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VOLATILITY FORECASTING AND VOLATILITY INDICES: A COMPREHENSIVE REVIEW AND PRACTICAL APPLICATIONS IN FINANCIAL DERIVATIVES PRICING AND PORTFOLIO MANAGEMENT

by
ELEFTHERIOS DIMOSTHENI IOANNIDIS

Supervising Professor
ACHILLEAS ZAPRANIS

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To my priceless family

I would like to thank my supervising professor Dr. Achilleas Zapranis for his valuable assistance

Abstract

This thesis provides a broad overview of the evolution that has been made in the fields of volatility forecasting and derivatives pricing. The most common volatility forecasting models are described, in addition with derivatives pricing methods. The academic bibliography of volatility indices is also presented.

The principal aim of this thesis is to provide for the first time a complete and comprehensive literature review of all publicly available volatility indices, which are considered the most important tools for volatility forecasts, but also for risk management and portfolio management as well. In the next pages, the kinds of volatility indices, their calculation processes and their practical applications are discussed.

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Introduction

Asset prices change frequently over time since the beginning of the first organized market. During the years scientists are trying to answer fundamental questions such as how and why prices evolve with an ultimate purpose of predicting future changes. Since forecasting is of great importance in risk management, the expansion of research in the fluctuations of prices was inevitable.

Over the last 35 years, the financial sector has flourished rapidly worldwide. One of the reasons was the introduction and dissemination of new innovative products, such as derivatives. Forward, features, swaps and options contribute significantly to hedging speculation and arbitrage.

As derivatives bloomed, the study in the fluctuations, or *volatility*, of asset prices became essential for many academics.

A landmark breakthrough in derivatives evolution was the Black – Scholes pricing model, firstly proposed in 1973, which gave a tremendous boost in derivatives trading. Likewise, it can be said that a similar boost in the field of volatility research was given by the introduction of the ARCH model by Robert Engle in 1982. It should be noted that both discoveries were awarded with the Nobel Prize.

Since then, there have been many proposals by various scientists in both fields. Each one contributes more or less to the evolution of financial science.

This thesis aims to briefly examine the developments in volatility forecasting and derivatives pricing. It also presents for the first time a comprehensive literature review of volatility indices.

More particularly, the paper is organized as follows.

In the first chapter, the most common volatility forecasting models are presented. They are divided in four major categories: a) Historical volatility models, b) ARCH family models, c) Stochastic volatility models and d) Implied volatility models.

In the second chapter, the pricing methods of derivatives are described. Derivatives are also divided in three major categories: Forward and futures contracts swaps and options. Implied volatility is further explained.

In the third chapter a comprehensive academic review of volatility indices is demonstrated. Most important theoretical developments and empirical insights of all kinds of both actually introduced and academically proposed volatility indices are presented, in addition with their calculation processes and their practical applications. It is shown that volatility indices are principally applied in forecasting. There is also significant research about their information content and their relations with their underlying indices. Further, it must be mentioned that many studies find spillover effects between volatility indices and different markets or sectors which seem to be uncorrelated at first sight, such as macroeconomic announcements, or the commodity market of gold.

Finally, concluding remarks and fields for future research are presented.

Chapter 1: Volatility Forecasting

Introduction

Volatility forecasting has become a hot topic, especially after the recent financial crisis which originated in the US, but spread throughout the world and continues to severely affect Europe.

The cornerstone in volatility forecasting research was undoubtedly the seminal work of Robert Engle in 1982, who proposed the Auto Regressive Conditional Heteroskedasticity (ARCH) Model. The model was originally proposed to study the variance of UK inflation, but proved to have countless applications in financial econometrics.

In this chapter the types of volatility forecasting models are discussed. The author follows Poon's and Granger's (2003) and Poon's (2005) categorization, who separate forecasting models in four categories.

The first category includes historical volatility models which estimate volatility directly, mainly based on past observations.

The second includes ARCH, GARCH models and their variations. These models do not use past standard deviations, but form a new conditional variance of asset returns via maximum likelihood procedures.

The third contains stochastic volatility models, which are firstly theoretical models and volatility is treated like a random process.

The final category includes option implied volatility models mostly based on the Black – Scholes pricing method and its variations, which are further presented in chapter 2.

Overall rankings suggest that the later category provides the best and most reliable forecasts, principally because these models use a larger information set.

1.1 Volatility Definition

In everyday language volatility refers to the fluctuations observed in some phenomenon over time. In finance, volatility is a measure for variation of price of an asset over time. According to Poon (2005), volatility refers to the spread of all likely outcomes of an uncertain variable. Statistically, it is often measured as the sample standard deviation. Taylor (2005) defines volatility as the measure of price variability over some period of time. It typically describes the standard deviation of returns in a particular context that depends on the definition used. Andersen et al. (2005) state that volatility within economics is used more formally to describe, without a specific implied metric, the variability of the random (unforeseen) component of a time series. More precisely, or narrowly, in financial economics, volatility is often defined as the (instantaneous) standard deviation (or σ) of the random Wiener – driven component in a continuous – time diffusion model.

It is more than obvious that forecasting volatility accurately may neutralize risk and generate profit.

1.2 Volatility Forecasting Models

If volatility can be forecasted the question arises is which method can provide the best forecasts?

Poon and Granger (2003) and Poon (2005) classify Volatility Forecasting Models into 4 major categories:

1. HISVOL: for historical volatility, which include random walk and historical averages of squared or absolute returns. Also included in this category are time series models based on historical volatility using moving averages, exponential weights, autoregressive models, or even fractionally integrated autoregressive absolute returns. It must be noted that HISVOL models can be highly sophisticated, such as the multivariate VAR realized volatility model in Andersen et al (2001) which is classified in this category by Poon. All models in this group estimate volatility directly, omitting the goodness of fit of the returns

distribution or any other variables such as option prices. Comparing with other types of volatility models, these are the easiest to manipulate and construct.

2. ARCH: These models (firstly proposed by Engle in 1982 and extended by many researchers), do not make use of the past standard deviations, in contrast to HISVOL models, but form a conditional variance h_t or σ_t^2 of asset returns via maximum likelihood procedures. Any member of the ARCH, GARCH and so forth family is included in this category.
3. SV: for stochastic volatility models which are first and foremost theoretical models rather than a practical and direct tool for volatility forecasts. In these models volatility is treated as a random process, governed by state variables such as the price level of the underlying security.
4. ISD: for option implied standard deviation, based on the Black – Scholes model and various generalizations, which is used to forecast volatility.

In Poon's survey of papers, 93 studies were included, conducted from 1976 until 2004. 25 of them did not involve comparisons between at least two of the aforementioned methods, so were not helpful for comparison purposes. Of the remaining, some compared just one pair of forecasting techniques, while other compared several. Combination of forecasts has a mixed picture. Two studies find it to be helpful, but another does not.

For those involving both HISVOL and GARCH models, 22 found HISVOL better at forecasting than GARCH (56%) and 17 the opposite result (44%).

SV was found better than both HISVOL and GARCH in 6 out of 7 studies. Unfortunately, there are only 7 studies involving SV, so a further research is required.

The overall ranking suggests that ISD provides the best forecasting with HISVOL and GARCH roughly equal. Out of 34 studies, only 8 found HISVOL better than ISD (24%) and out of 18 between GARCH and ISD, GARCH is found better in only 1 (6%). There is also one study finding ISD better than SV and no studies finding SV better than ISD. The success of the implied volatility should not be surprising, since these forecasts use a larger and more relevant information set than the alternative methods, as they use option prices. They are also less practical, not being available for all assets.

The next table summarizes data from Poon's research.

Table 1.1: Poon's research summary

	Number of Studies	Studies Percentage
HISVOL > GARCH	22	56%
GARCH > HISVOL	17	44%
HISVOL > ISD	8	24%
ISD > HISVOL	26	76%
GARCH > ISD	1	6
ISD > GARCH	17	94
SV > HISVOL	3	
SV > GARCH	3	
GARCH > SV	1	
ISD > SV	1	

(Source : Poon, 2005)

In the next paragraphs, the most common volatility forecasting models are presented and a comparison between them is attempted.

1.3 Historical Volatility Models

It has been shown that HISVOL models have good forecasting performance compared with other time series volatility models. In these models, conditional volatility is modeled separately from returns, unlike ARCH and SV models. Hence, they are less restrictive and more ready to respond to changes in volatility dynamic (Poon 2005).

For example, the simplest form of ARCH is $r_t = \mu + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma_t)$ with $\varepsilon_t = z_t\sigma_t$, $z_t \sim N(0, 1)$ and $\sigma_t^2 = \omega + a_1\varepsilon^2_{t-1}$. The conditional volatility σ_t^2 is modeled as a “byproduct” of the return equation. The estimation is done by maximizing the likelihood of observing ε_t, using the normal, or other chosen density. In contrast, the HISVOL models are built directly on conditional volatility such as $\sigma_t = \gamma + \beta_1\sigma_{t-1} + \upsilon_t$ (AR(1) model). The parameters γ and β_1 are estimated by minimizing in – sample forecast errors υ_t, and the forecaster has the choice of reducing mean square errors, mean absolute errors etc.

The historical volatility estimates σ_t can be calculated as sample standard deviations, if there are sufficient data for each t interval. If there is insufficient information, the $H - L$ method may be used, (highest and lowest prices, easily observed in newspapers).

1.4 *Types of Historical Volatility Models*

According to Poon (2005) there are two major categories of HISVOL models: *the single state* and the *regime switching models*. All models differ by the number of lag volatility terms included in the model and the weights assigned to them, reflecting the choice on the tradeoff between increasing the amount of information and more updated information.

Single state models

The simplest historical model is the *random walk* model, where the difference between consecutive period volatility is modeled as a random noise: $\sigma_t = \sigma_{t-1} + \upsilon_t$. The best forecast for tomorrow's volatility is today's, where σ_t alone is used as a forecast for σ_{t+1}.

In the *historical average* method, a forecast is made based on the entire history of σ. The formula is $\hat{\sigma}_{t+1} = \frac{1}{t}(\sigma_t + \sigma_{t-1} + \cdots + \sigma_1)$.

The *simple moving average*, method is similar, except that older information is discarded. $\hat{\sigma}_{\tau+1} = \frac{1}{\tau}(\sigma_t + \sigma_{t-1} + \cdots + \sigma_{t-\tau-1})$. The value of τ (i.e. the lag length to past information used) could be subjectively chosen or based on minimizing in – sample forecast errors. Multi period forecasts (for $\tau>1$) will be the same as the one step ahead forecast.

Similarly, the *exponential smoothing* model, $\sigma_t = (1-\beta)\sigma_{t-1} + \beta\widehat{\sigma_{t-1}} + \xi_t$, and $0\leq\beta\leq1$, $\widehat{\sigma_{t-1}} = (1-\beta)\sigma_t + \beta\widehat{\sigma_t}$, gives more weight to the recent past and less weight to the distant. The smoothing parameter β is estimated by minimizing the in – sample forecast errors ξ_t.

The exponentially *weighted moving average* method or EWMA is a moving average method with exponential weights. Again the smoothing parameter β is estimated by minimizing the in – sample forecast errors ξ_t. The equation is: $\widehat{\sigma_{t-1}} = \sum_{i=1}^{\tau}\beta^i\sigma_{t-i-1} / \sum_{i=1}^{\tau}\beta^i$.

All HISVOL models described above have a fixed weighting scheme or a weighting scheme that follows a declining pattern. There are also models without prespecified weighting schemes.

The simplest of these models is the *simple regression* method, which expresses volatility as a function of its past values and an error term. This model is principally autoregressive with a formula $\sigma_t = \gamma + \beta_1\sigma_{t-1} + \beta_2\sigma_{t-2} + \cdots + \beta_n\sigma_{t-n} + \upsilon_t$, $\widehat{\sigma_{t-1}} = \gamma + \beta_1\sigma_t + \beta_2\sigma_{t-1} + \cdots + \beta_n\sigma_{t-n+1}$.

If past volatility errors are also included, the model is transformed to an *Auto Regressive Moving Average* Model or *ARMA*. $\widehat{\sigma_{t+1}} = \beta_1\sigma_t + \beta_2\sigma_{t-1} + \cdots + \gamma_1\upsilon_t + \gamma_2\upsilon_{t-1+} \cdots$.

If data show evidence of non – stationarity, an initial differencing step *I(d)* can be applied to remove non – stationarity. The model is called *ARIMA* (where *I* corresponds to the “integrated” part of the model), if *d=1* and *ARFIMA*, if *d<1*.

If exogenous inputs are included, the model is called *ARMAX*.

Regime switching and transition exponential smoothing

In this category, there is the threshold autoregressive model, firstly introduced by Cao and Tsay (1992) with equation: $\sigma_t = \varphi_0^{(i)} + \varphi_1^{(i)}\sigma_{t-1} + \cdots + \varphi_p^{(i)}\sigma_{t-p} + \upsilon_t$, $i = 1,2,..k$, $\widehat{\sigma_{t-1}} = \varphi_0^{(i)} + \varphi_1^{(i)}\sigma_t + \cdots + \varphi_p^{(i)}\sigma_{t+1-p}$, where the thresholds separate volatility into states with independent simple regression and noise processes in each state. The prediction $\widehat{\sigma_{t+1}}$ could be based solely on current state information *i* assuming that future will remain on current state. Alternatively it could be based on information of all states weighted by the transition probability for each state. Cao and Tsay (1992) found that this model outperformed EGARCH and GARCH (explained below) in the forecasting of the 1 to 30 month volatility of the S&P value weighted index. EGARCH provided better forecasts for the S&P value equally weighted index, possibly because the second index gives more weights to small stocks where the leverage effect could be more important.

The *smooth transition exponential smoothing* model, proposed by Taylor in 2004, is …$\widehat{\sigma_t} = a_{t-1}\varepsilon_{t-1}^2 + (1 - a_{t-1})\widehat{\sigma_{t-1}^2} + \upsilon_t$, where $a_{t-1} = \frac{1}{1+\exp(\beta+\gamma V_{t-1})}$ and $V_{t-1} = a\varepsilon_{t-1} + b|\varepsilon_{t-1}|$ is the transition variable. One day ahead forecasting results showed that the model performs very well against several ARCH counterparts, but these rankings were not tested for statistical significance, so it is difficult to come to a conclusion given the closeness of many error statistics reported.

Summarizing, the random walk and historical average methods seem naïve at first, but they perform well for medium and long horizon forecasts. For longer than 6 months horizons, low frequency data over a period at least as long as the forecast horizon works best.

1.5 ARCH Models

Financial market volatility is known to cluster. A volatile period tends to persist for some time before the market returns to normality (Poon, 2005). The *Auto Regressive Conditional Heteroskedasticity* or *ARCH* model proposed by Engle (1982) was designed to capture volatility persistence in inflation. The model was later found to fit many financial time series and its widespread impact has led to a Nobel Prize for Engle in 2003. The ARCH effect has been shown to lead to high kurtosis which fits well with the empirically observed tail thickness of many asset return distributions.

In contrast to HISVOL models, ARCH do not use the past standard deviations, but formulate a conditional variance h_t or σ_t^2 of asset returns via maximum likelihood procedures. To illustrate this, returns $r_t = \mu + \varepsilon_t$, and $\varepsilon_t = \sqrt{h_t z_t}$, where $z_t \sim D(0, 1)$ is a white noise. The Distribution D is often taken as normal. The process z_t is scaled by h_t which in turn is a function of past squared residual returns. In the *ARCH(q)* process, $h_t = \omega + \sum_{j=1}^{q} a_j \varepsilon_{t-j}^2$, with $\omega > 0$ and $\alpha_j \geq 0$ to ensure h_t is strictly positive variance. Typically, q is of high order due to the volatility persistence phenomenon. h_t is known at time $t - 1$. So the one step ahead forecast is readily available. The multi step ahead forecasts can be formulated by assuming $E[\varepsilon_{t+\tau}^2] = h_{t+\tau}$. The unconditional variance of r_t is $\sigma^2 = \frac{\omega}{1-\sum_{j=1}^{q} a_j}$.

1.6 Types of ARCH Models

The most common models in this category are:

Generalised ARCH or GARCH

If an ARMA model is assumed for the error variance, ARCH becomes *GARCH*, due to Bollerslev (1986) and Taylor (1986), where additional dependencies are permitted on p lags of past h_t as shown: $h_t = \omega + \sum_{i=1}^{p} \beta_i h_{t-i} + \sum_{j=1}^{q} a_j \varepsilon_{t-j}^2$, with ω

> 0. For GARCH(1, 1) the constraints $\alpha_1 \geq 0$ and $\beta_1 \geq 0$ are needed to ensure h_t is strictly positive. The unconditional variance is $\sigma^2 = \frac{\omega}{1-\sum_{i=1}^{p}\beta_i - \sum_{j=1}^{q}a_j}$. The GARCH(p, q) model is covariance stationary only if $\sum_{i=1}^{p}\beta_i - \sum_{j=1}^{q}a_j < 1$.

Volatility forecasts from GARCH(1, 1) can be made by repeated substitutions. The estimation for expected squared residuals is $E[\varepsilon_t^2] = h_t E[z_t^2] = h_t$. h_t and the one step ahead forecast is known at time t, $\widehat{h_{t+1}} = \omega + a_1\varepsilon_t^2 + \beta_1 h_t$. As the forecast horizon lengthens $\widehat{h_{t+\tau}} = \frac{\omega}{1-(\alpha_1\beta_1)} + (\alpha_1+\beta_1)^\tau[a_1\varepsilon_t^2 + \beta_1 h_t]$.

Integrated GARCH or IGARCH

This model is a restricted version of GARCH, where the persistent parameters sum up to one, and therefore there is a unit root in the process. For a GARCH(p, q) process, when $\sum_{i=1}^{p}\beta_i - \sum_{j=1}^{q}a_j = 1$, the unconditional variance $\sigma^2 \to \infty$ is no longer definite. The conditional variance is then described as an IGARCH. An infinite volatility is a concept rather counterintuitive to real phenomena. Empirical findings suggest that GARCH(1, 1) is the most popular structure for many financial time series.

Exponential GARCH or EGARCH

The model, firstly proposed by Nelson in 1991, specifies conditional variance in logarithmic form, meaning that there is no reason to impose an estimation constraint to avoid negative variance. The formula is $lnh_t = a_0 + \sum_{j=1}^{q}\beta_j lnh_{t-j} + \sum_{k=1}^{p}\left[\theta_k\epsilon_{t-k} + \gamma_k\left(|\epsilon_{t-k}| - \sqrt{2/\pi}\right)\right]$ and $\epsilon_t = \varepsilon_t/\sqrt{h_t}$. Here h_t depends on both the size and sign of ε_t. With appropriate conditioning of the parameters, this specification captures the stylized fact that a negative shock leads to a higher conditional variance in the subsequent period, than a positive shock. The process is covariance stationery only if $\sum_{j=1}^{q}\beta_j < 1$.

Tsay (2002) showed how forecasts can be formulated with EGARCH(1, 0) and gave the one step ahead and multi step forecasts.

GARCH in Mean or GARCH–M

Where a heteroskedasticity term is added into the mean equation.

1.7 Other Nonlinear Models

Models which also allow for nonsymmetrical dependencies include the *Nonlinear GARCH (NGARCH),* firstly proposed by Engle and Ng in 1993, where a parameter θ is introduced, *Quadratic GARCH (QGARCH)* by Sentana (1995), used for modeling symmetric effect of positive and negative shocks, where $\epsilon_t = \sigma_t z_t$, *CJR–GARCH* by Glosten, Jagabbathan and Runkle in 1993, similar to QGARCH, modeling asymmetry in the ARCH process and *Threshold GARCH (TGARCH)* from Zakoian (1994), similar with CJR–GARCH, but formulated with absolute return instead (the specification is one on conditional standard deviation instead of conditional variance).

According to Poon and Granger (2003) and Poon (2005) in their survey of papers, ARCH models and their variants have many supporters. Models that allow for volatility asymmetry perform well in the forecasting contest due to the strong negative relationship between volatility and shock.

In general, unlike ARCH models, the "simpler" methods (including EWMA) do not separate volatility persistence from volatility shocks and most of them do not incorporate volatility mean reversion. "Simpler" methods tend to provide larger volatility forecasts most of the time because there is no constraint on stationarity or convergence to the unconditional variance and may result in larger forecast errors and less frequent VaR violations. As ARCH models assume variance stationarity, their forecasting performance suffers when there are changes in volatility level. Parameter estimation becomes unstable when data period is short or when there is a change in volatility level. This has led to a GARCH convergence problem in several studies. They all favor some form of exponential smoothing method, for forecasting volatility of a wide range of assets.

Linear and nonlinear long memory models

As mentioned, volatility persistence is a feature that many time series models are designed to capture. A GARCH model features an exponential decay in the autocorrelation of conditional variances. However, it has been noted that squared and absolute returns of financial assets typically have serial correlations that are slow to decay, similar to those of an $I(d)$ process. A shock in volatility seems to have very long memory and to impact on future volatility over a long horizon. The IGARCH captures this effect, but a shock in this model impacts upon future volatility over an infinite horizon and the unconditional variance does not exist for this model.

Taylor (1986) was the first to note that autocorrelation of absolute returns, is slow to decay compared with that of squared returns. Following the work of Granger and Joyeux (1980), where fractionally integrated series was shown to exhibit long memory property, Ding, Granger and Engle **(**1993) proposed a *fractional integrated model* on $|r_t|^d$, where d is a fraction.

The most common fractional integrated models are the *FIGARCH*, which is estimated based on the appropriate maximum likelihood techniques using the truncated ARCH representation. The equation is $h_t = \omega + [1 - \beta_1 L - (1 - \varphi_1 L)(1 - L)d\varepsilon t2+\beta 1ht-1$ (used in Baillie, Bollerslev and Mikkelsen, 1996).

Noting that an EGARCH can be represented as an ARMA process in terms of the logarithm of conditional variance and thus always guarantees that the conditional variance is positive, Bollerslev and Mikkelsen (1996) proposed the *fractionally integrated EGARCH* or *FIEGARCH*, which is truly a model for absolute return.

There has been a lot of research investigating whether long memory of volatility can improve volatility forecasts and explain anomalies in option prices. Much of this research has used fractional integrated models, but several studies have showed that a number of nonlinear short memory volatility models are capable of spurious long memory characteristics as well, such as the *brake model* (Granger and Hyung, 2004), the *volatility component model* or *CGARCH* (Engle, Lee, 1999), and the *regime – switching model* (Hamilton and Susmel, 1994, Diebold and Inoue, 2001). Each of these short memory nonlinear models provides a rich interpretation of financial market volatility structure, compared with the apparently myopic fractional integrated model which simply requires financial market participants to remember and react to shocks for a long time.

