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«TIME-VARYING BETAS AND THE ATHENS STOCK EXCHANGE MARKET»

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Abstract

It has been argued that the Capital Asset Pricing Model fails to price some forms of systematic risk. In addition, numerous studies have demonstrated evidence against beta constancy, proving that estimating time varying betas and including them in the CAPM model gives a better presentation of the market, as all economic prices vary over time. It is generally confirmed that Dynamic Model of Capital Pricing outperforms the static Capital Asset Pricing Model. Aim of this paper is to describe the theory of the CAPM model, report the new form of CAPM model, the conditional CAPM, which includes time varying betas, analyze the role of beta as risk in the market and report ways to estimate dynamic betas. Athens Stock Exchange Market and two stocks of bank sector are chosen, in order to estimate time varying betas and discuss their performance in the market, as main point of this coursework is to account for the parameter of time variation, in order to develop a pricing model that more accurately describes the behaviour of returns. Additionally, previous literature concerning dynamic betas is presented and finally, GARCH models are applied in the estimation of time varying betas, as OLS estimators seem to be inaccurate in our case, where variances are not static.

ΠΕΡΙΕΧΟΜΕΝΑ

INTRODUCTION

The capital asset pricing model (CAPM), first introduced by Sharpe (1964) and Lintner (1966), has made a profound impact on the way investors understand the relationship between price and risk of capital assets. The CAPM simply states that the systematic difference in security returns can be explained by a single measure of risk, beta. According to the CAPM, the expected return on any risky security or portfolio of risky securities can be measured by the risk-free rate and the market risk premium multiplied by the beta coefficient (beta).

Beta represents one of the most widely used concepts in finance. It is used by financial economists and practitioners to estimate a stock's sensitivity to the overall market, to identify mispricings of a stock, to calculate the cost of capital and to evaluate the performance of asset managers. In the context of the capital asset pricing model (CAPM) beta is assumed to be constant over time and is estimated via ordinary least squares (OLS). However, inspired by theoretical arguments that the systematic risk of an asset depends on microeconomic as well as macroeconomic factors, various studies over the last decades, (Fabozzi and Francis (1978)), (Sunder (1980), Bos and Newbold (1984)), (Collins et al. (1987)), have rejected the assumption of beta stability.

From the above, we can understand that this relationship between expected return on an asset and market risk premium is often questionable due to many obstacles, including non-stationarity of beta coefficient and risk premium, inadequate proxy of the market portfolio, and joint hypothesis test problems associated with unobservable expected returns. As a result, many of the tests, concerning simple CAPM model have failed to give a strong basis for evaluating beta as a reliable measure of systematic risk and this failure was attributed to inefficiency of markets (Fama and French (1992), Davis (1994), He and Ng (1994), Burnie and Gunay (1993), Pettengill et al (1995)), even though some of the earlier empirical tests concluded in favour of the CAPM (Black et al. (1973), Fama and Macbeth, (1973)). In particular, some other results from cross-sectional tests of the CAPM indicate that the cross-sectional variation in expected returns cannot be explained by the market beta alone (Fama and French (1992), Chan et al. (1991)).

On the other hand, other studies show that the market beta has a substantial explanatory power on the market return (Pettengill et al, 1995). Chan and Chen (1988), using both time-varying and stationary assumption of portfolio betas, concluded that the unconditional single-factor market model is a better alternative to the pricing model with a size variable. These mixed empirical results found for the CAPM are interpreted in the literature as either evidence against the CAPM itself (Fama and French (1992)) or indication that the testing methodology is inappropriate (Calvet and Lefoll (1989), Roll and Ross (1994)).

In particular, most of CAPM tests have focused on the cross-sectional aspects of data holding beta coefficients constant in the sense that CAPM was originally developed to explain differences in risks across capital assets (Jaganathan and McGrattan (1995)). However, it is well known that firms frequently change their risk structures in conjunction with the macroeconomic environment that is to say that the risk structure of any given firm will vary over time. Furthermore, it is more appropriate to examine the relationship between return and beta using time series tests in the sense that time-varying properties of beta coefficients seems more realistic than the non-stochastic beta assumption. Jagannathan and Wang (1994) point out that the constant beta assumption is not reasonable and Longstaff (1989) points out that the model allowing time-varying expected returns and betas can improves the description of return behaviour.

Aim of this coursework is to investigate a new form of CAPM model, which includes a beta that varies during time, in other words, a dynamic beta. Athens Stock Exchange Market and two stocks of two Greek Banks are selected, for a time horizon of 5 years, in order to search for their static and dynamic betas and compare them. The rest of the paper is organised as followes: introduction, in section 1 are presented the theory about classic CAPM model, the Market Risk Line, the Risk as beta, unconditional CAPM and techniques for estimation of dynamic beta, in section 2 previous literature is presented, in section 3 the econometric analysis is reported and in section 4 conclusions are presented.

CHAPTER 1

THEORETICAL BACKGROUND

1.1 Capital Asset Prising Model

In finance, the **capital asset pricing model (CAPM)** is used to determine a theoretically appropriate required rate of return of an asset, if that asset is to be added to an already well-diversified portfolio, given that asset's non-diversifiable risk. The model takes into account the asset's sensitivity to non-diversifiable risk (also known as systematic risk or market risk), often represented by the quantity beta (β) in the financial industry, as well as the expected return of the market and the expected return of a theoretical risk-free asset.

The model was introduced by Jack Treynor (1961, 1962), William Sharpe (1964), John Lintner (1965a,b) and Jan Mossin (1966) independently, building on the earlier work of Harry Markowitz on diversification and modern portfolio theory. Sharpe, Markowitz and Merton Miller jointly received the Nobel Memorial Prize in Economics for this contribution to the field of financial economics.

The model of CAPM, or Sharpe's model makes some assumptions concerning investors, that is to say all investors aim to maximize economic utility, they are rational and risk-averse, they are broadly diversified across a range of investments, they cannot influence prices, that is to say they are price takers, they can lend and borrow unlimited amounts under the risk free rate of interest, they trade without transaction costs, they deal with securities that are highly divisible into small parcels, they assume all information is available at the same time to all investors and finally the market is perfectly competitive.

The CAPM is a model for pricing an individual security or a portfolio. For individual securities, we make use of the security market line (SML) and its relation to expected return and systematic risk (beta) to show how the market must price individual securities in relation to their security risk class. The SML enables us to calculate the reward-to-risk ratio for any security in relation to that of the overall market.

Therefore, when the expected rate of return for any security is deflated by its beta coefficient, the reward-to-risk ratio for any individual security in the market is equal to the market reward-to-risk ratio, thus:

$$\frac{E(R_i) - R_f}{\beta_i} = E(R_m) - R_f$$

The market reward-to-risk ratio is effectively the market risk premium and by rearranging the above equation and solving for E(Ri), we obtain the Capital Asset Pricing Model (CAPM).

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

where:

$E(R_i)$ is the expected return on the capital asset

R_f is the risk-free rate of interest such as interest arising from government bonds

β_i (the *beta coefficient*) is the sensitivity of the expected excess asset returns to the expected excess market returns, or also $\beta_i = \frac{\mathrm{Cov}(R_i, R_m)}{\mathrm{Var}(R_m)}$,

$E(R_m)$ is the expected return of the market

$E(R_m) - R_f$ is sometimes known as the *market premium* or *risk premium* (the difference between the expected market rate of return and the risk-free rate of return).

Restated, in terms of risk premium, we find that:

$$E(R_i) - R_f = \beta_i(E(R_m) - R_f)$$

which states that the *individual risk premium* equals the *market premium* times *β*.

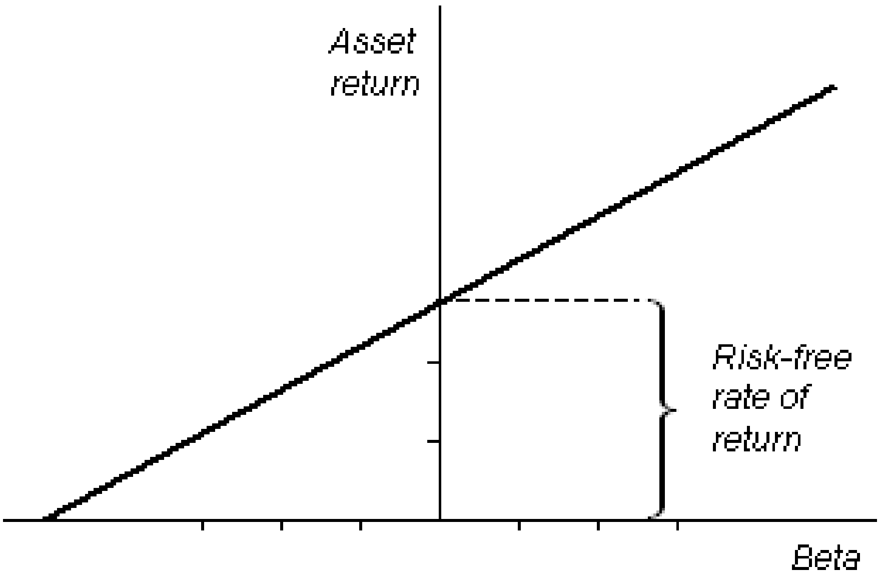
1.2 Security market line

The SML, consulting from CAPM model, essentially graphs the results from the capital asset pricing model (CAPM) formula. The *x*-axis represents the risk (beta), and the *y*-axis represents the expected return. The market risk premium is determined from the slope of the SML.

The relationship between β and required return is plotted on the *securities market line* (SML) which shows expected return as a function of β. The intercept is the nominal risk-free rate available for the market, while the slope is $E(R_m - R_f)$. The securities market line can be regarded as representing a single-factor model of the asset price, where Beta is exposure to changes in value of the Market. The equation of the SML is thus:

$$\mathrm{SML} : E(R_i) = R_f + \beta_i(E(R_M) - R_f).$$

It is a useful tool in determining if an asset being considered for a portfolio offers a reasonable expected return for risk. Individual securities are plotted on the SML graph. If the security's risk versus expected return is plotted above the SML, it is undervalued since the investor can expect a greater return for the inherent risk. And a security plotted below the SML is overvalued since the investor would be accepting less return for the amount of risk assumed.



