tilde{y}_t^2\right)/\,f_0 \\ \left(\bar{c}^2\kappa + (1-\bar{c})T^{-1}\tilde{y}_t^2\right)/\,f_0 \end{cases} \text{if} \quad x_t = \{1\} \quad \text{and} \quad x_t = \{1, t\}$$

respectively

where $\kappa = \sum_{\tau=2}^{\mathrm{T}} y_{t-1}^2 / \mathrm{T}^2$ and f_0 is an estimate of the residual spectral density at the zero frequency. The statistics MZ_α and MZ_t are efficient versions of the two PP tests (Z_a and Z_t) that have much smaller size distortions in the presence of negative moving average errors. Again the choice of the autoreggresive transaction, lag, p, is critical for correct calculation of f_0. Here p is chosen using the Modified Information Criteria (MIC(p)) of Ng-Perron (2001) as $p = p_{MIC} = \text{argmin}_p MIC(p)$ where

$$\tau_\tau(\text{p}) = (\hat{\sigma}_p^2)^{-1} \hat{\gamma}^2 \sum_{t=p_{\max}+1}^{T} \tilde{y}_{t-1}^2$$

$$\hat{\sigma}_p^2 = (\text{T} - \text{p}_{\max})^{-1} \sum_{t=p_{\max}+1}^{T} \hat{u}_t^2$$

4.4 KPSS Stationary Test

Unit root tests and mainly ADF and PP tests have been criticized due to that the power of the test is low if the process is stationary but with a root close to the non-stationarity boundary. One other way to get around this is to use a stationary test. The most commonly used stationary test is the KPSS test (Kwaitowski, Phillips, Schmidt and Shin, 1992).

They derive their test by starting with the model:

$$y_t = \beta' D_t + \mu_t + u_t \quad (7)$$
$$\mu_t = \mu_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$$

where D_t contains deterministic components (constant or constant plus time trend), u_t is I(0) and may be heteroskedastic. The μ_t is a pure random walk with innovation variance σ_ε^2. The null hypothesis H_0 is that y_t is stationary versus the alternative hypothesis H_1 that the y_t is non-stationary.Also this null hypothesis implies a unit moving average root in the ARMA representation of Δy_t. The KPSS test statistic is the Lagrange multiplier (LM) or score statistic for testing σ_ε^2=0 against the alternative that $\sigma_\varepsilon^2 > 0$ and is given by:

$$KPSS = \left(T^{-2} \sum_{t=1}^{T} \hat{S}_t^2 \right) / \hat{\lambda}^2$$

where $\hat{S}_t = \sum_{j=1}^{t} \hat{u}_j$, $\hat{u}_t$ is the residual of a regression of y_t on D_t and is a consistent estimate of the long-run variance of u_t using $\hat{u}_t$ .The stationarity test is a one-sided right-tailed test. And we can compare the results of these tests with the ADF procedure to see if we obtain the same conclusion.

4.5 Empirical results

We employ three test to test the variables in what order are integrated. The results of the unit and stationary tests are indicated in the tables 1,2,3,4,5 and 6. Table 1 is shown the results from the ADF unit root test. The null hypothesis is that there is a unit root in the time series. The null hypothesis of a unit root is rejected in favor of the stationary alternative in each case if the test statistics is more negative than the critical value. We test in levels and in first differences when a constant is considered and when a constant and a deterministic trend is considered. For the ADF test, the optimal lag length was taken to be the order selected by the Akaike Information Criterion (AIC) plus 2 (such as Koustas, Serletis 1999). We base our inferences on the 5% level of signifance. We cannot reject the null hypothesis of a unit root for all cases, except for the inflation of Japan and Germany in levels. In first differences we can reject the null hypothesis for all cases, consequently in first differences all the variables are I(0). So, we can conclude that all the interest rates are integrated of order one I(1) , the inflation rates of USA, UK, Canada, France and Italy are (1) while the inflation rates of Germany and Japan are stationary I(0).

Table 1: ADF Unit Root Test

Country	Sample period	Variables	Level		First Differences		Results
			ADF_c	ADF_t	ADF_c	ADF_t	
USA	1957M1-2014M10	i_t	-1.877313	-2.288680	-6.915793*	-6.995355*	I(1)
		π_t	-2.813923***	-2.946090	-11.94809*	-11.94230*	I(1)
UK	1988M1-2014M10	i_t	-1.805826	-2.835620	-9.680951*	-9.726661*	I(1)
		π_t	-1.987251	-2.070884	-8.420108*	-8.407029*	I(1)
Canada	1957M1-2014M10	i_t	-1.778590	-2.082276	-7.491318*	-7.561125*	I(1)
		π_t	-2.299114	-2.499282	-13.01978*	-13.01404*	I(1)
France	1970M1-2014M10	i_t	-1.527117	-3.083842	-8.083785*	-8.094059*	I(1)
		π_t	-1.345418	-2.819257	-9.364821*	-9.371423*	I(1)
Germany	1991M1-2007M08	i_t	-1.991555	-2.771736	-6.943661*	-6.999731*	I(1)
		π_t	-3.735103**	-3.655797**	-8.593032*	-8.648894*	I(0)
Italy	1977M3-2014M10	i_t	-0.917835	-2.548691	-10.09444*	-10.08290*	I(1)
		π_t	-1.119627	-1.813544	-8.453368*	-8.444458*	I(1)
Japan	1957M1-2014M10	i_t	-1.299570	-3.351019***	-5.933790*	-5.929044*	I(1)
		π_t	-3.038838**	-4.125621*	-9.666432*	-9.667970*	I(0)

***,**,* denote significance at the 10%, 5% and 1% significance levels. ADF_c is ADF with an intercept and ADF_t is ADF with an intercept and a deterministic trend. The critical values are for Mackinnon (1996). The critical values for ADF_c are -2,57 for 10%,-2,86 for 5% and -3,43 for 1% signifance level. The critical values for ADF_t are -3,13 for 10%, -3,42 for 5% and -3,97 for 1% signifance level. The results are based on 5% signifance level.

Table 2 is shown the results from the KPSS stationary test. We base our inferences on the 5% level of signifance. The null hypothesis is that there is stationarity in the time series. The null hypothesis of stationary is rejected in favour of the non- stationary alternative in each case if the test statistics is higher than the critical value. We test in levels and in first differences when a constant is considered and when a constant and a deterministic trend is considered. We can reject the null hypothesis of a stationary for all cases in levels for interest and inflation rates. In first differences we cannot reject the null hypothesis for all cases, consequently in first differences all the variables are I(0). So, we can conclude that all the interest rates and inflation are integrated of order one I(1).

Table 2: KPSS Stationary Test

Country	Sample period	Variables	Level		First Differences		Results
			$KPSS_c$	$KPSS_t$	$KPSS_c$	$KPSS_t$	
USA	1957M1-2014M10	i_t	0.869968*	0.576908*	0.107242	0.024886	I(1)
		π_t	0.635329**	0.419470*	0.181331	0.134447	I(1)
UK	1988M1-2014M10	i_t	1.512989*	0.577301*	0.149134	0.024118	I(1)
		π_t	0.961821*	0.423040*	0.046996	0.047300	I(1)
Canada	1957M1-2014M10	i_t	0.810429*	0.631613*	0.104001	0.028928	I(1)
		π_t	0.812711*	0.496384*	0.138141	0.115497	I(1)
France	1970M1-2014M10	i_t	1.512989*	0.335347*	0.043919	0.028751	I(1)
		π_t	2.094711*	0.295213*	0.071064	0.062375	I(1)
Germany	1991M1-2007M08	i_t	1.671361*	0.204882**	0.106112	0.041204	I(1)
		π_t	0.719000**	0.252786*	0.039622	0.039632	I(1)
Italy	1977M3-2014M10	i_t	2.330228*	0.239448*	0.044309	0.041199	I(1)
		π_t	1.965716*	0.401965*	0.106210	0.068030	I(1)
Japan	1957M1-2014M10	i_t	2.738290*	0.290544*	0.056810	0.041599	I(1)
		π_t	1.551058*	0.231182*	0.050561	0.032832	I(1)

***,**,* denote significance at the 10%, 5% and 1% significance levels. $KPSS_c$ is KPSS with an intercept and $KPSS_t$ is KPSS with an intercept and a deterministic trend. The critical values are for Mackinnon (1996). The critical values for $KPSS_c$ are 0,347 for 10%, 0,463 for 5% and 0,739 for 1% signifance level. The critical values for $KPSS_t$ are 0,119 for 10%, 0,146 for 5% and 0,216 for 1% signifance level. The results are based on 5% signifance level.

In next, tables 3,4,5,6 are showed the results from Ng-Perron unit root test. This test is unit root test, consequently we test the null hypothesis that there is a unit root in the time series and the null hypothesis of a unit root is rejected in favour of the stationary alternative in each case if the test statistics is higher in absolute values than the critical value. As the two previous test, we test in levels and in first differences when a constant is considered and when a constant and a deterministic trend is considered. We use GLS detrending estimation and modified AIC for selection lags. According to the results of Ng-Perron in levels and in first differences we can infer that all the variables of countries are I(1) except from the inflation of Japan.

