"PRICING CONSIDERATIONS OF CREDIT DEFAULT SWAPS AND CREDIT SPREAD OPTIONS"

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Abstract

One of the most daunting questions for credit derivatives is how to price them. Seminars and courses attended by specialists take place every year in order to introduce new methods and techniques for valuing credit derivatives. However, many practitioners still believe that there is no robust yet way of finding the fair value of a credit derivative. This is not to say no-one in the market has a pricing model but rather, there are several pricing models which give different values and this is the main problem of the market: the absence of a commonly accepted pricing model from all banks, institutions and specialists.

Keywords
Credit derivatives, Credit Default Swaps, Credit Spread Options, Total Return Swaps, Credit-Linked Notes

JEL Classification
G12

1. Introduction

According to financial analysts, practitioners and academicians, the two primary types of risk faced by firms engaged in financial transactions are market risk and credit risk. The former is the risk due to the movements in interest rates, exchange rates, stock prices or commodity prices and affects the firm's value. The latter is the risk due to the failure of counterparties engaged in financial transactions to make obliged payments. Credit risk is sometimes called default risk.

Even though the management of market risk is achieved by entering into offsetting or hedging transactions, the management of credit risk is not so simple issue. Bankers, managers and lenders have dealt with credit risk for years. However, the methodology used in the past is not very sophisticated and not well-suited for use in today's world of highly leveraged derivative transactions, often involving a multiplicity of parties and being determined by a potentially large number of market variables. Typical methods used in the past for controlling credit risk include: a) limiting the amount of business a party does with another party, b) requiring minimum counterparty credit ratings, c) periodically marking contracts to market, d) requiring collateral, and for some dealer firms, e) the establishment of separately capitalized derivatives's subsidiaries. While these methods undoubtedly reduce credit risk, they are not adequate to manage credit risk.

In the early 1990s, a new innovation in credit risk management appeared on the scene: credit derivatives. Credit derivatives are bilateral financial contracts that isolate specific aspects of credit risk from an underlying instrument and difuse that risk between two parties. Furthermore, credit derivatives are designed to segregate market risk from credit risk and to allow the separate trading of credit risk. As a result, credit derivatives as a new weapon in the risk management arsenal allow a more efficient allocation and pricing of credit risk.

Despite the wide use of credit derivatives in managing credit risk, a thorny and long-standing problem appeared:

how to price credit risk and consequently, credit derivatives. This issue sparkled a vivid debate among professionals of the market as well as academicians without reaching to a definite resolution.

The objective of this study is to present and discuss the various models proposed to price credit default swaps and credit spread options. Next, we apply some of these models in order to see how they work and what their drawbacks are.

This study is structured as follows: Section 2 describes the various types of credit derivatives. Section 3 presents the various models used to price credit default swaps and an application. Section 4 presents a pricing application of credit spread options, and Section 5 summarizes the conclusions.

2. Types of Credit Derivatives

Credit derivatives are swap, forward and option contracts that transfer risk and return from one counterparty to another without actually transferring the ownership of the underlying assets. However, similar products have been used in the past and include letters of credit, government export credit and mortgage guarantees, private sector bond reinsurance and spread locks. Many specialists claim that credit derivatives differ from their predecessors because they are traded separately from the underlying assets; in contrast, the earlier products were contracts between an issuer and a guarantor.

Historically, the market for credit derivatives started to operate in the early 1990s. In fact, the first credit derivative products were the credit default swaps (CDSs) on a basket of corporate names from Bankers Trust in 1993. Since then, the credit derivatives market has grown rapidly and more advanced and technical-based products were designed to meet the ever-changing needs of investors. To date, the main types of credit derivatives are credit default swaps (CDSs), total return swaps (TRSs), credit-linked notes (CLNs) and credit spread options.

2.1. Credit Default Swaps (CDSs)

The traditional or plain vanilla credit default swap (CDS) is an over-the-counter (OTC) bilateral financial contract, in which one counterparty (the protection buyer) pays a periodic (sometimes an upfront) fee, typically quoted in basis points (bps) per year (this fee is called the default swap spread or premium) paid on the notional amount in return for a contingent payment by the protection seller following a credit event with respect to a reference entity. The contract specifies a notional amount that is used to calculate the premium payments to the protection seller and to specify the principal balance of obligations used in settlement. In most cases, the CDS terminates either at maturity or when a credit event occurs.