Once the expected/required rate of return, $E(R_i)$, is calculated using CAPM, we can compare this required rate of return to the asset's estimated rate of return over a specific investment horizon to determine whether it would be an appropriate investment. To make this comparison, we need an independent estimate of the return outlook for the security based on either fundamental or technical analysis. In theory, therefore, an asset is correctly priced when its estimated price is the same as the required rates of return calculated using the CAPM. If the estimate price is higher than the CAPM valuation, then the asset is undervalued (and overvalued when the estimated price is below the CAPM valuation).

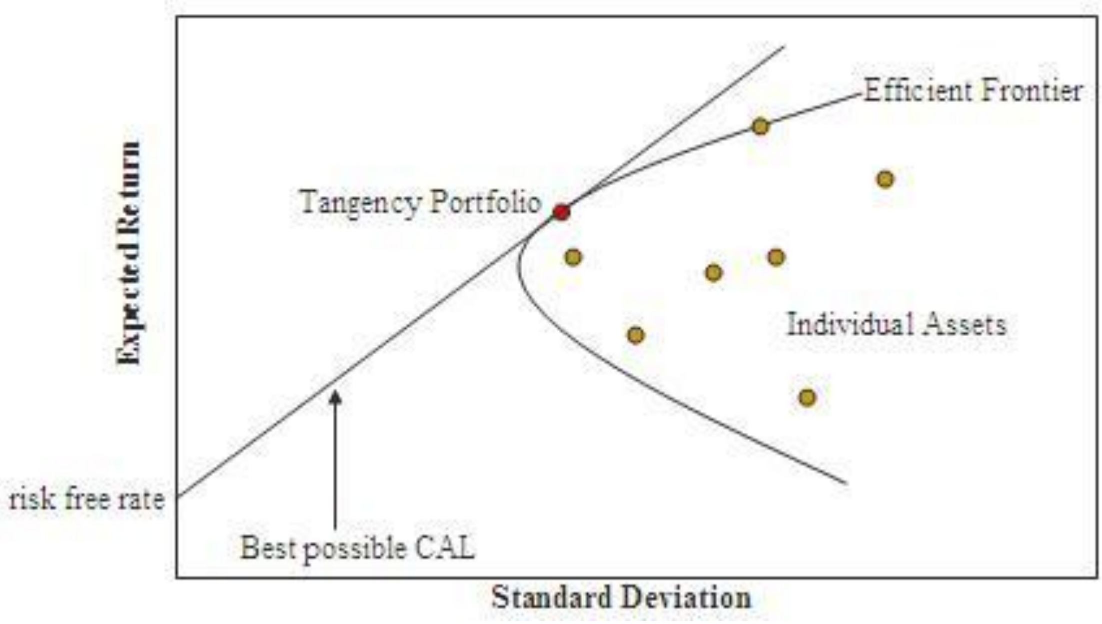
The CAPM returns the asset-appropriate required return or discount rate, that is to say the rate at which future cash flows produced by the asset should be discounted given that asset's relative riskiness. Betas exceeding one signify more than average "riskiness", betas below one indicate lower than average. Thus a more risky stock will have a higher beta and will be discounted at a higher rate; less sensitive stocks will have lower betas and be discounted at a lower rate. Given the accepted concave utility function, the CAPM is consistent with intuition - investors (should) require a higher return for holding a more risky asset.

Since beta reflects asset-specific sensitivity to non-diversifiable, i.e. market risk, the market as a whole, by definition, has a beta of one. Stock market indices are frequently used as local proxies for the market - and in that case have a beta of one. An investor in a large, diversified portfolio (such as a mutual fund) therefore expects performance in line with the market.

1.3 Risk

The risk of a portfolio comprises systematic risk, also known as undiversifiable risk, and unsystematic risk which is also known as idiosyncratic risk or diversifiable risk. Systematic risk refers to the risk common to all securities, while unsystematic risk is the risk associated with individual assets. Unsystematic risk can be diversified away to smaller levels by including a greater number of assets in the portfolio. The same is not possible for systematic risk within one market. In developing markets a larger number is required, due to the higher asset volatilities.

A rational investor should not take on any diversifiable risk, as only non-diversifiable risks are rewarded within the scope of this model. Therefore, the required return on an asset, that is, the return that compensates for risk taken, must be linked to its riskiness in a portfolio context - i.e. its contribution to overall portfolio riskiness - as opposed to its "stand alone riskiness." In the CAPM context, portfolio risk is represented by higher variance i.e. less predictability. In other words the beta of the portfolio is the defining factor in rewarding the systematic exposure taken by an investor. The CAPM assumes that the risk-return profile of a portfolio can be optimized - an optimal portfolio displays the lowest possible level of risk for its level of return. Additionally, since each additional asset introduced into a portfolio further diversifies the portfolio, the optimal portfolio must comprise every asset, (assuming no trading costs) with each asset value-weighted to achieve the above (assuming that any asset is infinitely divisible). All such optimal portfolios, i.e., one for each level of return, comprise the efficient frontier. Because the unsystematic risk is diversifiable, the total risk of a portfolio can be viewed as beta.



Sharpe's model assumes that either asset returns are normally distributed random variables or that investors employ a quadratic form of utility. It is however frequently observed that returns in equity and other markets are not normally distributed. As a result, large swings (3 to 6 standard deviations from the mean) occur in the market more frequently than the normal distribution assumption would expect. Additionally, it assumes that the variance of returns is an adequate measurement of risk. This might be justified under the assumption of normally distributed returns, but for general return distributions other risk measures (like coherent risk measures) will likely reflect the investors' preferences more adequately. Indeed risk in financial investments is not variance in itself, rather it is the probability of losing: it is asymmetric in nature.

Also the model assumes that all investors have access to the same information and agree about the risk and expected return of all assets (homogeneous expectations assumption) and that the probability beliefs of investors match the true distribution of returns. A different possibility is that investors' expectations are biased, causing market prices to be informationally inefficient. This possibility is studied in the field of behavioral finance, which uses psychological assumptions to provide alternatives to the CAPM such as the overconfidence-based asset pricing model of Kent Daniel, David Hirshleifer, and Avanidhar Subrahmanyam (2001).

The model does not appear to adequately explain the variation in stock returns. Empirical studies show that low beta stocks may offer higher returns than the model would predict and assumes that given a certain expected return investors will prefer lower risk (lower variance) to higher risk and conversely given a certain level of risk will prefer higher returns to lower ones. It does not allow for investors who will accept lower returns for higher risk. Casino gamblers clearly pay for risk, and it is possible that some stock traders will pay for risk as well. The market portfolio that concerns CAPM consists of all assets in all markets, where each asset is weighted by its market capitalization. This assumes no preference between markets and assets for individual investors, and that investors choose assets solely as a function of their risk-return profile. Additionally, all assets are infinitely divisible as to the amount which may be held or transacted. This market portfolio should in theory include all types of assets that are held by anyone as an investment but in practice, such a market portfolio is unobservable and people usually substitute a stock index as a proxy for the true market portfolio. Unfortunately, it has been shown that this substitution is not innocuous and can lead to false inferences as to the validity of the CAPM, and it has been said that due to the inobservability of the true market portfolio, the CAPM might not be empirically testable. Finally it is obvious that CAPM assumes that all investors will consider all of their assets and optimize one portfolio and this holds a contradiction with portfolios that are held by investors: humans tend to have fragmented portfolios.

1.4 CAPM and Time Variation

Financial markets are interrelated and increasingly global. When making decisions, traders incorporate information pertaining to price movements and volatility in the asset they are trading, including information about related assets. Thus, understanding how markets influence one another is important for pricing, hedging, and regulatory policy. A plethora of studies examine how price movements are correlated across asset and derivative security markets and international borders. The relation between stock index and stock index futures markets is of special concern because some suggest that trading in derivative securities causes instability in cash markets (Stoll and Whaley (1990)).

The capital asset pricing model (CAPM), originally proposed by Sharpe (1964) and Lintner (1965), following the suggestions of mean variance optimization in Markolvitz (1952), has provided a simple and compelling theory of asset market pricing for many years. In its simplest form the theory predicts that the expected return on an asset above the risk-free rate is proportional to the non diversifiable risk, which is measured by the covariance of the asset return with a portfolio composed of all the available assets in the market. But the assumptions implicit in the version of that model discussed are that (1) all investors choose mean-variance efficient portfolios for an one period horizon, although they need not have identical utility functions; (2) all investors have the same subjective expectations on the means, variances, and covariances of returns; and (3) the market is fully efficient in that there are no transaction costs, indivisibilities, taxes, or constraints on borrowing or lending at a risk-free rate. Empirical tests of the CAPM have tended to focus on the first assumption while strengthening the second to include the assumption that the common distributions are constant over time and that the entire market is the market for equities. These tests generally have found that the risk premium on individual assets can be explained by variables other than the estimated covariance. In particular, the own variance, firm size, and the month of January seem to be variables that help to explain expected returns. One interpretation for the failure of the CAPM to fully explain observed risk premia, due to Roll (1977), is that any empirical covariance is computed from an incomplete market for assets. Such an objection nearly makes the CAPM untestable. Another explanation is, of course, that alternative theories of asset pricing may be supportable such as the arbitrage pricing theory of Ross (1976) or the consumption beta formulation introduced by Breeden (1979).

Although the Capital Asset Pricing Model (CAPM) of Sharpe (1964) has constituted one of the cornerstones of modern finance theory for the last four decades, it posits a simple and stable linear relationship between an asset's systematic risk and its expected return. Many studies, notably Banz (1981), Basu (1983), Bhandari (1988), and Fama and French (1992), have found weak or no statistical evidence in support of this simple relationship. Stimulated by these findings, a number of researchers have sought to find alternative explanations for the risk and return trade off. One line of attack has been that of Fama and French (1993, 1995) who concluded that fundamental variables, namely Book-to-Market equity ratio and Market Equity, found to explain the variation in returns must be proxies for some unidentified risk factors. Another line has been advocated by Ferson (1989), Ferson and Harvey (1991, 1993), Ferson and Korajczyk (1995), and Jaganathan and Wang (1996), who argue that beta and market risk premium vary over time, therefore, static CAPM should be improved by incorporating **time variation** in beta in the model.

Although there is considerable empirical evidence on time variation in betas, it is not clear how this variation should be captured. Many researchers model the variation in betas using continuous approximation and the theoretical framework of the conditional CAPM. However, Ghysels (1998) shows that this approximation fails to capture the dynamics of beta risk. He argues that betas change through time very slowly and linear factor models like the conditional CAPM may have a tendency to overstate the time variation. Thus, they produce time variation in beta that is highly volatile, leading to large pricing errors.