Table 3: Ng-Perron Unit Root Tests in Levels when only constant is considered

Country	Sample period	Variables	MZa	MZt	MSB	MPT	Results
USA	1957M1-2014M10	i_t	-6.70500***	-1.77142***	0.26419***	3.86748***	I(1)
		π_t	-5.36236	-1.42758	0.27622	5.14967	I(1)
UK	1988M1-2014M10	i_t	-3.72835	-1.32081	0.35426	6.59805	I(1)
		π_t	-0.63425	-0.50983	1.05983	32.9200	I(1)
Canada	1957M1-2014M10	i_t	-6.14050***	-1.71919***	0.25998***	4.10191**	I(1)
		π_t	-4.46658	-1.48454	0.33237	5.50501	I(1)
France	1970M1-2014M10	i_t	-2.89496	-0.95587	0.33019	7.91869	I(1)
		π_t	-2.73567	-0.83567	0.82197	5.41013	I(1)
Germany	1991M1-2007M08	i_t	0.54193	0.54714	1.00962	64.6298	I(1)
		π_t	-0.95978	-0.44040	0.45886	14.4732	I(1)
Italy	1977M3-2014M10	i_t	0.68853	0.62076	0.90157	54.6306	I(1)
		π_t	-0.76706	-0.43418	0.56603	19.2507	I(1)
Japan	1957M1-2014M10	i_t	-2.33265	-0.85457	0.36635	9.18967	I(1)
		π_t	-10.8626**	-2.33051**	0.21454**	2.25547**	I(0)

***,**,* denote significance at the 10%, 5% and 1% significance levels. The critical values are from Ng-Perron (2001).The critical values for the Ng-Perron test for 10%, 5% and 1% are -5.7,-8.1 and -13.8 respectively for MZa, -1.62, -1.98 and -2.58 respectively for MZt , 0.27, 0.23, 0.174 respectively for MSB and 4.45, 3.17 and 1.78 respectively for MPT. The results are based on 5% signifance level.

Table 4: Ng-Perron Unit Root Tests in First Differences when only constant is considered

Country	Sample period	Variables	MZa	MZt	MSB	MPT	Results
USA	1957M1-2014M10	i_t	-16.9878*	-2.91414*	0.17154*	1.44332*	I(0)
		π_t	-10.5079**	-2.28484**	0.21744**	2.36127**	I(0)
UK	1988M1-2014M10	i_t	-62.6030*	-5.59052*	0.08930*	0.40139*	I(0)
		π_t	-125.509*	-7.92178*	0.06312*	0.19521*	I(0)
Canada	1957M1-2014M10	i_t	-106.709*	-7.30380*	0.06845*	0.23076*	I(0)
		π_t	-23.0139*	-3.38583*	0.14712*	1.08676*	I(0)
France	1970M1-2014M10	i_t	-16.8922*	-2.88430*	0.17075*	1.53355*	I(0)
		π_t	-217.960*	-10.4209*	0.04781*	0.13674*	I(0)
Germany	1991M1-2007M08	i_t	-57.6730*	-5.36996*	0.09311*	0.42482*	I(0)
		π_t	-14.1290*	-2.65564*	0.16796*	1.74300*	I(0)
Italy	1977M3-2014M10	i_t	-11.7095**	-2.37100**	0.20249**	2.28709**	I(0)
		π_t	-183.886*	-9.56134*	0.05200*	0.17226*	I(0)
Japan	1957M1-2014M10	i_t	-25.8679*	-3.59633*	0.13903*	0.94727*	I(0)
		π_t	-14.8267*	-2.66638*	0.17184*	1.77114*	I(0)

***,**,* denote significance at the 10%, 5% and 1% significance levels. The critical values are from Ng-Perron (2001).The critical values for the Ng-Perron test for 10%, 5% and 1% are -5.7,-8.1 and -13.8 respectively for MZa, -1.62, -1.98 and -2.58 respectively for MZt , 0.27, 0.23, 0.174 respectively for MSB and 4.45, 3.17 and 1.78 respectively for MPT. The results are based on 5% signifance level.

Table 5: Ng-Perron Unit Root Tests in Levels when constant and time trend are considered

Country	Sample period	Variables	MZa	MZt	MSB	MPT	Results
USA	1957M1-2014M10	i_t	-11.6710	-2.35164	0.20150	8.15682	I(1)
		π_t	-7.26144	-1.83024	0.25205	12.6985	I(1)
UK	1988M1-2014M10	i_t	-4.10286	-1.33709	0.32589	21.2162	I(1)
		π_t	-0.90721	-0.67170	0.74041	99.9870	I(1)
Canada	1957M1-2014M10	i_t	-8.19269	-1.96381	0.23970	11.3192	I(1)
		π_t	-3.74446	-1.35151	0.36094	24.0956	I(1)
France	1970M1-2014M10	i_t	-12.7049	-2.50835	0.19743	7.24209	I(1)
		π_t	-3.29911	-1.27756	0.38724	27.4825	I(1)
Germany	1991M1-2007M08	i_t	-5.82837	-1.62917	0.27952	15.5290	I(1)
		π_t	-0.70647	-0.60028	0.70025	91.6686	I(1)
Italy	1977M3-2014M10	i_t	-10.6200	-2.29466	0.21607	8.62987	I(1)
		π_t	-3.60688	-1.29788	0.35983	24.5521	I(1)
Japan	1957M1-2014M10	i_t	-16.6161***	-2.84945***	0.16981***	5.90411***	I(1)
		π_t	-19.8464**	-3.13265**	0.15784**	4.70084**	I(0)

***,**,* denote significance at the 10%, 5% and 1% significance levels. The critical values are from Ng-Perron (2001).The critical values for the Ng-Perron test for 10%, 5% and 1% are -14.2,-17.3 and -23.8 respectively for MZa, -2.62, -2.91 and -3.42 respectively for MZt , 0.185, 0.168 and 0.143 respectively for MSB and 6.67, 5.48 and 4.030 respectively for MPT. The results are based on 5% signifance level.

Table 6: Ng-Perron Unit Root Tests in First Differences when constant and time trend are considered

Country	Sample period	Variables	MZa	MZt	MSB	MPT	Results
USA	1957M1-2014M10	i_t	-18.4213**	-3.03490**	0.16475**	4.94673**	I(0)
		π_t	-21.9230**	-3.26987**	0.14915**	4.41033**	I(0)
UK	1988M1-2014M10	i_t	-43.9734*	-4.68839*	0.10662*	2.07547*	I(0)
		π_t	-125.509*	-7.92178*	0.09921*	0.06312*	I(0)
Canada	1957M1-2014M10	i_t	-266.383*	-11.5409*	0.04332*	0.34209*	I(0)
		π_t	-19.7351**	-3.13997**	0.15911**	4.62553**	I(0)
France	1970M1-2014M10	i_t	-52.2461*	-5.10874*	0.09778*	1.75578*	I(0)
		π_t	-50.4208*	-4.99601*	0.09909*	1.93304*	I(0)
Germany	1991M1-2007M08	i_t	-66.1695*	-5.75180*	0.08693*	1.37774*	I(0)
		π_t	-29.3888*	-3.81509*	0.12981*	3.20843*	I(0)
Italy	1977M3-2014M10	i_t	-32.5281*	-4.02828*	0.12384*	2.82796*	I(0)
		π_t	-29.3522*	-3.79007*	0.12912*	3.34571*	I(0)
Japan	1957M1-2014M10	i_t	-26.2436*	-3.62236*	0.13803*	3.47252*	I(0)
		π_t	-35.5664*	-4.21521*	0.11852*	2.57231*	I(0)

***,**,* denote significance at the 10%, 5% and 1% significance levels. The critical values are from Ng-Perron (2001).The critical values for the Ng-Perron test for 10%, 5% and 1% are -14.2,-17.3 and -23.8 respectively for MZa, -2.62, -2.91 and -3.42 respectively for MZt , 0.185, 0.168 and 0.143 respectively for MSB and 6.67, 5.48 and 4.030 respectively for MPT. The results are based on 5% signifance level.

The three procedures provide different inferences about the stationarity of the variables. However, the Ng-Perron procedure is more robust methd for testing the presence of unit roots in the variables, since the Ng-Perron is propelled by its superiority to Augmented Dickey Fuller (ADF), and furthermore, is built on the work by Elliott, Rothenberg, and Stock (1996) that yields substantial power gains over the standard ADF unit root test.

5. Cointegration tests

As said above, the majority if macroeconomic time series are usually non-stationary. So, if we apply traditional regression analysis such OLS on non-stationary time series then there is important possibility to have spurious regression and consequently no valid results. However, the non-stationary of macroeconomic time series has encouraged the development of various non-stationary time series analysis methods and techniques. The most famous is cointegration analysis. The concept of cointegration analysis, firstly introduced by Granger (1981) and later extended by Engle and Granger (1987), is based on the idea that if non- stationary variables are cointegrated, it means that a linear combination of them will be stationary. Cointegration, may also be seen as the long-term equilibrium or relationship between two series. So, a cointegration technique is an ideal analysis to validate the Fisher hypothesis: by ascertaining the existence of a long-term unit proportionate relationship between nominal interest rates and expected inflation, cointegration analysis can thereby establish if nominal interest rates and expected inflation are cointgrated. Two most known cointegration methods are the Engle-Granger Two-step method and Johansen Technique.

5.1 The Engle-Granger test

The Engle-Granger (1987) (EG) two-step test is the most commonly employed (single equation) approach to the analysis of cointegration in the econometrics literature. Given two variables interest $\{y_t, x_t\}$ the first stage procedure involves: a) we must make sure that all the variables are I(1) b) then we estimate the cointegrating (phenomenical) regression

$$y_t = \gamma_0 + \gamma_1 x_t + u_t \quad (8)$$

c) next we save the residuals of the cointegrating regression, $\hat{u}_t$ and d) we test if these residuals $\hat{u}_t$ are stationary I(0). If only the residuals are I(0) then there is cointegration and long-run relationship between the variables and we can continue in second step. In second step we use the residuals $\hat{u}_t$ as the equilibrium correction term where

$$\hat{u}_{t-1} = y_{t-1} + \hat{\gamma} x_{t-1} \quad (9)$$

And now we have the general model:

$$\Delta y_t = \beta_1 \Delta x_t + \beta_2 (\hat{u}_{t-1}) + u_t \quad (10)$$

and it can be estimated applying OLS.