Default protection can be purchased on a loan, a bond, sovereign risk due to cross-border commercial transactions, or even credit exposure in a cross-currency swap transaction. Credit protection can be linked to an individual credit or to a basket of credits.

Except for the traditional plain vanilla credit default swaps, there are other hybrid CDSs such as the binary credit default swaps, basket credit default swaps, contingent credit default swaps and dynamic credit default swaps. In a binary CDS, the payoff in case of default is a specific money amount. In a basket CDS, a group of reference entities are specified and there is a payoff, when the first of these reference entities defaults. In a contingent CDS, the payoff requires both a credit event and an additional trigger event. The additional trigger might be a credit event with respect to another entity, or a movement in equity prices, commodity prices or interest rates. Finally, in a dynamic CDS, the notional amount determining the payoff is linked to the mark-to-market value of a portfolio of swaps (Figure 1).

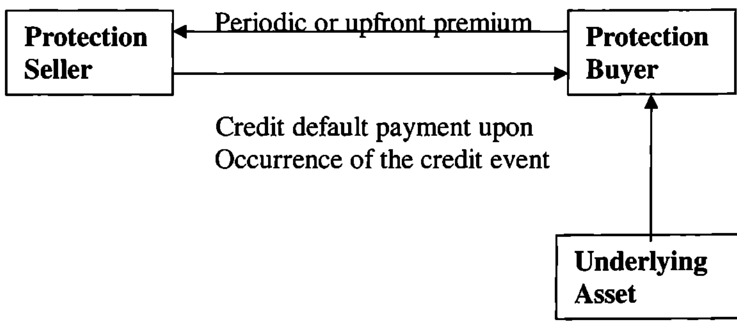

Figure 1. Credit Default Swap.

2.2. Total Return Swaps (TRSs)

A total return swap or total-rate-of-return swap (TRS) is also an over-the-counter bilateral financial contract designed to transfer credit risk between parties, but in contrast to a CDS a TRS involves the sale of not only the credit risk involved in an underlying asset, but also the market risk, caused by market value changes (such as interest rate changes). In a TRS, the total-return buyer (or protection buyer) who normally owns physically the underlying asset is paying all interest rate payments and possible positive market price changes (total economic performance) of the underlying asset. The other party involved, the total-return seller (or protection seller) is paying the LIBOR plus or minus a spread, the possible negative market price changes of the underlying and the loss occurring in case of a default. To compensate the counterparty for the credit risk taken, the LIBOR payment is applied at a lower notional amount than the coupon on the underlying asset.

There are a number of variations on the TRS. Some of them, the most known, are the following: capped total return swap, floored total return swap, fixed payout total return swap and asset switch swap. The capped TRS caps the gain at a pre-specified level. In return any spread paid by the total-return seller is reduced. If there is a loss in the floored TRS, this is limited to a pre-specified level. This potentially allows for an enhancement of the spread by the total-return seller. The fixed payout TRS is designed to avoid valuation

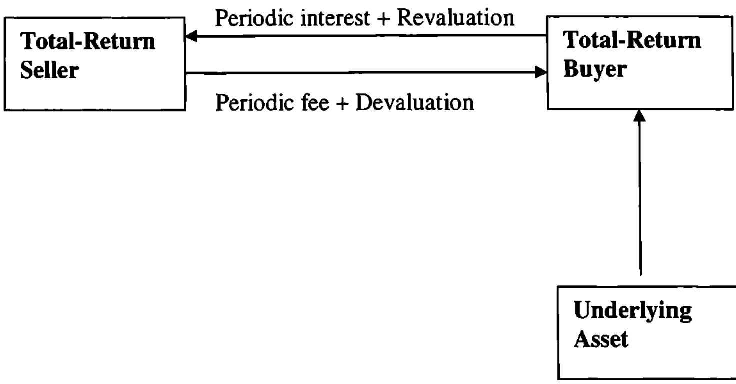

Figure 2. Total Return Swap.

problems in case of default of the underlying asset. In that case (of default), the value of the underlying asset is fixed at the outset of the transaction. Finally, regarding the asset switch swap, the total-return seller pays the total return on a different, second underlying asset. In such a case where the spread on the base swap is negative due to a market factors, the spread can be largely eliminated (Figure 2).

