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SUBJECT: The impact of small sample bias on impulse response analysis

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Abstract

This study investigates (i) the small sample bias of the coefficients of AR models, (ii) the effect of the bias on the estimated half-life of a shock and (iii) the ability of three alternative procedures proposed in the literature to account for the small sample bias. In addition, an empirical application to the Purchasing Power Parity Puzzle is provided.

Contents

1. INTRODUCTION

Investigating the degree of persistence in economic and financial time series has been the subject of many researches thus far. (see, for example, Campbell and Mankiw, 1987). It is mainly significant when testing for the validity of parity conditions in international economics. For instance, mean-reversion of real exchange rates is an essential requirement for the validity of the purchasing power parity (Rogoff, 1996).

Important information, when dealing with mean reversion, is provided by the set of impulse response coefficients. The coefficients of an estimated model cannot be easily interpreted in a direct way. Thus impulse responses are often computed in order to study the interrelationships within the variables of a system. Moreover, the half-life, defined as the number of periods required for the impulse response to a unit shock to a time series to dissipate by half, has emerged as a popular measure of persistence in this context.

The main problem which comes to surface when dealing with the aforementioned statistical tools is that of bias. Many econometric estimators (and among them the ordinary least squares (O.L.S.) estimators are significantly downward biased when the sample size is relatively small and especially when the autoregressive coefficients are near to unity. "Small sample bias" causes the aforementioned estimators to be a misleading approximation of the true autoregressive parameter. This failure of the estimators to approximate the true parameter spreads to both the impulse response function and the half-life estimation and, consequently, leads to misspecification of persistence of exogenous innovations.

Since measuring the degree of persistence in empirical research poses a serious problem when dealing with finite samples, it is natural to try to approximate or reduce bias and the idea of doing so is very old. For example, James Durbin confessed in an interview that he worked on the idea of reducing the bias by using computer-simulation in the early 1950s, but gave up because of the small computer capabilities at that time. Consequently, one would expect that the application of bias reduction methods to be more widespread than they are. Often small sample bias is neglected in empirical work, with a possible reason for this being that bias is not thought to be large. The purpose of this thesis is, consequently, to investigate whether the magnitude of bias poses a serious problem in empirical research. In addition, one should not forget that the apparently "small" bias quickly cumulates through the non-linear impulse response function, something which shows that bias might be increased when dealing with impulse response analysis.

The thesis is organized as follows. A brief presentation of half-life measurement can be found in Chapter 2. Chapter 3 reports the results of extensive Monte Carlo experiments which will reveal whether or not bias really poses a serious problem. If it turns out that bias appears only by exception or is insignificant, there would be little need to reduce the bias. Chapter 4 evaluates the ability of three existing methods to reduce bias, allowing for direct comparison between the bias-reduced and the original estimates. Chapter 5 presents an application on purchasing power parity. Chapter 6 concludes.

At this point, it should be mentioned that, for simplicity reasons, this research will pay attention only on the basis of linear univariate autoregressive AR(1) models. This simple case offers to the researcher the opportunity to investigate the “finite sample bias” in detail, without facing the programming difficulties of higher order or higher dimensional processes.

2. MEASUREMENT OF HALFLIFE: Historical Background and estimation methods

The measure of half-life is defined as the amount of time required for a quantity to fall to half its value as measured at the beginning of the time period. The original term, dating to Ernest Rutherford's discovery of the principle in 1907, was "half-life period" which was shortened to "half-life" in the early 1950s. Rutherford applied the principle of a radioactive elements' half-life to studies of age determination of rocks by measuring the decay period of radium.

Half-life is used to describe a quantity undergoing exponential decay, and is constant over the lifetime of the decaying quantity. It is a characteristic unit for the exponential decay equation. The term "half-life" may generically be used to refer to any period of time in which a quantity falls by half, even if the decay is not exponential.

Consider the AR(1) model of the form: $X_t = a + \beta X_{t-1} + \varepsilon_t$,

where $\varepsilon_t \sim iid\ (0, \sigma^2)$, and $-1 < \beta < 1$, (stationarity condition). . In this case, the half-life is estimated as:

$$\hat{h} = \begin{cases} \log(0.5) / \log(\hat{\beta}) & if \quad -1 < \hat{\beta} < 1. \\ \infty & otherwise. \end{cases}$$

where $\widehat{\boldsymbol{\beta}}$ is the least-squares estimator of $\boldsymbol{\beta}$. Estimations of half-life present the following statistical properties of $\hat{h}$. First, it has a completely unknown distribution. Second, it may not possess finite sample moments, since it takes extreme values as $\widehat{\boldsymbol{\beta}}$ approaches unity. Third, it may be intrinsically biased in small samples, as it is a non-linear function of $\widehat{\boldsymbol{\beta}}$ which may be biased too. The reason is that a tiny estimation error in $\hat{\beta}$ quickly cumulates thorough the non-linear half-life estimation and may result in an extremely biased $\widehat{\boldsymbol{h}}$. Note that the half-life estimation $\widehat{\boldsymbol{h}}$ is unbiased when the OLS estimator $\widehat{\boldsymbol{\beta}}$ is unbiased.

Recent studies which deal with the half-life estimation in the context of an AR model can be distinguished into two groups. The first group of studies is based on alternative asymptotic confidence intervals. The second group of studies uses the bootstrap method of Efron and Tibshirani (1993) combined with a bias-correction process for estimators. Such studies are those of Kilian (1998), Hansen (1999), Murray and Papell (2002), Rapach and Wohar (2004), Caporale et al. (2005) and Rossi (2005). For the purposes of this research, attention is paid to the second group of studies.

3. Monte Carlo Experiment

In this chapter we present the results of an extensive Monte Carlo experiment. The experiment was carried out for two reasons; Firstly, to assess the magnitude of bias and to investigate its dependence on the sample size and on persistence of AR(1) process. Secondly, to evaluate the performance of some bias–reduction methods proposed in the literature. Whereas the evaluation of those methods can be found in Chapter 4, the present chapter describes the experiments and the main results of the simulations.

3.1 An introduction to Monte Carlo simulations

As it was mentioned before, some properties of estimators are based on asymptotic theory, that is, these properties are valid only for infinite samples. However, all empirical applications use finite samples. For that reason, Monte Carlo simulations are used to examine the behavior of an estimator when dealing with a relative small sample size.

Monte Carlo simulations are based on artificial samples generated by a researcher with the help of a Random Number Generator.

Consider the AR(1) model: $X_t = a + \beta X_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim iid\ (0, \sigma^2)$, and $-1 < \beta < 1$

Initially, decisions about the sample size (T), the number of replications (r), the value of constant term (a), the value of the autoregressive parameter (β), and the initial value for variable under investigation (X), should be made. In the next step, the Random Number Generator is used to get random values for the error term ε_t. Afterwards, and after defining initial values, the values of error term from the previous step are used to calculate the values of X_t by using the following equation: $X_t = a + \beta X_{t-1} + \varepsilon_t$. Thereafter, the generated sample of X_t is used to estimate the autoregressive parameter $\hat{\beta}$ (using the Ordinary Least Squares estimation method). Next, the absolute bias of the estimation for the first generated sample is calculated from: $Bias_1 = \beta - \hat{\beta}$. Thereupon, the above steps are repeated as many times as the number of replications. Finally, the mean absolute bias is calculated across replications i.e. $(Bias_1 + Bias_2 + \cdots + Bias_r)/r$.

It is important to note that the results of Monte Carlo experiments are Data Generating Process-specific, that is, they are valid only for the specific DGPs used to generate the random samples and cannot be used to draw any general conclusions.

3.2 Experiment design

An AR(1) model of the form: $X_t = a + \beta X_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim iid\ (0, \sigma^2)$, and $-1 < \beta < 1$, has been simulated for parameter values $\beta = 0.75,\ 0.80,\ 0.86,\ 0.92,\ 0.95$. For each one of the 1.000 replications, autoregressive parameters were OLS estimated and after the final replication, means, medians and variances were computed. In addition, for each combination of parameter values and sample sizes, the mean absolute bias and the mean square bias have been computed. Moreover, percentage of cases where estimators overestimate and underestimate the true parameter is derived.

Regarding the effect of bias on the measurement of persistence, for each one of the 1.000 replications, the half-life has been estimated. After the final replication and for each combination of parameter values and sample size, the median was computed. Following this approach, it is easy to define the absolute bias of the half-life estimation by estimating: $|h - \hat{h}_{median}|$. In addition, percentage of half-life estimators above 100 (extreme values) is derived.

At this point, it should be mentioned that in the Monte-Carlo simulation, the constant term a was set equal to 0.01 and the variance of the error term equal to 0.001. The initial value X_0 which is required in order to create the sample X was set equal to 0. However, this decision did not affect the reliability of the experiment because during the creation of the sample, the first 25 observations were dropped.

3.3 Monte-Carlo results

3.3.1. True autoregressive parameter equal to 0.75

This section presents the results of our Monte Carlo experiment when β=0,75. As mentioned before, we examine five different sample sizes T (i.e. T=40. 70, 100, 150, 200) This will reveal the way that the sample size affects the magnitude of bias. Results are reported in Table 3.1:

Table 3.1: *O.L.S. and Half-life Estimates (β=0,75)*

SPECIFICATIONS: β=0,75 / h=2,409420					
	T=40	**T=70**	**T=100**	**T=150**	**T=200**
	Estimator of the autoregressive parameter ($\hat{\beta}$)				
Mean	0,663611	0,701303	0,718749	0,72727	0,734189
Median	0,687307	0,710240	0,725392	0,730494	0,738045
Variance	0,019281	0,008804	0,005301	0,003322	0,002443
Mean Absolute Bias	0,121883	0,079791	0,060453	0,048664	0,040335
Mean Square Bias	0,026744	0,011175	0,006278	0,003838	0,002693
Percentage Of Overestimation	0,313	0,338	0,361	0,368	0,405
Percentage Of Underestimation	0,687	0,662	0,639	0,632	0,595
	Half-life estimator ($\hat{h}$)				
Median	1,848520	2,025841	2,159048	2,207230	2,281966
Percentage Of extreme Values (over 100)	0	0	0	0	0

Starting with the mean and median of the estimates, it is clear that “small sample bias” poses a serious problem. With sample size equal to 40, both the mean and median differ from the true parameter. This fact accounts for the high value of mean absolute bias and mean square bias. The relative high variance indicates the “instability” of estimations, which is indicated from the presence of “extreme” cases where the estimated parameter is far away from its true value. In addition, it should be mentioned that in most cases (68.7%) the estimator tends to underestimate the real autoregressive parameter. This fact propagates to half-life estimation, that lies below the true half-life value.

The situation improves as the sample size increases. Estimators come closer and closer to the true parameter, as values of the mean and median reveal. For sample size equal to 200, estimates are very close to the true autoregressive parameter. However, it is noteworthy that the pace of bias reduction declines as the sample size increases. For example, an increase in sample size from T=40 to T=70 leads to a decrease in the mean absolute bias of about 0,042. In opposition, an increase in sample size from T=150 to T=200 leads to a decrease in the mean absolute bias of only 0.008. In addition, as the sample size increases, the percentage of underestimation tends to come closer to the percentage of overestimation, something which indicates that for sufficiently large sample size, “balance” of estimations settles down.

Regarding the half-life estimation, it is clear that “finite sample bias” poses a serious problem. For the smallest sample size (T=40), the median of the simulated Half-life is only 1,848, when the “true” half-life is equal to 2,4094. This result indicates that in cases where a researcher is interested in the half-life of a variable and he uses a relative small sample size, he will probably conclude that persistence of this variable is lower than the true one. As the sample size increases, the situation improves. However it should be mentioned that even for the highest sample size considered in this study (T=200), the bias in half-life remains high, even though the bias of the estimator of the autoregressive parameter has been reduced. And this fact makes the use of a bias- reduction method vital.

3.3.2. True autoregressive parameter equal to 0.80

Table 3.2: *O.L.S. and Half-life Estimates (β=0,80)*

SPECIFICATIONS: β=0,80 / h=3,106284					
	T=40	**T=70**	**T=100**	**T=150**	**T=200**
	Estimator of the autoregressive parameter ($\hat{\beta}$)				
Mean	0,707958	0,748433	0,765903	0,773818	0,781529
Median	0,723317	0,760084	0,773683	0,779603	0,786533
Variance	0,014877	0,007164	0,004502	0,002757	0,002081
Mean Absolute Bias	0,114177	0,074005	0,056678	0,044559	0,037439
Mean Square Bias	0,023349	0,009823	0,005665	0,003442	0,002422
Percentage Of Overestimation	0.240000	0,296	0,34	0,344	0,36
Percentage Of Underestimation	0.760000	0,704	0,66	0,656	0,64
	Half-life estimator ($\hat{h}$)				
Median	2,139955	2,526733	2,701344	2,784054	2,886657
Percentage Of extreme Values (over 100)	0	0	0	0	0

We now turn to the case where the "true" autoregressive parameter is equal to 0,80. It is clear that even for more persistent variables (with higher autoregressive parameters), the use of a relative small sample leads to biased-estimators. However, it seems that the effect of the "finite sample bias" is less than that of the previous case. For all sample sizes used in this research, both the mean absolute bias and the mean square bias are less than in the previous case. In addition, estimators come closer to the true parameter, as indicated by the values of the mean and the median. Nevertheless, the problem is still significant and bias continues to cause broblems. Moreover, it should be mentioned that in this case, estimators tend to underestimate the true parameter heavier than in the previous case.

Focusing on the half-life estimates, it seems that more persistent processes are more susceptible to "finite-sample bias". The median of the Monte-Carlo simulated half-life is far away from its true value for all sample sizes (more than in the previous case). This result looks strange at first look, as one would expect exactly the opposite (because the bias in parameter estimation is less for more persistent processes). However, results reveal the "sensitivity" of half-life estimation for AR(1) processes with high autoregressive parameters. The sample size still has an important role for the bias of estimations, when the sample size increases, the median of half-life estimations comes closer to its true value.

3.3.3. True autoregressive parameter equal to 0.86

Table 3.3: *O.L.S. and Half-life Estimates (β=0,86)*

SPECIFICATIONS: β=0,86 / h=4,59576913					
	T=40	**T=70**	**T=100**	**T=150**	**T=200**
	Estimator of the autoregressive parameter ($\hat{\beta}$)				
Mean	0,759414	0,807097	0,820646	0,836002	0,84262
Median	0,778416	0,821246	0,832287	0,840425	0,84817
Variance	0,01423	0,006084	0,003851	0,002219	0,001634
Mean Absolute Bias	0,116473	0,069905	0,054231	0,039324	0,032832
Mean Square Bias	0,024347	0,008882	0,005399	0,002795	0,001936
Percentage Of Overestimation	0,209	0,267	0,289	0,338	0,364
Percentage Of Underestimation	0,791	0,733	0,711	0,662	0,636
	Half-life estimator ($\hat{h}$)				
Median	2,767125	3,519712	3,775767	3,987102	4,209207
Percentage Of extreme Values (over 100)	0	0	0	0	0

Table 3.3 presents the results for the case where the true parameter value is equal to 0,86. The estimates of the autoregressive parameter confirm conclusions of **section 3.3.2**. The effect of the “small-sample bias” is weakens as the process becomes more persistent. Both mean absolute bias and mean square bias are lower here, and the mean and the median of simulated estimators are closer to the true value. The only exception arises when the sample size is really small (T=40), but the differences from previous cases are extremely small and they are not capable to change the general conclusions. In addition, the estimators tend to underestimate the true parameter more than in all previous cases. The reduction of bias as the sample size increases is still not linear, but it reduces as the size increases.

The half-life estimates are more affected from the “small-sample bias” than in the previous cases. The Medians are far from the true half-life and the problem becomes more serious. Even for the largest sample size considered in this study (T=200), the bias of half-life is still high and it would probably lead to wrong conclusions.

3.3.4. True autoregressive parameter equal to 0.92

Table 3.4: *O.L.S. and Half-life Estimates (β=0,92)*

SPECIFICATIONS: β=0,92 / h=8,312950					
	T=40	**T=70**	**T=100**	**T=150**	**T=200**
	Estimator of the autoregressive parameter ($\widehat{\beta}$)				
Mean	0,806769	0,862479	0,880089	0,895118	0,899063
Median	0,829695	0,874351	0,889490	0,900941	0,903622
Variance	0,012252	0,004931	0,002999	0,001523	0,001152
Mean Absolute Bias	0,12016	0,066308	0,050095	0,034532	0,030171
Mean Square Bias	0,025073	0,00824	0,004592	0,002142	0,00159
Percentage Of Overestimation	0,129	0,204	0,252	0,303	0,31
Percentage Of Underestimation	0,871	0,796	0,748	0,697	0,69
	Half-life estimator ($\widehat{h}$)				
Median	3,712672	5,162229	5,918907	6,644729	6,839526
Percentage Of extreme Values (over 100)	0,005	0,001	0	0	0

The discussion of results is continued with the case where the "true" autoregressive parameter is equal to 0,92. In connection to the conclusions of the previous section, both the mean absolute bias and the mean square bias continue their declining movement as the process becomes more persistent. In addition, it should be mentioned that the pace of the bias reduction as the sample size increases becomes lower for processes with a high autoregressive parameter. Moreover, similar to the previous cases, estimates of the autoregressive parameter tend to underestimate the true value more, as the process becomes more persistent.

Focusing on the half-life estimates, it becomes clear that the assumptions of the previous sections are now confirmed. The distance between the simulated estimates of the half-life and its true value is greater than any other case. In addition to this, for small sample sizes (T=40 and T=70), extreme values make their appearance. From the 1.000 replications of this Monte-Carlo experiment, 5 estimated half-lifes are above 100 for sample size equal to 40, and 1 for sample size equal to 70. This reveals that for really small sample sizes, the more persistent the process is, the more biased the half-life estimator becomes.

3.3.5. True autoregressive parameter equal to 0.95

Table 3.5: *O.L.S. and Half-life Estimates (β=0,95)*

SPECIFICATIONS: β=0,95 / h=13,513407					
	T=40	**T=70**	**T=100**	**T=150**	**T=200**
	Estimator of the autoregressive parameter ($\widehat{\beta}$)				
Mean	0,835195	0,888468	0,907437	0,920693	0,928998
Median	0,85411	0,902943	0,917297	0,927846	0,934758
Variance	0,011625	0,004577	0,002346	0,001256	0,000875
Mean Absolute Bias	0,119844	0,06719	0,047955	0,034323	0,026682
Mean Square Bias	0,024805	0,008363	0,004157	0,002115	0,001317
Percentage Of Overestimation	0,097	0,162	0,186	0,213	0,269
Percentage Of Underestimation	0,903	0,838	0,814	0,787	0,731
	Half-life estimator ($\widehat{h}$)				
Median	4,395475	6,789244	8,029605	9,25557	10,27383
Percentage Of extreme Values (over 100)	0,019	0,012	0,003	0	0

Finally Table 3.5 presents the results for the case where the "true" parameter is equal to 0,95. Conclusions about the estimates of the autoregressive parameter are exactly the same with those of the previous sections. The bias is less than in all previous cases, but still remains a serious problem. In addition, bias reduces as the sample size increases, however the pace of bias reduction continues to fall with the increase in the sample size. Moreover, in this case we observe the greatest percentages of underestimation of the true parameter value for all sample sizes considered in this study.

Regarding the half-life estimations, we observe the greatest deviation between the median of simulated Half-life and the true one. And this deviation increases as the sample size decreases. From now on, results do not allow any dispute: The more persistent the process is, the more "sensitive" to small sample bias the half-life estimation becomes. Furthermore, we observe the greatest percentage of extreme values for relative small samples (T=40, 70, 100) and as someone would expect, this percentage decreases as the sample size increases.

4. BIAS REDUCTION TECHNIQUES

4.1 Historical Background of Bias Reduction Techniques

The results from the previous chapter certify that small sample bias is a serious problem in empirical research. Thus, it is natural to try to reduce bias and appropriate methods have already been proposed in the literature. Existing methods can be divided into 2 categories:

The first category consists of analytical bias formulas which lead to adjustment for the "small sample bias". The main advantage of these methods is that they are extremely simple and easy to use. But, whether or not these methods have equally good properties compared to the second category poses a serious question, which we try to answer. Such methods have been proposed by, amongst others, Marriott and Pope (1954), Kendall (1954), Orcutt and Winokur (1969), Bhansali (1981), Tjostheim and Paulsen (1983), Pantula and Fuller (1985), Shaman and Stine (1988), Stine and Shaman (1989), Andrews (1993), who derived expressions for the first order bias for a least squares (LS) estimation of a general *p-th* order autoregression.

The second category consists of methods which are based on computer programming and they use simulation techniques. These methods are believed to perform better than the ones in the previous category, but they entail a serious disadvantage: they are computer intensive and involve many technicalities and subtleties. This means that they require knowledge of the appropriate programming language and results crucially depend on computer capabilities. For example, the usage of a "slow" computer for the purposes of bias reduction will make application of those methods extremely difficult. Such techniques are the "median unbiased estimation" method of Andrews and Chen (1994), the "bootstrap-after-bootstrap" method of Kilian (1998), the "grid-bootstrap" of Hansen (1999), the "based on Bonferroni inequality", method of Wright (2000) and the "Bayesian Method" of Sims and Zha (1999) which is based on Monte Carlo integration.

4.2 Review of the bias-reduction methods, under examination

This research will pay attention on both categories of bias-reduction methods, in order not only to examine their performance separately, but also to compare them and resolve whether analytical formulas have equally good properties compared to computer intensive methods.

Therefore, this research will examine the performance of one analytical formula and two computer intensive methods. The analytical bias correction methology for bias of Andrews (1993) will be examined from the the first category. From the second category, we pay attention to the performance of the median unbiased estimation method of Andrews and Chen (1994). In addition, the bootstrap-after-bootstrap method of Kilian (1998) will be examined, which is still the basis for many studies in this subject.

4.2.1. The "Exactly Median-Unbiased" Method of Andrews (1993)

This section introduces the median-unbiased estimators of the autoregressive parameter for an AR process of order 1.

A number m is a median of a random variable X if $P(X \geq m) \geq 1/2$ and $P(X \leq m \geq 1/2$. Let β be an estimator for the autoregressive parameter β. By definition, β is median- unbiased for β if the true parameter β is a median of $\hat{\beta}$ for each β in the parameter space. In other words, $\hat{\beta}$ is median-unbiased if and only if the distance between $\hat{\beta}$ and the true parameter on average is less than or equal to the distance between $\hat{\beta}$ and any other parameter value.

Median-unbiased estimates are used instead for mean-unbiased estimates, because of their features in cases where the parameter space is un-bounded. When the parameter space is un-bounded and estimators take values in the parameter space, it is impossible to have a mean-unbiased estimator, because estimates can take extreme values. Therefore, and because this correction method allows estimates (especially those of half-life) to take extremely low or high values, median-unbiasedness is more useful.

The "Exact median-unbiased" method can be described as follows: Suppose $\hat{\beta}$ is an estimator whose median function $m(\beta)$ is uniquely defined and is strictly increasing on the parameter space B which is a finite interval, say (-1,1]. Then $\hat{\beta}^{corrected}$

$$\hat{\beta}^{corrected} = \begin{cases} 1 & if\ \hat{\beta} > m(1) \\ m^{-1}(\hat{\beta}) & if\ m(-1) < \hat{\beta} < m(1) \\ -1 & if\ \hat{\beta} \leq m(-1) \end{cases} \qquad \textbf{(4.1)}$$

is a median-unbiased estimator of β, where: $m(-1) = \lim_{\beta \to -1} m(\beta)$ and $m^{-1}: (m(-1), m(1)] \to (-1,1]$ is the reserve function of $m(\cdot)$ that satisfies $m^{-1}\big(m(\beta)\big) = \beta$ for $\beta \in (-1,1]$.

The above bias-correction method can be applied to the estimator $\hat{\beta}$ of an autoregressive parameter for the AR(1) process $X_t = a + \beta X_{t-1} + \varepsilon_t$, which is under examination in the context of this research. In consequence, **(4.1)** can be used to obtain an median unbiased estimator given $\hat{\beta}$ (which is OLS estimated).

Andrews, in his paper, reports tables that provide the "exactly median-unbiased" estimator for every possible combination of sample size and OLS estimator of the autoregressive parameter ($\hat{\beta}$), using the formula described above. These tables offer the opportunity to compare efficiency of analytical formula with performance of computer-intensive methods.

4.2.2. The “based-on-simulation median unbiased estimation” of Andrews and Chen (1994).

Secondly, the “median unbiased estimation method” of Andrews and Chen (1994) will be described. According to their approach, simulation methods can be used to compute bias-corrected estimators and, as a consequence, biased corrected estimations of half-life, as follows:

For a given estimation of the constant term, say $\hat{\alpha}$, and for a given estimation of the autoregressive parameter, say $\hat{\beta}$, a Random Number Generator process is used to obtain random values for the error term say ε_t^*. Afterwards, a sample for the variable of interest is created (X^*) according to $X_t^* = \hat{\alpha} + \hat{\beta} X_{t-1}^* + \varepsilon_t^*$. At this point, it should be mentioned that the initial value for X^* (X_0^*) is set equal to 0. Thereafter, X_t^* is regressed on X_{t-1}^* to obtain another OLS estimation of $\hat{\beta}$, say $\hat{\beta}^*$. This process is repeated 1.000 times in order to be able to compute the median of those estimations (median of $\hat{\beta}^*$’s), say m^*. If this estimation for the median is different from the initial estimation of the autoregressive parameter, (that is $\hat{\beta}$), then we adjust the parameter which has been used to create the sample of the parameter of interest and repeat the same process until the parity condition holds.

