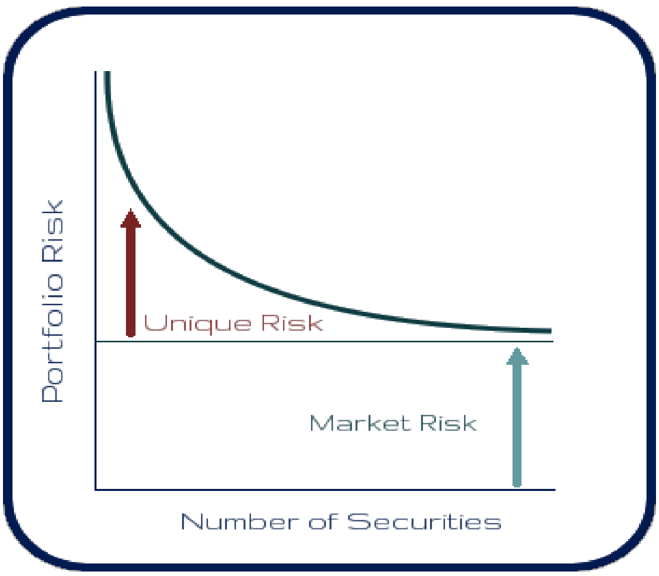
**UNIVERSITY OF MACEDONIA**
**DEPARTMENT OF ECONOMICS**

Thesis paper

# PORTFOLIO DIVERSIFICATION: APPLICATION & COMPARISON OF DIFFERENT STRATEGIES

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*Dedicated to my family*

"Give a portion to seven, or even to eight,
for you know not what disaster may happen on earth."
-ECCLESIASTES 11:2-

# Acknowledgments

The present thesis is a product of a lot effort. A significant amount of time was sacrificed so that the writer could collect the data, process them and apply the several diversification strategies, in order to reach to some conclusions. Hopefully, these results will be found useful by other researchers, who also attempt to contribute in the field of finance.

At this point, it is appropriate to express my gratitude to Theodore Panagiotidis, professor at the Department of Economics at University of Macedonia, for the assignment and supervision of the present thesis. His guidance was crucial during the whole procedure.

And of course, for one more time, I could never stress enough the continuous support of my family.

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# Prologue

Investing, by its very nature, always contains risk. The future returns are uncertain, which means that an asset’s value may grow slower than expected or even be diminished. One of the main problems in finance is the minimization of the financial risk, or alternatively the maximization of the portfolio returns. Through the years, various techniques have been developed which attempt to construct efficient investment portfolios.

Diversification is one of the major techniques for reducing investment risk. It means that an investor can reduce her portfolio risk by selecting a variety of assets. More specifically, diversification relies on the lack of a tight positive relationship among the assets' returns. If the asset values do not move up and down in perfect synchrony, meaning that there is no substantial positive correlation between the assets, any diversified portfolio will have less risk than the weighted average risk of the assets which constitute it and often less risk than the least risky of its constituent assets. Therefore, it is logical that any risk-averse investor will diversify to at least some extent. Also, more risk-averse investors will diversify more than less risk-averse investors.

In finance, diversification is applied with different methods. This paper’s purpose is to present some of those methods and compare their results. The data that will be used are collected for the five years period 2008-2013 and include the prices and returns of the stock indices of all the countries in the Eurozone.

As mentioned above, we will construct various portfolios using different techniques. Firstly, risk minimized portfolios will be made with the classical mean variance portfolio method. Next, going one step further, we will create portfolios that also minimize risk, but use long-run correlations, instead of simple correlations between the assets. The final method that we will test is the construction of portfolios via cointegration. All the above methods will be compared with each other and the benchmark.

At the first chapter, a literature review will be presented. At the second chapter, the theory of the portfolio construction techniques will be analyzed thoroughly. Finally, chapter three includes all the empirical part of the paper. We will present all the results, compare them and reach conclusions.

# Chapter 1

## Literature Review

In finance, as we mentioned earlier, diversification is applied with different methods. The most famous is modern portfolio theory. It is a mathematical formula which concept is the diversification in investing and aims to select a collection of investment assets that have in total lower risk than any individual asset. More technically, modern portfolio theory models an asset’ s return as a normally distributed function (or more generally as an elliptically distributed random variable), defines the risk as the standard deviation of return and constructs a portfolio as a weighted combination of assets, so that the return of a portfolio is the weighted sum of the assets' returns. By combining different assets whose returns are not perfectly positively correlated, modern portfolio theory seeks to reduce the total variance of the portfolio return. It also assumes that investors are rational, and markets are efficient. This model is also called, mean-variance model because it is based on expected returns (mean) and the standard deviation (variance) of the various assets and portfolios.

This theory introduced by Harry Markowitz in 1952 in its paper “Portfolio Selection”. His impact in the academic research was tremendous. The concepts that Markowitz analyzed were later essential to the development of the Capital Asset Pricing Model. So, during the later years, many academic papers applied this portfolio optimization theory.

Jorion (1992) applied monthly and annual data from various government bond returns and showed the benefits of diversification via the construction of portfolios with these investment vehicles. Additionally, Solnik (1995) in his paper “Why not Diversify Internationally Rather than Domestically?” supported that investors should not construct portfolios with domestic assets only. In order to reduce risk further, international diversification was recommended. His data were weekly from various US and European stocks and revealed that substantial benefits in risk reduction can be attained through portfolio diversification in foreign securities.

Several researchers attempted to locate negative correlations between countries. Alagidede, Panagiotidis & Zhang (2011) investigated the integration between African equity markets and the rest of the world. The paper employed parametric and nonparametric cointegration approaches using monthly data. Monthly series were used to circumvent the problem of nonsynchronous trading and to avoid the possible effects of autocorrelation in volatility, a feature of higher frequency data such as daily or weekly prices.

Also, Moss & Thuotte (2013) examined the market performance of several national stock indices against widely known international benchmarks to determine the potential of sub-Saharan African stock exchanges as an investment vehicle for enhancing international diversification. The data used in this study were sourced from Bloomberg LLP and included both daily and monthly observations for January 1990 to September 2012. Data from all major global indices in every region were included. The findings show that investors seeking international diversification via uncorrelated markets should consider sub-Saharan Africa as a destination.

The most recent papers, like Alagidede et al (2011), do not only attempt to locate simple correlations between the assets. For example, Alagidede et al (2011)’s results supported that there were few long –run relationships between African markets and also between Africa and the rest of the world. So, other concepts and techniques are also applied to examine the long-run relationships between the investment vehicles.

One concept is long-run correlation, which is not new in economics. It can be defined using the complex coherency function from spectral analysis. Granger and Weiss (1983), for example, noted that two I(1) series are cointegrated if and only if the differenced series have coherency equal to unity at frequency zero, meaning that their squared long-run correlation is equal to one. The long-run correlation is used to represent a measure of long-run relationship between two series.

Albuquerque (2001) proposed a nonparametric long-run correlation estimator. In his paper, the properties of the estimator were defined, including the asymptotic distribution and a simple test for the null of zero long-run correlation. Monte Carlo experiments showed that these asymptotic properties were a good approximation even in the case of small samples, meaning that the proposed estimator proved to be superior to commonly deployed alternatives. We will use this estimator to test if it leads to superior portfolios compared to the simple correlation mean-variance portfolios.

Additionally, a portfolio construction attempt will be made via cointegration. Cointegration was introduced by Engle & Granger (1987) and has become an important tool of time series econometrics. Several studies, such as Johansen's (2007) "Correlation, regression, and cointegration of nonstationary economic time series" highlighted the spurious correlation problem and supported the cointegration analysis.

Cointegration has been used as a powerful technique for examining common trends in time series with multiple variables, and provided a sound methodology for modeling both the long run and short run dynamics in a system. Even if models of cointegrated time series were common in the literature for years, until recently, their importance was mainly theoretical. This is because the portfolio risk management was, in the vast majority, applied via the correlation analysis of returns.

However, correlation is intrinsically a short run measure, and so correlation based strategies generally require frequent rebalancing. Therefore, portfolios that are based on cointegration, which measures long run co-movements in prices, may be more efficient. To sum up, investment management strategies which are based solely on volatility and correlation of returns cannot secure long-term performance. There is no mechanism to guarantee that the hedge will revert to the underlying, and nothing prevents the tracking error from behaving as a random walk.

Since high correlation alone does not suffice to guarantee the long-term performance of hedges, we need to augment traditional risk-return modeling strategies to take account of common (long-term) trends in prices. And this is precisely what cointegration provides.

In the last years, many important papers have created cointegration based portfolios with promising results. Each of these portfolios applied different methods and attempted to track an index, hedge or be market neutral. Alexander (1999) used several world indices, such as WIES[1] &EAFE[2], in order to create a global portfolio using cointegration and compare with the benchmark indices. Additionally, Alexander et al (2002) published another paper which presented the use of cointegration in portfolio optimization. A 75 stock hedge fund portfolio was constructed, in order to track the S&P100 benchmark, using cointegration. The results of both works demonstrated the ability of the model to capture market upswings whilst not compromising the downside protection.

Finally, Dunis & Ho (2005) used the concept of cointegration to design two quantitative European equities portfolios: a classic index tracking strategy and a long-short equity market neutral strategy. The designed portfolios were strongly cointegrated with the benchmark and indeed demonstrated good tracking performance. The long-short market neutral strategy generated steady returns under adverse market circumstances. Thus, the results were quite promising since all tracking portfolios produced much better returns and risk-adjusted returns, with less volatile Sharpe ratio profiles than those of the benchmark.

In this paper, alternative portfolios will be constructed using the methods mentioned above and will be compared, in order to select the best strategy for our collected data. Apart from the significant works that were referred above references section highlights some additional contributions which are worth mentioning. Some of them present portfolio creation methods which are not in the frames of the present thesis.

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**1. EAFE=European, Asian and Far East Morgan Stanley index.**
**2.WIES= World Integrated Equity Selector – for more information see Alexander (1999).**

# Chapter 2

# Theoretical background

## Introduction

In this chapter, we will present briefly all the mathematical terms and formulations which are used in the thesis. Firstly, some basic calculations will be shown. Then, we continue by analyzing the different portfolio construction methods (mean variance portfolio with simple correlations, mean variance portfolio with long-run correlations, cointegration optimized portfolios).

## 2.1 Basic financial calculations

### Variance

In probability theory and statistics, variance is the measure of how far a set of values is spread out. Variance is one of many descriptors of a probability distribution, describing how far the numbers lie from their mean (expected average). In particular, the variance is one of the moments of a distribution. In other words, it forms part of a systematic method to distinguishing between probability distributions. Although other such approaches have also been developed, those based on moments are better in terms of mathematical and computational simplicity.