Both HISVOL and ARCH models have been tested for fractional integration.

Multivariate Volatility models

There have been a number of studies that examined cross – border volatility spillover in stock markets, exchange and interest rates. The volatility spillover relationships are potential source of information for volatility forecasting, especially in the very short term and during global turbulent periods. Several variants of multivariate ARCH models exist for a long time while multivariate SV models are fewer and more recent. Truly multivariate volatility models, beyond two or three returns variables, are difficult to implement. The greatest challenges are parsimony, nonlinear relationships between parameters and keeping the variance – covariance matrix positive definite.

Most used multivariate model is the *asymmetric dynamic covariance model (ABC)* proposed by Kroner and NG in 1998, which encompasses many older multivariate ARCH models and unlike most of its predecessors, it allows for volatility asymmetry in the spillover effect.

Their future depends on their use. In long horizon forecasting, their use is restricted unless a more parsimonious factor approach is adopted (Sentana, 1998, Sentana and Fiorentini, 2001). They will continue to be useful for capturing volatility spillover.

1.8 Stochastic Volatility Models

The SV model is principally a theoretical method than a direct tool for volatility forecast, although recent developments in the volatility area should not be overlooked. As far as implementation is concerned, the SV estimation still poses a challenge to many researchers (Poon, 2005). SV and ARCH models are closely related and many ARCH models have SV equivalence as continuous time diffusion limit (Duan, 1997, Corradi, 2000, Fleming and Kirby, 2003).

The discrete time SV model is $r_t = \mu + \varepsilon_t$, $\varepsilon_t = z_t exp(0.5h_t)$, $h_t = \omega + \beta h_{t-1} + \upsilon_t$, where υ_t may or may not be independent of z_t.

The SV model has an additional innovative term in the volatility dynamics and hence, is more flexible than ARCH class models. It has been found to fit financial market returns better and has residuals closer to standard model. Modeling volatility as a stochastic variable leads to fat tail distributions for returns. The autoregressive term in volatility process introduces persistence and the correlation between the two innovative terms in the volatility and return processes, produce volatility asymmetry (Hull and White, 1987, 1988). Long memory SV models have also been proposed by allowing the volatility process to have a fractional integrated order (Harvey, 1998).

The volatility noise term makes the SV model a lot more flexible, but as a result the model has no closed form, therefore cannot be estimated directly by maximum likelihood. The *quashi – maximum likelihood estimation (QMLE)* approach of Harvey, Ruiz and Shepard (1994) is inefficient if volatility proxies are non – Gaussian (Andersen and Sorensen, 1997). The alternatives are the *generalized method of moments (GMM)* approach through simulations (Duffie and Singleton, 1993), or analytical solutions (Singleton, 2001) and the *likelihood approach* through numerical integration

(Fridman and Harris, 1998), or Monte Carlo integration using either importance sampling (Danielsson, 1994, Pitt and Shepard, 1997, Durbin and Koopman, 2000) or Markov chain (i.e. Jackuier, Polson and Rossi 1994, Kim, Shepard and Chib, 1998).

The MCMC approach

The *Markov Chain Monte Carlo* approach of modeling stochastic volatility was made popular by authors such as Jackuier, Polson and Rossi (1994).

Tsay (2002) describes well the procedure: in the simplest case $r_t = \alpha_t$, $\alpha_t = \sqrt{h_t \varepsilon_t}$, $\ln h_t = a_0 + a_1 \ln h_{t-1} + \upsilon_t$, where $\varepsilon_t \sim N(0, 1)$, $\upsilon_t \sim N(0, \alpha^2{}_t)$ and ε_t, υ_t are independent. The model estimation is complicated because the maximum likelihood function is a mixture over the volatility distributions.

Up till 2005, there were only 6 SV studies and a PhD thesis by Heynen (1995), according to Poon (2005). Heynen finds SV forecasts better for a number of stock indices across several continents. Heynen and Kat (1994) find that SV provides the best forecasts for indices but produces forecast errors that are 10 times larger than EGARCH's and GARCH's for exchange rates. Yu (2002) ranks SV top for forecasting New Zealand stock market volatility, but the margin is very small, partly because the evaluation is based on variance and not standard deviation. Lopez (2001) finds no difference between SV and other time series forecasts using conventional error statistics. All 3 papers have the 1987 crash in the in – sample period and the 1987 impact is not clear.

A further research is definitely needed.

1.9 Option Implied Standard Deviation for Volatility Forecasts

The term *implied volatility*, or *implied standard deviation (ISD)*, in financial mathematics refers to the volatility value which derives from an option contract. If that value is inserted to an option pricing model, will return a theoretical value equal to the current market price of the option. Option implied standard deviation has always been perceived as a market's expectation of future volatility and hence it is a market – based volatility forecast. It makes use of a richer and more updated information set and arguably it should be superior to time series volatility forecast.

On the other hand, option model – based forecast requires a number of assumptions to hold for the option theory to produce a useful volatility estimation. Moreover, option implied also suffers from many market – driven pricing irregularities.

Nevertheless, option implied volatility has superior forecasting capability, outperforming many HISVOL and ARCH models and matching the performance of forecasts generated from time series models that use a large amount of high frequency data. Indicatively mentioned, some studies supporting the implied volatility superiority are: Christensen and Prabhala, 1998, Fleming, 1998, Dumas et al., 1998 and Blair et al., 2001.

Once an implied volatility is obtained, it is usually scaled by $\sqrt{n}$ to get an n – day ahead volatility forecast. In some cases, a regression model may be used to adjust for historical bias, or the implied volatility may be parameterized within a GARCH/ARFIMA model with or without its own persistence.

A test on the forecasting power of ISD is a joint test of option market efficiency and a correct option pricing model. Since trading frictions differ across assets, some options are easier to replicate and hedge than others. It is therefore reasonable to expect levels of efficiency and different forecasting power for options on different assets.

While each historical price constitutes an observation in the sample used in calculating volatility forecast, each option price constitutes a volatility forecast over the option maturity and there can be many option prices at any one time. The problem of volatility smile and skew, discussed in chapter 2, means that options of different strike prices produce different implied volatility estimations.

The issue of a correct option pricing model is fundamental in finance. Option pricing has a long history and various extensions have been made to cope with dividend payments, early exercise, variety of underlying assets stochastic volatility and other problems.

Further explanations of options implied volatility are presented in chapter 2.

Chapter 2: Derivatives Pricing Methods

Introduction

In the last 30 years derivatives have become extremely important in finance. There are not any safe estimations of market's total value. According to Bank for International Settlements, at the end of 2011, only the OTC market reached $ 648 trillion. Some estimate total market value at $ 1,200 trillion, 20 times bigger than world GDP (www.dailyfinance, www.globalresearch).

In this chapter, the pricing methods of derivatives are discussed.

Derivatives are separated in three major categories: Forward and futures, swaps and options.

According to Hull (2008), the most common pricing methods use the assumption that all investors are risk – neutral. The expected payoff of a derivative is calculated in a risk – neutral world and then discounted at the risk – free interest rate. The approach gives the correct price in all other worlds.

An alternative pricing approach is to use historical data to calculate the expected payoff and then discount this expected payoff at the risk – free rate to obtain the price. This method is referred as the historical data approach. Historical data approach gives an estimate of the expected payoff in the real world and is correct only when the expected payoff of a derivative is the same in both worlds. When we move from the real to the risk – neutral world, the volatilities of variables remain the same, but the expected growth rates are liable to change. For example, the expected growth rate of a stock market index decreases when we move from the real to the risk – neutral world. The expected growth rate of a variable can be assumed to be the same in both worlds, only if the variable has zero systematic risk so that percentage changes in the variable have zero correlation with stock market returns. This means that a derivative valuation using the historical data approach is correct only if all underlying variables have zero systematic risk.

In addition, most methods for pricing interest rate options, such as caps or European bond and swap options, called standard market models, assume that the probability distribution of an interest rate, a bond price, or some other variable at a future point in time is lognormal. However, these models have limitations, since they do not provide a description of how rates evolve through time. Consequently, they cannot be used for the valuation of some types of interest rate derivatives, such as American swap options or callable bonds. These limitations are overcome by term structured models.

Undoubtedly, one of the reasons that derivatives bloomed was a major breakthrough in options pricing, achieved by Fischer Black, Myron Scholes and Robert Merton in the early 70s. Their pricing method, known as the Black – Scholes model, has had a huge influence, is used up till now and was the reason that Merton and Scholes were awarded the Nobel Prize in 1997. (Unfortunately, Black died in 1995). The Black

– Scholes model and its variations are applied to several pricing methods of different derivatives types.

For understanding purposes, the standard Black – Scholes model is presented below.

The pricing formula for a European option on a non-dividend stock, as described by Hull (2008), is:

$$c = S_0 N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} N(-d_2) - S_0 N(-d_1) \text{ where}$$

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \text{ and } d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

The variables c and p are the European call and put prices, S_o is the stock price at time zero, K is the strike price, r is the continuously compounded risk – free rate, σ is the stock price volatility and T is the time to maturity of the option.

2.1 *Other Pricing Methods*

When exact formulas are not available, Hull (2008) suggests three numerical procedures that can be used for valuing derivatives.

The first represent the asset price movements in the form of a tree and is called binomial trees. These methods assume that in each short interval of time Δt, a stock price can either move up by a multiplicative amount u or down by a multiplicative amount d. The sizes of u and d and their associated probabilities are chosen so that the change in the stock price has the correct mean and standard deviation in a risk – neutral world. Derivative prices are calculated by starting at the end of the tree and working backwards. For an American option, the value at a node is the greater of: a) the value if it is exercised immediately and b) the discounted expected value if it is held for a further period of time Δt. A similar procedure called trinomial trees is also used. Its difference lies at the assumption that a stock price can either move up, down or remain stable.

The second involves finite difference models which value a derivative by solving the differential equation that the derivative satisfies. The equation is converted into a set of difference equations and these are solved iteratively. They are similar to tree approaches, since the computations work back from the end of the derivative life to the beginning. The models are divided into two major categories: the implicit and the explicit method. Their main difference is that the first leads to an equation which gives a relationship between three different values of an option at time $i\Delta t$ and one value of an option at time $(i+1)\Delta t$, while the latter leads to an equation which gives a relationship between one value of an option at time $i\Delta t$ and three different values of an option at time $(i+1)\Delta t$. The explicit method is functionally the same as using a trinomial tree. The implicit finite difference method is more complicated but in usage, has the advantage of not having to take any special precautions to ensure convergence.

The first two methods are usually used for American options and other derivatives where the holder has decisions to make prior to maturity.

The third involves Monte Carlo simulation which is usually used where the payoff is dependent on the history of the underlying variable or where there are several underlying variables. The method uses random numbers to sample many different paths that the variables underlying the derivative could follow in a risk – neutral world. For each path the payoff is calculated and discounted at the risk – free interest rate. The arithmetic average of the discounted payoffs is the estimated derivative value.

In practice, the method chosen depends on the characteristics of the derivative under evaluation and the accuracy required. Monte Carlo works forward while the other two backwards. It can be used for European style derivatives and can cope with a great deal of complexity as far as the payoffs are concerned. It becomes relatively more efficient as the number of underlying variables increases. Tree approaches and finite difference methods can accommodate both American and European style derivatives. However, they are difficult to apply when the payoffs depend on the past history of the state variables as well as on their current values. They are also liable to become computationally very time consuming when three or more variables are involved.

The procedures discussed can be used to handle most of the so called plain vanilla products pricing problems, which have standard well defined properties. However, there is a significant number of nonstandard products trading at the markets, called exotics, which require different calculation procedures.

The major categories of derivatives are discussed below.

2.2 Forward and Futures Pricing

Forward and futures are the easiest to analyze, since the buyer is obligated to fulfill the terms of the contract. In fact, forward contracts are easier than futures because there is no daily settlement, only a single payment at maturity.

According to Cox et al. (1981) and Hull (2008), it can be shown that both forward and futures prices of a certain asset are usually very close when the maturities of the contracts are the same. It can be shown that in theory the two should be exactly the same when interest rates are perfectly predictable.

In order to value a forward or a future contract, some assumptions must be made for at least some key market participants such as large derivatives dealers (Hull, 2008):

1. The market participants are subject to no transaction costs when they trade.
2. The market participants are subject to the same tax on all net trading profits.
3. The market participants can borrow money at the same risk - free rate of interest as they can lend money.
4. The market participants take advantage of arbitrage opportunities as they occur.

It is convenient, for understanding purposes, to divide forward and futures contracts into two major categories: those in which the underlying asset is held for investment by a significant number of investors. These are contracts where the underlying assets are bonds, stocks, indices, interest rates or currencies. The second category includes all contracts where the underlying asset is held primarily for consumption purposes such as corn, or pork.

In the first category, there are three different situations:

1. The asset provides no income. Its price is $F_0 = S_o e^{rT}$ and the value of a long forward contract, as time passes, is $S_o – Ke^{-rT}$.
2. The asset provides a known income with present value I. Its price is $F_0 = (S_o – I)e^{rT}$ and the value of a long forward contract is $S_o – I – Ke^{-rT}$.
3. The asset provides a known yield q. Its price is $F_0 = S_o e^{(r-q)T}$ and the value of a long forward contract is $S_o e^{-qT} – Ke^{-rT}$.

T is time to delivery date in years, S_0 is the price of the underlying asset today, F_0 is the forward or futures price today, K is the delivery price or strike price mentioned in the contract and r is the zero – coupon risk – free rate of interest per annum, expressed with continuous compounding, for an investment maturing at the delivery date (Jarrow, Oldfield, 1981, Hull, 2008).

In the case of contracts on foreign currencies, r in the aforementioned formulas is substituted by $r – r_f$, where r is the local risk – free rate and r_f the foreign risk – free rate (i.e. Euribor and LIBOR). The equation becomes similar with the one in situation 3, since the foreign currency can be regarded as an investment asset paying a known yield q.

It must be noted that contracts on valuable metals such as gold or silver can be held either for investment or consumption purposes. Storage costs can be treated as negative income and added to S_0.

In the second category of consumption assets, contracts usually provide no income but can be subject to significant storage costs. It is not possible to obtain their future price as a function of the spot price and other observable variables, since individuals and companies who own the commodity usually plan to use it in some way and they are reluctant to sell it in the spot market and buy forward or futures contracts. In this case, the parameter known as the asset's convenience yield becomes important. This is the incremental value of spot prices over features prices after accounting for carrying costs (Milonas, Henker, 2001). It measures the extent to which users of the commodity feel that ownership of the physical asset provides benefits that are not obtained by the holders of the futures contract. These benefits may include the ability to profit from temporary local shortages or the ability to keep a production process running. An upper bound for the consumption assets future price can be obtained, but it cannot be nailed down as an equality relationship between futures and spot prices.

If the convenience yield can be calculated, the equation can be converted to $F_0e^{yT} = (S_0 + U)e^{rT}$, where y is the convenience yield and U the storage costs. If the storage costs per unit are a constant proportion u, then the equation becomes $F_0 = S_0e^{(r + u - y)T}$. For investment assets, y must be zero. Otherwise, there are arbitrage opportunities.

r, $r – q$, $r – rf$, and $r – q + u$ are also called the cost of carry, which is the cost of carrying or holding a position.

2.3 SWAPS

In general, a swap is an agreement between two companies to exchange cash flows in the future. There are five major categories of swaps: *interest rate swaps, currency swaps*, *credit default swaps*, *equity swaps* and *commodity swaps*.

In summary, there are two ways of valuation.

In the first, the swap is decomposed into a long position in one bond and a short position in another bond.

In the second, the swap can be characterized as a portfolio of forward rate agreements.

The main types of swaps and their pricing are described in the next paragraphs.

The simplest types of swaps, or *plain vanilla swaps* can be valued using the "assume that forward rates will be realized" approach. The first step is to calculate the swap's net cash flows, assuming that short term reference rates in the future equal the forward rates calculated today from reference rate / swap zero curve. The second step is to set the swap's value equal to the present value of the net cash flows using the reference rate / swap zero curve for discounting. (A reference rate / swap zero curve is a method of extending reference rates beyond 12 months. Usually, until 2 years eurodollar futures can be used for the zero curve extension. Beyond 2 years, swap rates are used for the extension)

Nonstandard types are more complicated and some of them require different valuation procedures. Generally, it is assumed that an adjusted rate, rather than the actual forward rate, is realized. (Hull, 2008).

2.4 Interest Rate SWAPS

This is an agreement where the two parties exchange cash flows in fixed and floating rates.

For example, one bond has a fixed rate and another floating. From the point of view of the floating rate player, the swap is regarded as a long position in a fixed rate bond and a short position in a floating rate bond. Its value is $B_{fix} - B_{fl}$, where B_{fix} is the net present value of the fixed rate bond (corresponding to payments that are made) and B_{fl} is the net present value of the floating rate bond (corresponding to payments that are received). Respectively, the exact opposite is regarded from the point of view of the fixed rate player and its value is $B_{fl} - B_{fix}$. The net present value of $B_{fix} = \sum_{i=1}^{n} ke^{-r_i t_i} + Le^{-r_n t_n}$ and $B_{fl} = k^* e^{-r_1 t_1} + Le^{-r_1 t_1}$, where L is the notional principal and k, k^* the cash flows of each bond. (Hull, 2008).

In the second way of valuation, the swap can be characterized as a portfolio of forward rate agreements. The first exchange of payments is known at the time the swap is negotiated and the remaining are considered as forward rate agreements. Its value is $(k - k^*)e^{-r_1 t_1} + \sum_{i=2}^{n}(k - 0{,}5\, \hat{r_i} L)e^{-r_i t_i}$.

Variations on the standard interest rate swap

These types involve variations to the structure described above. Instead of LIBOR, there is another type of rate such as Euribor, or the commercial paper rate. Sometimes floating for floating interest rates swaps are negotiated, with different types of reference rates. These are called the *basis swaps*. They can be valued using the "assume that forward rates will be realized" approach, but a zero curve other than LIBOR is necessary to calculate future cash flows on the assumption that forward rates are realized. The cash flows are discounted at LIBOR.

Amortizing swap, where the principal reduces in a predetermined way and *step - up swap* where the principal increases in a predetermined way.

Forward swaps, where the parties do not begin to exchange interest payments until a future date. Sometimes the principal applied to fixed payments is different from the principal applied to floating payments.

In a *LIBOR in arrears swap*, the reference rate observed on a payment date is used to calculate the payment on that date and not on the next. If the reset dates in the swap are t_i for $i = 0,1,....,n$ with $\tau_i = t_{i+1} - t_i$, R_i is the reference rate for the period between t_i and t_{i+1}, F_i is the forward value of R_i and σ_i is the volatility of this forward rate, the payment on the floating side of this swap at time t_i is based on R_i rather than R_{i-1}. It must be noted that wherever nonlinear functions occur, the expected future rate or yield does not equal the forward rate or yield, although future and forward prices are equal. In these cases, a *convexity adjustment* must be made. A swap rate is a par yield. In order to calculate a convexity adjustment, an approximation must be made, by assuming that the N – year swap rate at time T equals the yield at that time on an N – year bond with a coupon equal to today's forward swap rate. It is also noted that volatility's value is typically implied from caplet prices.

In this type of swap, is necessary to make a convexity adjustment to the forward rate when the payment is valued. The valuation is based on the assumption that the floating rate paid is not F_i, but $F_i + \frac{F_i^2 \sigma_i^2 \tau_i t_i}{1+F_i \tau_i}$.

In a *constant maturity swap*, or *CMS*, the floating rate equals the swap rate for a swap with a certain life (floating rate is exchanged for a swap rate). For example, the floating payments might be made every 6 months at a rate equal to the 5 year swap rate. Usually there is a lag so that the payment on a particular date is equal to the swap rate observed on the previous payment date. If rates are set at times t_0, t_1, t_2,...., payments are made at times t_1, t_2, t_3,.... and L is the notional principal, the floating payment at time t_{i+1} is $\tau_i L S_i$, where $\tau_i = t_{i+1} - t_i$ and S_i is the swap rate at time t_i. If y_i is the forward value of the swap rate S_i, for the payment valuation at time t_{i+1}, two adjustments must be made to the forward swap rate, so that the realized swap rate is assumed to be $y_i - \frac{1}{2} y_i^2 \sigma_{y,i}^2 t_i \frac{G_i''(y_i)}{G_i'(y_i)} - \frac{y_i \tau_i F_i \rho_i \sigma_{y,i} \sigma_{F,i} t_i}{1+F_i \tau_i}$, rather than y_i. In this equation, $\sigma_{y,i}$ is the volatility of the forward swap rate, F_i is the current forward interest rate between times t_i and t_{i+1}, $\sigma_{F,i}$ is the volatility of this forward rate and ρ_i is the correlation between the forward swap rate and the forward interest rate. $G_i(x)$ is the price at time t_i of a bond as a function of its yield x. The bond pays coupons at rate y_i and has the same life and payment frequency as the swap from which the CMS rate is calculated. $G'_i(x)$ and $G''_i(x)$ are the first and second partial derivatives of G_i with respect to x. The volatilities $\sigma_{y,i}$ can be implied from swaptions. The volatilities $\sigma_{F,i}$ can be implied from caplet prices. The correlation ρ_i can be estimated from historical data. This equation involves a convexity and a *timing adjustment*. The term $-\frac{1}{2} y_i^2 \sigma_{y,i}^2 t_i \frac{G_i''(y_i)}{G_i'(y_i)}$ is a convexity adjustment, based on the assumption that the swap rate S_i leads to only one payment at time t_i rather than to an annuity of payments. The term $-\frac{y_i \tau_i F_i \rho_i \sigma_{y,i} \sigma_{F,i} t_i}{1+F_i \tau_i}$ is a timing adjustment for the fact that the payment calculated from S_i is made at time t_{i+1} rather than t_i. Timing adjustments are made when a variable (S_i in the above case) is observed at time $T(t_i)$ but the payoff occurs at a later time $T^*(t_{i+1})$.

A *constant maturity Treasury swap (CMT)* works similarly to a CMS except that the floating rate is the yield on a particular Treasury bond with a specified life. The valuation is the same with S_i defined as the par yield on the Treasury bond. (Jamshidian, 1997, Hull, 2008).

In a *compounding swap*, interest on one or both sides is compounded forward to the end of the swap's life according to preagreed rules and there is only one payment date at the end.

2.5 Currency SWAPS

In this case the two parties exchange cash flows in different currencies with a fixed rate.

When domestic currency is received and foreign currency is paid, the value is $B_l - S_0B_f$, where B_l is the bond in local currency B_f is the bond in foreign currency and S_0 is the spot exchange rate. When local currency is paid and foreign currency is received the formula is $S_0B_f - B_l$.