Summarizing this method in few words, if the initial estimator $\hat{\beta}$ equals, say 0.9, then one does not use 0.9, but rather uses the value of β that yields the LS estimator to have a median of 0.9. Andrews and Chen concentrate on median-unbiased rather than mean-unbiased estimates for the same reasons described in the previous method.

4.2.3. The “bootstrap-after-bootstrap method” of Kilian (1998)

The third method under examination is the “bootstrap-after-bootstrap method” of Kilian (1998). Kilian’s method is based on the bootstrap techniques proposed by Bradley Efron in 1979. In statistics, bootstrapping is a method for assigning measures of accuracy to sample estimates. This technique allows the estimation of the sampling distribution of almost any statistic using only very simple methods. Generally, it falls in the broader class of resampling methods. Bootstrapping is the practice of estimating properties of an estimator by measuring those properties when sampling from an approximating distribution. One standard choice for an approximating distribution is the empirical distribution of the observed data. In the case where a set of observations can be assumed to be from an independent and identically distributed population, this can be implemented by constructing a number of resamples of the observed dataset (of equal size to the observed dataset), each of which is obtained by random sampling with **replacement** (an element may appear multiple times in one sample) from the original dataset.

Kilian’s method uses these techniques to reduce bias in the following way: firstly, the AR(1) model is OLS-estimated using the existing sample and estimations of the autoregressive parameter and the constant term (say $\hat{\beta}$ and $\hat{\alpha}$) are obtained. Secondly, bootstrap techniques are used to create a resample from the original sample of error terms (say ε^{res}). Next, values of the error term from the previous step (ε^{res}) are used to calculate the values of a new series (say Y) by using the equation: $Y_t = \hat{\alpha} + \hat{\beta} Y_{t-1} + {\varepsilon^{res}}_t$. This requires an initial value for Y to be defined (in this research

$Y_0 = 0$ is used). Thereafter, the generated sample of Y_t is used to estimate the autoregressive parameter $\hat{\beta}^{res}$ (using the Ordinary Least Squares estimation method). Afterwards, an estimation of the bias is obtained using the formula: $bias = \hat{\beta}^{res} - \hat{\beta}$. After repeating this procedure 1.000 times, the final approximation of bias is defined as the mean of the 1.000 bias estimations, and the bias-corrected estimator is defined as $\hat{\beta}^{corrected} = \hat{\beta} - bias$. Subsequently, this corrected estimation of the autoregressive parameter is used to obtain a bias-adjusted estimation for the half-life.

Because of the fact that the above bias-correction process can push parameter estimates to the non-stationary part of the parameter space, Kilian proposed a stationarity-correction method. This procedure can be described as follows: if $\hat{\beta}^{corrected}$ implies non-stationarity, then let $\delta_1 = 1$, $\Delta_1 = bias$ and $\widehat{\beta_\iota}^{corrected} = \hat{\beta} - \Delta_i$. Set $\Delta_{i+1} = \delta_i \Delta_i$ and $\delta_{i+1} = \delta_i - 0.01$ for i=1,2,3,...... . Iterate until $\widehat{\beta_\iota}^{corrected}$ satisfies the condition of stationarity and set $\hat{\beta}^{corrected} = \widehat{\beta_\iota}^{corrected}$.

4.3. Performance of bias-correction methods

This section pays attention to the performance of the bias-adjustment methods described above. The base for this research will be the Monte-Carlo simulated estimations, described in **Chapter 3** of this thesis. Specifically, in each one of the 1.000 replications of the Monte-Carlo experiment, the estimators of both the autoregressive parameter and the half-life are adjusted for bias according to the:

1. "Bootstrap-after-bootstrap method" proposed by Kilian (1998) (*We refer to this bias-corrected estimator as the "Kilian Estimator"*)
2. "Simulation-based", median unbiased estimation method" proposed by Andrews and Chen (1994). (*We refer to this bias-corrected estimator as the "Andrews & Chen Estimator"*).
3. "Exactly median-unbiased analytical formula" proposed by Andrews (1993). (*We refer to this bias-corrected estimator as the "Andrews Estimator"*).

For each correction method means, medians and variances are computed. In addition, for each combination of parameter values and sample size, the mean absolute bias and the mean square bias are derived. Moreover, percentage of cases where estimators overestimate and underestimate the true parameter are estimated. At this point it should be mentioned that the sum of these percentages may not be equal to unity, as there might be cases where the estimations are the same with the true parameters. In addition, a Kernel-Density Graph is used, in order to estimate the probability density function for each one of the aforementioned parameter-estimation methods.

Regarding the measurement of half-life, after the final replication and for each combination of parameter values and sample size, the median is computed. Moreover, appropriate histograms are used in order to reveal the distribution of half-life for each bias-correction method.

Results are reported in Tables 4.1-4.5. Each row refers to a specific bias-correction method. However, at the top of each table, the results from Chapter 3 are also reported, in order to be able to investigate the performance of each method easier. The rest of this Chapter will be separated in 5 sections, where each section corresponds to a different “true” autoregressive parameter value (β=0.75, 0.80, 0.86 0.92, 0.95). In each section, results for all the sample sizes under investigation (T=40, 70, 100,150, 200) are reported.

4.3.1. True autoregressive parameter equal to 0.75

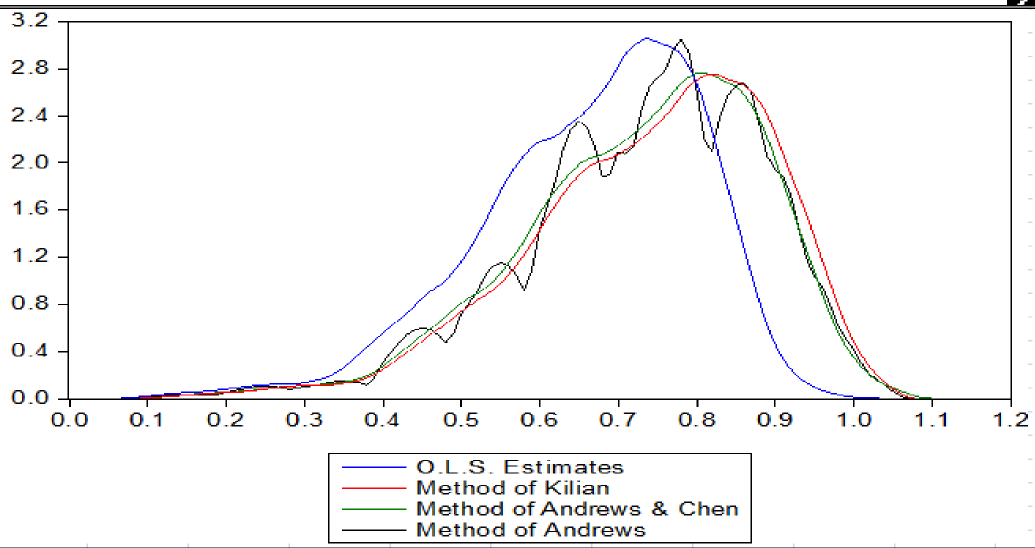
This section discusses results for the case where the true autoregressive parameter is equal to 0,75 and for all sample sizes. Starting with the estimator of the autoregressive parameter, we observe that the median of the Andrews estimator is equal to the true parameter value for all sample sizes. The other two computer intensive methods perform equally well. Values of the mean absolute bias confirm this conclusion. But the question about which method reduces the “small sample bias” more efficiently still remains. Looking only to the mean absolute bias and the mean square bias, one would conclude that “the best” bias reduction- method is the Kilian estimator. The mean of this method comes closer to the true parameter from any other method and, in addition, it has the smallest mean square bias. The Andrews estimator “works” better than the other the methods in most of the cases, but it has a serious disadvantage. There are cases where this correction pushes the estimation to extremely high or low areas. This is confirmed by the Kernel-Density Graph, in which the method of Andrews presents high density for parameter-values far away from its true one. The Andrews & Chen estimator performs better than the Andrews estimator and equally well to Kilian’s method. Furthermore, as the sample size increases, the performance of the Andrews Estimator and the Andrews & Chen Estimator gets better. However, the Kilian estimator has the opposite characteristics. Its performance deteriorates as the sample size increases, and the reason is the tendency of this method to overestimate the true parameter, and as a result, in most cases the corrected estimator surpasses its true value.

Turning to the Half-life estimator, our results confirm the aforementioned conclusions. The Andrews estimator seems to perform better than the other methods, as indicated by the value of the median. But this method has high frequency of extreme values (above 100) for really small sample sizes. And this fact confirms that the analytical formula of Andrews may perform well in most cases, but it may push the half-life estimator to extremely high or low areas, as indicated by the histograms (especially for sample size equal to 40). Comparing the two computer-intensive methods, in addition to what have been discussed above, the Andrews and Chen estimator seems to perform better than Kilian’s method. Its median comes close to the true half-life and in small samples it generates extreme values for the half-life less often than the other methods. The Kilian estimator overestimates the true half-life for small samples and as the sample size increases, its performance gets better, but even for the highest sample size of those under investigation (T=200), its median is higher than the true value.

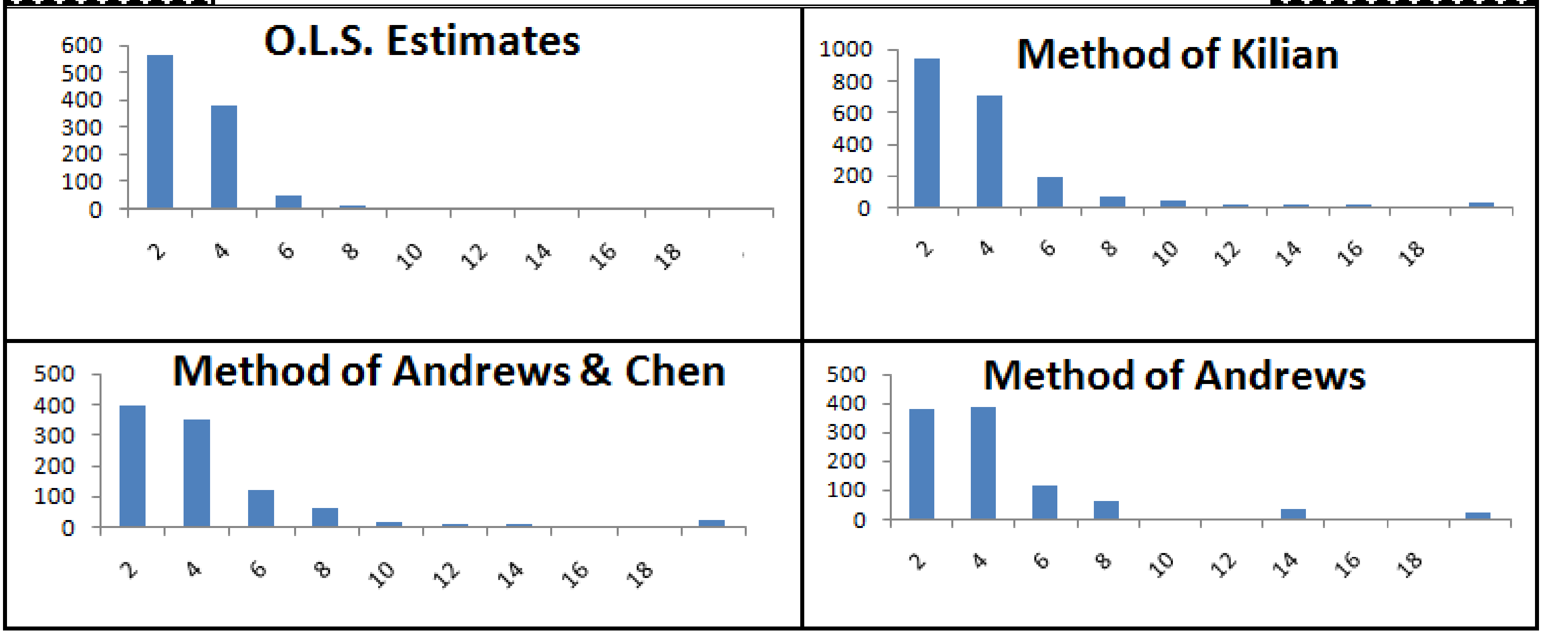
Table 4.1: *Bias-correction results (β=0.75)*

SPECIFICATIONS: T=40 / β=0,75 / h=2,409420									
	Estimator of the autoregressive parameter ($\hat{\beta}$)							**Half-life estimator ($\hat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,663611	0,687307	0,019281	0,121883	0,026744	0,313	0,687	1,848520	0,000
method of Kilian	0,741831	0,767324	0,022885	0,121082	0,022952	0,544	0,456	2,617174	0,013
method of Andrews and Chen	0,730681	0,754126	0,022549	0,119501	0,022923	0,509	0,491	2,456267	0,012
method of Andrews	0,730510	0,750000	0,022743	0,115510	0,023123	0,383	0,381	2,409421	0,014

Density Distribution of Estimators



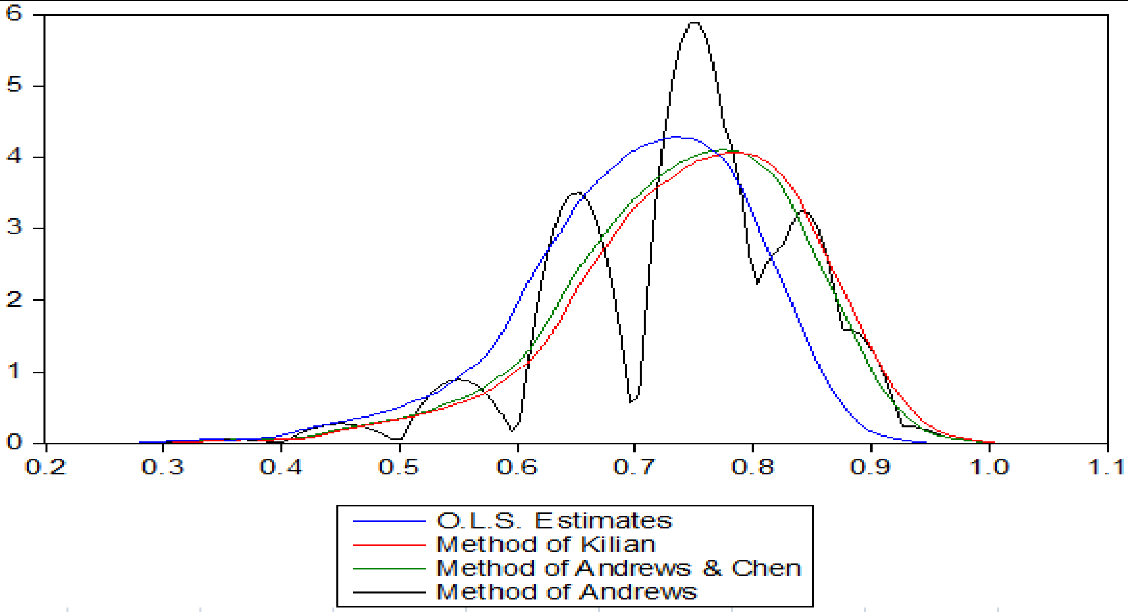
Histogram of Half-Life Estimators



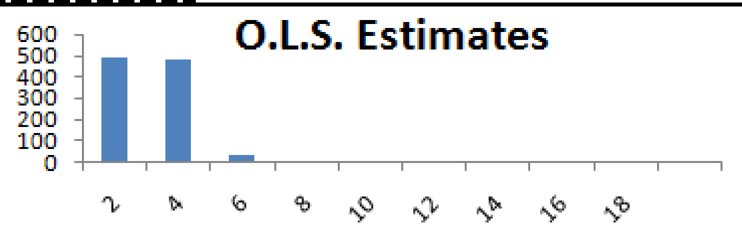
SPECIFICATIONS: T=70 / β=0,75 / h=2,409420

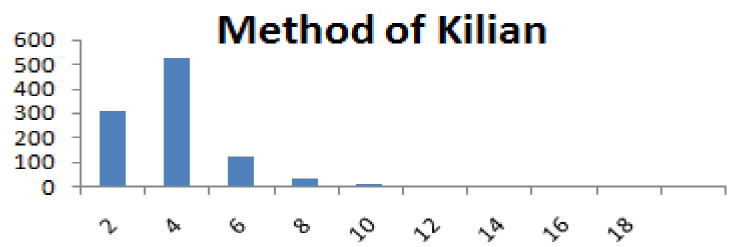
	Estimator of the autoregressive parameter ($\widehat{\beta}$)							Half-life estimator ($\widehat{h}$)	
	Mean	Median	Variance	Mean Absolute Bias	Mean Square Bias	% of over-estimation	% of under-estimation	median	% of extreme values
O.L.S. estimates	0,701303	0,710240	0,008804	0,079791	0,011175	0,338	0,662	2,025841	0,000
method of Kilian	0,747354	0,757214	0,009730	0,077571	0,009737	0,525	0,475	2,492358	0,000
method of Andrews and Chen	0,738803	0,747598	0,009573	0,076797	0,009699	0,495	0,505	2,382852	0,000
method of Andrews	0,737070	0,750000	0,009896	0,071070	0,010063	0,297	0,313	2,409421	0,000

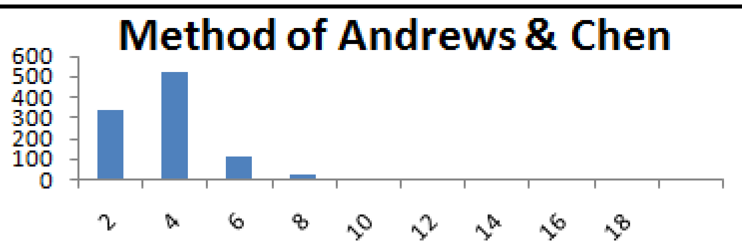
Density Distribution of Estimators

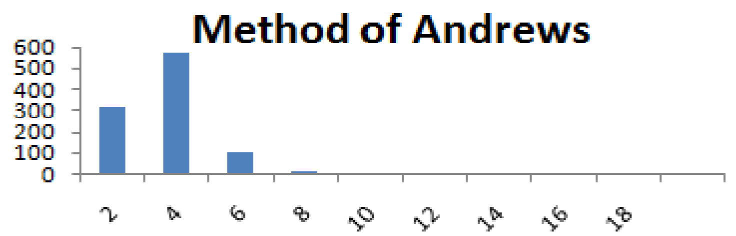


Histogram of Half-Life Estimators



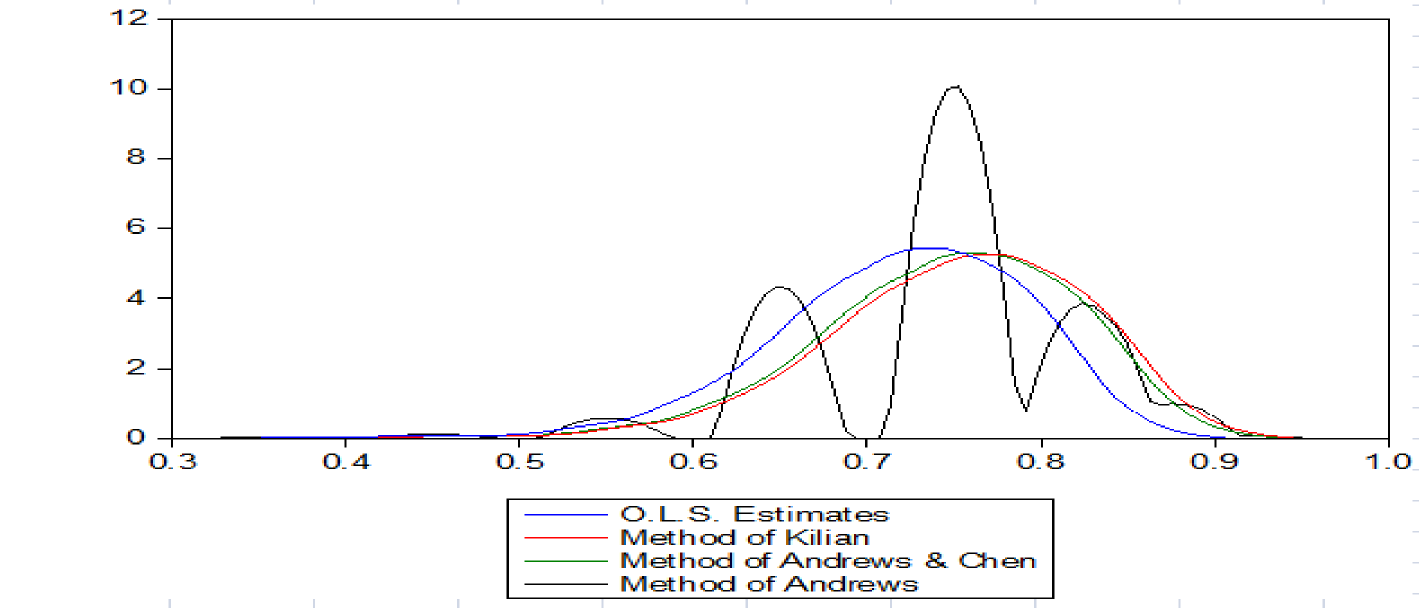




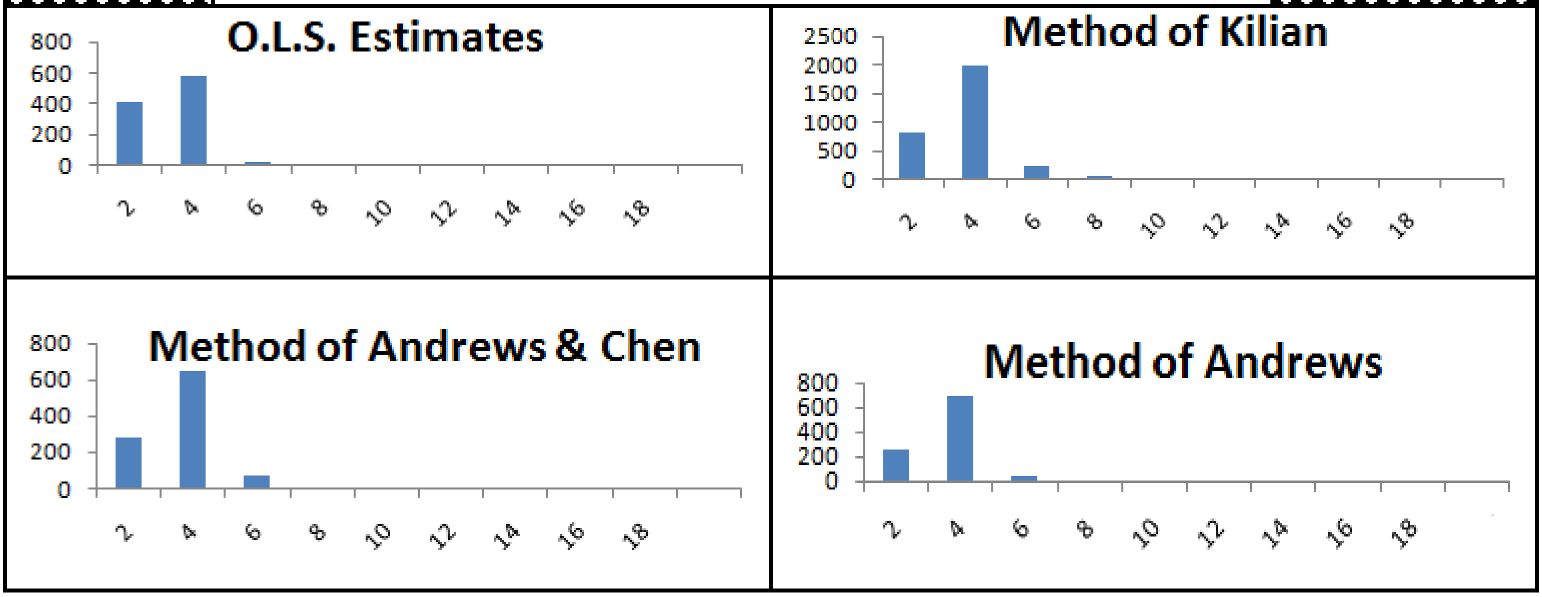


SPECIFICATIONS: T=100 / β=0,75 / h=2,409420									
	Estimator of the autoregressive parameter ($\widehat{\beta}$)							**Half-life estimator ($\widehat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,718749	0,725392	0,005301	0,060453	0,006278	0,361	0,639	2,159048	0,000
method of Kilian	0,751141	0,757964	0,005690	0,059626	0,005692	0,545	0,455	2,501265	0,000
method of Andrews and Chen	0,744899	0,752805	0,005617	0,058776	0,005643	0,511	0,489	2,441097	0,000
method of Andrews	0,741585	0,750000	0,005904	0,050785	0,005975	0,245	0,252	2,409421	0,000