2.3. Credit Spread Options

Credit spread options are over-the-counter put or call options on the price of either (a) a floating rate note, bond or loan, or (b) an asset swap which consists of a package of credit-risky instruments with any payment characteristics and a corresponding derivative contract that exchanges the cash flows of that instrument for a floating rate cash flow stream. We analyze the first case since it is the most common case.

A credit spread call option gives the option buyer the right but not the obligation, to buy from the option seller an underlying credit-sensitive asset at a predetermined price (strike price) for a predetermined period of time. Likewise, a credit spread put option gives the option buyer the right but not the obligation, to sell to the option seller an underlying credit-sensitive asset at a predetermined price for a predetermined period of time. Settlement may be on a cash or physical basis.

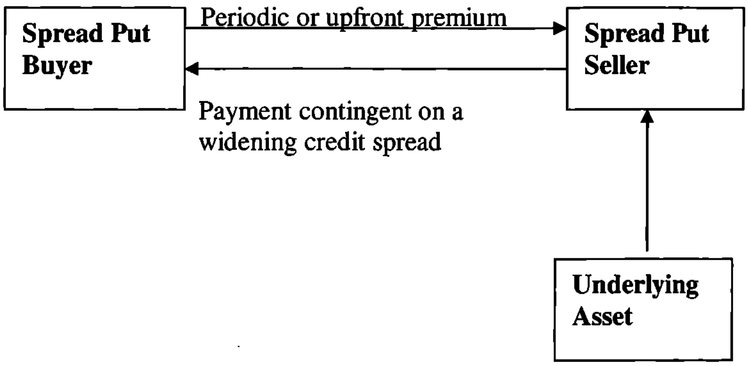

Figure 3. Credit Spread Put Option.

In general, credit spread options may be American, European or multi-European style. Unlike credit default swaps or total return swaps, in credit spread options counterparties do not have to define the specific credit events since the payout occurs regardless of the reasons for the credit spread movement. More specifically, credit spread puts usually involve the spread-put buyer paying an upfront fee to a spread-put seller in exchange for a contingent payment if the spread widens beyond a pre-agreed threshold level (Figure 3).

2.1.4 Credit-Linked Notes (CLNs)

The credit-linked note market is one of the fastest growing areas in the credit derivatives sector. Unlike credit spread swaps, CLNs are funded balance sheet securities that effectively embed default swaps within a traditional fixed income structure. In return for principal payment when the contract is made, they typically pay periodic interest plus, at maturity, the principal minus a contingent payment on credit event (Figure 4).

A variation of the CLNs is the first-to-default note (FTDN), which corresponds to a CLN bearing the credit risk of more than one reference entities. If one of the reference entities defaults during the life of the FTDN, the FTDN terminates and the protection buyer delivers the underlying asset of the defaulted debtor to the protection seller. FTDNs

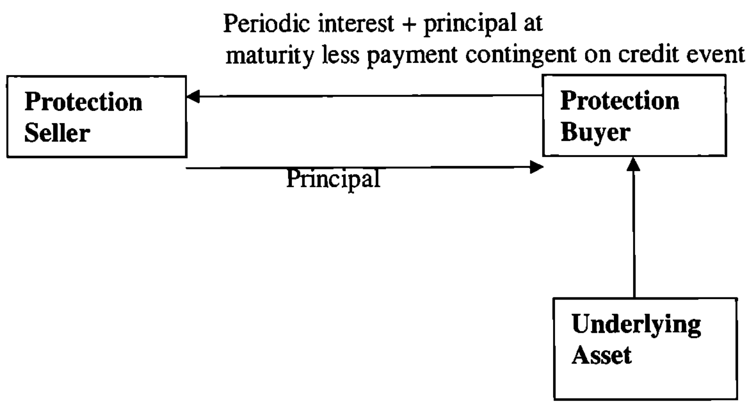

Figure 4. Credit-Linked Notes.

typically yield more than CLNs, as the higher credit risk is compensated with a higher premium.