The variance of a random variable X is its second central moment, meaning that it is the expected value of the squared deviation from the mean μ = E[X]:

$$Var(X) = E[(X - \mu)^2] \qquad (2.1)$$

In finance, variance (or standard deviation which is variance's square root) is the measure of investment risk.

## Covariance & correlation

Covariance is a measure of how much two random variables change together. If the higher values of one variable generally correspond with the greater values of the second variable, and the same holds for the smaller values, the variables tend to show similar behavior and their covariance is positive. In the opposite case, if the higher values of one variable generally correspond to the smaller values of the other, then the variables tend to show opposite behavior and the covariance is negative. So, the sign of the covariance states the tendency in the linear relationship between the variables. The magnitude of the covariance cannot be easily interpreted. The normalized version of the covariance, the correlation coefficient, however, shows by its magnitude the strength of the linear relation.

Thus, the covariance between two jointly distributed real-valued random variables X and Y with finite second moments is defined as:

$$Cov(X, Y) = E[(X - E[X])(Y - E[Y])] \qquad (2.2)$$

Also, the definition of correlation is:

$$Corr(X, Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} \qquad (2.3)$$

Covariance and correlation play an important role in financial economics, especially in portfolio theory. Covariances (or correlations) among various assets' returns are used to determine, under certain assumptions, the relative proportions of different assets that investors should choose to hold in their diversified portfolio.

### Returns

Asset returns can be computed in two different ways. The simple return is defined as:

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{2.4}$$

where $P_t$ denotes the asset price at time t. The other definition is the logarithmic return:

$$r_t = p_t - p_{t-1} \tag{2.5}$$

where $p_t$ is the natural logarithm of price $P_t$. The log-return is equal to the continuously compounded return. For small values, the two equations give equal results.

In empirical work in finance, the latter equation is primarily used. Log-return is often preferred because, under some assumptions, it has some convenient statistical properties that help simplify analysis. In the present thesis, log-return will be used. For more information, the book of Robert Sollis (see Bibliography) is proposed.

## 2.2 Modern portfolio theory

The fundamental concept behind MPT is that an investment portfolio's assets should not be selected individually, each on their own valuation. Rather, it is important to consider how each asset varies in price relative to how every other asset in the portfolio varies in price.

Investing is a tradeoff between risk and return. In general, assets with higher expected returns contain more risk. For a given amount of risk, MPT guides the investor and shows her how to choose a portfolio with the highest possible expected return. Or else, for a given expected return, MPT explains how to choose a minimum risk portfolio (the desirably expected return cannot exceed the highest-returning available asset, unless short sales of assets are possible.)

Therefore, Modern portfolio theory is a form of diversification. Under certain assumptions and for particular quantitative definitions of risk and return, MPT indicates the best possible diversification strategy.

MPT assumes that investors are risk-averse, which means that given two portfolios which offer the same expected return, investors will definitely prefer the less risky one. Thus, every investor will take on more risk only if compensated by better expected returns. Or to state it differently, an investor who wants higher expected returns has to accept more risk. The exact tradeoff will be equal for all investors, but different individuals will evaluate the tradeoff differently based on personal risk aversion characteristics. The result is that a rational investor will not invest in a portfolio if there exists another portfolio with a better risk-expected return profile. For example, if, for the same level of risk, an alternative portfolio exists that has better expected returns.

Note also that this theory uses standard deviation of return as a measure for risk, which is valid if the returns of the assets are jointly normally distributed or otherwise elliptically distributed.

Under the model:

1. Portfolio's total expected return is the proportion-weighted combination of the constituent assets' returns:

$$E(r_p) = \sum_i w_i E(r_i) \tag{2.6}$$

where $r_p$ is the return on the portfolio, $r_i$ is the return on asset i and $w_i$ is the weighting of component asset i (that is, the proportion of asset "i" in the portfolio).

2. Portfolio variance is a function that includes the correlations $\rho_{ij}$ of the component assets for all possible asset pairs (i, j):

$$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij} \tag{2.7}$$

where $\rho_{ij}$ is the correlation between the returns of assets i and j.

3. Portfolio return volatility (standard deviation):

$$\sigma_p = \sqrt{\sigma_p^2} \tag{2.8}$$

## 2.3 Long-run correlation

In MPT, simple correlations are widely used. However, this kind of estimation procedures is usually suboptimal. More specifically, the use of a simple correlation estimator to weekly or monthly aggregate returns, despite the fact that data is usually available at higher frequencies, leads to inefficiency. Additionally, simple correlation fails to capture permanent relationships between the time series.

Albuquerque suggests a better procedure that has the additional advantage of not significantly departing from common practice in time series correlation studies. He proposes long-run correlation which can be defined using the concept of complex coherency from spectral analysis and presents a nonparametric estimator of long-run correlation defined as:

$$\hat{\lambda}_k = \frac{\hat{\sigma}_{XY}(k)}{\sqrt{\hat{\sigma}_{XX}(k)\hat{\sigma}_{YY}(k)}} \quad (2.9)$$

where k is the number of lags and the covariance block estimator is defined as:

$$\hat{\sigma}_{XY}(k) = \sum_{t=k}^{T} \frac{[(X_t - X_{t-k}) - k\bar{\mu}_x][(Y_t - Y_{t-k}) - k\bar{\mu}_y]}{T-k} \quad (2.10)$$

where T is the size of the sample and $X_t$, $Y_t$ are the integrated versions of the original time series.

Once the long-run correlations are calculated, the same portfolio construction method (like simple correlations) is applied. For more information about long-run correlation estimator, the reader is advised to study Albuquerque (2001).

## 2.4 Portfolios based on cointegration

Traditional quantitative portfolio construction still heavily relies on the analysis of correlations for modeling the complex interdependencies between financial assets. However, the mean-variance criterion has nothing to ensure that portfolio deviations (errors) relative to a benchmark are stationary, in most cases being a random walk. As a consequence, the portfolio will drift virtually anywhere away from the benchmark unless is frequently rebalanced.

Portfolios based on cointegration rely on the long-term relationship between the time series. One method using cointegration is to use a number of assets to track a benchmark. If the asset allocations in a portfolio are constructed such that the portfolio tracks an index, then this portfolio should be cointegrated with the index. In the short run, it is likely that the portfolio will deviate from the index, but, in the long run, they should be tied together. Optimized portfolios via cointegration, as they are based on the long-run trends between asset prices, should, therefore, not require as much rebalancing. This dramatically reduces the transaction costs, which means a positive influence to portfolio returns.

## The mathematics of cointegration

When two or more series are individually integrated (in the time series sense), but some linear combination of these series has a lower order of integration, then the series are cointegrated. To state it differently, if two time series $x_t$ and $y_t$ are cointegrated, a linear combination of them must be stationary. In other words:

$$y_t - \beta x_t = u_t \qquad (2.11)$$

where $u_t$ is stationary.

For example, an integrated series of order 1 is the random walk (RW) function:

$$y_t = a + y_{t-1} + \varepsilon_t \qquad (2.12)$$

where $\varepsilon_t$ is stationary and independent and identically distributed (I.I.D.). This is a stochastic process with constant, finite variance and mean, and an autocorrelation that is independent of time (depending only on the lag).

If markets are strongly efficient, then log prices are random walks, $y_t = logP_t$, and $\varepsilon_t$ denotes the return at time t. It is not unusual to have some autocorrelation in returns, assuming they are I(0) but not I.I.D.. In this case, logarithmic prices are still integrated processes, but not pure RWs. When logarithmic asset price time series are integrated, over a period of time they may have wandered practically anywhere. There is little point in modeling them individually since the optimal forecast of any future value is the present value plus the drift. However, a multivariate model when there is cointegration between asset prices may be worthwhile because it reveals information about the long run equilibrium in the system. If a spread is found to be mean reverting it is known that, wherever one series is in several years, the other series should be right there along with the first one. Cointegrated asset prices have a common stochastic trend. They are ‘tied together’ in the long run because the spreads are mean reverting, even though there is a possibility of drifting apart in the short run. For more information about the mathematics of cointegration, the reader is advised to study Alexander (1999).

## Cointegration optimization algorithm

The cointegration process that will be used is that introduced by Engle and Granger (1987). In order to create a cloning portfolio, a portfolio that will try to track a benchmark index, we will use a similar algorithm like Ionescu (2002). The main reason is its simplicity. The optimization criterion is the minimizing of *t*-stat from the unit-root tests of residuals. The method consists of a few actions, by which we test all portfolio possibilities. The algorithm steps are:

- First we ensure that the asset price series are non-stationary, so that the concept of cointegration can be applied.

- We estimate the cointegrating regression using the whole set of assets, and perform the unit-root test on residuals. We will stress for one more time that we work with price series, unlike the MPT portfolios which use returns. The form of the OLS regression is:

$$P_{b,t} = c + \sum_{i=1}^{n} \beta_i \, P_{Ai,t} + \varepsilon_t \tag{2.13}$$

where $P_{b,t}$ is the time series of benchmark price and $P_{Ai,t}$ is the time series of asset i. The series are in logarithms. The coefficients of the regression, after normalization, will play the role of portfolio weights.

- We eliminate successively variable i from the regression, for every i=1,2,…,*n*. We extract for every case the residuals and perform the unit-root tests. From the *n* resulting values we choose the smaller one (the more negative) and we eliminate its variable from the portfolio. By doing so we obtain the most cointegrated portfolio at this round.

- We now have *n*-1 variables. We will proceed similarly and continue to eliminate variables from portfolio until we reach the optimal portfolio. The optimal portfolio is the one corresponding to the combination that leads to the minimum unit-root *t*-stat taking into account all steps possible. This means that when the minimum *t*-stat of a step is larger than the *t*-stat of the previous step we can conclude that the portfolio at the previous step is optimal. Once the *t*-stat starts to increase, we can no longer find more cointegrated portfolios. That is happening because we tend to move away from the common stochastic trend that made the series to move together in the long run.

Once the optimal regression coefficients are found, we normalize them so that their sum will be unity. The normalized coefficients are the optimal portfolio's weights.

# Chapter 3

## Data & results

### Introduction

In this chapter, we will introduce all the portfolio construction methods which were mentioned above. We will thoroughly analyze and compare them, in order to reach some useful conclusions regarding their returns, risk and overall success in capital accumulation. Firstly, the data which were used in our portfolios' creation will be presented. Next, we will show the results from all our portfolio strategies (mean variance portfolio with simple correlations, mean variance portfolio with long run correlations, cointegration optimized portfolios). Additionally, some tests will take place that check the validity of the results, especially for the cointegration optimized portfolios. Finally, we will conclude based on our findings.

### 3.1 Data

The goal of this paper is to compare different portfolios using stock equities. So, in order to construct and test the various approaches which mentioned above, we employ five years of daily data, from June 11 2008 to June 11 2013 (a total of 1305 observations). The data contain the equity indices of the countries which have the "Euro" as currency. Our benchmark is the equity index of the entire Europe (MSCI Europe Standard Large & Mid Cap). Both the prices and returns of the indices are given, in logarithms. The benefits of the use of logarithms were introduced earlier.

