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DYNAMIC ANALYSIS OF THE FTSE20, FTSE40 AND FTSE80 RETURNS SERIES OF THE ATHENS STOCK EXCHANGE MARKET

of

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Dedications

Especially dedicated to my mother for her unconditional love and endless support through all these years

Εξαιρετικά αφιερωμένη στην μητέρα μου, για την ανιδιοτελή αγάπη και την αδιάκοπη υποστήριξή της όλα αυτά τα χρόνια

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Abstract

During the past two decades academic researchers and economic analysts have been increasingly interested in the area of chaotic dynamics. In general it is widely accepted that financial markets are highly complex systems and both deterministic and stochastic descriptions are essential in order to identify main attributes of their dynamics. This paper examines the existence of a deterministic chaotic structure in the stock returns of the Athens Stock Exchange Market and in particular the FTSE20, FTSE 40 and FTSE 80 indeces. The overall result suggests that the returns series do not follow a random walk process. In order to test for chaos the diagnostic tools of correlation dimension and the Largest Lyapunov Exponent were calculated. The results reveal there is weak but clear evidence of high-dimensional chaotic structure in ASE high-frequency returns series.

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Introduction

In the past economists used to apply linear equations in an attempt to describe economic trends, since these were easier to handle and most of the times provided them with unique solutions. However as the sciences of mathematics and statistics evolved they offered to the economic analysts more complicated and sophisticated tools for analyzing and studying financial time series. Examples of phenomena where linear models could not be applied include depression and recession periods, extreme stock market price fluctuations, economic bubbles and corresponding crashes. In order to understand and explore such situations non-linear dynamics and chaos theory were employed.

During the last few decades there has been a growing interest in non-linear dynamic models in the field of economics. According to the Efficient Market Hypothesis (EMH), which was the cornerstone of numerous scientific articles and theories, market destabilization can only arise by the appearance of exogenous shocks such as technological evolution and altering policy decisions and furthermore all markets follow a random walk. On the other hand there is a vast amount of papers in literature that implies that markets are highly complex systems and non-linear economic models exhibit chaotic dynamics. So as to define key features of such dynamics both deterministic and stochastic descriptions are required. It has been proven that special classes of deterministic non-linear systems have the ability to produce self-sustained fluctuations without the appearance of exogenous shocks. Hence they can adequately explain large movements in financial data.

One of the most distinguishing attributes of chaotic processes is that these systems are extremely sensitive if initials conditions are changed even by a small amount. Furthermore chaotic processes are characterized by lack of long-term predictability whereas short-term forecasting is possible. During the last decades several tools for the detection of chaos in time series have been developed. Their aim was to distinguish between data produced by a deterministic system and data generated by random systems. It must be noted that it is essential in order to get accurate and reliable results concerning chaos that high quality and high frequency data are used. The most commonly used tests for the detection of chaos in time series are the Grassberger-Procaccia correlation dimension and the Largest Lyapunov Exponents. Correlation dimension can reveal the embedding dimension and can measure the convergence of all the trajectories towards the attractor. Largest Lyapunov

Exponent is appropriate for investigating the convergence or divergence of trajectories in phase space i.e. measures the rate of separation between two initially close trajectories. A positive Largest Lyapunov Exponent gives an indication of the sensitive dependence on initial conditions, and hence shows whether the system exhibits chaotic behavior.

During the past few years the presence of chaos in financial markets have been examined mostly in cases concerning major stock markets and world's developed economies. Little attention has been given in the examination of the chaotic structure in smaller stock markets and emerging economies. However it is likely that the latter exhibit different characteristics in their dynamics compared to major and mature markets. Something like that could be of great interest for investors, traders and policy makers. The aim of this study is to investigate and identify the nature of the underlying process of the Athens Stock Exchange returns series. In order to test for the existence of chaotic structure**,** the correlation dimension (Grassberger and Procaccia, 1983) and the Largest Lyapunov Exponent (Kantz, 1994) were calculated. High frequency daily data were used.

The rest of the paper is organized as follows: Chapter 1 presents a brief review of literature, Chapter 2 provides a brief account of the tests used in this study, Chapter 3 presents empirical results, Chapter 4 provides a discussion of the outcomes and the paper ends with some concluding remarks.

Chapter 1: Chaos theory. Tools and evidence

1.1 Complexity

1.1.1. General remarks and definitions

Up until the early decades of the twentieth (20^{th}) century the paradigm of order was well established and accepted by the majority of the scientific community. Furthermore the linear aspect of the world was adopted, not only in science, but also in the broader areas of social and political fields. The basic characteristics of this traditional paradigm were (a) **order**: when the causes are known, the effects can always be determined i.e. causes and effects can be linked, (b) **reductionism**: if a system is to be studied it is feasible be broken down to its smaller parts and by the examination of those conclusions can be drawn as far as the whole system is concerned, (c) **predictability**: by using a suitable model future behavior of the system can be predicted, and finally (d) **determinism**: the processes involved lead the system to unique and rational ends.

However not all phenomena, in fact the majority of them, do not comply with the rules of the paradigm of order. A significant number is characterized by complexity. But what does complexity mean? And how can somebody use it in order to explain the phenomena previously mentioned?

Over the past few years many have tried to come up with an appropriate and adequate definition of complexity, but all failed. The problem was that some definitions were only applicable to certain small areas of interest, while others were too general and vague. A system can be identified as complex when it is comprised by a vast number of elements and these elements are interconnected by numerous ways. These elements can not be separated unless the system is destroyed. Hence the method of decomposition can not be applied in this case, something that would have allowed a simpler analysis of the system. Additionally a reliable model of prediction for such system will be difficult to construct and these kinds of problems will be almost impossible to be solved.

One must always keep in mind that complexity is closely associated with the aspects of distinction and connection. The term distinction implies that the various elements of the system behave in different ways thus leading to disorder and chaos. On the other hand connection suggests that the different elements are related, hence knowing one part can lead to the establishment of characteristics of other parts and consequently to order. In other words complexity is a state between order and chaos or as it is more commonly known “at the edge of chaos”.

According to Lucas (2004) complexity can be divided in various ways, namely:

- **Static complexity**: This is the most simple type of complexity and its main characteristic is that the system under examination does not vary with time.
- **Dynamic complexity**: In this type the fourth dimension, that of time, is taken into account.
- **Evolving complexity**: In this case the manner through which systems evolve over time into different systems is considered.
- **Self-organizing complexity**: This is the most interesting type of all. Its main feature is that “a system as co-evolving with its environment so much so that classifications of the system alone, out of the context, are no longer regarded as adequate for a valid description.”

1.1.2. The theory of complexity

The theory of complexity deals with complex systems such as societies, stock markets and physical phenomena, which all resemble chaotic behavior as far as the factors that define and affect them are concerned, while it is quite difficult for someone to understand the way they are formed, function and evolve. Over the last few years the theory of complexity was used by sciences such as biology, chemistry, meteorology, physics and finance. It must be noted that the evolvement of computing engineering contributed the most to the development of the complexity theory since non-linear equations which are essential elements of complexity can only be dealt with strong computational techniques.

According to the complexity theory, the same system can exhibit different behavior under altering initial conditions. So sometimes under specific conditions a system can function regularly in predictable ways, whereas supposing other conditions present both regularity and predictability are lost. As American meteorologist Edward Lorenz (1961) observed during an experiment for modeling weather systems, even undetectable changes of the initial conditions would lead to chaotic behaviors, producing complexity and divergent outcomes. This is known as the “butterfly effect”, which states that under specific conditions, if a butterfly flaps its wings in China, this could produce the formation of a tornado in California.

In management the complexity theory treats a corporation more like a dynamic system that interacts with the environment in which it belongs, rather than a well designed machine where everything is programmed and predicted. The philosophy of such a system is that there is an inherent developed structure, which is not stable, but constantly changing, continuously adapting to the gained experience.

Summarising, it is clear that complexity theory has many applications in a wide range of sciences, from pure mathematics to social and humanitarian ones. How it has affected the way that financial markets work will be discussed in a following section of this study.

1.1.3. Complexity in economic and financial markets

The cornerstone of classical economics has been the efficient market hypothesis (EMH), which emphasizes that financial markets are characterized by order and rationality. For centuries economists deliberately avoided the complexities present in markets, by the justification that stock prices have the ability to adjust quickly in order to reflect any new information. In the case that stock prices appeared to move in a random way that was because of the random arrival of information and not due to heterogeneity and different beliefs of the crowd.

In Santa-Fe Institute a group of researchers have created an artificial stock market on the computer and studied its behavior. Their early findings seem to be in radical contradiction with what the efficient market hypothesis states. Economic agents take decisions based on their beliefs of what future prices will be and it is this procedure that forms the market. In other words beliefs can create economic behavior and economic results can influence beliefs. In most cases that could lead to complexity.

1.2 Theoretical background and analysis of articles relating to complexity and financial markets

In this section of the study articles, relative to the subject examined, will be briefly analyzed so as the reader can develop a more comprehensive view of the procedure followed in forthcoming sections. For reasons of simplicity the articles will be presented following the chronological order that they were written.

A vast number of models have been proposed over the years so as to accurately reflect the distribution of daily future returns. Unfortunately all of them have failed in one way or another.

In their paper, Yang and Brorsen (1994) have tried to find the most appropriate model among the diffusion jump, the GARCH process and the deterministic chaos processes. According to their methodology first the mixed diffusion jump process was tested against the changing variance model. Then the GARCH model was addressed under two distributional assumptions and market irregularities such as seasonality, day-of-the-week and maturity effects were taken into account. Furthermore market irregularities were more rigorously tested and finally the validity of the models was thoroughly examined.

In line with their results the case of deterministic chaos can not be excluded and no distribution was found to be normal. Finally the GARCH (1, 1) process with residuals following a t-student distribution was the one that fitted the data better but in the end it was proven not to be well calibrated.

As mentioned earlier the complexity theory refers to a state somewhere between order and chaos. In his paper Mouck (1998) presents an extensive analysis on how Mandelbrot's early findings of fractals affected other scientists and led to the development of a new paradigm which contradicts with the rules of the efficient market hypothesis and the capital market research paradigm.

In the paper the author states that Mandelbrot's beliefs that Newtonian classical mechanics can not be applied in real complex financial markets were constantly ignored. However as the world progressed and science evolved Mandelbrot's theories started gaining ground and helped in the establishment of the fractal market hypothesis stated by Peter in 1994. In the paper it is thoroughly described how chaos theory challenges basic methodological presumptions of an orderly world and how Mandelbrot's "Noah and Joseph effects" can be explained.

Mouck (1998) argues that stock prices behavior is highly influenced by heterogeneous expectations and various types of "herd behavior" and furthermore that the actual markets are responsible for any alterations in stock prices.

Asimakopoulos et al, (2000) in their paper provide an examination of the relationship between currency future returns. Their aim is to show that if volatility is not taken into account when tests are performed deceptive results may be obtained.

For their purposes they first used a vector autoregressive (VAR) model so as to remove any linear dependencies and after that the residuals were rested by applying a BDS test. Finally a non-linear Granger causality test was performed. The outcomes reveal that significant non-linear dependencies exist. The data used were daily currency future prices of the British Pound, the Deutsche Mark, the Japanese Yen and the Swiss Franc in US Dollars.

Although their early results indicate that future returns of one currency can be employed so as to predict changes in other currency future returns, further tests indicate much weaker relationships.

It is not unusual that statistical physics and thermodynamics are used in order to understand the complexity present in financial markets. In 2001 Mansilla used a quantity called physical complexity in order to study the behavior of markets in times before crashes and in times with no financial turbulence. He stated that financial data are quite useful and reliable since they are constantly monitored and thus a huge amount of data is available.

The main objective of his paper was to verify that physical complexity can be successfully applied to the time series of real financial markets. The data he used was from the US Stock Index (NASDAQ) and the IPC which is one the leading indexes of Mexican Stock Exchange. In order to use the data he first had to convert them to binary digits series.

Physical complexity can be defined as a number of binary digits that is meaningful with respect to the environment ε in a string η. In the paper the physical complexity of substrings of ε is examined. Furthermore the complexity of statistical ensembles of these substrings for different values of l time steps is studied.

For both indexes (NASDAQ and IPC) three different periods are selected. The authors propose the following type

$$C[l] = \delta l^{\alpha}$$

and C(l) is calculated for the binary sequences associated with the previously mentioned intervals. The exponent α provides a good indication of how close a market is to the situation described by the EMH.

According to his findings in the periods where nothing extraordinary was happening in markets i.e. there was a calm state, the informational contents of binary series were small indicating that the behavior of prices were random and hence all the available information was absorbed by the agents. However the informational contents of the binary series were high in periods before crashes indicating that the information available in the market was not fully integrated by the agents. As a conclusion it can be said that in periods where markets exhibit high volatility the informational content is bigger.

In his paper McMillan (2001) examines whether the use of linear or non-linear functional forms can provide more reliable results as far as the stock market returns predictability is concerned.

For the purposes of his paper he uses data from the S&P 500 stock market index. At first the author uses a linear model and shows that interest rates and macroeconomic variables indicate some kind of predictability. Following by employing model-free non-parametric methods he seeks for the presence of non-linear relationships between the variables.

A non-linear relationship is detected between returns and interest rates but not among returns and macroeconomic series. The author continues by building a smooth transition autoregressive (STAR) model from which the results obtained support a satisfactory in-sample performance and a marginally higher out-of-sample performance in comparison to the linear alternative.

The prediction of the volatility of stock series has been widely used by scientists in an attempt to evaluate and understand the dynamic behavior observed in financial markets. For this purpose a large number of generalized autoregressive conditional heteroskedasticity (GARCH) models have been developed. However these models' major drawback is their inability to detect any sign asymmetries, meaning that negative shocks to returns lead to higher prices of volatility than when equivalent positive ones occur. Furthermore the rate of persistence seems to be the same regardless of the magnitude of volatility (high/low) which contracts with the empirical results. Trends continue to exist for longer periods when the shocks imposed on returns are low and vice versa.

In order to overcome these difficulties Verhoeven et al.(2002), in their paper decided to use a non-linear non-parametric model based on canonical variate analysis and compare it with a "GJR-GARCH(1,1)-t" model. The data fed on the model were daily closing prices of the S&P 500 over a 4years period.