We can assume that though CAPM’s wide use implies acceptance by the finance community, it is clear that the model has limitations. Due to the assumption that all assets are tradeable, following Markowitz mean-variance optimization leads to the result that the only source of systematic risk is the market. In other words, the only factor that needs to be considered when using Arbitrage Pricing Theory is the excess market return. Empirical evidence suggests that this result may not hold. Chen, Roll, and Ross (1986) used macroeconomic variables to proxy for systematic factors not included in CAPM. Several of these variables had significant predictive power; indicating risk stemming from economic factors can be hedged. Fama and French (1993) adopted a different approach, using firm-specific variables as proxies. In addition to the excess return on the market index, the factors proposed relate to firm size and book-to-market ratio. The factors tested significant, and the intercepts from the regressions were close to zero, suggesting that the model effectively explained returns. The CAPM has a constant beta parameter estimated by OLS, but numerous studies have attacked this assumption of constant risk sensitivity. Fabozzi and Francis (1978) rejected the hypothesis of a static beta against the alternative of a random coefficient. Bos and Newbold (1984) extended the test, allowing for the evolution of beta to follow a first order autoregressive model. They found evidence to support the randomness of risk sensitivity, but did not find evidence favouring an autoregressive beta to a random one. Given the evidence against constant betas, Mergner and Bulla (2005) sought to explicitly model the time-varying behavior of betas for European industry portfolios. In addition to the standard OLS model, they considered a bivariate *t*-GARCH(1,1) model, a bivariate stochastic volatility model, two Markov switching models, and two state-space models and of the models estimated, the state-space models performed the best, because a state-space model with an autoregressive beta had the lowest in-sampling forecasting error, although it performed poorly in out-of-sample forecasting, having the same error as the OLS model. Given their results, Mergner and Bulla found that the state-space model with a random walk beta was the best model to describe their data.

It is possible that time-varying parameter models outperform the CAPM for the same reason that multifactor models do. The original efforts to test beta stability were motivated by arguments that systematic risk is sensitive to changing macroeconomic and microeconomic factors. So a static model that includes proxies for macroeconomic and microeconomic factors may capture the same sensitivity that a single factor time-varying beta model does. However, it is possible beta may be sensitive to changes that cannot be captured in a static multifactor model. If this were the case, then it would be reasonable to develop a multifactor model with a time-varying market beta. Or, more generally, one might specify a multifactor model where each parameter, or beta, is time varying. Johnson and Sakoulis (2003) did this to create a forecasting model, using lagged macroeconomic variables as factors. The developing an industry portfolio pricing model, which explains better returns, concerns capturing changes in systematic risk missed by static multifactor models, but allowing for parameters to follow a random walk.

Additionally, an extended form of CAPM with a dynamic beta, could also explain other market anomalies, like the phenomenon of momentum that has perplexed financial economists, generating both excitement and controversy. Since Jegadeesh and Titman (1993) first documented that a variety of strategies that buy past winners and sell past losers can produce significant profits, researchers have shown that the phenomenon is both robust for the "out-of-sample" U.S. data after 1993 (Jegadeesh and Titman (2001)), and pervasive in other parts of the world. According to traditional asset pricing theories, the momentum phenomenon should not exist. Recently, many behavioural, as well as rational, hypotheses have emerged to explain the source of momentum returns. But beyond the stylized empirical facts, the only consensus seems to be that traditional asset pricing models such as the Capital Asset Pricing model (CAPM) and the Fama and French (1993) three-factor model fail to explain momentum. Asset pricing factors do not themselves exhibit significant momentum; therefore, a constant loading of the factors should not result in momentum. The establishment of a new relationship between the CAPM beta and the risk premium, including time varying betas could explain this phenomenon and other market anomalies. A rational economy can have size, book-to-market, and several momentum-related phenomena because of this relationship. Traditional asset pricing models assume a static variance-covariance matrix for asset returns and, consequently, predict that a beta is static and that the expected risk premium is linear in beta. If a firm's investment opportunity set changes over time, however, the term's beta may, and usually are, dynamic. Although different model specifications can lead to dissimilar beta processes, in general beta dynamics reflect expected changes in the business opportunity set.

1.5 Unconditional CAPM and Dynamic Beta

Fama and French (1992) present compelling evidence that the unconditional CAPM does not account for returns on the size of sorted portfolios. Since then, the asset pricing literature has developed alternative theories, which depart from the original model along several dimensions. Promising avenues of research, which preserve the single factor structure, have been conditional versions of the CAPM. The idea behind this approach is that, although CAPM may hold conditionally on time *t* information, it may not hold unconditionally, as we have already mentioned before. Accordingly, the poor empirical performance of CAPM might be due to the failure to account for time-variation in conditional moments.

Dynamic beta reaching to an augmented CAPM model is calculated and presented as followed:

As a starting point, market risk is treated as being constant. The benchmark for time varying betas is the excess-return market model with constant coefficients where an asset's unconditional beta can be estimated via OLS:

$$R_{it} = \alpha_i + \beta_i R_{0t} + \varepsilon_{it}; \quad \varepsilon_{it} \sim (0, \sigma_i^2)$$

with $\beta = Cov(R_0, R_1) / Var(R_0)$,

where R_0t denotes the excess return of the market portfolio and R_{it} denotes the excess return to sector i for i = 1,….,I, each for period t = 1,…….,T.

The error terms ε_{it} are assumed to have zero mean, constant variance $\sigma^2{}_i$ and to be independently and identically distributed (IID). Following the Sharpe (1964) and Lintner (1965) version of the CAPM, where investors can borrow and lend at a risk-free rate, all returns are in excess over a risk-free interest rate and α_i is expected to be zero.

Among the many implementations of conditional CAPM, the ones which have proved most successful are proposed by Jagannathan and Wang (1996) and Lettau and Ludvigson (2001). In both cases, the authors explicitly model the evolution of the conditional distribution of returns as a function of lagged state variables. They specify the covariance between the market return and portfolio returns as functions of these variables. This specification is estimated as a multi-factor model, in which the additional factors are the interactions between the market return and the state variables.

Another paper by Lewellen and Nagel (2005) casts doubts on the empirical success of this approach. While acknowledging that betas vary considerably over time, these authors present evidence suggesting that the covariation between betas and the market return is not large enough to justify the deviations from the unconditional CAPM observed for value and momentum portfolios (Fama and French (1993) and Jegadeesh and Titman (1993)). They argue that the good empirical performance of previous conditional studies is due to their cross-sectional design - which ignores key theoretical restrictions on the estimated slope coefficients - and suggest time-series regressions instead. Low frequency movements of beta play a crucial role, because investors' inference about the long-run level of beta can cause a significant difference between the ex-ante expected level of risk and ex-post estimates from typical OLS regressions. This mechanism is particularly relevant for portfolios such as value and small stock portfolios that have experienced considerable long-run variation in beta (Franzoni (2002)). The wedge between investors' ex-ante expectation of beta and ex-post OLS estimates can account for a large fraction of the mispricing in standard OLS time-series regressions. In other words, the mismeasurement of expectations of beta and, hence, of equilibrium expected returns can be the source of the apparent mispricing.

Tobias Adrian and Francesco Franzoni (2005), price errors of an augmented CAPM comparing it in magnitude to the one from the Fama-French three factor model when standard conditioning variables are included. The intuition behind the empirical success of the augmented CAPM is pricing portfolios in a different way. They prove that the failure of CAPM to price value stocks in an OLS framework is due to the fact that these assets have high average returns but low estimated betas. Given the decrease in systematic risk of value stocks, the high level of the factor loading from the past affects today's estimates and makes them larger than OLS betas. A high estimate of beta is thus matched with high average returns and the estimated alpha of value stocks is reduced. A similar intuition applies to small stocks, which have also experienced a decline in systematic risk. The situation is reversed in the case of large and growth stocks, whose levels of risk have increased over time.

Their results provide support for the conditional CAPM. Consistent with their prediction, they find that the standard conditioning variables do not improve the performance of CAPM much when the model is tested in the time-series via OLS. However, the inclusion of these conditioning variables in their framework reestablishes the success of the conditional CAPM in pricing size and B/M sorted portfolios. In the context of conditional asset pricing models, this procedure implicitly assumes that investors price stocks according to the risk of the asset class to which they belong. Barra (2005), a provider of beta estimates, uses a fundamental measure of a stock's beta. Their estimate is the weighted average of the betas of a set of characteristic-based portfolios to which a stock belongs. Barra says that this fundamental beta is superior to the historical beta in predicting future risk.

1.6 Techniques for Dynamic Beta

There are several techniques for the estimation of time varying beta. The first technique for estimating time varying betas is based upon the multivariate generalized autoregressive conditional heteroskedasticity (M-GARCH) model, first proposed by Bollerslev (1990), which belongs to the class of GARCH models, introduced by Engle (1982) and Bollerslev (1986). The conditional variance estimates as produced by a GARCH(1,1) model are utilized to generate the series of conditional time varying betas. This approach has been applied in various studies to model time varying betas. For example, Giannopoulos (1995) uses weekly local stock market data over the period from 1984 until 1993 to estimate time varying country betas. Brooks et al. (1998) estimate conditional time dependent betas for Australian industry portfolios using monthly data covering the period from January 1974 to March 1996. Li (2003) studies the time varying beta risk for New Zealand sector portfolios by analyzing daily data from January 3, 1997 to August 28, 2002. Although the popular GARCH(1,1) model is able to describe the volatility clustering in financial time series as well as other prominent stylized facts of returns, such as excess kurtosis, the standard GARCH model does not capture other important properties of volatility, e.g. asymmetric effects on conditional volatility of positive and negative shocks. Therefore, nonlinear extensions of the basic GARCH model have been proposed and adopted to the modeling of time varying betas. For example, Braun et al. (1995) employ an exponential GARCH (EGARCH) model to test for predictive asymmetry in beta and Fama et al. (2000) estimate time varying systematic risk of UK industry indices by an EGARCH and a threshold ARCH (TARCH) specification.

