UNIVERSITY OF MACEDONIA

Interdepartmental Program of Postgraduate Studies in Economics

MASTER THESIS

Interdependence between the oil and natural gas markets

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Abstract

In this master thesis we investigate, using daily returns, the existence of volatility spillovers between two commonly traded indices the NYSE Arca natural gas index (XNG) and the NYSE Arca oil index (XOI). We use semi-parametric (Cheung-Ng (1996) and Hong (2001)) and parametric techniques (BEKK-GARCH model). Our findings dictate that there is causality in variance for both indices. We expand our analysis scrutinizing the dynamics of a shock in variance using Volatility Impulse Response analysis as proposed by Hafner and Herwartz (2006). We implement our results to compose an optimal portfolio allocated between these two indices.

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1. Introduction

Baneful consequences of Black Monday of 1987 triggered a vast interest and research in the area of financial economics. In particular, research focused on shock and volatility transmission and dynamics. Furthermore, integration and globalization of financial markets brought to surface the need for a deep understanding of market interactions. There are some prominent papers to that direction (Hamao et al. (1990), King and Wadhamis (1990), Theodosiou and Lee (1993), Kim and Rogers (1995), Goodhart (1998)). Engle et al. (1990) provided evidence that these spillovers have the form of 'meteor'showers.

The existence of volatility spillovers means that the impact of a shock does not remain within the asset or the market but spreads to other assets and markets. The literature suggests that volatility is related to information flow and that information flow is reflected on prices. The study of volatility may help us understand how news is transmitted within and among markets and about the reactions of market participants.

There are two major classes of research in the field of volatility transmission. The first is cointegration analysis which describes the co-movements of the underlying variables. The second examines volatility dynamics using the GARCH approach.

In this master thesis we investigate the existence and dynamics of volatility spillovers between two major energy indices, NYSE Arca natural gas index (XNG) and NYSE Arca oil index (XOI). In statistics and econometrics, there are, in general, three alternative approaches to deal with a problem, namely the parametric, the semi-parametric and the non-parametric one. For our purposes, we have adopted a semi-parametric and a parametric procedure. We first follow the methodology proposed by Cheung and Ng (1996) and later extended by Hong (2001). The parametric analysis is based on the popular BEKK-GARCH model of Engle and Kroner (1995). We then study the dynamics of volatility spillovers applying the Volatility Impulse Response Functions introduced by Hafner and Herwartz (2006).

The remainder of the thesis is organized as follows. In section 2 we present the literature review. In section 3 we present the three alternative approaches in statistics and econometrics that a researcher can employ. In section 4 we describe the dataset, we present some preliminary results and fully describe the methodology we apply. In section 5 we present our empirical results, while in section 6 we implement our findings in portfolio management and hedging. Finally, in section 7 we present our main conclusions.

2. Literature Review

Oil and natural gas have lion's share of fuel use worldwide. From an economic point of view, these two commodities are substitutes in demand and supplements in supply because both are used in plants. It is natural that their price relationship attracts the interest of many researchers. Several papers have shown the existence of a cointegrating relationship between oil and gas prices while others investigated the existence of Granger-causality. For instance, Panagiotidis and Rutledge (2004) and Villar and Joutz (2006) provide evidence that oil and natural gas prices are cointegrated. Other authors show the existence of volatility spillovers in prices. For example, Efimova and Serletis (2014), using a GARCH model for daily crude oil, natural gas and electricity prices, show that there is unidirectional spillover from oil to natural gas, and from natural gas to electricity. Pindyck (2004), using a GARCH model and daily data, shows the existence of unidirectional spillover from oil prices to natural gas prices. Other authors investigate the relationship between oil, natural gas and other sectors of the economy. To this direction, Malik and Hassan (2007), who use daily data and a BEKK model, examine the existence of volatility transmission between financial, industrial, consumer, health, energy and technology sectors. Alternative procedures and methodologies have also been used by Tonn et al. (2010). They use wavelet analysis and argue that oil and natural gas futures prices are highly correlated.

In the 1970's and thereafter oil crisis initiated a vast interest in investigating the relation between the various macroeconomic variables. Among others, Hamilton (1983, 2003), Hutchison (1993), Lee and Ni (2002), Barsky and Killian (2004) and Killian (2008), show that oil price increases have different effects depending on the activities of the industries.

A large body of the literature examines the impact of oil price shocks on stock returns (see, among others, Jones and Kaul (1996), Huong et al. (1996), Sadorsky (1999), Ciner (2001), Basher and Sadorsky (2006)). All the aforementioned studies agree that oil price shocks have a harmful impact on stock returns.

To the best of our knowledge only Ewing et al. (2002) investigate the existence of volatility transmission between XOI and XNG employing daily data from 1/4/1996 to 29/10/1999. They use a BEKK model and find evidence of spillovers from natural gas to oil. On the other hand, they are able to find spillover effects from oil to natural gas only for a 10% confidence level.

3. Parametric, non-parametric and semi-parametric approach

Generally, in statistics and econometrics, we are interested in some certain values of a population, but in most cases it is impossible to collect data for all the population. Instead, we collect data for a sub-sample and calculate estimates for the corresponding population. We usually focus on the mean and the variance. When these values are calculated from a sample data, they are called statistics. A crude distinction between the parametric and the non-parametric method is the preliminary assumptions of each approach about the data generating process. Below we present a brief description of these approaches.

3.1 The parametric approach

The parametric estimation and inference is based upon the assumption that the density function or probability model that provides the data generation process is known. To put it more formally, let us consider a scalar random variable "*y*" and a random vector "*x*", then the joint density function is defined as following: $f(y,x)=g(y|x,\beta) * h(x|\theta)$, where β and θ are the unknown parameters.

For instance let us consider a linear regression model where the error term is normally distributed. The density function of y conditional on x in this case is $y_i|x_i \sim N(x_i'\beta,\sigma^2)$. Our goal is to fully determine the parameters that describe the population. This can be accomplished by specifying the values of the unknown yet fixed parameters. If these parameters become available, the conditional distribution of y_i is fully specified and the mean, variance and other relevant probabilities can be derived.

In econometrics the most popular class of parametric estimators is the maximum likelihood estimators. These estimators use all available information provided by the sample. For example, let us consider a sample of observations with known density but unknown parameters. In this case, if the observations are independent, their joint density consists the likelihood function $f(y_1 \ldots y_n, x_1 \ldots x_n) = \prod_{i=1}^{n} f(y_i, x_i | \beta, \theta)$. Values of the unknown parameters that maximize the likelihood function are the maximum likelihood estimators.

3.2 The non-parametric approach

A different approach is the non-parametric one where no assumptions are made for the data generating process. This approach is based solely on the data and avoids the cost of selecting a wrong parametric model. The estimators obtained with a non-parametric method have a lower rate of convergence relatively to the parametric counterpart. In econometrics, non parametric methods can be useful in two cases. First, they can be used as preliminary analysis for the selection of an appropriate parametric model or class of models. Linearity and homoscedasticity can be tested using non-parametric estimates of the conditional mean and the conditional variance. They can also be used for assessment of fit. Although the non-parametric method provides the desirable property of not making assumptions about the data generating process, it has two major drawbacks. First, it is less statistically powerful than its parametric counterpart when data are approximately normal. Second, non-parametric tests provide results that are more difficult to interpret than the results of parametric tests.

3.3 The semi-parametric approach

The third approach which stands between the two aforementioned ones is the semi parametric methodology. Generally, there is no prominent definition of semi-parametric models. A general definition is that a semi-parametric model is a model for which some parts of the data generating process would be estimated parametrically and some others non-parametrically. Obviously semi-parametric models are hybrids and as such contain advantages and disadvantages from both ingredients.

4. Data, Preliminary Analysis and Methodology

4.1 Data

We examine the existence of volatility spillovers between oil and natural gas using two major and commonly observed indices that describe the behavior of major oil and natural gas companies. We have collected daily data for the NYSE Arca Natural Gas Index (XNG) and the NYSE Arca Oil Index (XOI). XOI is a price weighted index which was created to measure the performance of the oil industry. This is achieved through changes in the prices of a cross section of major corporations involved in various activities such as the exploration, production and development of petroleum. XOI index was introduced on 27/8/1984 with an initial value of 125. XOI is a price weighted index, which is an index estimated based on stock prices. As a result, a small company trading at a high price will have a greater impact in the XOI index than a large corporation with a lower stock price. Despite this limitation, the XOI index is a widely followed index[1]. The components of the index are given in the appendix.

The NYSE Arca Natural gas index is a weighted index designed to measure the performance of highly capitalized companies which act in the natural gas industry. This index contains companies involved not only in gas exploration but in the production of pipelines for the transformation and transmission of natural gas too. The XNG index is a more recent index compared to the XOI index and this might have to do with the rise of natural gas use. XNG was established with a benchmark value of 300 on 15/10/1993. The XNG index is reevaluated in every quarter based on closing prices on the third Friday in January, April, July and October to make sure that each component continues to represent equal weights in the index[2]. The components of XNG are presented in the appendix.