Based on the results the canonical variate analysis (CVA) model outperforms the GJR-GARCH (1, 1)-t model since it is shown that the asymmetry in the persistence of volatility is more efficiently detected. It verifies the observation that when large shocks are imposed volatility tends to be less persistent. It should be noted that the final remark of the authors mentions that the predicting ability of CVA models is less reliable during times of high volatility.

The main objective of .Kyrtsou and Terraza (2002) article is to validate the view that the interactions among agents with heterogeneous beliefs are the underlying causes of the excess irregularities of financial markets.

The authors suggest that it is essential to understand the way that investors think and act in order to be able to comprehend the changes in stock returns. Furthermore strong trends usually are the product of positive feedback loops regardless of how process move, up or down. In their study two kinds of investors exist, namely fundamentalists (rational) and noise traders. It is due to the interaction of those two types that price fluctuations are caused. In an attempt to show how the interactions among heterogeneous investors can lead to noisy chaotic evolution of price series the authors replicated the model of Chen et al., (2001). The data used for the purposes of the study were the daily index return series of the French Stock Exchange during a 12-years period. Several tests were applied namely:

i. The fractional integration test of Geweke and Porter-Hudak (GPH) in an attempt to deal with long memory properties
ii. The correlation dimension of Glassberger and Procaccia and the Lyapunov exponent method of Dencay and Dechert in order to handle chaotic structures
iii. The Principal Components Regression (PCR) and the Radial Basis Functions (RBF) were employed in order to predict the French stock returns.

According to the results the following were deduced. First the French stock returns series displayed short memory, second the series could be produced by either noisy chaotic or pure stochastic process and finally chaotic models performed better than the GARCH ones. However it is the authors' view that no matter what kind of sophisticated type of model is used forecast values will never be able to match real ones since complexity can not be fully depicted by standard econometric models.

It is widely accepted that stock prices experience non-linear dynamics. In order to evaluate these Shively (2003) uses a three-regime non-linear threshold random walk model. In general threshold models are widely used for the examination of non-linear time series. The main idea in this kind of models is that they separate the time series in specific regimes and they examine the properties of each one of them separately. In the paper data from France, Germany, United Kingdom, Japan, United States and Canada stock price indices are used.

Using the model mentioned earlier Shively (2003) tried to prove that while investors trade stocks in all three regimes they have larger motivation of selling stocks after unusually large positive returns and buying stocks after unusually large negative returns. In order to do so he had to illustrate that stock prices in outer two regimes are consistent with stationary mean reversing process.

Nevertheless according to the findings of the paper the stock prices are consistent with a regime revering process where on average, stock prices in the external two systems revert in the middle. This reveals that stock prices exhibit some kind of predictability and hence a violation of the efficient market hypothesis. The main outcome is that results have shown that stocks that performed badly in past experienced better returns compared to stock market averages in succeeding periods and vice versa.

In his paper Belaire-Franch (2004), tests for non-linear structures in an artificial financial market. In the first part of the paper he calculates Hinich's bi-spectrum test and White's neural network test so as to show that a more consistent rejection of the null of linearity is found. According to the findings, the latter outperforms Kaplan's test and the BDS test in terms of detected non-linearity.

In the second part of the paper the author examines a methodology borrowed from Physics, the recurrence quantification analysis in order to test for hidden structures in data. It is believed that this method offers greater information when compared to the analysis of univariate series only. The results verify that this method can be extremely useful for detecting complexity in financial data series.

In accordance with several studies it has become clear that because of the non-linear structure of financial markets the prediction of stock returns is highly complex in some cases even impossible. Thawornwong and Enke (2004) in their paper however support that the use of data-mining techniques and carefully selected neural networks can be of great help in this area. Their novelty lies on the fact that neural networks do not necessitate the early specification of the relationships between input and output variables.

In their work they specify two types of neural networks, namely the feed-forward and the probabilistic, which are in turn compared against the classical linear regression model. The data used are from the S&P 500 stock index.

Their main findings indicate that stock returns can be predicted but the whole process is still very complicated since the existence of other factors such as psychological, social or political events can significantly alter and influence the procedure. Nonetheless, the results demonstrate that the use of redeveloped neural network models lead to higher profits with lower risks in comparison with the buy and hold strategy, the classical linear regression and the random walk model.

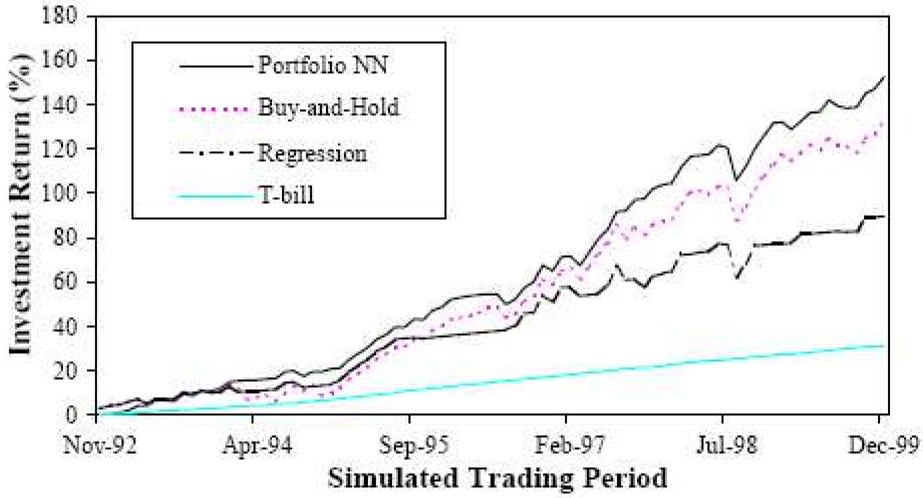


Figure 1.1: Graphs of the cumulative investment returns (Thawornwong and Enke, 2004)

Significant number of articles has been written concerning the behavior of financial markets when extreme fluctuations are observed. One of the most common problems encountered in non-linear models is that of over-fitting i.e. existence of in-sample behavior performs satisfactory but there is low out-of-sample performance.

In their paper Bradley and Jansen (2004) examine the likelihood that stock returns can be described by using a non-linear state-dependent model which has the ability to show different dynamics in periods following big sways in stock returns. Furthermore they explore if periods of high volatility in stock markets can be linked to real sector performance. In addition excess stock returns and industrial production growth are estimated using non-linear models and their forecasting performance is evaluated against that of linear models.

Authors test for linearity the BDS statistic whereas for non-linearity they use the logistic smooth transition autoregressive (LSTAR) model and the multiple regime smooth transition autoregressive (MRSTAR). An alternative non-linear model is also used which is based on the “Current Depth of Recession” (CDR) variable. The data used for the purposes of the paper were the monthly percentage change in seasonally adjusted industrial production and the monthly excess returns of the S&P 500 stock index.

Based on their findings, when excess stock returns are examined the linear model outperforms all non-linear models. However in the case of industrial production non-linear models seem to “behave” better than the linear ones and in particular CDR model outperforms the others.

Much has been written during the past decades about the reliability of both linear and non-linear models as tools for predicting performance of financial time series.

In their paper Clements et al. (2004), address several theoretical and empirical issues such as predictive density, loss functions, data mining and aggregation. In accordance with their results there still has not been discovered a non-linear model able to estimate and forecast procedures in a reliable and simple way.

According to the efficient market hypothesis, past price data can not be used in order to forecast future values since all the history of the asset is included in its current value. However empirical evidence states otherwise. Strozzi and Zaldivar (2005) in their paper use a new methodology based on a state space reconstruction technique attempting to qualify the amount of predictability in time series.

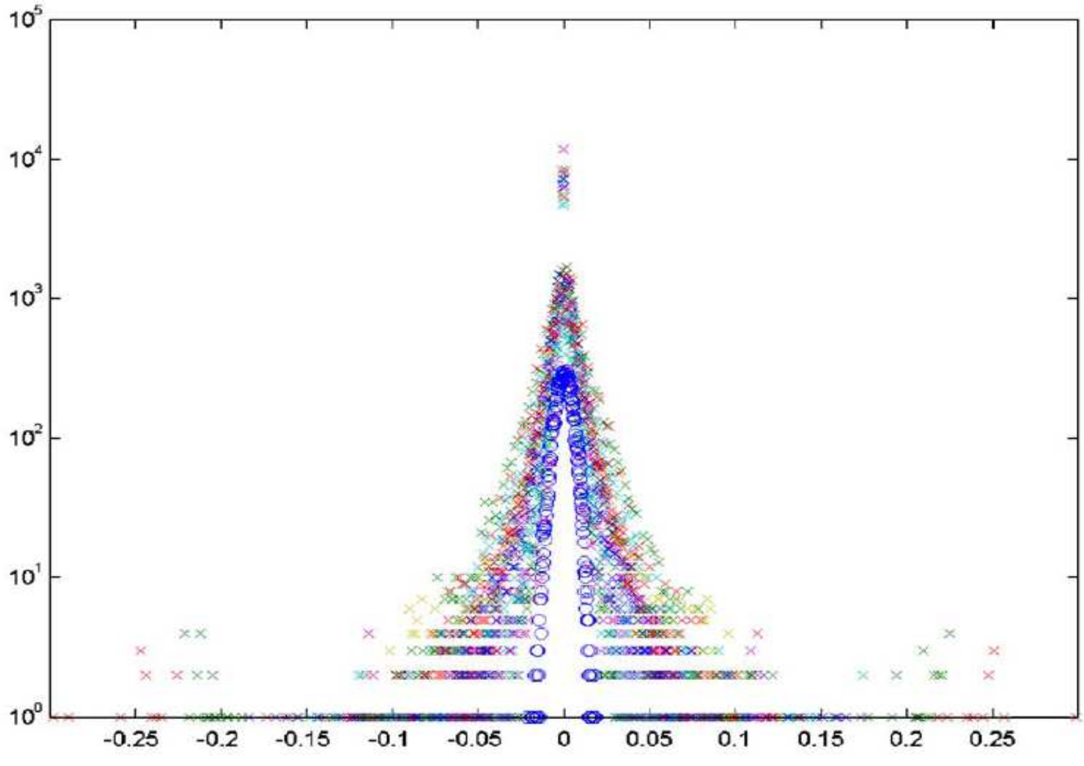


Figure 1.2: Functions of probability distribution. (Strozzi and Zaldivar, 2005)

They use exchange rates between US Dollars and eighteen other foreign currencies, both from the Euro zone and outside of it. Their findings reveal that noisy information about past values of time series can actually be proven profitable. Though, it must be mentioned, that the feedback mechanism present in the series has not been taken into account. The main conclusion of the paper is that high frequency foreign exchange time series experience dissimilar performance in comparison to the random walk, allowing prediction.

Numerous articles have been written concerning the non-linear dynamics of stock returns due to the interaction between heterogeneous agents, more specifically noise traders and fundamentalists. In his paper McMillan (2005) investigates the presence of these dynamics using stock market returns for France, Germany, Hong-Kong, Japan, Malaysia and Singapore. As a tool he uses a quadratic logistic smooth transition autoregressive (QLSTAR) model, which in reality, is a generalized form of the exponential smooth transition autoregressive (ESTAR) model. The selection of the model lies on the fact that the particular one is able to detect behavior that is in agreement with the noise traders models in periods where market dynamics switch between high and low returns.

According to the results, the non-linear models behave much more satisfactory compared to the linear ones. Furthermore predictability of returns is greater only when returns are of high magnitude, whereas in the case when small returns are presented it is not possible to forecast which group of traders (fundamentalists or noise traders) will step in and force the market towards equilibrium. In conclusion the predictability degree appears to be substantially higher in the case of the Asia-Pacific markets than in the European economies.

It has been observed that autocorrelation is present in stock index returns and additionally the first order autocorrelations are inversely related to the volatility present in financial markets.

Venetis and Peel (2005) suggest in their paper the feedback trading theory can be used since it relates autocorrelation to individual investors trading patterns. They use a non-linear in conditional mean and in conditional variance model which includes the possibility of disproportional shock impacts on volatility. Their data comprises of the daily close and high-low prices of six stock indexes, (Frankfurt, Paris, London, Hong-Kong, Tokyo, and the US) over a nine years period.

Based on their results the presence of feedback traders is confirmed. In three indexes, namely those of Hong-Kong, Tokyo and USA an inverse volatility-correlation relationship is documented. Furthermore it seems that volatility is increased by previous movements of the daily range series.

Chen and Yu (2005) address the problem of long term dependence and asymmetric behavior of financial time series.

They use data from Hong-Kong, Singapore, Taiwan, Korea, Japan and US stock exchanges. In their effort to prove the long memory of stock returns they use the fractionally integrate autoregressive moving average (ARFIMA) model, where as in order to examine asymmetries they employ a threshold generalized autoregressive conditional heteroskedasticity (TGARCH) model. According to their findings, there is strong evidence of long memory and asymmetry in volatility of the stock indexes under examination.

Jondean and Rockinger (2006) address the problem of finding the relationship between stock returns during high-volatility driven periods. Thereby they propose the usage of what is called the "Copula Functions". Their main advantage is that they can be used if only marginal distributions are known in order to derive a multi-variate distribution. Then by applying Hansen's model they build a skewed t-student distribution so as to assess for asymmetry and the existence of fat tails.

In this aim they use data from four major stock indexes, namely, the American S&P 500, the British FTSE 100, the German DAX and the French CAC over a twenty years period.

According to their outcomes, the skewed t-student distribution with time-varying volatility, skewness and kurtosis seems to fit well the distribution of daily returns. Furthermore in the case of the European markets their correlation appears to increase considerably when movements are of the same direction i.e. a boom or a crash. On the contrary, this dependency does not seem to be present between USA and European markets.

So far the presence of non-linear dynamics in financial markets has been examined, in most cases, in relation with the effect of volatility. McMillan (2007) suggests that the variable of volume should also be considered and tested for its forecasting power. There is a vast amount of evidence suggesting a negative relationship between volume and future returns and more specifically that low volume is closely related with momentum behavior and high volume with reverting behavior.