An alternative way of modelling the time varying behaviour of beta is based on the state space form of the CAPM. In contrast to volatility based models where time varying betas are calculated indirectly by utilizing estimated conditional variance series, the state space approach allows to model and estimate time varying betas directly by using the Kalman Filter (KF). Different models for the dynamic process of conditional betas have been proposed. For US data Fabozzi and Francis (1978) and Collins et al. (1987) modelled beta as a random coefficient. The RC model has also been applied by Wells (1994) for Swedish stocks and by Fama et al. (2000) for UK industry indices. Two of the most prominent alternatives to the model time varying betas are the random walk (RW) model, recently employed by Lie et al. (2000) for Australian financial stocks and by Li (2003) for New Zealand industry portfolios, and the mean reverting (MR) model which has been used by Bos and Newbold (1984) for US data, by Brooks et al. (1998) and by Groenewold and Fraser (1999) for Australian sectors. For their investigation of the systematic risk of Australian industrial stock returns Yao and Gao (2004) also considered an autoregressive moving average model (ARMA) as well as an MR model in which the mean beta is allowed to vary over time as proposed by Wells (1994). Another approach uses a Markov switching framework which belongs to the large class of Markov switching models introduced in the seminal works of Hamilton (1989, 1990). Although Markov switching regression models have been applied in many different settings, the literature dealing with time varying betas is relatively thin.

CHAPTER 2

PREVIOUS LITERATURE

The increasingly international investment environment has placed the need for accurate modelling of risk firmly on the research agenda. One of the issues is the extent to which the techniques used to model risk in a domestic context can be readily applied to the international context. One of the most popular measures of risk in a domestic context is beta. The beta measure has been to capture country risk in a number of papers.

There is a large literature that establishes that individual stock betas vary over time. This matter has also been studied for country betas. A number of these studies have used GARCH based approaches for the estimation of country betas. Giannopolous (1995) used a bivariate GARCH model to calculate betas for weekly stock returns data for 13 developed countries. Brooks, Faff and McKenzie (2002) also used a bivariate GARCH model to calculate betas for 17 developed countries.

Since Sharpe (1964) and Lintner (1965) proposed the Capital Asset Pricing Model (CAPM) to describe the risk-return relationship, substantial empirical work has been conducted to investigate the validity of the model. Many empirical studies, by using a different approach proposed by Fama and MacBeth (1973), show that the Sharpe-Lintner CAPM provides an inadequate explanation of the risk-return relationship due to the lack of evidence that supports a statistically significant relationship between risk and return (Fama and French 1992, He and Ng 1994). This unsuccessful empirical performance of the CAPM caused people to cast doubts on the model. Pettengill, Sundaram, and Mathur (1995) argued that the validity of the CAPM is not directly examined by the Fama-MacBeth methodology. The CAPM postulates a positive relationship between beta and expected return. To test for this relationship, empirical studies use observable returns in place of unobservable returns. CAPM shows that this procedure is biased against finding a significant relationship between beta and expected returns because the relationship between beta and realized returns is conditional on the market return. In up markets high beta securities should be rewarded for bearing risk with higher returns than low beta securities, but in down markets high-risk, high-beta securities experience lower returns than low beta securities. Thus, standard tests are biased against finding a relationship between beta and returns because these tests mix periods where the relationship between beta and returns is direct (up markets) with periods where the relationship between beta and returns is inverse (down markets).

Concluding, the basic criticism of the Fama-MacBeth methodology is the OLS regressions that assume a constant beta risk. Studies such as Harvey (1989) and Ferson and Harvey (1991, 1993) suggest that a constant beta estimated by OLS may not capture the dynamics of beta. Jagannathan and Wang (1996) and Lettau and Ludvigson (2001), show that conditional CAPM with a time-varying beta outperforms the unconditional CAPM with a constant beta. Adrian and Franzoni (2004, 2005) also argue that an econometric model that fails to mimic the investors' learning process of time evolving beta may lead to inaccurate estimates of beta. There is plenty of previous literature that reach to a new augmented model of CAPM, using time varying covariances, reaching to a dynamic beta, in order to explain the function of the market in a more reasonable way.

Tim Bollerslev Robert F. Engle and Jeffrey M. Wooldridge(1998) studied a market portfolio composed of bills (&month Treasury bills), bonds (20-year Treasury bonds), and stocks, because in broad terms, these three assets account for a good part, but certainly not all, of the liquid investment opportunities. The data they used were quarterly percentage returns from the first quarter of 1959 through the second quarter of 1984, for a total of 102 observations. In their paper a multivariate generalized autoregressive conditional heteroscedastic process was estimated for returns to bills, bonds, and stocks where the expected return was proportional to the **conditional** covariance of each return with that of a fully diversified or market portfolio. They proved that the conditional covariances are quite variable over time and are a significant determinant of the time-varying risk premia and simultaneously, the implied betas are also time-varying and forecastable. Overall, they showed evidence that other variable including innovations in consumption should also be considered in the investor's information set when estimating the conditional distribution of returns.

In their paper they focus attention on the possibility that agents may have common expectations on the moments of future returns but that these are *conditional* expectations and therefore random variables rather than constants.

The vector of covariances in the market is:

$$H_t\omega_{t-1} \text{ , (1)}$$

and the CAPM requires

$$\mu^t= \delta H_t\omega_{t-1.} \qquad (2)$$

In this formulation, as derived by Jensen (1972), δ is a scalar constant of proportionality, which in equilibrium is an aggregate measure of relative risk aversion given by the harmonic mean of the agents' degree of relative risk aversion weighted by the agents' share of aggregate wealth (Bodie, Kane, and McDonald 1983, 1984). Throughout the paper it is assumed δ to be constant.

The conditional variance of the market excess return is:

$$\sigma^2 M_t=\omega_{t-1}H_t\omega_{t-1}\text{,(3)}$$

and the conditional mean is

$$\mu M_t=\omega_{t-1}M_t, \qquad (4)$$

which from (1) can be written as

$$\mu M_t=\delta\, \sigma^2 M_t\text{,(5)}$$

so that δ is seen to be the slope of the market trade-off between mean and variance.

In this paper the conditional covariance matrix of a set of asset returns is allowed to vary over time following the generalized autoregressive conditional heteroscedastic (GARCH) process (Engle 1982; Bollerslev 1986). This essentially assumes that agents update their estimates of the means and covariances of returns each period using the newly revealed surprises in last period's asset returns. Thus agents learn about changes in the covariance matrix only from information on returns. There may, of course, be additional information relevant to agents' expectations that would lead to misspecification.

The approach is a multivariate generalization of Engle, Lilien, and Kobins (1987), which treated a single asset, and therefore estimates the time-varying risk premium as a function of the conditional variance of that asset return alone. A similar idea was employed in other papers by French, Schwert, and Stambaugh (1986) and Poterba and Summers (1986).The approach can also be seen as a statistical implementation of the intertemporal CAPM of Bodie et al. (1983, 1984), in which they had no unknown parameters and no statistical test of the model performance.

As a conclusion, the results reported in this paper supported the conclusion that the conditional covariance matrix of the asset returns is strongly autoregressive, that is to say that the data clearly reject the assumption that this matrix is constant over time. The expected return or risk premia for the assets were significantly influenced by the conditional second moments of returns and there is also evidence that the risk premia are better represented by covariances with the implied market than by own variances.
Peng Huang and James Hueng(2007) investigate the asymmetric risk-return relationship in a time-varying beta CAPM, stating a space model, estimated by the Adaptive Least Squares with Kalman foundations proposed by McCulloch (2006). They use S&P 500 daily data from November 1987 to December 2003, and more specific 385 stocks, which are classified into ten industries based on their Global Industry Classification Standard (GICS) codes, which includes energy, material, industrials, consumer discretionary, consumer staples, health care, financials, information technology, telecom services and utilities, finding a positive risk-return relationship in the up market (positive market excess returns) and a negative relationship in the down market (negative market excess returns). They initially support the argument by Pettengill, Sundaram and Mathur (1995), who use a constant beta model, but they conclude to a model, which outperforms theirs by eliminating the unexplained returns and improving the accuracy of the estimated risk price.

Purpose of their paper is to check the robustness of CAPM model by incorporating a time-varying beta, in order to see whether CAPM's success is based on an incorrectly specified constant beta and they re-examine the asymmetric risk-return relationships in the up and down markets with a time-varying beta model. They use the Adaptive Least Squares with Kalman foundations (ALSKF) proposed by McCulloch (2006), to estimate the time-varying beta model, as the ALSKF provides a better way of estimating time-varying coefficients and proxying investors' time evolving expectations by incorporating the learning process. This methodology nests the

Kalman solution of the elementary local level model and uses a simple way to setup a rigorous initial condition.

Overall, they reach to the conclusion that when the market excess return is positive, there exists a significant and positive risk-return relationship; when the market excess return is negative, there exists a significant and negative risk-return relationship. These results indicate that the ALSKF successfully improves the accuracy of the estimation of beta risk by mimicking the investors' learning process on the unobservable beta that the OLS cannot account for. To estimate a time-varying beta, studies have tried different modeling strategies. For example, Jagannathan and Wang (1996) and Lettau and Ludivigson (2001) treat beta as a function of several state variables in a conditional CAPM. Engle, Bollerslev, and Wooldridge (1988) model the movements of beta in a GARCH model. Adrian and Franzoni (2004, 2005) suggest a time-varying parameter linear regression model and use the Kalman filter to estimate the model. McCulloch (2006) considers a more general case where the covariance matrix is time-varying, in an attempt to better proxy agents' expectation through learning from prediction.

Peng Huang and James Hueng test CAPM's asymmetric risk-return relationship in a time-varying beta setup, by adopting Adrian and Franzoni's approach because a time-varying parameter linear regression model allows to use the same regression models as those in CAPM, with the only difference being the time-varying betas as opposed to the constant betas in CAPM.

Their time-varying beta model not only supports the asymmetric risk-return relationship, but also shows improvements of CAPM's constant-beta model in other aspects. Furthermore, their model also improves CAPM's constant beta model and the time-varying beta model eliminates the unexplained returns in the constant beta model. Secondly, the time-varying beta model obtains a more accurate estimate of the per unit risk price. Finally, the Adaptive Least Squares with Kalman foundations successfully improves the accuracy of the estimation of the beta risk by mimicking the investors' learning process on the unobservable beta.