Our data span from 7/1/1997 to 27/01/014 consisting of 4282 observations, all obtained from www.yahoofinance.com. During that time span, there have been many turbulent periods accompanied with high volatility i.e. terrorists attack (11th of September of 2001), wars (US invasion in Iraq), diplomatic crises, natural disasters, financial crises and oil and natural gas prices rallies.

[1] Description of XOI is obtained from http://www.amex.com/othProd/prodInf/OpPiIndMain.jsp?Product_Symbol=XOI
[2] Description of XNG is obtained from http://www.amex.com/othProd/prodInf/OpPiIndMain.jsp?Product_Symbol=XNG

For each index we construct the returns defined as the first logarithm difference i.e. $r_{i,t}$=log(i_t)-log(i_{t-1}), i=XOI, XNG. Before proceeding to the estimation, preliminary analysis indicated that the return series are stationary. Figures 1.a and 1.b present the XNG and XOI prices in logs for the whole sample.

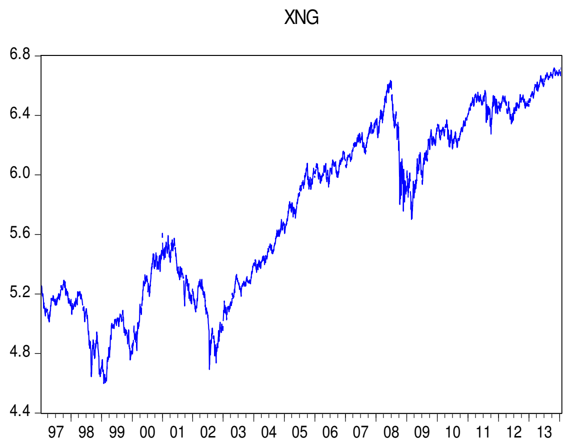

Figure 1.a. XNG prices in logs

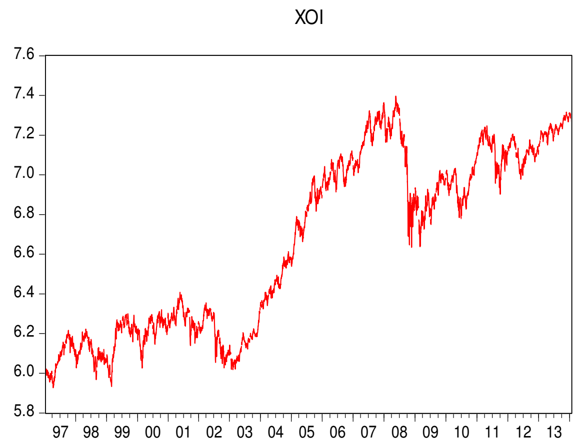

Figure 1.b. XOI prices in logs

4.2 Preliminary analysis

Following Dickey and Fuller (1981), we test for the existence of a unit root in our data. To be more specific, we use the Augmented Dickey Fuller approach to solve the problem of autocorrelation in the residuals. We use a parameterization of the form $\Delta Y_t=\alpha_0+\beta t+\gamma Y_{t-1}+\sum_{i=2}^{p}\Delta y_{t-i+1}+\varepsilon_t$ where y_t denotes the variable under examination i.e. XNG or XOI. The procedure we follow is the following. The parameter of interest is γ and the null hypothesis is H_0: γ=0, that is our data are non-stationary against the alternative that our data are stationary (γ<0)

In order to check the robustness of our results we use the KPSS test, developed by Kwitkowski et al. (1992) which tests nonstationarity against the null hypothesis of stationarity using the model $y_t=\alpha+\beta t+\gamma\sum_{t=1}^{T} z_i+\varepsilon_t$, where y_t is the variable under scrutiny, ε_t is a stationary series and z_i follows an independent and identically distributed process with zero mean and unity variance i.e. ε_t~i.i.d. (0,1). The coefficient of interest is now γ. KPPS tests the null hypothesis, H_0: γ=0 against the alternative that γ is nonzero. Table 1 presents the results of the unit root tests.

Table 1. ADF and KPPS unit root tests

t-statistics	*ADF test in level*	*ADF test in first difference*	*KPPS test in level*	*KPPS test in first differences*
XNG	-3.074	-48.474*	0.406*	0.053
p-values	(0.112)	(0.000)		
XOI	-2.408	-51.510*	0.607*	0.045
p-values	(0.3784)	(0.000)		

asterisk (*) denotes rejection of the null hypothesis for 5% significance level

The results of Table 1 clearly indicate that the series under examination become stationary after taking first differences.

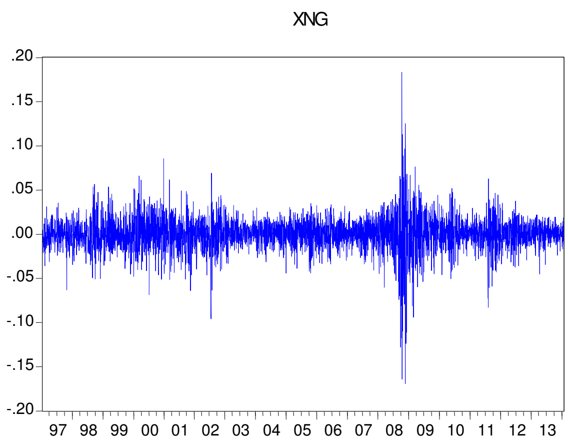

Figure 2.a. Returns of XNG series

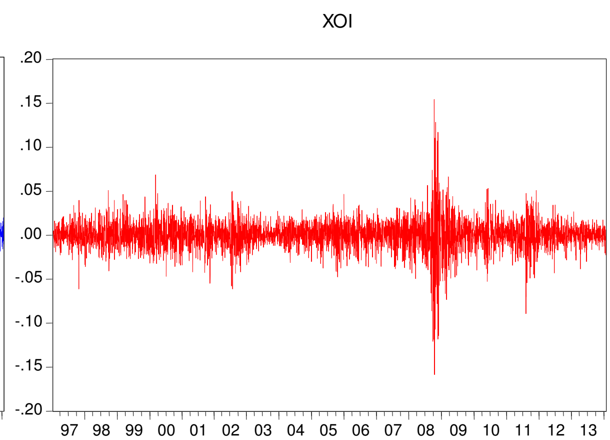

Figure 2.b. Returns of XOI series

Figures 2a and 2b display the returns of the two indices. The horizontal axis presents years from 1997 to 2014 and the vertical the returns of the corresponding series. Generally, there are some stylized facts about financial series. First, both plots reveal that periods of high (low) volatility are followed by periods of high (low) volatility. For example, a period of relatively low volatility spans from 1997 to 1998 and from 2003 to 2007 whereas a period of relatively high volatility is that from 1998 to 2002. This phenomenon is known as volatility clustering. Both series show the same volatility dynamics for the period under scrutiny. This raises the question of whether there are volatility spillovers between the two variables. This question cannot be answered by a simple visual inspection of figures 2.a and 2.b. Instead, we need to apply more sophisticated econometric techniques.

Table 2. Descriptive Statistics

Returns of	Mean	Median	Max.	Min.	Std. Deviation	Skewness	Kurtosis	Jarque-Bera	Probability	Obs.
XNG	0.0003	0.0006	0.183	-0.169	0.018	-0.433	12.418	15959	0.000	4282
XOI	0.0002	0.0006	0.154	-0.158	0.016	-0.336	12.972	17838	0.000	4282

Table 2 provides the basic descriptive statistics for the corresponding series. Both series have standard deviations of approximately the same magnitude. The descriptive statistics indicate that both series are leptokurtic and have negative kurtosis. Specifically, kurtosis for the returns of XNG (DXNG) and the returns of XOI (DXNG) is 12.418 and 12.972 respectively which is greater than 3. DXNG and DXOI skewness is -0.433 and -0.336 respectively, indicating a negative skewness for both series. Taking these facts into account, it might be appropriate to use a generalized autoregressive conditional heteroscedastisity (GARCH) parameterization. Descriptive statistics clearly reject the hypothesis of normality for both series.

The empirics of this master thesis, could be segmented into two parts. In the first part, we accommodate a semi-parametric approach. More specifically, we investigate the existence of Granger causality in mean and variance following the procedure suggested by Cheung and Ng (1996) and extended by Hong (2001). In the second part, we follow a parametric approach using a bivariate BEKK-GARCH model to examine for volatility spillovers between the two indices. In what follows we describe the two methods we employed.