Density Distribution of Estimators

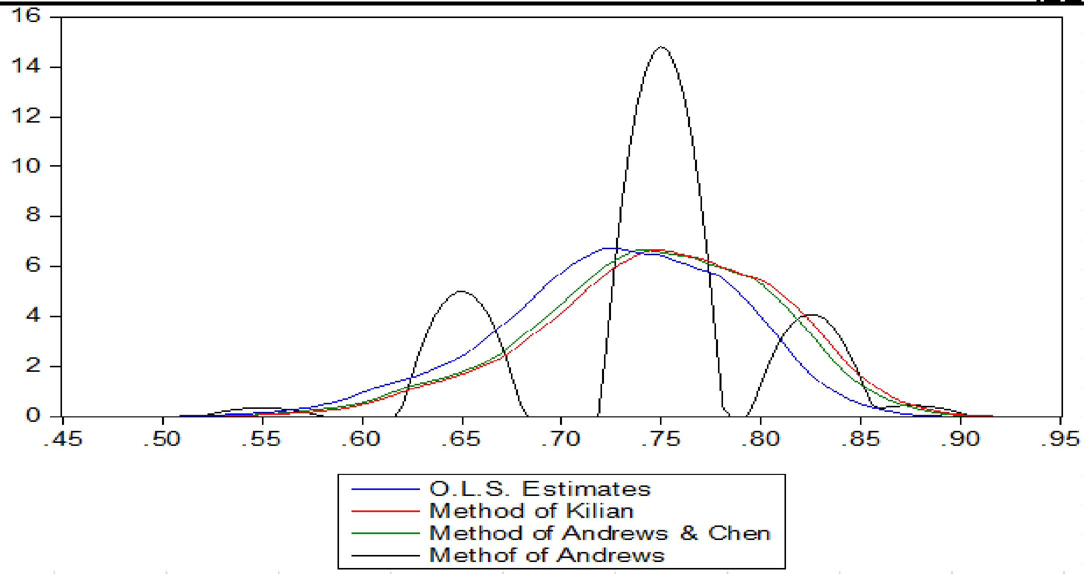


Histogram of Half-Life Estimators

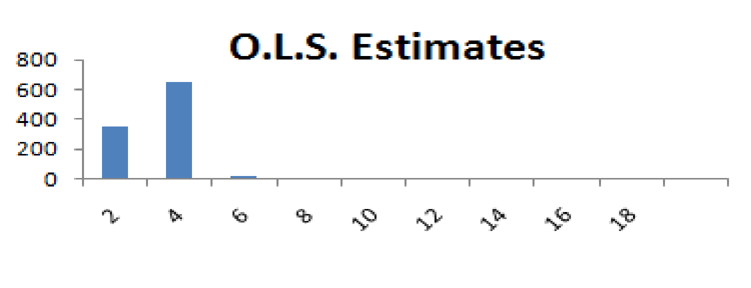


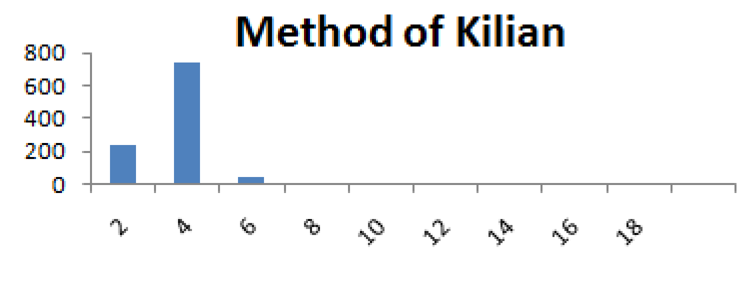
SPECIFICATIONS: T=150 / β=0,75 / h=2,409420									
	Estimator of the autoregressive parameter ($\hat{\beta}$)							**Half-life estimator ($\hat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,727270	0,730494	0,003322	0,048664	0,003838	0,368	0,632	2,207230	0,000
method of Kilian	0,748982	0,751828	0,003472	0,047130	0,003473	0,516	0,484	2,429978	0,000
method of Andrews and Chen	0,744225	0,746782	0,003442	0,046932	0,003476	0,479	0,521	2,373946	0,000
method of Andrews	0,741600	0,750000	0,003734	0,037800	0,003805	0,184	0,217	2,409421	0,000

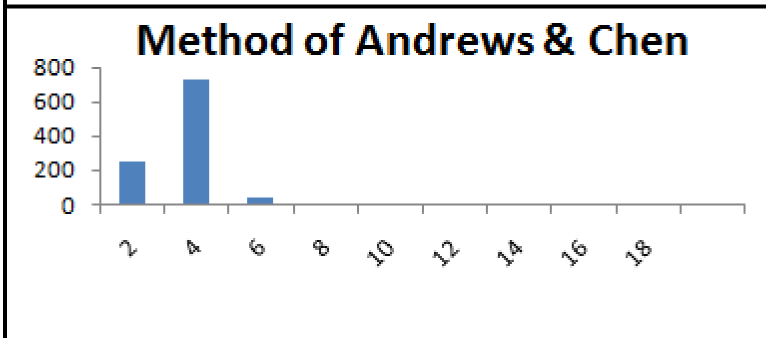
Density Distribution of Estimators

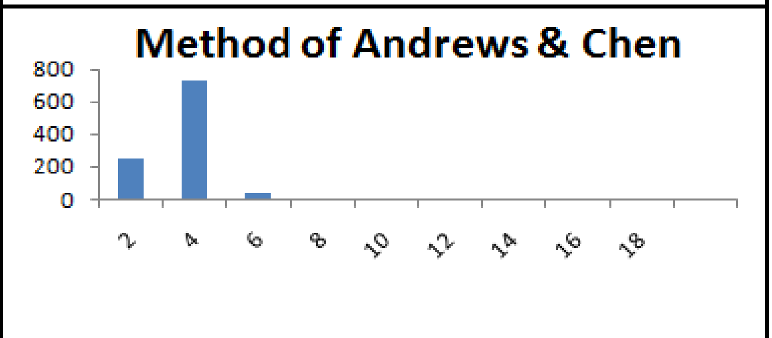


Histogram of Half-Life Estimators



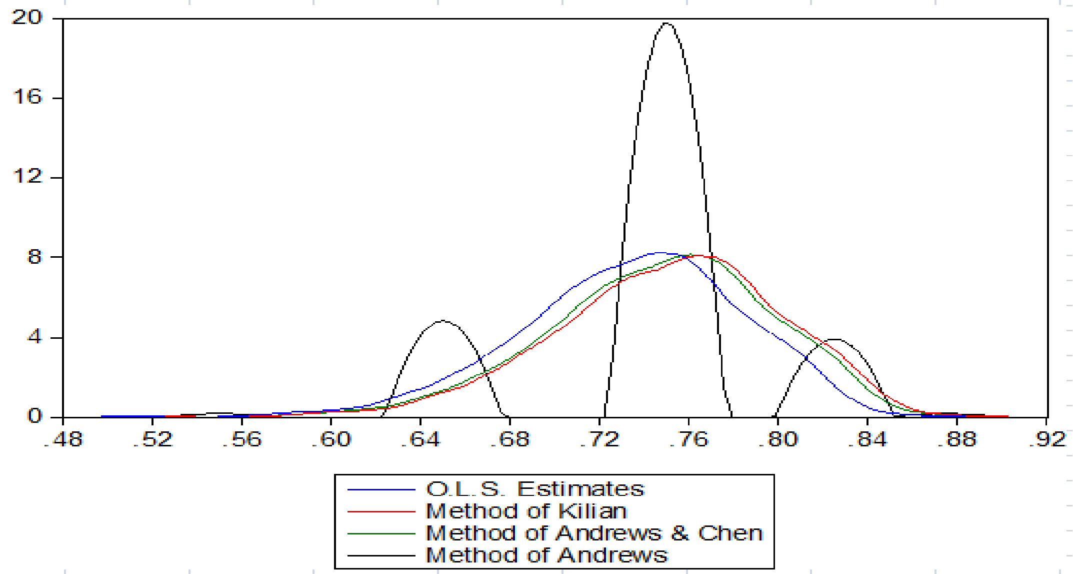






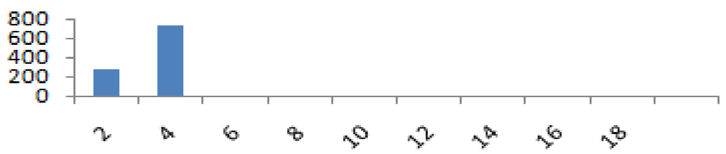
SPECIFICATIONS: T=200 / β=0,75 / h=2,409420									
	Estimator of the autoregressive parameter ($\widehat{\beta}$)							**Half-life estimator ($\widehat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,734189	0,738045	0,002443	0,040335	0,002693	0,405	0,595	2,281966	0,000
method of Kilian	0,750485	0,754365	0,002533	0,040115	0,002533	0,534	0,466	2,459032	0,000
method of Andrews and Chen	0,746804	0,750802	0,002501	0,039674	0,002511	0,506	0,494	2,418403	0,000
method of Andrews	0,742975	0,750000	0,002714	0,028775	0,002763	0,141	0,173	2,409421	0,000

Density Distribution of Estimators

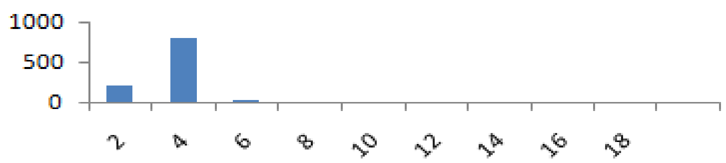


Histogram of Half-Life Estimators

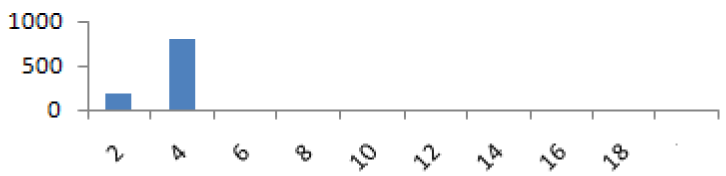
O.L.S. Estimates



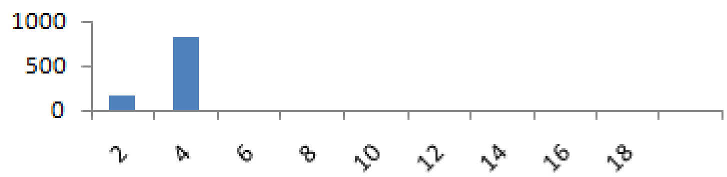
Method of Kilian



Method of Andrews & Chen



Method of Andrews



4.3.2. True autoregressive parameter equal to 0.80

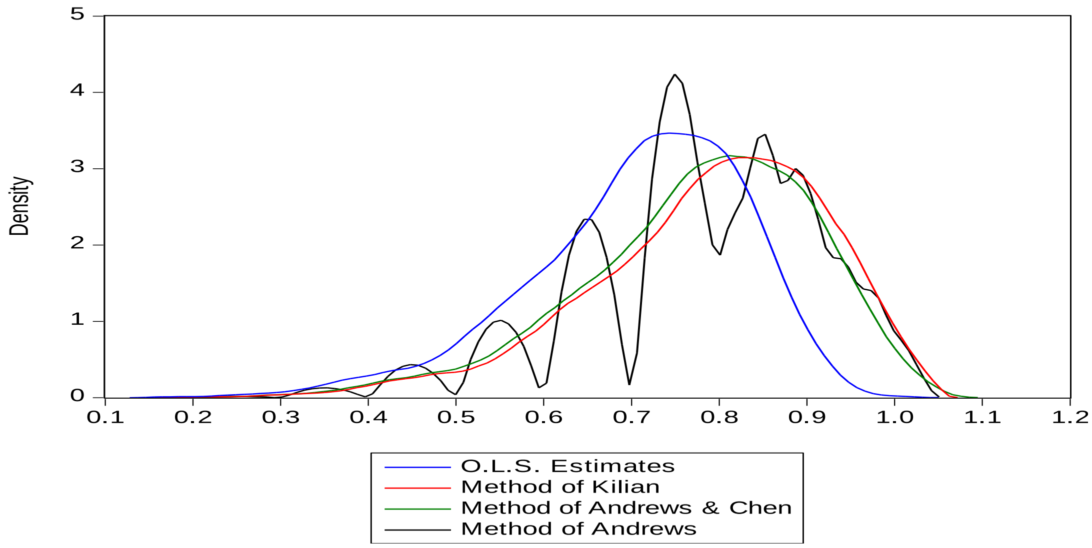
Table 5.2.presents the results for the case where the true autoregressive parameter is equal to 0,80. In this case, the computer intensive methods seems to perform really well, as both the mean and the median are very close to the true parameter value. In addition, the mean absolute bias and the mean square are very low. In contrast to the conclusions of the previous case, the Andrews and Chen estimator performs better here, but differences from the Kilian estimator are not significant. Moreover, it should be mentioned that the variance of simulated estimators for these two correction methods, is clearly lower than in the previous case. In respect to the analytical method of Andrews, it seems that its performance is the worst, in the context of this section. The values of the mean and the median are far from the true parameter, and the high variance indicates the presence of extreme values, which are confirmed by the Kernel-Density Graph, in which this method presents a high density for values that are far away from the true parameter. According to this graph, the other methods perform much better, but it is clear that they are not able to remove the bias completely. Furthermore, the Kilian estimator is the only one which tends to overestimate the true parameter and it is notable that the other two methods, have almost the same percentages of overestimation and underestimation.

If we focus on the half-life estimation, it is clear that the Andrews and Chen estimator performs better than the other two methods. The median are very close to the true half-life, and for sample size equal to 200, the estimated half-life is almost unbiased. This leads us to the conclusion that this method performs better as the sample size increases. Kilian's method performs equally well, but in cases where the sample size is really small it pushes the half-life to extremely high values. And this conclusion is confirmed by histograms, in which Kilian's method presents high frequency for values greater than 18, when the true half-life is 3,1062. In addition, this explains the fact that for all sample sizes, the median of corrected half-life is greater than its true value. The analytical formula of Andrews presents many problems and especially for large sample sizes, where the corrected half-life is more biased even from the uncorrected estimator. For T=40 and T=70 the use of this method seems to improve the situation, but still performs worst than the other methods. However, the half-life estimation of this method presents high frequency for values near to the true half-life (more than the other methods), as indicated by the histograms.

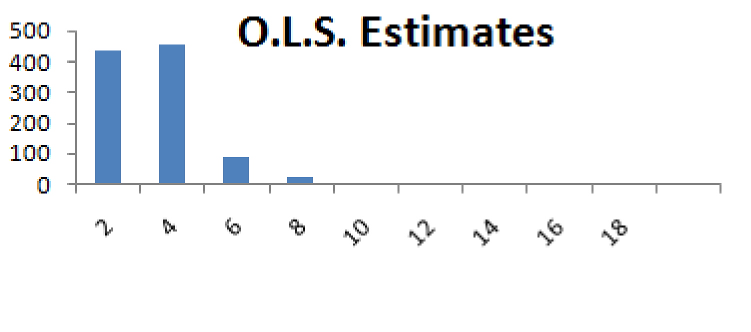
Table 4.2: *Bias-correction results (β=0.80)*

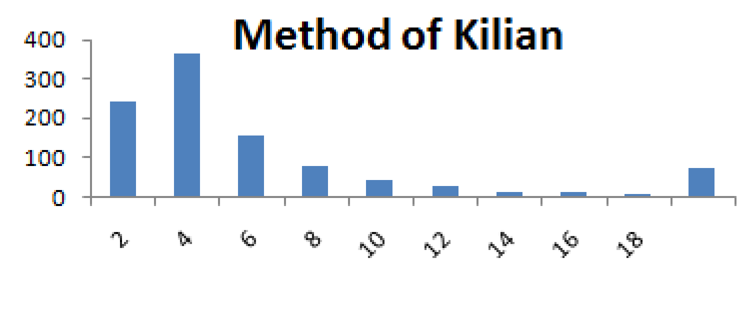
SPECIFICATIONS: T=40 / β=0,80 / h=3,106284									
	Estimator of the autoregressive parameter ($\widehat{\beta}$)							**Half-life estimator ($\widehat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,707958	0,723317	0,014877	0,114177	0,023349	0.240	0.760	2,139955	0,000
method of Kilian	0,789555	0,805903	0,017572	0,103671	0,017681	0,530	0,470	3,212104	0,031
method of Andrews and Chen	0,776593	0,792696	0,017378	0,103098	0,017925	0,476	0,524	2,983654	0,019
method of Andrews	0,778330	0,750000	0,017922	0,107430	0,018392	0,491	0,509	2,409421	0,036

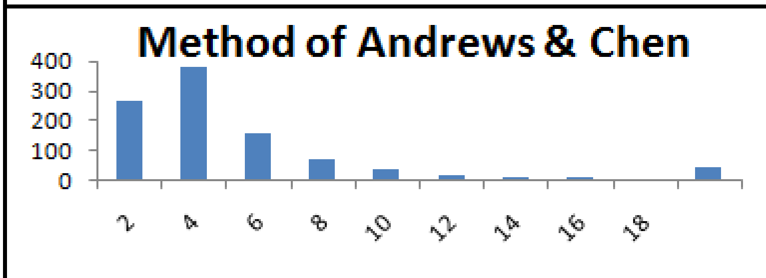
Density Distribution of Estimators

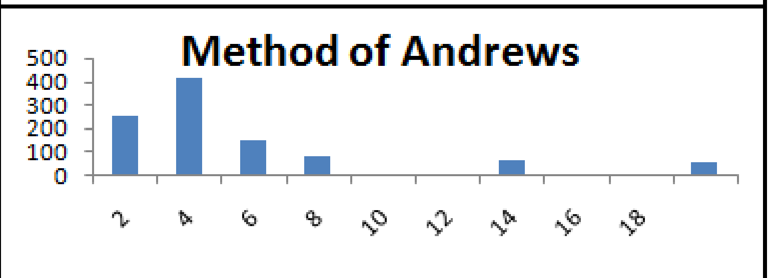


Histogram of Half-Life Estimators





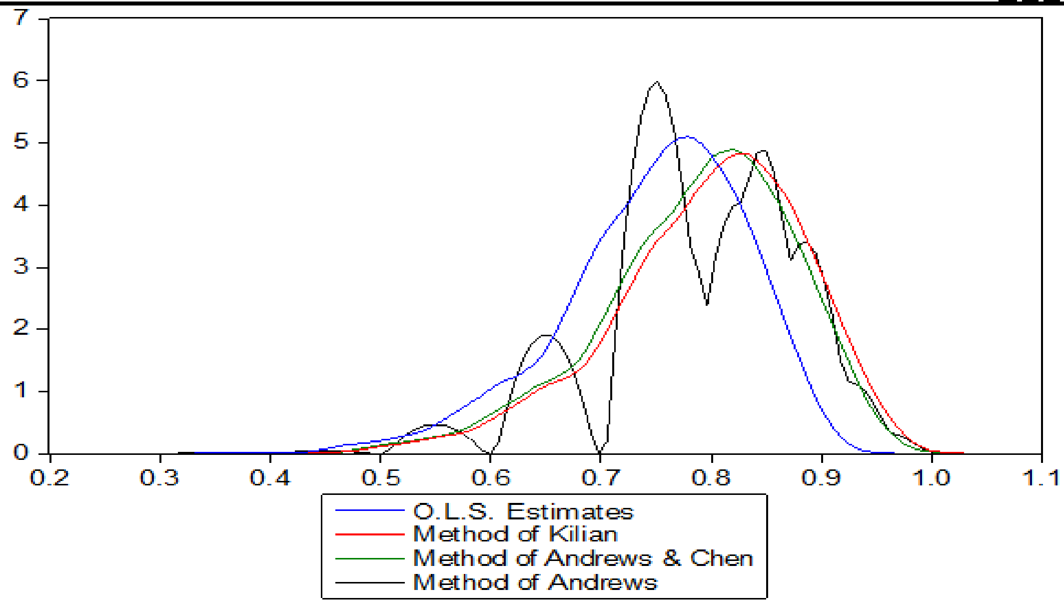




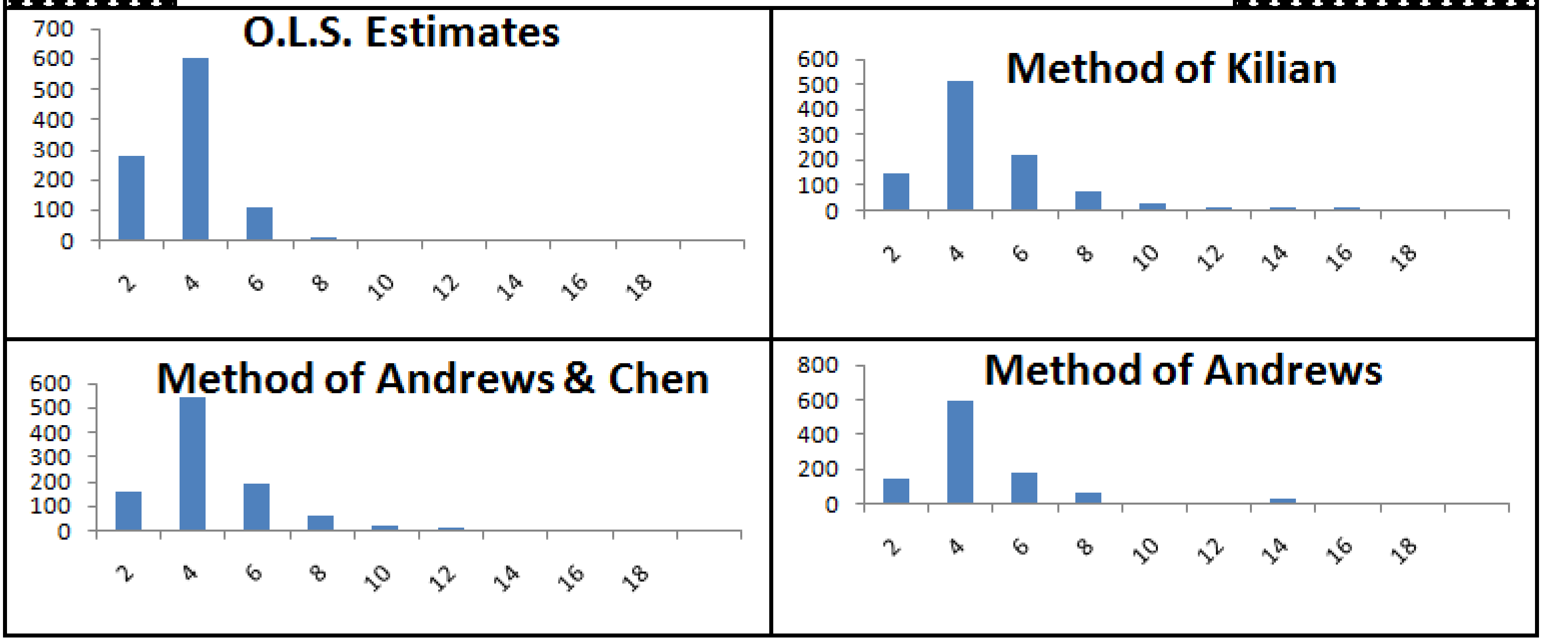
SPECIFICATIONS: T=70 / β=0,80 / h=3,106284

	Estimator of the autoregressive parameter ($\widehat{\beta}$)							Half-life estimator ($\widehat{h}$)	
	Mean	Median	Variance	Mean Absolute Bias	Mean Square Bias	% of over-estimation	% of under-estimation	median	% of extreme values
O.L.S. estimates	0,748433	0,760084	0,007164	0,074005	0,009823	0,296	0,704	2,526733	0,000
method of Kilian	0,796745	0,808355	0,007966	0,069624	0,007977	0,545	0,455	3,257976	0,001
method of Andrews and Chen	0,787753	0,799580	0,007835	0,068679	0,007985	0,498	0,502	3,098998	0,000
method of Andrews	0,786245	0,787500	0,008031	0,072845	0,008220	0,500	0,500	3,006292	0,001