The author used daily stock index and volume values for the U.K. the USA, France and Japan over a fourteen years period. The data were divided in two periods, with the first decade performing as the in-sample estimation period and the remainder as the out-of-sample period. Initially he considered a smooth transition regression (STR) model with lagged volume as a switching variable and tests its predictability, in comparison with a logistic smooth transition regression (LSTR) model with lagged returns as a switching variable, a linear autoregressive (AR) model and a linear random walk model. Furthermore he addressed the performance of other non-linear models, namely the threshold autoregressive (TAR) model, the momentum threshold autoregressive (M-TAR) model and the exponential smooth regression (ESTR) model.

Results imply that low volume is closely related with momentum behavior. Furthermore in three out of the four cases the smooth transition non-linear models that have volume as the switching variable outperform the linear autoregressive and random walk models, the alternative univariate LSTR, the TAR and the M-TAR models.

As mentioned before, lots of cases can be detected where properties or measures were borrowed from other sciences such as physics and mathematics and applied in economics. Araujo and Louca (2008) decided to use an “analog of the Gutenberg-Richter histogram of earthquake magnitude” in order to measure and classify the intensity of crises.

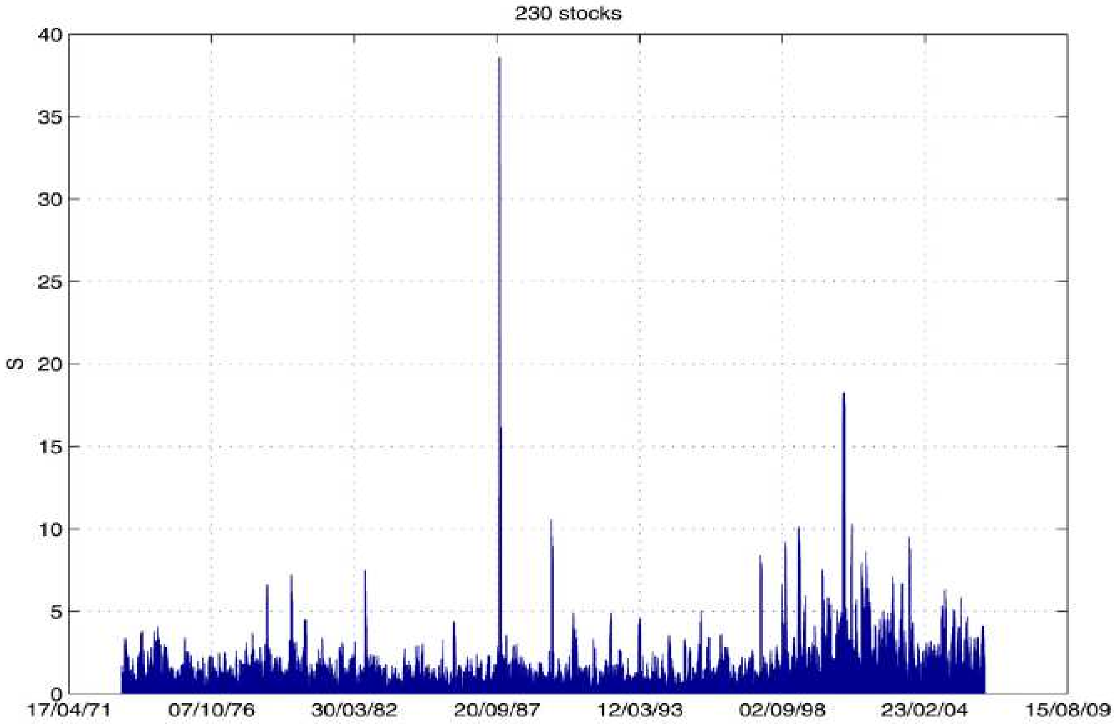


Figure 1.3: The graph of index S illustrating the development of the S&P 500 (Araujo and Louca 2008)

In times when markets experience great and violent anomalies, the way investors behave changes rapidly leading to dynamic alterations of the markets’ behavior and shape. The authors examine a thirty three years period by analyzing a sample of 230 stock values composing of the S&P 500. They choose for their scale of measurement as the highest value to be the Crash of 1987 and more specifically the Black Monday. By plotting the data they identify thirteen peaks which correspond to thirteen crashes.

The period following the 1987 Crash seems to be characterized by small turbulences and anomalies. However the years following 1997 appear to bring to play a new phase of high volatility which in turn alters the shape of the market.

There are various models that can be used in order to examine whether stock index returns are predictable. McPherson and Palardy (2007) chose to use the generalized spectral test for their paper. Their aim was to study if international stock returns are predictable in both a linear and a non-linear sense. The reasons for choosing the generalized spectral tests were:

i. They are robust to conditional heteroskedasticity and they do not have to assume a specific type of non-linearity.
ii. Both short run and long run behavior can be detected.
iii. Non-linear dependencies can be identified
iv. They help to define the frequencies where predictability arises.

The authors used data from nine stock market indexes, namely from Canada, France, Germany, Hong-Kong, Italy, Japan, the Netherlands, Switzerland and the United Kingdom.

In their findings it is stated that significant departures from normality is observed in all series examined. However, according to the tests only five out of the nine markets examined indicate some degree of predictability. More specifically, in the case of Canada, Germany, Italy and Switzerland there are indications of mean reversion, which is long run dependence process, whereas in the case of Hong-Kong there is evidence of short run dependence implying momentum or short run calendar effects.

Park et al. (2007), use computational mechanics in order to assess the complexity of stock markets. By using data from the S&P 500 stock index they construct deterministic finite automata known as “epsilon machine”.

They argue that with ε-machine the predictive behavior of stocks can be readily improved since the machine is able to detect trends and anomalies in a way that reveals the causal structure of the process. Finally based on their results it is deduced that nowadays information travels faster and the length of effective trends in histories has become shorter.

As Gil-Alana (2008) has proposed, long memory models, and in particular fractionally integrated processes, have been widely used during the past few years in an attempt to understand the dynamic behavior of financial time series. They have been known to help in assessing volatility properties of economic series.

The novelty of this paper lies upon the concurrent use of non-linear and long memory processes in a unified treatment. Two different sets of data were selected. The first set comprised of three stock market indexes (Nikkei 225, S&P 500 and Euro Stocks 50) and the second was the Spanish stock market index. For the first set when a non-linear sign model was assumed the unit root null was rejected for the Nikkei 225 and the Euro Stocks 50 but not for the S&P 500. In the case of the Spanish stock market index the unit root null could not also be rejected. The concluding remarks of the paper show that the unit root model will be rejected and higher orders of integration will appear when a simple non-linear sign structure is achieved in the data. Consequently it can be argued that there is strong indication of long memory in stock market returns.

Most of the literature so far has dealt with the detection of short-term non-linearities in stock returns. However, in the majority of papers dealing with long horizon forecasting of stock returns was examined by means of linear models. Kim et al. (2008), in their work attempt to address the long-term behavior of stock returns using non-linear models.

They use monthly stock market index data from the G-7 countries and employ smooth transition autoregressive (STAR) models, in particular the logistic (LSTAR) and the exponential (ESTAR) types. Besides, in order to examine the dynamic stability of the models they apply a non-linear impulse response function (NIRF).

In proportion to their empirical results the non-linear models undoubtedly outperform the linear ones “in-sample” and in most of the cases “out-of-sample” forecast. The NIRF indicates that non-linear models experience a strong stability of return dynamics.

In recent years many economists have dealt with the manner that information affects and alters the complex structure of financial markets. In accordance with the Efficient Market Hypothesis information is perfect and market perturbations can only be provoked by external shocks. Nonetheless this statement was proved to be invalid. However lots of past studies have concentrated on the way of removing seasonality and extreme observations from financial time series. In line with Kyrtsou and Malliaris (2009) work such phenomena should not be excluded, since because of the high non-linearity that these series experience, the impact of new information on the structure can not be determined in advance.

Their major intention is to find by means of simulation the effect on linear and non-linear time series of induced seasonal or irregular information. For their analysis they choose the noisy version of the Mackey-Glass model since by small alterations they can reproduce almost ideally the characteristics of real financial time series.

They show that when trading rules follow linearity, volatile behaviour cannot be reproduced. However in the case that investors trading rules are non-linear then the appearance of new information can lead to perturbations and instability of the financial market.

A great amount of researchers during the past few years have tried to establish the existence of predictability in financial market series. Guidolin et al. (2009), conduct a substantial survey of whether, when and where non-linear models are appropriate to give reliable predictions of financial returns.

They used monthly data on asset returns and macroeconomic variables for the G-7 countries. The econometric models employed in order to address the predictability of asset returns were:

- Linear models
- ARCH in mean models
- Markov switching models
- Heaviside threshold autoregressive (TAR) model
- Smooth transition regression models both logistic (LSTAR) and exponential (ESTAR)
- The simple random walk with drift model and
- A simple autoregressive (AR) model

Based on their outcomes the authors argue that there is not a unique model that can be applied across all countries and asset markets which outperforms all others. The suitability of each model alters due to different forecast horizons, from country to country, and also depends on the kind of market (stock or bond). In addition, the estimation results indicate that U.K. and US stock index returns can provide reliable forecasts with non-linear models and especially that of Markov's switching type. Furthermore, in five out of the seven countries examined, there is evidence that in highly volatile periods (specifically 1999-2002) a lower degree of predictability is detected. Finally UK and US are the only two countries in which the data indicate statistically substantial differences between forecasting models.

In an attempt to model non-linear co-movements between time series Kyrtsou and Vorlow (2009) proposed a new dynamic model with a bivariate Mackey-Glass in the mean equation and a BEKK-GARCH structure in the variance. Their aim was to explore the dynamics that seem to provoke the comovements of US short term interest rates and more precisely the Federal fund and the 3-month Treasury bill rates.

The results indicated that heteroskedasticity and non-linear behavior was detected in both the Federal Fund and the 3-month Treasury bill. Furthermore there was substantial evidence that non-linear determinism greatly affects the relationship between US short term interest rates.

Analyzing further real stock returns properties, Nainzada and Ricchiuti (2009) attempt to establish that as far as heterogeneity in beliefs increases, price fluctuations are seriously affected.

The authors assume two gurus that generate different expectations about future prices, which mean that two different fundamental values are created. Market agents can switch from one guru to the other. Furthermore two assets are examined one risky and one risk-free.

The main outcome of the paper is that fundamentalists can lead to market destabilization as well. Furthermore authors' findings show that as heterogeneity rises a "pitchfork bifurcation occurs and a period of doubling is created together with a bigger reaction to misalignment of both market makers and investors" (p. 1771). In conclusion, what they call "homoclinic" bifurcation forces the market prices to move between bull and bear markets.

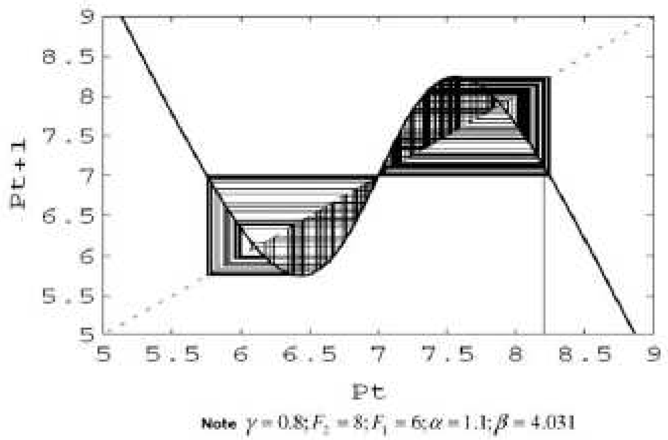


Figure 1.4: Illustration of homoclinic bifurcation (Nainzada and Ricchiuti 2009)

Hassani et al. (2010), argue about the significance of noise reduction when dependencies of financial series are computed. The authors use daily data from the stock indexes of

Greece, France, Germany, the United Kingdom, Portugal, Spain and the USA over a period of seventeen years.

In order to determine the dependencies, the authors consider two different approaches. In the first one, they evaluate the dependencies directly from the noisy time series while in the other they first filter the series from noise and finally calculate the dependencies. So as to address linear dependencies they compute the autocorrelation function (ACF) and λ the standard measure for the mutual information, the coefficient λ which can capture both linear and non-linear dependence. With the intension of addressing non-linear dependencies they use the detrended fluctuation analysis (DFA), which allows the exposure of long-range correlation hidden in appearing non-stationary series and the detrended moving average method (DMA), which reveals if the data are following a pattern and the way deviations from the pattern are linked. For reducing the noise of the data the authors used three filtering methods, namely: autoregressive moving average (ARMA), generalized autoregressive conditional heteroskedasticity (GARCH) and singular spectrum analysis (SSA)

In line with their findings it was proven that the ACF was not appropriate for addressing dependencies whereas λ was considered a reliable measure. Furthermore DMA was found to be sensitive in comparison with DFA. As far as the filtering process is concerned the SSA appeared to be a more suitable method for removing noise.

Zunino et al. (2010), proposed the use of what is called complexity entropy causality plane. The main benefits of this tool is that it can distinguish between Gaussian and non-Gaussian process and furthermore among different degrees of correlations.

The authors employed Shannon's logarithmic information measure and a statistical complexity measure (SCM). The latter has the ability to identify crucial details of the dynamics and differentiate various degrees of periodicity and chaos. It is defined through the product C_{fs} of the normalized Shannon entropy H_s for the evaluation of these quantifiers the Bandt and Pompe method (BPM) of assessing the probability distribution was used. Using provided evidence it was established that these are reliable tools for "the detection and quantification of noise induced temporal correlations in stochastic resonance phenomena" (p. 1894).

Based on the authors findings it has been shown that complexity-entropy causality plane can help in the identification of the stage of stock market development. Hence an emergent stock market can be easily distinguished from a developed one. In that context, an emergent stock market exhibits lower entropy and higher complexity, which indicates that significant time correlations and some degree of order are present. As a concluding remark the authors state that temporal correlations are the key factor of inefficiency.