4.3 The Cheung-Ng and the Hong tests for causality in mean and causality in variance

4.3.1 Causality in Mean

Let us assume two stationary and ergodic time series $\{Y_{1,t}, Y_{2,t}\}_{t=-\infty}^{\infty}$, $I_{i,t}$,i=1,2 denotes the corresponding information set for $\{Y_{i,t}\}$ available at time t. According to Granger (1980) $Y_{2,t}$ Granger-causes $Y_{1,t}$ with respect to the available information set $I_{i,t}$, i=1,2 if $Pr(Y_{1,t}|I_{1,t-1}) \neq Pr(Y_{1,t}|I_{t-1})$. Unfortunately this definition is too general to be practical. An alternative and more convenient definition is that $Y_{2,t}$ Granger-causes $Y_{1,t}$ in mean with respect to I_{t-1} if $E(Y_{1,t}|I_{1,t-1}) \neq E(Y_{1,t}|I_{t-1}) \equiv \mu_{1,t}^{0}$. Extending the definition of causality in mean, causality in variance is present if the volatility of $Y_{2,t}$ Granger-causes the volatility of $Y_{1,t}$ .This is true when $E\{(Y_{1,t}-\mu_{1,t}^{0})^2|I_{1,t-1}\} \neq E\{(Y_{1,t}-\mu_{1,t}^{0})^2|I_{t-1}\}=Var(Y_{1t})$. Formally the hypotheses are expressed as follows:

H_0: $E\{Var(Y_{1,t}|I_{t-1})\ |I_{1,t-1})\}=Var(Y_{1,t}|I_{t-1})$
H_1: $E\{Var(Y_{1,t}|I_{t-1})\ |I_{1,t-1})\} \neq Var(Y_{1,t}|I_{t-1})$

Acceptance of the null means that $Y_{2,t}$ does not Granger cause $Y_{1,t}$ in variance with respect to I_{t-1} whereas rejection of null means the opposite. That method relies on the sample cross-correlation functions of the standardized residuals and is employed in two steps. In the first step a univariate model having the following formulation is estimated for both series.

$$Y_{1,t}=\mu_{1,t}+h_{1,t}^{1/2} * \varepsilon_{1,t}$$
$$Y_{2,t}=\mu_{2,t}+ h_{2,t}^{1/2} * \varepsilon_{2,t} \qquad (1)$$

where $Y_{1,t}$, $Y_{2,t}$ are the variables under consideration and $\mu_{1,t}$, $\mu_{2,t}$ are the corresponding conditional means and $\varepsilon_{1,t}$, $\varepsilon_{2,t}$ are innovations which follow a t-distribution with v(·) degrees of freedom. In our study, we choose to model $y_{i,t}$, i=1,2 using an AR(m)-GARCH(p,q) model given in the general case by:

$$y_{t}= w_0+\lambda_1 y_{t-1}+\lambda_2 y_{t-2}+\ldots\lambda_m y_{t-m} +\varepsilon_t,\ \varepsilon_t \sim N(0,h_t^2)$$

$$h_t^2 = \alpha_0+\alpha_1\varepsilon_{t-1}^2+\alpha_2\varepsilon_{t-2}^2+\ldots+\alpha_p\varepsilon_{t-p}^2 + b_1h_{t-1}^2+ b_2h_{t-2}^2+\ldots+ b_qh_{t-q}^2 \qquad (2)$$

we use the Schwarz Information Criterion (SIC) to select the lag order. Afterwards, we take the standardized residuals from each model, denoted by $\hat{\varepsilon}_{1,t}$ and $\hat{\varepsilon}_{2,t}$.

Then, the sample cross-correlations of the standardized residuals are used to test for causality in mean.The sample cross-correlation function of $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$, $\hat{\rho}_{1,2}(k)$, is computed using the following formulas:

$$\hat{\rho}_{1,2}(k) = \frac{\hat{C}_{1,2}(k)}{\sqrt{\hat{C}_{1,1}(0) * \hat{C}_{2,2}(0)}}, \tag{3}$$

where

$$\hat{C}_{1,2}(k) = \frac{1}{T}\sum_{t=k+1}^{T}\left[\left(\hat{\varepsilon}_{1,t} - \bar{\varepsilon}_{1,t}\right)\left(\hat{\varepsilon}_{2,t-k} - \bar{\varepsilon}_{2,t-k}\right)\right] \quad k \geq 0 \tag{4}$$

$$\hat{C}_{1,2}(k) = \frac{1}{T}\sum_{t=-k+1}^{T}\left[\left(\hat{\varepsilon}_{1,t+k} - \bar{\varepsilon}_{1,t}\right)\left(\hat{\varepsilon}_{2,t} - \bar{\varepsilon}_{2,t-k}\right)\right] \quad k < 0 \tag{5}$$

where T is the sample size, $\bar{\varepsilon}_{1,t}$ is the sample mean of $\varepsilon_{1,t}$ and $\bar{\varepsilon}_{2,t}$ is the sample mean of $\varepsilon_{2,t}$. Cheung and Ng show that under certain assumptions

$$\sqrt{T}(\hat{\rho}_{1,2}(k_1) \dots \hat{\rho}_{1,2}(k_m)) \longrightarrow N(0,I) \tag{6}$$

Cheung and Ng proposed the following statistic for testing the existence of a causal relation in mean.

$$S = T * \sum_{k=j}^{M} \hat{\rho}_{1,2}(k) \tag{7}$$

which can be shown that asymptotically follows a chi-square distribution with M-j+1 degrees of freedom. Setting j equal to one, S can be used for testing if Y_2 Granger causes Y_1. The null hypothesis is that there is no Granger causality from Y_2 to Y_1. we can test the hypothesis of Granger causality from Y_1 to Y_2 by modifying the previous statistic,

$$S = T * \sum_{k=-M}^{-1} \hat{\rho}_{1,2}(k) \tag{8}$$

Finally,

$$S = T * \sum_{k=-M}^{-M} \hat{\rho}_{1,2}(k) \tag{9}$$

can be used for testing bidirectional causality in mean.

When the sample size T is small Cheung and Ng (1995) propose a modification of S. That is,

$S_m = T\sum_{i=1}^{k} w_i\, \hat{r}_{1,2}(i)^2$, where w_i =T/(T-i) or w_i =(T+2)/(T-i) (10)

The Hong's statistic is

$$Q_1 = \frac{T * \sum_{j=1}^{T-1} k^2\left(\frac{j}{M}\right) * \hat{\rho}_{1,2}^2(j) - C_{1T}(k)}{\sqrt{2 * D_{1T}(k)}} \qquad (11)$$

where $C_{1T}(k) = \sum_{j=1}^{T-1}\left(1 - \frac{j}{T}\right) * k^2\left(\frac{j}{M}\right)$ (12)

$$D_{1T}(k) = \sum_{j=1}^{T-1}\left(1 - \frac{j}{T}\right) * \left(1 - \frac{j+1}{T}\right) * k^4\left(\frac{j}{M}\right) \qquad (13)$$

and k(j/M) is a weighting function. Asymptotically Q_1 follows the standard normal distribution, In addition to Q_1, test statistics for other causality hypotheses can be immediately obtained.

When Hong calculates the cross covariance of the standardized residuals he sets $\bar{\varepsilon}_{1,t}$ and $\bar{\varepsilon}_{2,t}$ equal to their theoretical value which is zero [$E(\varepsilon_{i,t})=0$] . There are various functions that can be used as weighting functions such as the truncated, Bartlett, Daniell, Parzen, quadratic-spectral (QS), and Tuckey-Hanning kernels.

The null hypothesis is that of no Granger causality. Hong's statistic is one-sided test and upper-tailed critical values should be used.

4.3.2 Causality in variance

The previous statistics can also be used to test for causality in variance. In this case we don't use the standardized residuals for the estimation of autocorrelation function but the squared standardized residuals.

The S-statistic is asymptotically robust to distributional assumptions. Cheung and Ng (1995) point out that this method has some advantages compared to other alternative methods for testing causality in variance. Compared to a multivariate method, the Cross Correlation Function (CCF) method is superior since it does not need simultaneous model estimation of the variables such as the multivariate GARCH models. Under this perspective, this method is useful when many variables are examined. Moreover, it's a very easy to implement method. It is important to note that the S-statistic gives equal weights to the sample cross-correlations of the standardized residuals. As mentioned earlier, one of the most prominent stylized facts of financial time series is volatility clustering. In congruence with volatility clustering, cross-correlations are found to decay to zero as the lag order increases. Under this consideration, equal weighting of $\widehat{\rho}_{1,2}(k_i)$ in the estimation of the statistic can make S inefficient. Therefore, it is more reasonable to give more weight to small lags and less weight to large lags. On that basis, Hong (2001) proposed an alternative statistic for testing for Granger causality-in-mean that puts diminishing weights on the sample autocorrelations.