However, this method suffers from a number of problems. In general, unit root and cointegration tests have low power in finite samples. Also, we are forced to treat the variables asymmetrically and to specify one as the dependent and the other as independent variables. And we cannot perform any hypothesis tests about the actual cointegrating relationship estimated at first step.

5.2 Johansen test

Some drawbacks of EG two-step can be dealt with by Johansen test. The Johansen Technique is based on VAR (Vector Autoregression). So, the Johansen's methodology takes its starting point in the VAR of order k given by:

$$y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3} + \ldots + \beta_k y_{t-k} + u_t \quad (11)$$

where y_t is a nx1 vector of variables that are integrated of order one I(1) and u_t is a nx1 vector of error terms.

We need to turn this VAR into VECM (Vector Error Correction Model), which can be written as:

$$\Delta y_t = \Pi y_{t-k} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \Gamma_3 \Delta y_{t-3} + \ldots + \Gamma_{k-1} \Delta y_{t-(k-1)} + u_t \quad (12)$$

where $\Pi = (\sum_{i=1}^{k} \beta_i) - \mathrm{I}_g$ and $\Gamma_i = (\sum_{j=1}^{i} \beta_j) - \mathrm{I}_g$

Π is a long run coefficient matrix since all the $\Delta y_{t-i} = 0$ and states if there is cointegration and which is long-run relationship.

The test for cointegration between the y's is calculated by looking at the rank of the Π matrix via its eigenvalues. The rank of a matrix is equal to the number of its characteristic roots (eigenvalues) that are different from zero.
The eigenvalues denoted λ_i are put in order:

$$\lambda_1 \geq \lambda_2 \geq \ldots. \geq \lambda_g$$

If the variables are not cointegrated, the rank of Π will not be significantly different from zero, so $\lambda_i = 0$ and $\ln(1-\lambda_i) = 0 \ \forall i$ .If $\lambda_i \neq 0$ then $\ln(1-\lambda_i) < 0 \ \forall i > 1$.

Johansen proposes two different likelihood ratio tests for cointegration: the trace test and maximum eigenvalue test, shown in equations 4 and 5 respectively

$$\lambda_{trace}(\mathrm{r}) = -\mathrm{T} \sum_{i=r+1}^{g} \ln(1-\hat{\lambda}_i) \quad (13)$$

$$\lambda_{\max}(\mathrm{r}, \mathrm{r}+1) = -\mathrm{T} \ln(1-\hat{\lambda}_{r+1}) \quad (14)$$

where $\hat{\lambda}_i$ denotes the estimated value for i_{th} ordered eigenvalue from the Π matrix and T is the number of observations.

The first statistic tests the null hypothesis that the number of cointegrating vectors is equal to or less than r, against an unspecified alternative. $\lambda_{trace} = 0$ when all the $\lambda_i = 0$, so it is a joint test. The further estimated λ_i are from zero, the more negative is $\ln(1-\lambda_i)$, and the greater is λ_{trace}. The second statistic tests the null that the number of cointegrating vectors is r, against the alternative of $\mathrm{r}+1$.

Johansen and Juselius (1990) provide critical values for both of these. The distribution of the test statistics is non-standard. The critical values depend on the value of $g-r$, the number of non-stationary components and whether a constant and / or trend are included in the regressions.

Also one of the most interesting aspect of the Johansen procedure is that it allows for testing restricted forms of the cointegrating vectors. You can test a hypothesis about one or more coefficients in the cointegrating relationship by viewing the hypothesis as a restriction on the Π matrix. For example, in a linear model one might want to test $\beta_1=1$, $\beta_2<0$ and $\beta_3>0$. However, if there are r cointegrating vectors, only these r linear combinations are stationary. All other linear combinations are non-stationary.

5.3 Empirical results

The previous analysis we apply three unit root tests to test the order of integration of time series and to determine which variables are I(1) and consequently in which countries can apply cointegration techniques. Based on results of Ng-Perron we will use cointegration methods for the USA, UK, Canada, France, Italy and Germany. The Japan is rejected from our analysis for cointegration test, since one variable, its inflation is stationary and to apply cointegration test all variables should be non–stationary.

At first we apply Engle and Cranger 2-step. This involves regressing one variable against the other to obtain the OLS regression residuals $\hat{e}$. We test the null hypothesis of no-cointegration against the alternative of cointegration that is based on testing for a unit root in the regression residuals $\hat{e}$ using the ADF test and critical values which correctly take into account the number of variables in the cointegrating regression.

Table 7: Engle-Granger cointegration test. ADF unit root tests on residuals.

Country	Integration Order		Dependent Variable		R^2
	‹i_t›	‹π_t›	i_t	π_t	
USA	1	1	0.5232	0.0460	0.220878
UK	1	1	0.1024	0.4281	0.036307
Canada	1	1	0.4842	0.0773	0.183725
France	1	1	0.4429	0.5722	0.364653
Italy	1	1	0.3986	0.6217	0.500056
Germany	1	1	0.1862	0.0100	0.304385

Notes: ‹x› represents the order of integration x, based on results reported in tables 1-6. All tests use a constant and trend variable. Asymptotic p-values for the Engle-Granger cointegration test are computed using the coefficients in Mackinnon 1996.

The cointregration tests are first done with the inflation rate (π_t) as the dependent variable in the cointegrating regression and then repeated with the nominal interest rate (i_t) as the dependent variable. We should be wary of a result indicating cointegration using one time-series as the dependent variable, but no cointegration when the other time-series is used as the dependent variable. We use these tests a constant and a trend variable and the number of augmenting lags is chosen using the AIC+2 rule. Asymptotic p-values for the Engle-Granger test are computed using the coefficients in Mackinnon (1996).

Table 7 shows the results from the Engle-Granger cointegration test. The results suggest that the null hypothesis of no cointegration between the inflation rate and the short –term nominal interest rate cannot be rejected (our inferences are based on the 5% level), except for Germany and perhaps for Usa.

As said above the Engle-Granger 2-steps presents some drawbacks and that's why we will use a more recent method the Johansen procedure.

Table 8:Johansen Cointegration test when we allow for deterministic trend in data with constant.

Canada (Lags interval (in first differences): 1 to 7)

Variables	Eigenvalue	Trace statistics	Max-Eigen statistics	Critical value 5%		Cointegration Hypothesis	
				Trace	Max	H_0	H_1
i_t π_t	0.062523	47.49256 (0.0000)	44.22582 (0.0000)	15.49471	14.26460	r=0	0<r≤2
	0.004758	3.266745 (0.0707)	3.266745 (0.0707)	3.841466	3.841466	r=1	1<r≤2

Trace and Max test indicate 1 cointegrating eqn(s) at the 0.05 level.

France (Lags interval (in first differences): 1 to 3)

Variables	Eigenvalue	Trace statistics	Max-Eigen statistics	Critical value 5%		Cointegration Hypothesis	
				Trace	Max	H_0	H_1
i_t π_t	0.075693	44.59721 (0.0000)	41.95325 (0.0000)	15.49471	14.26460	r=0	0<r≤2
	0.004948	2.643958 (0.1039)	2.643958 (0.1039)	3.841466	3.841466	r=1	1<r≤2

Trace and Max test indicate 1 cointegrating eqn(s) at the 0.05 level.

Germany (Lags interval (in first differences): 1 to 1)

Variables	Eigenvalue	Trace statistics	Max-Eigen statistics	Critical value 5%		Cointegration Hypothesis	
				Trace	Max	H_0	H_1
i_t π_t	0.369032	96.34571 (0.000)	90.71841 (0.0000)	15.49471	14.26460	r=0	0<r≤2
	0.028161	5.627304 (0.0177)	5.627304 (0.0177)	3.841466	3.841466	r=1	1<r≤2

Trace and Max test indicate 2 cointegrating eqn(s) at the 0.05 level.

Italy (Lags interval (in first differences): 1 to 1)

Variables	Eigenvalue	Trace statistics	Max-Eigen statistics	Critical value 5%		Cointegration Hypothesis	
				Trace	Max	H_0	H_1
i_t π_t	0.146551	72.29381 (0.0000)	71.15288 (0.0000)	15.49471	14.26460	r=0	0<r≤2
	0.002538	1.140925 (0.2855)	1.140925 (0.2855)	3.841466	3.841466	r=1	1<r≤2

Trace and Max test indicate 1 cointegrating eqn(s) at the 0.05 level.

UK (Lags interval (in first differences): 1 to 6)

Variables	Eigenvalue	Trace statistics	Max-Eigen statistics	Critical value 5%		Cointegration Hypothesis	
				Trace	Max	H_0	H_1
i_t π_t	0.096859	35.80563 (0.0000)	31.81323 (0.0000)	15.49471	14.26460	r=0	0<r≤2
	0.012081	3.816463 (0.0507)	31.98917 (0.0507)	3.841466	3.841466	r=1	1<r≤2

Trace and Max test indicate 1 cointegrating eqn(s) at the 0.05 level.