The data is available online from Morgan Stanley Capital International (MSCI), a source very often quoted in financial literature. The indices are computed in Euro currency.

We will divide the five years of data in two sub-periods. The first four years will be our calibration period. We will use it to create our models. The last year will be the testing period. The models found with the data of the calibration period will be applied, in order to compare the performance of each methodology. During the testing period, both cases of portfolios with no rebalancing and portfolios with rebalancing will be presented.

## 3.2 Portfolios with no rebalancing

We will present the results of various portfolios below. Firstly, the mean-variance portfolios will be created (these portfolios were created using simple and long-run correlation matrices). Next, the index tracking portfolios using cointegration optimization will be shown. For all different methods, portfolios with and without short sales allowed will be constructed.

At this point, it is imperative to mention the tools we used. All the data were preprocessed using Microsoft Excel 2007. The mean-variance portfolios' problems were solved using Matlab R2008a (the created code tracked the portfolio with the minimum variance and gave its weights). Cointegration portfolios were analyzed with Eviews 7.

Finally, in this subchapter, our calibration period will be four years. Our testing period will be one year with no portfolio rebalancing.

### Mean-variance portfolios with simple correlations

We use four years daily data, from June 11 2008 to June 11 2012, in order to find the minimum variance portfolio weights and the expected portfolio return. In the next tables, all the results are shown. We calculate the correlations matrix first (Table 3.1). As we can observe, the Eurozone countries are generally positively correlated with each other. Some are extremely correlated (France and Netherlands have a positive correlation of 0.93) while others have more mediocre correlation values (Greece and Ireland have a positive correlation of 0.42).

**Table 3.1: Simple correlations matrix (4 years period)**

| | *Austria* | *Belgium* | *Finland* | *France* | *Germany* | *Greece* | *Ireland* | *Italy* | *Netherlands* | *Portugal* | *Spain* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Austria*** | 1 | 0.67392 | 0.6933 | 0.797605 | 0.755249 | 0.499644 | 0.618482 | 0.766461 | 0.781093 | 0.701296 | 0.749523 |
| ***Belgium*** | 0.67392 | 1 | 0.664932 | 0.779432 | 0.713463 | 0.438774 | 0.653948 | 0.718626 | 0.78978 | 0.647122 | 0.70916 |
| ***Finland*** | 0.6933 | 0.664932 | 1 | 0.832604 | 0.7843 | 0.43244 | 0.60308 | 0.790157 | 0.784274 | 0.673539 | 0.735497 |
| ***France*** | 0.797605 | 0.779432 | 0.832604 | 1 | 0.927304 | 0.508014 | 0.6986 | 0.926537 | 0.933861 | 0.777213 | 0.886827 |
| ***Germany*** | 0.755249 | 0.713463 | 0.7843 | 0.927304 | 1 | 0.473289 | 0.635084 | 0.860504 | 0.869246 | 0.697254 | 0.80554 |
| ***Greece*** | 0.499644 | 0.438774 | 0.43244 | 0.508014 | 0.473289 | 1 | 0.423934 | 0.502117 | 0.50046 | 0.511305 | 0.503775 |
| ***Ireland*** | 0.618482 | 0.653948 | 0.60308 | 0.6986 | 0.635084 | 0.423934 | 1 | 0.643671 | 0.703957 | 0.565346 | 0.635977 |
| ***Italy*** | 0.766461 | 0.718626 | 0.790157 | 0.926537 | 0.860504 | 0.502117 | 0.643671 | 1 | 0.878787 | 0.77099 | 0.891573 |
| ***Netherlands*** | 0.781093 | 0.78978 | 0.784274 | 0.933861 | 0.869246 | 0.50046 | 0.703957 | 0.878787 | 1 | 0.746532 | 0.832988 |
| ***Portugal*** | 0.701296 | 0.647122 | 0.673539 | 0.777213 | 0.697254 | 0.511305 | 0.565346 | 0.77099 | 0.746532 | 1 | 0.78391 |
| ***Spain*** | 0.749523 | 0.70916 | 0.735497 | 0.886827 | 0.80554 | 0.503775 | 0.635977 | 0.891573 | 0.832988 | 0.78391 | 1 |

Next, using the simple correlations matrix we will calculate the asset allocation of the minimum variance portfolios.

**Table 3.2: Portfolios' asset allocation**

| | *Weights no short sales* | *Weights with short sales* |
|---|---|---|
| ***Austria*** | 0 | -0.210630697 |
| ***Belgium*** | 0.24 | 0.283867519 |
| ***Finland*** | 0 | 0.073294038 |
| ***France*** | 0 | -0.568952928 |
| ***Germany*** | 0.13 | 0.512094098 |
| ***Greece*** | 0 | 0.019242705 |
| ***Ireland*** | 0 | -0.057861931 |
| ***Italy*** | 0 | -0.22203375 |
| ***Netherlands*** | 0.03 | 0.503404082 |
| ***Portugal*** | 0.6 | 0.737893392 |
| ***Spain*** | 0 | -0.070316529 |
| ***Sum*** | 1 | 1 |

As we can observe, when short sales are prohibited, our options are limited. The algorithm impels us to invest in only four countries, in contrast with the portfolio in which the short sales are allowed. In the next table expected portfolio returns, portfolios' minimum variances and standard deviations will be presented, alongside with our benchmark's values.

**Table 3.3: Expected returns, variances & standard deviations (annual)**

| | *Europe* | *No short sales* | *Short sales allowed* |
|---|---|---|---|
| ***Returns*** | -5.62% | -13.84% | -8.15% |
| ***Variance*** | 0.066 | 0.05 | 0.047 |
| ***Standard deviation*** | 25.7% | 22.4% | 21.6% |

Our findings support that the mean-variance portfolios found will have a lower risk (standard deviation) than the benchmark. However, their returns will be worse. We will test all the models during the testing period later.

## Mean-variance portfolios with long-run correlations

Again, we use four years daily data, from June 11 2008 to June 11 2012, in order to find the minimum variance portfolio's weights and the expected portfolio return. We follow the same methodology, but we use long-run correlations. In the next tables, all the results are shown. We calculate the long-run correlations matrix (Table 3.5). As we can observe, the Eurozone countries are positively correlated with each other. Additionally, it can be stressed that long-run correlations have generally larger values than the simple correlations.

Using the long-run correlations matrix we calculate the asset allocation of the minimum variance portfolios.

**Table 3.4: Portfolios' asset allocation**

| | *Weights no short sales* | *Weights with short sales* |
|---|---|---|
| ***Austria*** | 0 | -0.297913217 |
| ***Belgium*** | 0.2695 | 0.363135409 |
| ***Finland*** | 0 | 0.086269471 |
| ***France*** | 0 | -0.319457237 |
| ***Germany*** | 0.1156 | 0.533436971 |
| ***Greece*** | 0 | -0.071380031 |
| ***Ireland*** | 0 | -0.139073059 |
| ***Italy*** | 0 | -0.236005649 |
| ***Netherlands*** | 0 | 0.38107359 |
| ***Portugal*** | 0.6149 | 0.666786197 |
| ***Spain*** | 0 | 0.033127555 |
| ***Sum*** | 1 | 1 |

**Table 3.5: Long-run correlations matrix (4 years period)**

| | *Austria* | *Belgium* | *Finland* | *France* | *Germany* | *Greece* | *Ireland* | *Italy* | *Netherlands* | *Portugal* | *Spain* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Austria*** | 1 | 0.7257 | 0.723 | 0.8524 | 0.8193 | 0.6983 | 0.6681 | 0.8165 | 0.8242 | 0.7126 | 0.7757 |
| ***Belgium*** | 0.7257 | 1 | 0.6128 | 0.7667 | 0.7201 | 0.6072 | 0.7182 | 0.7164 | 0.8176 | 0.6243 | 0.6784 |
| ***Finland*** | 0.723 | 0.6128 | 1 | 0.829 | 0.7999 | 0.5104 | 0.6179 | 0.7633 | 0.7546 | 0.6543 | 0.6851 |
| ***France*** | 0.8524 | 0.7667 | 0.829 | 1 | 0.9381 | 0.6525 | 0.7442 | 0.9245 | 0.9222 | 0.7605 | 0.8551 |
| ***Germany*** | 0.8193 | 0.7201 | 0.7999 | 0.9381 | 1 | 0.6336 | 0.7132 | 0.8613 | 0.8707 | 0.6955 | 0.7886 |
| ***Greece*** | 0.6983 | 0.6072 | 0.5104 | 0.6525 | 0.6336 | 1 | 0.4747 | 0.674 | 0.6181 | 0.6323 | 0.6813 |
| ***Ireland*** | 0.6681 | 0.7182 | 0.6179 | 0.7442 | 0.7132 | 0.4747 | 1 | 0.6731 | 0.7682 | 0.5459 | 0.6153 |
| ***Italy*** | 0.8165 | 0.7164 | 0.7633 | 0.9245 | 0.8613 | 0.674 | 0.6731 | 1 | 0.8899 | 0.7606 | 0.8761 |
| ***Netherlands*** | 0.8242 | 0.8176 | 0.7546 | 0.9222 | 0.8707 | 0.6181 | 0.7682 | 0.8899 | 1 | 0.7465 | 0.7976 |
| ***Portugal*** | 0.7126 | 0.6243 | 0.6543 | 0.7605 | 0.6955 | 0.6323 | 0.5459 | 0.7606 | 0.7465 | 1 | 0.7679 |
| ***Spain*** | 0.7757 | 0.6784 | 0.6851 | 0.8551 | 0.7886 | 0.6813 | 0.6153 | 0.8761 | 0.7976 | 0.7679 | 1 |

As we can observe, when short sales are prohibited, our options are limited. The algorithm impels us to invest in only three countries, in contrast with the portfolio in which the short sales are allowed. In Table 3.6 expected portfolio returns, portfolios' minimum variances and standard deviations will be presented, alongside with our benchmark's values.

**Table 3.6: Expected returns, variances & standard deviations (annual)**

| | *Europe* | *No short sales* | *Short sales allowed* |
|---|---|---|---|
| ***Returns*** | -5.62% | -14% | -1.37% |
| ***Variance*** | 0.066 | 0.055 | 0.044 |
| ***Standard deviation*** | 25.7% | 23.4% | 21% |

Our findings support again that the mean-variance portfolios found will have a lower risk (standard deviation) than the benchmark. For one more time, the portfolio with no short sales has worse returns. However, the portfolio which allows negative weights promises superior returns, with a significant decrease in risk, making it so far the best possible strategy. Of course, that needs to be verified at the testing period. We will test all the models during the testing period later.