We use these standardized residuals and we estimate the cross-correlation functions in the same manner. Hong uses the theoretical value of $\bar{\varepsilon}_{1,t}$, $\bar{\varepsilon}_{2,t,}$ which is unity, when he calculates the cross covariance of the squared standardized residuals. Again, the null hypothesis suggests no Granger causality in variance.

It is important to note that before testing for causality in variance, causality in mean must be filtered out. Otherwise, the tests suffer from severe size distortions, especially when causality in mean is strong. Therefore, it is crucial to select a correct specification in the conditional mean before proceeding in testing for causality in variance. Finally, it is also important to have a correct specification in the conditional variance, since the asymptotic results about the behavior of the statistics assume that the conditional variance is correctly specified.

4.4 The BEKK model

Engle (1982) proposed a method for jointly modeling conditional mean and conditional variance depending on the available information set denoted by I_{t-1}.

$\varepsilon_t|I_{t-1}\sim N(0,h_t^2)$

$h_t^2 = \alpha_0+\alpha_1\varepsilon_{t-1}^2+\alpha_2\varepsilon_{t-2}^2+...+\alpha_p\varepsilon_{t-p}^2$. This is an ARCH(p) model.

The ARCH(p) was extended by Bollerslev (1986) allowing the conditional variance to depend on past values of itself. In that way, the above model becomes:

$h_t^2 = \alpha_0+\alpha_1\varepsilon_{t-1}^2+\alpha_2\varepsilon_{t-2}^2+\ldots+\alpha_p\varepsilon_{t-p}^2 + b_1h_{t-1}^2+ b_2h_{t-2}^2+\ldots+ b_qh_{t-q}^2$ and this is the Generalized ARCH process GARCH(p,q).

In order to examine the existence of volatility spillovers between two series we must employ a multivariate GARCH model. There are two commonly used parameterizations for the multivariate GARCH models. The first one is that introduced by Bollerslev et al. (1988), known as the VECH model. Extending the univariate case, Engle and Kroner (1993) propose the multivariate version of a GARCH process. For this model, the variance covariance matrix of the n-dimensional zero mean random variables depend on the available information set. Let us suppose that $\varepsilon_t|I_{t-1} \sim N(0, H_t)$. Expressed alternatively $\varepsilon_t=H_t^{1/2}Z_t$, where $Z_t\sim i.i.d(0,I_n)$. Then the conditional variance equals:

$H_t=C_0+A_1\varepsilon_{t-1}+\ldots+A_q\varepsilon_{t-p}+G_1h_{t-1}+\ldots+G_ph_{t-q}$, where C_0 is a nx1 vector of parameters and A,G nxn parameter matrices. The equivalent in matrix notation is:

$H_t =[C_0 \quad A_1 \ldots A_p \quad G_1 \ldots G_q]\begin{bmatrix} 1 \\ \varepsilon_{t-1} \\ . \\ . \\ . \\ \varepsilon_{t-p} \\ h_{t-1} \\ . \\ . \\ . \\ h_{t-q} \end{bmatrix}$ this is the VECH parameterization. Finally,

$$VECH(H_t)= \omega+\sum_{k=1}^{q} A_k VECH(\varepsilon_{t-k}\varepsilon_{t-k}^{'})+\sum_{k=1}^{p} B_k VECH(H_{t-k}), \qquad (14)$$

where ω is m(m+1)/2 vector column with positive elements. A and B m(m+1)/2 rows and columns. VECH is the operator that stacks the lower triangular part of a matrix into a column vector.

For a bivariate variable, the VECH parameterization of a GARCH(1,1) model is:

$$H_t=\begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix}=\begin{bmatrix} c_{01} \\ c_{02} \\ c_{03} \end{bmatrix}+\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix}+\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}\begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{13,t-1} \end{bmatrix}$$

Clearly, for the bivariate GARCH(1,1) case a total of 21 coefficients need to be estimated. Also, during the estimation all parameters must be constrained to be positive to ensure that a positive semi-definite covariance matrix will be obtained.

Obviously the VECH parameterization implies some constraints. In order to solve the problems implied by the VECH, Engle and Kroner (1995) propose an alternative presentation, the so-called Baba-Engle-Kraft-Kroner or BEKK-GARCH model: $H_t=\Omega_0\Omega_0^{'} + \sum_{k=1}^{K}\sum_{i=1}^{q} A_{ki}^{'}\,\varepsilon_{t-i}\varepsilon_{t-i}^{'}A_{ki} + \sum_{k=1}^{K}\sum_{i=1}^{p} B_{ki}^{'}\,H_{t-i}B_{ki}$ (15)

Ω is an mxm lower triangular matrix and A,B are nxn parameter matrices. Usually, K=p=q=1 yielding $H_t=\Omega\Omega'+A'\varepsilon_{t-1}\ \varepsilon_{t-1}'$ A+$B'H_{t-1}$B. The BEKK model for the bivariate case GARCH(1,1) is given by:

$$H_t=\begin{bmatrix}\omega_{11} & 0\\ \omega_{21} & \omega_{22}\end{bmatrix}\begin{bmatrix}\omega_{11} & 0\\ \omega_{21} & \omega_{22}\end{bmatrix}'$$

$$\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}'\begin{bmatrix}\varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1}\\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2\end{bmatrix}\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}+\begin{bmatrix}b_{11} & b_{21}\\ b_{12} & b_{22}\end{bmatrix}'$$

$$\begin{bmatrix}h_{11,t-1} & h_{12,t-1}\\ h_{21,t-1} & h_{22,t-1}\end{bmatrix}\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix} \quad (16)$$

Clearly, the BEKK model needs less parameters to be estimated relatively to VECH. Engle and Kroner provide the necessary conditions for covariance stationarity for the two versions of the model. For the VECH version of the model if H_t is positive definite then the covariance stationarity condition holds if and only if all the eigenvalues of $\sum_{i=1}^{q} A_i$+$\sum_{i=1}^{p} G_i$ are less than unity in modulus. For the BEKK version of the model, the condition that should hold in order to be covariance stationary is that all the eigenvalues of $\sum_{i=1}^{q}\sum_{k=1}^{K}(A_{ik}\otimes A_{ik})$+$\sum_{i=1}^{p}\sum_{k=1}^{K}(G_{ik}\otimes G_{ik})$ are less than one in modulus. The estimation of the bivariate GARCH-BEKK is accomplished under the assumption that innovations follow a t-distribution with v(·) degrees of freedom. This assumption generates more efficient estimates for conditional errors than the normal distribution. We maximize the following likelihood function:

$L(\theta)=\sum_{i=1}^{T}\ln(l_t\theta)$ with (17)

$L_t=\frac{\Gamma(\frac{(T+v)}{2})}{\Gamma\left(\frac{v}{2}\right)[\pi(v-2)]^{T/2}}|H_t|^{-1/2}$ x $[1+\frac{1}{v-2}E_t^{'}H_t^{-1}E_t]^{-(T+v)/2}$, (18)

where (v) stands for the degrees of freedom and Γ(·) is the gamma function. Maximization of the likelihood function is achieved using the BHHH (1974) algorithm. The initial values for the BHHH algorithm is obtained by the corresponding values of the univariate GARCH models. Diagonal elements of the matrices were obtained by univariate GARCH models while off diagonal were set equal to zero.

4.5 Volatility Impulse Response Functions

We follow the methodology proposed by Hafner and Herwartz (2006) which regards a shock as being created by the data generating process. As mentioned earlier in section 4.4, Engle and Kroner (1995) show that under certain conditions every BEKK model can be expressed into an equivalent VECH representation

$$VECH(H_t)=Q+R*VECH(E_{t-1}*E_{t-1}^{'})+P*VECH(H_{t-1}) \tag{19}$$

where for the bivariate case Q is a 3x1 matrix of constants and R, P are matrices of coefficients. VECH is the operator that stacks the lower triangular elements of a matrix into a vector. The Q, R and P matrices are connected to the BEKK model through the following relationships

$$Q=\begin{bmatrix} \omega_{11}^2 \\ \omega_{11}\omega_{21} \\ \omega_{21}^2+\omega_{22}^2 \end{bmatrix}, \tag{20}$$

$$R=\begin{bmatrix} a_{11}^2 & 2a_{11}a_{12} & a_{12}^2 \\ a_{11}a_{21} & a_{11}a_{22}+a_{12}a_{21} & a_{22}a_{12} \\ a_{21}^2 & 2a_{21}a_{22} & a_{22}^2 \end{bmatrix} \textit{and} \tag{21}$$

$$P=\begin{bmatrix} b_{11}^2 & 2b_{11}b_{12} & b_{12}^2 \\ b_{11}b_{21} & b_{11}b_{22}+b_{12}b_{21} & b_{22}b_{12} \\ b_{21}^2 & 2b_{21}b_{22} & b_{22}^2 \end{bmatrix} \tag{22}$$