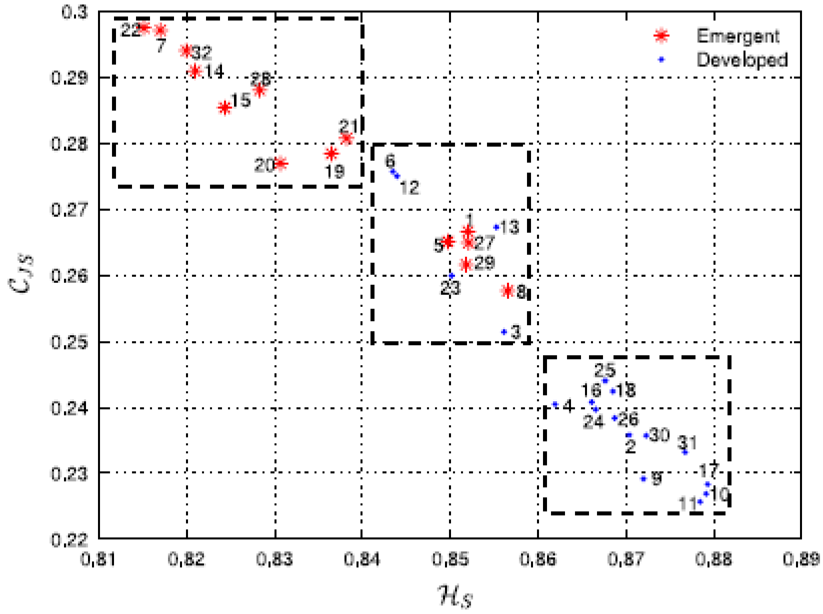


Figure 1.5: Groups of the different stock markets as they appear in the complexity-entropy causality plane (Zunino et al. 2010).

1.3 Chaos Theory

As previously stated a lot of systems in the natural world are govern by chaos and can exhibit non-linear behavior. For many years, due to their high degree of complexity these systems were considered random. The birth of chaos theory can be traced back in 1890 when Henry Poincare stated the three body problem. What Poincare achieved was to examine a new manner of studying dynamical systems which was concentrated on qualitative and geometric features, rather than just mathematical formulae. Later on, in early 1960's Edward Lorenz was the first to describe chaotic behavior of weather systems.

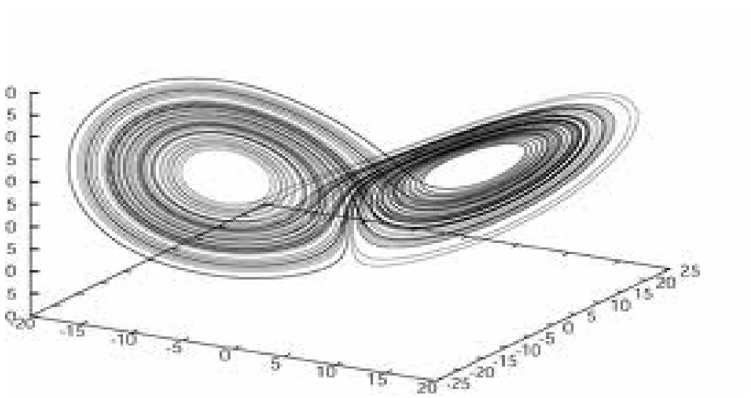


Figure 1.6: Image of the Lorenz attractor (http://www.mizuno.org/c/la/index.shtml)

In general, it can be said that chaos theory is based on the observation that when simple rules are iterated, this can give rise to apparently complex behavior. If the initial conditions are changed by a small amount, the resulting sequence will be very different after some iterations. However, there has still not been found a single and adequate definition of chaos accepted by all researchers and scientists.

In most cases, chaos theory is adopted in order to examine highly non-linear systems and has helped in the analysis and interpretation of economics and financial problems. One of its main advantages is it reveals system information and relationships without having to specify the equations or laws governing the underlying dynamics. Although, it does not offer the researchers with equations for the development of a traditional model, it can give valuable information involving all system dynamics for general application.

1.4 Taken's Theorem

In 1981, Floris Taken found a simple method for analyzing chaotic series called Taken's delay embedding theorem. A brief description of the theorem and some key definitions are mentioned as follows.

Generally with the term dynamical systems we characterize systems that change over time. Dynamical systems are comprised of two significant components: a set of points called states and a specific arrangement of how states alter with time. If time is continuous, differential equations are used to describe how states change, whereas if time is discrete a set of difference equations are applied. In most cases, discrete time systems are more straightforward and easy to work with, so even continuous time systems are transformed and treated as discrete ones.

Consider a dynamical system that consists of a set M and a transformation T: M→ M mapping M to itself. Then M is defined as the state space and its points are states. T shows how the states vary in one time step meaning $x \in M$ changes to Tx. When T is applied successively it can be found that after n steps, the state becomes equal to $T^n x$. So the set of points that were created for each state x is called the *trajectory* of x.

$$T^n x : n = 0,1,...... \quad (1.1)$$

M is usually $\Re^n$ or a subset of it and it is known as a *differential manifold*. One of manifold's main features is that around each point there is an area which can be given a coordinate system. In figure 1.7 a graphical representation of the above statement is given.

Therefore, it must be noted that differentiable functions can exist between manifolds and one of their most significant type is the *deffeomorphisms*, which are invertible functions where both the function and its inverse are differentiable.

Scientists observed that n increases three scenarios unfold:

1. As T is applied some points remain in the same position so that $Tx = x$, called *fixed points*.
2. As T is applied some points move but return to the original state after some time steps so as $T^k x = x$, called *periodic points*.
3. As T is applied some points move but their orbits never return to the original plane i.e. *chaotic trajectories.*

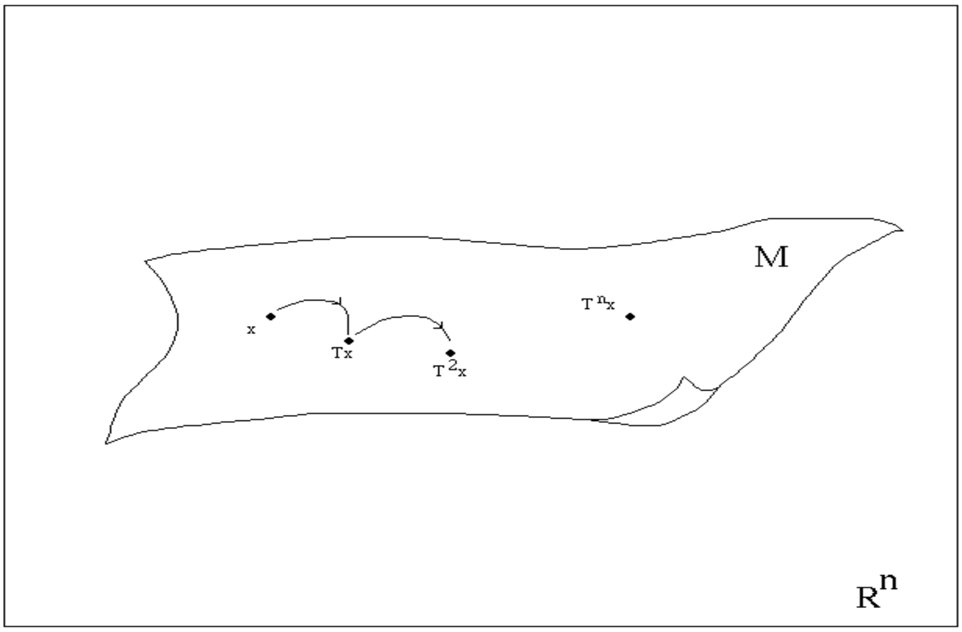


Figure 1.7 A differentiable dynamical system
(Huke, http://personalpages.manchester.ac.uk/staff/jerry.huke/intro.pdf)

Consider an observation function h, which represents the arrangement of an experimental procedure. Every trajectory of the dynamical system corresponds to an array of real numbers, which is called *time series*. Delay vectors can be formed by using consecutive values.

$$h_0 = (h_0, h_1, \ldots h_{d-1})$$

$$h_1 = (h_1, h_2 \ldots h_d)$$

$$\vdots$$

$$h_n = (h_n, h_{n+1}, \ldots h_{n+d-1})$$

$$= \left(h(x_n), h(x_{n+1}) \dots h(x_{n+d-1})\right)$$

$$= \left(h(x_n), h(Tx_n) \dots h(T^{d-1}x_n)\right) \quad (1.2)$$

The function $\Phi(T,h)$ is called the delay map and is defined as

$$\Phi(T,h): M \to \Re^d$$

$$\Phi_{(T,h)}(x) = \left(h(x), h(Tx), \dots, h(T^{d-1}x)\right) \quad (1.3)$$

This shows that the time series of the delay vectors is in reality a copy of the initial trajectory. This can be clearly shown in Figure 1.8.

The connection between dynamical systems and time series is known as **Taken's Theorem** which states:

"Let M be a compact manifold of dimension m. For pairs (T,h), with T a diffeomorphism of M and h a smooth, real valued function on M, it is a generic property that $\Phi(T,h): M \to \Re^d$ *is an **embedding** if* $d \geq 2m$ *"* (Huke, p.10)

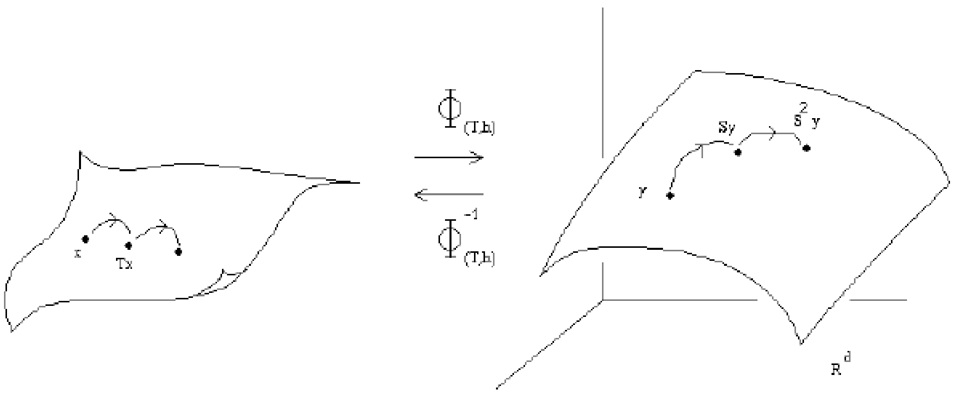


Figure 1.8: The original dynamical system and its reconstruction in delay space.
(Huke, http://personalpages.manchester.ac.uk/staff/jerry.huke/intro.pdf)

In simpler words, if for a given T and h, where $\Phi(T,h)$ is an embedding, then the delay map will also be an embedding for all T' and h' sufficiently close to T and h. So, if d is large enough, the vector series reproduces a lot of the significant dynamic characteristics of the original series. Careful choices for the lag and the embedding dimension are vital in order to get reliable results. Finally, according to Taken's theorem 2m+1 delays should be used, where m is the dimension of the state space M.

1.5 Correlation Dimension

In Euclidean geometry any object can be characterized by a set of points on the condition that m is big enough so as to include all points of the object. Each set in $\Re^m$ can be described by a topological dimension d, which is an integer that varies in the range $[0, m]$. If the set is the whole of $\Re^m$ then consequently $d = m$. Thus in Euclidean geometry a point has dimension $d = 0$, a line has dimension $d = 1$, a plane has dimension $d = 2$, a solid has dimension $d = 3$, etc. Furthermore according to theory, a *fractal dimension* is a dimension that can take non-integer values and a *fractal* is a set with a non-integer fractal dimension.

It is widely accepted that a significant sign of the existence of low dimensional chaos is the presence of a strange attractor. With the intention of verifying that chaos can be detected in experimental data an estimation of the dimension of this attractor, embedded in some space, must be made. This dimension is an indication of the minimum number of variables required to model the process. Through the years, several methods have been established in an attempt to define and measure the attractor's dimension.

One of the most commonly used is the *correlation dimension*, which belongs to the family of fractals. Due to its computational simplicity, correlation dimension is one of the most popular techniques. In essence, it helps researchers quantify self-simplicity. A big value of correlation dimension implies a high degree of complexity and low self-similarity. Correlation dimension can be calculated by the Grassberger and Procaccia (1983) method. They used the correlation integral $C(r)$ which denotes the probability that a randomly selected pair of points in the reconstructed phase space is separated by a distance less than r. If N is the number of points in the reconstructed vector time series the correlation integral will be approximately equal to

$$C_N(r) = \frac{2}{N(N-1)} \sum_{j=1}^{N} \sum_{i=j+1}^{N} \Theta\left(r - \left|x_i - x_j\right|\right) \quad (1.4)$$

Where Θ is the Heaviside function with

$$\Theta(x) = \begin{matrix} 0 & for \;\; x < 0 \\ 1 & for \;\; x \geq 0 \end{matrix} \qquad (1.5)$$

And $\left|x_i - x_j\right|$ is the distance between points x_i and x_j. Then the correlation dimension D_c will be equal to

$$D_c = \lim_{r \to 0} \lim_{N \to \infty} \frac{\log C_N(r)}{\log r} \quad (1.6)$$

When the time series are of finite length the sum of $C_N(r)$ will also depend on the embedding dimension m. For this reason, the correlation dimension D_c can be calculated by inspecting the slope of the linear portion of the diagram of $\log C_N(r)$ versus $\log r$ for successively bigger values of m. In cases where $m < D_c$ the structure of the dynamical state can not be resolved because the dimension of the reconstructed phase state is low and thus the slope is approximately equal to the embedding dimension. For higher values of m the resolution of the dynamical state is improved. In most cases, the slope of the diagram of $\log C_N(r)$ versus $\log r$ increases with m until it reaches an area of stability called *plateau*. In the value that this occurs that is taken as the best approximation of D_c. It can be shown that for an accurate estimation of the correlation dimension, the set N must be in accordance with the following inequality.

$$D_c < 2\log_{10} N \qquad (1.7)$$

1.6 Lyapunov Exponents

As previously mentioned one of the most intriguing features of chaotic systems is their high sensitivity to initial conditions. In order to prove the existence of chaos the previously mentioned feature needs to be displayed. A common method of quantification of sensitivity dependence on the initial conditions is the Lyapunov exponents. These can be used so as to measure the averaged divergence or convergence rate of two neighboring trajectories in a phase space. Lyapunov exponents' sign indicates the existence of chaos and their value measures how chaotic a system is. Positive values are considered evidence of chaos; negative exponents imply stochastic processes whereas values near zero indicate the existence of noisy chaos (Kyrtsou & Terraza, 2002). There is a big variety of different Lyapunov exponents for a dynamical system. The most commonly used and important one is the *maximum Lyapunov exponent* which gives an estimation of the degree of chaos in the underlying dynamical system.