## Portfolio construction using cointegration

For one more time, in order to construct the cloning strategy we use four years daily data, from June 11 2008 to June 11 2012. As mentioned above, for the creation of these portfolios, we will use logarithmic prices instead of asset returns. The first step is to test for stationarity. The series must be nonstationary, which means integrated of order one, so we can proceed with the next steps. We will use Augmented Dickey-Fuller (ADF) test of Eviews (with trend & intercept option). For the series that the findings are not clear we will run also a Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test. Table 3.7 shows the ADF tests:

**Table 3.7: Summary of ADF unit-root tests (* means stationarity)**

| | *ADF (Levels)* | *ADF ($1^{st}$ Differences)* |
|---|---|---|
| ***Austria*** | -2.75 | -24.03*** |
| ***Belgium*** | -3.36* | -30.17*** |

| | | |
|---|---|---|
| ***Finland*** | -2.54 | -32.41*** |
| ***France*** | -2.78 | -24.99*** |
| ***Germany*** | -2.79 | -31.52*** |
| ***Greece*** | -1.088 | -31.91*** |
| ***Ireland*** | -3.46** | -31.9*** |
| ***Italy*** | -2.42 | -31.83*** |
| ***Netherlands*** | -2.62 | -32.09*** |
| ***Portugal*** | -1.73 | -30.93*** |
| ***Spain*** | -1.93 | -24.47*** |
| ***Europe*** | -2.83 | -24.73*** |

All values were compared with MacKinnon (1996) one-sided *p*-values

We can observe that at price levels all the series are clearly nonstationary, with the exceptions of Belgium and Ireland. For these two countries we also run a KPSS test which showed that they are nonstationary, so we will accept our results. Also, the null hypothesis of unit-root can be rejected only after taking the first difference. Thus, we will conclude that all series are non-stationary, being integrated of order one.

We are ready to implement the index tracking strategy. We will try to construct and test a portfolio that clones the benchmark. We will employ Engle-Granger cointegration method, and then we will apply the optimization algorithm presented in a previous section. We start by estimating the cointegrating regression of the benchmark with all our assets (see equation 2.13).

Then we test the stationarity of this regression's residuals. We find that the unit-root *t*-stat equals -5.81. So, we can conclude that the residuals are stationary. Although we have found cointegration, we are not going to stop here. We will try, using the algorithm, to find the most cointegrated portfolio. Below we will estimate Engle-Granger cointegrating regressions, by eliminating successively one series at a time, and extracting residuals. We will eliminate completely, at every step, one single series according to the minimum ADF *t*-stat criterion. The results are presented in Table 3.8.

We can observe that in order to obtain a portfolio more and more cointegrated (an error more and more stationary) we need to eliminate several countries. Finally, after five eliminations, a further attempt to optimize the portfolio composition will end up in obtaining a suboptimal portfolio, because eliminating the Portugal equity index from the portfolio will lead to an error less stationary (ADF *t*-stat of –6.8) comparing to the previous round (ADF *t*-stat of –7.078). We will conclude that previous round gives us the most cointegrated portfolio.

**Table 3.8: Cointegration optimization (elimination algorithm[1])**

| | *t*-statistics | | | | | |
|---|---|---|---|---|---|---|
| | **Round 1** | **Round 2** | **Round 3** | **Round 4** | **Round 5** | **Round 6** |
| ***Austria*** | -5.56 | -5.73 | -5.24 | -6.35 | -6.48 | -6.23 |
| ***Belgium*** | -5.64 | -5.81 | -6.58 | -5.42 | -5.55 | -5.52 |
| ***Finland*** | -5.65 | -5.97 | -6.78 | **-7.072** | | |
| ***France*** | **-6.041** | | | | | |
| ***Germany*** | -5.88 | **-6.7** | | | | |
| ***Greece*** | -5.69 | -5.91 | -5.97 | -5.63 | -5.62 | -5.6 |
| ***Ireland*** | -5.56 | -5.67 | -5.67 | -4.79 | -5.56 | -5.69 |
| ***Italy*** | -5.86 | -6.01 | -6.12 | -5.25 | -4.47 | -4.03 |
| ***Netherlands*** | -5.85 | -6.16 | **-6.96** | | | |
| ***Portugal*** | -5.41 | -5.66 | -6.28 | -6.74 | -6.89 | -6.8 |
| ***Spain*** | -5.51 | -5.94 | -6.73 | -7.03 | **-7.078** | |

The optimal composition of the index tracking portfolio includes Austria, Belgium, Greece, Ireland, Italy and Portugal. All the coefficients are statistically significant (at level 1%) and as shown above the ADF test indicates a strong relationship of cointegration. In the figure below, we depict the course of the benchmark and the cointegration portfolio during the calibration period.

**1. Elimination algorithm procedure: First we regress with all countries on the table (*t*-stat=-5.81). At round 1 we observe that the regression with France excluded is more stationary than the previous regression (*t*-stat=-6.041). Thus, we start round 2 excluding France permanently. At round 2 we can see that the regression with Germany excluded is more stationary than the regression from the previous round (*t*-stat=-6.7). We exclude Germany permanently and proceed to round 3. We continue in a similar way until round 6. At round 6 any remaining country exclusion will not lead to a more stationary regression. So, we have the most stationary regression (*t*-stat=-7.078) which contains only the countries that reached round 6. The countries are Austria, Belgium, Greece, Ireland, Italy and Portugal.**

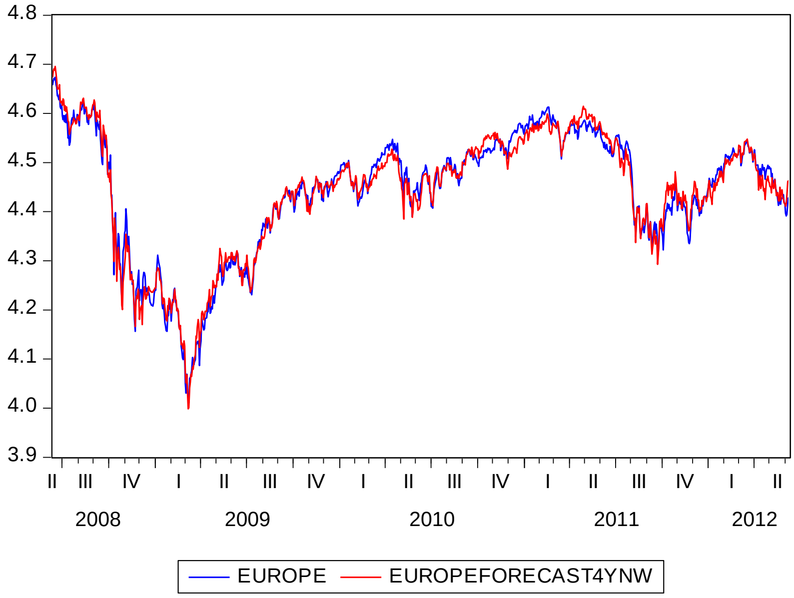



**Figure 3.1: Benchmark & Portfolio depiction (Short sales allowed)**

The cointegration portfolio seems to achieve cloning the benchmark during the calibration period. Also, the correlation between the two series is close to 1 (0.989). Of course, it is wise to expect the results during the testing period to see the level of our success in cloning the benchmark.

Now we will determine the weights of the component assets from the coefficients of the regression. After normalizing the coefficients so that they sum to unity, we have found the cloning portfolio weights (see Table 3.10).

The above portfolio allowed short sales. Next we will attempt to construct a portfolio with no short sales. First, we eliminate all the negative coefficients from the regression (see equation 2.13). Several elimination steps may be needed. Every time we regress, and then we eliminate every country with negative coefficient. In our four years data, the elimination of negative coefficients leaves us with only three countries, Belgium, Germany and Netherlands. The stationarity test of this regression's residuals gives an ADF *t*-stat of -5.046. So, this regression is less stationary than the regression that includes all countries (unit root *t*-stat of -5.81). However, we will proceed in order to check the performance of a cointegrated portfolio with only positive weights. So we apply the cointegration optimization algorithm like before.

**Table 3.9: Cointegration optimization (elimination algorithm[1])**

| | *t*-statistics |
|---|---|
| | **Round 1** |
| ***Belgium*** | -4.72 |
| ***Germany*** | -4.11 |
| ***Netherlands*** | -4.14 |

An attempt to optimize the portfolio composition will end up in obtaining a suboptimal portfolio, since no value is algebraically greater than -5.046. So no further optimization will be made and we proceed with the weight calculation. All the coefficients are statistically significant (at level 1%) and as shown above the ADF test indicates a strong relationship of cointegration. In the figure below, we depict the course of the benchmark and the cointegration portfolio during the calibration period.

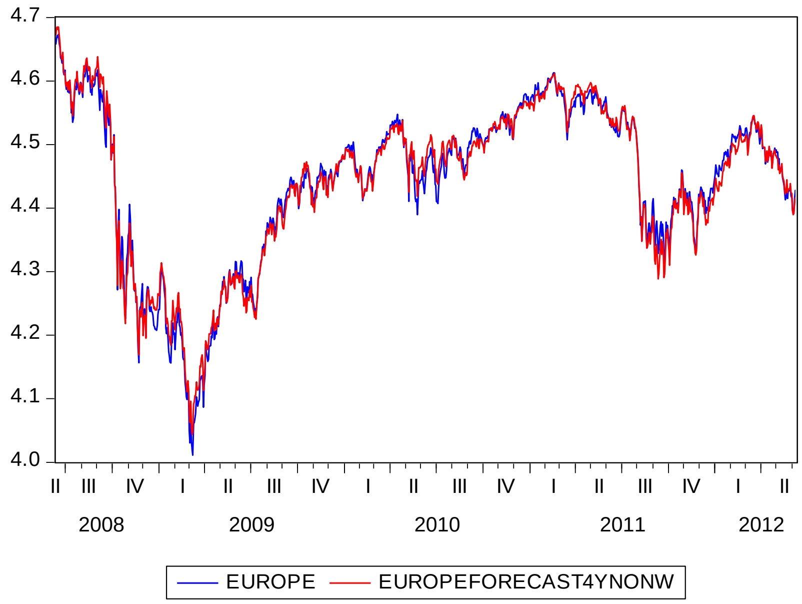



**Figure 3.2: Benchmark & Portfolio depiction (No short sales allowed)**

The cointegration portfolio with no short sale seems to achieve cloning the benchmark during the calibration period. Also, the correlation between the two series is close to 1 (0.994). Of course, it is wise to expect the results during the testing period to see the level of our success in cloning the benchmark.

In Table 3.10, we present the asset allocation of both techniques.