Robert D. Brooks (2003) extent the research of the literature to examine emerging or developing markets. In their paper they study the Chinese market, which allows the trading of two broad classes of shares, A shares for domestic residents and B shares for foreigners across two different markets, Shanghai and Shenzhen. They considered that it is worthwhile to extend the literature by considering the behaviour of betas calculated for the indices of A and B shares on the respective markets and they proved that the variability in the betas across the different exchanges reflect differences in the variability of returns for that index relative to a world index, fact that provides insights into information transfer and market integration. They use data collected on a daily basis for the following stock price indices: Shanghai A; Shanghai B; Shenzhen A; Shenzhen B; and MSCI Global for the period from January 1994 to December 2001. They did their research by the application of the autoregressive conditional heteroscedastic (ARCH) family of models introduced by Engle (1982). In particular, the analysis utilises the standard GARCH model introduced by Bollerslev (1986). The model used in this study is a GARCH (1,1) model where the conditional volatility is estimated as follows: $\sigma^{2t}= \omega+ \alpha\varepsilon^2_{t-1}+\beta\sigma^2_{t-1}$.(1)

The conditional variance equation given is the one period ahead forecast variance based on past information. The conditional variance equation is a function of the mean, ω, news about volatility from the previous period, ε_{t-1}^2 (ARCH term), and last periods forecast variance σ_{t-1}^2 (GARCH term).

The next stage of their analysis involves using the multivariate GARCH (M-GARCH) approach introduced by Bollerslev (1990). This modelling approach requires the covariance between an asset (in this case one of the Chinese indices) and the market (in this case the world index), and conditional variance information derived from the GARCH model for both the asset and the market proxy. A practical problem however, of the M-GARCH approach is that the number of coefficients that need to be estimated are quite large. Bollerslev (1990) proposed that the complications related to the number of coefficients could be reduced by setting the off-diagonal elements of the past error and variance coefficient matrices equal to zero. This restriction reduces the number of coefficients for a GARCH (1,1) model and allows a much simpler estimation procedure.

Although this model produces a time varying beta series, the constant correlation assumptions of Bollerslev (1990) necessarily implies that the model used in this analysis is a restricted version of the full M-GARCH model. This also implies that shocks to the beta in a particular time period are caused by shocks to the volatility of that index.

Finally, they concluded that in contrast to developed equity markets the correlations between the Chinese indices and the world index is very low and consistent with this low correlation, the betas relative to the world index are also low, although the pattern of time variation is consistent with the timing of many of the regulatory developments in the Chinese markets.

Gloria González-Rivera (1997), generalized an asset pricing model based on the Arbitrage Pricing Theory (APT) allowing beta to be time-varying. Making beta a random variable, she added flexibility to the model by permitting a non-linear relation between individual returns and the set of factors, and accounts for the effect of possible omitted variables. She also integrated the conditional APT with a general linear stochastic process for beta and she analyzed the behaviour of the conditional expected return, the conditional variance and conditional covariance of individual asset returns as functions of the conditional moments of beta. She also introduced another source of uncertainty (risk) independent of the factors in order to disentangle if this extra risk is systematic or non-systematic. To this end, she introduced a modified conditional APT model that rationalizes why the time variation of beta may represent extra systematic risk and tested the hypothesis of time-varying beta and the feasibility of the modified conditional APT, presenting a test for time-varying beta based on the conditional second moments of returns. She found that there is strong evidence against constancy of betas in favour of a random coefficient model, and that the time variation of beta is due to non-systematic behaviour of the firms and investors should be able to diversify this risk away.

Alexander Stremme (2007), documented the failure of the static-beta CAPM to explain the cross-section of returns on portfolios sorted on firm size, book-to-market ratio, momentum, and even portfolios sorted on past CAPM betas and in his paper he showed that the model's performance dramatically improves when portfolio betas are allowed to be time-varying functions of (lagged) business cycle variables. He used an approach based on Hansen and Richard (1987) to construct a candidate stochastic discount factor (SDF), using the excess return on the market portfolio as the single factor, scaled by a time-varying coefficient and he resulted to a model in which the conditional factor risk premium is a non-linear function of the business cycle variables. He assessed the performance of his model by computing the R2 of the cross-sectional regression of realized on model-implied expected returns, as for example in Jagannathan and Wang (1996) and although this is not a formal test of the model's ability to price the assets correctly, it does provide an informative summary statistic that allows to compare the performance of this scaled model with that of the static version.

While CAPM is known to perform particularly poorly, this scaled model explains around 60% of the cross-sectional variation in returns on beta and book-to-market portfolios, and 87% for momentum portfolios and additionally the model captures 70% of the value premium (the return spread between the highest and lowest book-to-market deciles portfolios), and 75% of the momentum premium (the spread between the past 'winner' and 'loser' portfolios). These results confirmed the crucial importance of time-varying risk premiums in explaining the cross-section of average returns on these sets of portfolios, although the conditional market risk premium and hence also the betas implied by this model exhibits considerable non-linearity in the business cycle instruments.

Hong Zhang (2003) proposed a rational model for several CAPM anomalies while focusing on the economic foundation of momentum, that is to say that when a firm achieves economies of scale by dynamically adjusting its business according to forecasted business uncertainties, investors face a beta risk in addition to the market risk. These two risks jointly create a nonlinear risk premium, which leads to several momentum-related phenomena in the equilibrium and empirically, the risk premium contributes a leading portion of stock momentum profits. Furthermore, the risk premium helps identify stocks that are likely to generate more momentum profits and market-to-book, size, or two proxies of mispricing do not absorb the effect.

His article contributes to the momentum literature by introducing a new relationship between the CAPM beta and the risk premium, as a rational economy can have size, book-to-market, and several momentum-related phenomena because of this relationship. He proved that the traditional CAPM model and the Fama-French model do not sufficiently describe the risk premium, as an econometrician fails to separate the risk premium properly from the true residual, then he observes residual momentum.

Traditional asset pricing models assume a static variance-covariance matrix for asset returns and, consequently, predict that a firm's risk exposure, or beta, is static and that the expected risk premium is linear in beta. However, if a firm's investment opportunity set changes over time, the firm's beta may be dynamic (See, for example, Berk, Green, and Naik (1999)). More generally, a firm's optimal investment decisions should correlate with its anticipation of business risks: a rational firm wants to explore good business opportunities and achieve economies of scale whenever possible. To demonstrate this intuition, his paper starts from a general observation that the future cash flow for any production project can be affected by different types of risks. But instead of specifying a set of systematic factors, he decomposes the cash flow risks into two components: one that is predictable from the firm's perspective and one that is not. Examples of predictable risks include uncertainties related to a firm's local business environment, such as demand changes or technology innovations.

Although different model specifications can lead to dissimilar beta processes, in general beta dynamics reflect expected changes in the business opportunity set. Since it is risk related, a firm's beta might have subtle impacts on the asset risk premium. Specifically, when investors do not directly observe business risks, they view the behaviours of the firm as a beta risk. Consequently, investors' asset allocation decisions depend on their inference about the firm's investment dynamics, as well as that about the systematic risk. This two-fold uncertainty creates nonlinear risks for investors. This paper demonstrates that the investors' stochastic control problem leads to two new components in the risk premium, in addition to the traditional CAPM element. To investors, the conditional CAPM model fails to hold at each period and furthermore, this paper, shows that the equilibrium price system already reflects all the available public information, that is to say that the existence of momentum or other anomalies does not necessarily mean a failure of the efficient market hypothesis, but it is the linear pricing formula that fails.

The model proposed by this paper is coherent for several CAPM anomalies, including momentum, and is able to explain a number of documented momentum phenomena in the literature. Although the focus of this paper is a simple dynamic CAPM model, this model, together with its closed-form solution, can be generalized, because the linear latent factor can be augmented to incorporate more specific information, such as analyst forecast, financial constraints, or conditional instruments, while the market factor could include more observable systematic risks such as macro factors (Chen, Roll, and Ross (1986)).

Concluding, this article reconciles several CAPM anomalies by pointing out that different anomalies capture different aspects of the risk premium. On this evidence, this paper proves that it is premature to equal the existence of momentum and other asset pricing anomalies to irrationality and market inefficiency, as momentum, for example, is likely to contain both rational and irrational components. To the extent that models usually provide a simplified description of the real world, this article suggests that CAPM and a possible theory for momentum might belong to different levels of approximations, including dynamic and time varying betas and covariances.

Sascha Mergner and Jan Bulla (2005) investigated the time varying behaviour of systematic risk for eighteen pan-European sectors. Using weekly data over the period 1987 to 2005, they applied four different modeling techniques in addition to the standard constant coefficient model: a bivariate t-GARCH(1,1) model, two Kalman filter based approaches, a bivariate stochastic volatility model estimated via the efficient Monte Carlo likelihood technique as well as two Markov switching models. A comparison of the different models' ex ante forecast performances indicated that the random walk process in connection with the Kalman filter is the preferred model to describe and forecast the time varying behavior of sector betas in a European context.

The data used in this paper were weekly excess returns calculated from the total return indices for eighteen pan European industry portfolios, covering the period from 2 December 1987 to 2 February 2005. The sample forecast performances of the various techniques suggested that independent from the utilized modelling approach, the extent to which sector returns can be explained by movements of the overall market is always higher for time varying betas than in connection with standard OLS. This implied confirmation of previous literature findings that sector betas are not stable over time and the results of this study indicated in general that time varying sector betas are best described by a random walk process, estimated by the use of the Kalman Filter.

Mark (2004) forecasted the weekly time varying beta of 20 UK firms by means of four different GARCH models and the Kalman filter method. The four GARCH models applied were the bivariate GARCH, BEKK GARCH, GARCH-GJR and the GARCH-X model and he also compared the forecasting ability of the GARCH models and the Kalman method. He mainly pointed that, given that the beta is time-varying, empirical forecasting of the beta has become important and forecasting time-varying beta is important for few reasons. Since the beta (systematic risk) is the only risk that investors should be concern about, prediction of the beta value helps investors to make their investment decisions easier and also the value of beta can also be used by market participants to measure the performance of fund managers through Treynor ratio. Additionally, for corporate financial managers, forecasts of the conditional beta not only benefit them in the capital structure decision but also in investment appraisal.

The data applied in this paper is weekly ranging from January 1989 to December 2003. Twenty UK firms are selected based on size (market capitalization), industry and the product/service provided by the firm, while the stock returns are created by taking the first difference of the log of the stock indices, in order to avoid differences in scales, and the excess stock returns are created by subtracting the return on a risk free asset from the stock returns. The risk-free asset applied is the UK Treasury Bill Discount 3 Month. Mark declares that it is important to point out that the lack of benchmark is an inevitable weak point of studies on time-varying beta forecasts, since the beta value is unobservable in the real world. Although the point estimation of beta generated by the market model is a moderate proxy for the actual beta value, it is not an appropriate scale to measure a beta series forecasted with time variation. As a result, evaluation of forecast accuracy based on comparing conditional betas estimated and forecasted by the same approach cannot provide compellent evidence of the worth of the approach.