Imagine a dynamical system specified by a map with x_n a k-dimensional vector

$$x_{n+1} = G(x_n) \quad (1.8)$$

Assume a displacement from the initial orbit $x_n \to x_n + \delta x_n$ where δx_n is an infinitesimal vector. Then the evolution of δx_n can be described by the differentiation of equation 3.8

$$\delta x_{n+1} = DG(x_n)\delta x_n \quad (1.9)$$

Where $DG(x)$ is the $\kappa \times \kappa$ Jacobian matrix of partial derivatives of $G(x)$ with respect to the κ components of x.

Assume $y_n = \frac{\delta x_n}{|\delta x_0|}$ with y_n defined as tangent vector. Similarly the evolution of y_n is

$$y_{n+1} = DG(x_n)y_n \quad (1.10)$$

It is clear that the evolution of y_n depends on both the orbit x_n and the initial orientation of the unit tangent vector y_0. Of great interest is the exponential rate at which y grows or shrinks per iterate of the map. So, the Lyapunov exponent can be defined as

$$\lambda = \lim_{n \to \propto} \frac{1}{n} In|y_n| \quad (1.11)$$

The Lyapunov exponents are m in number as many as the components of vector y_n. So the Lyapunov number is defined as

$$L = e^{\lambda} \quad (1.12)$$

Similarly for the case of continuous time dynamical systems assume a k-dimensional system of first order ordinary differential equations $\dot{x} = F(x)$ and an infinitesimally displacement of the orbit $x(t) + \delta x(t)$ and a tangent vector $y(t) = \frac{\delta x(t)}{|\delta x(0)|}$. So, $\dot{y} = DF(x(t))y$. The Lyapunov exponents will be defined as

$$L = \lim_{t \to \propto} \frac{1}{t} In|y(t)| \quad (1.13)$$

Chapter 2: Application and Empirical Results

2.1 Data and descriptive statistics

The data used for the purposes of this thesis are daily returns series of the Athens Stock Exchange (ASE) index and in particular those of FTSE 20, FTSE 40 and FTSE 80, during the period from 03-06-2002 to 30-09-2004 , giving 11369 observations each. In order to examine whether the stock returns follow the Efficient Market Hypothesis (hereafter EMH), i.e. the price levels resemble a random walk, descriptive statistics were applied. The computational program of E-Views was used and the following graphs and information was derived for the three cases.

2.1.1 FTSE 20

In Figure 2.1 the graphical representation of the return series of FTSE 20 is given. It is clear that extreme observations are detected. The reference point of the EMH is the presence or not of autocorrelation. The EMH is valid, when autocorrelation is equal to zero that means that the information is instantly absorbed and each day is completely independent from the next one. Besides, EMH still holds in core where even short-term significant autocorrelations appear. From the correlogram (Appendix A) it can be seen that the first few values of the probability column are greater than 0,05, which implies that autocorrelation is zero. However, descriptive statistics must be considered before reliable conclusions are deduced.

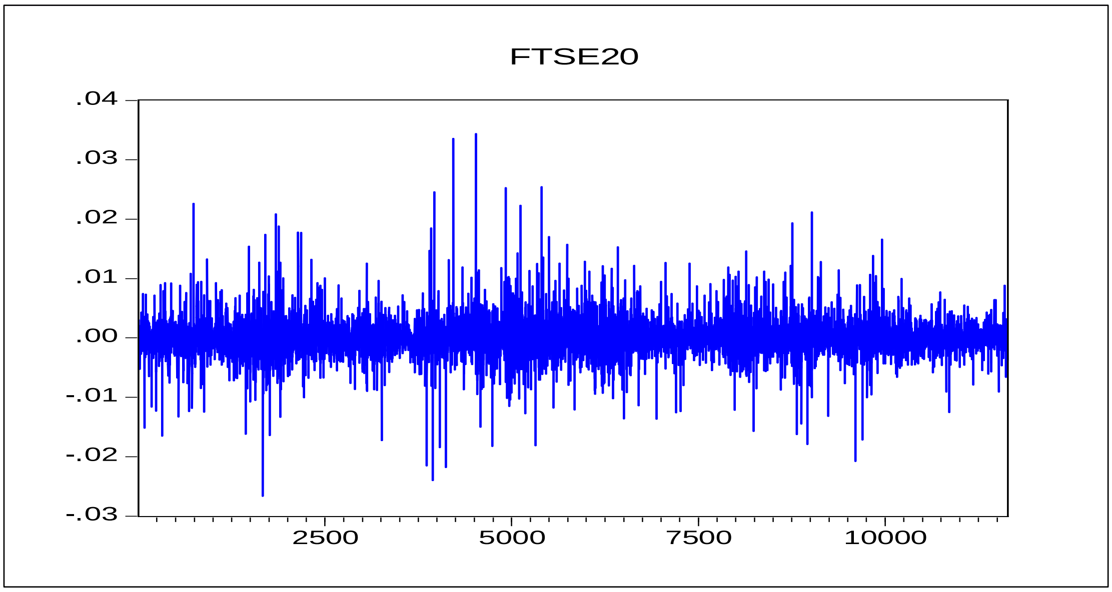


Figure 2.1: Return series of the FTSE 20

It is clearly shown that the EMH is not valid since the return series of FTSE 20 is leptokurtic (kurtosis = 19.29) and non-normal (Jarque-Bera = 129551.7). The presence of skewness also indicates the existence of fat tails and its positive value shows that abnormal values are located at the tails of the distribution. These results are confirmed by Jarque-Bera measure.

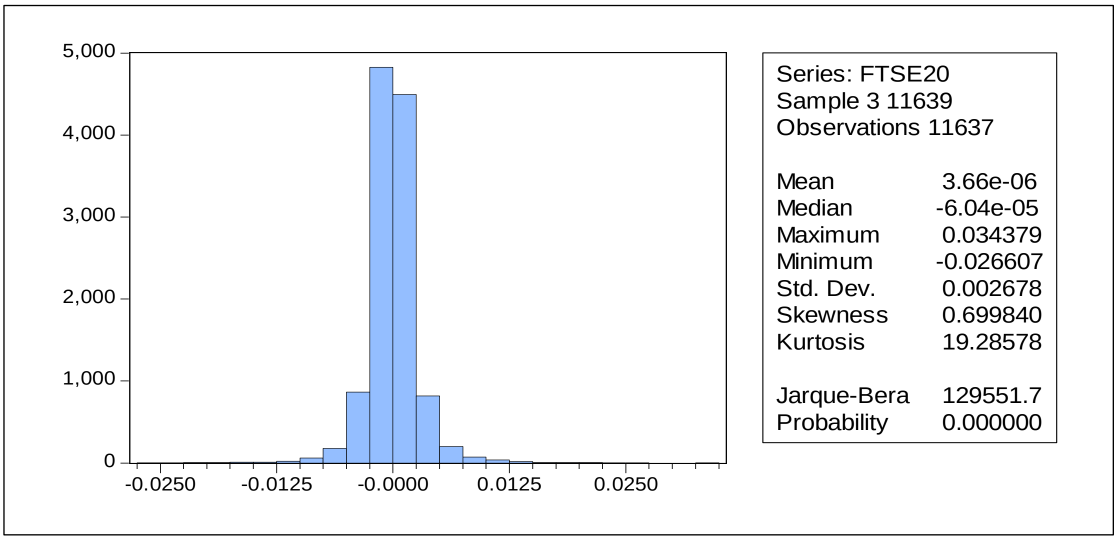


Figure 2.2: Descriptive statistics of the FTSE 20 returns

2.1.2 FTSE 40

In Figure 2.3 the graphical representation of the return series of FTSE 40 is given. Again

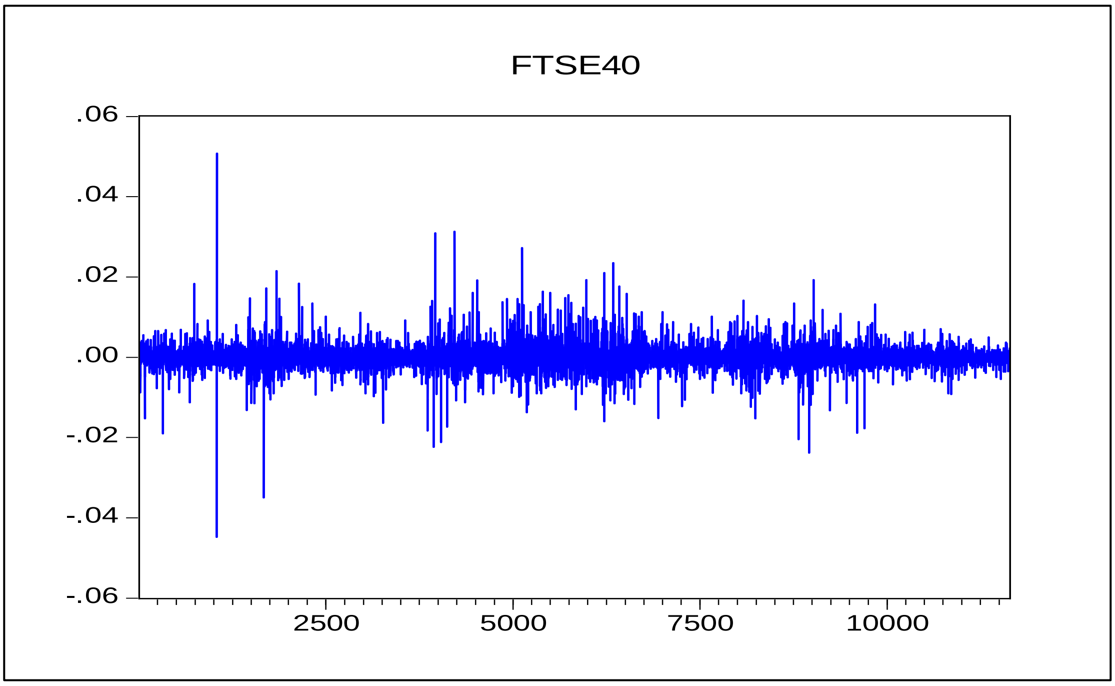
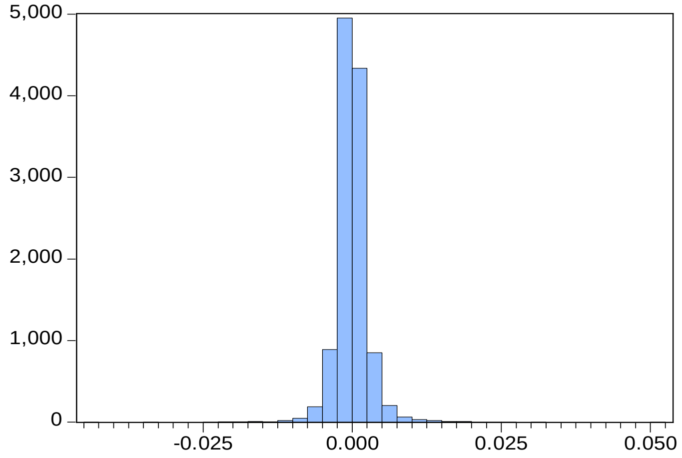


Figure 2.3: Return series of the FTSE 40

it is clear that extreme observations are detected. In this case from the correlogram (Appendix A) it can be seen that the values of the probability column are all less than 0,05, which implies that autocorrelation is not zero. Hence there is evidence against the EMH.

Similarly Figure 2.4 sums up the results from descriptive statistics. In this case as well it is shown that the EMH is not valid since the returns series of FTSE 40 is leptokurtic (kurtosis = 34.49) and non-normal (Jarque-Bera = 481493.7). The fact the value of kurtosis is so high implies that no white noise is present in the series. The presence of skewness also indicates the existence of fat tails and its positive value shows that irregular values are found at the tails of the distribution. Furthermore the value of Jarque-Bera is greater compared to that of FTSE 20 implying an even more speculative framework.



Series: FTSE40	
Sample 3 11639	
Observations 11637	
Mean	-2.48e-05
Median	-0.000121
Maximum	0.050814
Minimum	-0.044822
Std. Dev.	0.002701
Skewness	0.651146
Kurtosis	34.48540
Jarque-Bera	481493.7
Probability	0.000000

Figure 2.4: Descriptive statistics of the FTSE 40 returns

2.1.3 FTSE 80

In Figure 2.5 the graphical representation of the returns series of FTSE 80 is given. Extreme observations are detected in these return series as well as the previous two. Again from the correlogram (Appendix A) it can be seen that the values of the probability column are all less than 0,05, which implies that autocorrelation is not zero. Hence there is evidence that EMH is not valid.

Figure 2.6 gives the results for descriptive statistics. For one more time it is clear that the EMH is not valid and the returns series of FTSE 80 is leptokurtic (kurtosis = 25.99) and non-normal (Jarque-Bera = 256335.0). The presence of skewness again indicates the existence of fat tails and its positive value shows that at the tails of the distribution anomalous values can be detected. However the value of Jarque-Bera is smaller compared to that of FTSE 40 implying smaller divergences from the equilibrium point of the market.