USA (Lags interval (in first differences): 1 to 11)

Variables	Eigenvalue	Trace statistics	Max-Eigen statistics	Critical value 5%		Cointegration Hypothesis	
				Trace	Max	H_0	H_1
i_t π_t	0.024539	22.80249 (0.0033)	16.91926 (0.0186)	15.49471	14.26460	r=0	0<r≤2
	0.008602	5.883222 (0.0153)	5.883222 (0.0153)	3.841466	3.841466	r=1	1<r≤2

Trace and Max test indicate 2 cointegrating eqn(s) at the 0.05 level.

Note: MacKinnon-Haug-Michelis (1999) p-value in parentheses. The VAR lag length is based on the Akaike and set to p-1. And the r denotes the number of cointegrating relationship

Table 9:Johansen Cointegration test when we allow for no deterministic trend in data with constant.

Canada (Lags interval (in first differences): 1 to 7)

Variables	Eigenvalue	Trace statistics	Max-Eigen statistics	Critical value 5%		Cointegration Hypothesis	
				Trace	Max	H_0	H_1
i_t π_t	0.062523	47.54200 (0.0000)	44.22588 (0.0000)	20.26184	15.89210	r=0	0<r≤2
	0.004829	3.316114 (0.5233)	3.316114 (0.5233)	9.164546	9.164546	r=1	1<r≤2

Trace and Max test indicate 1 cointegrating eqn(s) at the 0.05 level.

France (Lags interval (in first differences): 1 to 3)

Variables	Eigenvalue	Trace statistics	Max-Eigen statistics	Critical value 5%		Cointegration Hypothesis	
				Trace	Max	H_0	H_1
i_t π_t	0.075734	44.92637 (0.0000)	41.97675 (0.0000)	20.26184	15.89210	r=0	0<r≤2
	0.005519	2.949612 (0.5901)	2.949612 (0.5901)	9.164546	9.164546	r=1	1<r≤2

Trace and Max test indicate 1 cointegrating eqn(s) at the 0.05 level.

Germany (Lags interval (in first differences): 1 to 1)

Variables	Eigenvalue	Trace statistics	Max-Eigen statistics	Critical value 5%		Cointegration Hypothesis	
				Trace	Max	H_0	H_1
i_t π_t	0.369150	98.58469 (0.0000)	90.75547 (0.0000)	20.26184	15.89210	r=0	0<r≤2
	0.038963	7.829220 (0.0891)	7.829220 (0.0891)	9.164546	9.164546	r=1	1<r≤2

Trace and Max test indicate 1 cointegrating eqn(s) at the 0.05 level.

Italy (Lags interval (in first differences): 1 to 1)

Variables	Eigenvalue	Trace statistics	Max-Eigen statistics	Critical value 5%		Cointegration Hypothesis	
				Trace	Max	H_0	H_1
i_t π_t	0.146867	74.55559 (0.0000)	71.31920 (0.0000)	20.26184	15.89210	r=0	0<r≤2
	0.007182	3.236396 (0.5375)	3.236396 (0.5375)	9.164546	9.164546	r=1	1<r≤2

Trace and Max test indicate 1 cointegrating eqn(s) at the 0.05 level.

UK (Lags interval (in first differences): 1 to 6)

Variables	Eigenvalue	Trace statistics	Max-Eigen statistics	Critical value 5%		Cointegration Hypothesis	
				Trace	Max	H_0	H_1
i_t π_t	0.097399	37.43551 (0.0001)	32.17691 (0.0001)	20.26184	15.89210	r=0	0<r≤2
	0.016608	5.258606 (0.2563)	5.258606 (0.2563)	9.164546	9.164546	r=1	1<r≤2

Trace and Max test indicate 1 cointegrating eqn(s) at the 0.05 level.

USA (Lags interval (in first differences): 1 to 11)

Variables	Eigenvalue	Trace statistics	Max-Eigen statistics	Critical value 5%		Cointegration Hypothesis	
				Trace	Max	H_0	H_1
i_t π_t	0.024541	22.83860 (0.0216)	16.92115 (0.0344)	20.26184	15.89210	r=0	0<r≤2
	0.008652	5.917448 (0.1972)	5.917448 (0.1972)	9.164546	9.164546	r=1	1<r≤2

Trace and Max test indicate 1 cointegrating eqn(s) at the 0.05 level.

Note: MacKinnon-Haug-Michelis (1999) p-value in parentheses. The VAR lag length is based on the Akaike and set to p-1.And the r denotes the number of cointegrating relationship.

In Johansen's method, the specification of a constant and a trend (either restricted or unrestricted) is very important. The inclusion of a constant term is unambiguously supported by economic theory since Fisher hypothesizes a real interest rate represented by the constant. So, tables 8 and 9 that show the results from the cointegration test we consider an intercept but the first table we allow for deterministic trend while the second we assume no deterministic trend the data.

We use the Akaike Information Criteria (AIC) to select the lag length of the VAR(p) (8 lags for Canada VAR(8), 4 lags for France VAR(4), 2 lags for Italy and Germany VAR(2), 7 lags for UK VAR(7) and 12 lags for USA VAR(12))and set to be p-1 for the Johansen test.

The Johansen cointegration test is a serial test that we keep increasing the value of r until we no longer reject the null. The results from both tables suggest that all the countries exists a cointegrating relationship between the nominal interest rate and inflation rate except for USA and Germany if assume deterministic trend in data the test suggests that there are 2 cointegrating relations.

Since the Johansen test has detected cointegration between the two series we can infer that it exists a long-run equilibrium relationship between them, so we apply VECM in order to evaluate the short run properties of the cointegrated time series. The regression equation form for VECM is as follows:

$$\Delta i_t = a_1 + A_{11}(\mathrm{L})\Delta\pi_{t-1} + A_{12}(\mathrm{L})\Delta i_{t-1} + \gamma(\mathrm{i}_t - \alpha_0 - \alpha_1\pi_t) + \varepsilon_{1t} \quad (15)$$
$$\Delta\pi_t = a_2 + A_{21}(\mathrm{L})\Delta\pi_{t-1} + A_{22}(\mathrm{L})\Delta i_{t-1} + \delta(\mathrm{i}_t - \alpha_0 - \alpha_1\pi_t) + \varepsilon_{2t} \quad (16)$$

Table 10: Vector Error Correction Matrix (Estimated Cointegrated Vector and Error Correction Term in Johansen Estimation)

Canada **Cointegrating vector: i_{t-1}=- 0.3631 +1.6465π_{t-1}**

Error correction:	Δi_t	$\Delta\pi_t$
ECT_t	-0.016359*	0.226714*
	[-3.18555]	[5.06808]
Δi_{t-1}	0.225372	0.240708
	[5.88038]	[0.72101]
Δi_{t-2}	0.132352	0.314863
	[3.36973]	[0.92030]
Δi_{t-3}	-0.092889	-0.272093
	[-2.35653]	[-0.79245]
Δi_{t-4}	-0.035586	0.101098
	[-0.90003]	[0.29354]
Δi_{t-5}	0.038875	-0.139701
	[0.98315]	[-0.40559]
Δi_{t-6}	-0.095418	-0.282587
	[-2.42079]	[-0.82305]
Δi_{t-7}	-0.032452	0.210457
	[-0.82735]	[0.61597]

Δi_{t-8}	0.072804	-0.040577
	[1.90299]	[-0.12176]
$\Delta \pi_{t-1}$	-0.013923	-0.419939
	[-1.61014]	[-5.57509]
$\Delta \pi_{t-2}$	-0.010804	-0.352483
	[-1.27735]	[-4.78440]
$\Delta \pi_{t-3}$	-0.013230	-0.294510
	[-1.61851]	[-4.13609]
$\Delta \pi_{t-4}$	-0.007779	-0.199787
	[-1.00024]	[-2.94903]
$\Delta \pi_{t-5}$	-0.005164	-0.218310
	[-0.70499]	[-3.42146]
$\Delta \pi_{t-6}$	-0.005422	-0.168518
	[-0.81373]	[-2.90338]
$\Delta \pi_{t-7}$	-0.004641	-0.146982
	[-0.80257]	[-2.91793]
$\Delta \pi_{t-8}$	-0.001906	-0.065616
	[-0.42322]	[-1.67263]
Intercept	-0.003582	-0.006548
	[-0.19888]	[-0.04174]

France **Cointegrating vector: i_{t-1}= 0.006362 +1.343410π_{t-1}**

Error correction:	Δi_t	$\Delta \pi_t$
ECT_t	-0.016228*	0.187252*
	[-3.27275]	[4.89944]
Δi_{t-1}	0.298572	0.573964
	[6.90420]	[1.72200]
Δi_{t-2}	-0.038489	0.113739
	[-0.85935]	[0.32948]
Δi_{t-3}	0.141775	-0.343981
	[3.19282]	[-1.00507]
Δi_{t-4}	-0.037992	0.017934
	[-0.88581]	[0.05425]
$\Delta \pi_{t-1}$	-0.013453	-0.456025
	[-1.76391]	[-7.75780]
$\Delta \pi_{t-2}$	0.000814	-0.350014
	[0.10571]	[-5.89609]
$\Delta \pi_{t-3}$	-0.009375	-0.125428
	[-1.32523]	[-2.30026]
$\Delta \pi_{t-4}$	-0.005790	-0.085994
	[-0.99906]	[-1.92524]
Intercept	-0.012246	-0.013440
	[-0.67493]	[-0.09610]