In the second way of valuation, each exchange of payments in a fixed for fixed currency swap is a forward foreign exchange contract and can be valued by assuming that forward exchange rates are realized (Hull, 2008).

Other currency swaps

There are also fixed for floating rate currency swaps which are a combination of a fixed for floating interest rate swap and a fixed for fixed currency swap, also known as *cross - currency interest rate swaps*.

In a *floating for floating currency swap*, a floating rate in one currency is exchanged with a floating rate in another. For example the USD LIBOR with the GBP LIBOR.

Both types can be valued using the "assume that forward rates will be realized" rule. Future reference rates in each currency are assumed to equal today's forward rates, so cash flows can be determined.

In a *differential swap, or diff swap* or *quanto*, a floating interest rate observed in one currency is applied to a principal amount in another currency. Essentially, the interest rates are associated with two different currencies yet the actual interest payments are denominated in a single currency and a *quanto adjustment* must be made (Wei, 1994). Suppose that LIBOR for the period between t_i and t_{i+1} in currency Y is applied to a principal in currency X with a payment at time t_{i+1}. Define V_i as the forward interest rate between t_i and t_{i+1} in currency Y and W_i as the forward exchange rate for a contract with maturity t_{i+1} (expressed as the number of units of currency Y that equal one unit of currency X). If LIBOR in currency Y was applied to a principal in currency Y, the cash flows at time t_{i+1} would be valued on the assumption that LIBOR at time t_i equals V_i. But in this case, a correction in the valuation of cash flow must be made on the assumption that LIBOR equals to $V_i + V_i\rho_i\sigma_{w,i}\sigma_{V,i}t_i$, where $\sigma_{V,i}$ is the volatility of V_i, $\sigma_{w,i}$ is the volatility of W_i and ρ_i is the correlation between V_i and W_i.

2.6 Credit Default SWAPS

These are contracts where the payoff depends on the credit worthiness of one or more companies or countries. Essentially, the buyer buys protection from a possible credit event of a reference entity. They can be categorized as *single name,* where the payoff depends on what happens to one company or country and *multi name*, where the payoff depends on more than one underlying reference credit.

The most popular single name credit derivative is a *Credit Default Swap*, or *CDS*, where the buyer makes periodic payments and the seller pays only in case of a credit event of the reference entity. The payoff is usually the difference between the face value of a bond issued by the reference entity and its value immediately after a default. Their valuation can be achieved by the present value of both the expected payments and expected payoff in a risk – neutral world, multiplied by the probabilities of payments and default to be realized respectively. The recovery rate is also embedded to the expected payoff.

In the case of a default at time *t*, the payoff is $L - RL[1 + A(t)] = L[1 - R - A(t)]$ where L is the notional principal, R is the recovery rate, and $A(t)$ is the accrued interest on the reference obligation at time *t* as a percent of its face value. (Hull, White, 2000).

A *binary credit default swap* is structured similarly except that the payoff is a fixed amount.

There are also *basket credit default swaps*, where there are a number of reference entities. *Add - up basket CDS* provides a payoff when any of the entities default and *Kth - to - default CDS* provides a payoff only when the Kth default occurs.

In a *contingent credit default swap,* the payoff requires both a credit default and an additional trigger, such as a credit event on another reference entity, or a specified movement in some market variable.

In a *dynamic credit default swap*, the notional amount determining the payoff is linked to the mark – to – market value of a swaps portfolio (Hull, White, 2000).

A *total return swap* is an agreement to exchange the total return on a bond, or any portfolio of assets, for an interest rate (LIBOR) plus a spread. Total return includes coupons, interest and the gain or loss on the asset over the swap's life.

Collateralized debt obligations or *CDOs,* are a type of *asset - backed securities* (created from a portfolio of bonds, loans or other financial assets). This is an instrument where its creator acquires a portfolio of bonds, passes them to a special purpose vehicle (SPV or trust, or conduit), which passes the income generated by the portfolio to a series of tranches. The bonds income is firstly used to provide the promised return to the most senior tranche, then to the next most senior tranche and so on. The structure is designed so that most senior tranche is rated AAA. The most junior (equity) tranche is sometimes retained by the CDO arranger. The objective of the CDO originator is to make money by selling the tranches to investors higher than the amount paid for the bonds.

Since a long position in a corporate bond has essentially the same risk as a short position in the corresponding credit default swap, an alternative way of creating CDOs is to form a portfolio consisting of short positions in credit default swaps. Credit risks are then passed on to tranches. This is called a *synthetic CDO*, in which default losses are allocated to tranches. The value of the tranche is $C - sA - sB$, where $C = \sum_{j=1}^{m}(E_{j-1} - E_j)\upsilon(0.5\tau_{j-1} + 0.5\tau_j)$, $A = \sum_{j-1}^{m}(\tau_j - \tau_{j-1})E_j\upsilon(\tau_j)$, $B = \sum_{j-1}^{m} 0.5(\tau_j - \tau_{j-1})(E_{j-1} - E_j)\upsilon(0.5\tau_{j-1} + 0.5\tau_j)$, E_j is the expected tranche principal at time τ_j, $\upsilon_{(\tau)}$ is the present value of 1 unit of currency received at time τ, and s is the spread on a particular tranche per year.

In general, the time to default is estimated by using the Gaussian One Factor method. In the standard market model, it is assumed that the time to default distribution is the same for all portfolio companies. This is a homogeneous model, but a heterogeneous model can also be used, assuming that each company has a different default probability. Since this is more complicated, different numerical procedures must be used such as those described in Hull and White (2004).

2.7 Equity SWAPS

This is an agreement to exchange the total return (both dividends and capital gains) realized on a share of stock or an equity index for either a fixed or a floating interest rate. These swaps can be used by portfolio managers to convert returns from a fixed or floating investment to returns from a share or an equity index investment and vice versa, or to increase – reduce an exposure to a share or an index without buying or selling stocks. The reference rate cash flow can be valued easily. The value of the equity cash flow is LE/E_0, where L is the principal, E is the current value of the index and E_0 is its value at the last payment date (Chance, Rich, 1998).

2.8 Commodity SWAPS

These are in essence, a series of forward contracts on a commodity with different maturity dates and the same delivery prices. At least one stream of payments is calculated on the basis of a commodity price or index applied at given times to a commodity quantity. A floating price based on the underlying commodity is exchanged for a fixed price over a specified period (Henderson, 1989).

Their valuation is similar to a fixed for floating interest rate swap.

2.9 Other types of SWAPS

There are also some other types worth mentioning such as:

Swaps with embedded options

In these cases, options are embedded in a swap agreement.

In options on swaps or *swaptions*, one party has the right to enter into a swap at a future date where a predetermined fixed rate is exchanged for floating. A swaption provides benefits from favorable interest rate movements and protection from unfavorable interest rate movements. For a swaption valuation, it must be considered a case where the holder has the right to pay a rate s_K and receive LIBOR on a swap that will last n years, starting at T years. There are m payments per year under the swap and the notional principal is L. For the moment, day count convention effect which leads to slightly different fixed payments on each date, is ignored. Each fixed payment on the swap is the fixed rate times L/m. Suppose that the swap rate for an n – year swap starting at time T proves to be s_T. By comparing cash flows with the fixed rates s_T and s_K, it can be seen that the swaption payoff consists of a series of cash flows equal to $L/m \max(s_T - s_K, 0)$. The cash flows are received m times per year for the n years of the swap's life. Suppose that swap payment dates are $T_1, T_2, \ldots, T_{mn}$, measured in years from today. Each cash flow is the payoff from a call option on s_T with strike price s_K. Whereas a cap is a portfolio of options on interest rates, a swaption is a single option on the swap rate with repeated payoffs. The standard market model gives the value of a swaption where the holder has the right to pay s_K as $\sum_{i=1}^{mn} \frac{L}{m} P(0, T_i)[s_0 N(d_1) - s_K N(d_2)]$, where $d_1 = \frac{\ln(s_0/s_K) + \sigma^2 T/2}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$. s_0 is the forward swap rate at time zero calculated as $s_t = \frac{P(t-T_0)-P(t,T_N}{A_{(t)}}, A_{(t)} = \sum_{i=0}^{N-1}(T_{i+1} - T_i)P(t,T_{i+1})$. This is an extension of Black's model (which is a variation of BS, for bond prices, caplets, floorlets and swaptions valuation). The $\sum_{i=1}^{mn} P(0,T_i)$ term is the discount factor for the *mn* payments. Defining A as the value of a contract that pays *1/m* at times *Ti(1* $\leq i \leq$ *mn),* the swaption's value becomes $LA[s_0 N(d_1) - s_K N(d_2)]$, where $A = \frac{1}{m}\sum_{i=1}^{mn} P(0,T_i)$.

If the swaption gives the holder the right to receive a fixed rate of s_K instead of paying it, the swaption payoff becomes *L/m max* $(s_K - s_T, 0)$. This is a put option on s_t and its value becomes $LA[s_K N(-d_2) - s_0 N(-d_1)]$.

The above formulas can be made more precise by considering day count conventions. If α_i is the accrual fraction corresponding to the time period between T_{i-1} and T_i, the formulas are correct with the annuity factor A, defined as $A = \sum_{i=1}^{mn} a_\iota P(0,T_i)$. (Chance, Rich, 1998, Hull, 2008).

In an *accrual swap*, the interest on one side accrues only when the floating reference rate is in a certain range. For example, if the reference cutoff rate is R_K and payments are exchanged every τ years, there is a day i during the swap's life and t_i is the time until day i. Suppose that the τ – year reference rate on day i is R_i and interest accrues only when $R_i<R_K$. Define F_i as the forward value of R_i and σ_i as the F_i volatility (estimated from caplet volatilities). Using the usual lognormal assumption, the probability that the rate is greater than R_K in a forward risk – neutral world with respect to a zero – coupon bond maturing at time $t_i + \tau$ is $N(d_2)$, where $d_2 = \frac{\ln(F_i/R_K)-\sigma_i^2 t_i/2}{\sigma_i\sqrt{t_i}}$. The payoff from the binary option is realized at the swap payment date following day i. Suppose that this time is at time s_i. The probability that the rate is greater than R_K in a forward risk neutral world with respect to a zero – coupon bond maturing at time s_i is $N(d_2^*)$, where d_2^* is calculated as d_2, but with a small timing adjustment to F_i reflecting the difference between times $t_i + \tau$ and s_i. The binary option value corresponding to day i is $\frac{QL}{n_2}P(0,s_i)N(d_2^*)$. The total value of binary options is obtained by summing this formula for every day in swap's life. The timing adjustment (causing d_2 to be replaced by d_2^*) is so small that is frequently ignored in practice (Hull, 2008).

In a *cancelable swap*, one party has the option for termination on one or more payment dates. Termination is the same as entering into an offsetting (opposite) swap. If there is only one termination date, the cancelable swap is the same as a regular swap plus a position in a European swaption. If the swap can be terminated on a number of different dates, it is a regular swap plus a Bermudan – style swaption. Sometimes compounding swaps can be terminated on specified payment dates. These are called *cancelable compounding swaps*. On termination, both parties exchange compounded, up to the time of termination, cash flows.

In an *extendable swap*, one party has the option to extend the swap's life beyond the specified period.

In a *puttable swap*, one party has the option to terminate the swap early.

Variance swaps

This is an agreement to exchange the realized variance rate $\bar{V}$ for a prespicified rate. The variance rate is the square of volatility ($\bar{V} = \bar{\sigma}^2$). The variance rate can be replicated using a portfolio of put and call options. The valuation equation is $L_{var} = \left[\hat{E}(\bar{V}) - V_K\right]e^{-rT}$, where L_{var} is the notional principal and V_K is the fixed variance rate. For any value S^* of an asset price, the expected value of the average variance between times *0* and T is given by $\hat{E}(\bar{V}) = \frac{2}{T} ln\frac{F_0}{S^*} - \frac{2}{T}\left[\frac{F_0}{S^*} - 1\right] + \frac{2}{T}\left[\int_{K=0}^{S^*}\frac{1}{K^2}e^{rT}p(K)dK + \int_{K=S^*}^{\infty}\frac{1}{K^2}e^{rT}c(K)dK\right]$, where F_0 is the forward price and $c(K)$, $p(K)$, are call and put prices of European options, all maturing at time T.

Volatility swaps

In this case there is an agreement to exchange the realized volatility of an asset for a prespecified fixed volatility. There are a series of time periods. At the end of each period, one side pays a preagreed volatility σ_K, while the other side pays the historical volatility realized during this period $\hat{E}(\bar{\sigma})$ (symbolized as expected for valuation purposes). Both volatilities are multiplied by the same notional principal L_{vol} in calculated payments. The formula is $L_{vol} = \left[\hat{E}(\bar{\sigma}) - \sigma_K\right]e^{-RT}$, where $\hat{E}(\bar{\sigma}) = \sqrt{\hat{E}(\bar{V})}\left\{1 - \frac{1}{8}\left[\frac{var(\bar{V})}{\hat{E}(\bar{V})^2}\right]\right\}$. $var(\bar{V})$ is the variance of $\bar{V}$, so the valuation requires an estimate of the variance of the average variance rate during the contact's life.

2.10 Options: The Black – Scholes Model

The two main types are European options, which have one maturity date and American options that can be exercised at any time.

European bond options pricing formula is based on the standard market model. The assumption made is that the forward bond price has a constant volatility σ_B and this allows the use of Black's model which becomes:

$$c = P(0,T)[F_B N(d_1) - KN(d_2)],$$

$$p = P(0,T)[KN(-d_2) - F_B N(-d_1)] \ ,$$

where $d_1 = \frac{\ln(F_B/K)+\sigma_B^2 T/2)}{\sigma_B\sqrt{T}}$ and $d_2 = d_1 - \sigma_B\sqrt{T}$.

c, p are call and put prices, F_B is the forward bond price, K the strike price and T its time to maturity. $F_B = \frac{B_0 - I}{P(0,T)}$, where B_0 is the bond price at time zero and I the present value of the coupons paid during option's life.

The Black – Scholes pricing formulas for a European option on a non – dividend stock, as mentioned above, are:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$p = Ke^{-rT} N(-d_2) - \ S_0 N(-d_1) \text{ where}$$

$$d_1 = \frac{\ln(S_0/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

The variables c and p are the European call and put prices, S_o is the stock price at time zero, K is the strike price, r is the continuously compounded risk – free rate, σ is the stock price volatility and T is the time to maturity of the option.

The Black – Scholes model assumptions are (Poon, 2005):

1. Short sale is permitted with full use of proceeds.
2. No transaction costs or taxes. Securities are infinitely divisible.
3. No dividend before option maturity.
4. No arbitrage.
5. Continuous trading (so that portfolio rebalancing is done instantaneously).
6. Constant risk – free interest rate r.
7. Constant volatility σ.

In Black – Scholes model, the volatility of a stock price can be defined as the standard deviation of the return provided by the stock in 1 year when the return is expressed using continuous compounding. Volatility is estimated from historical data or more sophisticated methods such as GARCH models. However, this parameter cannot be directly observed in the formulas. In practice, traders usually work with implied volatilities. These are the volatilities implied by option (not stock) prices observed in the market. Unfortunately, it is not possible to invert the call equation so that the implied volatility σ is expressed as a function of S_0, K, r, T and c, according to Hull (2008). However, an iterative search procedure can be used to find σ, inserting different values of σ into the call equation until the correct value of c is found. DerivaGem can be also used to calculate implied volatilities. A similar procedure, in conjunction with binomial trees, can be used to find implied volatilities for American options.

The Black – Scholes model results can be extended to cover European options on dividend – paying stocks, by using the Black – Scholes formula with the stock price reduced by the present value of the dividends anticipated during the life of the option and the volatility equal to the volatility of the stock price net of the present value of these dividends.

Regarding American call options, in theory it is optimal to exercise them immediately before any ex – dividend date. In practice, it is often only necessary to consider the final ex – dividend date. Black suggested setting the American call price equal to the greater of two European call prices. The first European call expires at the same time as the American call and the second expires immediately prior to the final ex – dividend date.

Afore mentioned formulas can be applied to options in stock indices and currencies, since both of them are analogous to stocks paying dividend yields.

Future options can be priced either using binomial trees or formulas similar to those of BS method. A future price behaves in the same way as a stock that provides a dividend yield equal to the risk – free rate. So the formulas can be applied if the stock price is replaced by the futures price and set the dividend yield equal to the risk – free interest rate.

The Black – Scholes model and its extensions assume that volatility is constant and probability distribution of the underlying asset at any given time is lognormal (Black, 1989). In practice, traders assume that probability distribution of an equity price has a heavier left tail and a less heavy right tail than lognormal distribution. They also assume that probability distribution of an exchange rate has a heavier right tail and a heavier left tail than lognormal distribution (Kon, 1984). To allow for nonlognormality, traders use volatility smiles which define the relationship between the implied volatility of an option and its strike price. For equity options, the volatility smile tends to be downward sloping (Bakshi et al., 2000). For foreign currency options, the volatility smile is U – shaped (Xu, Taylor, 1996). They also use a volatility term structure often. The implied volatility of an option then depends on its life. When volatility smiles and volatility term structures are combined, they produce a volatility surface which defines implied volatility as a function of both the strike price and time to maturity (Hull, 2008).

2.11 Explanations about Volatility and other Pricing Methods

If the Black – Scholes method is the correct option pricing model, then there can be only one Black – Scholes implied volatility regardless of the option's strike price, or whether the option is a call or a put. The Black – Scholes implied volatility smile (also called smirk) and skew are clear evidence that market option prices are not priced according to the BS formula. Traders' practices were discussed in the previous paragraph The important question is about the relationship between Black – Scholes implied volatility and the true volatility.

The Black – Scholes option price is a positive function of the volatility of the underlying asset. If Black – Scholes model is correct, then market option price should be the same as the Black – Scholes option price and the Black – Scholes implied volatility derived from market option price will be the same as true volatility. If the Black – Scholes price is incorrect and lower than market price, then Black – Scholes implied volatility overstates true volatility. The reverse is true if the Black – Scholes price is higher than market price. The problem becomes more complicated by the fact that implied volatility differs across strike prices. All theories that predict the relationship between Black – Scholes model and market option prices are contingent on the proposed alternative option pricing model or the proposed alternative pricing dynamic being correct. Given that the Black – Scholes implied volatility has been proven overwhelmingly to be the best volatility forecast, as presented in chapter 1, it will be useful to understand the links between Black – Scholes implied and true volatilities.

There have been a lot of efforts made to solve the Black – Scholes anomalies. The *stochastic volatility option pricing model* is one of the most important extensions of Black – Scholes model. This model is motivated by the widespread evidence that volatility is stochastic and the fact that risky asset returns distribution has longer tail than that of normal distribution. An SV model with correlated price and volatility innovations can address both anomalies. The model was developed roughly over a decade with contributions from Johnson and Shanno (1987), Wiggins (1987), Hull and White (1987, 1988), Scott (1987), Stein and Stein (1991) and Heston (1993). In the last, a closed form solution was derived using the characteristic function of the price distribution.

The Heston stochastic volatility option pricing model

Heston (1993) specifies the stock price and volatility price processes as follows: $dS_t = \mu S d_t + \sqrt{\upsilon_t} S dz_{s,t}$, $d\upsilon_t = \kappa[\theta - \upsilon_t]d_t + \sigma_v \sqrt{\upsilon_t} S dz_{\upsilon,t}$, where υ_t is the instantaneous variance, κ is the speed of mean reversion, θ is the long run level of volatility and σ_v is the volatility of volatility. The two Wiener processes, $dz_{s,t}$, $dz_{\upsilon,t}$ have constant correlation ρ. The assumption that consumption growth has a constant correlation with spot – asset returns generates a risk premium proportional to υ_t. Given the volatility risk premium, the risk – neutral volatility process can be written as $d\upsilon_t = \kappa^*[\theta^* - \upsilon_t]d_t + \sigma_v \sqrt{\upsilon_t} dz^*_{\upsilon,t}$, where $\kappa^* = \kappa + \lambda$ and $\theta^* = \kappa\theta/(\kappa + \lambda)$. Here λ is the market price of (volatility) risk, κ^* is the risk – neutral mean reverting parameter and θ^* is the risk – neutral long run level of volatility. The pricing formula for European calls is $c = SP_1 - Ke^{-r(T-t)}P_2$, $P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{e^{-i\varphi lnK} f_i}{i\varphi}\right] d\varphi$, and $f_i = exp\{C(T-t;\varphi) + D(T-t;\varphi)\upsilon + i\varphi x\}$, for $j = 1{,}2$. For further information, see Heston (1993).

According to Poon (2005), empirical findings from the model show that the thick tail and nonsymmetrical distribution could be a result of volatility being stochastic.

2.12 Exotic Options Pricing Methods

Exotics have been developed by financial engineers for a number of reasons. Sometimes they meet a genuine hedging need in the market or their design reflect a view on potential future movements in particular market variables; other times there are tax, accounting, legal or regulatory reasons. Occasionally, they are designed to appear more attractive than they really are to attract unwary investors. Although they are usually a relatively small part of a portfolio their importance is high, since they are generally much more profitable.

Hull (2008) categorizes exotics similarly with a series of articles written by Reiner and Rubinstein for Risk magazine during 1991 – 1992. In brief, the main categories are:

Packages

A portfolio consisting of plain vanilla products, forward contracts, cash and the underlying asset itself (i.e. bull spreads, bear spreads etc).

Nonstandard American Options

The American options traded in the OTC markets which have nonstandard features. For example, early exercise is restricted to certain dates (*Bermuda options*), or the strike price changes during the life of the option. These options are usually valued using binomial trees. At each node, the test for early exercise is adjusted to reflect the terms of the option.

Forward Start Options

Options that will start at some time in the future (i.e. some employee stock options). Using risk – neutral valuation, the formula is $e^{-rT_1}\hat{E}\left[c\,\frac{S_1}{S_0}\right]$

Compound options

These are options on options, such as a call on a call. This type has two strike prices and two exercise dates.

Chooser Options

Where the holder can choose whether the option is a call or a put, after a specified period of time.

Barrier Options

Where the payoff depends on whether the underlying asset's price reaches a certain level during a certain period of time. They can be classified as either knock – out or knock – in options. The first ceases to exist when the underlying asset price reaches a certain barrier, while the latter comes into existence only when underlying asset price reaches a certain barrier.

Binary Options

These options have with discontinuous payoffs such as a cash– or– nothing call which pays off nothing if the asset price ends up below the strike price at time T and pays a fixed amount Q, if it ends up above the strike price.