Taufiq Choudhry (2003) empirically investigated the effects of the terrorist attacks and the period after on the time-varying beta (risk) of a few companies in the United States, as the tragic events of September 11, 2001 in the United States is said to have adversely affected the global economy and the financial markets around the world. He used daily data from 1991 to 2002 and the bivariate MA-GARCH model to create the time-varying betas for the firms and his results indicate that September 11 events and the period after affected most of the United States companies under investigation, although the size and direction of the effect varies according to the firms and not all companies did not experience an increase in the beta.

The same writer investigated the effects of the Asian financial crisis of 1997–1998 on the time-varying beta of 10 firms from each of Malaysia and Taiwan, using daily data from 1990 to 2001 and the bivariate MA-GARCH model (BEKK) to create the time-varying betas for the firms. His results provided ample evidence of the influence of the financial crisis and the period after on the time-varying betas of the twenty firms and also he provided evidence of rising in the beta in some cases and a fall in other cases, while the 10 Malaysian firms applied were proved to be more affected than the Taiwanese firms.

Kwang Woo Park and Myeong Hwan Kim (2009) examined financial linkage of systematic risks for 20 industry portfolio returns between Korean and US stock markets, applying time-varying beta coefficients of Capital Asset Pricing Model and Granger-causality tests, in order to identify the significance of the industrial relations between the two stock markets. Their empirical findings showed that the strength and the causality of international financial linkage vary depending on the types of industry and the shocks in the systematic risk, while some Korean industries, including financing industries, iron and metal industries, service, and textile and wearing industries were found relatively vulnerable to systematic risk associate with US industries.

Kwang Woo (2004) examined the conditional relationship between returns and varying betas by using Kalman filter technique that is the special case of the state-space model stating that it is more appropriate to examine the conditional relationship between beta and return by using time-series analysis, as it is well known that the beta is not stable over time. His paper provided evidence from U.S. stock market that there is a significant and systematic relationship between return and Kalman filtered beta when the conditional nature of the CAPM is analyzed. Following time series properties of non-stochastic time-varying beta, he examined the conditional relationship between returns and betas by using a time series analysis, Kalman filter technique and since the change of the systematic risk, or beta, is the fundamental premise of the time-series analysis, the time-series test is less affected by data selection bias than the case for the related cross-sectional studies, as Black (1993) argues that previous findings of a flat relationship between beta and return (Fama and French, 1992) may be attributed to data mining bias.

More particularly, this paper is closely related to the notion of conditional testing approach proposed by Pettengill et al (1995) and the subsequent works by Fletzer (1997), Hodoshima (2000), Elsas et al (1999) in the sense of having the conditional relation between beta and return, although this paper departs from these in considering beta as time-varying coefficient using the estimated Kalman Filter technique. In addition, Woo states in this paper that the advantage of using time series for CAPM tests is that returns and betas do not need to be averaged out over time to compare among different portfolios, as is the case for the most of cross-sectional studies. Another advantage is that one can avoid using too few observations, as with cross-sectional studies, this often becomes problematic especially when researchers try to draw a conclusion from examining the cross-sectional relationships among less than fifty portfolios. Fama and French (1992) used the twenty-two portfolio average returns to conclude that there is a flat relationship between return and beta. This paper estimates the time-varying betas for twenty-two selected portfolios based on size and industry and comparison among the traditional constant betas and Kalman filtered time-varying betas indicates that use of constant beta estimates may cause serious pricing error, as initial tests of beta stability show that the beta is non-stationary. From the initial examination of unconditional relationship between return and beta, he found weak and inter-temporarily inconsistent results. However, when positive and negative realized market returns are considered, the results showed that there is a significant and systematic relationship between beta and return and that is, because empirical results are consistent with the implication that beta is a useful measure of systematic risk over time.

Young-Hye Cho and Robert F. Engle (1999) investigated whether or not a dynamic beta increases with bad news and decreases with good news, just as does volatility. Using daily returns for nine stocks in a double beta model with EGARCH specifications, they showed that news asymmetrically affects the betas of individual stocks. They found that betas depend on two source of news: market shocks and idiosyncratic shocks, while some stock betas depend on both and others depend on one. They categorized each stock return as belonging to one of three beta process models, a joint, an idiosyncratic, and a market model based on the role of market shocks and idiosyncratic shocks and their conclusions differed from those of Brown, Nelson, and Sunnier (1995) who worked with monthly aggregated data in a bivariate EGARCH model, as they stated that stock price aggregation in this previous research resulted in a loss of cross sectional variation and consequently lead to weak results. If the asymmetric effect is more readily apparent in daily data, then this may again explain previous researchers' inability to detect asymmetric effects. Their findings shed light on the controversy as to whether abnormalities in stock returns result from overreaction to information or from changes in expected returns in an efficient market and by finding an asymmetric effect in betas led them to conclude that abnormalities can, at least partially, be explained by changes in expected returns through a change in beta.

Argyrios Volis, George Karathanasis and Panayiotis Diamandis (2003) investigated time varying risk premium for the stocks traded on the Athens Stock Exchange. Their research methodology utilised two well known empirical findings; the time varying beta risk and the day-of-the-week effect, and especially the Monday Effect. For that purpose, they introduced a multivariate model, based on the research paper of Faff and Brooks (1998). Using a set of dummy variables, they examined the stability of the beta coefficient, and further they investigated the impact that the findings might have on portfolio theory, by re-evaluating the steps that are necessary, when constructing a portfolio. For that purpose, the sample period, that they chose to analyze, is divided into 3 sub-periods (each one having specific characteristics, as the first period doesn't exhibit any significant volatility, while the second and third one is described by increasing and decreasing returns of the market respectively), and in this way, they explored the behaviour of the beta risk of the sectors, as well as the companies included in the data set. The main findings were that the sub-periods play an important role to the beta risk formation, and that the beta risk is a function of the direction of the market, and the magnitude of the market returns.

Tourani-Rad, A., Choi, D. & Wilson, B. (2006) estimated New Zealand's country level risk using a time-varying country beta market model, as country beta is allowed to vary as a function of several macro-economic variables, including the net government overseas borrowing, 90-day bill rate, ten-year bill rate, wool price, trade-weighted index, manufacturers' price index, retail trade, current account balance, and money supply. Multivariate regression analysis was used in this paper to test the relation between country volatility and the macro-economic variables for the period September 1985 to March 2000 and it was found that the US dollar exchange rate (USD) and the monetary conditions index (MCI) have a significant impact on New Zealand's country dynamic beta.

R.D. Brooks (2006) examined post time-varying beta estimation, modelling and asset pricing tests. In particular, he investigated these issues using a sample of monthly data on Australian industry portfolios over the nineteen-year period 1974 to 1992. While primarily illustrative in nature, the industry betas were modelled, estimated and tested with reasonable success in terms of regimes related to periods of regulation/deregulation/imputation, the level of market returns and a measure of volatility on the risk-free rate of interest. However, univariate and multivariate tests reported in the paper provided mixed evidence concerning the applicability of a time-varying beta CAPM that incorporates these variables.

Ralf Ostermark (1993) in his paper showed that, based on equally weighted portfolios of continuously listed Finnish and Swedish stocks, a Dynamic Model of Capital Asset Pricing (DCAPM) outperforms the static Capital Asset Pricing Model (CAPM) and he demonstrated that the portfolio efficiency of the dynamic model is improved, when using a properly defined transition matrix in the Kalman Filtering Algorithm.

He firstly estimated the weekly returns series for a number of Finnish and Swedish portfolios by the Fama-MacBeth procedure and then he used weekly returns of portfolio aggregated data in order to overcome some of the criticism levelled against empirical testing of the CAPM with daily returns of individual stocks in Scandinavian conditions. The main idea of the Fama- MacBeth method is to combine asset returns into portfolios, each consisting of an exclusive set of q assets. Finally he estimated the CAPM of the portfolio returns of stage by Katrnan Filtering and determined the portfolio efficiency of the dynamic CAPM (DCAPM) of stage by the Super Criterion Test.

More specifically, he proved that while the APT framework still outperforms the competing models, the strength of domination is weakened in both countries, when confronting the APT with a dynamic nonstationary CAPM, and this evidence is based on a fairly crude search procedure for the transition process: whereas the true transition process may be expected to vary over time as well as over assets, practicality has forced him to assume constancy of the transition matrix elements. The evidence further justified and motivated a thorough analysis of the mechanism by which capital asset prices react to new information and changing economic conditions over time. A dynamic framework incorporating investor reactions to new information in the beta-risk assessment process seemed theoretically appealing. Finally, the dynamic model was proved more powerful with Finnish than with Swedish data.

CHAPTER 3

EMPIRICAL ANALYSIS

3.1 Data

The market data employed in this study are the daily stock return of Athens Stock Exchange Market and returns of two stocks of Athens Stock Exchange, concerning the sector of banking. These two stocks are the stock of Bank of Eurobank and the stock of Bank of Piraeus. Our market data, which are daily, concern the period from 02 January of 2002 to 29 December of 2006, they are transformed in logarithms in order to avoid differences in scales, they are taken from databasis of Athens Stock Exchange Market and will be run, so that dynamic beta is investigated. The econometric programme that is used is e-Views 5.

3.2 Stationarity and Descriptive Statistics

The following step is to assess the most appropriate technique to test the hypothesis of stock returns, by examining if the variables examined are stationary or not. Stationarity means that a variable, although fluctuating, tends to return to a constant mean (hence, it is called “mean reverting”). A non-stationary variable, on the other hand, would exhibit apparent changes in mean, or appear highly persistent.