**1. Elimination algorithm procedure: See explanation of table 3.8 (page 29)**

**Table 3.10: Portfolios' asset allocation**

| | *Weights no short sales* | *Weights with short sales* |
|---|---|---|
| ***Austria*** | 0 | 0.21 |
| ***Belgium*** | 0.08 | 0.43 |
| ***Finland*** | 0 | 0 |
| ***France*** | 0 | 0 |
| ***Germany*** | 0.46 | 0 |
| ***Greece*** | 0 | -0.2 |
| ***Ireland*** | 0 | -0.16 |
| ***Italy*** | 0 | 0.82 |
| ***Netherlands*** | 0.46 | 0 |
| ***Portugal*** | 0 | -0.1 |
| ***Spain*** | 0 | 0 |
| ***Sum*** | 1 | 1 |

We can observe that cointegration optimization leads to more parsimonious portfolios. In the next table expected portfolio returns, portfolios' minimum variances and standard deviations will be presented, alongside with our benchmark's values.

**Table 3.11: Expected returns, variances & standard deviations (annual)**

| | *Europe* | *No short sales* | *Short sales allowed* |
|---|---|---|---|
| ***Returns*** | -5.62% | -6.95% | -8.04% |
| ***Variance*** | 0.066 | 0.07 | 0.108 |
| ***Standard deviation*** | 25.7% | 26.7% | 32.8% |

Our findings show that our clone portfolios try to track the benchmark. However, the expected returns are in both cases worse than the benchmark, as well as the risk indicators (standard deviations). Of course, that needs to be verified at the testing period. We will test all the models during the testing period in the next section.

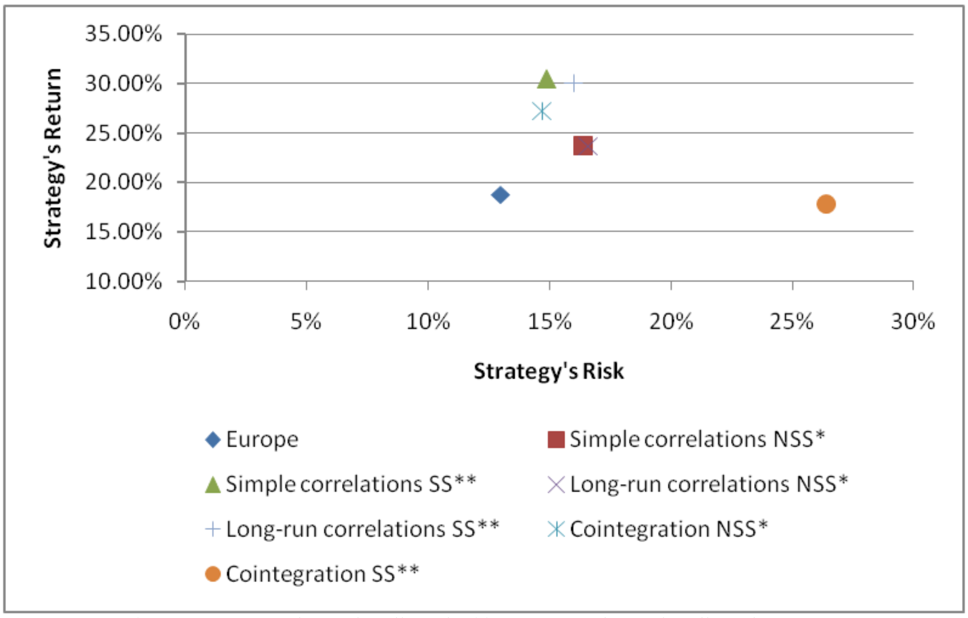
## Testing & comparing the strategies

In this section, we will test the different methodologies with future data. The testing period employs one year data, from June 12 2012 to June 11 2013. It is stressed that we construct portfolios at the beginning of the testing period and check their performance at the end of it. No rebalancing will take place. We will use the asset allocations found above. The returns and risk of the strategies are presented in Table 3.12 below, and are depicted in Figure 3.3.

**Table 3.12: Returns & risk during the testing period no rebalancing (annual)**

| | *Returns* | *Standard deviations* |
|---|---|---|
| ***Europe*** | 18.73% | 13% |
| ***Simple correlations NSS**** | 23.69% | 16.4% |
| ***Simple correlations SS***** | 30.45% | 14.9% |
| ***Long-run correlations NSS**** | 23.65% | 16.6% |
| ***Long-run correlations SS***** | 30.03% | 16% |
| ***Cointegration NSS**** | 27.24% | 14.7% |
| ***Cointegration SS***** | 17.80% | 26.4% |

*NSS means “No short sales allowed”, **SS means “Short sales allowed”




*NSS means “No short sales allowed”, **SS means “Short sales allowed”

**Figure 3.3: Returns & Risk depiction (No rebalancing)**

The tests made for the cointegration portfolios are available at the Appendix section.

First, we observe that during the testing period equity indices generally have an upward trend, so returns seem to be satisfying. The best results, regarding the portfolios' returns, come from the mean-variance strategies with short sales allowed, which by far outperformed the benchmark with only a slight risk tradeoff. Clearly the mean-variance portfolios with no short sales are inferior to the latter methods, since they dramatically reduce our strategy options. When short sales are allowed even the portfolio's risk is less than its corresponding portfolio with no short sales.

From cointegration optimization strategies, the strategy allowing short sales achieved to track satisfactorily the benchmark's course. However, the risk of the portfolio ranges to high levels. In the next subchapter, we will test the strategies with a rebalancing, to see if our results change significantly.

## 3.3 Portfolios with rebalancing

We will present the results of several portfolios below. Firstly, the mean-variance portfolios will be created (these portfolios were created using simple and long-run correlation matrices). Next, the index tracking portfolios using cointegration optimization will be shown. For all different methods, portfolios with and without short sales allowed will be constructed.

We use the same tools and software that was used in section 3.2. All the data were preprocessed using Microsoft Excel 2007. The mean-variance portfolios' problems were solved using Matlab R2008a (the created code tracked the portfolio with the minimum variance and gave its weights). Cointegration portfolios were analyzed with Eviews 7.

Finally, in this subchapter, our calibration period will be again four years. Our testing period will be one year with a rebalancing. So, for the first part of the testing period, the weights found at section 3.2 will be applied, while, for the last part, weights found in this subchapter will be applied.

### Mean-variance portfolios with simple correlations

In order to rebalance our portfolio we use a period of daily data longer than four years, from June 11 2008 to January 11 2013. We will find the minimum variance portfolio's weights and the expected portfolio return during this calibration period. In the next tables, all the results are shown. We calculate the correlations matrix first (Table 3.13). We can observe that the Eurozone countries are generally positively correlated with each other.

**Table 3.13: Simple correlations matrix (4.5 years period)**

| | ***Austria*** | ***Belgium*** | ***Finland*** | ***France*** | ***Germany*** | ***Greece*** | ***Ireland*** | ***Italy*** | ***Netherlands*** | ***Portugal*** | ***Spain*** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Austria*** | 1 | 0.672533 | 0.696211 | 0.800564 | 0.760775 | 0.481483 | 0.615288 | 0.767094 | 0.782544 | 0.699098 | 0.745727 |
| ***Belgium*** | 0.672533 | 1 | 0.659715 | 0.779241 | 0.71453 | 0.423022 | 0.647862 | 0.709594 | 0.789259 | 0.635676 | 0.696087 |
| ***Finland*** | 0.696211 | 0.659715 | 1 | 0.832438 | 0.785134 | 0.4073 | 0.59595 | 0.788316 | 0.78317 | 0.678198 | 0.728142 |
| ***France*** | 0.800564 | 0.779241 | 0.832438 | 1 | 0.927861 | 0.485494 | 0.690287 | 0.921722 | 0.93255 | 0.773023 | 0.875929 |
| ***Germany*** | 0.760775 | 0.71453 | 0.785134 | 0.927861 | 1 | 0.455511 | 0.63111 | 0.855882 | 0.870526 | 0.697684 | 0.796412 |
| ***Greece*** | 0.481483 | 0.423022 | 0.4073 | 0.485494 | 0.455511 | 1 | 0.407766 | 0.478041 | 0.482343 | 0.483957 | 0.475433 |
| ***Ireland*** | 0.615288 | 0.647862 | 0.59595 | 0.690287 | 0.63111 | 0.407766 | 1 | 0.628379 | 0.697884 | 0.554792 | 0.616148 |
| ***Italy*** | 0.767094 | 0.709594 | 0.788316 | 0.921722 | 0.855882 | 0.478041 | 0.628379 | 1 | 0.869091 | 0.771557 | 0.892158 |
| ***Netherlands*** | 0.782544 | 0.789259 | 0.78317 | 0.93255 | 0.870526 | 0.482343 | 0.697884 | 0.869091 | 1 | 0.739213 | 0.816166 |
| ***Portugal*** | 0.699098 | 0.635676 | 0.678198 | 0.773023 | 0.697684 | 0.483957 | 0.554792 | 0.771557 | 0.739213 | 1 | 0.776238 |
| ***Spain*** | 0.745727 | 0.696087 | 0.728142 | 0.875929 | 0.796412 | 0.475433 | 0.616148 | 0.892158 | 0.816166 | 0.776238 | 1 |

Next, using the simple correlations matrix we will calculate the asset allocation of the minimum variance portfolios.

**Table 3.14: Portfolios' asset allocation**

| | *Weights no short sales* | *Weights short sales allowed* |
|---|---|---|
| ***Austria*** | 0 | -0.208633089 |
| ***Belgium*** | 0.264767792 | 0.297908898 |
| ***Finland*** | 0 | 0.063872418 |
| ***France*** | 0 | -0.551138346 |
| ***Germany*** | 0.108793169 | 0.510454264 |
| ***Greece*** | 0.002272542 | 0.026156751 |
| ***Ireland*** | 0 | -0.051357141 |
| ***Italy*** | 0 | -0.249557152 |
| ***Netherlands*** | 0.07383701 | 0.53521634 |
| ***Portugal*** | 0.550329486 | 0.689625626 |
| ***Spain*** | 0 | -0.062548569 |
| ***Sum*** | 1 | 1 |

As we can observe, yet another time, when short sales are prohibited, our options are limited. The algorithm impels us to invest in only five countries, in contrast with the portfolio in which the short sales are allowed. In the next table expected portfolio returns, portfolios' minimum variances and standard deviations will be presented, alongside with our benchmark's values.

**Table 3.15: Expected returns, variances & standard deviations (annual)**

| | *Europe* | *No short sales* | *Short sales allowed* |
|---|---|---|---|
| ***Returns*** | -1.39% | -7.01% | -1.56% |
| ***Variance*** | 0.06 | 0.052 | 0.044 |
| ***Standard deviation*** | 24.5% | 22.8% | 20.9% |

Our findings support that the mean-variance portfolios found will have a lower risk (standard deviation) than the benchmark. However, their returns will be worse (although the short sales portfolio is quite near the benchmark). Similarly as above we will test all the models during the testing period in a later section.