3. Pricing of Credit Default Swaps

In order to be able to derive a price for a credit default swap, we need to know three things: a) the default probability, b) the recovery rate in case of a credit event, and c) the payment amount. These three parameters are simultaneously needed to compute the price of a credit default swap (the most popular credit derivative product). Even though the third parameter can be easily understood as either a fixed or floating cash-flow, the other two need further analysis.

There are several methods to obtain the default probability of an institution on its obligations. The following paragraphs present a brief list of some methods used today.

3.1. Rating-Based Default Probabilities Models

These models approximate the probability of default or downgrade of a given underlying asset based on its credit rating and on published data on default losses, such as the Altman dataset, or transition matrices. Credit ratings can be taken from leading rating agencies such as the Moody's

Investors Service or the Standard & Poor's. To supplement the default or downgrade data, we also need to make assumptions about what the likely rate will be in the event of a default. Some of these models use fixed recovery rates, while others rely on random stochastic rates. An example of the first approach is the model used by Jarrow, Lando and Turnbull (1994), which models the default process based on credit ratings. Particularly, the model assumes that the credit rating of a risky bond follows a Markov chain, and it employs a matrix of probabilities for the transition between credit ratings. Das and Tufano (1996) extend the above approach and develop a model that allows for stochastic recovery in the event of default. The advantage of this approach is that it does not require intensive data but, instead, it is based on aggregate statistics. Moreover, this approach is a good solution to the problem of inadequate (or missing) issuer-specific data. The major weakness of this model is that all the companies are not rated.

3.2. Credit-Spread Based Default Probabilities Models

These models use the term structure of an issuer's credit spread over default-free assets (e.g T-bond or T-bills) of similar maturity to estimate the default probability, or recovery rate in default. Once this term structure is established, it is used to estimate the default probability of the issuer for a specific term. The strength of this model is that it allows for the use of issuer-specific data. A weakness of this model is that a complete term structure of credit spreads for most issuers is unavailable. Another weakness is that the model assumes the entire spread over treasuries to be due to credit risk implicitly, even though some other factors can affect this spread (e.g tax, liquidity).

3.3. Pricing Based on Guaranteed Product Market

This model is probably the simplest approach, but it is very limited in that it requires comparison to a credit default instrument already priced in the market. For instance, two counterparties reach an agreement whereby one party is paying the other 50bps to guarantee the debt of a third party, and then any similar default products on the third party should be priced similarly. The strength of this approach is that it is easily used while the weakness is that it is only available for a limited number of names and product structures.

3.4. Replication / Cost of Funds Models

These models price credit derivatives in terms of hedging costs. The dealer decides, using probability models, default ratings etc. what portfolio of assets he requires to hedge the payments under and how much it costs the dealer to enter into hedging. The net hedging cost (plus reserves and dealer's profit and loss) is the price of the credit derivative. The advantage of this approach is that it is the most straightforward approach for cases when a hedge can be constructed. The problem with this approach is that, for many structures, a complete hedge is not available, or would be very expensive.

3.5. KMV Expected Default Frequency (EDF) Models

These models are based on the idea that a company goes bankrupt and defaults when its liabilities exceed its value as calculated by its shares. That means, using a market variable such as the share price, we can estimate approximately the probability of default. The appeal of this model is the use of the share price as an indicator of the probability of default, not least because the default indicator is a market variable, affected by the same factors that are expected to influence the probability of default, but it assumes all changes in share price are caused by changes in default probability, which is not true. Share prices are affected by a huge different number of variables and this makes difficult to extract the probability of default directly.

3.6. Application of Pricing a Credit Default Swap

As it has been already mentioned, the valuation of a credit default swap requires as parameter inputs both default probability and recovery rates for each period in order to compute both the expected value of costs of default and the standard deviation or "volatility" of value. Many investment banks (such as Morgan Stanley Dean Witter, JP Morgan, UBS) and financial institutions use the credit-spread based default probability models in order to construct the term structure of an issuer's credit spread over default-free instruments and estimate the probability of default. In our study, we use the same model in order to estimate the probability of default.