Germany **Cointegrating vector:** $i_{t-1}=0.864984+1.530457\pi_{t-1}$

Error correction:	Δi_t	$\Delta\pi_t$
ECT_t	-0.009696**	0.760632*
	[-2.29338]	[8.54250]
Δi_{t-1}	0.133352	0.576473
	[1.86948]	[0.38374]
Δi_{t-2}	0.139481	0.166205
	[1.95751]	[0.11076]
$\Delta\pi_{t-1}$	-0.010661	0.044740
	[-2.05139]	[0.40876]
$\Delta\pi_{t-2}$	-0.005672	0.032645
	[-1.63680]	[0.44729]
Intercept	-0.016877	-0.013488
	[-1.30837]	[-0.04965]

Italy **Cointegrating vector:** $i_{t-1}=0.6411+1.402051\pi_{t-1}$

Error correction:	Δi_t	$\Delta\pi_t$
ECT_t	-0.023747*	0.202876*
	[-4.46040]	[5.90927]
Δi_{t-1}	-0.017925	0.211142
	[-0.38610]	[0.70527]
Δi_{t-2}	0.073169	0.540497
	[1.57564]	[1.80494]
$\Delta\pi_{t-1}$	-0.007655	-0.350676
	[-0.90050]	[-6.39729]
$\Delta\pi_{t-2}$	-0.006414	-0.171800
	[-0.86469]	[-3.59178]
Intercept	-0.032074	-0.010981
	[-1.32627]	[-0.07042]

UK **Cointegrating vector:** $i_{t-1}=-5.493+3.989861\,\pi_{t-1}$

Error correction:	Δi_t	$\Delta\pi_t$
ECT_t	0.003358***	0.208806*
	[1.79252]	[5.19806]
Δi_{t-1}	0.444703	1.828099
	[7.79471]	[1.49432]
Δi_{t-2}	-0.091978	-0.661215
	[-1.47573]	[-0.49474]
Δi_{t-3}	0.091293	-0.062676

	[1.47691]	[-0.04729]
Δi_{t-4}	0.108898	-0.297678
	[1.80891]	[-0.23060]
Δi_{t-5}	-0.030342	-0.054316
	[-0.51061]	[-0.04263]
Δi_{t-6}	0.126389	-2.243651
	[2.14651]	[-1.77701]
Δi_{t-7}	-0.134039	0.562957
	[-2.58312]	[0.50594]
$\Delta \pi_{t-1}$	0.013548	-0.126812
	[1.89817]	[-0.82856]
$\Delta \pi_{t-2}$	0.014215	-0.164358
	[2.08373]	[-1.12357]
$\Delta \pi_{t-3}$	0.013725	-0.284916
	[2.25615]	[-2.18417]
$\Delta \pi_{t-4}$	0.013387	-0.295003
	[2.50363]	[-2.57284]
$\Delta \pi_{t-5}$	0.006418	-0.347152
	[1.42183]	[-3.58666]
$\Delta \pi_{t-6}$	0.004052	0.021384
	[1.08086]	[0.26602]
$\Delta \pi_{t-7}$	0.002046	-0.009387
	[0.75544]	[-0.16163]
Intercept	-0.019238	-0.054047
	[-1.55766]	[-0.20408]

USA **Cointegrating vector:** $\mathbf{i_{t-1}=-2.081512+1.84636\,\pi_{t-1}}$

Error correction:	Δi_t	$\Delta \pi_t$
ECT_t	-0.007827***	0.130495*
	[-1.71257]	[3.75776]
Δi_{t-1}	0.367881	0.357113
	[9.45831]	[1.20830]
Δi_{t-2}	-0.187811	0.876094
	[-4.56533]	[2.80264]
Δi_{t-3}	0.062273	0.071007
	[1.48174]	[0.22235]
Δi_{t-4}	-0.097257	0.346222
	[-2.33926]	[1.09592]
Δi_{t-5}	0.160041	-0.015861
	[3.84151]	[-0.05010]
Δi_{t-6}	-0.233844	0.184063
	[-5.58432]	[0.57846]

Δi_{t-7}	-0.048088	0.147413
	[-1.14599]	[0.46232]
Δi_{t-8}	0.077719	0.763152
	[1.87509]	[2.42310]
Δi_{t-9}	0.117322	0.446550
	[2.83860]	[1.42187]
Δi_{t-10}	-0.086631	0.233253
	[-2.11318]	[0.74878]
Δi_{t-11}	0.095159	0.070939
	[2.34368]	[0.22993]
Δi_{t-12}	-0.110585	0.014806
	[-2.90513]	[0.05119]
$\Delta \pi_{t-1}$	-0.002120	-0.332952
	[-0.22979]	[-4.75051]
$\Delta \pi_{t-2}$	-0.010954	-0.353687
	[-1.19227]	[-5.06637]
$\Delta \pi_{t-3}$	-0.004534	-0.397673
	[-0.50142]	[-5.78811]
$\Delta \pi_{t-4}$	-0.019497	-0.344330
	[-2.20865]	[-5.13332]
$\Delta \pi_{t-5}$	0.000170	-0.327481
	[0.01992]	[-5.04400]
$\Delta \pi_{t-6}$	-0.004560	-0.336753
	[-0.54990]	[-5.34383]
$\Delta \pi_{t-7}$	0.004569	-0.301395
	[0.57565]	[-4.99765]
$\Delta \pi_{t-8}$	0.000338	-0.328447
	[0.04454]	[-5.70378]
$\Delta \pi_{t-9}$	0.005159	-0.334767
	[0.71943]	[-6.14394]
$\Delta \pi_{t-10}$	0.000392	-0.236713
	[0.05906]	[-4.69158]
$\Delta \pi_{t-11}$	0.003873	-0.130674
	[0.64736]	[-2.87436]
$\Delta \pi_{t-12}$	-0.002888	0.053949
	[-0.55156]	[1.35602]
Intercept	-0.002066	0.005073
	[-0.13409]	[0.04334]

Notes: [] represents test statistic and * , **, ***denotes significant at 1%, 5% and 10% signifance level respectively. Critical-values are -2.58, -1.96, and 1.64 for 1%, 5% and 10% signifance level, respectively.

Table 11: LR Test - Full Fisher Effect H_0: [1 -1] is a cointegrating vector.

	Test-stat	p-value
Canada	9.915041	0.001639
France	4.217686	0.040005
Germany	6.725396	0.009505
Italy	10.29721	0.001332
UK	19.65013	0.000009
USA	5.478630	0.019250

Notes: x^2 with one degree of freedom.

Table 10 shows the estimated cointegrating vector and the error correction model. The estimated vector therefore represents the regression estimation $i_t = a_0 + a_1\pi_t + e_t$, which follows the form of Fisher Hypothesis $i_t=r_t+\pi_t$. All countries have the correct sign (+) in the coefficient of inflation and this implies that exists a positive relationship in the long-run between nominal interest rate and inflation rate in all countries. However, the coefficient of inflation is more than one in all countries. This indicates the possible existence of a tax effect. Especially in Canada, France, Germany, Italy and USA the coefficient ranges from 1.34 to 1.84 while in UK is quite high almost 4.

If the interest income is taxed, lenders are concerned about the expected real rate of interest after taxes. But, when the expected rate of inflation increases then an equal rise in the nominal rate maintains only the before tax expected real rate of interest. This increase will not be sufficient to preserve the expected real rate of interest after taxes because part of the increase on the nominal interest income will be taxed away. The nominal rate would have to rise by more than any increase in expected inflation to keep the after- tax real rate unchanged. So, taxes on lender's incomes tend to boost the size of changes in nominal rates associated with changes in expected inflation.

In addition, we tested for the full Fisher effect by applying the restriction of β=[1 -1]. Table 11 presents the LR statistic test for to examine if there is the full Fisher Effect. The LR statistic is distributed as a x^2 variate with one degree of freedom (given by the total number of restrictions minus the number of just identifying i.e 2-1=1). This table suggests that the full Fisher effect applies only for France and USA for 1% level significance.

Table 10 also shows the error correction models. The coefficients of the error correction terms have the correct sign (- for the coefficient of Δi_t and + for the coefficient of $\Delta\pi_t$) for all the countries besides from the UK which has positive coefficient of Δi_t .The coefficients of ECM of $\Delta\pi_t$ are statistically significant at 5% level for all countries since their t-statistic is bigger than critical value of 1.96. The coefficients of Δi_t are statistically significant only for the countries Canada, France, Italy and Germany at 5% level significance while for USA and UK present statistically significance at 10% level significance because their t-statistic is bigger than critical value of 1.65. Nonetheless, we want only one variable to be statistically important and consequently for all the countries the ECM is important.

The magnitude is fairly significant, meaning that it will take a normal amount of time for the equations to return to their equilibrium after a shock occurs. In particular, for $\Delta\pi_t$ in Canada, France, Italy and UK the coefficients range from 0.187 to 0.227 and this implies that inflation on these countries moves to eliminate close to 20% of the long-run disequilibrium within one month. While in Germany the coefficient is near 0.75 and in USA is almost 0.13 and this means that the inflation moves 75% and 13% approximately of the long-run equilibrium to turn back in Germany and USA respectively. Likewise for Δi_t in Canada and France the coefficient is near 0.016, in Italy is almost 0.024, in Germany is near to 0.009, UK is close to 0.0034 and in USA is near 0.0078 and this means that the nominal interest rate moves to eliminate 1.6%, 1.6%, 2.4%, 0.9%, 0.34% and 0.7% of the long-run disequilibrium within one month in Canada, France, Italy, Germany, UK and USA respectively. The direction of movement is given by the sign of the coefficients, with respect to the cointegrating relation.