Density Distribution of Estimators

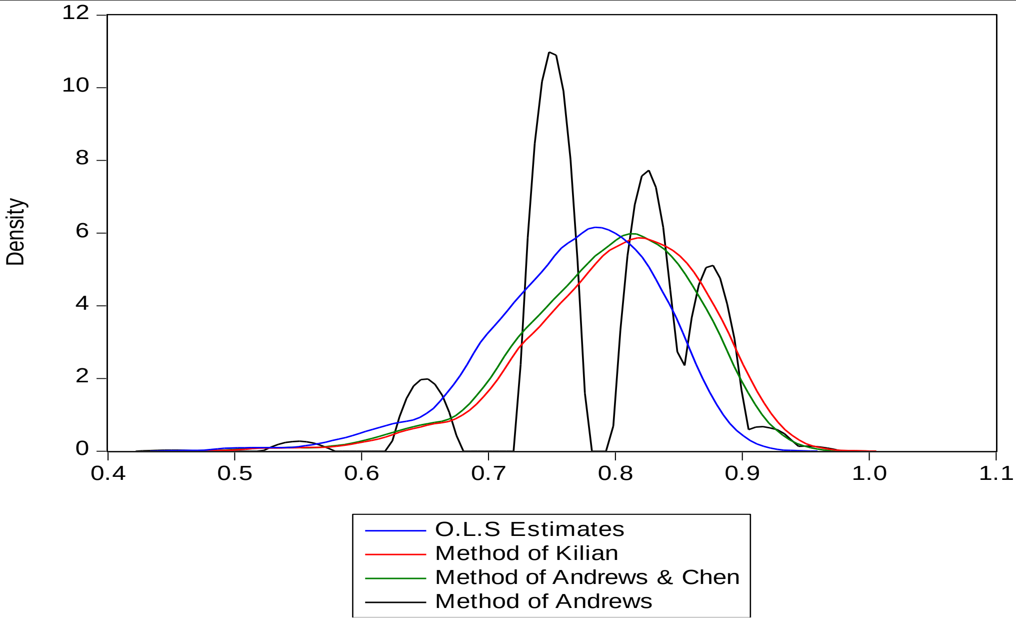


Histogram of Half-Life Estimators

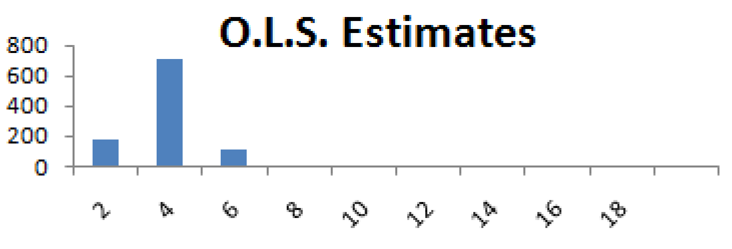


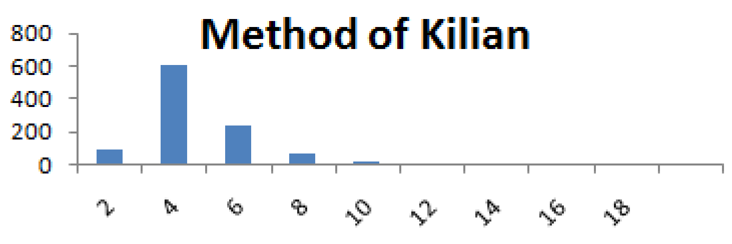
SPECIFICATIONS: T=100 / β=0,80 / h=3,106284									
	Estimator of the autoregressive parameter ($\widehat{\beta}$)							Half-life estimator ($\widehat{h}$)	
	Mean	Median	Variance	Mean Absolute Bias	Mean Square Bias	% of over-estimation	% of under-estimation	median	% of extreme values
O.L.S. estimates	0,765903	0,773683	0,004502	0,056678	0,005665	0,340	0,660	2,701344	0,000
method of Kilian	0,800138	0,806999	0,004861	0,054857	0,004861	0,547	0,453	3,232476	0,000
method of Andrews and Chen	0,793133	0,800410	0,004807	0,054178	0,004854	0,502	0,498	3,113432	0,000
method of Andrews	0,790625	0,825000	0,005038	0,059425	0,005126	0,506	0,494	3,603162	0,000

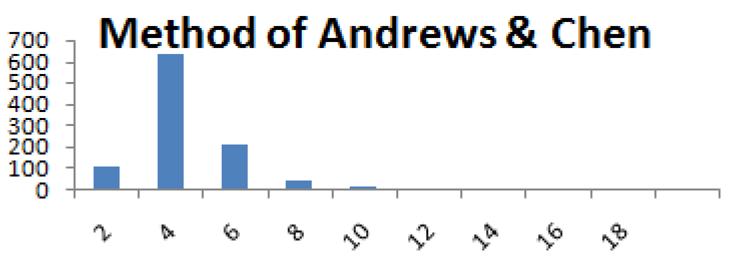
Density Distribution of Estimators

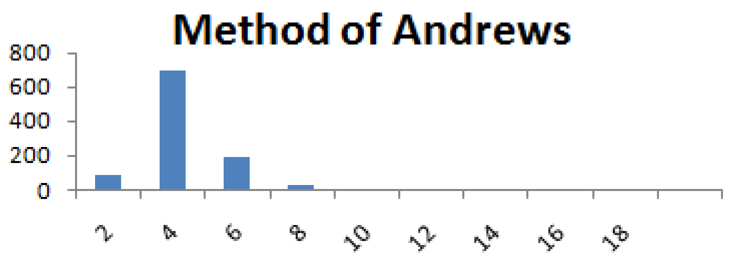


Histogram of Half-Life Estimators



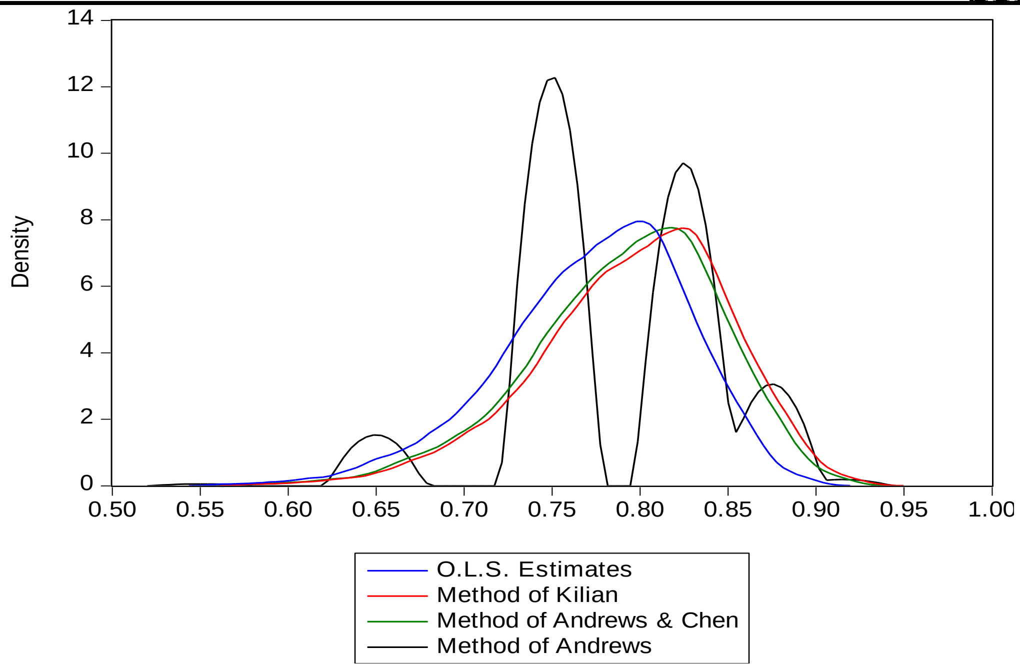




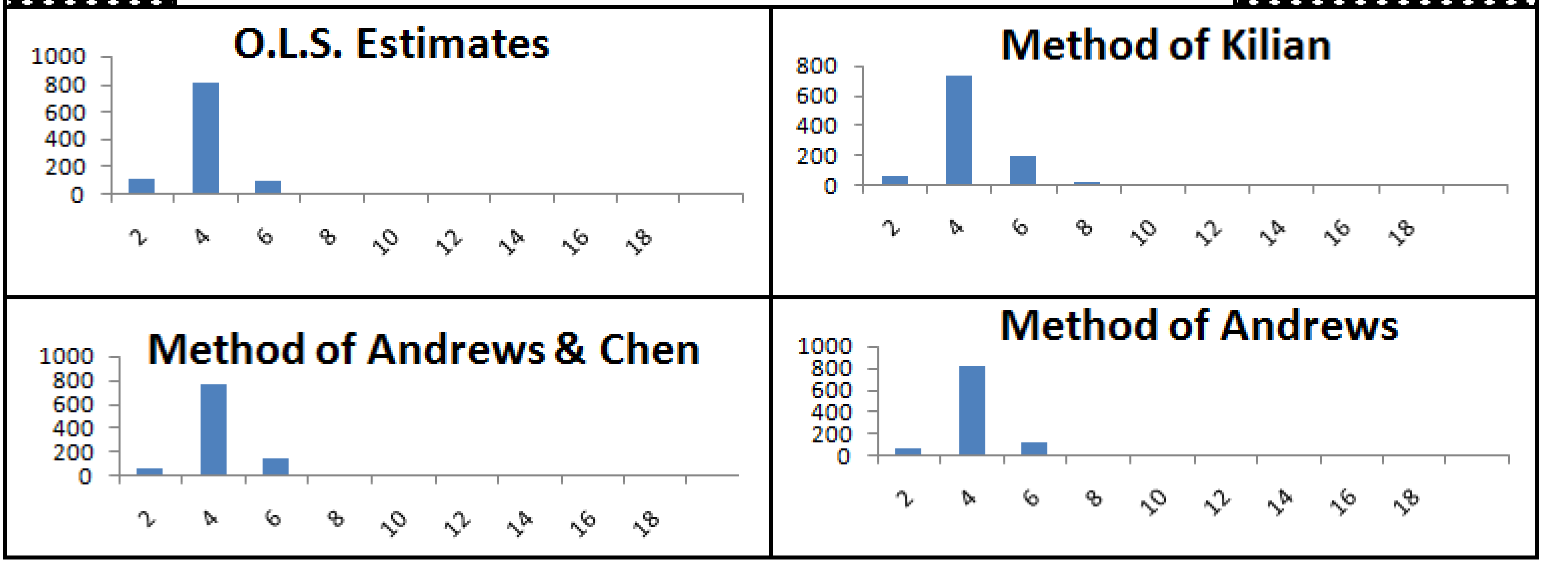


SPECIFICATIONS: T=150 / β=0,80 / h=3,106284									
	Estimator of the autoregressive parameter ($\widehat{\beta}$)							**Half-life estimator ($\widehat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0.773818	0.779603	0.002757	0.044559	0.003442	0.344	0.656	2,784054	0,000
method of Kilian	0.796508	0.802453	0.002892	0.042635	0.002905	0.515	0.485	3,149494	0,000
method of Andrews and Chen	0.791608	0.797135	0.002860	0.042317	0.002931	0.480	0.520	3,057125	0,000
method of Andrews	0.786380	0.750000	0.003329	0.050380	0.003514	0.482	0.518	2,409421	0,000

Density Distribution of Estimators



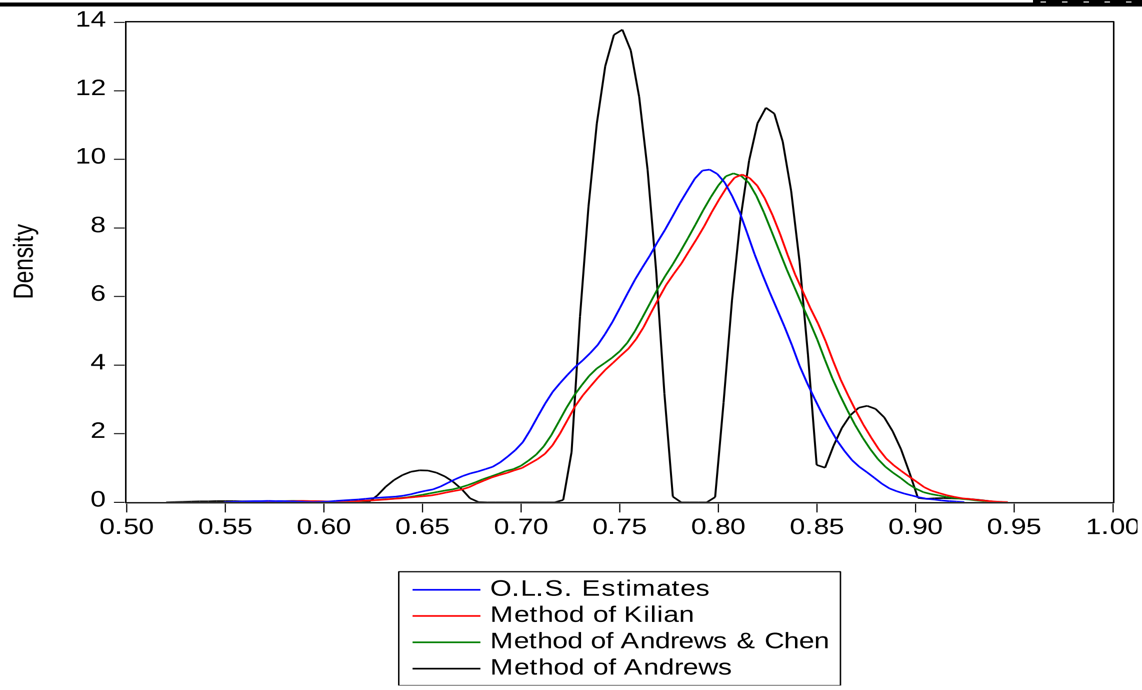
Histogram of Half-Life Estimators



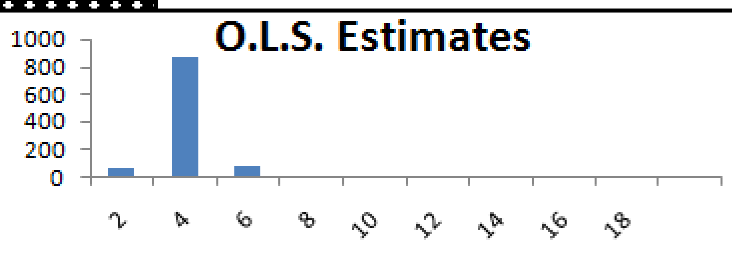
SPECIFICATIONS: T=200 / β=0,80 / h=3,106284

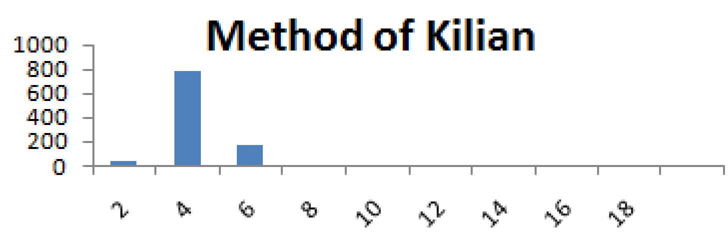
	Estimator of the autoregressive parameter ($\widehat{\beta}$)							Half-life estimator ($\widehat{h}$)	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,781529	0,786533	0,002081	0,037439	0,002422	0,360	0,640	2,886657	0,000
method of Kilian	0,798586	0,803646	0,002162	0,036486	0,002164	0,536	0,464	3,170899	0,000
method of Andrews and Chen	0,794659	0,799973	0,002132	0,036066	0,002161	0,500	0,500	3,105807	0,000
method of Andrews	0,788810	0,750000	0,002679	0,046210	0,002804	0,494	0,506	2,409421	0,000

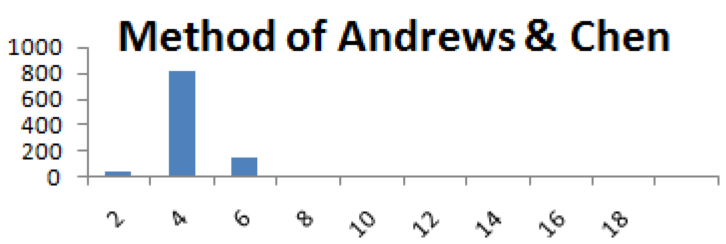
Density Distribution of Estimators

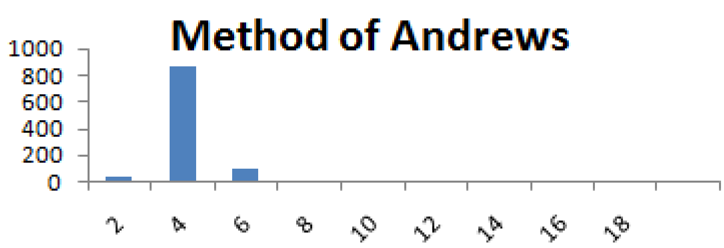


Histogram of Half-Life Estimators









4.3.3. True autoregressive parameter equal to 0.86

We now turn to the case where the true parameter value is equal to 0,86. We first consider the estimation of the autoregressive parameter. The two computer intensive methods continue to perform equally well. Values of the mean and the median are very close to the true one, and both the mean absolute bias and the mean square bias are low. However, these two methods have an important difference. On the one hand, the estimator of Kilian performs better for small sample sizes and as the sample size increases its performance deteriorates. On the other hand, the Andrews and Chen estimator has exactly the opposite characteristics as its performance gets better for increasing sample size. In addition the variance of the corrected estimators (for both correction-methods) is higher than the variance of the estimators without correction. This observation leads us to the conclusion that in some cases both methods can push the estimator to extremely high values. Turning to the Andrews estimator, it is clear that its performance is not satisfactory, as the high variance, the mean absolute bias and the mean square bias indicate. Moreover, this method presents high density for values far from the true parameter, and especially in cases where the sample size is large. However, it is the only method that has really high density at regions close to the true parameter.

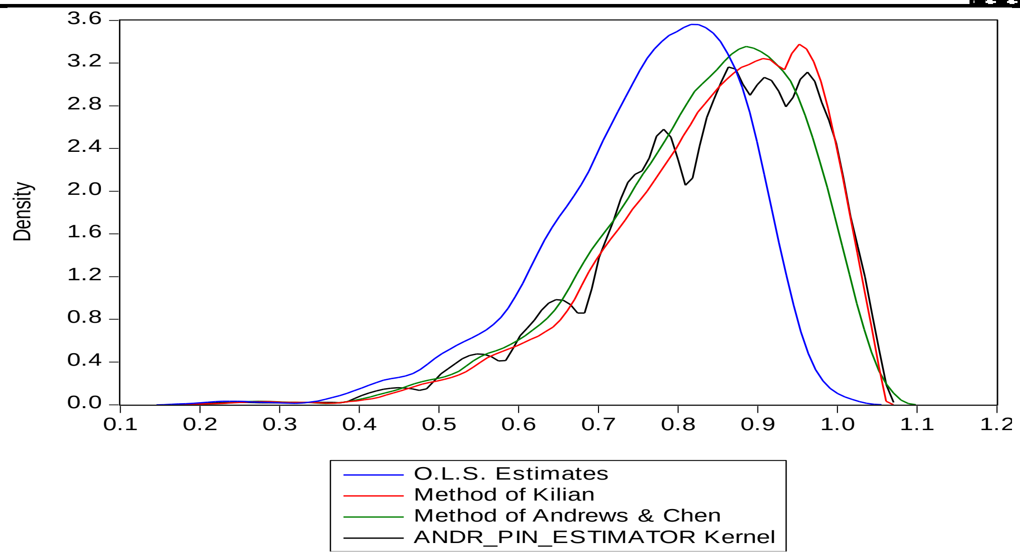
Regarding the half-life estimation, the two simulation-based methods have the same characteristics as to the previous cases: they perform equally well but the method of Andrews and Chen performs better as the sample size increases, when Kilian's method does not "work" well for large sample sizes. Moreover, the Kilian estimator seems to lead the half-life estimation to extremely high values more often than the method of Andrews and Chen (especially for small samples), as indicated by the percentages of extreme values and by the histograms. The analytical formula of Andrews does not "work" well neither in this case. The main problem of this method arises from the fact that it tends to overestimate the true half-life and, in addition, it tends to push the half-life to extreme values more often than every other method (for sample size equal to 40 this method leads the estimated half-life to extreme values in 126 out of 1000 replications).

Table 4.3: *Bias-correction results (β=0.86)*

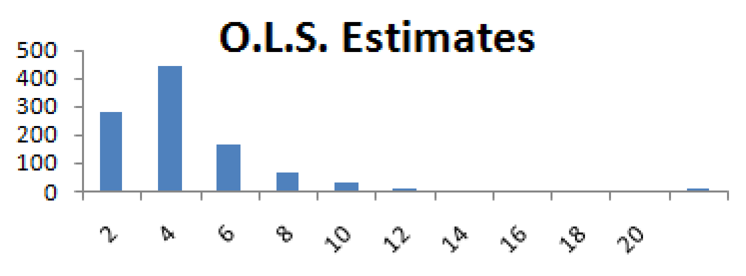
SPECIFICATIONS: T=40 / β=0,86 / h=4,59576913

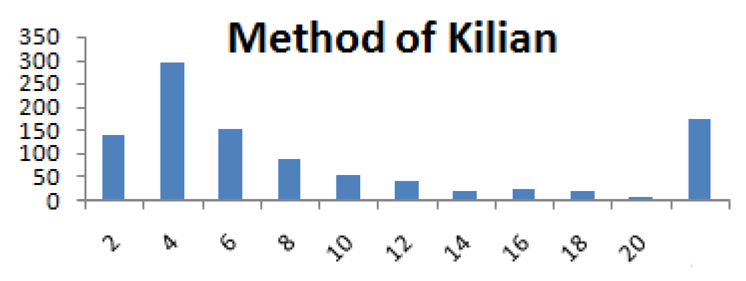
	Estimator of the autoregressive parameter ($\hat{\beta}$)							Half-life estimator ($\hat{h}$)	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	median	% of extreme values
O.L.S. estimates	0,759414	0,778416	0,014230	0,116473	0,024347	0,209	0,791	2,767125	0,000
method of Kilian	0,840891	0,861271	0,016249	0,099840	0,016614	0,503	0,497	4,641232	0,110
method of Andrews and Chen	0,826364	0,847439	0,016117	0,099667	0,017249	0,457	0,543	4,187275	0,048
method of Andrews	0,834565	0,875000	0,017580	0,104925	0,018227	0,522	0,478	5,190893	0,126

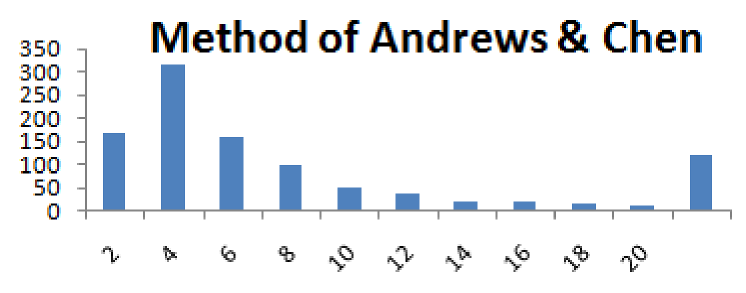
Density Distribution of Estimators

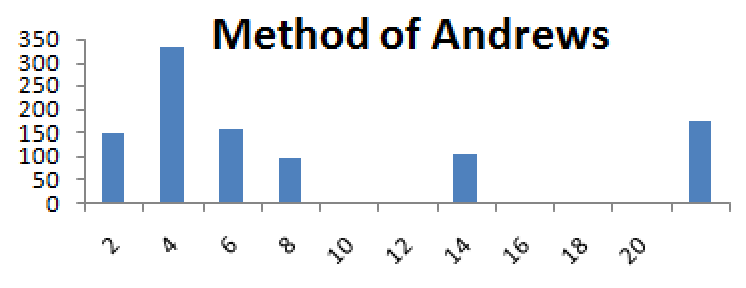


Histogram of Half-Life Estimators



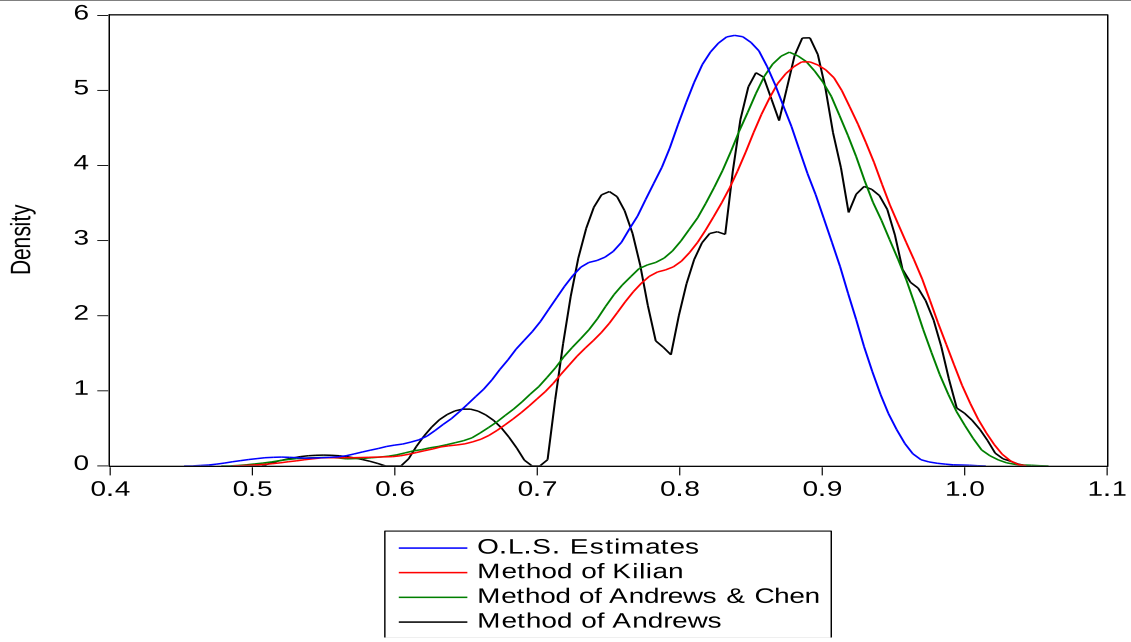




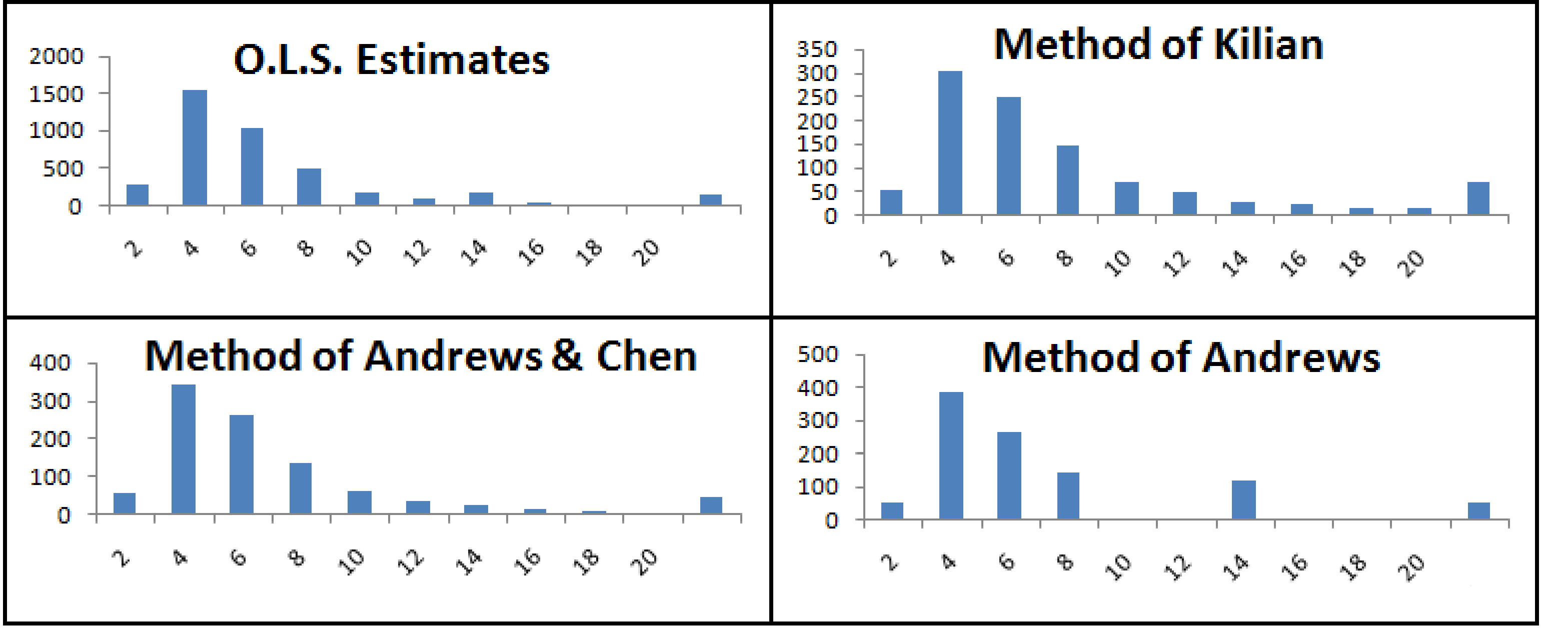


SPECIFICATIONS: T=70 / β=0,86 / h=4,59576913									
	Estimator of the autoregressive parameter ($\widehat{\beta}$)							**Half-life estimator ($\widehat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,807097	0,821246	0,006084	0,069905	0,008882	0,267	0,733	3,519712	0,000
method of Kilian	0,858014	0,872264	0,006851	0,065342	0,006855	0,555	0,445	5,071922	0,010
method of Andrews and Chen	0,847627	0,861043	0,006697	0,063965	0,006851	0,503	0,497	4,633017	0,004
method of Andrews	0,847290	0,875000	0,007360	0,068240	0,007521	0,565	0,435	5,190893	0,010