### Mean-variance portfolios with long-run correlations

Again, we use a period of daily data longer than four years, from June 11 2008 to January 11 2013, in order to find the minimum variance portfolio's weights and the expected portfolio return. We follow the same methodology, but we use long-run correlations. In the next tables, all the results are shown. We calculate the long-run correlations matrix (Table 3.16). As we can observe, the Eurozone countries are positively correlated with each other. Additionally, it can be stressed that long-run correlations have generally larger values than the simple correlations.

Using the long-run correlations matrix we calculate the asset allocation of the minimum variance portfolios (Table 3.17).

As we can observe, when short sales are prohibited, our options are limited. The algorithm impels us to invest in only four countries, in contrast with the portfolio in which the short sales are allowed. In Table 3.18, expected portfolio returns, portfolios' minimum variances and standard deviations will be presented, alongside with our benchmark's values.

Our findings support again that the mean-variance portfolios found will have a lower risk (standard deviation) than the benchmark. For one more time, the "no short sales" portfolio has worse returns. However, the portfolio which allows negative weights promises superior returns and for the first time in our data the strategy presents positive expected returns. Also, there is a significant decrease in risk, making it again so far the best possible approach. Of course, that needs to be verified at the testing period. We will test all the models during the testing period later.

**Table 3.16: Long-run correlations matrix (4.5 years period)**

| | *Austria* | *Belgium* | *Finland* | *France* | *Germany* | *Greece* | *Ireland* | *Italy* | *Netherlands* | *Portugal* | *Spain* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Austria* | 1 | 0.7191 | 0.6962 | 0.853 | 0.8218 | 0.6845 | 0.6642 | 0.8019 | 0.823 | 0.699 | 0.7634 |
| *Belgium* | 0.7191 | 1 | 0.6058 | 0.7617 | 0.7201 | 0.5947 | 0.7124 | 0.6971 | 0.8158 | 0.6206 | 0.667 |
| *Finland* | 0.6962 | 0.6058 | 1 | 0.8304 | 0.7998 | 0.4978 | 0.6135 | 0.7618 | 0.7537 | 0.6543 | 0.6864 |
| *France* | 0.853 | 0.7617 | 0.8304 | 1 | 0.9363 | 0.6352 | 0.7276 | 0.9176 | 0.9222 | 0.7558 | 0.8473 |
| *Germany* | 0.8218 | 0.7201 | 0.7998 | 0.9363 | 1 | 0.6158 | 0.7101 | 0.8549 | 0.8719 | 0.6884 | 0.7725 |
| *Greece* | 0.6845 | 0.5947 | 0.4978 | 0.6352 | 0.6158 | 1 | 0.4603 | 0.6626 | 0.6147 | 0.6127 | 0.6465 |
| *Ireland* | 0.6642 | 0.7124 | 0.6135 | 0.7276 | 0.7101 | 0.4603 | 1 | 0.6551 | 0.7605 | 0.5388 | 0.5935 |
| *Italy* | 0.8019 | 0.6971 | 0.7618 | 0.9176 | 0.8549 | 0.6626 | 0.6551 | 1 | 0.8749 | 0.7665 | 0.8795 |
| *Netherlands* | 0.823 | 0.8158 | 0.7537 | 0.9222 | 0.8719 | 0.6147 | 0.7605 | 0.8749 | 1 | 0.7138 | 0.7707 |
| *Portugal* | 0.699 | 0.6206 | 0.6543 | 0.7558 | 0.6884 | 0.6127 | 0.5388 | 0.7665 | 0.7138 | 1 | 0.7604 |
| *Spain* | 0.7634 | 0.667 | 0.6864 | 0.8473 | 0.7725 | 0.6465 | 0.5935 | 0.8795 | 0.7707 | 0.7604 | 1 |

**Table 3.17: Portfolios' asset allocation**

| | *Weights no short sales* | *Weights short sales allowed* |
|---|---|---|
| ***Austria*** | 0 | -0.282275182 |
| ***Belgium*** | 0.25679439 | 0.288089016 |
| ***Finland*** | 0 | 0.057913847 |
| ***France*** | 0 | -0.378419193 |
| ***Germany*** | 0.100936146 | 0.549702411 |
| ***Greece*** | 0 | -0.057824991 |
| ***Ireland*** | 0 | -0.143892589 |
| ***Italy*** | 0 | -0.353383743 |
| ***Netherlands*** | 0.091104507 | 0.606015532 |
| ***Portugal*** | 0.551164957 | 0.643108718 |
| ***Spain*** | 0 | 0.070966174 |
| ***Sum*** | 1 | 1 |

**Table 3.18: Expected returns, variances & standard deviations (annual)**

| | *Europe* | *No short sales* | *Short sales allowed* |
|---|---|---|---|
| ***Returns*** | -1.39% | -6.91% | 5.46% |
| ***Variance*** | 0.06 | 0.052 | 0.042 |
| ***Standard deviation*** | 24.5% | 22.8% | 20.4% |

## Portfolio construction using cointegration

For one more time, in order to construct the cloning strategy we use a period of daily data longer than four years, from June 11 2008 to January 11 2013. As mentioned above, for the creation of these portfolios, we will use logarithmic prices instead of asset returns. The first step is to test for stationarity. The series must be nonstationary, which means integrated of order one, so we can proceed with the next steps. We will use Augmented Dickey-Fuller (ADF) test of Eviews (with trend & intercept option). For the series that the findings are not clear we will run also a Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test. Table 3.19 shows the ADF tests:

**Table 3.19: Summary of ADF unit-root tests (* means stationarity)**

| | *ADF (Levels)* | *ADF (1st Differences)* |
|---|---|---|
| ***Austria*** | -2.94 | -25.61*** |
| ***Belgium*** | -3.6** | -32.46*** |
| ***Finland*** | -2.77 | -34.63*** |
| ***France*** | -3.08 | -22.91*** |
| ***Germany*** | -3.05 | -33.74*** |
| ***Greece*** | -1.78 | -34.56*** |
| ***Ireland*** | -3.76** | -34.03*** |
| ***Italy*** | -2.73 | -34.04*** |
| ***Netherlands*** | -2.88 | -34.50*** |
| ***Portugal*** | -2.29 | -32.61 *** |
| ***Spain*** | -2.45 | -33.31*** |
| ***Europe*** | -3.1 | -17.14*** |

All values were compared with MacKinnon (1996) one-sided *p*-values

We can observe that, at levels, all the series are clearly nonstationary, with the exceptions of Belgium and Ireland. For these two countries, we also run a KPSS test. It showed that Ireland is nonstationary, but again Belgium was shown as stationary so Belgium will be excluded from the regression. Additionally, for all the series the null hypothesis of unit-root can be rejected only after taking the first difference. Thus, we will conclude that all series, except Belgium, are non-stationary, being integrated of order one.

We are ready to implement the index tracking strategy. We will try to construct and test a portfolio that clones the benchmark. We will employ Engle-Granger cointegration method, and then the optimization algorithm presented in a previous section. We start by estimating the cointegrating regression of the benchmark with all our assets (see equation 2.13).

Then we test the stationarity of this regression's residuals. We find that the unit-root *t*-stat equals -5.75. So, we can conclude that the residuals are stationary. Although we have found cointegration, we are not going to stop here. We will try, using the algorithm, to find the most cointegrated portfolio. Below we will estimate Engle-Granger cointegrating regressions, by eliminating successively one series at a time, and extracting residuals. We will eliminate completely, at every step, one single series according to the minimum ADF *t*-stat criterion. The results are presented in Table 3.20.

We can observe that in order to obtain a portfolio more and more cointegrated (an error more and more stationary) we need to eliminate several countries. Finally, after three eliminations, a further attempt to optimize the portfolio composition will end up in obtaining a suboptimal portfolio, because eliminating the Spain equity index from the portfolio will lead to an error less stationary (ADF *t*-stat of –7.02) comparing to the previous round (ADF *t*-stat of –7.062). We will conclude that the previous round gives us the most cointegrated portfolio.

**Table 3.20: Cointegration optimization (elimination algorithm[1])**

| | *t*-statistics | | | |
|---|---|---|---|---|
| | **Round 1** | **Round 2** | **Round 3** | **Round 4** |
| ***Austria*** | -5.06 | -5.10 | -4.54 | -4.68 |
| ***Finland*** | -5.73 | -6.11 | **-7.062** | |
| ***France*** | -6.018 | **-7.021** | | |
| ***Germany*** | **-6.107** | | | |
| ***Greece*** | -5.67 | -5.32 | -6.24 | -6.25 |
| ***Ireland*** | -5.77 | -5.97 | -6.08 | -6.12 |
| ***Italy*** | -5.91 | -6.17 | -6.54 | -6.51 |
| ***Netherlands*** | -5.06 | -5.41 | -5.99 | -6.001 |
| ***Portugal*** | -5.36 | -5.72 | -6.56 | -6.55 |
| ***Spain*** | -5.60 | -5.94 | -7.02 | -7.02 |

The optimal composition of the index tracking portfolio includes Austria, Greece, Ireland, Italy, Netherlands, Portugal and Spain. All the coefficients are statistically significant (at level 1%) and as shown above the ADF test indicates a strong relationship of cointegration. In the figure below we depict the course of the benchmark and the cointegration portfolio during the calibration period.

**1. Elimination algorithm procedure: See explanation of table 3.8 (page 29)**

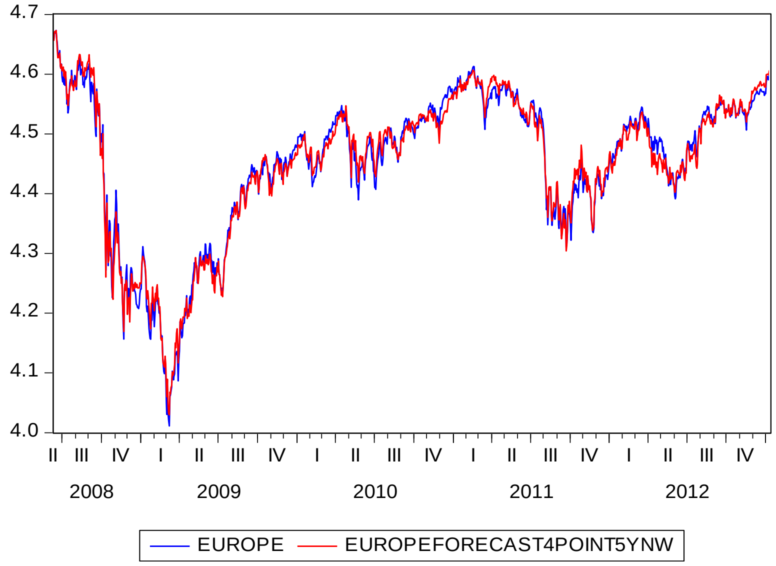



**Figure 3.4: Benchmark & Portfolio depiction (Short sales allowed)**

The cointegration portfolio seems to achieve cloning the benchmark during the calibration period. Also, the correlation between the two series is close to 1 (0.994). Of course it is wise to expect the results during the testing period to see the level of our success in cloning the benchmark.