We start with a simple example in order to understand how this approach operates. Suppose that a two-year zero-coupon Treasury bond with a face value of 100 yields 6% and a similar two-year zero-coupon bond issued by a corporation yields 7% (both rates are expressed with continuous compounding). A naïve formula to estimate the present value of cost of defaults is the following:

Value of T-bond – Value of corporate bond = PV of costs of defaults

By using this formula to calculate the present value of the cost of defaults on a range of different bonds and making an assumption about the recovery rates, we are able to estimate the probability of a corporation defaulting at different future times. In our example, we assume a zero recovery rate. Therefore, the value of T-bond is equal to $100e^{-0.06*2}$ =88.69 and the value of corporate bond is equal to $100e^{-0.07*2}$ =86.93. The present value of cost of defaults is equal to 88.69 – 86.93=1.76. However, if we define the risk-neutral default probability as P^d and from the relationship between P^d and PV of cost of defaults which is given as:

$$PV = e^{-Rf*n} (P^d * 100 + (1 - P^d) R \qquad (1)$$

where,
PV is the present value of cost of default,
Rf is the risk-free rate (6% in our example),
P^d is the risk-neutral probability of default and
R is the recovery rate (zero in this example)
Therefore, from Equation (1) we have: $1.76 = e^{-0.06*2} P^d *100 + 0$, that is, P^d =1.98%

However, there are two main reasons why such calculations for deriving default probabilities from bond prices are, in practice, more complicated than this. First, the assumption of zero recovery rates is usually out of reality and second, most corporate bonds are not zero-coupon bonds. If the recovery rate is non-zero, it is necessary to make an assumption about the claim made by bondholders in the event of default. There are three assumptions regarding the claim made by bondholders until recently. The first assumption made by Jarrow and Turnbull (1995) and Hull and White (2000) assumes that the claim equals the no-default of the bond. The second assumption made by Duffie and Singleton (1997) assumes that the claim is equal to the value of the bond immediately prior to default. The third assumption made by Jarrow and Turnbull (2000) and J.P. Morgan (1999) is that the claim made in the event of a default equals the face value of the bond plus accrued interest.

The payoff from a CDS in the event of a default at time t is usually the face value of the reference obligation[1] minus its market value just after time t. Using the third assumption, the market value of the reference obligation just after default is the recovery rate times the sum of its face value and accrued interest. Therefore, the payoff from a typical CDS is:

$$N - R\,N\,(1 + A(t)) = N[1 - R\,A(t)] \tag{2}$$

where
N is the notional principal,
R is the recovery rate and
A(t) is the accrued interest on the reference obligation at time t as a percent of its face value.

The next step is to present a general analysis[2] for default probabilities with alternative assumptions about the claim amount. We assume a set of N bonds issued by a reference entity and we also assume that defaults can occur on any of the bond maturity dates. We also suppose that the maturity of the *i*th bond is t_i with $t_1<t_2<t_3\ldots<t_N$. We define:

1. Reference obligation as previously defined is the bond, which is protected again credit default and its par value that can be delivered or sold in the credit event is known as notional principal.
2. To find the risk neutral probability of default and the value of the CDS, we follow the approach introduced by Hull and White (2000)

H_j: Price of the *j*th bond today,
K_j: Price of the *j*th bond today if there were no default probability (that is, the price of a T-bond promising the same cash flow as the *j*th bond),
$F_j(t)$: Forward price of the *j*th bond for a forward contract maturing at time t assuming the bond is default-free ($t< t_j$),
$V(t)$: Present value of 1$ received at time t with certainty,
$C_j(t)$: Claim made by holders of the *j*th bond if there is a default at time t ($t<t_j$),
$R_j(t)$: Recovery rate for holders of the *j*th bond in the event of a default at time t ($t<t_j$),
A_{ij}: Present value of the loss, relative to the value the bond would have if there were no possibility of default, from a default on the *j*th bond at time t_i and
P_i: The risk-neutral probability of default at time t_i

We also assume that interest rates are deterministic and that both recovery rates and claim amounts are known with certainty. Because interest rates are assumed to be deterministic, the price at time t of the no-default value of the *j*th bond is $F_j(t)$. If there is a default at time t, the bondholder makes a recovery at rate $R_j(t)$ on a claim of $C_j(t)$. Therefore we have:

$$A_{ij} = V(t_i)\,[F_j(t_i) - R_j(t_i)\,C_j(t_i)] \tag{3}$$

There is a probability P_i of the loss A_{ij} being incurred. The total present value of the losses on the *j*th bond is :

$$K_j - H_j = \Sigma P_i\, A_{ij} \tag{4}$$

Therefore the risk-neutral probability of default is equal to:

$$P_j = (K_j - H_j - \Sigma P_i\, A_{ij})\,/A_{jj} \tag{5}$$

So far, we have assumed that interest rates are constant and both recovery rates and claim amounts are known. Our analysis continues by examining the two assumptions about the claim made, that is, first the claim amount equals the no-default value of the bond at the time of the default and second it equals the face value plus accrued interest