Density Distribution of Estimators



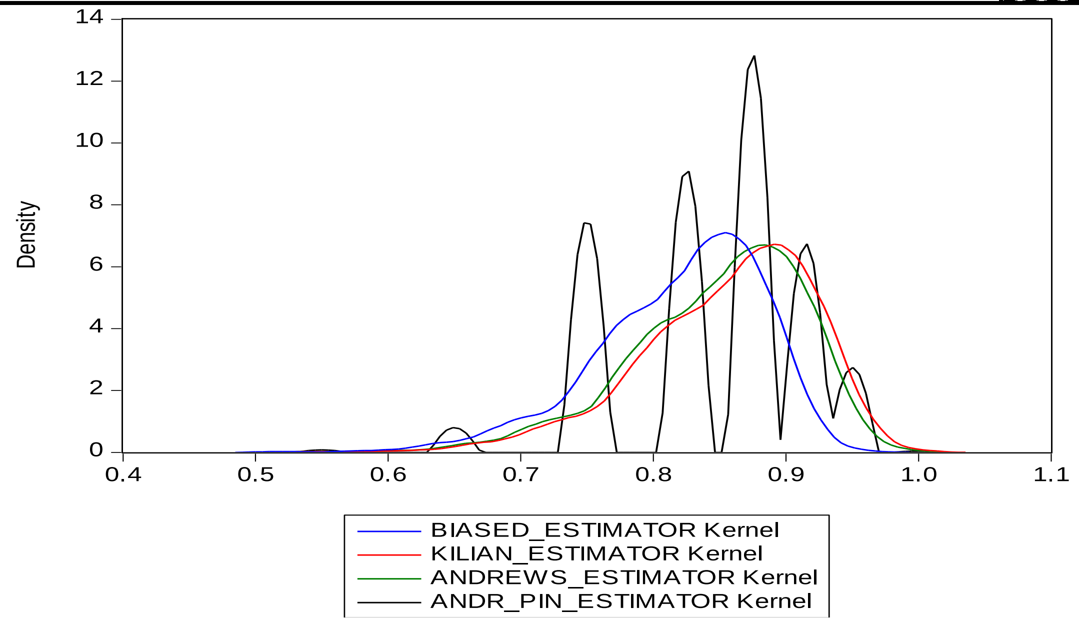
Histogram of Half-Life Estimators



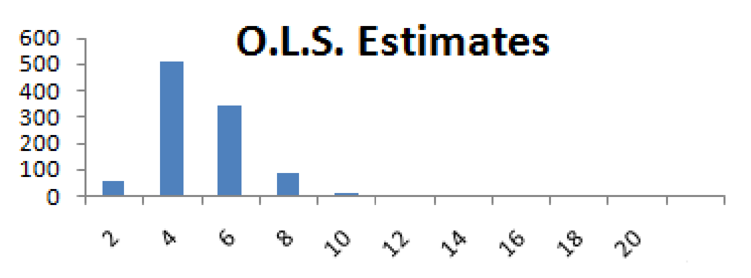
SPECIFICATIONS: T=100 / β=0,86 / h=4,59576913

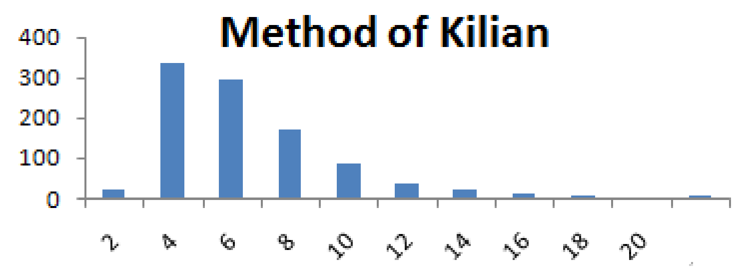
	Estimator of the autoregressive parameter ($\widehat{\beta}$)							Half-life estimator ($\widehat{h}$)	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,820646	0,832287	0,003851	0,054231	0,005399	0,289	0,711	3,775767	0,000
method of Kilian	0,856331	0,868171	0,004201	0,051192	0,004215	0,544	0,456	4,903195	0,001
method of Andrews and Chen	0,848921	0,859331	0,004153	0,050544	0,004276	0,495	0,505	4,572179	0,000
method of Andrews	0,846670	0,875000	0,004616	0,053990	0,004794	0,559	0,441	5,190893	0,001

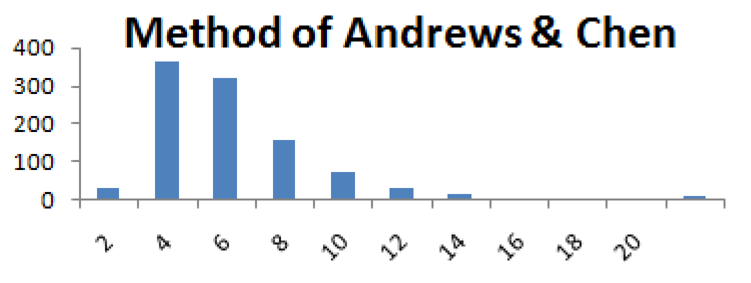
Density Distribution of Estimators

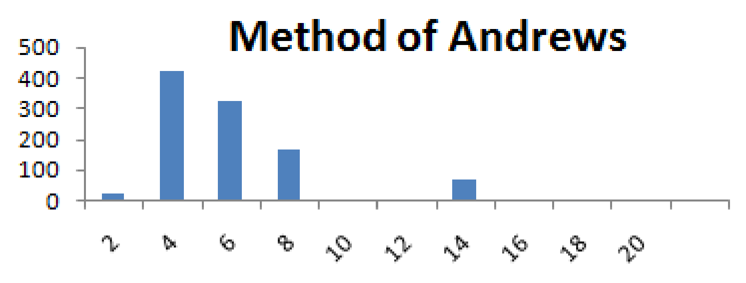


Histogram of Half-Life Estimators





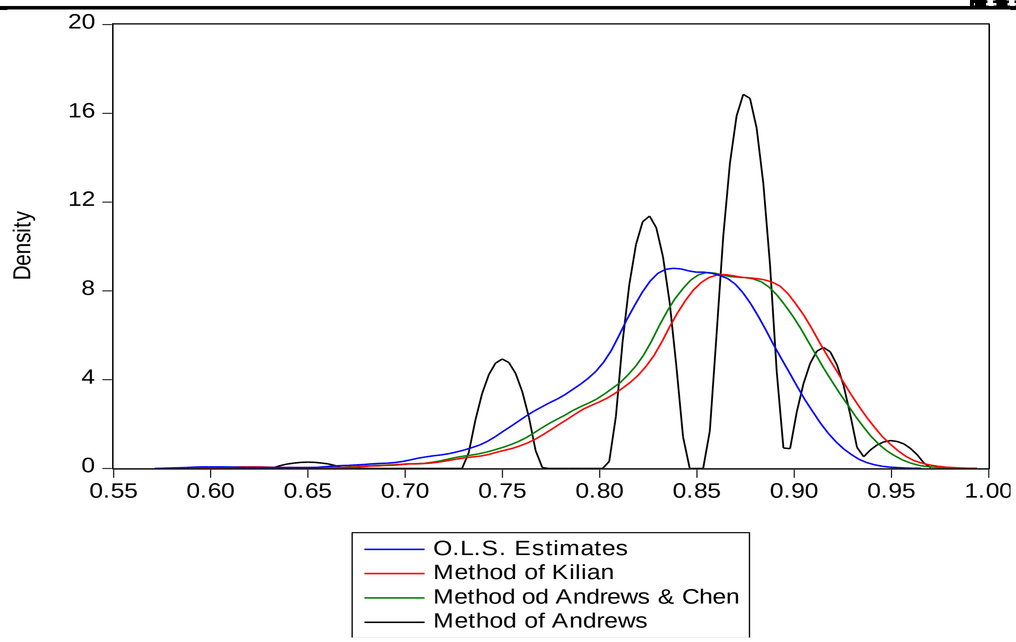




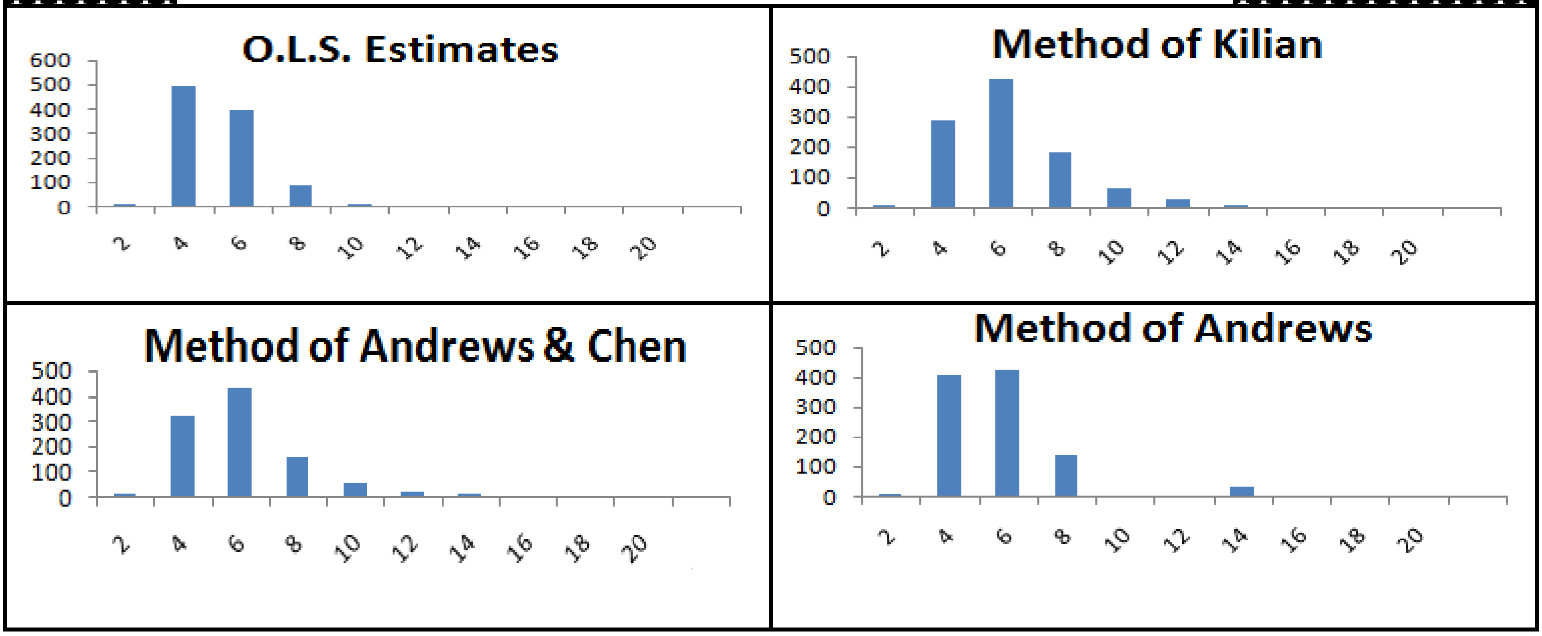
SPECIFICATIONS: T=150 / β=0,86 / h=4,59576913

	Estimator of the autoregressive parameter ($\widehat{\beta}$)							Half-life estimator ($\widehat{h}$)	
	Mean	Median	Variance	Mean Absolute Bias	Mean Square Bias	% of over-estimation	% of under-estimation	median	% of extreme values
O.L.S. estimates	0,836002	0,840425	0,002219	0,039324	0,002795	0,338	0,662	3,987102	0,000
method of Kilian	0,860183	0,864869	0,002340	0,037429	0,002340	0,540	0,460	4,774501	0,000
method of Andrews and Chen	0,854702	0,859462	0,002319	0,037071	0,002347	0,496	0,504	4,576790	0,000
method of Andrews	0,851750	0,875000	0,002818	0,041340	0,002886	0,588	0,412	5,190893	0,000

Density Distribution of Estimators



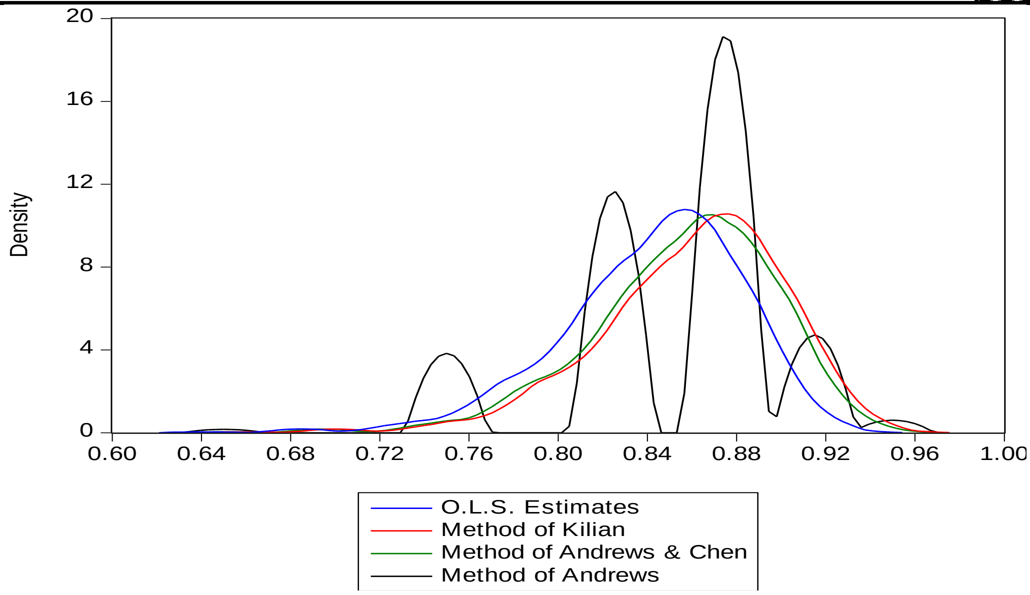
Histogram of Half-Life Estimators



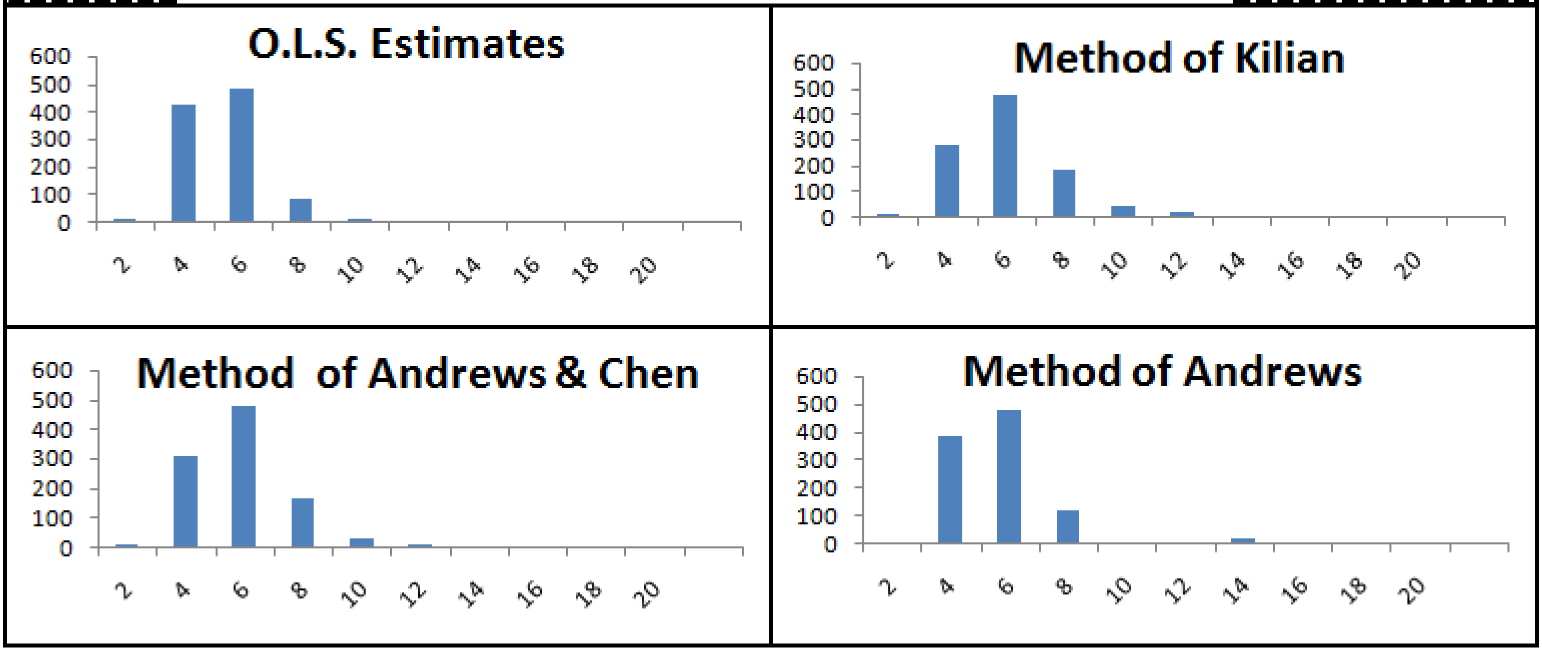
SPECIFICATIONS: T=200 / β=0,86 / h=4,59576913

	Estimator of the autoregressive parameter ($\widehat{\beta}$)							Half-life estimator ($\widehat{h}$)	
	Mean	Median	Variance	Mean Absolute Bias	Mean Square Bias	% of over-estimation	% of under-estimation	median	% of extreme values
O.L.S. estimates	0,842620	0,848170	0,001634	0,032832	0,001936	0,364	0,636	4,209207	0,000
method of Kilian	0,860752	0,866165	0,001695	0,032292	0,001695	0,560	0,440	4,824263	0,000
method of Andrews and Chen	0,856515	0,861519	0,001687	0,031816	0,001699	0,516	0,484	4,650167	0,000
method of Andrews	0,853530	0,875000	0,002222	0,036410	0,002264	0,611	0,389	5,190893	0,000

Density Distribution of Estimators



Histogram of Half-Life Estimators



4.3.4. True autoregressive parameter equal to 0.92

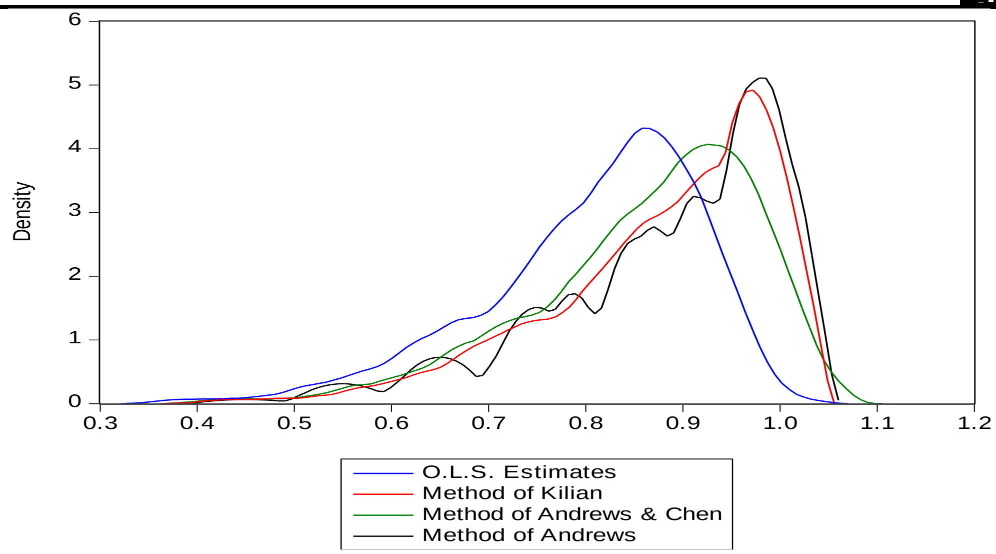
Table 4.4 reports the results for the case where the true autoregressive parameter is equal to 0,92. In this case, the analytical formula of Andrews seems to perform equally well to the computer intensive methods, as the values of mean and median indicates. Especially for small sample sizes, this method performs extremely well and in some cases it surpasses the performance of the other two methods. However, it presents the highest values of variance, something that indicates the tendency of this method to over-correct the estimators. This conclusion is confirmed by the Kernel-Density Graph, as this method presents really high density for values much higher than the true parameter. As for the other two methods, results confirm the conclusions of the previous sections, according to which Kilian's method "work" better for small samples, and the method of Andrews and Chen for large sample sizes. In addition, the Kilian estimator is the only one which tends to overestimate the true autoregressive parameter.

Regarding the measurement of half-life, it is clear that the Andrews estimator has weaknesses to correct the half-life estimator efficiently, for really small sample sizes (T=40). The distance between the median of estimated half-lifes and its true value is really high. However, as the sample size increases, the performance of this method gets better. Based on the conclusions of previous sections one would expect that Kilian's method would not be able to correct the bias in large sample sizes. But this is not the case. The performance of the Kilian estimator may fall as the sample size increases, but it never presents such weaknesses. Besides, it is the only case where the correction method of Andrews and Chen pushes the half-life estimator to extremely high values so often (for sample size equal to 70, this method leads to extreme values for 47 out of 1.000 replications). In addition, it should be mentioned that both computer-intensive methods lead half-life to much higher values than the real one in many cases, as indicated by the histograms. However, it is noteworthy the fact that in most cases these methods continues to perform sufficiently well even for more persistent processes, which are more sensitive to "finite sample bias". The analytical formula, on the other hand, does not "work" effectively neither in this case. It leads to biased half-life estimations very frequently, as indicated by the histograms. Moreover, its median is far away from the true half-life, and it leads to extremely high values more often than any other method (especially for small sample sizes).