Additionally, we can observe that the coefficients of the error correction terms for inflation equations are generally very larger than the corresponding nominal interest rate equations. This means that the inflations or prices are more flexible than interest rates. And it seems not to be consistent with standard rational expectations models which assume efficient financial markets and sticky goods. The reason for this may be that the countries do not choose the interest rates as monetary policy instrument but they rather choose other monetary policy instrument like as exchange rate or they implement fiscal policy.

6. Granger-causality test and Impulse Response Function (IRF)

The Granger-causality test is a statistical hypothesis test for determining whether one time series is useful in predicting another. The basic "Granger-causality" definition is quite simple and implies that if we have a bivariate model (X, Y) like ours,

$$X_t = \sum_{i=1}^{p} A_{11,i} X_{t-i} + \sum_{i=1}^{p} A_{12,\mathrm{i}}\, \mathrm{Y}_{t-i} + \gamma(\mathrm{i}_t - \alpha_0 - \alpha_1 \pi_t) + \varepsilon_{1t} \quad (17)$$

$$Y_t = \sum_{i=1}^{p} A_{21,i}\, \mathrm{X}_{t-i} + \sum_{i=1}^{p} A_{22,\mathrm{i}}\, \mathrm{Y}_{t-i} + \gamma(\mathrm{i}_t - \alpha_0 - \alpha_1 \pi_t) + \varepsilon_{2t} \quad (18)$$

where p are the lags

then if X causes Y, the lags of X should be significant in the equation for the Y(18).In this case and not vice versa, it would be said that X 'Granger causes' Y or that there exists unidirectional causality from X to Y. On the other hand, if Y causes X, lags of Y should be significant in the equation for X (17). If both sets of lags were significant, it would be said that there was 'bi-directional causality' or 'bi-directional feedback'. Specifically, If the variance of ε_{1t} (or ε_{2t}) is reduced by the inclusion of the Y (or X) terms in the first (or second) equation, then it is said that Y (or X) Granger-(G)-causes X (or Y). In other words, Y G-causes X if the coefficients in A_{12} are jointly significantly different from zero. This can be tested by performing an F-test of the null hypothesis that all coefficient of $A_{12} = 0$, given assumptions of covariance stationarity on X and Y. Likewise, we test and if all other coefficients in A_{11}, A_{21} and A_{22} are statistical different from zero by performing an F-test.

Also, the word 'causality' for Granger-causality test really means only a correlation between the current value of one variable and the past values (lags) of other and it does not mean that movements of one variable cause movements of another.

However, the Granger-causality test may not explain us the interaction between the variable of a system completely. That's why, it is good to know the response of one variable to an impulse in another variable. So, it is often used in analysis the Impulse Response Function IRF which traces the effect of a one-time unit shock to one of the innovations on current and future values of the endogenous variables. This is derived by first expressing a VAR in a Vector MA (∞) form such as: $y_t = \alpha + \sum_{i=1}^{\infty} \Psi_i \varepsilon_{t-i}$ (19)

The IRF is:

$$y_{t+n} = \alpha + \sum_{i=1}^{\infty} \Psi_i \varepsilon_{t+n-i}$$

where the matrix $\{\Psi_n\}_{i,j} = \frac{\partial y_{i,t+n}}{\partial \varepsilon_{j,t}}$ is a vector such that row *i* column *j* of Ψ matrix dentifies the response of $y_{i,t+n}$ to a one-time impulse in $y_{j,t}$ with all other variables dated *t* or earlier held constant.

6.1 Empirical results

Using the VECM system estimated, our analysis will be extended to examine the Granger-causality test and to generate impulse response functions.

Table 12: Granger Causality test

Country	Null Hypothesis (H_0)	Wald(x^2)	Probability	Decision
Canada	i does not Granger cause π	3.371598	0.9089	Do not reject
	π does not Granger cause i	4.231434	0.8357	Do not reject
France	i does not Granger cause π	4.456843	0.3477	Do not reject
	π does not Granger cause i	10.06018**	0.0394**	Reject
Germany	i does not Granger cause π	0.229852	0.8914	Do not reject
	π does not Granger cause i	4.380224	0.1119	Do not reject
Italy	i does not Granger cause π	3.737811	0.1543	Do not reject
	π does not Granger cause i	1.018725	0.6009	Do not reject
UK	i does not Granger cause π	6.891037	0.4403	Do not reject
	π does not Granger cause i	9.586664	0.2132	Do not reject
USA	i does not Granger cause π	26.32586*	0.0096*	Reject
	π does not Granger cause i	24.83575**	0.0156**	Reject

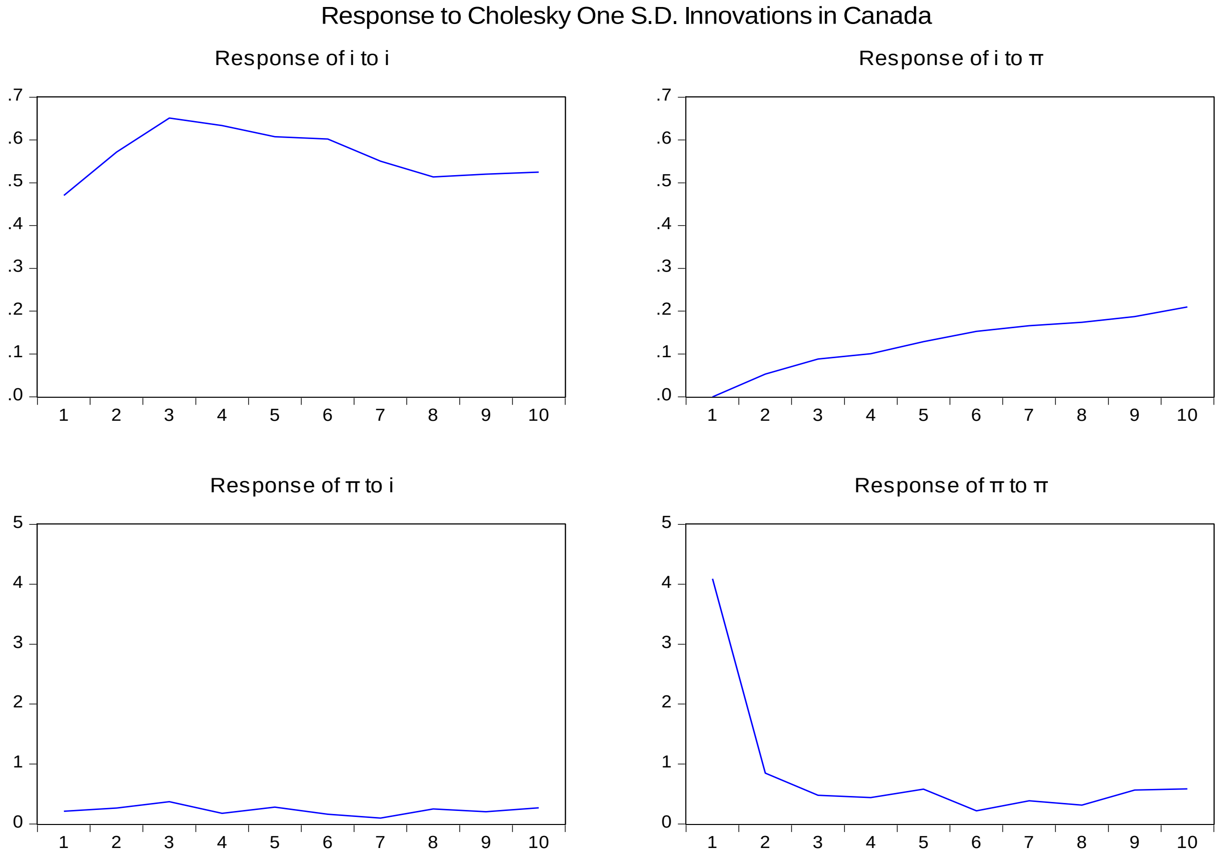
Notes: * and **denotes significant at 1% and 5% signifance level, respectively.