Now we will determine the weights of the component assets from the coefficients of the regression. After normalizing the coefficients so that they sum to unity, we have found the cloning portfolio weights (see Table 3.22).

The above portfolio allowed short sales. Next we will attempt to construct a portfolio with no short sales. First, we eliminate all the negative coefficients from the regression (see equation 2.13). Several elimination steps may be needed. Every time we regress, and then we eliminate every country with negative coefficient. In our current data period the elimination of negative coefficients leaves us with only two countries, Germany and Netherlands. The stationarity test of this regression's residuals gives an ADF *t*-stat of -4.85. So this regression is less stationary than the regression that includes all countries (unit root *t*-stat of -5.75). However, again we will proceed in order to check the performance of a cointegrated portfolio with only positive weights. So we apply the cointegration optimization algorithm like before.

**Table 3.21: Cointegration optimization (elimination algorithm[1])**

| | *t*-statistic |
|---|---|
| | **Round 1** |
| ***Germany*** | -4.17 |
| ***Netherlands*** | -3.68 |

An attempt to optimize the portfolio composition will end up in obtaining a suboptimal portfolio, since no value is algebraically greater than -4.85. So no further optimization will be made and we proceed with the weight calculation. All the coefficients are statistically significant (at level 1%) and as shown above the ADF test indicates a strong relationship of cointegration. In the figure below we depict the course of the benchmark and the cointegration portfolio during the calibration period.

The cointegration portfolio with no short sale seems to achieve cloning the benchmark during the calibration period. Also, the correlation between the two series is close to 1 (0.993). Of course it is wise to expect the results during the testing period to see the level of our success in cloning the benchmark.

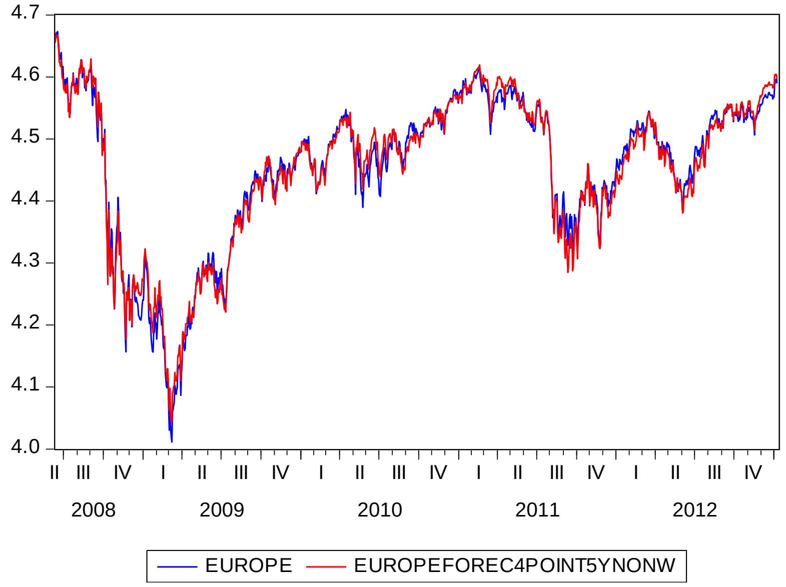



**Figure 3.5: Benchmark & Portfolio depiction (No short sales allowed)**

**1. Elimination algorithm procedure: See explanation of table 3.8 (page 29)**

In Table 3.22, we present the asset allocation of both methods.

**Table 3.22: Portfolios' asset allocation**

| | *Weights no short sales* | *Weights with short sales* |
|---|---|---|
| ***Austria*** | 0 | 0.28 |
| ***Belgium*** | 0 | 0 |
| ***Finland*** | 0 | 0 |
| ***France*** | 0 | 0 |
| ***Germany*** | 0.46 | 0 |
| ***Greece*** | 0 | -0.13 |
| ***Ireland*** | 0 | -0.07 |
| ***Italy*** | 0 | 0.26 |
| ***Netherlands*** | 0.54 | 0.74 |
| ***Portugal*** | 0 | -0.14 |
| ***Spain*** | 0 | 0.06 |
| ***Sum*** | 1 | 1 |

We can observe that cointegration optimization leads to more parsimonious portfolios. In the next table expected portfolio returns, portfolios' minimum variances and standard deviations will be presented, alongside with our benchmark's values.

**Table 3.23: Expected returns, variances & standard deviations (annual)**

| | *Europe* | *No short sales* | *Short sales allowed* |
|---|---|---|---|
| ***Returns*** | -1.39% | -1.19% | -0.63% |
| ***Variance*** | 0.06 | 0.066 | 0.091 |
| ***Standard deviation*** | 24.5% | 25.8% | 30.3% |

Our findings show that our clone portfolios try to track the benchmark. Unlike to the four years data, here the expected returns are in both cases better than the benchmark. However, the risk indicators (standard deviations) indicate for one more time a higher expected risk than the benchmark. The above makes the anticipated results of the testing period quite interesting. We will test all the models during the testing period in the next section.

## Testing & comparing the strategies

In this section we will test the different methodologies with future data. The testing period employs one year data, from June 12 2012 to June 11 2013. It is stressed that we construct portfolios at the beginning of the testing period, we rebalance them on January 14 2013 and check their performance at the end of it. We will use the asset allocations found above, for both the calibration periods. The returns and risk of the strategies are presented in Table 3.24 and are depicted in Figure 3.6 below.

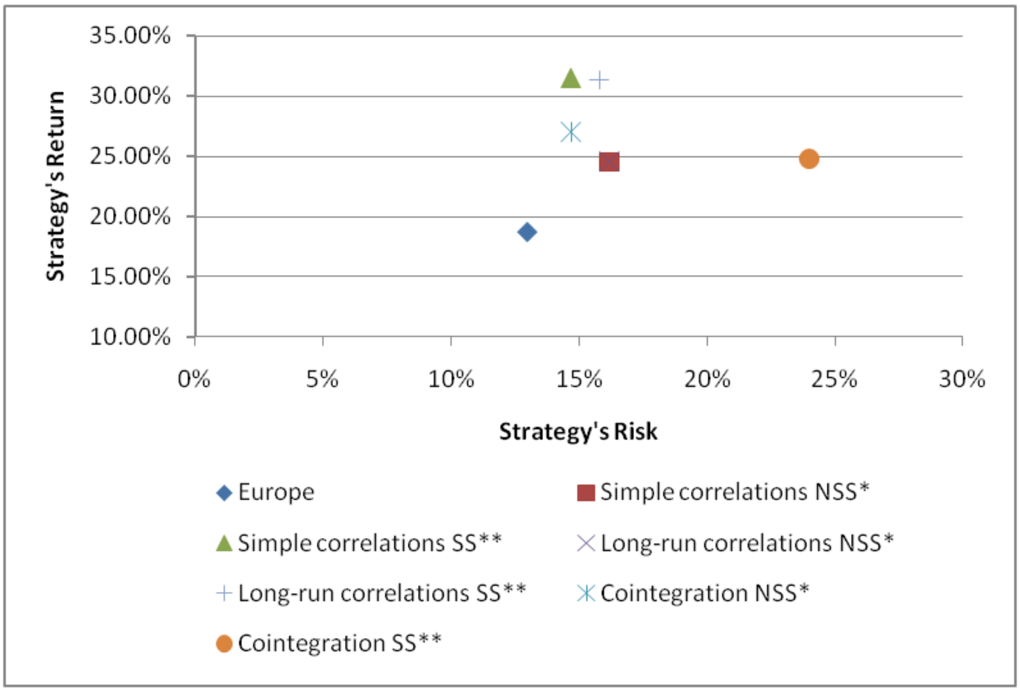
**Table 3.24: Returns & risk during the testing period with rebalancing (annual)**

| | *Returns* | *Standard deviations* |
|---|---|---|
| ***Europe*** | 18.73% | 13% |
| ***Simple correlations NSS**** | 24.52% | 16.2% |
| ***Simple correlations SS***** | 31.52% | 14.7% |
| ***Long-run correlations NSS**** | 24.56% | 16.2% |
| ***Long-run correlations SS***** | 31.33% | 15.8% |
| ***Cointegration NSS**** | 27.01% | 14.7% |
| ***Cointegration SS***** | 24.82% | 24% |

*NSS means "No short sales allowed", **SS means "Short sales allowed"

The tests made for the cointegration portfolios are available at the Appendix section.

First, we observe that during the testing period equity indices generally have an upward trend, so returns seem to be satisfying. As we can see, with rebalancing, again the best results, regarding the portfolios' returns, come from the mean-variance strategies with short sales allowed, which by far outperformed the benchmark with only a slight risk tradeoff. Clearly the mean-variance portfolios with no short sales are inferior to the latter methods, since they dramatically reduce our strategy options. When short sales are allowed even the portfolio's risk is less than its corresponding portfolio with no short sales.




*NSS means "No short sales allowed", **SS means "Short sales allowed"

**Figure 3.6: Returns & Risk depiction (With rebalancing)**

From cointegration optimization strategies, this time with rebalancing, the strategy allowing short sales to track the benchmark's course did not go so well. Also the risk of the portfolio ranges to high levels. In the next chapter we will conclude and propose what method is the best, according to our data.

# Conclusions

Several different portfolio construction methods were applied in the present thesis, using daily data. The data period was the years 2008-2013, a turbulent time for the global economy. This fact made the strategies' goals in achieving positive results quite challenging. However, during the testing period, all the portfolios achieved to overperform the benchmark, with the sole exception of cointegration portfolio with short sales allowed and no rebalancing.

More specifically, the mean-variance portfolios with simple correlations had an excessive return of 5%-6% (no short sales portfolios) and 11.5%-12.5% (short sales portfolios). These levels are quite satisfying and with a small risk tradeoff. The better performances were achieved by the portfolios that allowed short sales, and with less risk compared to the corresponding no short sales portfolios. So, it is quite clear that, according to the data, negative weights should be allowed to the investor's portfolio, since they achieve superior returns with lower risk. Regarding rebalancing, the rebalanced portfolios had a return optimization of about 1%-1.5% and a minor risk reduction of two decimals. This optimization is not astonishing, and we should take account the additional transaction costs in order to decide clearly whether portfolio rebalancing deserves our attention.

The results about the mean-variance portfolios with long-run correlations are quite similar. In fact, we actually reach the same conclusions. The portfolios had an excessive return of 5%-6% (no short sales portfolios) and 11.5%-12.5% (short sales portfolios) and the risk fluctuated at the same levels as above. Again, regarding rebalancing, it is unknown if the minor return and risk optimization overcome the additional costs.

If we compare the portfolios regarding the correlation method (simple or long-run), we observe that they give quite similar results and it is in the discretion of each investor to choose and apply the strategy of her choice.