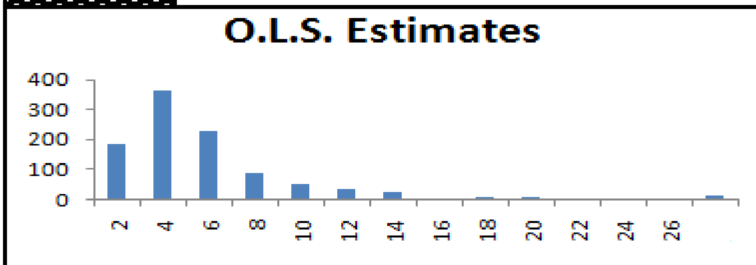
Table 4.4: *Bias-correction results (β=0.92)*

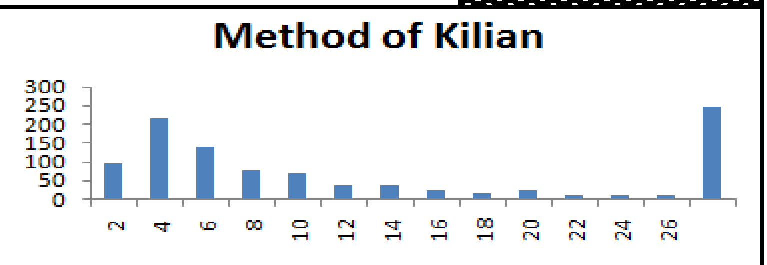
SPECIFICATIONS: T=40 / β=0,92 / h=8,312950									
	Estimator of the autoregressive parameter ($\hat{\beta}$)							**Half-life estimator ($\hat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,806769	0,829695	0,012252	0,120160	0,025073	0,129	0,871	3,712672	0,005
method of Kilian	0,879344	0,906838	0,012920	0,088758	0,014573	0,462	0,538	7,088031	0,181
method of Andrews and Chen	0,862369	0,885606	0,013031	0,093064	0,016353	0,368	0,632	5,705702	0,085
method of Andrews	0,885140	0,915000	0,013866	0,091520	0,015081	0,457	0,543	7,802969	0,250

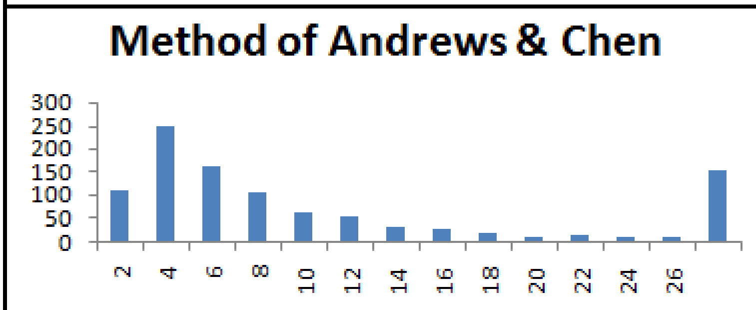
Density Distribution of Estimators

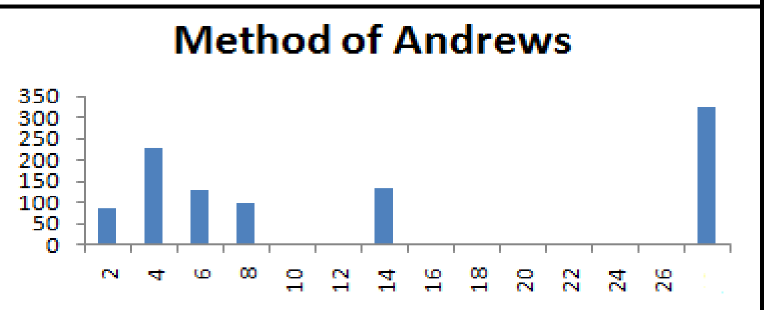


Histogram of Half-Life Estimators



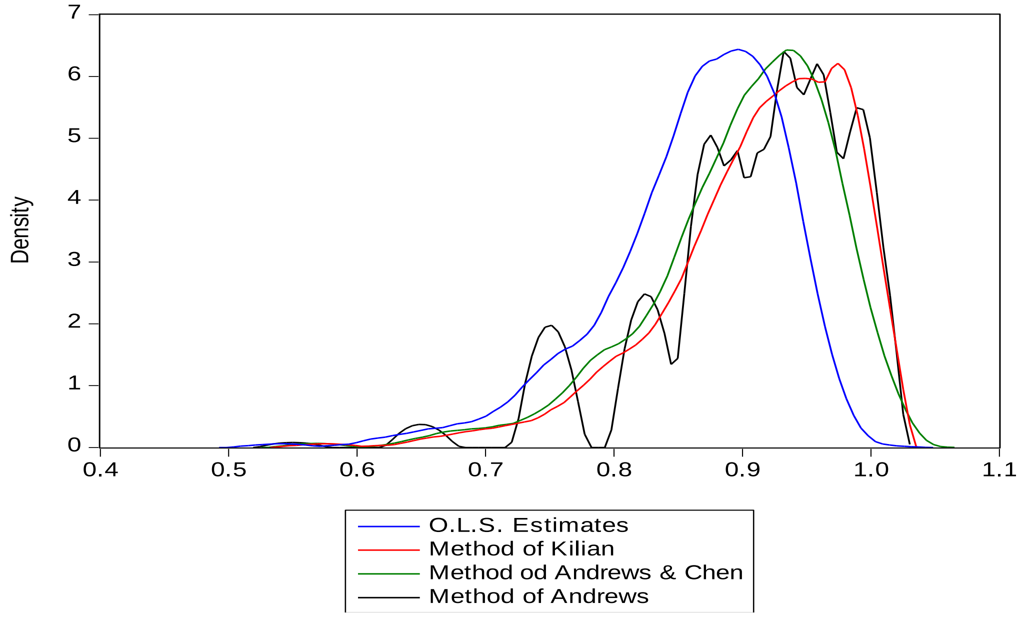




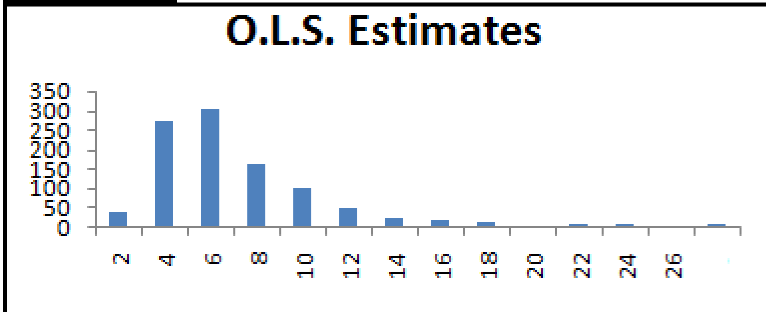


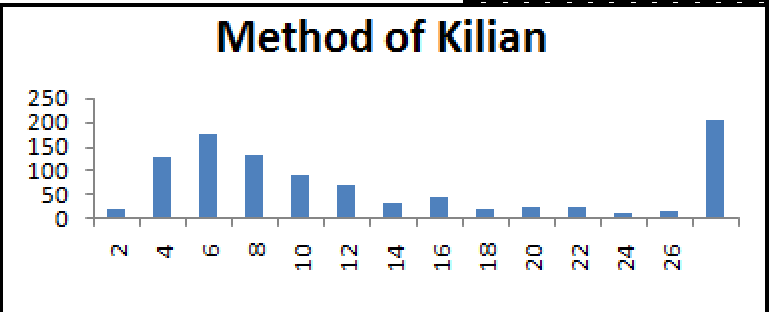
SPECIFICATIONS: T=70 / β=0,92 / h=8,312950									
	Estimator of the autoregressive parameter ($\hat{\beta}$)							**Half-life estimator ($\hat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,862479	0,874351	0,004931	0,066308	0,008240	0,204	0,796	5,162229	0,001
method of Kilian	0,911187	0,925198	0,005318	0,055658	0,005396	0,528	0,472	8,915330	0,000
method of Andrews and Chen	0,899239	0,912387	0,005249	0,055205	0,005680	0,451	0,549	7,559632	0,047
method of Andrews	0,907305	0,915000	0,006024	0,058645	0,006185	0,447	0,553	7,802969	0,140

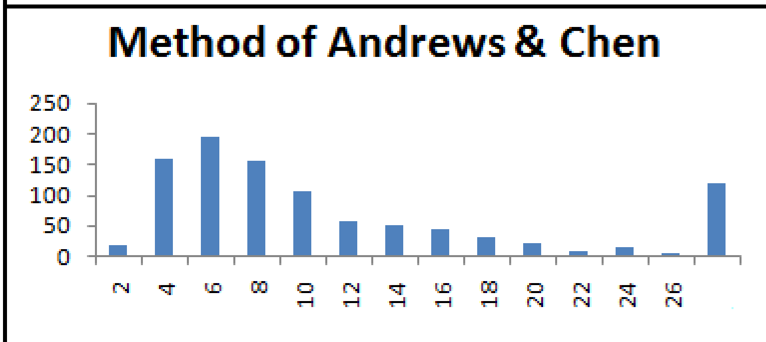
Density Distribution of Estimators

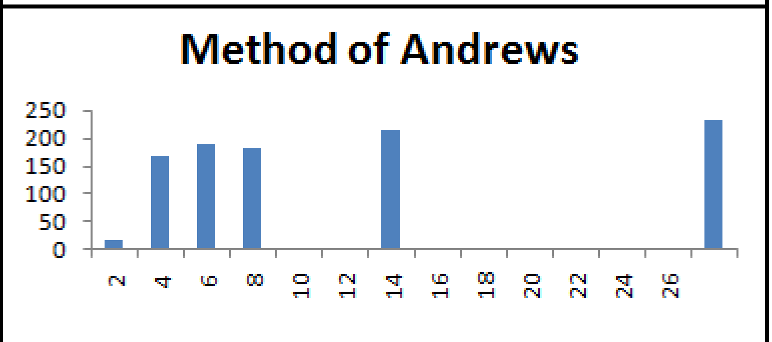


Histogram of Half-Life Estimators



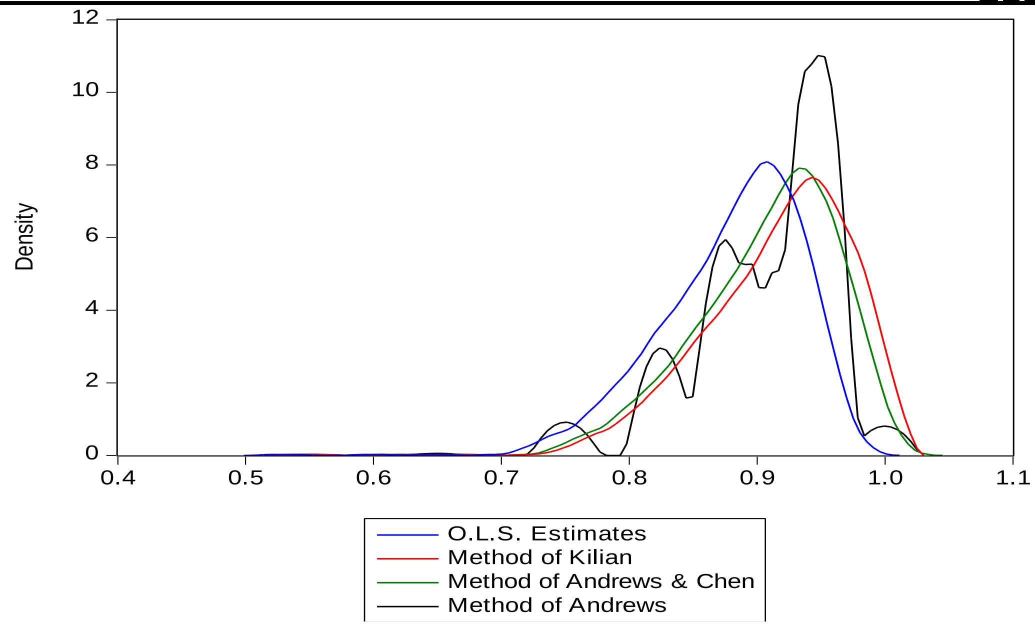




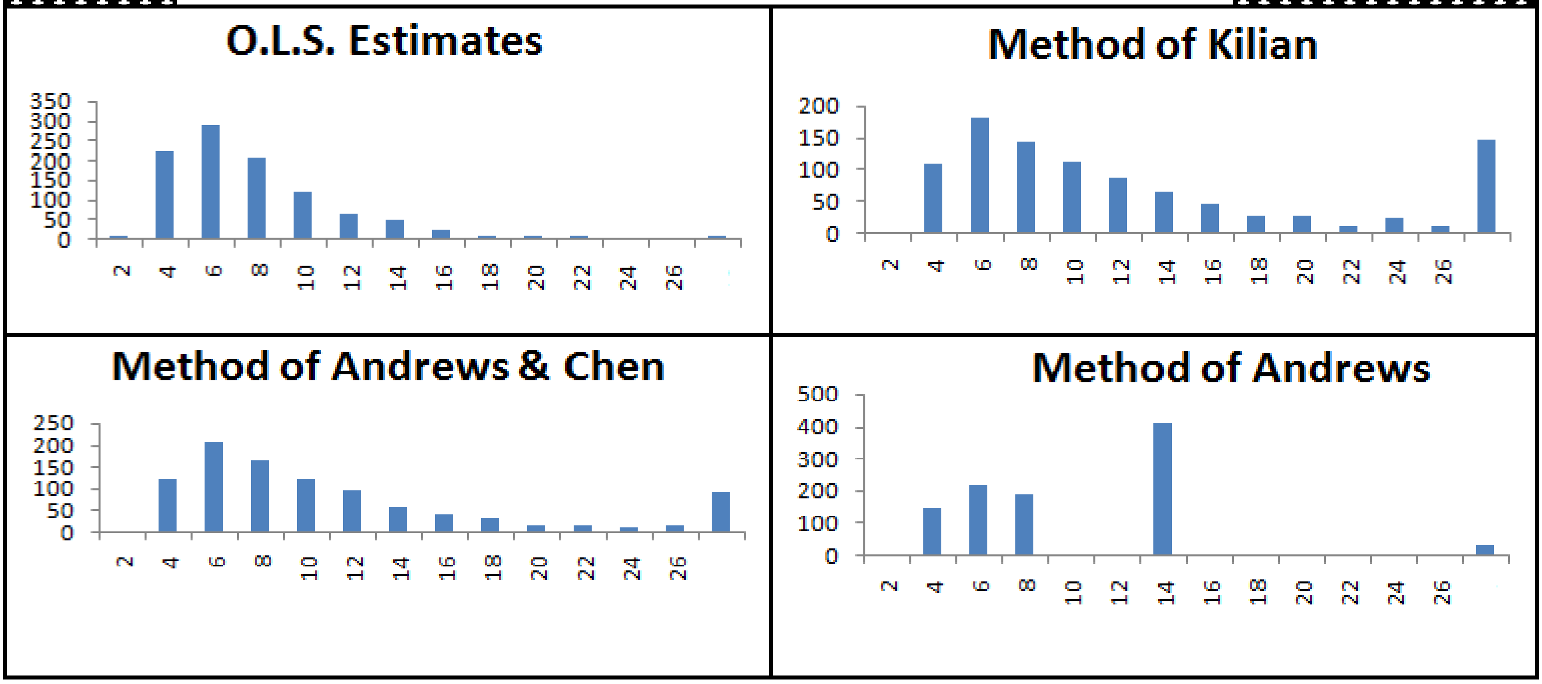


SPECIFICATIONS: T=100 / β=0,92 / h=8,312950									
	Estimator of the autoregressive parameter ($\widehat{\beta}$)							**Half-life estimator ($\widehat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,880089	0,889490	0,002999	0,050095	0,004592	0,252	0,748	5,918907	0,000
method of Kilian	0,916092	0,926318	0,003254	0,044950	0,003269	0,547	0,453	9,056256	0,052
method of Andrews and Chen	0,907314	0,917365	0,003174	0,043926	0,003335	0,479	0,521	8,036529	0,024
method of Andrews	0,906800	0,915000	0,003098	0,042780	0,003272	0,443	0,557	7,802969	0,030

Density Distribution of Estimators



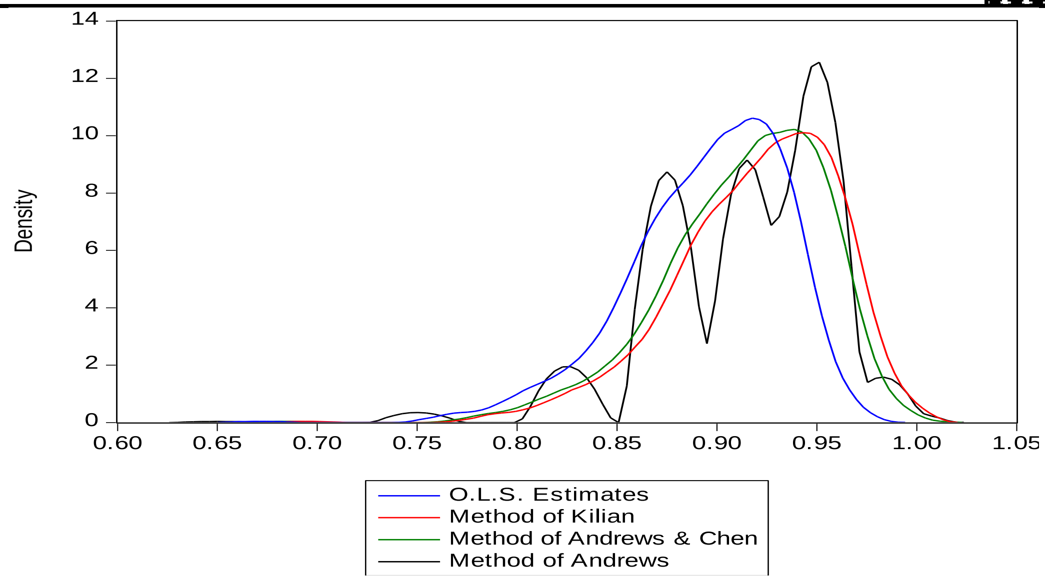
Histogram of Half-Life Estimators



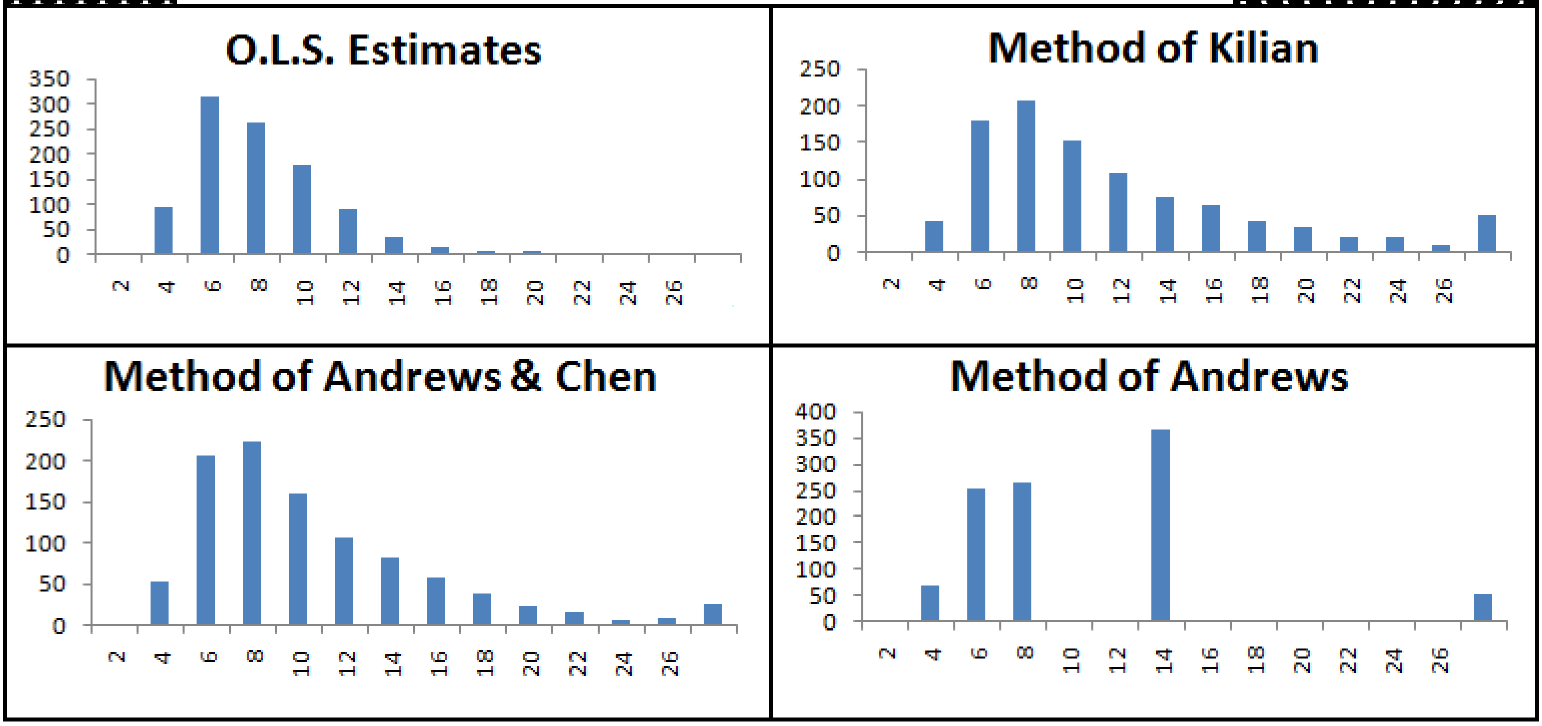
SPECIFICATIONS: T=150 / β=0,92 / h=8,312950

	Estimator of the autoregressive parameter ($\widehat{\beta}$)							Half-life estimator ($\widehat{h}$)	
	Mean	Median	Variance	Mean Absolute Bias	Mean Square Bias	% of over-estimation	% of under-estimation	median	% of extreme values
O.L.S. estimates	0,895118	0,900941	0,001523	0,034532	0,002142	0,303	0,697	6,644729	0,000
method of Kilian	0,919922	0,925164	0,001635	0,032132	0,001635	0,555	0,445	8,911127	0,010
method of Andrews and Chen	0,913888	0,919568	0,001601	0,031448	0,001638	0,494	0,506	8,266404	0,002
method of Andrews	0,914080	0,915000	0,001887	0,034170	0,001922	0,416	0,584	7,802969	0,009

Density Distribution of Estimators

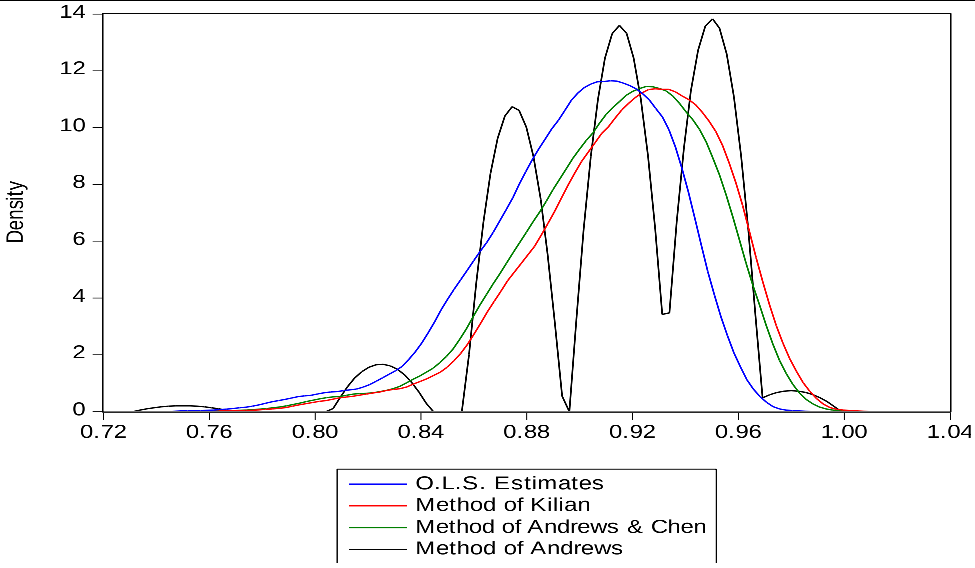


Histogram of Half-Life Estimators

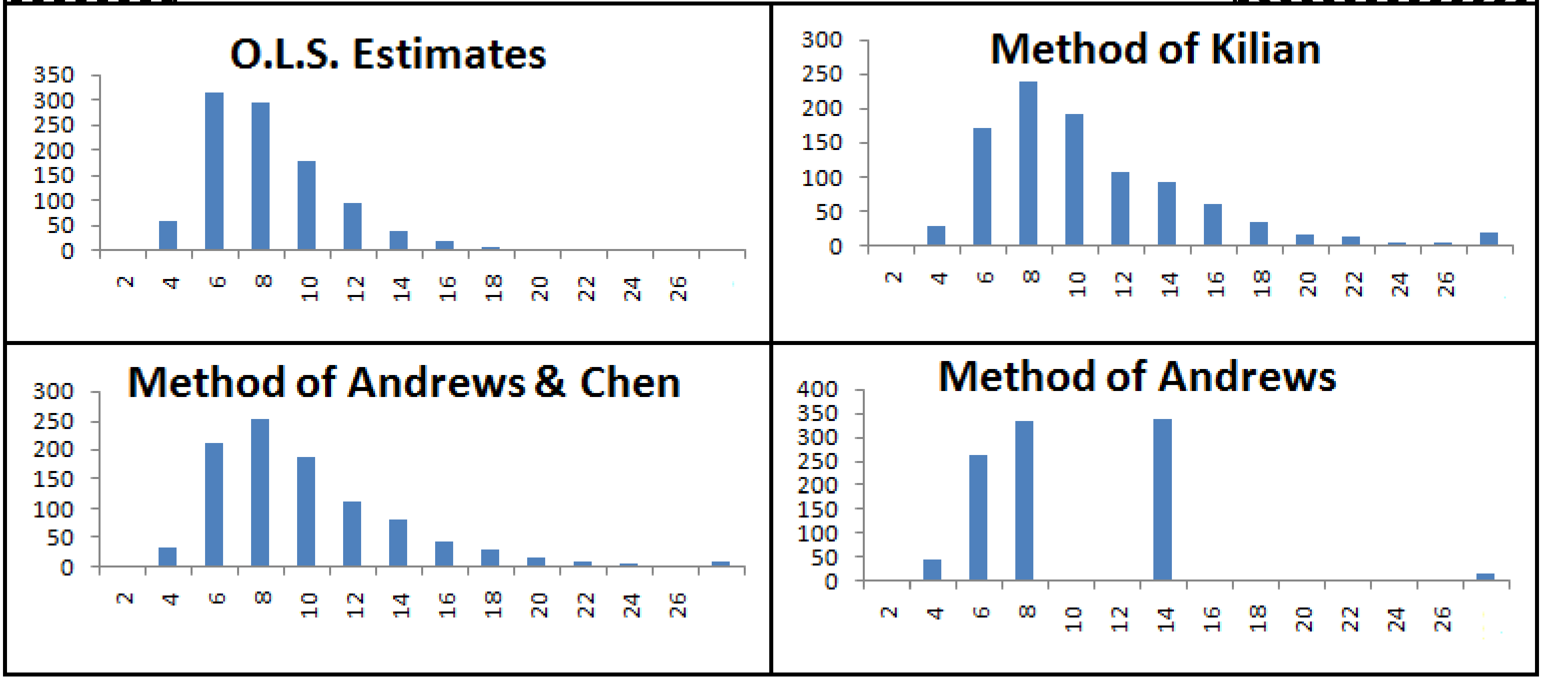


SPECIFICATIONS: T=200 / β=0,92 / h=8,312950									
	Estimator of the autoregressive parameter ($\hat{\beta}$)							**Half-life estimator ($\hat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,899063	0,903622	0,001152	0,030171	0,001590	0,310	0,690	6,839526	0,000
method of Kilian	0,917694	0,922254	0,001213	0,027590	0,001218	0,530	0,470	8,564358	0,000
method of Andrews and Chen	0,913198	0,917354	0,001208	0,027604	0,001254	0,470	0,530	8,035388	0,000
method of Andrews	0,912960	0,915000	0,001378	0,029540	0,001427	0,357	0,643	7,802969	0,000