Lookback Options

In this type, payoffs depend on the maximum and minimum asset price reached during the life of the option. The payoff from a *floating lookback call* is the amount that the final asset price exceeds the minimum asset price and the put payoff is the amount of the maximum asset price achieved, minus the final asset price.

Shout Options

Where the holder can "shout" to the writer at one time during its life. At the end of option's life, the holder receives either the usual payoff from a European option or the intrinsic value at the time of the shout, whichever is greater. This type is valued by constructing a binomial or trinomial tree, therefore the procedure is similar to this for valuing an American option. At each node, the value of the option if the holder shouts and the value if the holder does not shout are calculated.

Asian Options

Where the payoff depends on the average price of the underlying asset during at least some part of the option's life.

Options to exchange one asset for another (or exchange options) and

Options involving several assets (or rainbow options)

Which involve two or more risky assets such as a basket options where the payoff depends on the value of a portfolio (basket) of assets. This can be valued with Monte Carlo simulation by assuming that assets follow correlated geometric Brownian motion processes.

From the aforementioned is obvious that derivatives are only limited by the imagination of financial engineers and the desire of corporate treasurers and fund managers for exotic structures (Hull, 2008).

Chapter 3: Volatility Indices

Introduction

A volatility index is a financial instrument that tracks the value of implied volatility of other derivative securities. In essence, it is a key measure of market expectations of near term – volatility, conveyed by listed option prices. Futures and options contracts are available on some of these indices (www.cboe.com).

The one that started it all is the Chicago Board Options Exchange (CBOE) which is today the world's largest options exchange with annual trading volume that hovered around one billion contracts at the end of 2007.

On April 26, 1973, in a celebration of the 125th birthday of the Chicago Board of Trade, CBOE introduced the trading of the first option contracts.

In 1993, CBOE became the first organized exchange that officially introduced an *implied volatility index*, the renowned *VIX*. The CBOE Volatility Index (VIX), often referred to as the *fear index* or the *fear gauge*, is the world's most widely followed barometer of investor sentiment and market volatility.

On March 24, 2004, CBOE introduced the first exchange – traded VIX futures contract. Two years later in February 2006, CBOE launched VIX options, the most successful new product in Exchange history. In less than five years, the combined trading activity in VIX options and futures has grown to more than 100,000 contracts per day.

Nowadays, CBOE calculates and updates the values of more than 20 volatility indices, designed to measure the 30 – day implied volatility of different securities. It also offers options on over 2,200 companies, 22 stock indices and 140 exchange traded funds (www.cboe.com).

VIX very quickly became the benchmark risk measure and the most widely followed volatility barometer. Following the extremely successful example of CBOE, other exchanges across the world developed their own respective indices. Indicatively, Deutsche Börse introduced in 1994 the VDAX and French Marchė des Options Nėgociables de Paris (MONEP) introduced, in 1997, two implied volatility indices, VX1 and VX6.

In 2003, CBOE re – launched VIX (the *new VIX*), changing its calculation methodology.

The latest additions to the implied volatility indices family is the *CBOE Interest Rate Volatility Index (SRVX)*, introduced on June 18, 2012 and a benchmark, volatility – related, index introduced on November 30, 2012, the *Low Volatility Index (LOVOL)* (www.cboe.com).

Although there are innumerable papers, from acclaimed economists and other scientists dealing with various subjects of both volatility and volatility indices there is no literature review in the field of volatility indices.

There are only two recent remarkably integrated papers: the first is from Siriopoulos and Fassas (2009), which tests and documents the information content, regarding both the realized volatility and the returns of the underlying equity index, of all publicly available implied volatility indices. They also investigate the volatility indices forecasting performance and the relationship between implied volatility changes and the corresponding underlying equity index returns.

The second paper is from Lopez and Navarro (2012) which deals with the aforementioned subjects and presents some of the volatility indices applications.

However, there is a significant number of volatility indices, not presented in the above studies and many academic papers expanding research in the field of volatility indices.

The author's attempt is to enrich the already mentioned issues and to present more fields of study relevant to volatility indices, thus creating for the first time, a complete and comprehensive literature review. It should be mentioned that this thesis does not include studies where volatility indices are used exclusively as data. For example, there are infinite studies comparing HISVOL / ARCH / SV models forecasting performance and use volatility indices prices. Others use this data for explanation purposes or for introducing new financial, stochastic, bivariate or other models. Indicatively mentioned studies excluded are: Fernandes et al., 2007, Ahoniemi, 2008 and Degiannakis, 2008.

In the following pages, the features of all publicly available volatility indices are presented, in addition with the historical background, their calculation methods, their uses and applications, via published articles. It is also noted that some unpublished papers are included, that are either of significant importance and mentioned in many other studies, or written recently from prestigious economists and are likely to be published in the next years.

3.1 The original VIX (VXO)

According to Carr, Madan (1998), Gastineau in 1977, was the first academic who created a volatility index based on option market prices. In the following years, several researchers followed Gastineau’s paradigm (indicatively mentioned: Galai, 1979, Cox and Rubinstein, 1985, Brenner and Galai, 1989), but it was the exceptional work of Whaley (1993) that basically established the field of indices based on the volatility implied by option prices. In particular, Whaley developed an innovative methodology of calculating an implied volatility index, since he was the first to consider index options, instead of individual stock options, emphasizing in that way on the systematic, non – diversifiable risk. Furthermore, he used both call and put options, in contrast to the previous works that solely used call options, increasing in that way the information content captured by the index (Fleming et al., 1995).

In 1993, the Chicago Board Options Exchange Volatility Index, (CBOE VIX) was introduced, which was originally based on the methodology of Whaley and very quickly became the benchmark risk measure of the US equity market. Designed to measure the market's expectation of 30 – day volatility implied by at–the–money S&P 100 Index (OEX) option prices, VIX soon became the most followed benchmark for U.S. stock market volatility.

The calculation of the original VIX (from now on referred to the old VIX as VXO, which is the ticker symbol that CBOE currently uses) is described in detail by Whaley (1993, 2000) and Fleming et al. (1995). It is based on the Black – Scholes – Merton option valuation formula and it is constructed from the volatility implied by four pairs of call and put options on the S&P100. Specifically, the index is calculated as a weighted average of two near–the–money (one above and one below the at–the–money strike price) call options and two near–the–money put options of the nearby monthly expiry and respectively four near–the–money options (two calls and two puts) of the second nearby monthly expiry.

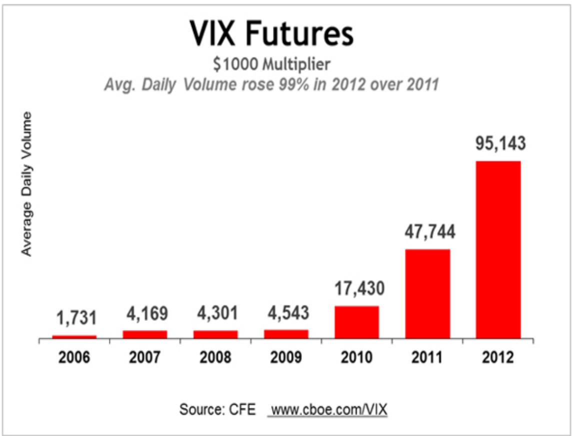
According to the Black – Scholes model, the theoretical price of an option is a function of the spot underlying price, the strike price, the time to maturity, the interest rate and the volatility of the underlying asset during the maturity of the option. Since many options are listed in organized exchanges they have a market price determined by market forces of demand and supply, besides their theoretical price. Therefore, assuming that the market price of an option is its "fair" price, which excludes possible arbitrage movements and since the other inputs are objectively known, using the Black – Scholes formula, the volatility that market participants are expecting for the period until the expiration of the option can be implied. The calculated implied volatility of each option is weighted is such a way that VXO represents the implied volatility of a hypothetical at–the–money option on S&P100 with a constant maturity of 30 – calendar days (22 trading days). The formula of the CBOE original volatility index (VXO) is the following: $VXO = \sigma_1 \left(\frac{N_{t2}-22}{N_{t2}-N_{t1}}\right) + \sigma_2 \left(\frac{22-N_{t1}}{N_{t2}-N_{t1}}\right)$, where $\sigma_1 = \sigma_1^{X_l} \left(\frac{X_u-S}{X_u-X_l}\right) + \sigma_1^{X_u} \left(\frac{S-X_l}{X_u-X_l}\right)$ and $\sigma_2 = \sigma_2^{X_l} \left(\frac{X_u-S}{X_u-X_l}\right) + \sigma_1^{X_u} \left(\frac{S-X_l}{X_u-X_l}\right)$. S is the spot underlying price, X_l is the lower exercise price, X_u is the upper exercise price, N_{t1} is the number of trading days to expiration of the nearby contract and N_{t2} is the number of trading days of the second contract. Although the new volatility index was officially introduced in 1993, minute by minute values (dating back to the beginning of January 1986), were calculated using index option prices, so comparisons between VXO levels and historical volatility levels could be facilitated. (Siriopoulos and Fassas, 2009, www.cboe.com).

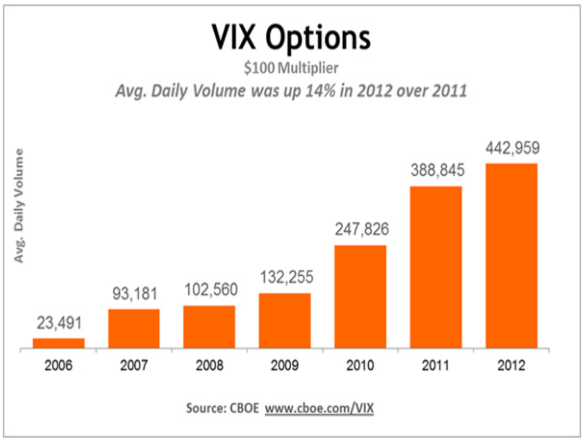
Shortly after the successful launch of VXO, two European markets followed its paradigm. In 1994, Deutsche Börse launched the German volatility index *VDAX*, which looked ahead 45 days, while the Marché des Options Négociables de Paris (MONEP) started to calculate two volatility indices, in 1997: *VX1* and *VX6*, for two constant terms to maturity (31 and 185 calendar days, respectively).

The negative correlation of volatility to stock market returns is well documented and suggests a diversification benefit to including volatility in an investment portfolio. VIX futures and options are designed to deliver pure volatility exposure in a single, efficient package. CBOE Features Exchange (CFE) provides a continuous, liquid and transparent market for VIX products that are available to all investors from the smallest retail trader to the largest institutional money managers and hedge funds (www.cboe.com).

Figure 3.1 presents the average daily volume growth of VIX features and options, since 2006.

Figure 3.1: Average daily volume growth of VIX features and options

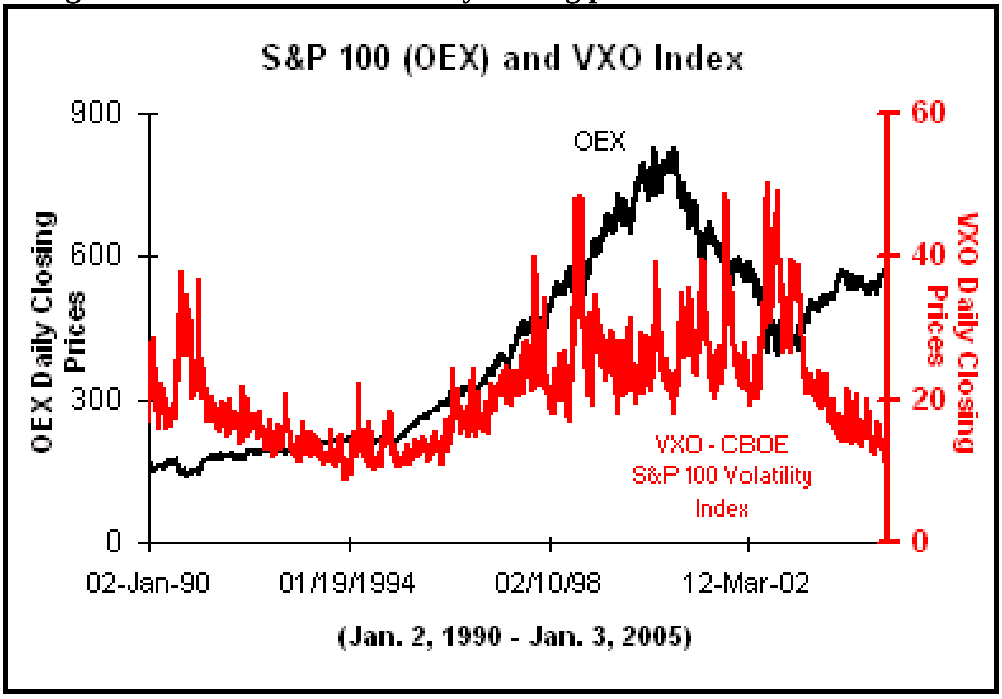




Perhaps one of the most valuable features of VIX is the existence of more than 20 years of historical prices. This extensive data set provides investors with a useful perspective of how option prices have behaved in response to a variety of market conditions. Price history for the original CBOE Volatility Index (Ticker – “VXO”) based on OEX options is available from 1986 to the present. CBOE has created a similar historical record for the new VIX dating back to 1990 so that investors can compare the new VIX with VXO, which reflects information about the volatility “skew” or “smile.” (Siriopoulos and Fassas, 2009, www.cboe.com).

Figure 3.2 presents VXO and OEX daily closing prices for a 15 – year period.

Figure 3.2: VXO and OEX daily closing prices



(Source: CBOE)

Despite of its success, the original VIX suffered from two problems: it was depended to the Black – Scholes pricing method and it lacked a theoretical interpretation with implications from a practical point of view (Lopez and Navarro, 2012).

3.2 *The new VIX*

Ten years later, in September 2003, CBOE together with Goldman Sachs updated the VIX, using a new methodology for pricing variance swaps that was essentially based on the work of Demeterfi et al. (1999). The concept was that the new VIX should reflect a new way of measuring expected volatility, one that continues to be widely used by financial theorists, risk managers and volatility traders alike. The new VIX is based on the S&P 500 Index (SPXSM), the core index for U.S. equities, and estimates expected volatility by averaging the weighted prices of SPX puts and calls over a wide range of strike prices. By supplying a script for replicating volatility exposure with a portfolio of SPX options, this new methodology transformed VIX from an abstract concept into a practical standard for trading and hedging volatility (www.cboe.com).

The new VIX has three essential differences compared to the old VXO. To begin with, the two indices have different underlying assets: in particular, the new VIX calculation is based on options written on the S&P 500 index, while VXO uses options on the S&P 100. The rationale of the change was that, although the two indices are well correlated, the S&P 500 is considered to be the benchmark of the U.S. stock market return. The second difference is that the new VIX, uses information from out–of–the–money call and put options of a wider range of strike prices, whereas VXO uses only eight at–and near–the–money options. The third difference is that the two indices use a different method of calculation of the implied volatility. The new VIX is independent of any model (model – free), whereas VXO is a weighted average of the Black – Scholes implied volatility. (Siriopoulos and Fassas, 2009, Poon, 2005).

Up till then, volatility indices were computed by using the implied volatilities derived from option market prices based on an option pricing model. However, the model could be misinterpreted given the assumptions that underlie its derivation. Particularly, it is documented that the implied volatility of options with the same to maturity varies across different strike prices giving rise to volatility smiles and skews (Rubinstein, 1994, Dumas et al., 1998, Foresi and Wu, 2005), while the Black – Scholes model assumes constant volatility (Hull, 2008), as described in chapter 2.

The new VIX is based on the concept of the fair delivery value of the future realized variance of a variance swap as proposed by Demeterfi et al. in 1999. A variance swap is a forward contract on the realized variance of the underlying index over the life of the contract. At maturity, the holder of the contract pays a fixed variance rate (the variance swap rate), determined at the contract initiation, and in exchange receives the realized variance rate of the underlying index. The difference between these two rates is multiplied by the notional amount of the swap. Since the contract has zero value at the time of entry, by no–arbitrage, the variance swap rate equals the risk – neutral expected value of the realized variance. Thus, the variance swap rate is the fair delivery value of future realized variance.

The concept of a model – free implied variance appeared first in Dupire (1994) and Neuberger (1994). Subsequently, it was enhanced by various researchers, including Carr and Madan (1998), Demeterfi et al. (1999a, 1999b), Britten – Jones and Neuberger (2000), Carr and Wu (2005) and Jiang and Tian (2005, 2007). The calculation of the new VIX is based on the work of Demeterfi et al. (1999), who derive a formula to replicate the variance swap rate by means of a static position in a portfolio of European call and put options over a wide range of strikes (not only near–the–money options) and a dynamic position in futures trading. Jiang and Tian (2005, 2007) prove that this formula is theoretically equivalent to the model – free implied variance formulated in Britten – Jones and Neuberger (2000). It is model – free since it involves using directly option prices instead of their respective implied volatilities and it is consistent with a very general class of continuous asset price processes. Britten – Jones and Neuberger (2000) and Jiang and Tian (2005) also showed that the information content of the model – free methodology volatility is superior to that of the Black – Scholes volatility.

CBOE uses a discrete approximation of this formula since it involves integrating option prices over an infinite range of strike prices. Then, VIX is calculated as the squared root of the weighted average of two variance swap rates computed by using near – term and next – term S&P 500 options (that is, options with less and more than 30 days to expiration, respectively). Thus, the constructed volatility index has a nice theoretical interpretation under the new algorithm: once it is squared, it approximates the variance swap rate. Furthermore, since VIX is based on a wide range of option prices, VIX represents a market consensus view of the expected volatility of the S&P 500. Carr and Lee (2003) also provide a theoretical interpretation for VXO. They show that the at–the–money implied volatility approximates the volatility swap rate of a volatility swap. However, unlike variance swaps, the payoff on a volatility swap is notoriously difficult to replicate. Given the explicit meaning of volatility indices constructed according to the new methodology and its direct link to a portfolio of options, the launch of derivative products (futures and options) on volatility indices becomes the natural next step. In this way, VIX futures began trading on the CBOE Futures Exchange (CFE) in 2004, while VIX options traded on the CBOE were launched two years later (www.cboe.com).

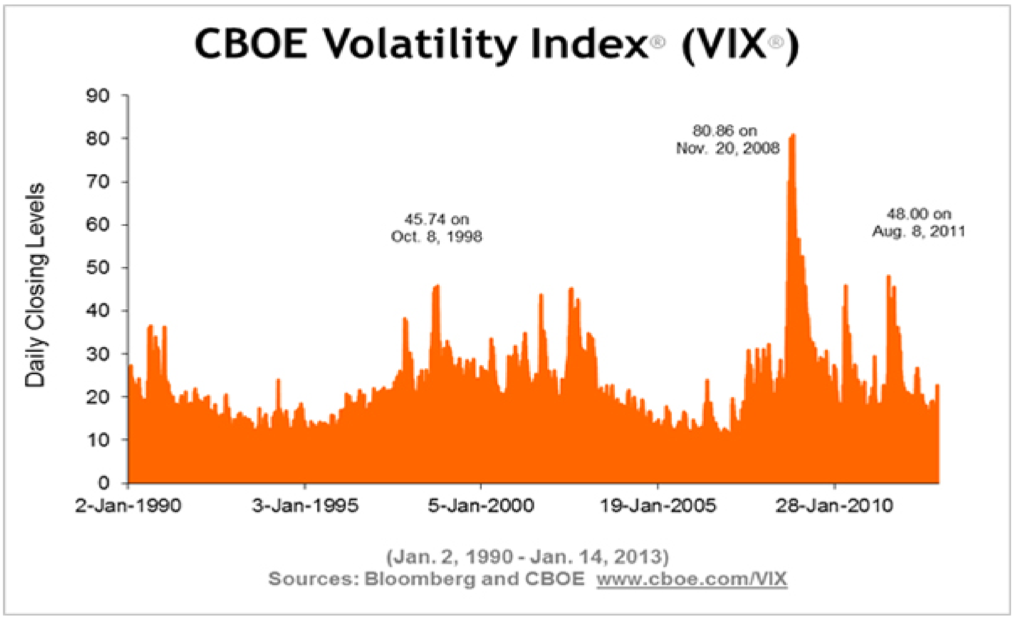
In essence, the new VIX is calculated directly from market observables, which are independent of any pricing model, such as the market prices of call and put options and interest rates. The new VIX represents a model – free approximation of the one – month variance swap rate.

According to CBOE, the generalized calculation formula of the new VIX is: $VIX = 100\sqrt{\frac{2}{T}\sum \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left(\frac{F}{K_0} - 1\right)^2}$, where T is the time to expiration, or $T = \frac{(M_{Current\ day} + M_{Settlement\ day} + M_{Other\ days}}{Total\ number\ of\ minutes\ in\ a\ year}$. Note that $M_{Current\ day}$ = number of minutes until midnight of the current day, $M_{Settlement\ day}$ = number of minutes from midnight until 08:30 a.m. on SPX settlement day and $M_{Other\ days}$ = total minutes in the days between current day and settlement day. It is important mentioning that Siriopoulos and Fassas (2009) refer to $M_{Settlement\ day}$ as the number of minutes until 14:00 on settlement day. The author of this thesis follows the description of the CBOE official site (copyright 2009). F is the forward index level derived from index option prices, K_0 is the first strike below F, K_i is the strike price of i^{th}, out–of–the–money option; a call if $K_i > K_0$, a put if $K_i < K_0$ and both put and call if $K_i = K_0$. ΔK_i is the interval between strike prices – half the difference between the strike on either side of K_i meaning $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$ (ΔK for the lowest strike is simply the difference between the lowest strike and the next higher strike and vice versa). R is the risk – free interest rate to expiration and $Q(K_i)$ is the midpoint of the bid – spread for each option with strike K_i.

The first step in its calculation is to select the options to be used. The next step is to calculate volatility for both near – term and next – term options. The final step is to calculate the 30 day weighted average of σ_1^2 and σ_2^2, where $\sigma_1^2 = \frac{2}{T_1} \sum \frac{\Delta K_i}{K_i^2} e^{RT_1} Q(K_i) - \frac{1}{T_1} \left[\frac{F_1}{K_0}\right]^2$ and $\sigma_2^2 = \frac{2}{T_2} \sum \frac{\Delta K_i}{K_i^2} e^{RT_2} Q(K_i) - \frac{1}{T_2} \left[\frac{F_2}{K_0}\right]^2$. Then the square root of that value is taken and is multiplied by 100 to get the index: $VIX = 100 * \sqrt{\left\{T_1 \sigma_1^2 \left[\frac{N_{T2} - N_{30}}{N_{T2} - N_{T1}}\right] + T_2 \sigma_2^2 \left[\frac{N_{30} - N_{T1}}{N_{T2} - N_{T1}}\right]\right\} * \frac{N_{365}}{N_{30}}}$, where $N_{T1} =$ number of minutes to settlement of the near–term options, $N_{T2} =$ number of minutes to settlement of the next – term options, $N_{30} =$ number of minutes in 30 days and $N_{365} =$ number of minutes in 365 – day year (www.cboe.com).