Following Hafner and Herwartz (2006), in order to calculate the Variance Impulse Response Function (VIRF) a BEKK model has to be expressed into the equivalent VECH representation. For the estimation of volatility impulse response function we consider that at time t=0 a shock occurs without knowing if that shock is ‘good’ or ‘bad’ (as indicated by the sign) or how big the shock is (as shown by the size). In nonlinear models (like the GARCH model), impulse responses are not symmetric to the shock in contrast to traditional impulse response functions. It must be highlighted that the conditional covariance matrix

H_t is a function of the initial shock and the initial covariance matrix and the innovations. From that point of view, Hafner and Herwartz (2006) define the VIRF as the difference between the expectation of the variance conditional on the initial shock and the available information set and the expectation of the variance conditional on the available information set i.e. $V_t(\xi_0)=E[VECH(H_t)|Z_0,I_{t-1}]-E[VECH(H_t)|I_{t-1}]$. In that formulation if we consider a bivariate GARCH model, as in our case, the first and third element of V_t ($v_{1,t}$ and $v_{3,t}$ respectively) capture to the reaction of the conditional variance of first and second variables respectively whereas $v_{2,t}$ represents the effect on the covariance. As noted by Panopoulou and Pantelidis (2003) an indicator of the persistence of a shock can be obtained by calculating the eigenvalues of the R+P matrix. The closer the eigenvalues are to unity the more persistent the impact of a shock would be. In the case of an eigenvalue that is greater than unity the VIRF would be explosive.

It is important to note that VIRFs depend on the initial variance H_0. This initial volatility can be either the volatility state the time the shock occurred or any other time chosen arbitrarily by the researcher depending on the desired analysis. VIRFs also depend on the unexpected returns vector when the shock occurs. As a result, the asymmetric response of volatility on negative and positive shocks can be easily accommodated. Negative shocks i.e. unexpected returns in one market can result in a different volatility profile than positive shocks ceteris paribus.

For a better understanding of our empirical results we elaborate on three separate VIRFs cases of interest, the case of no volatility transmission, the case of one way volatility transmission and the case of bidirectional transmission. As a measure of the decay of persistence of the volatility shocks we employ the half-life of a volatility shock defined as the time required for the impact of the shock to reduce to half of its initial value.

Let the 3x1 matrix Ψ be $\Psi=[\psi_{i,1}]:=vech(H_0^{1/2}Z_0Z_0'H_0^{1/2})-vech(H_0)$ where i=1,2,3. Clearly the elements of Ψ are functions of the elements of the initial state H_0 and the elements of the shock Z_0.

Case 1: Diagonal BEKK model ($\alpha_{12}=\alpha_{21}=b_{12}=b_{21}=0$). This is the case where R and P and consequently R+P are diagonal matrices. It is not a difficult task to show that:

$v_{1,1}=\alpha_{11}^2\psi_{1,1}$ *and* $v_{1,t} = (\alpha_{11}^2+b_{11}^2)^{t-1}v_{1,1}$ *, for t>1*

$v_{2.1}=\alpha_{11}\alpha_{22}\psi_{2,1}$ *and* $v_{2,t}=(\alpha_{11}\alpha_{22}+b_{11}b_{22})^{t-1}v_{2,1}$ *for t>1*

$v_{3,1}= \alpha_{22}^2\psi_{3,1}$ *and* $v_{3,t}=(a_{22}^2+b_{22}^2)^{t-1}v_{3,t}$ *for t>1.*

Therefore, in this special case there are no volatility spillovers given that $v_{1,t}$ and $v_{3,t}$ depend solely on their own history. It must be noted that in this case of a diagonal BEKK model, the half life of a volatility shock does not depend either on the initial shock, $Z_{0,}$ or on the initial state H_0.

Case 2: $\alpha_{12}=b_{12}=0$, while $\alpha_{21}\neq0$ and/or $b_{21}\neq0$. This is when both R and P and consequently R+P are lower triangular matrices.

$v_{1,1}=a_{11}^2\psi_{1,1}$ *and* $v_{1,t}=(a_{11}^2+b_{11}^2)^{t-1}v_{1,t}$ *for t>1*

$v_{2,1}=\alpha_{11}\alpha_{21}\psi_{2,1}$ *and* $v_{2,t}=f(v_{1,1},v_{2,1})$ *for t>1*

$v_{3,1}=a_{21}^2\psi_{1,1}+2\alpha_{21}\alpha_{22}\psi_{2,1}+\alpha_{22}^2\psi_{3,1}$ and $v_{3,t}=g(v_{1,1} , v_{2,1} , v_{3,1})$ for t>1,

where f is a function of $v_{1,1}$, $v_{2,1}$ $\alpha_{i,j}$ and $b_{i,j}$, j=1,2 and g is a function of $v_{1,1}$, $v_{2,1}$, $v_{3,1}$, $\alpha_{i,j}$ and $b_{i,j}$, i,j=1,2. In this particular case, there is one way volatility transmission from the first to the second variable of the model. Let us assume a shock in the conditional variance of the second variable of the system. Then, it is transmitted to the first variable through the coefficients of $h_{11,t-1}$, $\varepsilon_{1,t-1}^2$ and that of the covariance $h_{12,t-1}$. Volatility spillover still exist even in the case $\alpha_{21}=0$ or $b_{21}=0$ through the coefficients of $\varepsilon_{1,t-1}$, $\varepsilon_{2,t-1}$. Consequently, the effect of the shock on the conditional variance of the first variable of the system does not depend on the behavior of the second variable of the system. We should note that even if $\alpha_{21}=0$ or $b_{21}=0$, there are still volatility spillovers from the first to the second variable of the system. Finally, in this particular case, the half life of a volatility shock in $h_{11,t}$ is independent of the initial shock, Z_0, and the initial variance H_0, while the half life of a volatility shock in $h_{22,t}$ and $h_{12,t}$ depends on both the initial shock, Z_0, and the baseline state H_0.

Case 3: $\alpha_{12}\neq0$ and/or $b_{12}\neq 0$, while $\alpha_{21}\neq0$ and/or $b_{21}\neq 0$. In this general case, it is not a difficult task to confirm that volatility spillovers exist between the variables of the system.

As expected, the half-life of a volatility shock in $h_{11,t}$ and $h_{22,t}$ depends on both the initial shock, Z_0 and the initial variance H_0.

5. EMPIRICAL RESULTS

5.1 Cheung-Ng and Hong tests for causality in mean and causality in variance

We now examine whether there exists causality in mean between our series using the methodology proposed by Cheung and Ng as described in section 4.3.

As mentioned before, it is important to select an appropriate model for the estimation of the conditional mean. We made our selections using the Schwartz information criterion which gives a more parsimonious parameterization of the model. Thus, we have selected an AR(1)-GARCH(1,1) for both series. The estimation results are presented in Table 3.α.

Table 3.a. XNG AR(1)-GARCH(1,1) estimation

Mean Equation	*Variable*	*Coefficient*	*Standard error*
	w_0	$0.006x10^{-1}$	$0.002x10^{-1}$
	λ_1	0.066	$0.158x10^{-1}$
Variance Equation	α_0	$0.293x10^{-4}$	
	α_1	0.073	0.009
	b_1	0.918	0.011

Table 3.b. XOI AR(1)-GARCH(1,1) estimation

Mean Equation	*Variable*	*Coefficient*	*Standard error*
	w_0	$0.006x10^{-1}$	$0.001x10^{-1}$
	λ_1	*-0.008*	*0.015*
Variance Equation	α_0	$0.310x10^{-4}$	
	α_1	*0.075*	*0.008*
	b_1	*0.913*	*0.009*

It is clear that in case of XNG the sum of α and β is 0.991 and in the case of XOI 0.988. This fact could be an indicator of an I-GARCH process. Our next task is to use the standardized residuals of the above equations. Then we estimate the sample autocorrelation functions as described by equations (3)-(5). For robustness, we choose to estimate the autocorrelation function for 3, 5 and 10 lags. The cross correlations of the standardized residuals are reported in Table 4.a.

Table 4.a. Cross-correlations of standardized residuals

lag	From DXOI to DXNG	From DXNG to DXOI
1	-0.0048	0.0359
2	-0.0108	-0.0094
3	-0.0150	-0.0215
4	-0.0214	-0.0219
5	-0.0091	-0.0176
6	-0.0111	-0.0117
7	-0.0095	-0.0137
8	-0.0106	0.0112
9	0.0295	0.0240
10	0.0166	0.0176

We also calculated the squared standardized residuals and the cross correlations of the squared standardized residuals in order to test whether there exist any Granger causality in variance. Table 4.b presents the cross correlations of the squared standardized residuals.