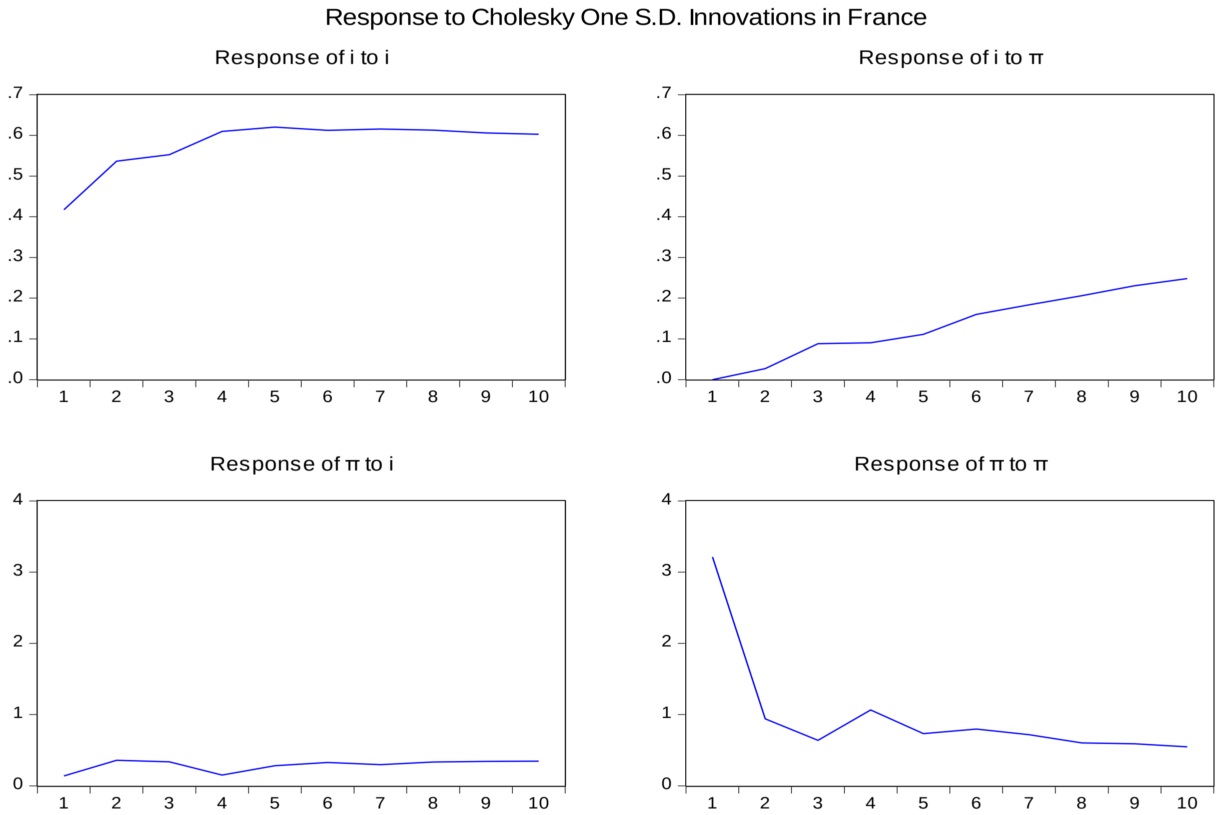
Table 12 provides the results of Granger-causality tests. The study used Chi-Square statistics and probability values to measure causality between the variables. x^2 and p-values constructed under the null hypothesis of non causality.

Table 12 suggests that there is causality between the variables only for France and USA. In France exists a unidirectional causality from inflation to nominal interest rate since the coefficient is statistical significant at 5% while in USA there is a bi-directional feedback since and both coefficient are jointly significant at 5% and 1% level of signifance respectively.

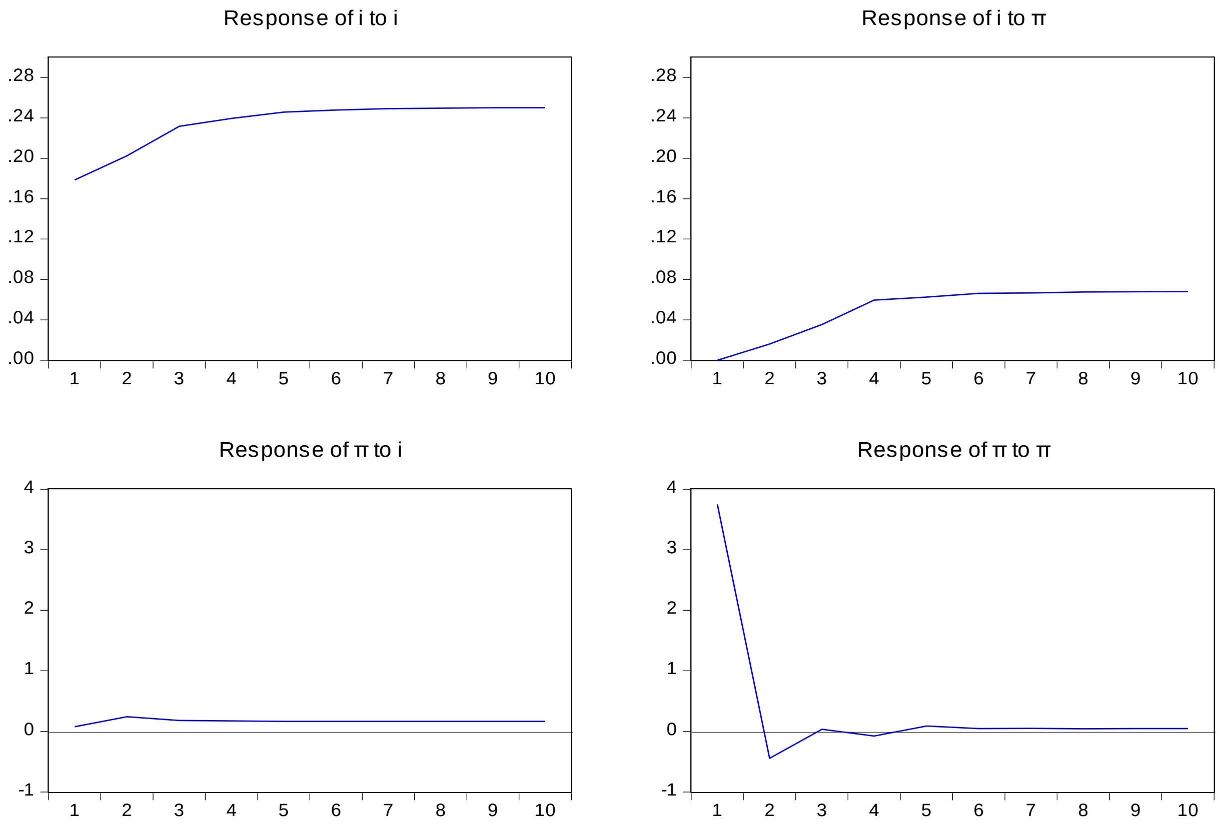
In other words, the test suggests that for Canada, Germany, Italy and UK there is not short term linkage between the nominal interest rates and inflation rates while for France and USA seems to exist a short-run relation among the two variables. However, the VECM suggests a long-run causality between nominal interest rate and inflation rate and in six countries.