Figure 3.3 demonstrates the VIX daily closing levels for a 23 – year period (1990 – 2013).

Figure 3.3: VIX daily closing prices



Reasons for the calculation changes

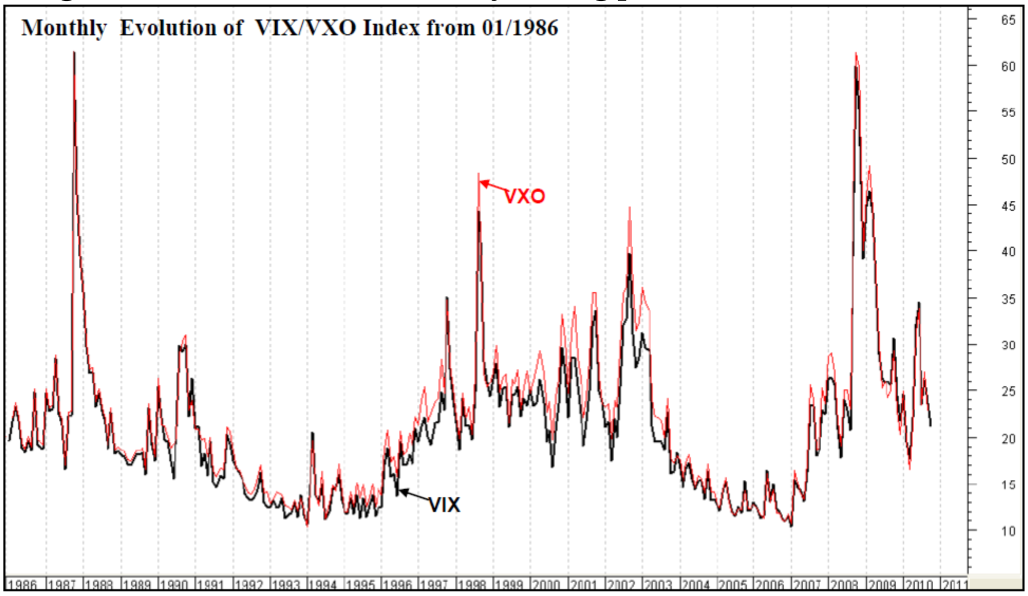
According to Poon (2005), there are a lot of reasons for this change: first of all the new index is hedgeable, while the old one is not. The new VIX can be replicated with a static portfolio of S&P 500 options or S&P 500 futures, so hedging is allowed. CBOE argues that the new VIX reflect information in a broader range of options rather than just the few at–and near–the–money options. More importantly, the new VIX is aiming to capture the information in the volatility skew. It is linked to the broader – based S&P 500 index (instead of the S&P 100 index), which is the primary index for most portfolio benchmarking so derivative products that are more closely linked to S&P 500 will facilitate risk management.

Similarities between VIX and VXO

Although the two volatility indices are compiled very differently, their statistical properties are very similar. The new VIX has a smaller mean and is more stable than the old VXO (Poon, 2005). As also shown in the chart below, the new VIX behaves very much like the original, tending to increase during stock market declines and decrease when the market advances.

Figure 3.4 demonstrates the VIX/VXO similarities in monthly prices for a 25 – year period (1986 – 2011).

Figure 3.4: VXO and VIX monthly closing prices



(Source: Bellini and de la Torre – Gallegos, 2011)

3.3 Other CBOE Indices

After the widespread success and acceptance of VIX, the number of volatility indices calculated by using a similar formula has noticeably increased.

CBOE calculates volatility indices on three other broad – based indexes representing different segments of the U.S. stock market:

CBOE Nasdaq–100 Volatility Index (VXN) based on Nasdaq–100 Index (NDX) options, launched in 2001,

CBOE DJIA Volatility Index (VXD) based on options on the Dow Jones Industrial Average (DJX), launched in 2005 and

CBOE Russell 2000 Volatility Index (RVX) based on Russell 2000 Index (RUT) options.

For each of these indices, the selection of component options and calculation are identical to the method detailed in VIX.

The *CBOE S&P 500 3–Month Volatility Index (VXV)* measures the market's expectation of 3–month volatility implied by SPX options that bracket a 93–day maturity. Comparing VIX and VXV provides investors with information about the SPX volatility term structure in the four near – term contract months (www.cboe.com).

In 2008, CBOE pioneered the use of the VIX methodology to estimate expected volatility of certain commodities and foreign currencies:

CBOE Crude Oil Volatility Index (OVX) based on United States Oil Fund, LP (USO) options, measuring the market's expectation of 30 – day volatility of crude oil prices,

CBOE Gold Volatility Index (GVZ) based on the, SPDR Gold Shares (GLD) options, reflecting 30 – day implied volatility of gold prices and lastly

CBOE EuroCurrency Volatility Index (EVZ) based on Currency Shares Euro Trust (FXE) options, measuring the market's expectation of 30 – day volatility of the $/€ exchange rate.

All three underlying assets are exchange traded funds (ETFs) that intend to reflect, as closely as possible, the return of the corresponding assets, less the fund expenses.

For each of the non – equity volatility indexes, the method of selecting component options and the formula are identical to that used for VIX and other broad–based volatility indexes. However, there is a slight difference in the methodology that accounts for the fact that USO, GLD and FXE options expire at the close rather than at the open. As before, the time to expiration is given by the aforementioned expression with the difference: $M_{Settlement\ day}$ = number of minutes from midnight until 03:00 p.m on expiration day. All option price quotes are derived from CBOE exclusively (www.cboe.com).

At the end of 2007, CBOE launched three new indices that attempt to capture the additional premium that market participants, who are inherently short on volatility due to the negative relationship between stock returns and volatility, seem willing to pay in order to own implied or realized volatility.

Specifically, the three new indices are the CBOE VIX Premium Strategy Index (VPD), the CBOE Capped VIX Premium Strategy Index (VPN) and the CBOE S&P 500 VARB-X Strategy Benchmark (VTY).

According to the CBOE official description of the indices:

The S&P 500 VARB–X Strategy Benchmark is based on selling realized variance while the VIX Premium Strategy Index and Capped VIX Premium Strategy Index are based on selling implied volatility proxied by the VIX. The capped version of the VIX Premium Strategy Index limits the risk of a short VIX exposure with long out–of–the–money VIX calls.

The volatility–related indices group has bloomed since then, including today 15 different kinds of indices. The latest introduction is the *Low Volatility Index (LOVOL),* consisting of 60% of the S&P 500 BuyWrite Index and 40% of the VIX Tail Hedge

IndexSM. LOVOL is designed for investors whose preferences have shifted from investing in riskier assets to lower volatility assets and aims to provide the ability of an investment strategy replication that is subject to less downside volatility, while still preserving the bulk of market gains (www.cboe.com).

On March 16 2011, CBOE announced the extension of its successful volatility franchise, by applying the VIX methodology to six active ETFs. Particularly, CBOE launched:

The *Brazil ETF Volatility Index (VXEWZ)*, which measures the market's expectation of 30 – day implied volatility of the EWZ ETF by applying the VIX methodology to the options of the iShares of MSCI Brazil Index Fund (EWZ) spanning a wide range of strike prices.

The *China ETF Volatility Index (VXFXI)*, based on the options of iShares of Trust FTSE China 25 Index Fund and also

The *Emerging Markets Volatility Index Options (VXEEM)*, based on the iShares of MSCI Emerging Markets Index Fund, the *Gold Miners ETF Volatility Index (VXGDX)*, based on Market Vectors Gold Miners Fund, the *Silver ETF Volatility Index (VXSLV)*, based on the iShares of Silver Trust and the *Energy Sector ETF Volatility Index (VXXLE)*, based on Energy Select Sector Fund.

There are also other *equity volatility indices* for options on the stocks of *Amazon*, *Apple*, *Goldman Sachs*, *Google* and *IBM*. Futures and options are available for many of the indices (www.cboe.com).

The latest additions in the CBOE family worth mentioning are:

In March 2012, the *CBOE VIX of VIX Index (VVIX)* was introduced, which tracks the volatility of VIX, as its name implies and uses the same calculation methodology which is derived from the price of a portfolio of liquid at–and–out–of–the–money VIX option puts and calls.

In June 2012, the *CBOE Interest Rate Volatility Index (SRVX)* was introduced, designed to offer fixed income option traders and portfolio managers a standardized and transparent measure of interest rate swap volatility. According to CBOE, the index measures expected basis – point volatility in the interest rate swap market. It is based on one – year / ten – year US dollar – swaptions, which are the most actively traded contracts in the $ 14.5 trillion notional OTC US dollar interest rate option market. Specifically, the index measures the fair market value of future volatility implied by the swaption market, for any swaption maturity and tenor of the underlying swap. The index differs from at–the–money implied volatilities as it incorporates additional information contained in the entire skew of out–of–the–money swaption prices, serving as the swap rate counterpart to the VIX for equity volatility. For additional information about the newly disseminated indices, see CBOE's white papers. (www.cboe.com).

Tables 3.1 – 3.4 show all CBOE volatility indices with closing levels of 28 January 2013 (www.cboe.com).

Table 3.2: CBOE Volatility Indices

Implied Volatility Indexes

TICKER	INDEX	WEBSITE	DELAYED QUOTES		
			SYM	LAST	PT. CHANGE
VIX®	CBOE Volatility Index®	www.cboe.com/VIX	VIX	14.28	-0.04
VXN	CBOE NASDAQ Volatility Index	www.cboe.com/VXN	VXN	14.83	0.45
VXO	CBOE S&P 100 Volatility Index	www.cboe.com/VXO			
VXD	CBOE DJIA Volatility Index	www.cboe.com/VXD	VXD	13.31	0.16
RVX	CBOE Russell 2000 Volatility Index	www.cboe.com/RVX	RVX	17.42	-0.08
VXV	CBOE S&P 500 3-Month Volatility Index	www.cboe.com/VXV	VXV	15.55	0.13
VVIX	CBOE VIX of VIX Index	www.cboe.com/VVIX	VVIX	76.72	-0.62
SRVX	CBOE Interest Rate Swap Volatility Index	www.cboe.com/SRVX	SRVX	84.73	0.00

Table 3.3: CBOE ETF Volatility Indices

ETF Volatility Indexes

OVX	CBOE Crude Oil ETF Volatility Index	www.cboe.com/OILVIX	OVX	22.54	-0.34
GVZ	CBOE Gold ETF Volatility Index	www.cboe.com/GVZ	GVZ	13.76	0.55
EVZ	CBOE EuroCurrency ETF Volatility Index	www.cboe.com/EVZ	EVZ	8.20	-0.19
VXEEM	CBOE Emerging Markets ETF Volatility Index	www.cboe.com/VXEEM	VXEEM	19.24	0.56
VXSLV	CBOE Silver ETF Volatility Index	www.cboe.com/VXSLV	VXSLV	24.83	0.55
VXFXI	CBOE China ETF Volatility Index	www.cboe.com/VXFXI	VXFXI	20.19	0.43
VXGDX	CBOE Gold Miners ETF Volatility Index	www.cboe.com/VXGDX	VXGDX	28.00	-0.16
VXEWZ	CBOE Brazil ETF Volatility Index	www.cboe.com/VXEWZ	VXEWZ	19.71	-0.39
VXXLE	CBOE Energy Sector ETF Volatility Index	www.cboe.com/VXXLE	VXXLE	18.52	0.65

Table 3.4: CBOE Equity Volatility Indices

Equity Volatility Indexes

VXAZN	CBOE Equity VIX® on Amazon	www.cboe.com/VXAZN	VXAZN	32.10	0.87
VXAPL	CBOE Equity VIX® on Apple	www.cboe.com/VXAPL	VXAPL	26.87	-1.33
VXGS	CBOE Equity VIX® on Goldman Sachs	www.cboe.com/VXGS	VXGS	24.62	0.91
VXGOG	CBOE Equity VIX® on Google	www.cboe.com/VXGOG	VXGOG	18.53	-0.32
VXIBM	CBOE Equity VIX® on IBM	www.cboe.com/VXIBM	VXIBM	15.83	-0.11

Table 3.5: CBOE Volatility–related Indices

Other Volatility-related Indexes

TICKER	INDEX	WEBSITE	DELAYED QUOTES		
			SYM	LAST	PT. CHANGE
VPD	CBOE VIX Premium Strategy Index	www.cboe.com/VPD	VPD	226.90	0.57
VPN	CBOE Capped VIX Premium Strategy Index	www.cboe.com/VPN	VPN	224.74	0.56
VXTH	CBOE VIX Tail Hedge Index	www.cboe.com/VXTH	VXTH	150.85	-0.45
LOVOL	CBOE Low Volatility Index	www.cboe.com/LOVOL	LOVOL	142.62	-0.24
SKEW	CBOE S&P 500 SKEW Index	www.cboe.com/SKEW	SKEW	121.92	0.00
VXTH	CBOE VIX Tail Hedge Index	www.cboe.com/VXTH	VXTH	150.85	-0.45
KCJ	CBOE S&P 500 Implied Correlation Index (fixed maturity)	www.cboe.com/KCJ			
ICJ	CBOE S&P 500 Implied Correlation Index (fixed maturity)	www.cboe.com/ICJ	ICJ	47.10	0.00
JCJ	CBOE S&P 500 Implied Correlation Index (fixed maturity)	www.cboe.com/JCJ	JCJ	62.87	1.45
VIN	CBOE SPX Near-term VIX Index	www.cboe.com/VIN	VIN	13.48	-0.22
VIF	CBOE SPX Far-term VIX Index	www.cboe.com/VIF	VIF	14.50	-0.03
VWB	CBOE VIX Indicative Bid Index		VWB	13.67	0.01
VWA	CBOE VIX Indicative Ask Index		VWA	14.87	-0.09
VBEEM	CBOE VXEEM Indicative Bid Index		VBEEM	17.74	0.41
VAEEM	CBOE VXEEM Indicative Ask Index		VAEEM	20.94	0.65

(Sourse: CBOE)

3.4 *Other Indices measuring Volatility globally*

According to Siriopoulos, Fassas (2009), and Lopez, Navarro (2012), the successful paradigm of VIX disseminated throughout the globe. In most of the cases the calculation methodology is similar to that employed by CBOE.

Europe

Firstly, Deutsche Börse launched in 1994 a series of implied volatility indices based on the DAX – 30 index options. In particular, it introduced eight volatility sub–indices, which corresponded to the various DAX options expiry months (from 2 to 24 months) and were calculated through two pairs of options (one pair with a strike price above and one pair with a strike price below the calculated forward price) of the respective expiry month. The *VDAX*, the main German implied volatility index, was calculated through a linear interpolation of the two sub–indices with a remaining life closest to 45 days. The rationale of the sub–indices was that the use of only four at–the–money options for each maturity makes the replication of each index easier and thus, allows them to be used as an underlying asset in a derivatives product. The calculation of VDAX differed from the original VIX (VXO) in two ways: it approximated the implied volatility of a theoretical at–the money option with a constant maturity of forty five days (and not thirty days which is the reference period for VXO) and additionally, the underlying price considered was not the spot price of the underlying index, as it was in VXO, but its forward price.

The MONEP (Marché des Options Négociables de Paris) launched on October 8, 1997 two volatility indices: *VX1* and *VX6*, based on the implied volatilities of around at–the–money CAC40 call option (4 call options are used for each index). The method used by the MONEP is based on the proposal by Brenner and Galai (1989) who observed a quasi – linear relationship between the premium and the volatility of the options around the at–the–money price. The main difference with VIX was that VX1 and VX6 used only call options, which were used to calculate the theoretical price of an at–the–money option with 31 days or 185 days (VX1 and VX6 respectively) to expiration, then using a binomial model adjusted for the daily dividends, its implied volatility was calculated. Finally, each VX1 day was adjusted to a trading day basis by multiplying the ratio of the square root of the number of calendar days to the square root of the number of trading days.

In 2007, New York Stock Exchange (NYSE) Euronext12, redesigned VX1 and VX6 calculation methodology, in order to match with the new VIX's and discontinued the dissemination of the original indices. NYSE Euronext on September 3, 2007, also introduced three new indices: *AEX* (Amsterdam Euronext) Volatility Index *(VAEX)*, *BEL 20* (Brussels Euronext) Volatility Index *(VBEL)* and *CAC 40* (Paris Euronext) Volatility Index *(VCAC)*. These indices follow the new VIX methodology and are calculated based on the implied variance of the call and put options available on London International Financial Futures and Options Exchange (LIFFE). Additionally, at the beginning of June 2008, NYSE Euronext announced the launch of a fourth volatility index, namely the *FTSE 100 Volatility Index (VFTSE)*, which represents the implied volatility embedded in options prices (again available on LIFFE) of the UK benchmark equity index, that is FTSE 100. The FTSE 100 Volatility Index is calculated by NYSE Euronext, in the same way with the other implied volatility indices of the exchange, using the new VIX methodology. Historical daily values for all four NYSE Euronext implied volatility indices are available retroactively since January 2000 (www.nyxdata.com).

European Exchange (EUREX), the third of the "big three" derivatives exchanges worldwide, which is jointly operated by Deutsche Börse and the Swiss Exchange, has also created a consistent family of volatility indices in 2007: *VDAX–*

NEW based on the DAX, *VSTOXX* based on the Dow Jones EURO STOXX 50 and *VSMI* based on the SMI (Switzerland's blue chips index). These indices follow the same model – free methodology, which is used for the calculation of the new VIX. Their calculation does not need an option pricing model, such as Black – Scholes, but only involves summations over a strip of out–of–the–money options prices and measures the square root of the implied variance for a rolling fixed maturity of 30 days. They are calculated on the basis of eight expiry months with a maximum time to expiry of two years. A sub–index is calculated for each option expiry, based on the implied variance across all options of the given time to expiry. Then, the main index, that is VDAX–NEW, VSTOXX or VSMI, is calculated through linear interpolation using the two sub–indices that include the remaining time to expiry of 30 days.

In particular for the VDAX–NEW, its main difference with VDAX in calculation, is that the new index is based on a more recent model (similar to the new VIX), developed jointly by Goldman Sachs and Deutsche Börse. The implied volatilities of at–the–money options have been replaced by the square root of implied variance across at–and out–of–the–money options. This model offers great advantages in terms of creating, trading and hedging derivative products on this index. The fixed remaining time to expiration has been reduced from 45 to 30 days. Thus, the new index has a broader volatility surface than VDAX. The new index and its various sub–indices have been calculated on a continuous basis with effect from 18 April 2005. Historical time series for the main index and 5 out of 8 sub–indices were calculated based on daily settlement prices dating back to January 1992 (www.dax-indices.com). For more detailed information about German Volatility Indices, see Guide to the Volatility Indices of Deutsche Börse (2007).

On December 2010 the Russian Trading System Stock Exchange (RTS), introduced the *Russian Volatility Index (RTSVX)*, which is based on the options of the RTS Index futures contract. According to RTS, for the calculation, it is applied the Black – Scholes option pricing formula for futures – style options with futures as the underlying asset. For the calculation, the option's volatility is determined on the basis of standard exchange's volatility curve with two truncated parameters in use. Truncation is applied in order to use range of strikes with the stable option's quotes in calculation (www.rts.ru).

America

Except of the CBOE ETF volatility index for Brazil, the TMX Group (Toronto and Montréal Exchanges) computed and disseminated an implied volatility index, named *MVX*, calculated from the prices of the at–the–money options on the iShares of the CDN S&P/TSX 60 Fund (an Exchange Traded Fund (ETF) on the Canadian benchmark stock index). The MVX Index was calculated from 4 pairs of at–the–money calls and puts of the two nearest expiries on the XIU that were traded on the Montréal Exchange. MVX used the Black – Scholes model to obtain the implied volatility for each option, which was then averaged for each strike and expiry. Afterwards, through linear interpolation, a 30 – day calendar implied volatility was created. MVX had been in existence since December 2002. (http://www.m-x.ca).

In October 2010 another volatility index was introduced: the *S&P/TSX 60 VIX or VIXC*, which is based on the cost of index options traded on the Montréal Exchange and uses the VIX methodology (www.bloomberg.com, http://www.m-x.ca).

In 2004, the Mexican Derivatives Exchange (MexDer) released the *Mexico Volatility Index (VIMEX)*, calculated from the IPC Mexican Index of at–the–money option prices, using the methodology described by Fleming et al. (1995). A 90 – day volatility is calculated (www.mexder.com).

Asia

In the last years, several volatility indices have also been developed for the Asian stock markets using the VIX methodology.

Except of the CBOE index for China, *Taiwan's* Futures Exchange (TAIFEX) calculates and disseminates the *TAIEX Options Volatility Index (TVIX)*, based on the VIX methodology. Unfortunately, the website does not provide additional data (www.taifex.com).

The *Korea* Exchange KRX disseminates a domestic version of the CBOE volatility index, named *VKOSPI–200*, based on options on the Korean stock index KOSPI 200, since April 13 2009. Future (30–day maturity) volatility index is derived from the constituent of the recent and second month in the KOSP 200 market. In estimation, only the constituent of the immediate delivery month is used, since most transactions are concentrated in this month. The Black – Scholes option pricing model is used to complement the estimated option, if this option for listing is considered to be lacking (http://eng.krx.co.kr).

In 2010, National Stock Exchange of India Limited (NSE) started to compute the *India VIX* from the NSE's key index known as the S&P CNX NIFTY (National Stock Exchange Fifty, or NIFTY) options, using the CBOE new VIX methodology (http://vix.co).

The same year, Nikkei Inc. launched the *Nikkei Stock Average Volatility Index (VXJ/CSFI–VXJ)*, based on the Nikkei Stock Average (Nikkei 225). Its calculation formula is based on VIX and has replaced Nikkei Stock Average implied volatility, while retroactive calculation has been made since June 1989 (Nikkei Press release, 2010). On February 27 2012, the Osaka Securities Exchange Co. started offering Nikkei 225 Volatility Index futures (http://vix.co).

Hang Seng Indexes Company Limited distributes the *HIS Volatility Index (VHSI)* for the Hong Kong equity market since 2011, aiming to measure the 30 – calendar day expected volatility of the Hang Seng Index (HSI) implicit in the prices of near – and next – term options which are now trading on the Hong Kong Exchange. The calculation process follows VIX methodology. Historical values were calculated since January 2001 (www.hsi.com).