Table 1. Recovery Rates on Corporate Bonds

Recovery Rates on Corporate Bonds
From Moody's Investor's Service (2000).

Class	Mean (%)	Standard Deviation (%)
Senior Secured	52.31	25.15
Senior Unsecured	48.84	25.01
Senior Subordinated	33.17	20.78
Subordinated	33.17	20.78
Junior Subordinated	19.69	13.85

at the time of the default. It can be shown that, for either of these two assumptions, if default occurs, treasury interest rates and recovery rates are mutually independent, Equations (3) and (4) are still true for stochastic interest rates, uncertain recovery rates, and uncertain default probabilities providing the recovery rate is set equal to its expected value in a risk-neutral world. It is also worth mentioning that systematic risk in recovery rates are very rare, which means that expected recovery rates observed in the real world are also expected recovery rates in the risk-neutral world. This allows the expected recovery rate to be estimated from historical data. Table 1 shows some estimates on recovery rates produced by Moody's.

As might be expected, the mean recovery rate is heavily dependent on the seniority of the bond.

If we extend our analysis to situations where defaults can happen at any time and not only at discrete times as we have assumed so far, and define Q(t) as the probability of defaul[3]t between times t and t + Δt, we can derive another relationship for the present value of the loss[4] (B_{ij}) in case of defaulting at any time. This is the following:

$$B_{ij} = \int_{t-1}^{ti} V(t)[F_j(t) - RC_j(t)]dt \qquad (5)$$

3. Q(t) is known as the Default Probability Density.

4. B_{ij} can be estimated using standard procedures, such as Simpson's rule for evaluating a definite integral.

Table 2. Hypothetical Example of Bonds Issued by Reference Entity.

Bond Life (years)	Coupon (%)	Bond Yield (spread over treasury par Yield in bps)
1	7	160
2	7	170
3	7	180
4	7	190
5	7	200
10	7	220

Using the above relationship, we have:

$$Q_j = (K_j - H_j - \Sigma Q_i B_{ij}) / B_{jj} \quad (6)$$

Next, we present a numerical example in order to examine the impact of different assumptions about the claim amount. More specifically, we investigate the results for the default probability density (Q_j) using the first assumption of no-default value as stated by Jarrow and Turnbull (1995) and Hull and White (2000) which implies that $C_j(t) = F_j(t)$. Second, we use the third assumption described earlier that $C_j(t)$ equals the face value of bond *j* plus accrued interest at time t. Table 2 provides some hypothetical data on six bonds issued by a reference entity. The maturity of bonds ranges from one to ten years and the spreads of their yields over Treasury yields are similar to those of BBB-rated bonds. The coupons are paid semiannually, the Treasury zero curve is assumed to be flat at 10% (semiannually compounded) and the expected recovery rate is assumed to be 30%.

Table 3 calculates the default probability density for the two alternative assumptions about the claim amount.

The next step is to price the CDS. We assume a plain vanilla CDS with a $1 notional principal. We also assume that default events, Treasury interest rates and recovery rates are mutually independent. The claim in the event of default is the face value plus accrued interest. We define:

T: Life of credit default swap,
Q(t): Risk-neutral default probability density at time t,

Table 3. Implied Probabilities of Default.

Implied Probabilities of Default for Data in Table 2

Time (years)	Default probability density: Claim = No-default value	Default probability density: Claim = Face value + accr.int.
0 – 1	0.0220	0.0219
1 – 2	0.0245	0.0242
2 – 3	0.0269	0.0264
3 – 4	0.0292	0.0285
4 – 5	0.0315	0.0305
5 – 10	0.0295	0.0279

R: Expected recovery rate on the reference obligation in a risk-neutral world,
U(t): Present value of payments at the rate of 1$ per year on payment dates between time zero and time t,
E(t): Present value of an accrual payment at time t equal to $t - t^*$ where t^* is the payment date immediately preceding time t,
V(t): Present value of $1 received at time t,
W: Total payments per year made by credit default swap buyer,
S: Value of W that causes the CDS to have a value of zero,
Π: The risk-neutral probability of no credit event during the life of the swap, and
A(t): Accrued interest on the reference obligation at time t as a percent of face value.