Table 4.b. cross-correlations of squared standardized residuals

lag	From DXOI to DXNG	From DXNG to DXOI
1	0.0224	0.0107
2	0.0444	0.0486
3	0.0001	0.0021
4	-0.0037	0.0068
5	0.0048	0.0388
6	-0.0009	-0.0059
7	0.0134	-0.0013
8	0.0223	0.0265
9	0.0017	0.0039
10	0.0246	-0.0007

For the estimation of the Hong statistic the selection of the weighting function doesn't affect the results. As a weighting function we use the Bartlett kernel.

Table 5.a. p-values of the Cheung-Ng statistic for causality-in-mean and causality-in-variance

	MEAN		VARIANCE	
Number of lags, M	H_0: No Granger causality from XOI to XNG	H_0: No Granger causality from XNG to XOI	H_0: No Granger causality from XOI to XNG	H_0: No Granger causality from XNG to XOI
3	0.6684	0.0485	0.0142	0.0139
5	0.5668	0.0457	0.0566	0.0040
10	0.4231	0.0750	0.0926	0.0249

Table 5.b. p-values of Hong statistic testing causality in mean and variance

HONG statistic				
	MEAN		VARIANCE	
Number of lags, M	H_0: No Granger causality from XOI to XNG	H_0: No Granger causality from XNG to XOI	H_0: No Granger causality from XOI to XNG	H_0: No Granger causality from XNG to XOI
3	0.7593	0.0014	0.0194	0.0112
5	0.7518	0.0035	0.0120	0.0047
10	0.7100	0.0100	0.0050	0.0004

The results of Table 5.a and Table 5.b clearly indicate that there is no causality-in-mean from the oil index to the natural gas index. This result holds using both statistics. All p-values are greater than the critical values for all conventional levels. Examining the existence of causality in variance, we find that it is significant for M=3 at a 5% significance level and for M=5, 10 at a 10% significance level. When we employ the Hong statistic causality-in-variance is accepted for all values of M.

When we examine the existence of mean causality from XNG to XOI using the Cheung and Ng statistic, we find that there is causality in mean for M=3 and M=5 for a 5% significance level and M=10 for a 10% significance level. These results also hold using the Hong statistic. In the examination of volatility spillover, it seems that there is also volatility transmission from gas to oil. To encapsulate the results, there is Granger causality from XNG to XOI and bidirectional volatility spillover.

Results of Table 5.a and 5.b indicate that there is Granger-causality in mean from XNG to XOI. Literature suggests a cointegrating relationship between oil and natural gas prices with oil prices driving natural gas prices. Our empirical finding can be attributed to the fact that we examine the relationship between indices and to the “shale revolution”. XNG index is composed not only with companies involved to gas exploration but also to natural gas infrastructures (pipelines, rail, processing, storage etc). During the 2000’s there was a rise of natural gas use, productivity and consequently infrastructures. This fact in combination with the "shale revolution" (usage of new more advanced and sophisticated methods such as hydraulic fracturing and horizontal drilling, allowed to drill in areas where was unprofitable before) increased investments in natural gas infrastructures.

Tables 5.a and 5.b indicate the existence of bidirectional volatility spillovers under the existence of causality in mean from natural gas to oil. It has been shown by Pantelidis and Pittis (2004) that when causality in variance is examined under the existence of neglected causality in mean, the statistic suffers from severe size distortions.

In order to verify our results, we proceed by filtering out the causality in mean. We accomplish that by inserting lags of XNG into the specification of XOI. The model selection is based on the standard Box-Jenkins techniques. We estimate a specification of the form:

$XOI_t=c+\alpha_1 XOI_{t-1}+\beta_1 XNG_{t-1}+\beta_2 XNG_{t-2}+\varepsilon_t$, $\varepsilon_t \sim N(0,h_t^2)$

$h_t^2=\gamma+a_1\varepsilon_{t-1}^2+b_1 h_{t-1}^2$

Table 6. Estimation of XOI equation inserting lags of XNG

Mean Equation	Variable	Coefficient	Standard error
	c	$0.0617\text{x}10^{-2}$	$0.0175\text{x}10^{-2}$
	α_1	-0.0810	0.0273
	β_1	0.0783	0.0243
	β_2	-0.0272	0.0150
Variance Equation	γ	$0.0310\text{x}10^{-4}$	
	a_1	0.0754	0.0083
	b_1	0.9131	0.0099

Having inserted lags of XNG into the XOI equation, we calculate again the cross-correlation function of the residuals of the two indices.

Table 7.a. cross-correlation of standardized residuals after filtering out causality in mean

lag	From DXOI to DXNG	From DXNG to DXOI
1	-0.0050	0.0091
2	-0.0112	0.0082
3	-0.0140	-0.0178
4	-0.0224	-0.0219
5	-0.0088	-0.0167
6	-0.0111	-0.0133
7	-0.0096	-0.0134
8	-0.0106	0.0110
9	0.0287	0.0245
10	0.0155	0.0166

Table 7.b. cross-correlation of squared standardized residuals after filtering out causality in mean

lag	From DXOI to DXNG	From DXNG to DXOI
1	0.0245	0.0066
2	0.0436	0.0511
3	0.0004	0.0036
4	-0.0040	0.0067
5	0.0044	0.0394
6	0.0008	-0.0061
7	0.0146	0.0005
8	0.0213	0.0251
9	0.0005	0.0041
10	0.0251	-0.0013

We repeat the procedure to obtain the standardized and squared standardized residuals. We then calculate the Cheung and Ng and Hong statistics again. The results are presented in Table 8.a.

Table 8.a. p-values of the Cheung-Ng for testing causality in mean and variance after filtering out causality in mean

	MEAN		VARIANCE	
Number of lags, M	H_0: No Granger causality from XOI to XNG	H_0: No Granger causality from XNG to XOI	H_0: No Granger causality from XOI to XNG	H_0: No Granger causality from XNG to XOI
3	0.6868	0.5724	0.0134	0.0097
5	0.5551	0.3861	0.0542	0.0025
10	0.4475	0.3575	0.0884	0.0197

Table 8.b. p-values of the Hong statistic after filtering out causality in mean

	MEAN		VARIANCE	
Number of lags, M	H_0: No Granger causality from XOI to XNG	H_0: No Granger causality from XNG to XOI	H_0: No Granger causality from XOI to XNG	H_0: No Granger causality from XNG to XOI
3	0.7556	0.7139	0.0107	0.1173
5	0.7501	0.7000	0.0071	0.0027
10	0.7116	0.5984	0.0043	0.0014×10^{-1}

Our results have not changed much relatively to our previous results. Obviously, we have filtered out the causality in mean as confirmed by the third column of Tables 8.a and 8.b. All p-values are greater than all conventional levels. What is important is that our initial results of bidirectional volatility transmission between the two series hold after removing the causality in mean. Specifically, in Table 8.a the fourth column reveals that there is volatility transmission from XOI to XNG when M=3 for a 5% significance level and for a 10% significance level when M=5 and 10. Moreover, there is Granger causality in variance from XNG to XOI for all conventional levels. Table 8.b displays no Granger causality in variance from both directions for M=3 whereas when M=5 and 10 bidirectional causality is present.

5.2 BEKK-GARCH estimation

Following Panopoulou and Pantelidis (2009), we use a bivariate VAR(1)-GARCH(1,1) model for our analysis. Assuming that $Y_t=(y_1,y_2)'$ is the vector representing the returns of XOI and XNG respectively. The adoption of a VAR-GARCH model captures the causality in mean between the two series. The model can be described in matrix form as following:

$Y_t=C+M*Y_{t-1}+E_t$

where Y_t is the vector of series under scrutiny, C is a 2x1 vector of constants, M is a 2x2 matrix of parameters and E_t is the 2x1 vector of zero-mean time-varying error terms. In other words, our model for the conditional mean is as follow:

$y_{1,t}=c_1+\mu_{11}y_{1,t-1}+\mu_{12}y_{2,t-1}+\varepsilon_{1,t}$

$y_{2,t}=c_2+\mu_{21}y_{2,t-1}+\mu_{22}y_{2,t-1}+\varepsilon_{2,t}$

We further assume that the conditional heteroscedasticity H_t of E_t follows the BEKK parameterization as proposed by Engle and Kroner (1995)

$E_t=H_t^{1/2}*Z_t$

$H_t=\Omega*\Omega'+A*E_{t-1}*E_{t-1}'*A'+B*H_{t-1}*B'$, where Ω is a lower triangular matrix of constants and A,B are 2x2 matrices of coefficients.