An important aspect of analyzing the time series process of our variables is to have a unit root test that is able to identify a nonstationary property. Unfortunately, unit root tests are notoriously low power tests. To overcome this, we present three different unit root tests, which are: the augmented Dickey-Fuller test (ADF), the DF-GLS unit root tests and the Kwiatkowski, Phillips, Schmidt, and Shin test (KPSS). Consider the time series with serial correlation in errors described as

$$y_t = a+\beta y_{t-1}+\varepsilon_t \qquad (10)$$

and

$$\varepsilon_t = \varphi\varepsilon_{t-1}+e_t+\theta e_{t-1} \qquad (11)$$

The ADF test is carried out by estimating

$$\Delta y_t = a+\alpha y_{t-1}+\sum\nolimits_{j=1}^{k} \beta_j \Delta y_{t-j}+\varepsilon_t \qquad (12)$$

where α = ρ−1 and t = 1, …., T. The augmented terms Δy_t of higher order lags are included into equation (12) to correct the serial correlations of the disturbances ε_t . The number of k lags are selected by the Schwartz Information Criteria. The null hypothesis of a unit root (α= 0) is tested against the alternative hypothesis of stationarity (α<0). The test statistic is evaluated using the conventional t-ratio for α and the critical value is obtained by MacKinnon's updated version of Dickey-Fuller critical values.

The DF-GLS unit root test is developed to solve the problem of low power. Elliott et al. (1996) propose a simple modification of the ADF tests in which the data are detrended so that explanatory variables are removed from the data prior to running the test regression. The GLS detrending of the data yields substantial power gains. After obtaining the GLS detrended data, say y_t^d ,the DF-GLS test involves estimating the standard ADF test by substituting the GLS detrended y_t^d for the original y_t :

$$\Delta y_t^d = \alpha + \alpha y_{t-1}^d + \sum_{j=1}^{k} \beta_j \Delta y_{t-j}^d + \varepsilon_t \qquad (13)$$

where the lag length k in this equation is selected using a modified Akaike information criteria (MAIC), which is

$$MAIC = -2(l/T) + 2(k+\tau)/T \qquad (14)$$

where l is the bandwidth parameter for the kernel-based estimators of the residual spectrum at frequency zero and $\tau = \alpha^2 \sum_t \tilde{y}_{t-1}^2 / \hat{\sigma}^2$ for $\tilde{y}$ is the autoregressive spectral density estimator. Perron and Ng (1996) suggested the use of MAIC and found substantial size improvements over standard information criteria in the unit root testing.

Lastly, we use the KPSS test to test the null of stationarity against the alternative hypothesis of a random walk. The KPSS test starts with

$$y_t = \delta t + \zeta_t + \varepsilon_t \qquad (15)$$

Where ζ_t is a stationary process and ε_t is a random walk given by

$$\zeta_t = \zeta_{t-1} + u_t, \quad u_t \sim \text{iid}(0, \sigma_u^2) \qquad (16)$$

The null hypothesis of stationarity is formulated as

$$\text{H}_0 : \sigma_u^2 \text{ or } \zeta_t \text{ is a constant}$$

and the alternative hypothesis is that the parameter follows a random walk. The test statistic for this hypothesis is given by

$$LM = \frac{\sum_{t=1}^{T} S_t^2}{\hat{\sigma}_e^2} \qquad (17)$$

where $S_t = \sum_{i=1}^{T} e_i$, t = 1, ..., T is a cumulative residual function for e_t are the residuals from the regression of y_t on a constant and a time trend, and $\hat{\sigma}_e^2$ is the residual variance. We use the Bartlett spectral window kernel-based estimator to obtain a consistent estimate of the variance and select the bandwidth by using the Newey-West method. The test is an upper-tailed test.

Table 1: ADF Unit Root Test

Variable	**ADF**	**Critical value 5%**	**Probability**
Level			
Athens Stock Exchange	-17.52790*	-2.872370	0.067
Bank of Pireaus	-9.260444* (T)	-3.427070	0.073
Bank of Eurobank	-8.530712* (T)	-3.427070	0.078

Notes:
* Implies significance at 5%. (T) indicates the trend is included as indicated by the significance of the trend terms in the estimation.

The unit root tests in table 1 indicate that overall the variables appear not to be stationary. In other words, based on the conventional ADF unit root tests, nearly all of the variables examined appear not to be stationary, as the null hypothesis of a unit root are significantly not rejected at 0.05 level.

Table 2: DF-GLS Unit Root Test

Variable	**DF-GLS**	**Critical value 5%**
Level		
Athens Stock Exchange	-1.739482*	-1.942040
Bank of Pireaus	-0.9241114* (T)	-2.917600
Bank of Eurobank	-0.7513289* (T)	-2.917600

Notes:
* Implies significance at 5%. (T) indicates the trend is included as indicated by the significance of the trend terms in the estimation.

Table 12 indicates the same hypothesis as above, that all variables are not stationary in the level. In other words, the DF-GLS test confirms the results from the unit root for all the variables.

Table 3: KPSS Unit Root Test

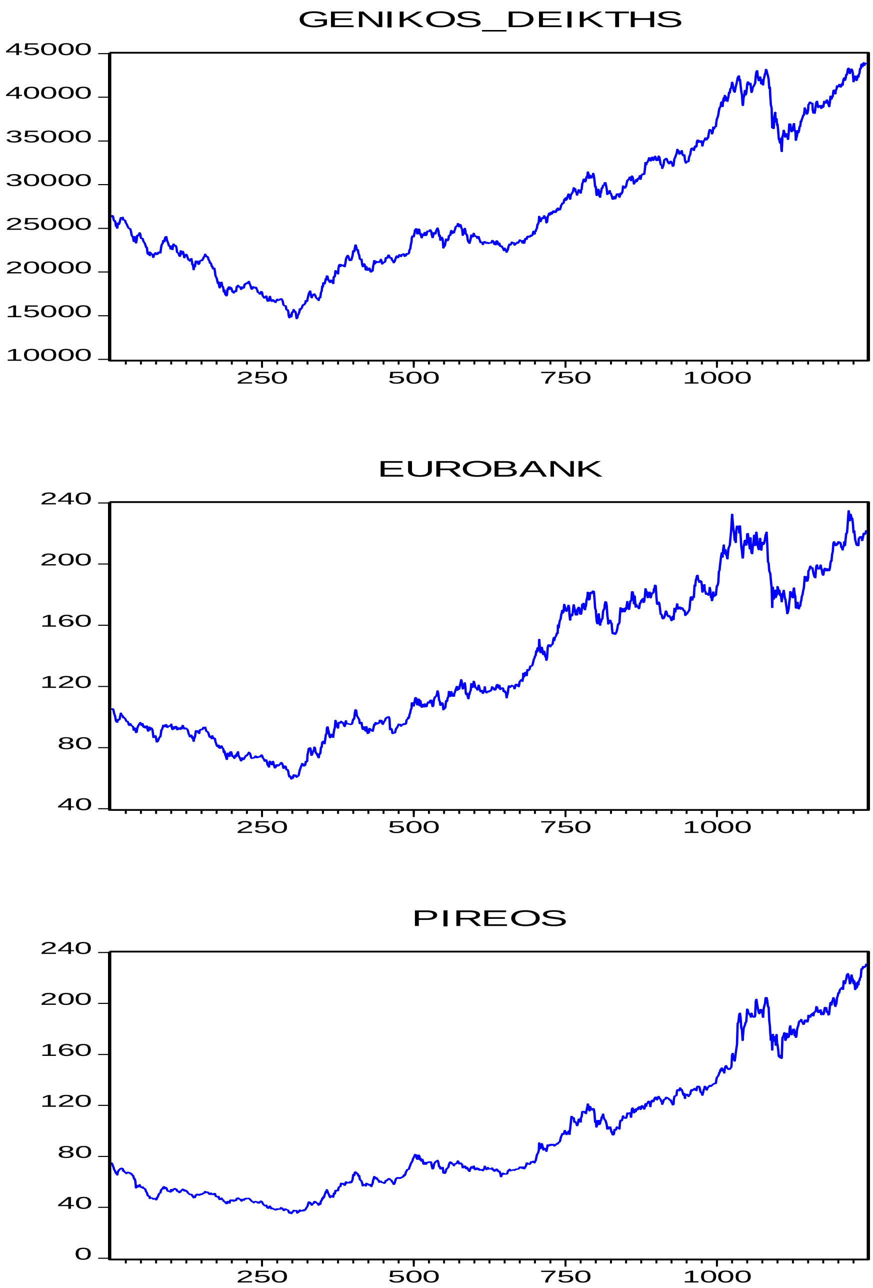
Variable	**KPSS**	**Critical value 5%**
Level		
Athens Stock Exchange	0.518634	0.463000
Bank of Pireaus	0.167413 (T)	0.146000
Bank of Eurobank	0.283654* (T)	0.146000

Notes:
* Implies significance at 5%. (T) indicates the trend is included as indicated by the significance of the trend terms in the estimation.

In contrast, KPSS test, where the null hypothesis is switched to be one of stationarity, leads to other results. In particular, this unit root test indicates that our variables are not stationary in the level.

Our finding that nearly all variables are not I(0)-according to the results of ADF and DF-GLS unit root tests- deserves some discussion. Theory suggests that if two series, such as stock returns, are stationary, means that there does not exist some linear combination of them; in other words, we cannot test for cointegration for the two series. In our case we can say that we have linear relationship between our variables, and that is why we have transformed them into returns (logarithms), in order to gain in symmetry and lessen the covariance between them. In Graphs 1,2 and 3 below, trend and stationarity is presented.

Graph 1, 2, 3 Trend in the series



Additionally, in table 4 below, all descriptive statistics are presented for Athens stock Exchange, Bank of Pireaus and bank of Eurobank. As we know from theory, concerning descriptive statistics, skewness shows symmetry in the market, as when skewness equals to zero market is symmetric and when skewness does not equal to zero the market is not symmetric. Additionally, when skewness is bigger than zero the distribution of the market has a big right tail and when skewness is smaller than zero the market has a big left tail, proportionally. Statistic of kurtosis presents the kurtosis, that is to say if the serie has a pick or not. If kurtosis is bigger than three, the distribution has bigger pick than normal and when this number is smaller than three, the distribution has a pick that is smaller than normal. Concerning Jacque-Bera statistic, this is a test for normality of distribution, with a null hypothesis that the serie is normally distributed (Jacque-Bera<5.99) and alternative hypothesis that the serie is not normally distributed (Jacque-Bera>5.99).

Concerning our series, Athens Stock Exchange has a skewness of -0.117778, that is not equal to zero and smaller than zero and so we have negative asymmetry, and additionally we have platokurtosis, as kurtosis is 4.925101>3. Jacque-Bera equals to 195.1279 and is bigger than 5.99, resulting to the conclusion that the market is not normally distributed, and so it is not efficient.