Density Distribution of Estimators



Histogram of Half-Life Estimators



4.3.5. True autoregressive parameter equal to 0.95

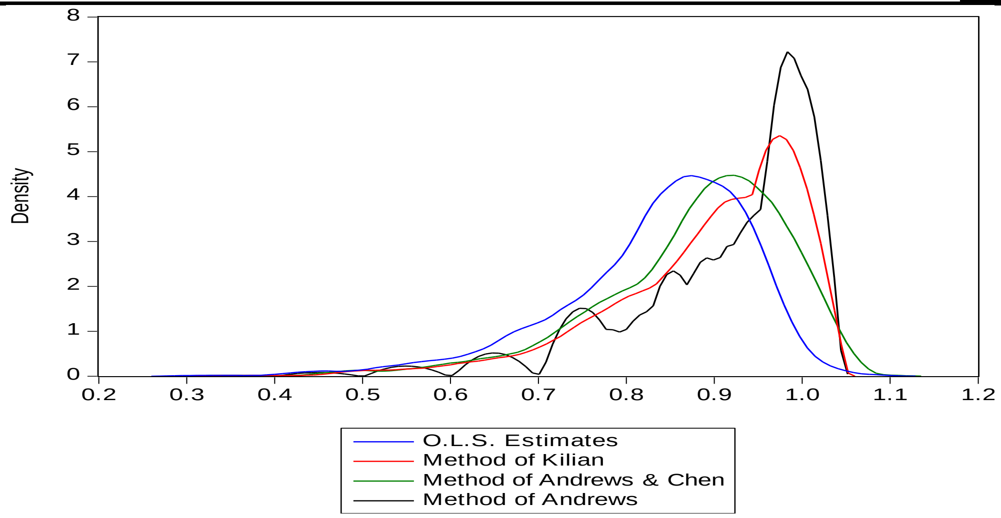
Finally, we report the results for the case where the true parameter value is equal to 0.95 are reported. Focusing on the estimator of the autoregressive parameter, it is clear that the more persistent the process is, the more effective the Andrews estimator becomes. For all sample sizes under investigation, the median of the estimates is exactly the same with the true value. In addition, it presents the lowest mean absolute bias, especially for very low sample sizes. Moreover, it presents the highest density at value 0.95 from all the other methods, as indicated by the Kernel-Density Graph. However, this method has the greatest variance, something which indicates that this method leads to over-correction very often. As for the computer-intensive methods, their performance is sufficiently good. More precisely, the Kilian estimator still “works” better for small sample sizes, but as the sample size increases, the mean and the median of Monte-Carlo simulated estimators surpass the true parameter. The method of Andrews and Chen has exactly the opposite characteristics. Moreover, it should be mentioned that the method of Andrews is the only one which has so high percentages where the estimations are the same with the true parameter, and especially for large sample sizes.

Turning to the half-life estimator, conclusions remain the same with the aforementioned cases. The analytical formula corrects the half-life very efficiently. Its median is almost the same with the true half-life. However, this method is the most “dangerous”, as it pushes the estimator to extreme values more often than any other method. It is noteworthy that this method leads the half-life estimations to “wrong” values very often, as the histograms show. Regarding the computer intensive methods, it is clear that Kilian’s method perform better for small samples, when the Andrews & Chen estimator “works” better when the sample size is large. But in the case where the sample size is really small (equal to 40), none of the two methods is able to reach the effectiveness of the analytical formula.

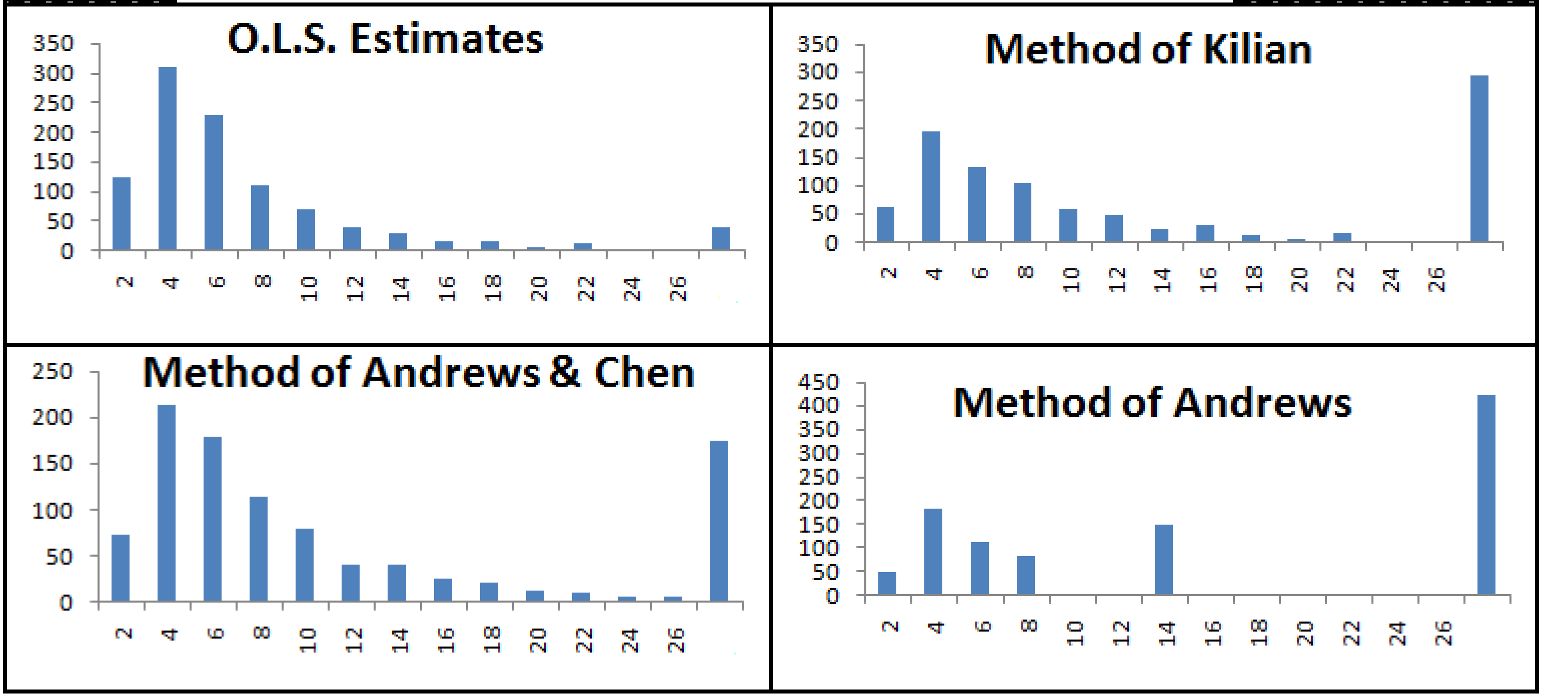
Table 4.5: *Bias-correction results (β=0.95)*

SPECIFICATIONS: T=40 / β=0,95 / h=13,513407									
	Estimator of the autoregressive parameter ($\widehat{\beta}$)							**Half-life estimator ($\widehat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,835195	0,854110	0,011625	0,119844	0,024805	0,097	0,903	4,395475	0,019
method of Kilian	0,892616	0,917932	0,011548	0,084385	0,014841	0,378	0,622	8,094497	0,202
method of Andrews and Chen	0,878070	0,898657	0,011703	0,092749	0,016877	0,270	0,730	6,486930	0,113
method of Andrews	0,910100	0,950000	0,011962	0,079280	0,013554	0,419	0,432	13,51341	0,363

Density Distribution of Estimators

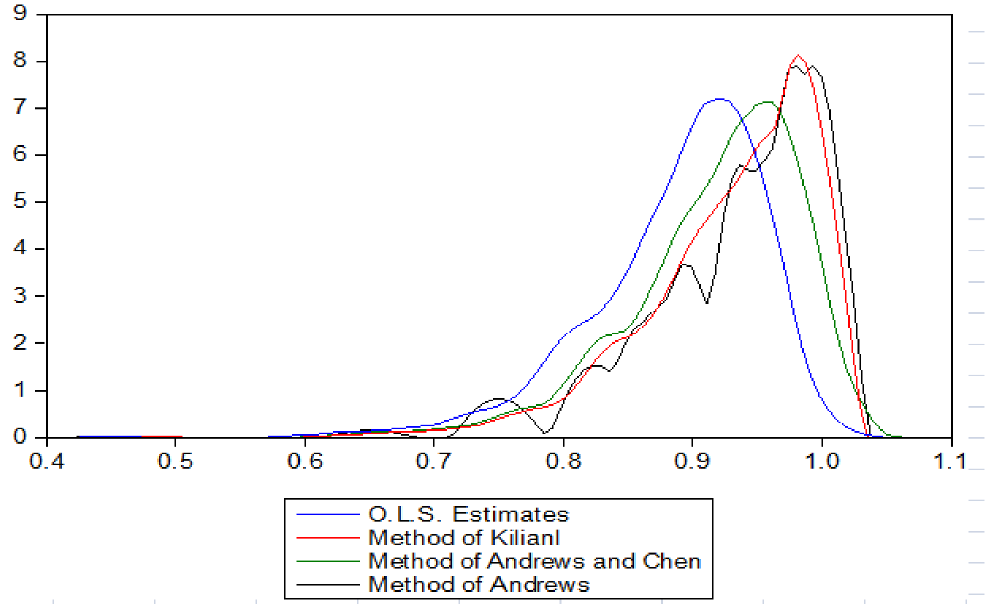


Histogram of Half-Life Estimators

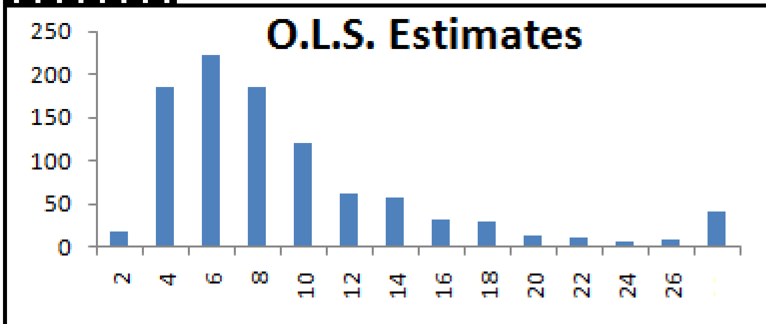


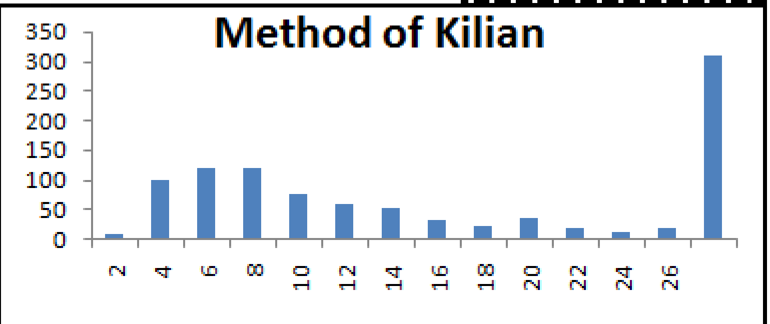
SPECIFICATIONS: T=70 / β=0,95 / h=13,513407									
	Estimator of the autoregressive parameter ($\widehat{\beta}$)							Half-life estimator ($\widehat{h}$)	
	Mean	Median	Variance	Mean Absolute Bias	Mean Square Bias	% of over-estimation	% of under-estimation	median	% of extreme values
O.L.S. estimates	0,888468	0,902943	0,004577	0,067190	0,008363	0,162	0,838	6,789244	0,012
method of Kilian	0,929187	0,945543	0,004598	0,051259	0,005032	0,467	0,533	12,37872	0,202
method of Andrews and Chen	0,917313	0,931588	0,004601	0,053344	0,005670	0,368	0,632	9,781304	0,072
method of Andrews	0,933540	0,950000	0,004993	0,051130	0,005264	0,395	0,378	13,51341	0,287

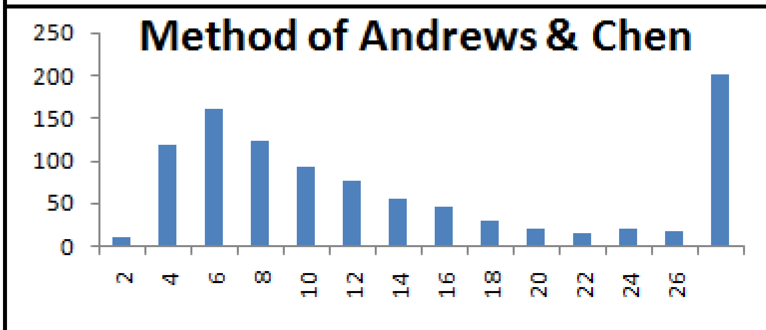
Density Distribution of Estimators

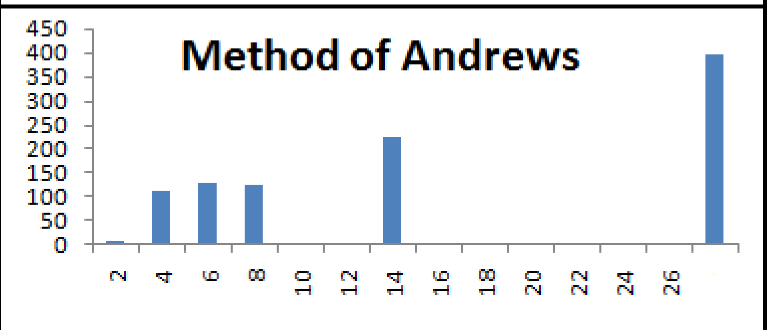


Histogram of Half-Life Estimators



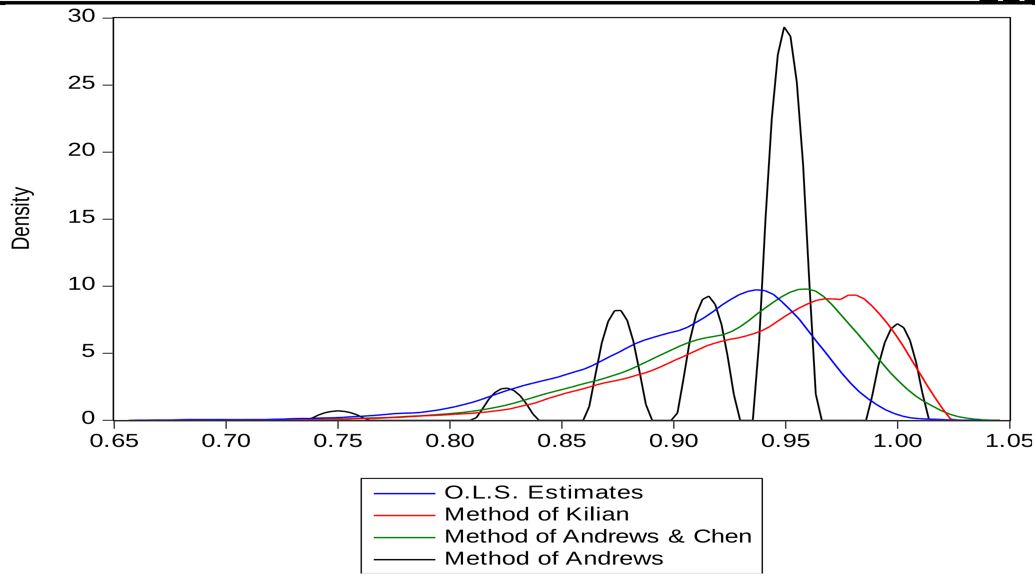




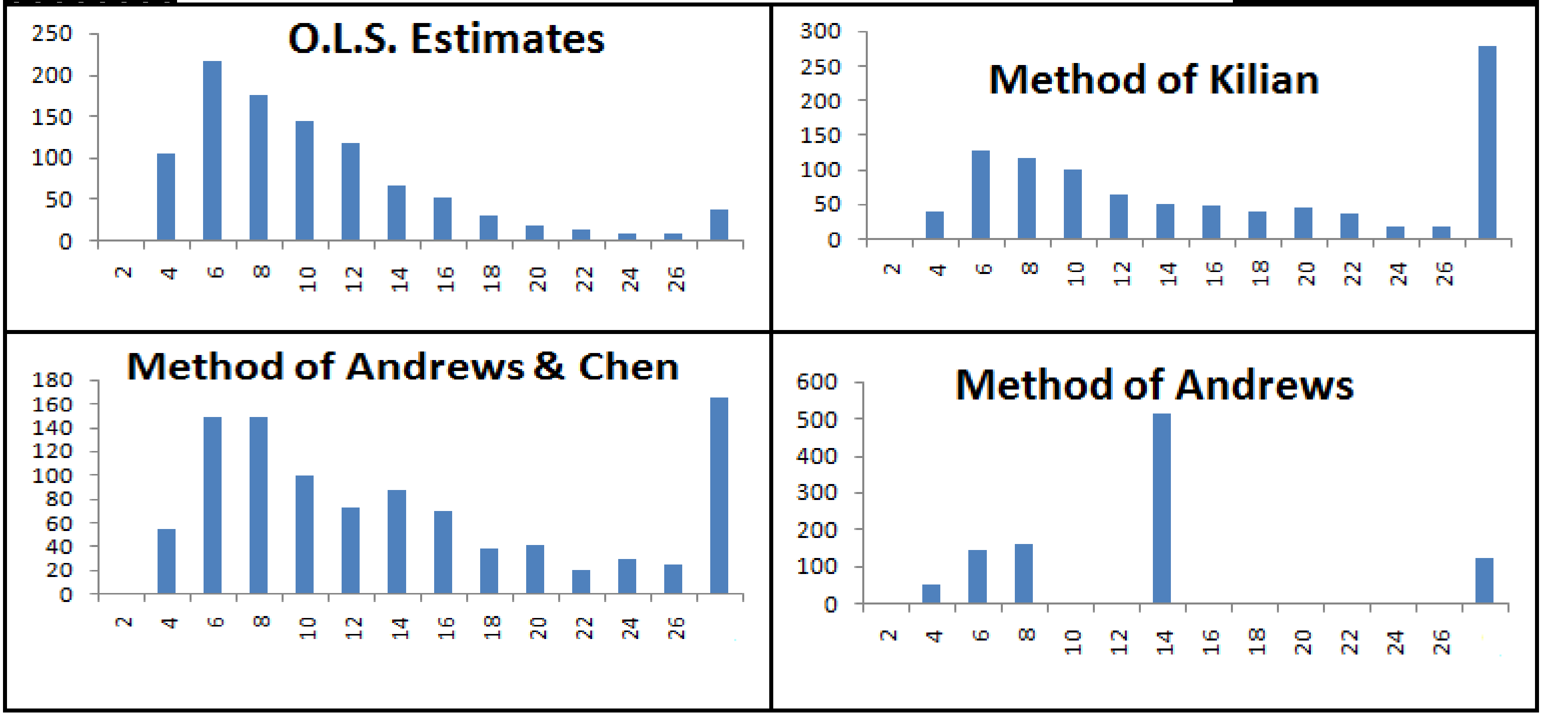


SPECIFICATIONS: T=100 / β=0,95 / h=13,513407									
	Estimator of the autoregressive parameter ($\hat{\beta}$)							**Half-life estimator ($\hat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,907437	0,917297	0,002346	0,047955	0,004157	0,186	0,814	8,029605	0,003
method of Kilian	0,938892	0,950329	0,002437	0,039020	0,002560	0,504	0,496	13,60519	0,125
method of Andrews and Chen	0,929282	0,940482	0,002372	0,039015	0,002801	0,407	0,593	11,29586	0,053
method of Andrews	0,932055	0,950000	0,002141	0,030445	0,002463	0,125	0,361	13,51341	0,125

Density Distribution of Estimators



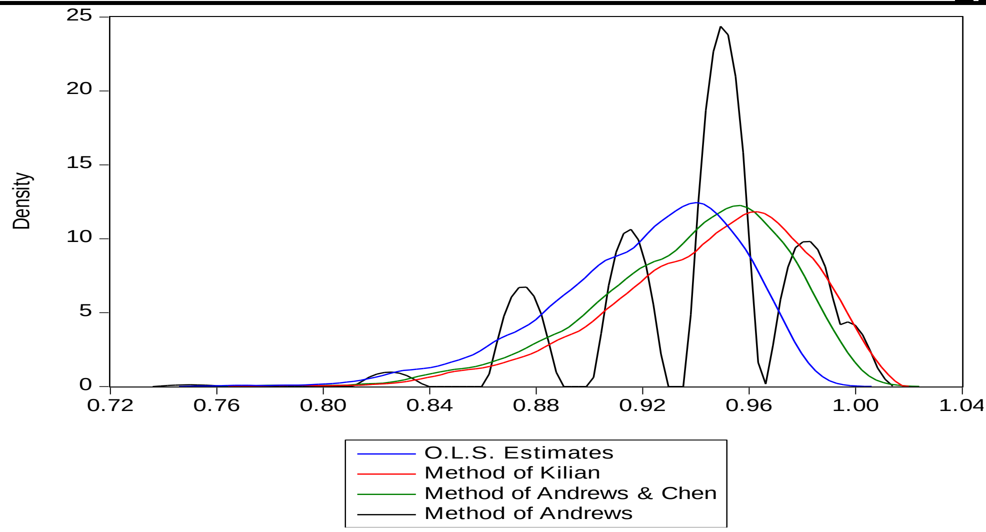
Histogram of Half-Life Estimators



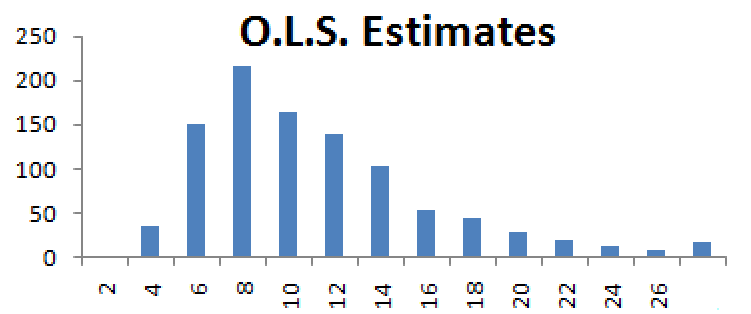
SPECIFICATIONS: T=150 / β=0,95 / h=13,513407