$Z_t=(z_{1,t},z_{2,t})'\sim i.i.d\left(\begin{bmatrix}0\\0\end{bmatrix},\ \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right)$

For the calculation of volatility spillovers, we focus on matrices A and B. Matrix A demonstrates the extent to which conditional variances are affected by previous period error terms. Matrix B indicates the effect of previous period conditional variance and covariance on current conditional variance. Clearly our interest focuses on the values of the matrices A and B. The off-diagonal elements of matrix A are indicative of what Engle et al. (1988) call 'meteor' showers, that is volatility spillovers among the markets. That term is borrowed from astronomy and is used to show the interdependencies between markets. Diagonal elements of matrix B capture what Engle et al. (1988) call 'heat waves', which is the conditional variance of one index depends solely upon the past shocks in this index. It is a term borrowed from meteorology to indicate that the conditional variance shows a process like a heat wave, so that a hot day in New York is likely to be followed by another hot day in New York but not typically by a hot day in Tokyo. The sum of the diagonal elements of the matrices A and B provides a crude measure of the persistence of conditional volatility of each index.

An illustration of the conditional variance in the bivariate GARCH(1,1) is as follows:

$h_{11,t}=\omega_{11}^2+\alpha_{11}^2\varepsilon_{1,t-1}^2+2a_{11}a_{12}\varepsilon_{1,t-1}\varepsilon_{2,t-1}+a_{12}^2\varepsilon_{2,t-1}^2+b_{11}h_{11,t-1}+2b_{11}b_{22}h_{12,t-1}+$
$b_{12}^2h_{22,t-1}$

$h_{22,t}=\omega_{21}^2+\omega_{22}^2+\alpha_{21}^2\varepsilon_{1,t-1}^2+2\alpha_{21}\alpha_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1}+a_{22}^2\varepsilon_{2,t-1}^2+b_{21}^2h_{11,t-1}+$
$+2b_{21}b_{22}h_{21,t-1}+b_{22}^2h_{22,t-1}$

$h_{12,t}=\omega_{11}\omega_{21}+\alpha_{11}\alpha_{21}\varepsilon_{1,t-1}^2+(\alpha_{11}\alpha_{22}+\alpha_{12}\alpha_{21})\varepsilon_{1,t-1}\varepsilon_{2,t-1}+\alpha_{12}\alpha_{22}\varepsilon_{2,t-1}^2+b_{11}b_{21}h_{11,t-1}$
$+(b_{11}b_{22}+b_{12}b_{21})h_{12,t-1}+b_{12}b_{22}h_{22,t-1}$

We continue our analysis employing the bivariate BEKK-GARCH(1,1) model we described earlier. In this context we model both the mean and volatility dynamics. However, our interest focuses on the variance dynamics between the two indices. We firstly estimate an unrestricted version of the model i.e. without setting any constraints on the parameters. The results are reported in Table 9.

Table 9. Estimation of unrestricted BEKK-GARCH

	Coefficient	Std. Error
ω_{11}	0,0259x10^{-2}	0,0394x10^{-1}
α_{11}	0,2158	0,0268
α_{12}	0.0690	0.0333
b_{11}	0.9751	0.0078
b_{12}	-0.0231	0.0110
ω_{21}	-0.0848x10^{-2}	0.0170
ω_{22}	0.0010	0.0152
α_{21}	0.0714	0.0259
α_{22}	0.2120	0.0302
b_{21}	-0.0135	0.0092
b_{22}	0.9647	0.0094

Table 10. Estimation of restricted BEKK-GARCH

	Coefficient	Std. Error
ω_{11}	0.0938×10^{-5}	0.9324
α_{11}	0.1979	0.0268
α_{12}	0.0911	0.0309
b_{11}	0.9835	0.0083
b_{12}	-0.0326	0.0108
ω_{22}	0.0019	0.0003
α_{21}	0.0690	0.0287
α_{22}	0.2154	0.0330
b_{22}	0.9513	0.0066

Obviously some estimated coefficients are statistically insignificant. Taking into consideration that some of the estimated coefficients of our model may obscure the impulse response analysis, we remove them from the model one by one.

To be more specific, we re-estimate the model dropping out the most statistically insignificant coefficient. We stop when all estimated parameters are statistically significant. The unrestricted and restricted estimates are presented in Tables 9 and 10 respectively. The results indicate that the conditional variance of the XNG index is directly affected by its own past conditional volatility and by past conditional volatility of XOI. Higher past volatility of XNG and XOI lead to higher current volatility as illustrated by the coefficients of $h_{11,t-1}$ and $h_{22,t-1}$. Furthermore, the coefficient of covariance between XNG and XOI is statistically significant implying indirect volatility spillover through the covariance term (h_{12}) from XOI to XNG. In addition, our results suggest that the conditional variance of XNG is affected by shocks originated in the oil sector and the natural gas sector. Moreover, it seems that the conditional variance of XNG is indirectly affected by shocks in the oil sector as shown by the coefficient of $\varepsilon_{1,t-1}^2 \varepsilon_{2,t-1}^2$.

The behavior of the XOI conditional volatility is similar to that of XNG. The current conditional variance of XOI directly depends on the conditional variance of XOI at time t-1 as indicated by the coefficient of $h_{22,t-1}$. The major difference between the behavior of conditional volatility of XOI and XNG is the absence of direct dependence of $h_{22,t}$ on $h_{11,t-1}$. On the other hand, the conditional volatility of XOI is affected by shocks originated in both natural gas and oil sectors as indicated by the statistically significant coefficients of ε_1,ε_2. At the same time, $h_{22,t}$ is indirectly affected by shocks in the natural gas sector as indicated by the coefficient of $\varepsilon_{1,t-1}^2\varepsilon_{2,t-1}^2$.

Similarly to the univariate models, the persistence of volatility is obtained by adding the coefficients of previous periods variance, covariance squared error terms and cross-product of error terms. The volatility persistence is a value which indicates the degree to which past shocks and volatility should be taken into consideration when forecasting conditional future conditional variance. Specifically, when the persistence is found to be significant it would be better to put more weight to recent observations in order to explain future volatility. Conversely, if low persistence is found, less weight should be put to recent observations. The reasoning is that in the absence of persistence the conditional variance will return faster to its unconditional variance compared to persistent process.

5.3 VIRFs Analysis

We continue our analysis investigating the reaction of the conditional variance of one index to a shock to the other index, using VIRFs as described earlier in section 4.5.

For our purposes we select two observed historical shocks. This makes the analysis realistic providing better comprehension of the magnitude, the pattern and persistence of volatility transmission. We employ as a measure of a shock persistence its half life expressed in days. Half-life is defined as the time needed for the impact of a shock to be half its initial value. We have selected a military intervention (US invasion in Iraq in 2003) and a natural disaster (hurricane Katrina in 2005). Historical shocks are by construction the standardized residuals which have the desired property of news.

We first estimate the initial shocks for both cases denoted as $\hat{Z}_0$=$(\widehat{H}_t^{1/2})^{-1}\widehat{E}_t$. As it was mentioned in section 4.5, the calculation of VIRFs depends on the estimated parameters of the BEKK model as well as the initial shocks, and the baseline state of volatility. We have used in both cases the corresponding volatility H_t for the time the shock occurred. Taking into consideration that for each case some days or weeks earlier both the invasion and the hurricane had been anticipated, we selected the day when the largest shock (in absolute value) was observed.

After the quantification of the two historical shocks we continue with the impulse response analysis. Our preliminary analysis using semi-parametric and parametric methods has provided strong evidence for the existence of bidirectional spillover effects between the two indices but it does not provide us with any helpful insights about the dynamic adjustment of the system to volatility shocks. All that information is obtained by VIRFs. The following figures present the estimated VIRFs for both cases.