Australia and Africa

There are also two volatility indices reflecting expected volatility of the Australian and South African equity markets:

Since 2010, *S&P/ASX 200 VIX (XVI)* is distributed by the Australian Securities Exchange (ASX) using S&P/ASX 200 options and the VIX methodology. Two maturities are used with the nearby having at least a week until expiry. The volatility of the options closest to maturity is interpolated with that of the options farthest from maturity to arrive at a constant 30 – day indication of expected volatility in S&P/ASX 200 (www.asx.com).

Finally, in the Johannesburg Stock Exchange (JSE), the South African Futures Exchange (SAFEX) follows a similar methodology in order to calculate the volatility index of the TOP40 index, from 2007. The *South African Volatility Index* (*SAVI)* is computed by options on the FTSE/JSE Top 40. The basis for the calculation is the Whaley (1993) methodology, but because of the limited liquidity of the SAFEX, only at–the–money options are considered in the index construction process. The other noticeable difference is that the rolling fixed maturity is three months (ninety one calendar days), instead of the typical one month period (www.jse.co.za).

It is important to mention that out of 19 existing volatility indices throughout the globe, created by different exchange markets, today only 2 use other than the new VIX methodology (Russia and Mexico).

In the following table some of the world's major volatility indices are presented, in addition with non–equity and strategy indices.

Table 3.6: Major Global Volatility Indices

Volatility Indexes (Updated on 22-Jan-2013)

Symbol	Price	1 week chng	Description	30 Days
ASCNCHIX:IND			AlphaShares Chinese Volatility Index	
INVIXN:IND	13.89	+0.31	India NSE Volatility Index	
RTSVX:IND	20.92	-0.17	Russian Volatility Index	
RVX:IND	15.59	-1.06	Russell 2000 Volatility Index	
SAVIT40:IND			JSE Securities South African Volatility Index	
SPAVIX:IND	10.96	+0.05	S&P/ASX Volatility Index	
V1X:IND	13.97	+0.25	Deutsche Borse VDAX-NEW Volatility Index	
V2X:IND	16.06	+0.09	Dow Jones EURO STOXX 50 Volatility Index	
V3X:IND	13.77	+1.63	Deutsche Borse VSMI Volatility Index	
VAEX:IND	11.87	-0.28	AEX Volatility Index	
VCAC:IND	15.41	-0.28	CAC 40 Volatility Index	
VFTSE:IND	12.15	+1.08	FTSE 100 Volatility Index	
VHSI:IND	13.46	-1.07	HSI Volatility Index	
VIMEX:IND	10.92	-0.90	Volatility Index Mexico	
VIX:IND	12.98	-0.57	CBOE S&P 500 Volatility Index	
VIXC:IND	11.84	-0.55	S&P/TSX 60 VIX	
VKOSPI:IND	14.28	-0.86	KOSPI 200 Volatility Index	
VNKY:IND	22.48	+0.66	NIKKEI Volatility Index	
VXD:IND	11.94	-0.41	Dow Jones Industrial Average Volatility Index	
VXN:IND	14.86	-1.26	NDX Volatility Index	
VXV:IND	15.40	-0.93	CBOE S&P 500 3-Month Volatility Index	

Non-Equity Indexes (Updated on 22-Jan-2013)

Symbol	Price	1 week chng	Description	30 Days
BBOX:IND	75.90	+0.76	Barclays Swaption Volatility	
CIV:IND	19.93	-1.38	CBOE/CBOT Corn Volatility Index	
EVZ:IND	8.83	+0.21	CBOE EuroCurrency Volatility Index	
GVXX:IND	13.17	-1.08	CBOE/COMEX Gold Volatility Index	
GVZ:IND	12.88	-0.88	CBOE Gold ETF VIX Index	
JPMVXYEM:IND	7.00	+0.09	JPMorgan Emerging Market Volatility Index	
JPMVXYG7:IND	8.48	+0.09	JPMorgan G7 Volatility Index	
JPMVXYGL:IND	8.25	+0.09	JPMorgan Global FX Volatility	
MOVE:IND			Merrill Lynch Option Volatility Estimate MOVE	
OIV:IND	21.72	-4.68	CBOE/NYMEX WTI Oil Volatility Index	
OVX:IND	21.87	-3.83	CBOE Oil ETF VIX Index	
SAVIWMAZ:IND			JSE Securities White Maize Volatility Index	
SIV:IND	21.27	+1.15	CBOE/CBOT Soybean Volatility Index	
SRVX:IND	81.90	+1.29	CBOE Interest Rate Volatility Index	
VXSLV:IND	24.49	-2.20	CBOE Silver ETF VIX Index	
VPN:IND	225.29	+3.23	CBOE VIX Premium Strategy Index	

Volatility Strategy Indexes (Updated on 22-Jan-2013)

VXTH:IND	149.85	+0.63	CBOE VIX Tail Hedge
VPD:IND	227.45	+3.48	CBOE Capped VIX Premium Strategy Index
SKEW:IND	124.79	+3.89	CBOE SKEW Index
VTY:IND	85.58	+0.46	CBOE S&P 500 VARB-X Index
MLHFEV1:IND			Merrill Lynch Equity Volatility Arbitrage Index
DBVEHUT:IND	261.42	-0.63	DB US Volatility Harvest
ICJ:IND			CBOE SPX Implied Correlation 2013
JCJ:IND	63.39	+0.38	CBOE SPX Implied Correlation 2014
KCJ:IND	69.96	-0.15	CBOE SPX Implied Correlation 2012
VVIX:IND	76.50	-1.34	CBOE VIX of VIX
VXEEM:IND	17.17	-0.08	CBOE Emerging Markets ETF Volatility Index
VXEWZ:IND	18.66	+0.23	CBOE Brazil ETF Volatility Index
VXFXI:IND	19.64	-0.24	CBOE China ETF Volatility Index
VXGDX:IND	25.09	-0.54	CBOE Gold Miners ETF Volatility Index
VXXLE:IND	15.66	-0.43	CBOE Energy Sector ETF Volatility Index
VXAPL:IND	43.62	-1.35	CBOE Apple VIX
VXAZN:IND	38.09	-2.22	CBOE Amazon VIX
VXGOG:IND	30.21	-0.65	CBOE Google VIX
VXGS:IND	23.66	-3.23	CBOE Goldman Sachs VIX
VXIBM:IND	23.25	+0.96	CBOE IBM VIX

3.5 *Academic Volatility Indices*

In addition to the volatility indices officially created and calculated by organized exchanges and other institutions, there are a number of volatility indices created in an academic content. Specifically, there are 10 academic papers that propose an implied volatility index for an international stock market, using both the Black – Scholes and the model – free methodologies.

There are also 11 other studies, identifying problems and proposing solutions to the currently existing volatility indices or presenting new ideas. These are presented to the following paragraphs.

One of the first academic indices was proposed by Skiadopoulos (2004), who constructed a *Greek Volatility Index (GVIX)* for the local market. His proposed calculation method is based on the Black's (1976) model, but he differentiates from Whaley's method, mainly due to liquidity issues. In particular, he uses four options, instead of eight (two for each of the two expiries). The main difference though is that he takes into consideration only the first out–of–the money call and the first out–of–the money put for each expiry and calculates, through linear interpolation, at–the–money option implied volatility. Then, the thirty – day rolling fixed maturity implied volatility is derived by interpolating between the implied volatility of the two expiries.

Siripoulos and Fassas (2012) proposed also a *Greek Volatility Index (GRIV)*, based on FTSE/ATHEX–20 options, using the model – free methodology. Their method is applied for the first time in a peripheral and illiquid market.

According to Siriopoulos and Fassas (2009), Giner and Morini (2004) developed a volatility index *(VIBEX)* for the *Spanish* IBEX–35 following a methodology similar to Whaley (unpublished paper).

5 years before the Australian volatility index (XVI) was officially introduced, Dowling and Muthuswamy (2005), proposed a measure of the Australian stock market volatility, named *Australian Market Volatility Index (AVIX)*, which was based on the implied volatility of S&P/ASX 200 Index options and constructed with the Whaley methodology (1993). In order to bypass any potential calculation issues arising from the quarterly expiration cycle of S&P/ASX options, which requires a large extrapolation of the implied volatility data, they also calculated AVIX with a 66 trading-day horizon, besides the typical 22 trading – day expiry. Additional calculation issues arose from the lack of all eight required option contracts in order to calculate AVIX during certain days. In that case, the implied volatility of the same class of option contract from the previous day was used as a substitute.

At the same period, Jiang and Tian (2005), introduce a model – free implied volatility measure (MFIV) using observed S&P 500 option prices across different strikes, as an alternative way to the formula employed by the CBOE. The difference lies in using trapezoidal integration for the option prices calculation.

An attempt dealing with the Spanish equity market and using the model – free methodology can be found in Gonzalez and Novales (2007: working paper, 2011) who proposed the *VIBEX–NEW* for the Spanish market, using the methodology used by EUREX, to estimate the German (VDAX–NEW) and Swiss (VSMI) volatility indices. The simplicity of their methodology makes it especially suitable to estimate a volatility index in less than perfectly liquid markets, as it is the options market on the IBEX–35 index. The information requirements are weaker than for a previous methodology used to estimate volatility indices in international markets, and that enable them to compute the volatility index for a significantly higher percentage of market days than under the old methodology.

Maghrebi, Nishina and Kim (2006: unpublished with first author Nishina and 2007 with first author Maghrebi), developed a new model – free benchmark of implied volatility for the Japanese and the Korean stock markets, similar in construction to the new VIX. Especially for the Korean market, the index is based on the market prices of options on the Korean KOSPI 200 index. It is reminded that both indices were launched in 2009 and 2010.

Another proposal for an implied volatility index comes from Areal (2008), who constructed a UK implied volatility index (VFTSE) using high–frequency data on FTSE–100 index options. He suggested three different methods to construct his implied volatility index; specifically a modified version of the original VXO methodology adapted for a market less liquid than the US market, a methodology that uses only out–of–the–money options and finally the new VIX model – free methodology.

Frijins et al. (2008) proposed a volatility index for the New Zealand stock market, the NZVIX. Since there were no equity options in New Zealand, they proposed a new approach that uses four stock options.

Ninanussornkul et al. (2009), using VIX and VSTOXX, propose one volatility index for both Europe and US. They suggest the construction by using conditional volatility models either by fitting a univariate model to the portfolio returns, or by using a multivariate volatility model to forecast the conditional variance of each asset in the portfolio, as well as the conditional correlations between all asset pairs, in order to calculate the forecasted portfolio variance.

The same year, they also suggest (using the same methodology) the construction of a volatility index for the South East Asian (ASEAN) countries (unpublished). They use data of the three highest–volatilities countries: Indonesia, Philippines and Thailand.

Muzzioli (2010 unpublished) analyses and empirically tests how to unlock volatility information from option prices. The information content of three option based forecasts of volatility: Black – Scholes implied volatility, model – free implied volatility and corridor implied volatility is addressed, with the ultimate plan of proposing a new volatility index for the *Italian* stock market. As for model – free implied volatility, two different extrapolation techniques are implemented. As for corridor implied volatility, five different corridors are compared. The results, which point to a better performance of corridor implied volatilities with respect to both Black – Scholes implied volatility and model – free implied volatility, are in favour of narrow corridors. The volatility index proposed is obtained with an overall 50% cut of the risk neutral distribution.

McAleer and Wiphatthananananthakul (2010) propose a *Simple Expected Volatility Index (SEV)*, supporting that the new VIX methodology seems to be based on a very complicated formula. With the use of Thailand's SET50 Index Options data, they modify the VIX formula to a very simple relationship and they show that their model seem to be better than TVIX as a hedging diversification tool.

Andersen et al. (2011, 2012) propose the construction of the *Corridor Implied Volatility Index (CX)*. They argue that a number of fundamental questions regarding the equity index return dynamics are difficult to address due to the latent character of spot volatility. In practice, the high frequency VIX series is plagued by idiosyncratic biases and noise which severely distort the measure. Instead, they exploit high frequency reliable option quotes from a strike range, covering an "economically invariant" proportion of the future S&P 500 index values, in order to compute the proposed index. They find striking new empirical findings regarding the nature of volatility jumps and the return volatility asymmetries. Comparing time series properties of these alternative volatility indices, they find that their model is superior in terms of filtering out the noise and avoiding large artificial jumps, so they document that their proposal performs better than VIX, especially during turbulent market conditions.

Bozdog et al. (2011) propose a new methodology for an alternative calculation of market volatility index based on a multinomial tree approximation by implementing a stochastic volatility technique. The estimation is performed by constructing synthetic options with consistent properties. They believe that this model brings more information about the markets and the methodology has the potential to produce market indicators each indicative of a certain aspect of the financial market.

Cont and Kokholm (2011) propose and study a flexible modelling framework for the joint dynamics of an index and a set of forward variance swap rates written on this index, allowing options on forward variance swaps and options on the underlying index to be priced consistently. Their model reproduces various empirically observed properties of variance swap dynamics and allows for jumps in volatility and returns. An affine specification using Levy processes, such as building blocks, leads to analytically tractable pricing formulas for options on variance swaps as well as efficient numerical methods for pricing of European options on the underlying asset. The model has the convenient feature of decoupling the vanilla skews from spot/volatility correlations and allowing for different conditional correlations in large and small spot/volatility moves. They show that their model can simultaneously fit prices of European options on S&P 500 across strikes and maturities as well as options on the VIX. The calibration of the model is done in two steps, first by matching VIX option prices and then by matching prices of options on the underlying index.

Fukasawa et al. (2011), as the VXJ Research Group on behalf of the Centre for the Study of Finance and Insurance (CSFI) from the Osaka University propose a modification to Japan's existing volatility index CSFI–VXJ, using options on the Nikkei 225. They introduce a new interpolation scheme for the volatility surface, designed to be consistent with arbitrage bounds, in order to reduce approximation errors (apparently, severely affected by the recent global turbulence).

Lopez (2001, unpublished) and Lopez and Navarro (2011, unpublished), suggest the construction of a set of *interest rate volatility indices (IRVIXs)* (in the first paper only for the US), that measure the future volatility of three–month tenor forward rates over horizons ranging from one to ten years ahead. This is a very important difference with respect to other indices such as VIX or VDAX–NEW in the equity market or the MOVE Indices for interest rates, which measure volatility over very short horizons (from one to six months ahead). Data are extracted from the cap (floor) market which is one of the most liquid interest rate derivatives markets. The idea came from the current financial crisis which has had a severe impact on both short– and long–term IRVIXs. The potential uses of these indices are broad and include the introduction of derivative contracts, the estimation of the volatility term structure or their usage as leading indicators of the business cycle.

Tzang et al. (2011) investigate whether liquidity and sampling methods play an important role in the construction of volatility indices. They propose four methods by which to sample option prices using proxies for liquidity.1–, 2–, 3–, 7–, and 8–day rollover rules—for option trades in order to construct volatility index series. Based on the sampling method using the average of all midpoints of bid and ask quote option prices, the volatility indices constructed by one– minute tick data have less missing data and are at least as efficient in volatility forecasting as the method suggested by the CBOE.

Finally, according to Siriopoulos and Fasas (2009), an effort that is neither official nor academic is the one by Bank Clariden Leu, a Swiss private bank, which calculates and disseminates, since 1996, an implied volatility index called the Real Time Volatility Index VCL, which represents the expected volatility of the Swiss stock market. The novelty of the particular index is that its underlying assets are individual stocks and not on a particular index.

Particularly, VCL is calculated from the prices of six at–the–money call options with 45 days to maturity (two expiries are chosen) on 24 big Swish stocks that trade on Eurex17. From the prices of these options, using the Black – Scholes formula, a sub–index is calculated for each stock. Then, these sub–indices are aggregated, according to a weighting scheme based on the trading volume of the last ten trading days, to an overall index, the VCL.

3.6 Correlations of Volatility Indices

All the papers examined, which compared data between volatility indices and stock markets suggest that there is a strong negative correlation between implied volatility and stock returns, reaffirming the determination of implied volatility as the "fear factor".

In general, the literature suggests that market returns have an asymmetric contemporaneous relationship with volatility, meaning that, negative returns have larger effects on volatility than positive returns. In simple words, the correlation increases much more with bad news than with good news of the same magnitude (Black, 1976, Christie, 1982, Pindyck, 1984 and Schwert, 1989).

Figures 3.5 and 3.6 show this correlation between VIX and its underlying stock market index S&P 500, for a 12 – year and an 8 – year period respectively (www.cboe.com).

Figure 3.5: VIX and S&P 500 closing levels

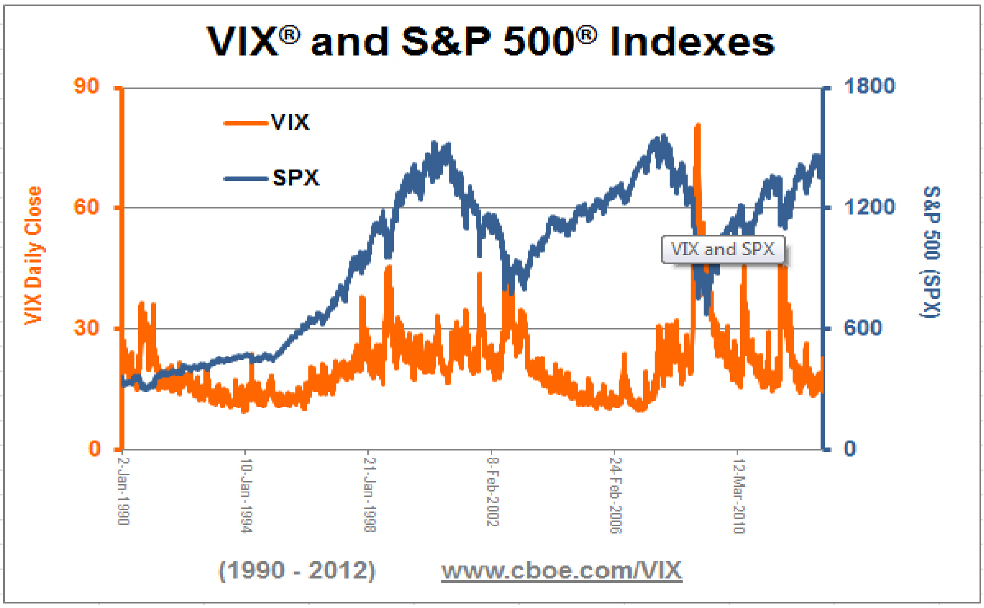
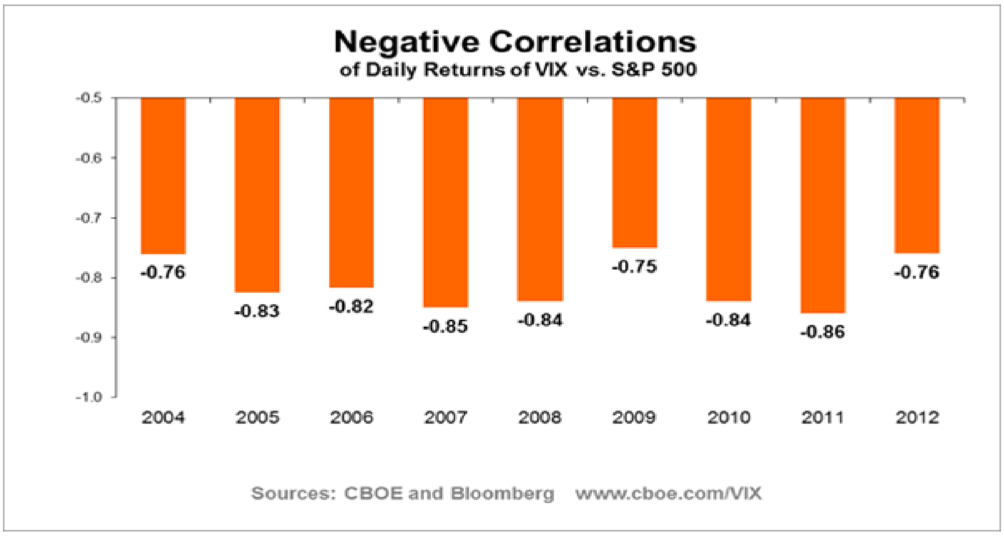
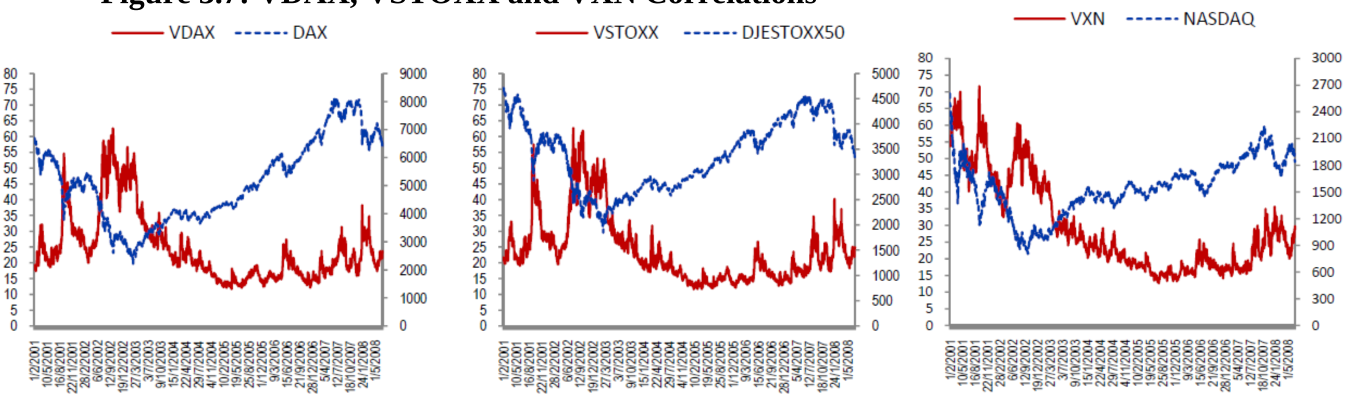


Figure 3.6: VIX and S&P 500 Correlation



In figure 3.7, the same relation between VDAX, VSTOXX, VXN and their underlying indices is also presented for a 7 – year period (2001 – 2008) (Badshah, 2009).

Figure 3.7: VDAX, VSTOXX and VXN Correlations



The pioneers in this field were Markowitz (1952) and Tobin (1958), who establish the basis of the modern portfolio theory by relating the expected return of a portfolio with its standard deviation, while Sharpe (1964) formalized the celebrated capital asset pricing model (CAPM). The first to document negative relationship between stock market returns and changes in ex post calculated future volatility were Black (1976) and Christie (1982), while Pindyck (1984) and Schwert (1989) refer to this relationship as asymmetric (the negative return associated with an increase in volatility is larger than the positive return associated with a decrease in volatility).