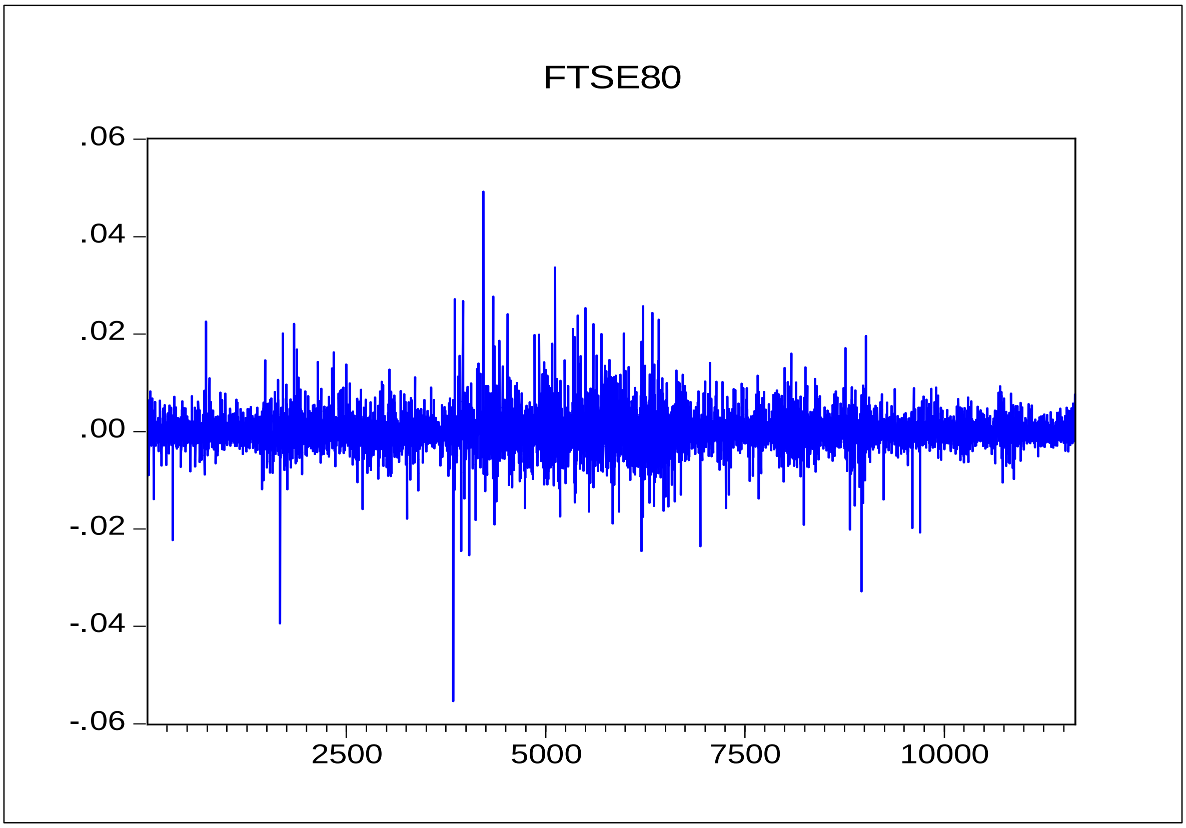


Figure 2.5: Return series of the FTSE 80

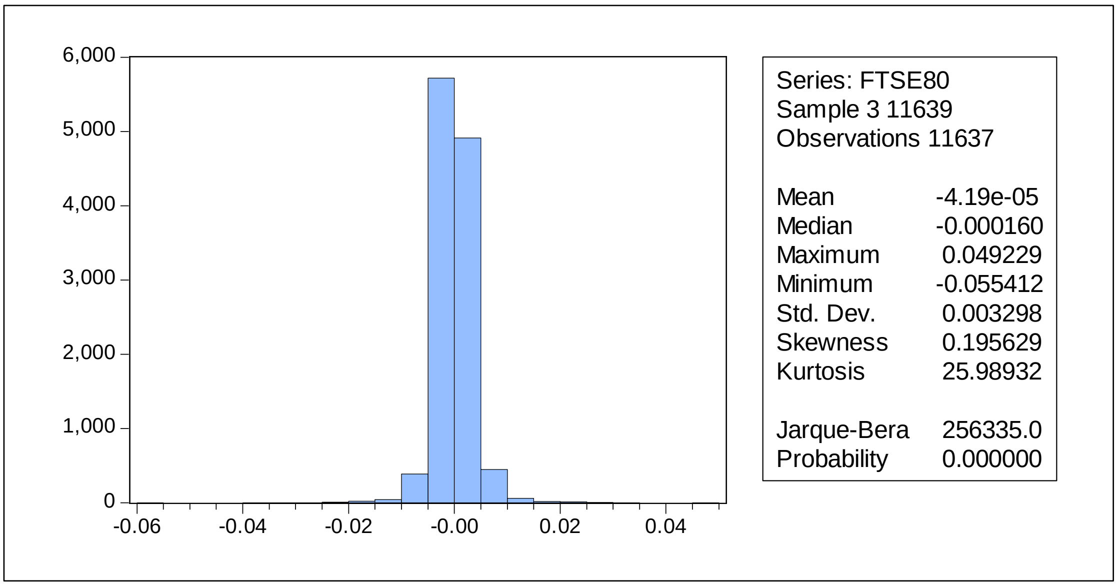


Figure 2.6: Descriptive statistics of the FTSE 80 returns

2.2 Application findings

In order to estimate and analyze the correlation dimension and the maximum Lyapunov exponents the program “Non-Linear Time Series Analysis (NLTSA v. 2.0) by Syriopoulos and Leonditsis (2000) was used. The results are presented as follows.

2.2.1 FTSE 20

Table 2.1 sums up the results for the correlation dimensions in consecutive embedding dimensions. It seems that the correlation dimension keeps on increasing for higher values of the embedding dimension and there is no sign that it levels off at some point. It can be observed that correlation dimension values are relatively high, but not as high as the ones described in the paper of Kyrtsou and Terraza (2003) for the case of Paris Stock Exchange returns series CAC40 where values were around 6.

Table 2.1 Correlation dimension estimates for the FTSE 20 series.

FTSE 20									
Embedding dimension m	2	3	4	5	6	7	8	9	10
Correlation dimension D_c	0,656	1,05	1,48	1,93	2,39	2,88	3,39	3,92	4,48

The graph of the correlation dimension versus the embedding dimension is illustrated in Figure 2.7. The fact the correlation dimension does not converge to a single value implies nothing conclusive can be stated about the chaotic or non-chaotic nature of the series generator. However other useful information about the system can be deduced from the graph. According to the theory when white noise is present the correlation dimension increases at the same rate as the embedding dimension, meaning that the slope is equal to 1 and the correlation dimension equals the embedding dimension. This can be explained by the fact that because white noise is random fills whatever space is available to it. In the case that the correlation dimension increases in slower rate as the embedding dimension increases, meaning that slope is less than 1, this indicates the existence of a deterministic system. From Table 2.1 it is clear that the correlation dimension stays well below the embedding dimension even as the embedding dimension increases (correlation dimension equals 3.39, 3.92, 4.48 for embedding dimensions 8, 9, 10 respectively). This implies that there are signs of inherent non-linear determinism in the FTSE 20 returns.

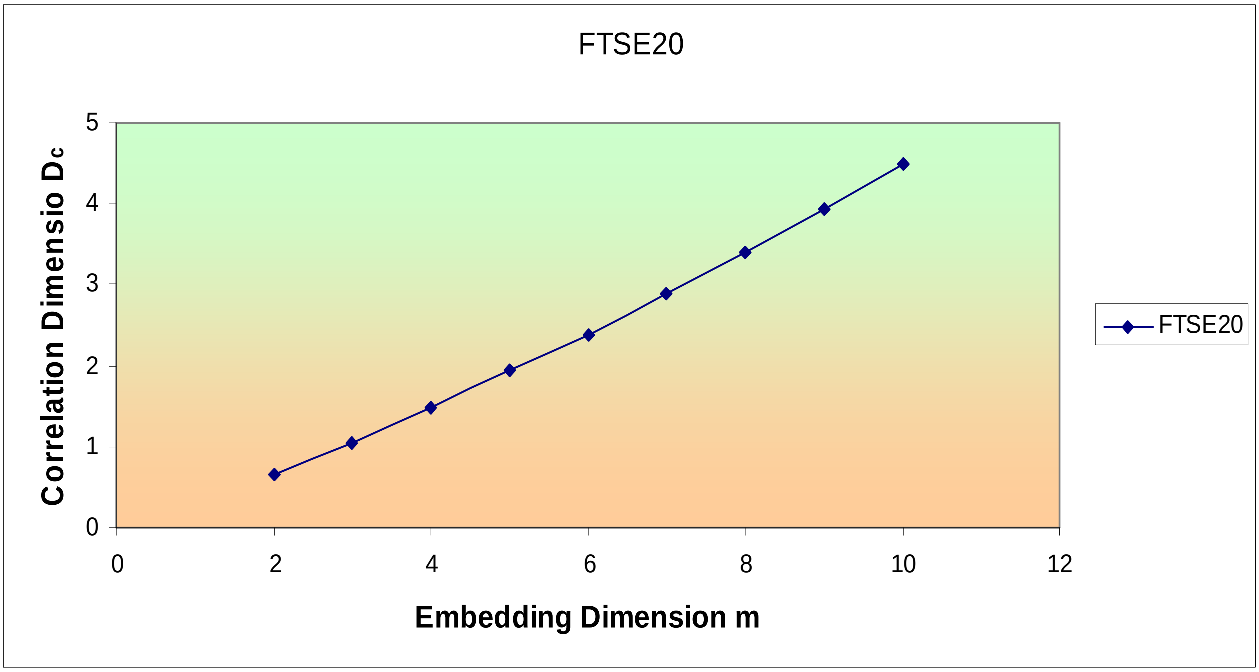


Figure 2.7 Graph of the correlation dimension for FTSE 20 series.

Figure 2.8 illustrates the logarithmic plot of the correlation integral versus the correlation distance. The correlation dimension is the slope of each graph for the corresponding embedding dimensions. It is clear that as the embedding dimension increases the slope of the equivalent graph becomes steeper indicating a higher correlation dimension. That graphically verifies the results of table 2.1

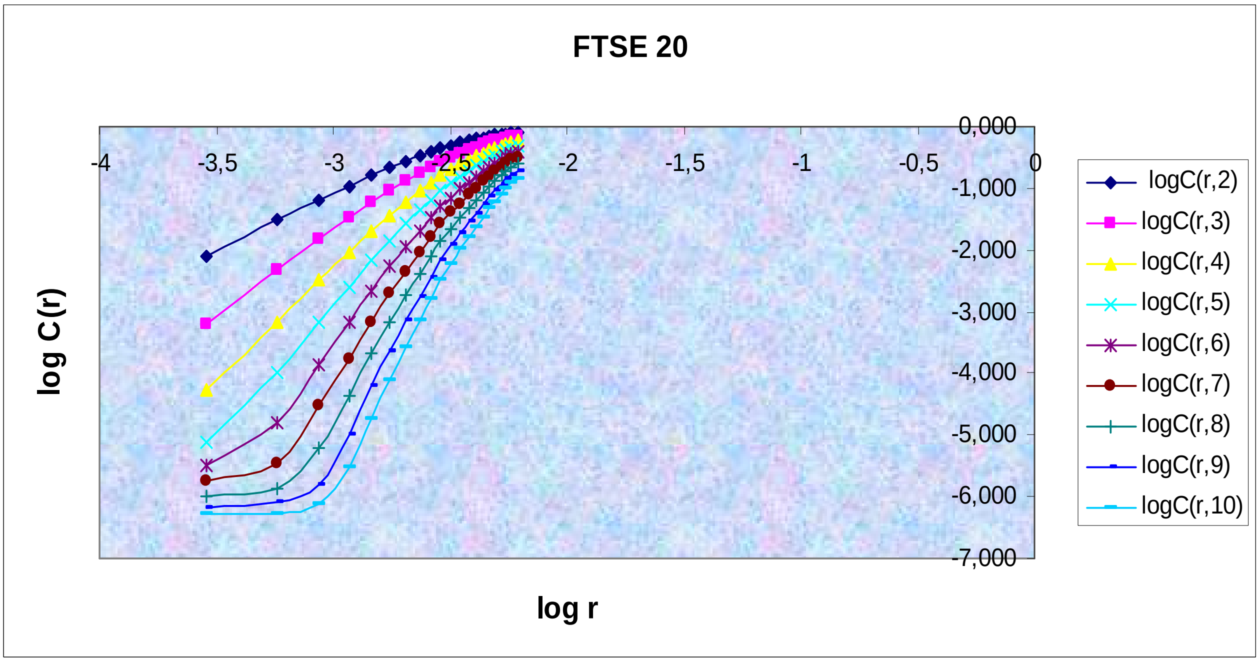


Figure 2.8: Plot of log r versus log C(r) for FTSE 20 series

In order to understand better the dynamic behavior of the series the Lyapunov exponents were computed. Their values for the FTSE 20 series, are summarized in Table 2.2 while Figure 2.9 shows the graph of evolution time a versus S(a). We notice that these exponents can distinguish between low-dimensional chaos and stochastic processes. From Table 2.2 it can be seen all Lyapunov exponents are positive and the Larger Lyapunov exponent equals 0,00902 corresponding to an embedding dimension of 2. The fact that all exponents are close to zero (0) could indicate the existence of an underlying high-dimensional chaotic system.