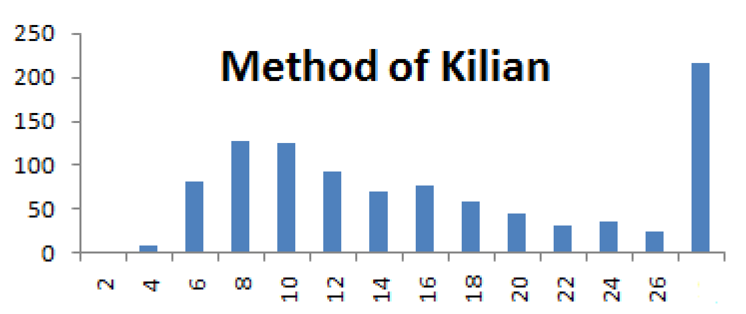
	Estimator of the autoregressive parameter ($\widehat{\beta}$)							Half-life estimator ($\widehat{h}$)	
	Mean	Median	Variance	Mean Absolute Bias	Mean Square Bias	% of over-estimation	% of under-estimation	median	% of extreme values
O.L.S. estimates	0,920693	0,927846	0,001256	0,034323	0,002115	0,213	0,787	9,255570	0,000
method of Kilian	0,943811	0,950960	0,001347	0,028998	0,001385	0,508	0,492	13,78499	0,058
method of Andrews and Chen	0,937028	0,944378	0,001295	0,028715	0,001463	0,421	0,579	12,11179	0,012
method of Andrews	0,940845	0,950000	0,001486	0,026765	0,001570	0,249	0,324	13,51341	0,079

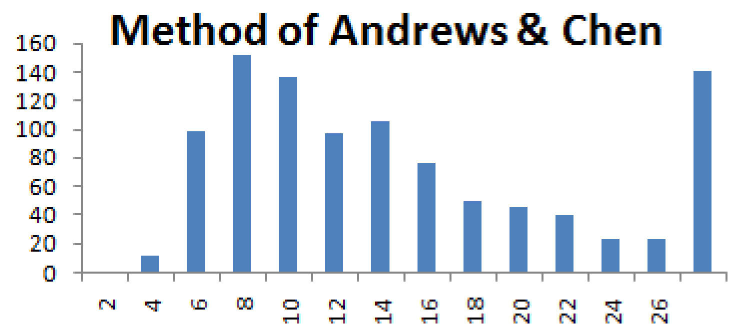
Density Distribution of Estimators

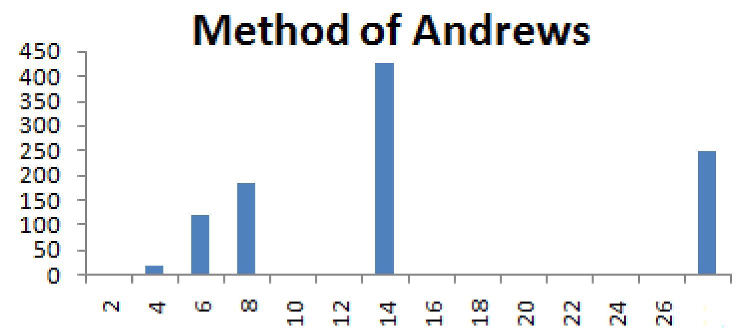


Histogram of Half-Life Estimators



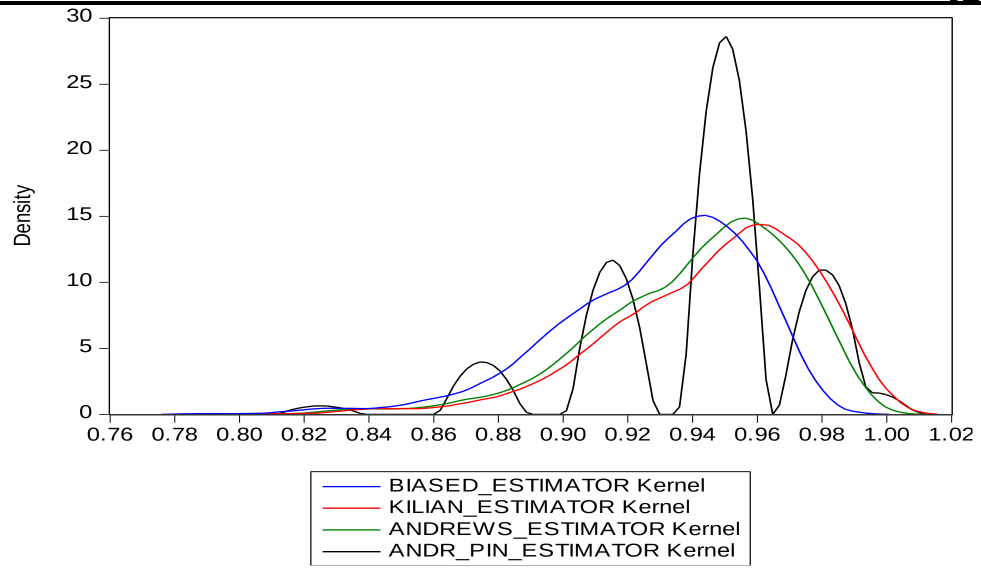




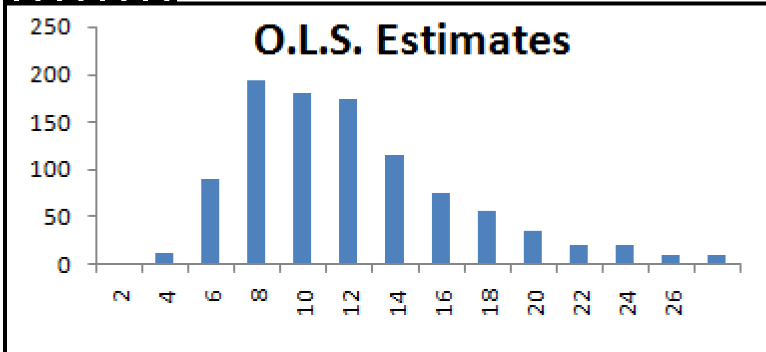


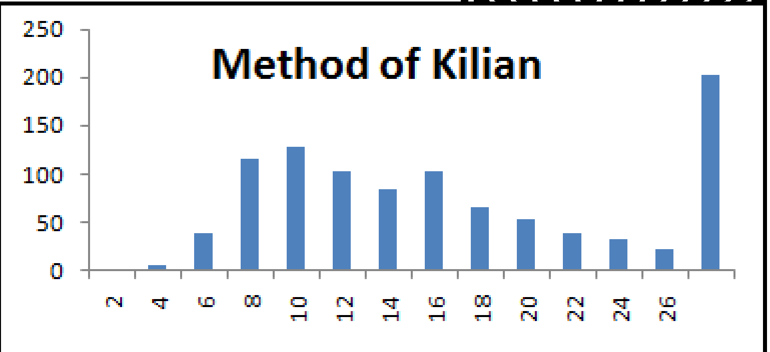
SPECIFICATIONS: T=200 / β=0,95 / h=13,513407									
	Estimator of the autoregressive parameter ($\hat{\beta}$)							**Half-life estimator ($\hat{h}$)**	
	Mean	**Median**	**Variance**	**Mean Absolute Bias**	**Mean Square Bias**	**% of over-estimation**	**% of under-estimation**	**median**	**% of extreme values**
O.L.S. estimates	0,928998	0,934758	0,000875	0,026682	0,001317	0,269	0,731	10,27383	0,000
method of Kilian	0,947007	0,952845	0,000932	0,023909	0,000941	0,542	0,458	14,34996	0,016
method of Andrews and Chen	0,941943	0,947494	0,000897	0,023371	0,000962	0,457	0,543	12,85157	0,002
method of Andrews	0,943410	0,950000	0,000999	0,020720	0,001042	0,220	0,283	13,51341	0,029

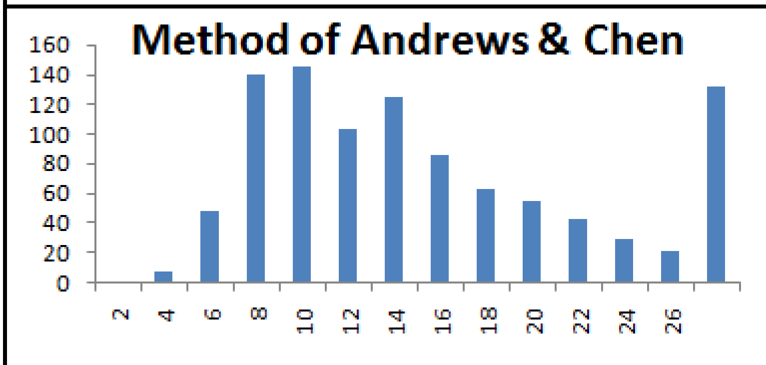
Density Distribution of Estimators

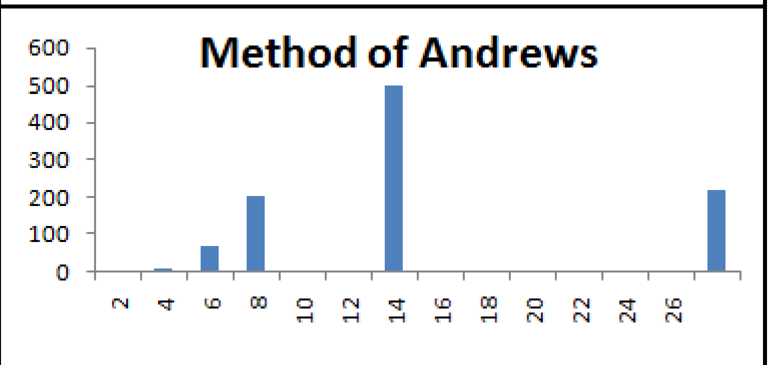


Histogram of Half-Life Estimators









5. PURSHASING POWER PARITY: AN EMPIRICAL APLICATION

5.1. Introduction to the "Purchasing Power Parity" Theory

In this chapter the ideas put forward in the previous chapters will be tested on a real data set considering the Purchasing Power Parity. Purchasing Power Parity (PPP) is a theory about the exchange rate between two countries. More precisely, this theory implies that the price for a given basket of services and goods should be the same in two countries, if measured in the same currency. Because of the fact that PPP is considered as the base for many applications in international economics, its validity has been investigated by a great number of studies. As the real exchange rate is the nominal exchange rate adjusted for the relative price level, it has been proved to be a useful tool for the purposes of those studies. Specifically, the PPP theory simply implies that the real exchange rate is mean reverting

Essentially, recent empirical works have shown that PPP theory does not seem to hold. Specifically, real exchange rates are appear extremely persistent. Financial factors, such as monetary or financial shocks, affect the real exchange rate, and their impact does not last long. Rogoff (1996) used the phrase "Purchasing Power Parity Puzzle" to describe the aforementioned fact. This puzzle continues to attract considerable attention in the literature, and this provides part of the motivation for this empirical research.

In the context of this empirical puzzle, the half-life has been proved to be a useful tool for measuring the persistence of the deviation from PPP. Among others, Abuaf and Jorion (1990), Glen (1992) and Cheung and Lai (1994) deal with this issue.

5.2. Testing the validity of the Purchasing Power Parity Condition

The nominal exchange rate (E_t) is defined as the relative price of two currencies. The purchasing power parity, in consequence, requires that in equilibrium, E_t should reflect the relative purchasing powers of those countries. So, supposing that P_t is the price level in the domestic country and P_t^* is the price level in the foreign country, then PPP requires:

$$E_t = \frac{P_t}{P_t^*}$$

Thus, the logarithm of the real exchange rate, defined as $y_t = \ln(E_t * P_t^*/P_t) = \ln(E_t) + \ln(P_t^*) - \ln(P_t)$, should be constant if PPP holds at every point in time.

The half-life is a commonly used measure of the persistence of real exchange rates. Supposing that the real exchange rate follows an autoregressive process of order one:

$$Y_t = \beta Y_{t-1} + \varepsilon_t$$

Then the half-life of real exchange rates is defines as:

$$h = \ln(0.5) / \ln(\beta)$$

The purpose of this empirical research is to reveal the interaction between the "finite sample bias" and the Purchasing Power Parity Puzzle, mentioned before. This examination will be accomplished using the bias-reduction method, mentioned in section 4.2. More precisely, we consider the real exchange rate of the Australian Dollar, the Canadian Dollar and the Swiss Franc relative to the U.S. Dollar. Afterwards, an AR(1) model for the real exchange rate of those countries will be estimated. Next, the estimate of the autoregressive parameter will be used to compute the half-life. For PPP parity condition to hold, the half-life should be really small. Thereafter, the three bias-correction methods, described in chapter 4, will be used in order to compute a bias-adjusted half-life. This will allow us to investigate the effect of small sample bias on the Purchasing Power Parity Puzzle.

5.3. Data Specification

The data used in this empirical research are from Datastream (IMF Database). Data on the nominal exchange rate are end-of-period and data on prices are **seasonally-adjusted**, which is a statistical method for removing the seasonal component of a time series. Many economic phenomena have seasonal cycles, such as consumer consumption (e.g. grater consumption leading up to Christmas). It is necessary to adjust for this component in order to understand the underlying trends. The nominal exchange rate is expressed as national currency units in terms of 1 U.S. dollar. Data are quarterly from 1975:3 to 2012:4 (150 observations) for: Australia, Canada, and Switzerland. The price indices are consumer price indexes (CPI), so they do not distinguish between tradeables and non-tradeables. The log of the real exchange rate is constructed as the log of the bilateral nominal exchange rate plus the log of CPI in the U.S. minus the log of CPI in the reference country.

5.4. Results

Results of this empirical research are summarized in **Table 5.1.** This table consists of three panels, one for each country under investigation. The first row of each panel contains the O.L.S. estimator of the AR(1) process and its relevant half-life. The other three rows in each panel contain he results for the "Kilian", the "Andrews & Chen and the "Andrews" estimators.

Table 5.1: *O.L.S. and Half-life estimates*

AUSTRALIAN DOLLAR/U.S. DOLLAR		
	Estimator of the autoregressive parameter	**Half-life estimator**
O.L.S. estimates	0,986690	51,72958
method of Kilian	0,996298	186,9101
method of Andrews and Chen	0,994190	118,9542
method of Andrews	1,000000	∞
CANADIAN DOLLAR/U.S. DOLLAR		
	Estimator of the autoregressive parameter	**Half-life estimator**
O.L.S. estimates	0,989767	67,38740
method of Kilian	0,998098	364,0999
method of Andrews and Chen	0,997267	253,2482
method of Andrews	1,000000	∞
SWISS FRANC/U.S. DOLLAR		
	Estimator of the autoregressive parameter	**Half-life estimator**
O.L.S. estimates	0.970660	23,27673
method of Kilian	0.974885	27,25117
method of Andrews and Chen	0.978160	31,39033
method of Andrews	0,98	34,30962

In the case of Australia, the estimated autoregressive coefficient (without any small bias corrections) is 0.986 and the corresponding half-life is 51,72 quarters. The “Kilian” and the “Andrews & Chen” estimators “push” the estimated value closer to unity, increasing substantially the calculated half-lifes to 186,91 and 118,95 respectively. On the other hand, the “Andrews” estimate is 1, suggesting a non-stationary process. These results indicate that the Purchasing Power Parity condition does not hold, especially when the small sample bias is taken into account.

In the case of the real exchange rate between the Canadian dollar (CAD) and the U.S. dollar (USD), results lead to the same conclusions with the aforementioned case. Specifically, the estimated autoregressive coefficient (without correction) is high (equal to 0.989), and the corresponding half-life equal to 67.38. The two simulation-based methods lead the estimation of the autoregressive coefficient very close to unity and the calculated half-life to extremely high regions (the “Kilian” estimator leads the half-life to 364.09 and the “Andrews & Chen” estimator to 253.24). On the other hand, the “Andrews” estimator suggests a non-stationary process and a half-life equal to infinity.

The case of real exchange rate between the Swiss Franc (CHF) and the U.S. dollar (USD) does not differ. The estimated autoregressive coefficient (without any small-sample bias corrections) is 0.97 and the corresponding half-life is 23.27 quarters. The “Kilian” and the “Andrews & Chen” estimators lead the estimators closer to unity, increasing the estimated half-life to 27.25 and 31.39 respectively. The method of "Andrews" leads the estimation for the autoregressive coefficient to 0.98, and the relative half-life to 34.30.

Summarizing this empirical research, the Purchasing Power Parity condition does not seems to hold. Real exchange rates are very persistent, and the adjustment for small sample bias leads the estimated half-life to even higher levels.

Finally, we should mention that there are numerous more sophisticated models available in the literature that try to address the PPP puzzle.

6. CONCLUDING REMARKS

This study examines, (i) the small-sample bias of the coefficients of AR Models, (ii) the effect of the bias on the estimated half-life of a shock and (iii) the ability of the three alternative procedures proposed in the literature to account for the small sample bias when calculating half-life. To be more specific, we consider the following three procedures: First, the analytical median unbiased estimator of Andrews, second the median-unbiased estimation method of Andrews & Chen, and finally the bootstrap-after-bootstrap method of Kilian. The analysis is based on extensive Monte-Carlo simulations. The main findings can be summarized as follows:

First of all, our findings clearly reveal that the small-sample bias of the parameters in AR models is a serious problem in empirical research. More precisely, 1.000 replications of the autoregressive parameters and their relative half-life estimations were simulated, in order to compute the means, the medians the variances, the mean absolute bias, the mean square bias and the percentages of over and under-estimation. Results indicate that for really small sample sizes, estimations are far away from the true values especially for the half-life. In addition, we conclude that the more persistent the process is, the more “sensitive” to finite sample bias the estimators become. Moreover, it becomes clear that the estimators tend to under-estimate the true values.

Secondly, regarding the performance of the three bias-adjustment methods, it becomes clear that the analytical formula of Andrews “works” well in most cases, but it suffers from a serious disadvantage: there are cases where this method leads the estimations to extremely high regions and, as a coincidence to false conclusions conclusions. On the other hand, Kilian’s method “works” better for small samples, but as the sample size increases this method tends to over-estimate the true value. The Andrews and Chen estimator has exact the opposite characteristics: Its performance is not satisfactory for small sample sizes, but it improves as the sample size increases. Nevertheless, one should not forget that the estimations continue to suffer from small sample bias, even though bias-adjustment reduce it in a sufficiently level.

Finally, the study also contains a small empirical application to the Purchasing Power Parity Puzzle. According to PPP theory, the price for a given basket of services and goods should be the same in two countries, if measured in the same currency. However, studies have shown that this theory does not hold in reality. And this conclusion arises from the fact that the real exchange rate is presented extremely persistent. The purpose of this thesis was to examine the interrelationship between the small sample bias and the PPP Puzzle. For this reason, the real exchange rate between the Australian Dollar (AUD), the Canadian Dollar (CAD), and the Swiss Franc (CHF), relative to one U.S. Dollar (USD) was investigated. Results indicate that, in reality, Real Exchange Rate may be even more persistent than what initial estimates suggest. Specifically, all bias-correction methods increase the time which is required for real exchange rate to fall to half its initial value.

To sum up, the small sample bias should not be ignored in empirical research. The results of this thesis indicate that its magnitude is large and the use of a bias correction method is vital. However, it should be mentioned that none of the three bias-correction methods under investigation, offer sufficiently good performs well in all cases, as their efficiency changes depending on the characteristics of the data.

REFFERENCES

- F.H.C. Marriott and J.A. Pope (1954), **"Bias in the Estimation of Autocorrelations,"** *Biometrika, Vol. 41, No. ¾, 390-402.*

- M. G. Kendall (1954), **"Note on Bias in the Estimation of Autocorrelation,"** *Biometrika, Vol. 41, 403-404.*

- Guy Orcutt and Herbert Winokur (1969), **"First Order Autoregression: Inference, Estimation, and Prediction,"** *Econometrica, Vol. 37, No. 1, 1-14.*

- Dac Tjostheim and Jostein Paulsen (1983), **"Bias of some Commonly-Used Time Series,"** *Biometrika, Vol. 70, No. 2, 389-399.*

- Gregory Mankiw and Matthew Shapiro (1985), **"Do We Reject Too Often? Small Sample Bias in Tests of Rational Expectations Model,"** *Economic Letters 20, 139-145.*

- Sastry Pantula and Wayne Fuller (1985), **"Mean Reversion Bias in Least Squares Estimation of Autoregressive Processes,"** *Journal of Econometrics, Vol. 27, Issue 1, 99-121.*

- Karl-Heinz Jockel (1986), **"Finite Sample Properties and Asymptotic Efficiency of Monte-Carlo Tests,"** *The Annals of Statistics Vol.14, No 1, 336-347.*

- John Campbell and Gregory Mankiw (1987), **"Are Output Fluctuations Transitory?,"** *Quarterly Journal of Economics, Vol. 102, Issue. 4, 857-880.*

- Paul Shaman and Robert Stine (1988), **"The Bias of Autoregressive Coefficient Estimators,"** *Journal of the American Statistical Association, Vol. 83, Issue 403, 842-848.*

- Robert Stine and Paul Shaman (1989), **"A Fixed Point Characterization for Bias of Autoregressive Estimators,"** *The Annual of Statistics, Vol. 17, No.3, 1275-1284.*

- Griffiths, William and Helmut Lütkepohl (1990), **"Confidence Intervals for Impulse Responses from VAR Models: A Comparison of Asymptotic Theory and Simulation Approaches,"** unpublished manuscript, Institut für Statistik und Ökonometrie, Humboldt- Universitıt.

- Niso Abuaf and Philippe Jorion (1990), **"Purchasing Power Parity in the Long Run,"** *The Journal of Finance Vol XLV, No.1.*

- Bradley Efron and Robert Tibshirani (1993), **"An Introduction to the Bootstrap,"** *Monographs on Statistics and Applied Probability 57.*

- Yin-Wong Cheung and Kon S. Lai (1994), **"Mean Reversion in Real Exchange Rates,"** *Economic Letters 46, 251-256.*

- Christian Gourieroux, Eric Renault and Nizar Touzi (1995), **"Calibration by Simulation for Small Sample Bias Correction,"** *in Simulation-Based Inference in Econometrics: Methods and Applications, eds. R. Mariano, M. Weeks and T. Schuermman, Cambridge, Cambridge University Press.*

- Kenneth Rogoff (1996), **"Purchasing Power Parity Puzzle,"** *Journal of Economic Literature, Vol. 34, No. 2, 647-668.*

- James MacKinnon and Anthony Smith (1997), **"Approximate Bias Correction in Econometrics,"** *Journal of Econometrics 85, 205-230*.

- Lutz Kilian (1998a), **"Small Sample Confidence Intervals for Impulse Response Functions,"** *Review of Economics and Statistics* 79, 107-129.

- Lutz Killian (1998b)**, "Confidence Intervals for Impulse Responses under Departures from Normality,"** *Econometric Reviews 17(1), 1-29***.**

- Lutz Kilian (1999), **"Finite-sample properties of percentile and percentile-t bootstrap confidence intervals for impulse responses,"** *Review of Economics and Statistics 81, 652–660.*

- Hansen BE. (1999), **"The Grid Bootstrap and the Autoregressive Model,"** *Review of Economics and Statistics 81, 594-607.*

- Sims, C. A., and Zha, T. (1999)**, "Error Bands for Impulse Responses,"** *Econometrica, 67, 1113-1155.*

- K.D. Patterson (1999), **"Bias Reduction in Autoregressive Models,"** *Economics Letters 68, 135-141.*

- Lutz Kilian and Pao-Li Chang (2000), **"How accurate are confidence intervals for impulse responses in large VAR models?,"** *Economic Letters 69, 299–307.*

- Jonathan Wright (2000), **"Confidence Intervals for Univariate Impulse Responses with a near Unit Root,"** *Journal of Business & Economic Statistics, Vol. 18, No. 3, 368-373.*

- Lutz Kilian (2002), **"Quantifying the Uncertainty about the Half-Life of Deviations from PPP,"** *Journal of Applied Econometrics, Vol. 17, Issue 2, 107-125.*

- Cristian Murray and David Papell (2002), **"The Purchasing Power Parity Persistence Paradigm,"** *Journal of International Economics, Vol. 56, Issue 1, 1-19.*

- Nikolay Gospodinov (2004), **"Asymptotic Confidence Intervals for Impulse Responses of Near Integrated Processes,"** *The Econometrics Journal, Vol. 7, Issue 2, 505-527.*

- David Rapach and Mark Wohar (2004), **"The Persistence in International Real Interest Rates,"** *International Journal of Finance & Economics, Vol. 9, Issue 4, 339-346.*

- Elena Pesavento and Barbara Rossi (2005), **"Small Sample Confidence Intervals for Multivariate Impulse Response Functions at Long Horizons,"** *Journal of Applied Econometrics, Vol. 21, Issue 8, 1135-1155.*

- Barbara Rossi (2005), **"Confidence Intervals for Half-Life Deviations from Purshasing Power Parity,"** *Journal of Business & Economic Statistics Vol.23, Issue 4, 432-442*

- Qian Chen and David E. Giles (2007), **"A Saddlepoint Approximation to the Distribution of the Half-Life Estimator in an Autoregressive Model: New Insights into the PPP Puzzle,"** *University of Victoria, Econometrics Working Paper EWP0703.*

- Jae H. Kim, Param Silvapulle and Rob J. Hyndman (2007), **"Half-Life Estimation based on the Bias-Corrected Bootstrap: A Highest Density Region Approach,"** *Computational Statistics & Data Analysis, Vol. 51, Issue 7, 3418-3432.*

- Colin Gallagher and Ferebee Tunno (2008), **"A Small Sample Confidence Interval for Autoregressive Parameters,"** *Journal of Statistical Planning and Inference 138, 3858 – 3868.*

- Tom Engsted and Thomas Pedersen (2011), **"Bias-correction in Vector Autoregressive Models: A Simulation Study,"** *Working Paper, Aarhus University.*