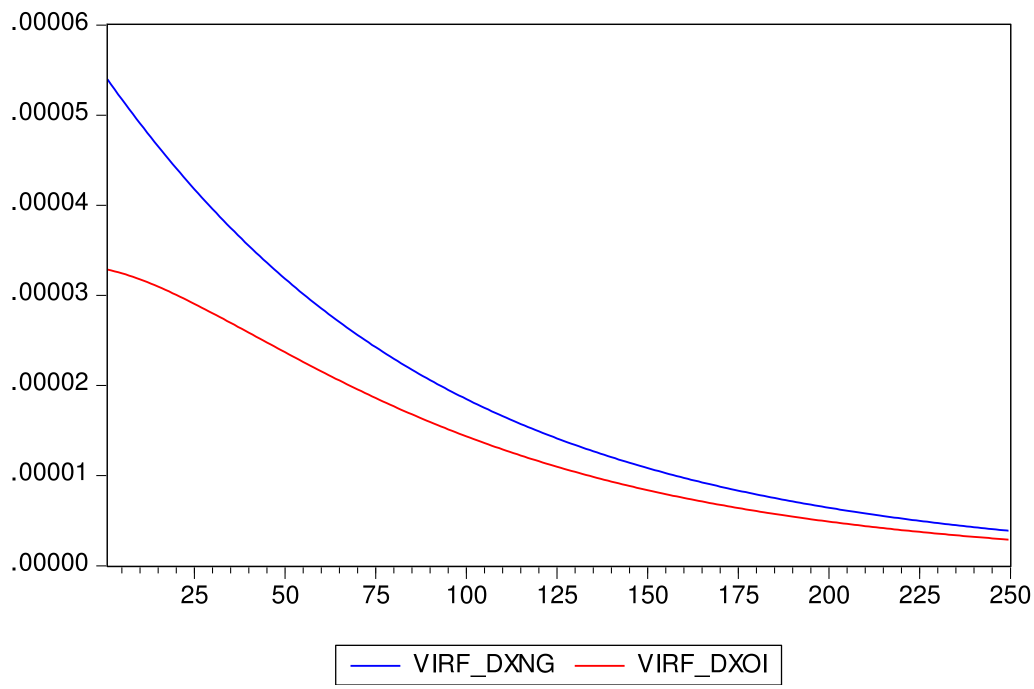

Figure 3. VIRFs for the US invasion in Iraq in 2003

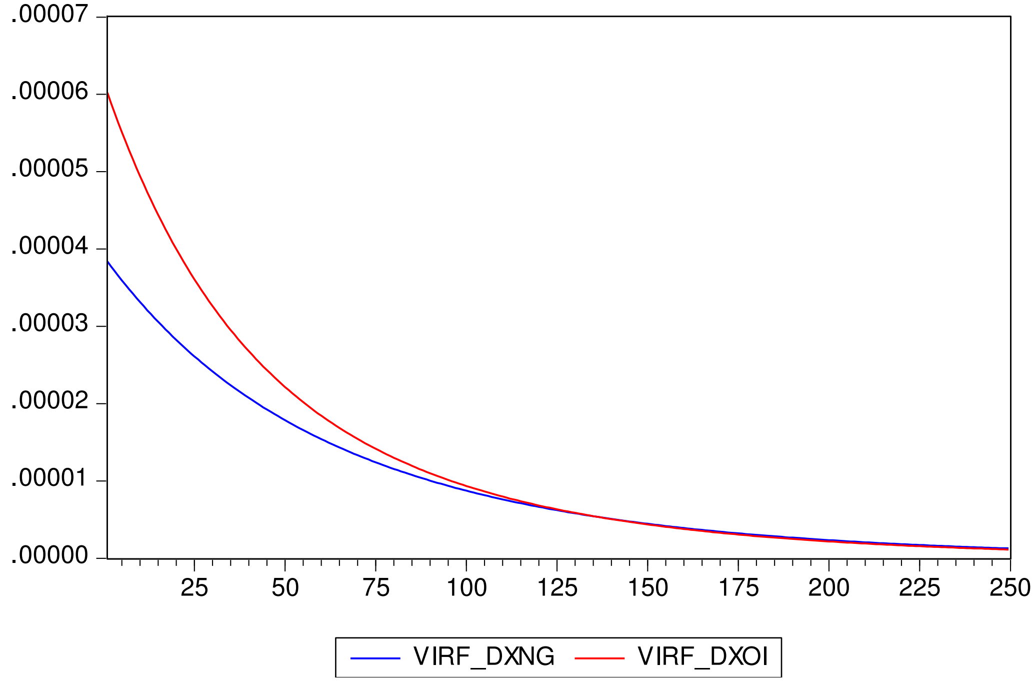

Figure 4. VIRFs for hurricane Katrina

In both cases the effect of the shock decreases to zero. For the the first case (US invasion of Iraq), there is a smoother absorbance of the shock. In this case, the XNG index needs 79 days to absorb half the shock and XOI 110 days. In the case of hurricane Katrina, there is a more abrupt absorbance of the shock. Specifically, XNG needs 71 days to absorb half of the shock and XOI needs 47 days.

6. IMPLICATIONS

After having identified that there exist volatility spillovers between the two indices it would be reasonable to investigate the practical implications of that finding. Investors of these indices are primarily interested in how the volatility of the indices behaves over time and the factors that cause volatility changes and spillovers.

Two financial issues that a market participant has to tackle with is portfolio selection and risk management. Let us assume an investor allocating his portfolio between the XNG and XOI indices. Volatility spillovers require the investor to estimate the optimal weights and hedge ratios in order to sufficiently tackle with risk. Assuming that the expected returns are zero, it makes the problem equivalent to estimating risk minimizing portfolio weights. Kroner and Ng (1998) define $\omega_t = \frac{h_{22,t} - h_{12,t}}{h_{11,t} - 2h_{12,t} + h_{22,t}}$. (23)

They continue assuming a mean variance utility function and show that:

$$\omega_t = \begin{cases} 0 \; if \; \omega_t < 0 \\ \omega_t \; if \; 0 \leq \omega_t \leq 1 \\ 1 \; if \; \omega_t > 1 \end{cases} \tag{24}$$

Consequently the optimal weight for the other asset is $1-\omega_t$. The average value of ω_{xng} is reported in Table 11. For that reason, we have calculated the average values of $h_{11,t}$, $h_{22,t}$ and $h_{12,t}$ using the values obtained by the estimation of the bivariate BEKK-GARCH and implemented them in equation (23).

Table 11. optimal portfolio weights for XNG and XOI

Optimal portfolio weights	
XNG	27.9%
XOI	72.1%

The results in Table 11 indicate that an investor allocating his portfolio among XNG and XOI should hold 27,9% of his portfolio on XNG and the remaining budget of 72,1% should be invested in XOI in order to minimize the risk without reducing the expected returns.

The second task is the computation of hedge ratios. Following Kroner and Sultan (1993) we assume a portfolio consisted of two assets (in our case XNG and XOI). Aiming to the minimization of the risk of a portfolio which is long \$1 in XNG the portfolio manager should short \$β of XOI where the coefficient β_{xng} is defined as following:

$$\beta_{xng} = \frac{h_{12,t}}{h_{22,t}} \tag{25}$$

We have calculated the averages of h_{12} and h_{22} using the values of h_{12} and h_{22} we obtained from the estimated restricted BEKK-GARCH model. Then we implemented our sample averages of h_{12} and h_{22} in equation (25). Using the simple above formula we have estimated $\beta_{xng,t} = 0.834$. That means that an investor allocating his portfolio between these two indices when they have a long position of 1\$ in XNG should have a short position of 0.834\$ in XOI.

7. Conclusion

In this master thesis we have examined the existence of volatility spillovers between the NYSE Arca natural gas index (XNG) and the NYSE Arca oil index using daily data spanned from 1997 to 2014. For our purposes we used two methods, a semi-parametric and a parametric. The semi parametric method was that proposed by Cheung and Ng (1996) and extended by Hong (2001). The parametric method was based on the bivariate BEKK-GARCH model of Engle and Kroner (1996). Moreover, we examine the volatility transmission dynamics of a shock from one index to the other using the Volatility Impulse Response Functions (VIRFs) proposed by Hafner and Herwartz (1996). Finally, we implement our findings in portfolio weighting and portfolio hedging.

Our analysis using the semi-parametric approach has indicated a causality-in-mean from XNG to XOI and a bidirectional causality-in-variance. This causality-in-variance remained after filtering out causality-in-mean. The BEKK-GARCH model indicated that the conditional variance of XNG depends on the previous realizations of its conditional volatility and from the conditional volatility of XOI. On the other hand, volatility of XOI depends on its past realizations and shocks originated in XNG.

The volatility dynamics of XNG and XOI have a different behavior. When we examined the volatility dynamics of the U.S military invasion of Iraq, XNG needed less days to absorb half of the shock than the XOI, whereas the opposite occurred when we examined the volatility dynamics when hurricane Katrina hit on the U.S.

Having found evidence of volatility transmission between the two indices, we implemented our findings in a portfolio allocated between the two indices and we calculated the optimal weights and hedge ratios.

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APPENDIX

Components of XOI

Anadarko Petroleum Corporation	Petr
BP plc	Phillips 66
ConocoPhillips	Total SA
Chevron Corporation	Valero Energy Corporation
Hess Corporation	Exxon Mobil Corporation
Marathon Oil Corporation	Occidental Petroleum Corporation

Source: *www.yahoofinance.com*

Components of XNG

National Fuel Gas Company	Apache Corp.
NiSource Inc.	Anadarko Petroleum Corporation
QEP Resources, Inc.	Chesapeake Energy Corporation
Range Resources Corporation	Cabot Oil & Gas Corporation
Questar Corporation	Devon Energy Corporation
Southwestern Energy Co.	Encana Corporation
TransCanada Corp.	EQT Corporation
Ultra Petroleum Corp.	AGL Resources Inc.
Williams Companies, Inc.	Kinder Morgan, Inc.
WPX Energy, Inc.	Noble Energy, Inc.

Source: *www.yahoofinance.com*