Table 2.2: Lyapunov Exponent estimates for the FTSE 20 series

FTSE 20									
Embedding dimension m	2	3	4	5	6	7	8	9	10
Laypunov Exponent	0,00902	0,00691	0,00633	0,00652	0,0054	0,00652	0,00558	0,00446	0,00534

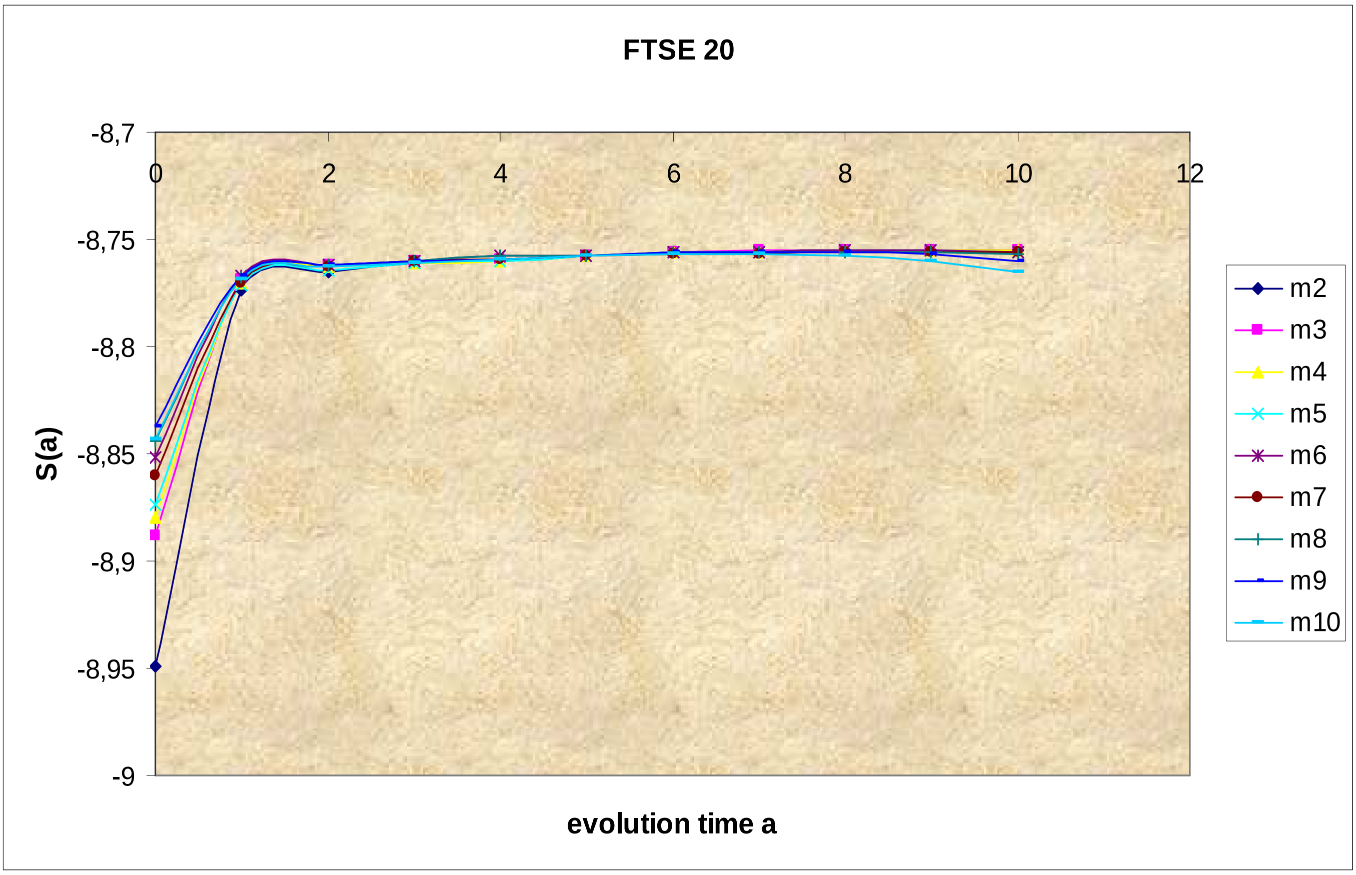


Figure 2.9: Graph of evolution time a versus S(a) for FTSE 20 series

2.2.2 FTSE 40

In Table 2.3 the results for the correlation dimensions of the FTSE 40 series are given. It can be observed that as the values of the embedding dimension take higher values so as the correlation dimension values rise. Furthermore there is no indication that correlation dimension levels off at any point.

Table 2.3 Correlation dimension estimates for the FTSE 40 series.

FTSE 40									
Embedding dimension m	2	3	4	5	6	7	8	9	10
Correlation dimension D_c	0,304	0,533	0,805	1,11	1,45	1,83	2,23	2,66	3,08

The graph of the correlation dimension versus the embedding dimension is illustrated in Figure 2.10. In this case as well the fact the correlation dimension does not level off to a single value does not give firm information about he chaotic or non-chaotic nature of the series generator. In this case, as well as in FTSE 20, the FTSE 40 time series are found to be governed by a high-dimensional non-linear deterministic and not a pure stochastic underlying structure since the correlation dimension remains below the embedding dimension, as the latter increases and the slope of the graph is less than 45^o. However, comparing the correlation dimension of FTSE 20 (D_c = 4,48) with those of FTSE 40 (D_c = 3,08) series it can be observed that in the case of the former the correlation dimension is higher implying a lower degree of non-linear determinism in these series

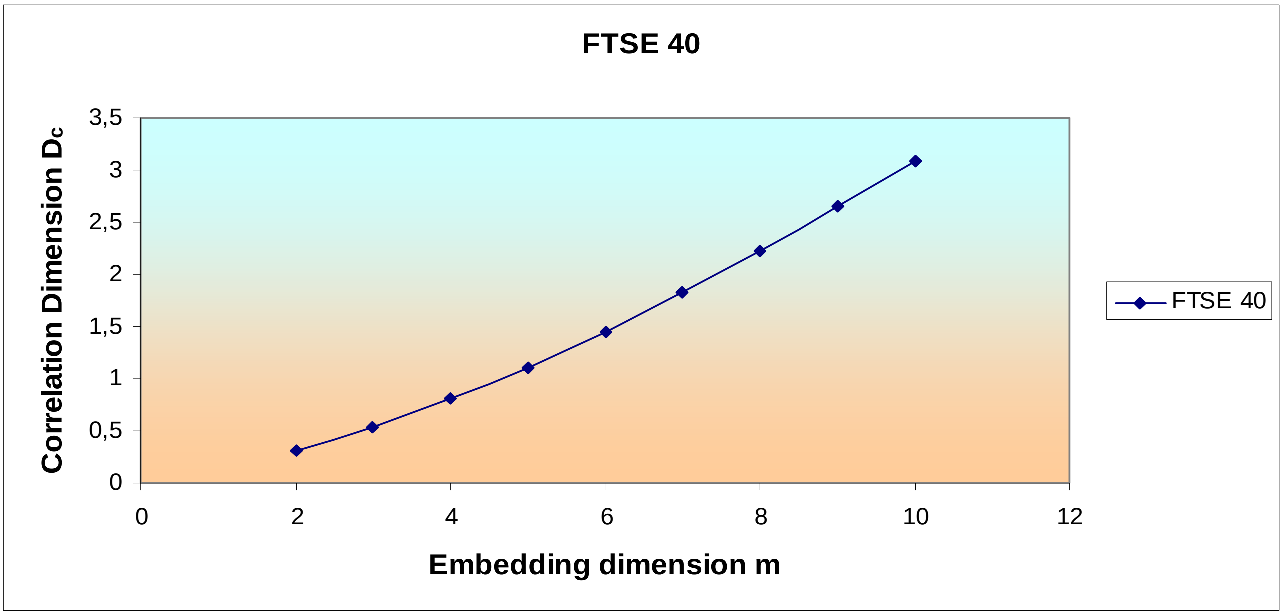


Figure 2.10 Graph of the correlation dimension for FTSE 40 series.

The results from Table 2.3 can be graphically validated. Figure 2.11 shows the logarithmic graph of the correlation dimension against the correlation distance. The correlation dimension is determined as the slope of each data set. The slope increases as the embedding dimensions become larger, implying higher correlation dimensions.

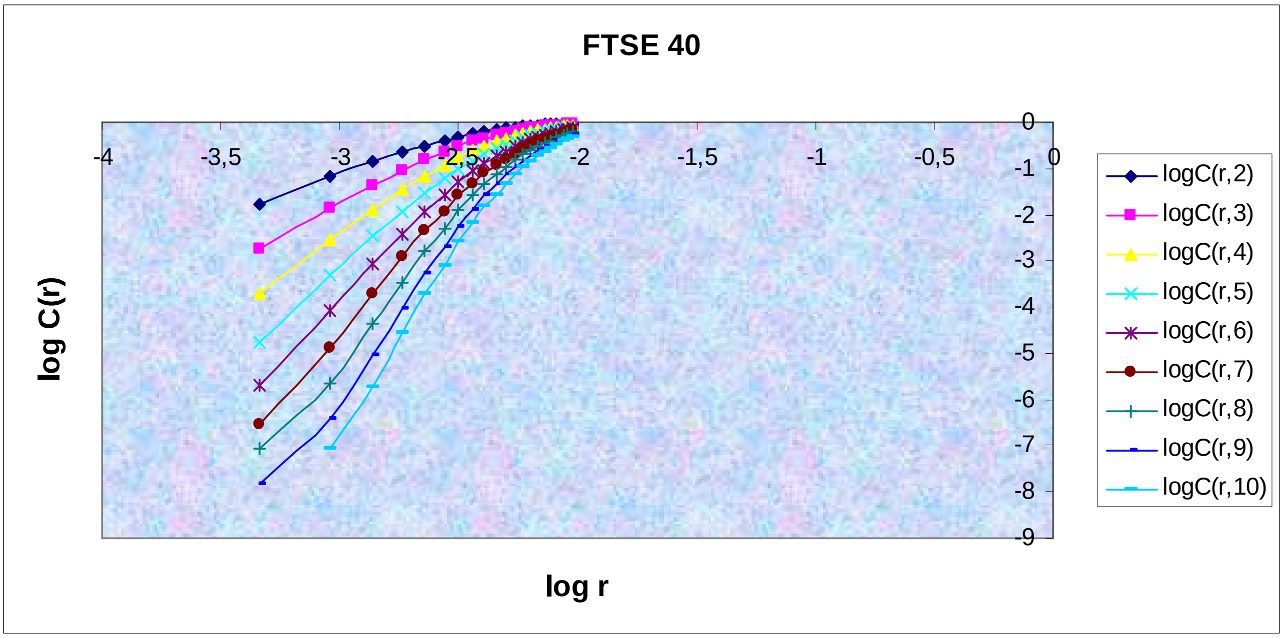


Figure 2.11: Plot of log r versus log C(r) for FTSE 40 series

In order to decide between low-dimensional chaos and stochastic processes the Lyapunov exponents are employed. Their values for the FTSE 40 series are summarized in Table 2.4 while Figure 2.12 shows the graph of evolution time a versus S(a).

Table 2.4: Lyapunov Exponent estimates for the FTSE 40 series.

FTSE 40									
Embedding dimension m	2	3	4	5	6	7	8	9	10
Laypunov Exponents	0,00487	0,00596	0,00581	0,00416	0,00391	0,0041	0,00409	0,00355	0,00246

From Table 3.4 it is clear that all Lyapunov exponents are positive and the Larger Lyapunov exponent equals 0,00596 corresponding to an embedding dimension of 3. The fact that all exponents are close to zero (0) could indicate the existence of an underlying high-dimensional chaotic system. Comparing the Largest Lyapunov Exponent with the equivalent ones from FTSE 20 and FTSE 40 series it can be seen that in the latter the exponent has a lower value.

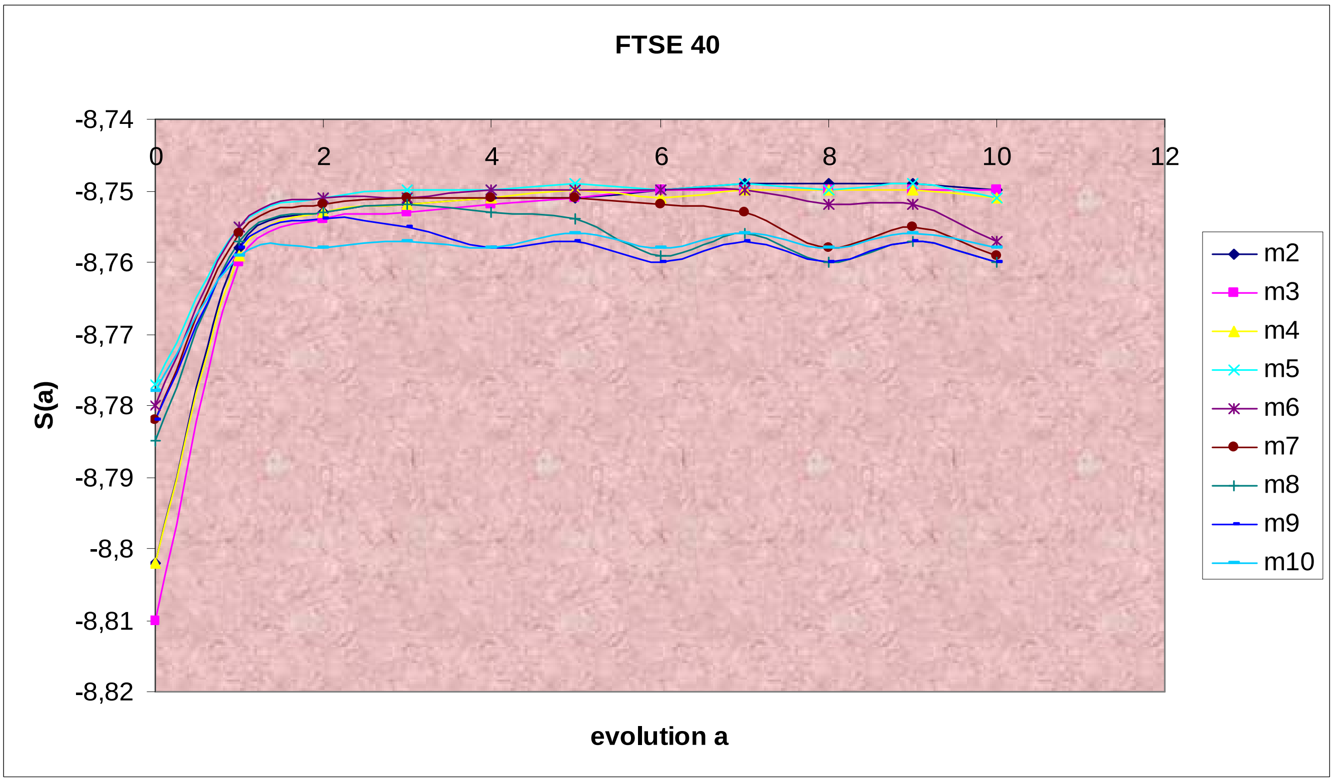


Figure 2.12: Graph of evolution time a versus S(a) for FTSE 40 series

2.2.3 FTSE 80

In the case of the FTSE 80 series correlation dimensions keeps on increasing as the embedding dimension becomes bigger. The results are given in Table 2.5. As in the previous two time series correlation dimension does not reach a stable value.

Table 2.5: Correlation dimension estimates for the FTSE 80 series.