Finally, the portfolios that were constructed based on cointegration did not give clear results. All the portfolios should have cloned the returns of the benchmark. However, the returns were quite divergent, with the exception of the portfolio with short sales allowed and no rebalancing which was quite close to the benchmark. Additionally that portfolio and its corresponding with rebalancing had almost double risk than the benchmark, making quite uneasy rides for a risk-averse investor. The failure of the cointegration portfolios to track the benchmark may be due to the data of the testing period which had a steep upward slope. The problem could be solved with the use of a broader calibration period and more frequent rebalancing. Anyway, the application of cointegration based portfolios has a lot of potential and should be tested further at the future works in this scientific field.

To conclude, the traditional mean-variance portfolios still achieve satisfying results, but in the future, researchers should also give weight to other portfolio construction methods such as cointegration. The present thesis attempted to contribute in this field and to provide the investor with some useful data in order to assist her in the struggle to diversify adequately her portfolio.

# Appendix

## A.1 Cointegration portfolios with no rebalancing tests

We run some tests in order to check the power of the cointegration portfolios. In figure A.1, we depict the portfolio's course (no rebalancing and no short sales) along with the benchmark during the testing period. As we can see, their paths are quite similar. More specifically, the correlation between the series during the testing period is still very high (value of 0.991).

Next we calculate the ADF *t*-stat of the two series residual to check if the unbalanced portfolio is still cointegrated with the benchmark after a whole year. We check for a four years period (June 11 2009 to June 11 2013) and find an ADF *t*-stat -3.45 which is statistically significant at level 5%. Additionally a KPSS test showed that the residual series is clearly stationary. So we can accept that our portfolio is still cointegrated with the benchmark. This is quite promising since the portfolio does not need a frequent rebalancing, meaning that we can diminish the transaction costs. However, the *t*-stat has dropped meaning that a rebalancing may be necessary to keep it in higher levels.

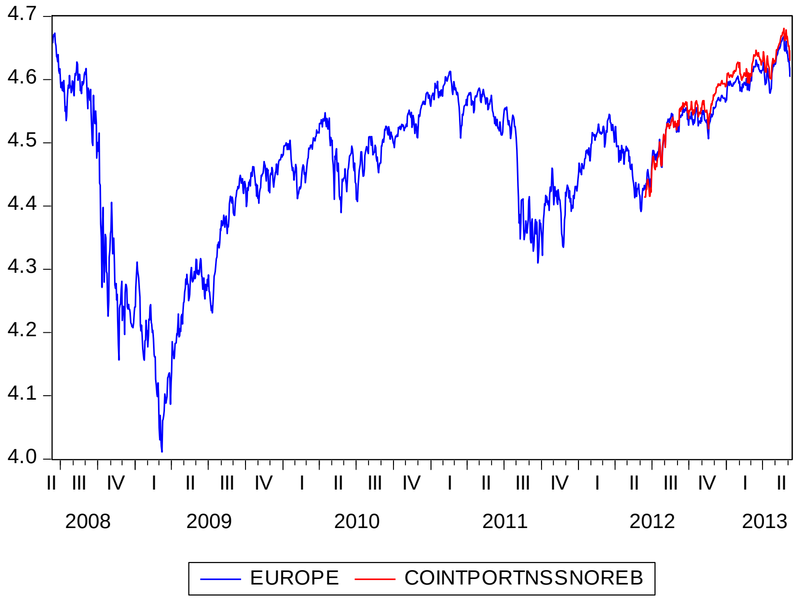



**Figure A.1: Benchmark & Portfolio depiction during the testing period (No short sales allowed)**

The same tests take place for the other portfolio (no rebalancing with short sales allowed). In figure A.2, we depict the portfolio's course along with the benchmark during the testing period. As we can see, their paths are similar but not as much as the previous portfolio. More specifically, the correlation between the series during the testing period is considerably diminished (value of 0.76).

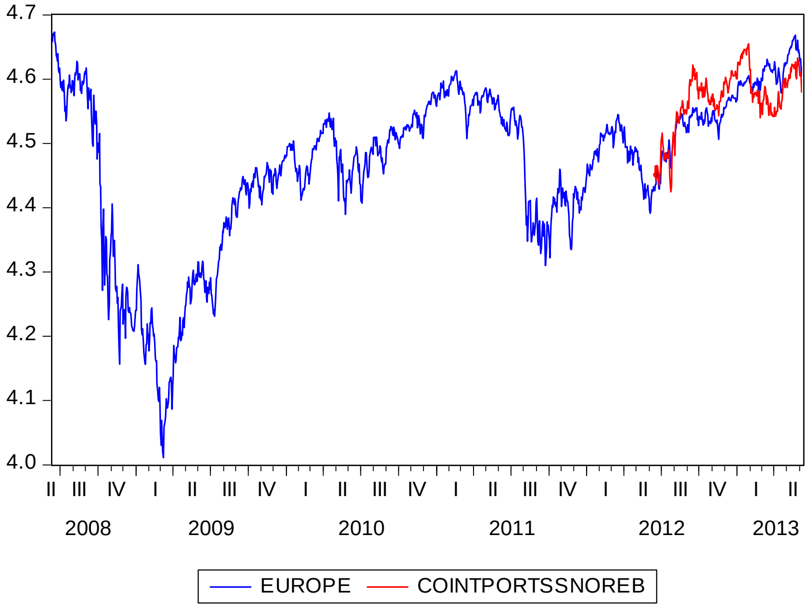



**Figure A.2: Benchmark & Portfolio depiction during the testing period (Short sales allowed)**

Finally, we calculate the ADF *t*-stat of the two series residual to check if the unbalanced portfolio with short sales is still cointegrated with the benchmark after a whole year. We check for a four years period (June 11 2009 to June 11 2013) and find an ADF *t*-stat -4.57 which is statistically significant at level 1%. We can see that the short sales portfolio kept its cointegration relationship with the benchmark to higher levels than the portfolio with no short sales. Again this is positive since the portfolio does not need a frequent rebalancing, meaning that we can diminish the transaction costs. For one more time, the *t*-stat has dropped meaning that a rebalancing may be necessary to keep it in even higher levels.

## A.2 Cointegration portfolios with rebalancing tests

In figure A.3, we depict the portfolio's course (with one rebalancing and no short sales) along with the benchmark during the testing period. As we can see, their paths are quite similar. More specifically, the correlation between the series during the testing period is still very high (value of 0.982).

Next we calculate the ADF *t*-stat of the two series residual to check if the rebalanced portfolio is still cointegrated with the benchmark after a whole year. We check for a four years period (June 11 2009 to June 11 2013) and find an ADF *t*-stat -3.79 which is statistically significant at level 5%. Additionally a KPSS test showed that the residual series is stationary with a significance of 5%. So we cannot fully accept that our portfolio is still cointegrated with the benchmark. This means that the rebalancing did not actually help in improving the cointegration, but rather affect it negatively. The reasons for this will not be investigated in the present thesis. An additional rebalancing may solved the problem.

The same tests take place for the other portfolio (rebalancing with short sales allowed). In figure A.4, we depict the portfolio's course along with the benchmark during the testing period. As we can see, their paths are similar but not as much as the previous portfolio. More specifically, the correlation between the series during the testing period is considerably diminished (value of 0.80).

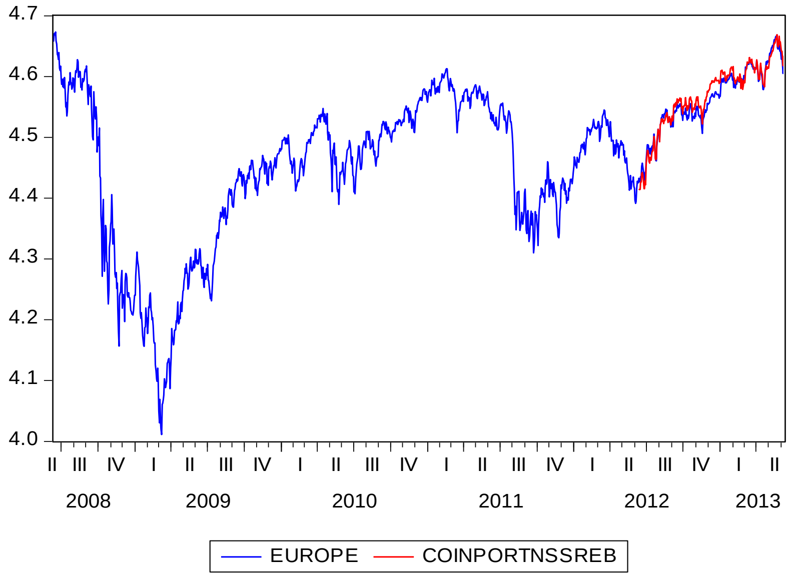



**Figure A.3: Benchmark & Portfolio depiction during the testing period (No short sales allowed)**

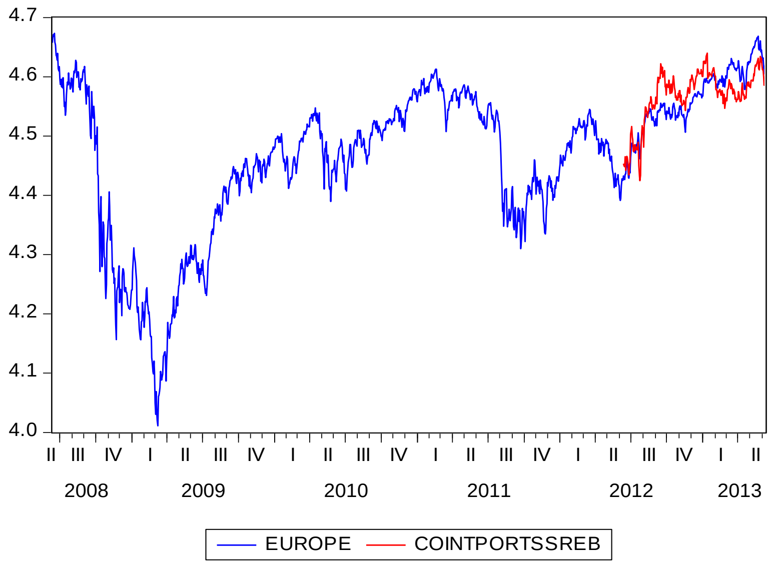



**Figure A.4: Benchmark & Portfolio depiction during the testing period (Short sales allowed)**

Finally, we calculate the ADF *t*-stat of the two series residual to check if the rebalanced portfolio with short sales is still cointegrated with the benchmark after a whole year. We check for a four years period (June 11 2009 to June 11 2013) and find an ADF *t*-stat -4.75 which is statistically significant at level 1%. We can see that the short sales portfolio kept its cointegration relationship with the benchmark to higher levels than the portfolio with no short sales. Again this is positive since the portfolio does not need a frequent rebalancing, meaning that we can diminish the transaction costs.

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