Indicatively, other studies verifying this relationship are: Bollen and Whaley (2004), Low (2004), Dennis et al. (2006) and Hibbert et al. (2008).

As far as implied volatility indices are concerned, a negative (and asymmetric in most of the cases) contemporaneous association between changes in volatility indices and the underlying stock indices returns has been extensively documented. Over their history, VIX and VXO have acted reliably as a fear gauge. High levels of both indices are coincident with high degrees of market turmoil, whether the turmoil is attributable to stock market decline, the threat of war, unexpected change in interest rates, or a number of other newsworthy events. For this reason, volatility indices are often referred to as being investor's gauges of fear (Whaley, 2000).

More specific, Whaley (2000) firstly mentions the asymmetric negative correlation for the old VIX.

Especially for Giot (2002, 2005a, 2005b) it must be mentioned that he investigates further, emphasizing the strong negative correlation between volatility indices and stock index returns, by comparing in all three papers VXO and VXN with the S&P100 and NASDAQ100 stock indices. He shows that when the VXO or VXN volatility indices increase, the stock indices exhibit on average negative returns. Particularly, for the S&P100 index, this relationship is asymmetric as negative stock index returns yield bigger changes in VXO than do positive returns. The VXO's response to negative stock index returns is sharper in low volatility periods, which suggests that option traders react aggressively to negative returns in low volatility periods by strongly bidding up implied volatility. For the NASDAQ100 index, the asymmetric effect is rather weak but the VXN response to the index is also somewhat muted in high–volatility trading environments. However, attractive (from a mean – variance perspective) positive returns should then be expected on average in the immediate short – term. In this framework, very high levels of implied volatility can on a statistical basis be viewed as signalling an imminent increase in stock indices, at least on a short – term basis. His analysis also shows that average to moderately high levels of implied volatility lead to unfavourable (from a mean – variance perspective) returns.

Simon (2003) for VXN, Whaley (2008) and Simlai (2010) for VIX, also confirm the same relationship between the aforementioned indices and their underlying equity indices, respectively.

Hibbert et al. (2008) confirm the same using VIX and quantile regression.

As for European volatility indices, Siriopoulos and Fassas (2008) document a negative and asymmetric relationship between VFTSE changes and FTSE–100 returns.

For the Greek market, they report an inverse and asymmetric correlation based on GRIV in 2012.

González and Novales (2009) provide evidence of a strong negative contemporaneous relationship between changes in VDAX–NEW, VSMI and VIBEX–

NEW and their respective stock indices returns, although there is no evidence of asymmetry.

Badshah (2009) investigates this phenomenon with VIX, VXN, VDAX–NEW VSTOXX and their underlying indices. He finds pronounced contemporaneous negative and asymmetric return – volatility relations between each stock market index and its corresponding volatility index. He also reports that VIX presents the highest asymmetric return – volatility relationship followed by the VSTOXX, VDAX and VXN, respectively. He argues that findings do not support either leverage or volatility feedback hypotheses. Instead the results could be explained by investors' heterogeneity, i.e., that there are clusters of pessimist investors (who overestimate volatility underestimate returns) and cluster of optimist investors (who underestimate volatility and overestimate returns) that leads to the strong observed short – term negative and asymmetric return – volatility relation.

In 2012, he uses quantile regression to investigate the same relationship with the same data. He again argues that neither the leverage hypothesis nor the volatility feedback hypothesis effectively explains the asymmetric return – volatility relation. Instead, behavioral explanations, such as the affect and representativeness heuristics, are supported, particularly in the short–term. The affect heuristic plays an important role. Moreover, the context of an extreme volatility change distribution, the affect heuristic and time–pressure dominate. Thus, a strong negative and asymmetric relation between each volatility index and its corresponding stock market index is observed. Finally, the observed asymmetry is more pronounced with the new volatility index measure than with the old, at–the–money volatility index measure.

Regarding other markets: Ting (2007) and Ryu (2012) for Korea, Kumar (2012) and Dhananjhay et al. (2012) for India and Frijns et al (2010) for Australia, mention the findings of the same asymmetric negative contemporaneous relationship. McAleer and Wiphatthananthakul (2010) find that their proposed model (SEV) has a higher negative correlation between the VIX for Thailand (TVIX) and SET50 index options.

Recently, Sarwar (2012a) analyzes the role of VIX as investor’s gauges of fear in foreign and US equity markets. Particularly, the relationship between VIX and stock market returns in US, Brazil, Russia, India and China is investigated. The study documents a contemporaneous negative relation between VIX and US which is stronger when VIX is higher and more volatile. The same relation exists between VIX and China, Brazil and India, while evidence of asymmetry is reported only for the first two markets. However, asymmetry is much weaker when VIX is large and more volatile.

The same year (b) he also examines the relationships between VIX and returns of the S&P 100, 500 and 600 indexes among three subperiods during 1992 – 2011 to account for structural shifts in VIX and to investigate if the role of VIX as an investor fear gauge and indicator of portfolio insurance price has strengthened in periods of high market anxiety and turbulence. He finds again a strong negative contemporaneous relation between daily changes (innovations) in VIX and S&P 100, 500 and 600 returns. Results suggest that the strength of contemporaneous VIX–returns relation depends on the mean and volatility regime of VIX, and that this relation is much stronger when VIX is both high and more volatile. In fact, during 2004 – 2011 the negative contemporaneous VIX–returns relation was the most dominating and the only significant relation. It is also indicated a strong asymmetric relation between daily stock market returns and innovations in VIX, suggesting that VIX is more of a gauge of investor fear and portfolio insurance price than investor positive sentiment. The response of VIX to negative changes in market returns was the highest during 2004–2011 when VIX was most volatile. This result is consistent with rising portfolio insurance premiums in periods of high market anxiety and turbulence.

Other correlations

Pan et al. (2003), using VIX, examine how volatility and the futures risk premium affect trading demands for hedging and speculation in the S&P 500 Stock Index futures contracts. The empirical results show a positive relation between volatility and open interest for both hedgers and speculators, suggesting that an increase in volatility motivates both hedgers and speculators to engage in more trading in futures markets. They also find that the demand to trade by speculators is more sensitive to changes in the futures risk premium than is the demand to trade by hedgers.

Bandopadhyaya and Jones (2008), based on the fact that recent finance literature suggests that shifts in non–economic factors, such as investor sentiment, may explain short–term movements in asset prices better than any other set of fundamental factors, investigate for investor sentiment, as possible determinant of asset prices. By comparing two measures of investor sentiment, the Put–Call Ratio (PCR) and the Volatility Index (VIX), which are computed daily by CBOE, and using daily data for a 2 – period (2004 – 2006), they find that the PCR is a better explanatory variable than is the VIX for variations in the S&P 500 index that are not explained by economic factors. This supports the argument that, if one were to choose between these two measures of market sentiment, the PCR is a better choice than VIX.

In 2010, an interesting study was conducted by Symeonidis et al. Among other tests, they use implied volatility indices VIX and VXO observations of an 18–year period (1990 – 2008), VXN observations of a 7–year period, (2001 – 2008), VXD observations of an 11–year period (1997 – 2008) and realized S&P 500 index returns, to investigate the empirical association between stock market volatility and investor mood – proxies related to the weather (cloudiness, temperature and precipitation) and the environment (nighttime length). Overall, results suggest that cloudiness and length of nighttime are inversely related to historical, implied and realized measures of volatility. Specifically, volatility measures tend to be negatively related with cloudiness and variation in nighttime hours. However, the underlying coefficients are statistically significant only in a pooled sample of four implied volatility indices. Their results are consistent with the explanation that good mood is associated with increased trading and volatility, respectively.

Kozyra and Lento (2011) use 10 – year period data (1999 – 2009) from VIX, S&P 500, NASDAQ and DJIA, to investigate for relationships between technical analysis and implied market volatility by calculating technical trading rules with the VIX price data, as opposed to the stock prices. Three trending trading rule signals are calculated on the prices of three major US indices and the VIX prices. The results reveal that the trading signals calculated with the VIX level provide large, statistically significant profits that are in excess of the profits from the traditional computation. Sub–period analysis reveals that technical trading rules were most (least) profitable during the period with the highest (lowest) volatility levels.

In 2011, Bellini and de la Torre – Gallegos, suggest that volatility indices, such as VIX, can be used for determining stock market direction. They analyze the relationship between changes in the VIX direction and changes in the turning point of S&P 500 and the MSCI Latin America Emerging Market Index, in order to see whether they anticipate the changes. Their conclusion is that turning points, or peaks and troughs, in the VIX are coincident with peaks and troughs in the opposite direction for the S&P 500 index and in emerging markets.

Füss et al. (2011) study a different subject. While many papers analyze the impact of scheduled macroeconomic announcements on equity market volatility, few focus on the impact on option implied volatilities, so they examine the link between German and US macroeconomic events and the respective implied volatility indices VDAX–NEW and CBOE VIX. They find that both indices fall on announcement days, with the strongest reactions occurring during the financial crisis from 2008 to 2009.

The same year, Gospodinov and Jamali examine a similar subject. In particular, they investigate the effects on implied volatility, as measured by VIX and VXO indices, of both expected and surprise changes in federal funds rate. By using daily and monthly data over a 17 – year period (1990 – 2007), their analysis suggest that implied volatility responds positively and significantly to surprise changes, while it does not respond to expected changes of federal funds rate. The results also hold when they account for macroeconomic news releases or when they change the rate surprise measure. An interesting finding is that macroeconomic variables, such as industrial production growth and inflation, also tend to affect volatility.

However, Vähämaa (2009) although supporting the correlation between macroeconomic events and the respective implied volatility indices, he argues that findings in several papers are contradictory. While some studies, examining the impact of macroeconomic news on most kinds of volatilities implied by stock, bond, interest rate and foreign exchange options, indicate a decrease in implied volatility, others present opposite results. Using VXO and its underlying market index, he suggests that a potential explanation for the contradictory results is that the direction of impact depends more on the methodological approach applied on each study.

Finally, Bagchi (2012) examines the direct and cross – sectional relationship of India VIX with portfolio returns. He constructs value weighted portfolio sorted on the basis viz., stock beta, market to book value of equity and market capitalization and finds that the index has a positive and significant relationship with the returns of the value – weighted high – low portfolios.

3.7 Applications of Volatility Indices: Forecasting

In chapter 1 of this thesis, it was emphasized that Poon, Granger (2003) and later Poon (2005), have compiled a comprehensive review of 93 papers dealing with the volatility forecasting performance of 4 categories of approaches (historical, ARCH modeled, stochastic and implied volatility based forecasts). Their general conclusion is that the implied volatility forecasts often outperform alternative models. The same conclusion is also reached by Hull (2008).

Furthermore, Siriopoulos, Fassas (2009) and Lopez Navarro (2012), present more recent papers, which were subsequent of the endeavor of Poon and Granger. In general, their conclusion is again that implied volatility is a superior estimator of future volatility.

In the literature review presented to the next section, some of the aforementioned studies, in addition with others which examine exclusively volatility indices are presented. The forecasting performance of various volatility indices is compared to alternative volatility models.

The empirical findings confirm the forecasting superiority of volatility indices, since out of 28 studies presented below, 17 find ISD forecasting models better than alternative volatility forecasting models and 9 find that comparison tests indicate no winner. Only in 2 papers volatility indices are outperformed by alternative models as predictors of future. Both of those indices are academically proposed (in 2004 and 2005 for the Greek and Australian markets respectively) and not actually introduced in real markets. It must be also noted that the majority of studies indicating no winner are more recently published (8 out of 9).

More particularly, Fleming et al. (1995) were the first to document that VXO is an outperforming predictor.

Moraux et al. (1999) examine the French VX1's ability to forecast future realized market volatility and show that these forecasts are reasonably accurate predictors even over different horizons.

Aboura and Villa (1999, 2003), find that VX1, VIX and VDAX are good tools for predicting future realized volatility and also show that past implied volatility informs more about future implied volatility than past realized volatility. As a second step, they embed each of the implied volatility indexes as an exogenous term in the GARCH variance equation and find that all of them dominate the GARCH terms.

Blair et al. (2001) compare the information content of implied volatilities and intraday returns via VXO and ARCH models respectively. In – sample estimates shows that nearly all relevant information is provided by VXO. For out – of – sample forecasting, VXO provides the most accurate forecasts for all forecast horizons and performance measures considered.

Bluhm and Yu (2002, unpublished) compare the German VDAX's implied volatility with various time series techniques (historical mean, EWMA, 4 ARCH – type models and an SV). Although the clear winner is difficult to be stated, due to sensitivity to error measurements and forecast horizons, if option pricing is the primary interest, the SV model and implied volatility should be used. A trading strategy also suggests that the time series models are not better than the implied volatility in forecasting.

Claessen and Mittnik (2002), using also VDAX, conclude that implied volatility is a biased but highly informative predictor for future volatility. Moreover, implied volatilities are informationally efficient relative to other historic volatility information sources. Findings support the efficient market hypothesis for the DAX–index options market.

Giot (2002, 2005a, 2005b) evaluates the performance of the VaR models by using a wide range of tests on the VXO, VXN, S&P100 and NASDAQ100 indices data. The results show that straightforward volatility forecasts based on the implied volatility indexes provide meaningful results when market risk must be quantified. However this is also true for the competing models in his analysis, RiskMetrics and GJR – GARCH. Furthermore, the models' performances do not deteriorate in challenging trading environments.

Skiadopoulos (2004), using FTSE/ASE – 20 returns and his proposed volatility index GVIX, finds that GVIX cannot forecast the future of the Greek stock market, therefore this volatility index cannot be treated as a leading indicator for the underlying stock market.

Corrado and Miller (2005) show that the CBOE implied volatility indices (VXO, VIX, and VXN) outperform historical volatility or the GARCH family, as estimators of future realized volatility of the corresponding underlying indices (S&P100, S&P500, and Nasdaq–100) for different time periods. Furthermore, they reported that the significant forecast errors that the three indices appeared to contain in the period before 1995 faded away in the latter period (1995–2003) of their sample.

Jiang and Tian (2005, 2007) compare the forecasting power between lagged realized volatility, their model (MFIV) and a Black – Scholes implied volatility measure. Data is obtained from S&P 500 options across different strikes. They find that the MFIV subsumes all information contained in the other two volatility measures. They also demonstrate that the model – free implied variance, reflected by the new VIX, subsumes all information contained in the Black – Scholes implied volatility and historical volatility and is a more efficient forecast for future volatility.

On the contrary, Dowling and Muthuswamy (2005) find that their version of an implied volatility index for Australian stock market underperforms historical volatility as a predictor of future realized volatility.

Maghrebi et al (2006: unpublished, 2007) indicate that their implied volatility indices for Japan and Korea, despite their upward bias, are good estimators of the changes in realized volatility and outperform historic volatility and GARCH models. In

2010 (unpublished), they conclude the same, using their model for Japan, VDAX–NEW and VIX.

In a more recent study, Corrado and Truong (2007) suggest that the intraday high – low price range offers volatility forecasts similarly efficient to high – quality volatility indices published by CBOE and especially the VIX, VXO, VXN and VXD. Their examination of in–sample and out–of–sample volatility forecasts reveals that neither implied volatility nor intraday high – low range volatility consistently outperforms the other.

The same conclusions are also reached by Becker et al. (2007) and Becker and Clements (2008) who compare VIX not to a single model but to a combination of forecasting models (GARCH, SV, ARFIMA and MIDAS models).

Banerjee et al. (2007) also find that VIX–variables have strong predictive ability.

The empirical results of Areal (2008), for the UK equity market, indicate that historic volatility is outperforming in forecasting the future realized volatility in all three of his alternative implied volatility indices.

Bekiros and Georgoutsos (2008) attempt to predict the direction of change of the S&P 500 index over a 4 – year period (1998 – 2002) by means of a recurrent neural network (RNN). They demonstrate that the incorporation of the VIX changes strongly enhances its profitability during "bear" market periods. This improvement is measured in comparison with a RNN including changes of estimated conditional volatility measures, a linear autoregressive model as well as to a buy–and–hold strategy. They suggest a number of theories that are consistent with their findings.

Konstantinidi et al. (2008) examine the possibility of forecasting the evolution of implied volatility, using a VAR analysis based on a set of eight US and European volatility indices: VIX, VXO, VXN, VXD, VDAX, VX1, VX6 and VSTOXX. The economic significance of the predictions is also assessed by trading games in the recently inaugurated CBOE volatility futures markets. Regarding the point forecasts, the in–sample statistical evidence suggest that the degree of predictability differs among indices and also depends on the choice of model and forecasting horizon. However, the considered models do not outperform the random walk model in an out–of–sample setting. The interval forecasts have no predictive power either. The trading games reveal that no economically significant profits can be attained.

Siriopoulos and Fassas (2008) find that VFTSE contains information about future volatility in the UK stock market beyond the one period lagged realized volatility.

In 2012, they extend the empirical evidence in favour of implied volatility comparing again with the one period lagged realized volatility. This time, they use their volatility index proposal for the Greek market (GRIV).

González and Novales (2009 and 2011), who proposed the VIBEX–NEW for the Spanish stock market, support an alternative interpretation. Using data from their proposed index in addition with data from VIX, VDAX–NEW and VSMI, conclude that the volatility index plays a good role in short term horizons, while not being very useful to advance the future behaviour of volatility, at least over long periods of time. According to this view, stock market participants seem to pay more attention to current conditions than to anticipating future fluctuations when trading options. This leads to implied volatilities which are more closely related to current and past than to future market conditions. Forecasts of future realized volatility obtained from volatility indices are as good as those obtained from historical volatility, but not good enough to be used for risk management.

Hung et al. (2009) suggest that VIX has better information content for improving volatility forecasting performance.

Li and Yang (2009) find the implied volatility forecasting performance better than historical volatility, using data from the Australian market.

Regarding emerging markets, McAleer and Wiphatthananthakul (2010) show that Taiwan VIX provides more accurate forecasts of option prices than their proposed SEV index, but the SEV index outperforms TVIX in forecasting expected volatility.

Tzang et al. (2011), based on different rollover rules, find that illiquidity in Taiwan's options market does not lead to substantial errors in the forecasting effectiveness of the volatility indices. The forecasting ability of VIX based on different sampling methods is found to be superior to that of VIX in Taiwan.

Yang and Liu (2012) also investigate Taiwan stock market and suggest that TVIX outperforms GARCH and HISVOL models

Kumar (2012) shows that the India VIX also outperforms the one period lagged realized volatility to forecast future volatility.

The same year, Ryu analyzing Korea's VKOSPI, finds that the volatility index outperforms the Black – Scholes implied volatility the RiskMetrics approach, and the GJR–GARCH model, although it provides slightly biased forecasts.

Forecasting using the volatility term structure

An interesting application of volatility indices is the calculation of forward volatilities by using the volatility term structure estimations which means that the implied volatility is used as a function of maturity. Similarly with the way forward interest rates are inferred from the term structure of interest rates, volatility indices constructed for different terms to maturity allow the calculation of forward volatility rates. The information content of forward volatilities is valuable since it offers insights into what the market expects to happen in the future (Lopez and Navarro, 2012).

Referring to this field, the following studies have been found:

Mixon (2007) tests the expectations hypothesis of the term structure of implied volatility for several stock market indices. The results indicate predictive ability, although not to the extent predicted by the expectations hypothesis.

Äijö (2008) presents new evidence on stock market integration by investigating the implied volatility term structure linkages between VDAX–NEW, VSMI, and VSTOXX volatility indices, calculated from options expiring in two, six, nine and eighteen months. The results of his study demonstrate that the estimated volatility term structures are highly correlated and vary over time, indicating that they are closely linked to each other. The results of the variance decomposition analysis further show that a large proportion of the forecast variance of the term structure of the SMI and the STOXX can be explained by the term structure of the DAX. Thus, volatility prediction methods can be improved by taking into account the innovations of the implied volatility term structure of the DAX.

3.8 Spillover Effects

Volatility plays the key role in the pricing of derivative contracts, so volatility spillover is particularly important for option portfolio managers. According to Badshah (2009), there are many studies which have investigated volatility transmissions using historical volatilities (indicatively: Hamao et al, 1990, Koutmos and Booth, 1995, Koutmos, 1996, Cifarelli and Paladino, 2005).

However, fewer studies have examined the dynamic behaviour of implied volatility and volatility transmission across markets (for example, Nikkinen and Sahlström, 2004 and Nikkinen et al., 2006).

This thesis focus on studies which use implied volatility indices to analyze whether there is a transmission of volatility across different markets or across different volatility indices.

Aboura and Villa (1999, 2003) were the first to examine spillovers between VX1, VDAX and VIX. They find that the French index is more sensitive to a shock than the US volatility index, since there is a strong correlation between them. They also observe that VIX is clearly the most influential since it explained around 11% of the error variance of VX1 and VDAX, while each one if the two European indexes does not explained more than 1% of the US error variance.

Skiadopoulos (2004) finds a contemporaneous spillover of change between his proposed volatility index GVIX and the US volatility indices (VXO and VXN). However, no lead – lag effects are present.

Wagner and Szimayer (2004) investigate the behavior of implied market volatility indices for the U.S. and Germany under a straightforward mean reversion model that allows for Poisson jumps. Their findings for daily data in the period 1992 to 2002 provide evidence of significant positive jumps, i.e. situations of market stress with positive unexpected changes in ex ante risk assessments. Jump events are mostly country–specific with some evidence of volatility spillover. Analysis of public information around jump dates indicates two basic categories of events. Crisis events occurring under spillover shocks, is the first category. Second, information release events which include three subcategories, namely–worries about as well as actual–unexpected releases concerning U.S. monetary policy, macroeconomic data and corporate profits. Additionally, foreign exchange market movements may cause volatility shocks.

Äijö(2008) investigates the relation between the new European volatility indexes (VDAX–NEW, VSMI and VSTOXX), examining term structure linkages. He finds that VDAX–NEW is the dominant source of information. VDAX–NEW Granger–causes both VSMI and VSTOX, and the variance of the forecast errors of the implied volatility term structure of the VSTOXX and VSMI explain 65% and 35%, respectively, of the implied volatility term structure of the VDAX.

Konstantinidi et al. (2008) perform a vector autoregressive analysis (VAR) based on a set of seven volatility indices: VIX, VXO, VXN, VXD, VDAX–NEW, VCAC and VSTOXX. They find that the coefficients of the one–day lagged differences of some of the CBOE volatility indices are statistically significant for explaining the changes in the European volatility indices, while only the coefficient on the French VCAC is statistically significant for explaining movements in the US volatility indices.