The value of Π is one minus the probability that a credit event will occur by time T. It is calculated from Q(t):

$$\Pi = 1 - \int_0^T Q(t)dt \qquad (7)$$

The payments last until a credit event or until time T, whichever is sooner. If a default occurs at time t ($t < T$), the present value of the payments is W[U(t) + E(t)]. If there is no default prior to time T, the present value of the payments is WU(T). The expected present value of the payments is, therefore:

$$W\int_0^T Q(t)[U(t) + E(t)]dt + W\Pi U(T) \qquad (8)$$

Given our assumption about the claim amount, Equation 2 shows that the risk-neutral expected payoff from the CDS is:

$$1 - [1 + A(t)]R = 1 - R - A(t)R$$

The present value of the expected payoff from the CDS is:

$$\int_0^T [1 - R - A(t)R]Q(t)V(t)dt \qquad (9)$$

and the value of the CDS to the buyer is the present value of the expected payoff minus the present value of the payments made by the buyer, or equal to:

$$\int_0^T [1 - R - A(t)R]Q(t)V(t)dt - W\int_0^T Q(t)[U(t) + E(t)]dt + W\Pi U(T) \qquad (10)$$

The CDS spread S is the value of W that makes this expression zero:

$$S = \frac{\int_0^T [1 - R - A(t)R]Q(t)V(t)dt}{\int_0^T Q(t)[U(t) + E(t)]dt + \Pi U(T)} \qquad (11)$$

The variable S is referred to as the Credit Default Swap Spread or CDS spread. It is the total of the payments per year, as a percent of the notional principal, for a newly issued CDS. Considering our hypothetical data from Table 3 and the assumption of 30% recovery rate; Equation (9) gives the value of S for a five-year CDS with semiannual payments to be 1.944% annually or 0.972% in six months.

Moreover, we quote an alternative, less sophisticated and simultaneously less complex relationship between default probability and credit spread that is currently used by banks to price CDS:

$$P^d = (\text{Credit spread}) / 1 - R \qquad (12)$$

where

$$\text{Credit Spread} = \frac{\sum_i D_{i+1} - \text{Cum P}(ND_i) * (1 - R) * \text{Marg P}(Def_{I+1})}{\sum_I D_{i+1} - \text{Cum P}(ND_i)} \tag{13}$$

and

$$\frac{\sum_i D_{i+1} - \text{Cum P}(ND_{i+1})}{\sum_i D_{i+1} - \text{Cum P}(ND_i)}\text{, which is equal to one} \tag{14}$$

Credit Spread = Credit Swap Spread,

where

R = Recovery rate in the event of default,
Marg P (ND_i) = Marginal probability of default at time I,
D_{i+1}= Risk-free discount factor at time i+ 1, and
Cum P (ND_{i+1}) = Cumulative probability of default at time i + 1

The disadvantage of this formula is that cumulative probability of default in the future may not equal the cumulative probability of default today. The product of these two expressions almost certainly is not one except for very high investment grade credits. However, most of the transactions dealing with credit derivatives do not involve high investment grade credits.

4. Pricing of Credit Spread Options

To price a credit spread option, we need to know the forward credit spread and the volatility of this parameter. There are two ways[5] to estimate the forward credit spreads. The implied forward rates for both the risk-free curve and the risky curve can be calculated and the forward yield spread determined by subtracting the risk free forward rate from the

5. Das S (2001) "Credit derivatives and Credit-linked notes", *John Wiley & Sons.*

risky forward rate. Alternatively, a zero coupon spread can be determined by calculating the spot credit spread from the two zero coupon rates. The forward credit spreads can then be calculated from the zero coupon derived spot credit spread curve. The difference between the spot credit spread and the implied forward spread is that the spot credit spread is for the securities to the final maturity date, while the implied forward spread is for a slightly shorter maturity out of the forward date. Once we know these two parameters, we can use one of the three models that exist for the valuation of credit spread options. These are: a) modeling the spread itself as an asset price, b) modeling the option as an exchange option, and c) utilizing multi-(two) factor options models.