From the correlogramme we can report that the signs change and the statistical significance as well, in a way that we have a special case of median-long memory in the serie, where there is not efficiency in the market. This is one reason why we cannot apply OLS estimation models, and we apply GARCH models.

Concerning the descriptive statistics of the series of the Bank of Eurobank and of the Bank of Pireaus, we can report that they present skewness which does not equal to zero and additionally is a positive number, fact that leads us to the result that we have a positive asymmetry and a leptokurtosic distribution. Descriptive statistics of kurtosis show that the distribution of both series have a pick bigger than normal and prices of Jacque-Bera for both series confirms the fact that the two series are not normally distributed, comparing once more the price of Jacque-Bera with typical price of 5.99. The rest of the statistics are presented in the table below.

Table 4 Descriptive Statistics

	Mean	Median	Max	Min	Std. Dev.	Skewness	Kurtosis	Jarque-Berra
Athens Stock Exchange	0.000413	0.000471	0.049736	-0.061067	0.010644	-0.117778	4.925101	195.1279
Bank of Pireaus	0.000902	0.000000	0.094004	-0.089921	0.016617	0.0285740	6.403170	617.7356
Bank of Eurobank	0.000603	0.000791	0.074517	-0.075454	0.015515	0.088180	4.673559	146.9050

3.3 Regression

Running firstly the regression, by applying OLS estimators, as in this case the relationship between the variables is linear, between the stock of Eurobank and Athens Stock Exchange Market and then the regression between the stock of Bank of Pireaus and the Athens Stock Exchange Market, we can report the static betas, which are 1.072113 for Eurobank and 1.135208 for the Bank of Pireaus. In this point we can say that these are two stocks that are aggressive, since the price of beta is quite bigger than the unit, in a way that these two stocks change in a more intense way than the stock market.

For the evaluation of dymamic beta, we can report that the covariance is heteroskedastic, and that is the reason why we apply GARCH models. The type for the evaluation of time varying beta is transformed as below:

$$b_{it} = \frac{Cov(r_{it}, r_{Mt})}{s^{2}_{Mt}} \quad (17)$$

where Cov (r_{it} , r_{Mt}) is the covariance between every stock and the stock market and where σ^2_{Mt} is the variance of the stock market. Both variance and covariances change during time, and that is why we have a dynamic beta and not a static one. In this point we can say that the real difference between dynamic and static beta is the covariance that change during time in the first case and remains the same during time for the second case, proportionally.

By this way mentioned above we calculate the covariance between every stock and the stock market and we find that the covariance between Eurobank and the stock market is 0.000121 and the covariance between the Bank of Pireaus and the stock market is 0.000129.

The model of GARCH that we apply has the form of:

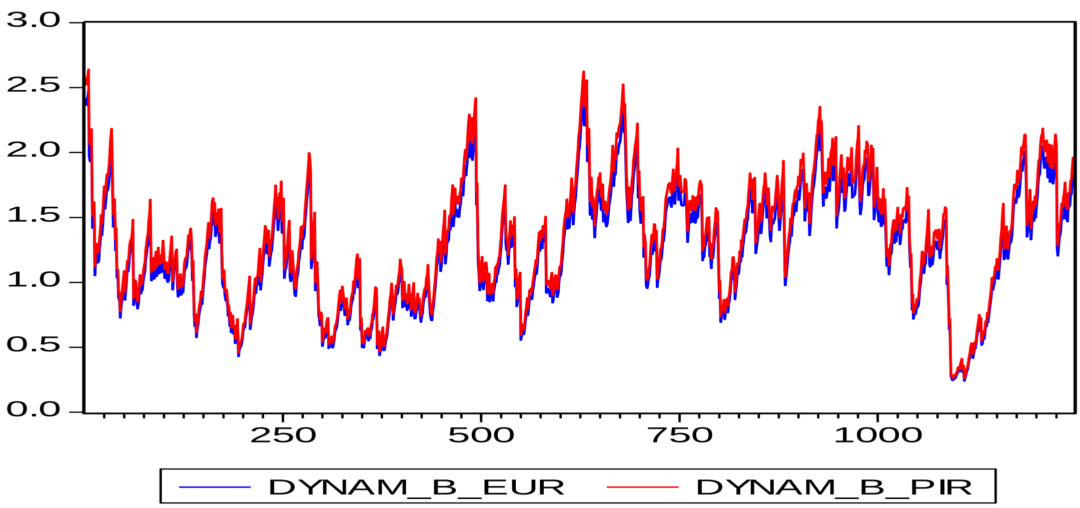
$$\sigma^2_t = \alpha + \beta_{1*}\varepsilon^2_{t\text{-}1} + \gamma_{1*}\sigma^2_{t.\text{-}1}\,, (18)$$

where α, β_1, γ_1 are positive numbers and they are statistically significant for the level of 5% and 10%. Additionally, $\beta_1+\gamma_1$ is smaller than the unit as 0,072383+0,896521=0,968904<1.

Having calculated the heteroscedastic variance of stock market and the covariance of the stock market with every stock individually, we apply equation (17), finding the dynamic betas for every stock proportionally. The dynamic betas of every stock are 0.000121 for the stock of Eurobank and 0.000129 for the Bank of Pireaus.

In the multi-graph below is presented the variance of the stock market in relationship with the dynamic betas of the two stocks. The variance of the stock market is multiplied by its returns. In the second half of the graph the stock market presents stability for a very small period and in the rest of the period presents a very intense variance, fact that proves that Greek Exchange Market is not stable.

Graph 4 Dynamic Betas



In the point where the stock market has a big through that is followed by a big pick a month later, proves that this period is characterized by big variance, which maybe happened because of a big shock. This same period, the dynamic betas are getting smaller and actually they have very low prices. This means that in periods of high variance of the stock market, these stocks have stability, and if we could include them in a portfolio they could absorb this high variance of the market.

Moreover, we can add that when we report a high variance of the market we see a big through of the market, at 22/05/2006 (fall of 5.92%) and a high pick, at 15/06/2006 (pick of 5.10%). Nevertheless, during the period of stability of the stock market, the stocks present dynamic betas with high variance, since they change prices in a very intense way. Additionally, we can report that dynamic betas of both stocks follow the same paths, although the stock of Bank of Pireaus present higher dynamic beta in contrast with the dynamic beta of Eurobank, fact that leads us to the conclusion that this stock is riskier. The periods when the dynamic betas are higher are the periods when the total exchange market presents a relative stability. These prices are 2.5%, that is to say that if the market changes 1%, these stocks will change 2.5%. This behaviour is opposite to the behaviour of these stocks when the market is in shock, since during this period these stocks are less risky, while during the periods of stability, they can be extremely risky and aggressive.

The fact that the two dynamic betas during time present the same behaviour is because they are both of the same sector of the market, banking, sector which has a high proportion of capitalization and faces the same risks. Even if these two stocks had a big proportion of a portfolio and absorbed a high proportion of its riskiness, this portfolio would not be diversified, since these two stocks have similar dynamic betas. In order to state if these stocks have a good behaviour, we should compare them with the dynamic betas of the banking branch of the economy or the time varying betas of the branch of high capitalization.

Finally, comparing statistics of these two dynamic betas with the static ones, we can report that the dynamic betas have similar prices with the static ones, fact that does not contradict previous literature and theory, which report that these two kinds of betas follow same paths.

As a conclusion, dynamic betas are a form of investigation of riskiness of stocks and stock market and they are the basis for an augmented CAPM model, which explains the market in a better way, because includes in its formation time variance. This augmented form of CAPM is the basis of the formulation of Value At Risk of the stocks, subject that needs further investigation.

CONCLUSIONS

There is a large literature that establishes that individual stock betas vary over time. This matter has also been studied for country betas. A number of these studies have used GARCH based approaches for the estimation of country betas. Giannopolous (1995) used a bivariate GARCH model to calculate betas for weekly stock returns data for 13 developed countries. Brooks, Faff and McKenzie (2002) also used a bivariate GARCH model to calculate betas for 17 developed countries. Since Sharpe (1964) and Lintner (1965) proposed the Capital Asset Pricing Model (CAPM) to describe the risk-return relationship, substantial empirical work has been conducted to investigate the validity of the model. Many empirical studies, by using a different approach proposed by Fama and MacBeth (1973), show that the Sharpe-Lintner CAPM provides an inadequate explanation of the risk-return relationship due to the lack of evidence that supports a statistically significant relationship between risk and return (Fama and French 1992, He and Ng 1994). This unsuccessful empirical performance of the CAPM caused people to cast doubts on the model. Pettengill, Sundaram, and Mathur (1995) argued that the validity of the CAPM is not directly examined by the Fama-MacBeth methodology.

Aim of this paper was to describe the theory of the CAPM model, report the new form of CAPM model, the conditional CAPM, which includes time varying betas, analyze the role of beta as risk in the market and report ways to estimate dynamic betas. Athens Stock Exchange Market and two stocks of bank sector were chosen, in order to estimate time varying betas and discuss their performance in the market, as main point of this coursework was to account for the parameter of time variation, in order to develop a pricing model that more accurately describes the behaviour of returns. Additionally, previous literature concerning dynamic betas was presented and finally, GARCH models were applied in the estimation of time varying betas, as OLS estimators cannot be applied when the covariance between the variables is heteroscedastic. Having calculated the heteroscedastic variance of stock market and the covariance of the stock market with every stock individually, we apply equation of GARCH model, finding the dynamic betas for every stock proportionally which were 0.000121 for the stock of Eurobank and 0.000129 for the Bank of Pireaus.

Additionally, we reported that dynamic betas of both stocks follow the same paths, although the stock of Bank of Pireaus presented higher dynamic beta in contrast with the dynamic beta of Eurobank, fact that led us to the conclusion that this stock is riskier. The periods when the dynamic betas are higher are the periods when the total exchange market presents a relative stability. This behaviour is opposite to the behaviour of these stocks when the market is in shock, since during this period these stocks are less risky, while during the periods of stability, they can be extremely risky and aggressive. Finally, comparing statistics of these two dynamic betas with the static ones, we reported that the dynamic betas have similar prices with the static ones, fact that does not contradict previous literature and theory, which report that these two kinds of betas follow same paths. As a conclusion, dynamic betas is a key point of modern literature, concerning economics and should be taken into account when Value at Risk of assets is estimated, field that needs further investigation.

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