Badshah (2009), using VIX, VXN, VDAX–NEW and VSTOXX finds significant spillover effects across the indices and bi–directional causality running between them. He mentions that a shock to VIX increases considerably the levels of all the other volatility indices, in a contemporaneous manner and the effects persist on average for 4 to 6 days. In Europe, VDAX–NEW is the dominant source of information. Conversely, a shock to VDAX–NEW to some extend affects the US indices as well.

In 2010, Cohen and Qadan make a unique study. They investigate relations between the fear gauge index (VIX) and gold price, in order to test whether gold price is motivated by VIX or vice versa. Using daily prices for a 56 – month period covering 2004 – 2009 and a GARCH model, they find that VIX is positively affected by previous day gold return, implying that the rising in gold prices lead to higher fear level. Additionally, in stable, low volatile periods, where there are small VIX fluctuations and the capital market exhibits stable and/or rising in prices, a bi–directional causality exists between VIX and gold prices, whereas during non – stable periods the gold price returns were found to be Granger causes the VIX. Both findings support the claim that gold is still the shelter to fear.

More recently, in 2012, Badsah et al. take the research further. They examine the contemporaneous spillover effects among the CBOE implied volatility indices for stocks (VIX), gold (GVZ) and the exchange rate (EVZ). They use the "identification through heteroskedasticity" approach of Rigobon (2003) using a multivariate GARCH(1, 1) model to decompose the contemporaneous relationship between these implied volatility indices into causal relationships. Their data covered a 43 – month period between 2008 and 2011, since GVZ and EVZ were back then newly introduced. Their findings suggest that there is strong unidirectional, spillover from VIX to GVZ and EVZ, where increases in stock market volatility lead to increases in gold and exchange rate volatility; and bi–directional spillover between GVZ and EVZ. They emphasize the implications of their model by comparing the impulse – responses generated by their structural VAR with the impulse – responses of a traditional VAR. The results show that responses to shocks originating in GVZ and EVZ are seriously overestimated in the traditional VAR. These findings on the direction and magnitude of spillover and the long – run impact on volatility have important implications for portfolio and risk management.

Kumar (2011) analyzes the spillover effect between VIX and India VIX, finding that only changes in the US market affect India market.

In 2012, he searches for volatility transmission between three developed stock markets from three different geographic regions (US, UK and Japan) and an emerging market, India. Results based on a VAR analysis show that the coefficients of past changes in VIX are statistically significant for explaining changes in VFTSE, VXJ and India VIX. The coefficients of past changes in VFTSE, are statistically significant for explaining changes in VXJ, suggesting that there is volatility contagion between US, European and Asian stock markets.

Füss et al. (2011) study a different effect. Searching for the impact of scheduled macroeconomic announcements on option implied volatilities, they examine the link between German and US macroeconomic events and the respective implied volatility indices VDAX–NEW and CBOE VIX. They identify a volatility spillover effect and significant covariance clustering between VDAX–NEW and VIX.

Siriopoulos and Fassas (2012) show that there is a leading volatility spillover effect from the German and US stock markets, as measured by VDAX–NEW and VIX, to the developing Greek market, as measured by their GRIV.

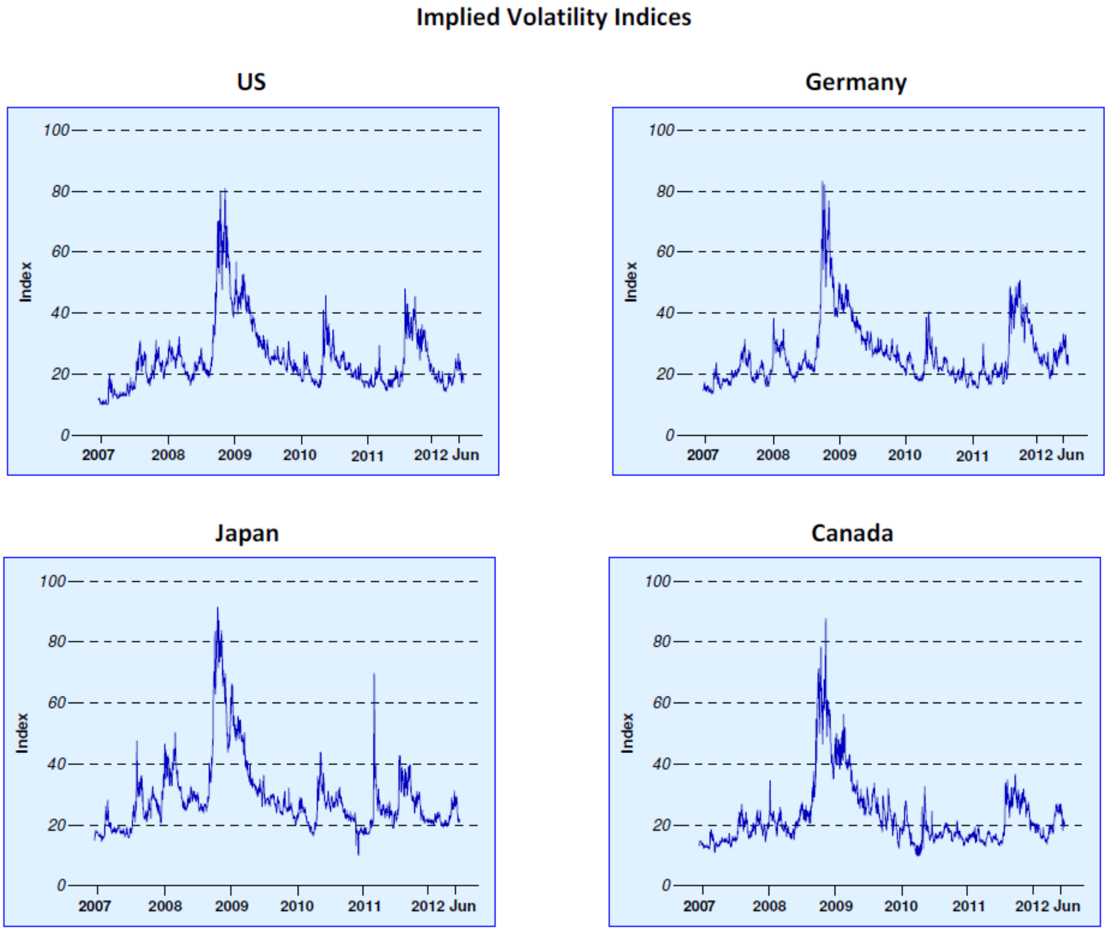
Peng and Ng (2012) explore the cross – dependence between five equity indices (S&P 500, NASDAQ 100, DAX 30, FTSE–100 and Nikkei 225) and their corresponding volatility indices (VIX, VXN, VDAX–NEW, VFTSE and VXJ). They propose a dynamic mixed copula approach which is able to capture the time – varying tail dependence coefficient (TDC). Their findings indicate the existence of financial contagion and significant asymmetric TDCs for major international equity markets. Although contagion cannot be clearly detected by stock index movements in some situations, it can be captured by dependence between volatility indices. The results imply that contagion is not only reflected in the first moment of index returns, but also in the second moment, the volatility. Results show also that dependence between volatility indices is more easily influenced by financial shocks and reflects instantaneous information faster than the stock market indices.

The same year, Narwal et al. search for spillovers from India to US, France, Germany and Switzerland, through India VIX, VIX, VDAX–NEW, VCAC and VSMI, respectively. The results obtained from a bivariate BEKK–GARCH were surprising, since they show outward volatility spillovers from the emerging market to the developed. However, the authors mention that in the real world mature markets are the dominant players.

On the other hand, Chiang (2012), adopting a bivariate GARCH model with TAR, examines the extent of volatility as well as return transmission between S&P 500 (NASDAQ 100) and VIX (VXN) since the introduction of VIX and VXN. Results show that the performance of VIX index is the best among the four indices during the whole sample period. But the volatility of VIX is also higher than other indices. Further, only lagged negative return (change) has a bi–directional casual effect in the low–fear regime for the SP500/VIX series. The results also indicate that VIX index market has a stronger pricing effect on S&P 500. However, there is no obvious lead–lag relationship between NASDAQ 100 and VXN. Moreover, the return and volatility responses to high–fear and low–fear gauge are asymmetrical.

From the aforementioned is concluded that never before global markets were so closely integrated. Especially during the recent financial crisis which began in 2008 from US and is still evolving, spillover effects are observed throughout the globe. A characteristic example supporting this argument is figure 3.8, which presents implied volatility indices of both developed and emerging markets. It is easily observed the sharp increase of risk which quadrupled in US in the fall of 2008 and spread to other economies. Figure 3.8 includes a 5 – year period (2007 – 2012) of 12 economies.

Figure 3.8: Implied Volatility Indices of 12 Economies



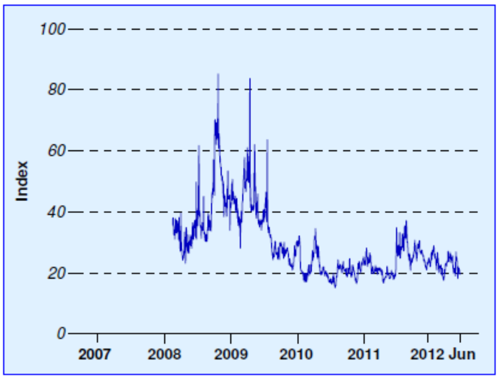
France

Switzerland

Belgium

Netherlands

India



Mexico

South Korea

South Africa

Source: Datastream and local stock markets

3.9 *Various Studies including Volatility Indices*

Arisoy et al. (2007) examine whether volatility risk is a priced risk factor in securities returns, using VIX. Zero–beta at–the–money straddle returns of the S&P 500 index are used to measure volatility risk. It is demonstrated that volatility risk captures time variation in the stochastic discount factor. The results suggest that straddle returns are important conditioning variables in asset pricing, and investors use straddle returns when forming their expectations about securities returns. One interesting finding is that different classes of firms react differently to volatility risk. For example, small firms and value firms have negative and significant volatility coefficients, whereas big firms and growth firms have positive and significant volatility coefficients during high-volatility periods, indicating that investors see these latter firms as hedges against volatile states of the economy.

Dotsis et al. (2007) explore the ability of alternative popular continuous–time diffusion and jump diffusion processes to capture the dynamics of implied volatility over time. Data is employed from the rapidly growing CBOE volatility futures market and eight US and European volatility indices: VIX, VXN, VXO,VXD, VDAX, VX1, VX6 AND VSTOXX. The performance of the volatility processes is assessed under both econometric and financial metrics. They find that the simplest diffusion/jump diffusion models perform best under both metrics. Mean reversion is of second order importance. The results are consistent across the various markets.

Becker et al. (2009) suggest that VIX subsumes historical information on the contribution of price jumps to volatility and it reflects all incremental information relative to model based forecasts pertaining to future jump activity. Their study differs from others, since others do not consider whether volatility is caused by continuous or non–continuous price changes.

Psychoyios et al. (2010), based on the fact that jumps are widely considered as a salient feature of volatility, although their implications for pricing implied volatility options and futures are not yet fully understood, they provide evidence indicating that the time series behavior of the VIX equity implied volatility index is well approximated by a mean reverting logarithmic diffusion with jumps. This process is capable of capturing stylized facts of VIX dynamics such as fast mean–reversion at high levels, level effects of volatility and large upward movements during times of market stress. Based on this process, they develop closed form valuation models for volatility futures and options and show that incorrectly omitting jumps may cause considerable problems to pricing and hedging.

Duan and Yeh (2010), develop an estimation method for extracting the latent stochastic volatility from VIX. Their model specification encompasses all mean–reverting stochastic volatility option pricing models with a constant–elasticity of variance and those allowing for price jumps under stochastic volatility. The approach is made possible by linking the latent volatility to the VIX index via a new theoretical relationship under the risk – neutral measure. Because option prices are not directly used in estimation, they can avoid the computational burden associated with option valuation for stochastic volatility/jump option pricing models. The empirical findings are: a) incorporating a jump risk factor is critically important; b) the jump and volatility risks are priced; c) the popular square – root stochastic volatility process is a poor model specification irrespective of allowing for price jumps or not. Their simulation study shows that statistical inference is reliable and not materially affected by the approximation used in the VIX index construction.

3.10 Why do we need Volatility Indices?

From the literature review presented above, it is concluded that implied volatility indices have a number of applications. Overall, these findings have important implications for risk management, portfolio formation and management, trading and hedging strategies, pricing and hedging volatility derivatives. Specifically:

1. First of all, the highly negative asymmetric correlation between volatility indices and their underlying equity indices make them important financial instruments for hedging stock portfolios, or portfolio diversification. Derivatives exchanges all over the world provide liquid markets for futures and options underlying these indices. Therefore, the position in futures or options on a volatility index can more accurately hedge the stock portfolio position without considering complicated stock index option trading strategies. For example, as Arora (2010) suggests using VIX data, being able to meaningfully interpret movements in the VIX and its reaction to market events can give investors an edge in managing the risk and profitability of their trading book and in designing portfolio strategies using VIX derivatives to capitalize on the dynamic and time – varying correlation of the VIX with its underlying S&P 500 Index.

2. The same correlation makes volatility indices useful not only for assessing potential risks, but also for speculative transactions by risk – seeking investors.
3. Since most of the volatility indices are based on the robust model – free methodology and provide better tradability, it is easier for derivatives issuers to engineer structured products on these indices.
4. Trading strategies could be exploited for profit generation. For instance, a volatility long position in decreasing volatility markets while a volatility short position in increasing volatility markets.
5. Their forecasting ability, also make volatility indices unique tools. As demonstrated, volatility indices are among the most reliable predictors of future. And prediction reduces risk and generates profits. For example, as Giot (2005b) suggests extremely high volatility levels may signal attractive buying opportunities for traders or according to Bellini and de la Torre – Callegos (2011) high readings of VIX usually occur after a market sell–off and indicate that a long–term position should be taken. Low readings usually occur after a rally and imply a short–term position. This data can help managers create market timing strategies.
6. Spillover effects between equity markets and volatility indices indicate that investors' expectations about the future realized volatilities are robustly related across the globe. Therefore, portfolio managers and option traders could incorporate correlated expectations using the volatility indices' implied correlations into their models. These correlations can be also incorporated into value–at–risk estimations.
7. From the studies examining relations between volatility indices and macroeconomic announcements, it is concluded that if lower volatility is a desirable outcome for policy makers, then central banks should become more effective at communicating not only their current policy stand, but their expected future course of action as well.
8. Finally, the greatest feature of volatility indices is their unique information content which allows research on various fields, from investors' psychology to forecasting and engineering new derivatives products. Their information content, which is beyond that conveyed by historical returns even under greater uncertainty, is also indicative of the anticipated risk and magnitude of crises. Therefore, understanding the dynamics of volatility expectations constitutes an integral part of the solution to crises.

Summary and Conclusions

This thesis was divided in three chapters.

The first presented the most common volatility forecasting models, divided in four major categories: historical volatility models, ARCH family models, stochastic volatility models and implied volatility models. It was shown that implied standard deviation (ISD) models are superior estimators of future. A further research is definitely required between stochastic volatility and other kinds of models, since there were found only 8 papers referring to implied volatility.

The second chapter described the most common pricing methods of derivatives. They were also divided in three major categories: forward and futures contracts swaps and options. Implied volatility was further explained. It was shown that the Black – Sholes pricing method is the cornerstone of derivatives pricing and one of the main reasons of derivatives bloom.

Finally, the third chapter presented for the first time a complete comprehensive academic review of volatility indices, consisting of approximately 100 studies. Most important theoretical developments and empirical insights of all kinds of both actually introduced and academically proposed volatility indices were presented, in addition with their calculation processes and their practical applications. It was shown that volatility indices are principally applied in forecasting, since 28 studies examined this subject. Significant research, including 30 papers, was also found studying the information content of volatility indices and their relations with their underlying indices. Furthermore, many studies found spillover effects between volatility indices and different markets or sectors which seem to be uncorrelated at first sight, such as macroeconomic announcements, or the commodity market of gold.

Concluding, it was found a number of applications for volatility indices.

It is obvious that volatility indices are among the most practical and reliable tools in finance. The majority of the studies discussed used as data the CBOE indices and especially VIX. It would be interesting if future research focus on the information content of other equity markets volatility indices, such as Russia’s, South Africa’s or Australia’s, since there were found no papers referring to these indices.

Regarding spillover effects a further research between emerging and developed markets and more importantly, between equity and commodity markets such as silver, or crude oil is proposed.

Since many indices are newly introduced, it is sure that research will continue presenting fascinating results with the use of implied volatility indices.

As an epilogue of this thesis, table 3.7 is presented containing a summary of all papers examining volatility indices.

Table 3.7: Summary of Papers and Volatility Indices examined

Volatility Index	Studies on the information content and correlations	Forecasting performance	Spillover effects and other studies
US VXO	Whaley (1993, 2000), Fleming et al. (1995), Carr and Madan (1998), Britten-Jones and Neuberger (2000), Pan et al. (2003), Carr and Lee (2003), Carr and Wu (2005), Poon (2005), Giot (2002, 2005a, b), Vähämaa, S., (2009), Symeonidis et al. (2010), Gospodinov and Jamali (2011)	Fleming et al. (1995), Blair et al. (2001), Giot (2002, 2005a, b), Corrado and Miller (2005), Corrado and Truong (2007), Konstantinidi et al. (2008)	Skiadopoulos (2004), Wagner and Szimayer (2004), Konstantinidi et al. (2008), Dotsis et al. (2007), Psychoyios et al. (2010)
US VIX	Demeterfi et al. (1999a, b), Poon (2005), Jiang and Tian (2005, 2007), Whaley (2008), Hibbert et al. (2008), Bandopadhyaya and Jones (2008), Badshah (2009, 2012), Simlai (2010), Symeonidis et al. (2010), Kozyra and Lento (2011), Bellini and de la Torre-Callegos (2011), Füss et al. (2011), Gospodinov and Jamali (2011), Sarwar (2012a, b)	Aboura and Villa (1999, 2003), Corrado and Miller (2005), Corrado and Truong (2007), Banerjee et al. (2007), Becker et al. (2007), Becker and Clements (2008), Bekiros and Georgoutsos (2008), Konstantinidi et al. (2008), González and Novales (2009, 2011), Hung et al. (2009), Maghrebi et al. (2010), Tzang et al. (2012)	Aboura and Villa (1999, 2003), Konstantinidi et al. (2008), Badshah (2009, 2012), Cohen and Qadan (2010), Kumar (2011, 2012a), Füss et al. (2011), Siriopoulos and Fassas (2012), Peng and Ng (2012), Narwal et al. (2012), Chiang (2012), Badshah et al. (2012) Arisoy (2007), Dotsis et al. (2007), Becker et al. (2009), Duan and Yeh (2010), Arora (2010),
US VXN	Giot (2002, 2005a, b), Simon (2003), Badshah (2009, 2012), Symeonidis et al. (2010)	Giot (2002, 2005a, b), Corrado and Miller (2005), Corrado and Truong (2007), Konstantinidi et al. (2008)	Skiadopoulos (2004), Konstantinidi et al. (2008), Badshah (2009, 2012), Peng and Ng (2012), Chiang (2012), Dotsis et al. (2007)
US VXD	Symeonidis et al. (2010)	Corrado and Truong (2007), Konstantinidi et al. (2008)	Konstantinidi et al. (2008), Dotsis et al. (2007)
US GVZ			Badshah et al. (2012)
US EVZ			Badshah et al. (2012)
German VDAX		Aboura and Villa (1999, 2003), Bluhm and Yu (2002), Claessen and Mittnik (2002), Konstantinidi et al. (2008)	Aboura and Villa (1999, 2003), Wagner and Szimayer (2004), Dotsis et al. (2007)
German VDAX–NEW	González, and Novales (2009), Badshah (2009, 2012), Füss et al. (2011)	Äijö (2008), González and Novales (2009, 2011), Maghrebi et al. (2010)	Äijö (2008), Konstantinidi et al. (2008), Badshah (2009, 2012), Füss et al. (2011), Siriopoulos and Fassas (2012), Peng and Ng (2012), Narwal (2012)

French VX1 – VX6 (old and new)	Brenner and Galai (1989)	Moraux et al. (1999), Aboura and Villa (1999, 2003), Konstantinidi et al. (2008)	Aboura and Villa (1999, 2003), Dotsis et al. (2007)
French VCAC			Konstantinidi et al. (2008), Narwal (2012)
UK VFTSE	Siriopoulos and Fassas (2008), Areal (2008)	Siriopoulos and Fassas (2008), Areal (2008)	Kumar (2012a), Peng and Ng (2012)
Swiss VSMI	González, and Novales (2009)	Äijö (2008), González and Novales (2009, 2011)	Äijö (2008), Narwal et al. (2012)
EU VSTOXX	Badshah (2009, 2012)	Konstantinidi et al. (2008), Äijö (2008)	Konstantinidi et al. (2008), Äijö (2008), Badshah (2009, 2012), Dotsis et al. (2007)
Spanish VIBEX and VIBEX–NEW (unofficials)	Giner and Morini (2004), González and Novales (2007, 2009, 2011)	González and Novales (2009, 2011)	
Greek GVIX and GRIV (unofficials)	Skiadopoulos (2004), Siriopoulos and Fassas (2012)	Skiadopoulos (2004), Siriopoulos and Fassas (2008)	Skiadopoulos (2004),
Italian Index (unofficial)	Muzzioli (2010)		
Japanese VXJ	Maghrebi et al. (2006, 2007), Fukasawa et al. (2011)	Maghrebi et al. (2006, 2007, 2010)	Kumar (2012a), Peng and Ng (2012)
Korean VKOSPI	Maghrebi et al. (2006, 2007),Ting (2007), Ryu (2012)	Ryu (2012)	
India VIX	Kumar (2012b), Dhananjhay et al. (2012), Bagchi (2012)	Kumar (2012a, b)	Kumar (2011), Narwal et al. (2012)
Taiwan TVIX	McAleer and Wiphatthananthakul (2010)	McAleer and Wiphatthananthakul (2010), Tzang et al. (2012), Xang and Liu (2012)	
Australian AVIX and XVI	Dowling, S. and Muthuswamy (2005), Frijins et al. (2010)	Dowling, S. and Muthuswamy (2005), Li and Yang (2009)	
New Zealand NZVIX (unofficial)	Frijins et al. (2008)		
SEV (unofficial)	McAleer and Wiphatthananthakul (2010)		
CX (Cor.Impl.) unofficial)	Andersen et al. (2011, 2012)		
IRVIXs (unofficial)	Lopez (2001), Lopez and Navarro (		
Europe, US and ASEAN Vol. Indices (unofficials)	Ninanussornkul et al. (2009a, b)		

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