The first model is quite simple, even though it creates problems assuming that the probability that the spread will ever become negative is nil. A formula that is used to price credit spread options is the following:

$$C_t = e^{-rt}\,[(S_t - K)\ N(h) + \sigma \sqrt{}\, t\, N'(h)] \tag{15}$$

where

$$h = (1/\sigma \sqrt{t}\,)(S_t - K) \tag{16}$$

where

K = strike yield spread,
S_t = forward yield at time t,
Σ = volatility (standard deviation of yield spread),
R = risk free rate,
t = time to option maturity,
N'(h) = standard normal distribution, and

$$N(h) = 1 - N'(h)\ (b_1 z + b_2 z^2 + b_3 z^3 + b_4 z^4 + b_5 z^5) \tag{17}$$

where

z = (1/ 1 + ah)

and

$a = 0.2316419$
$b_1 = 0.319381530$
$b_2 = -0.356563782$
$b_3 = 1.781477937$
$b_4 = -1.821255978$
$b_5 = 1.330274429$

Next, we quote a numerical example in order to show how this formula operates. Suppose that the trade date is 14 September 2001 and an option expires on September 2003. The current forward spread is 90 bps and the option is struck at a spread of 95 bps. The spread volatility is 55% and the risk free rate is 5% per annum.

$$C_t = e^{-0.05*2}[(0.009 - 0.0095)\,N(-0.0006) + 0.55\sqrt{2}\,N'(0.39894)]$$

Since $N(h) = (1/\,0.55\sqrt{2})(0.009 - 0.0095) = N(-0.0006) = 0.5$

Therefore, $C_t = 84.34\%$

The equivalent pricing for a put option is:

$$P_t = e^{-rt}\,[(K - S_t)\,(1 - N(h)) + \sigma\sqrt{t}\,N'(h)] \qquad (18)$$

where

$$h = (1/\sigma\sqrt{t}\,)\,(S_t - K) \qquad (19)$$

Credit spread options can be estimated by using the exchange option pricing model under certain circumstances. Margrabe (1978) provides a valuation model for valuing an exchange option where both assets are determined in the same currency. This is:

$$S_2\,e^{-q2t}\,N(d1) - S_1^{-q1t}\,N(d2) \qquad (20)$$

where

$$d1 = [\ln(S_2/S_1) + (q1 - q2 + \sigma^2/2)\,t]\,/\,\sigma\sqrt{t} \qquad (21)$$

$$d2 = d1 - \sigma\sqrt{t} \tag{22}$$

$$\sigma = \sqrt{(\sigma 1^2 + \sigma 2^2 - 2\rho\, \sigma 1\, \sigma 2)} \tag{23}$$

where

S, S_2 = spot price of assets 1 and 2,
q1, q2 = yields of assets 1 and 2,
$\sigma 1$, $\sigma 2$ = volatility of assets 1 and 2,
ρ = correlation between asset 1 and 2, and
t = time to expiry

5. Concluding Remarks

Credit derivatives have grown rapidly since the beginning of 1990s and the potential seems, at least, huge. Current estimations evidenced a market size of $1.6 trillion by 2002 and $5 trillion by 2005. While these numbers seem tiny, relative to the size of the global credit markets and other derivative markets, credit derivatives are bound to grow due to the need for managing credit and market risk.

Credit derivatives have almost all the characteristics of other derivative instruments and they exist in the form of swaps, options, notes and forward contracts. Their payout can be linked to loans, bonds, credit spreads or credit default events and these enable them to be in the forefront of banks' risk management tools.

We examined pricing considerations of credit derivatives. However, the pricing issue of credit derivatives is not adequately developed compared to that of other derivatives. The pricing of credit derivatives is complex because it is based on a series of variables that should be known in advance and this is the source of the problem. Market data sometimes are not available or do not reflect the reality, making difficult the pricing of credit derivatives. Consequently, no commonly accepted pricing model is used by banks and financial institutions, leading to different results and discrepancies and increasing the need for more research on the specific topic.

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