Graph 3: Impulse Response Functions

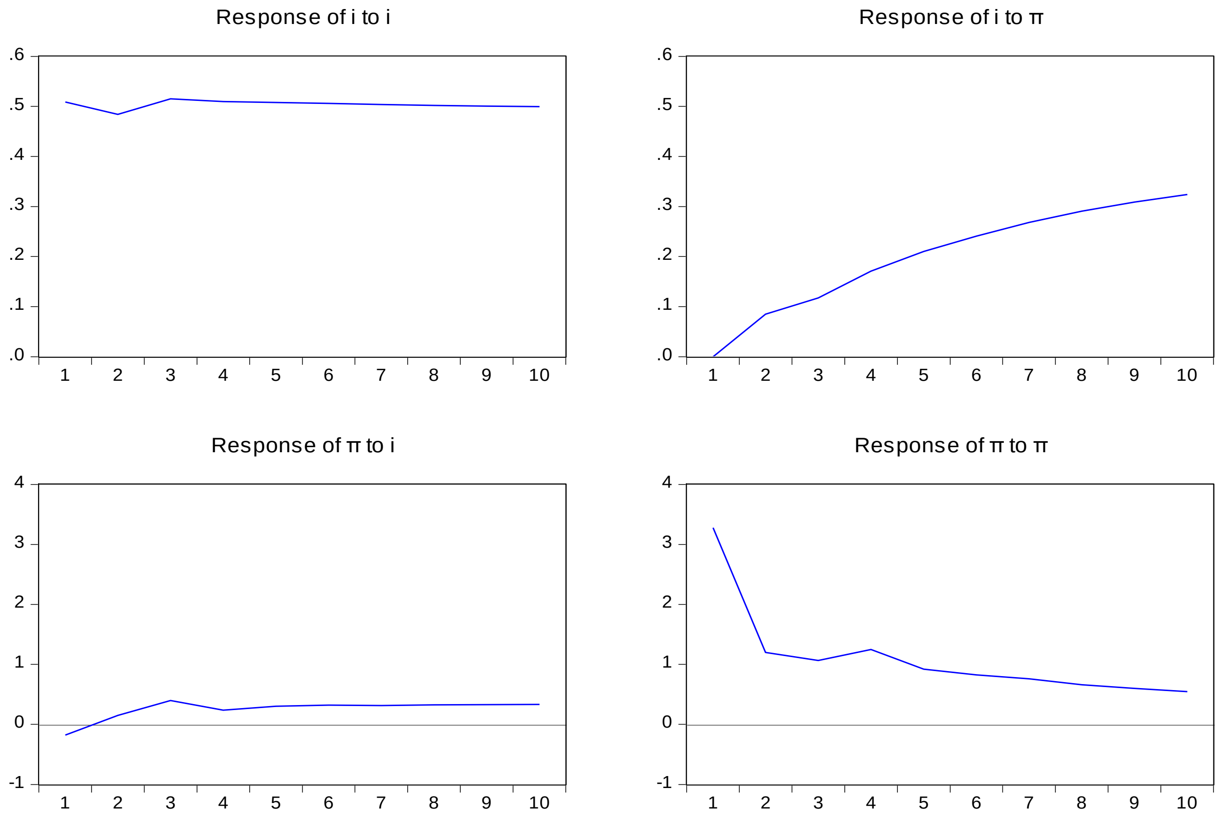


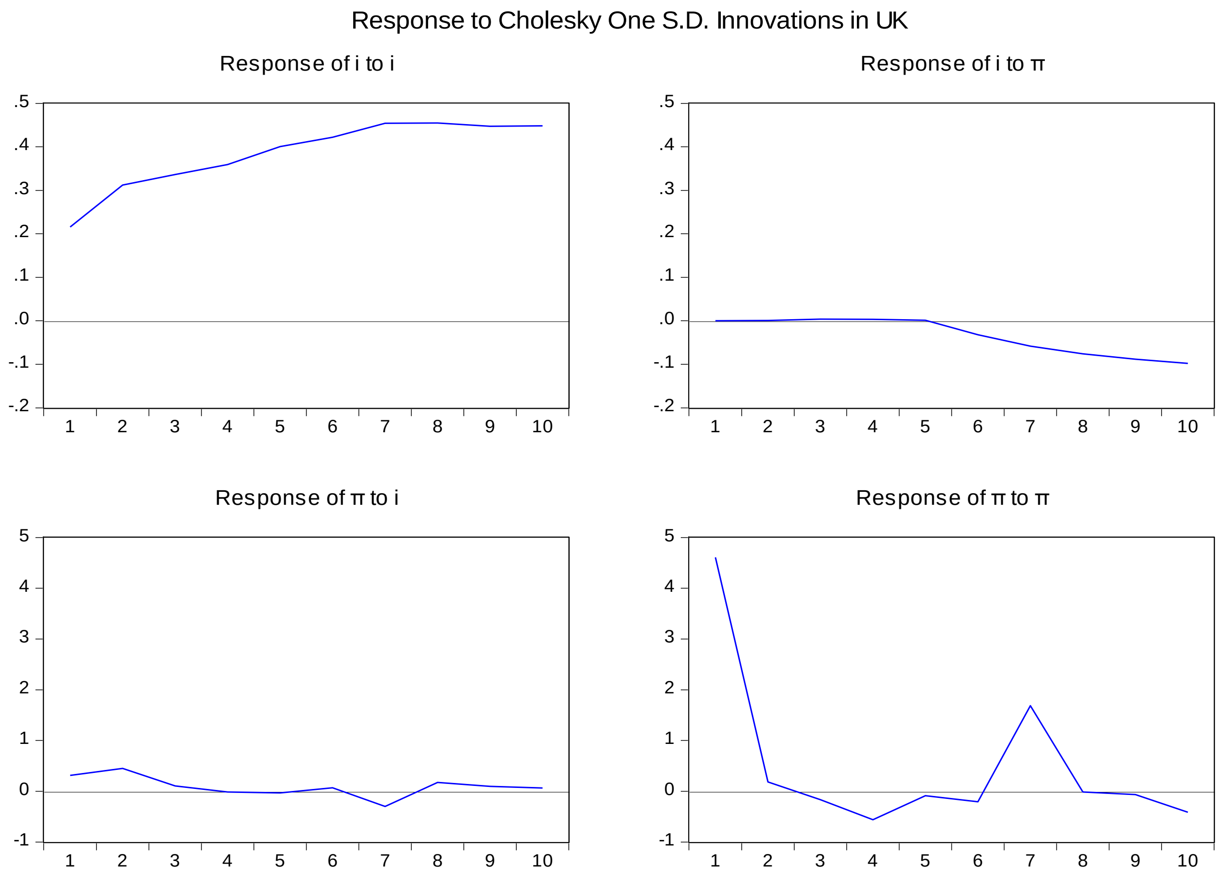


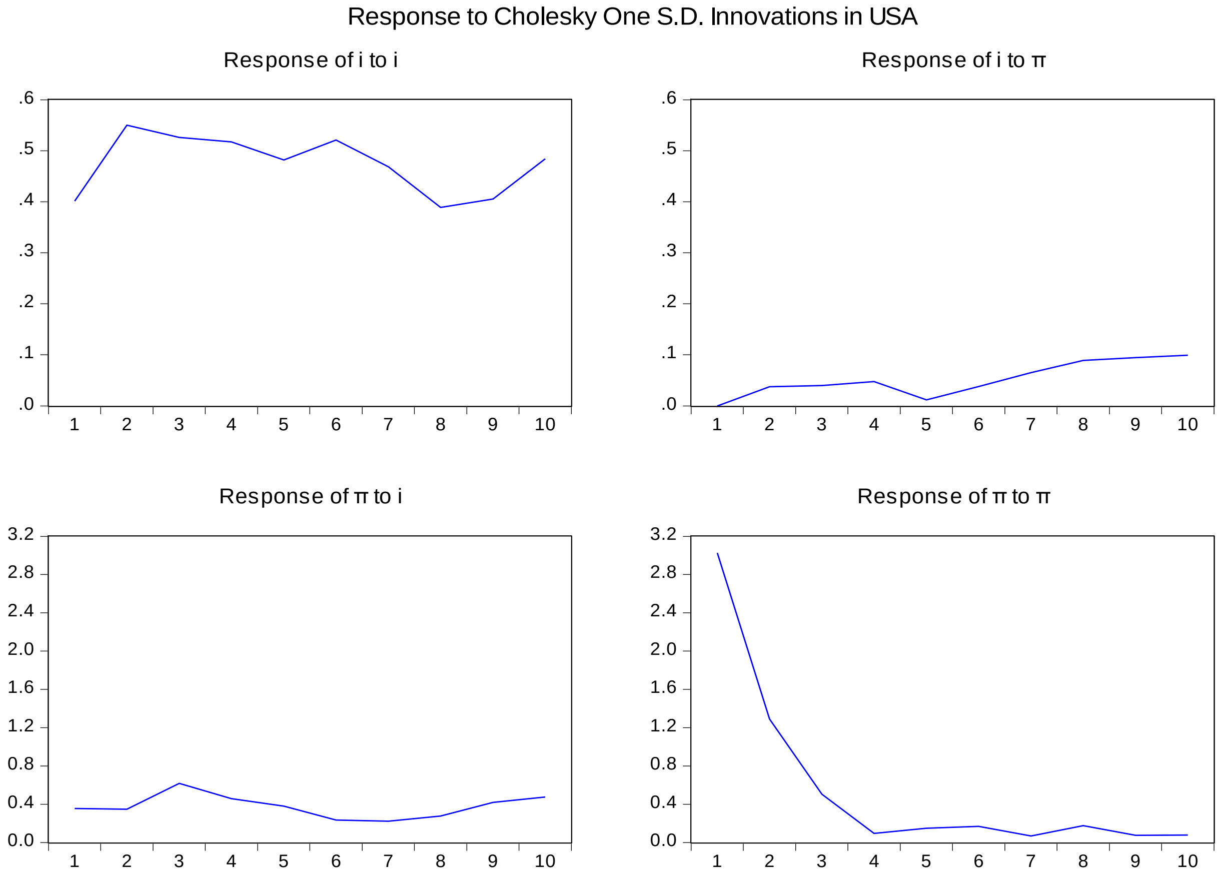
Response to Cholesky One S.D. Innovations in Germany



Response to Cholesky One S.D. Innovations in Italy




Response to Cholesky One S.D. Innovations in UK
Response of i to i
Response of i to π
Response of π to i
Response of π to π


Response to Cholesky One S.D. Innovations in USA
Response of i to i
Response of i to π
Response of π to i
Response of π to π

Graph 3 shows the impulse response functions. In Canada, France, Germany and Italy the impulse response function of i to a unit shock in π suggest that a shock increase in inflation leads to a permanent increase in nominal interest rates. Likewise, in USA a shock increase in inflation firstly leads to a temporary increase in nominal interest rates, then for a short period to a decrease and finally it leads to a permanent increase like and the four previous countries. However, in UK the impulse response function of i to a unit shock in π at first is neutral for a period and then the shock rise in inflation leads to a permanent reduction in nominal interest rates. So, in Canada, France, Italy, Germany and USA there is a positive relationship from the inflation to nominal interest rate and this agrees with the Fisher Effect but the UK there is a negative relationship.

The impulse response function of π to a unit shock in i over in all countries suggests that firstly the inflation rises after a shock in interest rates. Specifically, in Canada, France, UK and USA firstly there is positive response of inflation to a shock in interest rates and a slow adjustment to a new steady state. While in Germany and Italy exists a faster adjustment to new stable point. The finding is counter intuitive as monetary theory suggests that inflation falls for a given shock increase in nominal interest rate.

Our findings are similar with Crowder and Hoffman's (1996). They apply same methodology like us, using US data. Following the Johansen procedure, found that there is a more one to one long-run relationship between the nominal interest rate and inflation. Also applying VECM and Impulse Response Function suggested a long-run causality from the inflation rate to the nominal interest rate.

7. Summary and Conclusions

This study investigates whether the Fisher hypothesis holds in the G7 countries. The hypothesis involves a long-run one-to-one relation between the nominal interest rate and expected inflation. It has important policy implications since, if it holds, the monetary policy will have no influence on real interest rate as, in this case, any change in expected inflation will be offset by a change in the nominal interest rate, leaving the real interest rate unchanged.

Firstly, we use unit root and stationarity tests to determine the order of integration and end up that all time series are I(1) besides from the inflation of Japan. So, then we apply Engle-Granger 2-step and Johansen cointegration test to examine whether there is cointegration among of the time-series for G7 beside from Japan. We found that for these six countries there is long-run, positive and more than one to one relationship between the nominal interest rate and inflation. Evidence for the full Fisher effect applies only for France and USA. The dynamic relationship between the two variables is also examined from the error correction model derived. We find that the two ECTs are statistical significant but inflation is more flexible in restoring long-run relationship equilibrium.

Finally using the VECM applied Granger-causality test and Impulse Response Functions to investigate the interaction among the variables. Although the VECM there is a dynamic relationship among of two variables in all countries the Granger-causality test suggests feedback only for France and USA. And The IRF shows that there is positive relationship from inflation to nominal interest rate in Canada, France, Germany, Italy and USA which agrees with the theory of Fisher effect.

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