FTSE 80									
Embedding dimension m	2	3	4	5	6	7	8	9	10
Correlation dimension D_c	0,412	0,693	1,01	1,36	1,74	2,15	2,58	3,03	3,48

The graph of the correlation dimension versus the embedding dimension is illustrated in Figure 2.13. For another time the fact the correlation dimension does not level off to a single value does not help up us to identify the existence of chaotic processes or not. In this case, as well as in the previously two examined time series, the FTSE 80 again is found to be deterministic and not stochastic since the correlation dimension remains below the embedding dimension, as the latter increases and the slope of the graph is less than

45°. However the correlation dimension of FTSE 80 (D_c = 3,48) exhibits a lower value when compared to that of the FTSE 20 (D_c = 4,48) and appears a small difference when compared to FTSE 40 (D_c = 3,08). This means that FTSE 40 and FTSE 80 series are coherent and their lower value of correlation dimension implies a higher degree of determinism.

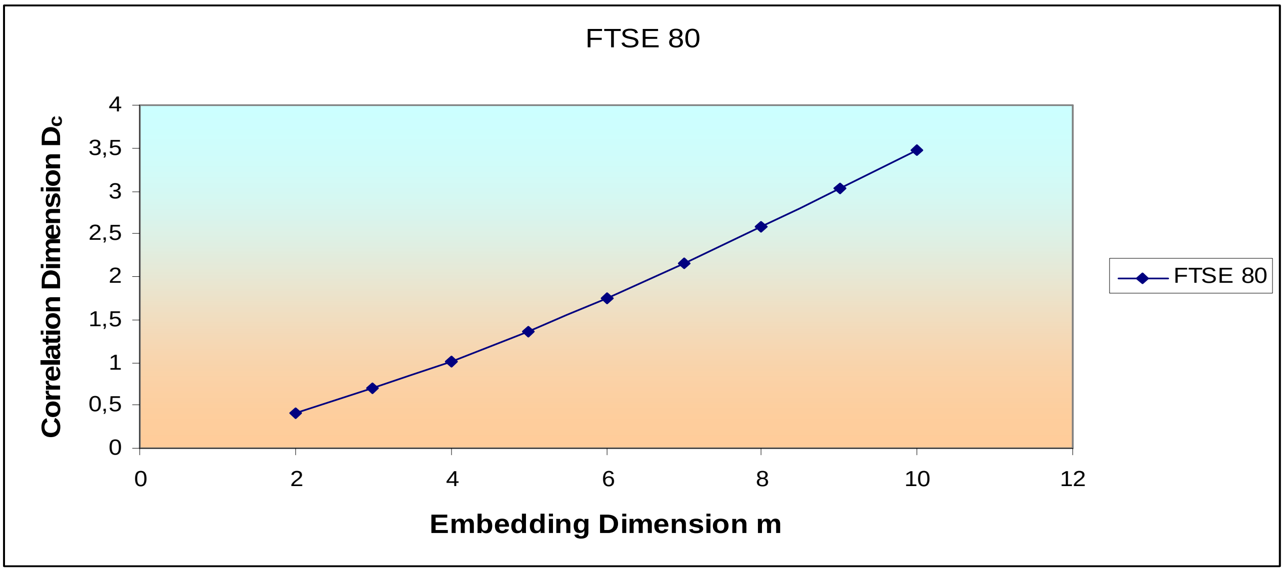


Figure 2.13 Graph of the correlation dimension for FTSE 80 series.

Similarly with the previous cases, the logaritmic plot of the corralation integral versus the correlation dimensions, shown in Figure 2.14, confirms the outputs of Table 2.5. Sharper slopes correspond to higher correlation dimensions as embedding dimensions increase.

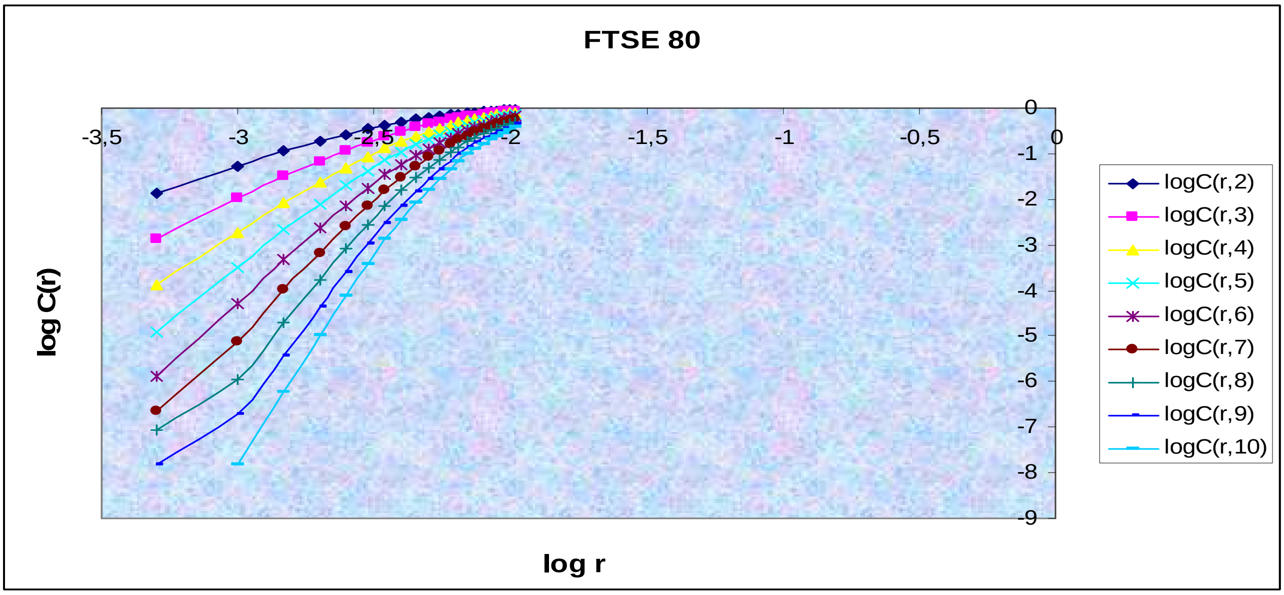


Figure 2.14: Plot of log r versus log C(r) for FTSE 80 series

The values of the Lyapunov exponents for the FTSE 80 series are summarized in Table 2.6 and Figure 2.15 shows the graph of evolution time a versus S(a).

Table 2.6: Lyapunov Exponent estimates for the FTSE 80 series.

FTSE 80									
Embedding dimension m	2	3	4	5	6	7	8	9	10
Laypunov Exponents	0,00837	0,0092	0,00447	0,00483	0,00485	0,00437	0,00427	0,003	0,00273

All Lyapunov exponents are found to be positive and the Larger Lyapunov exponent equals 0,0092 corresponding to an embedding dimension of 3. Since exponent values are all near zero an underlying high-dimensional chaotic system may be present. When the Largest Lyapunov Exponent is compared with the one found previously for FTSE 20 returns series it found to have the same value, therefore indicating the existence of noisy chaos.

The application of the correlation dimension and the Lyapunov exponents for all three series allows us to conclude that the series are characterized by noisy chaos which is a combination of determinism and dynamic noise (Kyrtsou & Terraza, 2003).

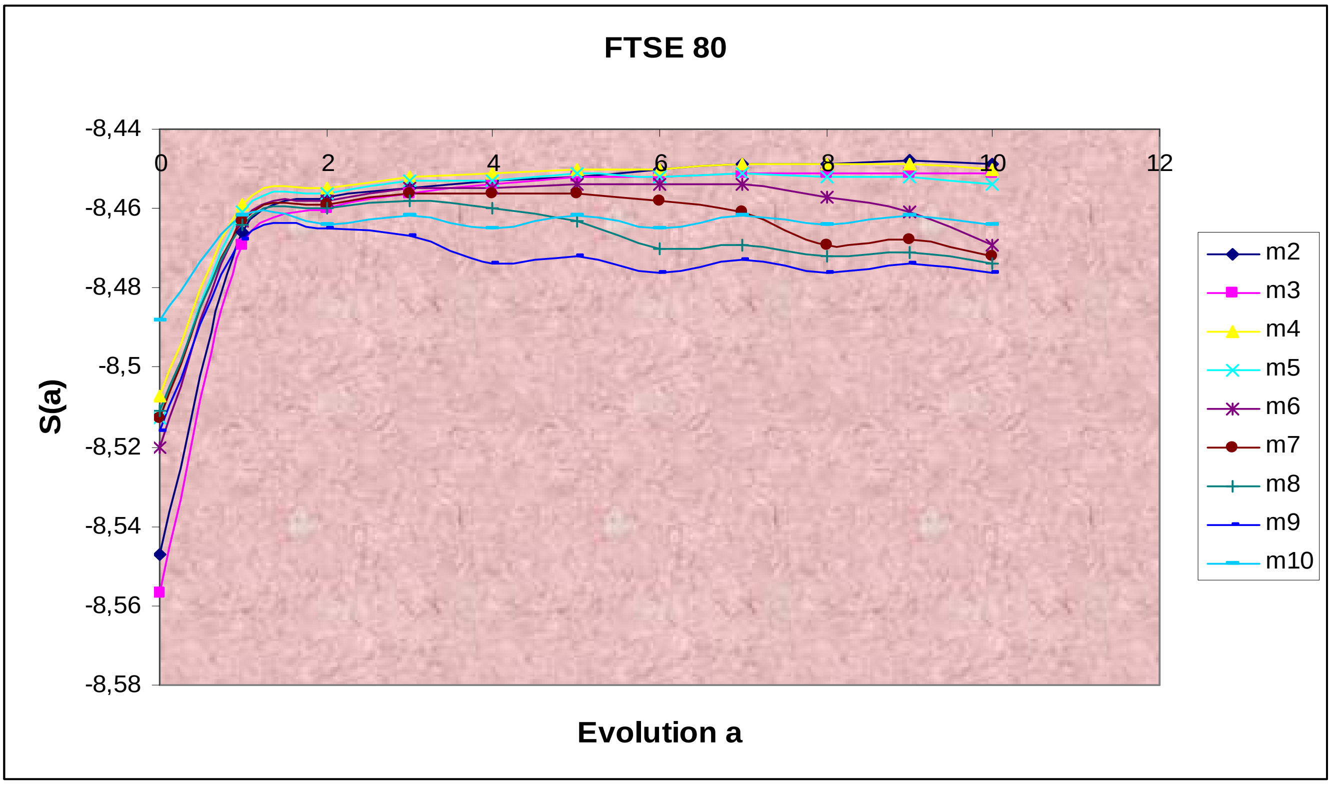


Figure 2.15: Graph of evolution time a versus S(a) for FTSE 80 series

2.3 Discussion

In general the role of financial markets is to make sure that financial resources are efficiently distributed to productive uses. Based on this statement economists created an ideal model of an optimally functioning market and weighted reality against this model. The most widely accepted idea about how financial markets work is the Efficient Market Hypothesis (EMH), which was conceived in the 1960's by Samuelson and Eugene Fama.

However reality appears to be much more different that what the Efficient Market Hypothesis describes. Its assumptions of the rational use of information and homogenous agents have been contradicted by many empirical observations and common experience of market agents. More specifically, imperfections of EMH include:

- All information may not be reflected in current prices because of time delays.
- Not all information is public.
- Investors can gain experience which they do not reveal through their transactions in the market.
- Investors may not use probabilities in the way that Bayes theorem states.
- Investors are not independent from each other. They can appear to have herd behavior.
- Traders can have different transaction costs.
- Traders can work with different time scales and investing horizons.

Chaotic systems have the ability to produce sudden and seemingly random fluctuations similar to the ones often observed in stock markets. Based on stochastic models these disturbances are usually caused by external random shocks. Conversely in chaotic systems these fluctuations are due to internal processes as part of the deterministic procedure. It must be noted that even if chaos is characterized by high unpredictability its deterministic nature allows for short-term forecasts, though not for long-term predictability. So the existence of noisy chaos in financial stock markets can be of great importance.

All the above led to the adoption of a new approach where heterogeneity dominates in stock markets. In relation to this the new models were characterized by heterogeneous interacting agents who were grouped in different populations based on their different beliefs or expectations. The most common types of traders are fundamentalists and noise traders. The first believe that an asset's price is only determined by its fundamental value, whereas the second consider that asset prices can be determined by simple technical trading rules, extrapolation of past trends and other patterns of past prices. A vast amount of recent research papers haw implied that because of this heterogeneity in beliefs market instability and complicated dynamics are produced. In these non-linear models the fluctuations in prices and returns are caused by "*the interaction between a stabilizing force driving prices back towards their fundamental value when the market is dominated by fundamentalists and a destabilizing force driving prices away from their fundamental value when the market is dominated by speculative noise traders*" (Kyrtsou & Terrazza, 2002, p.408). So in the cases, where heterogeneity is adopted the obtained results are observed to be closer to the empirical ones of stock prices, i.e. volatility, fat tails, e.t.c. Hence it can be said that the system itself "fluctuates".

In general financial markets can be characterized as highly complex systems feedback systems where all traders, either collectively or independently, interact with information. Dynamics of non-linear systems are characterized by such feedback processes. Thus non-linear dependency implies the existence of deeper structural forces, most likely noisy chaotic dynamics, which have an effect on financial market outcomes. The main benefit from adopting noisy chaotic dynamics is that it combines structural factors and external noise in order to explain market fluctuations, contrary to traditional stochastic theory where only exogenous forces exist.

This paper examines whether the behavior of Athens Stock Exchange (ASE) indexes, namely FTSE 20, FTSE 40 and FTSE 80 are governed by chaotic dynamics. Descriptive statistics tests for all three series were performed. In all three cases autocorrelation values are not zero indicating that the information is not constantly absorbed and implying some kind of dependence between past and present values. It must be noted that only in the case of FTSE 20 series in the first three lags autocorrelation appears to be zero. Furthermore descriptive statistics reveal for all three time series significant departures from normality as all of them appear to be positively skewed and leptokurtic. Significantly high values of the Jarque-Bera coefficient confirm that stock returns are not i.i.d.. Hence the Efficient Market Hypothesis does not hold.

In order to test for the existence of chaos in the series the diagnostic tools of correlation dimension and Largest Lyapunov Exponents were employed. Correlation dimension results for all three series lack of strong convergence in a single value but reveal that the series appear to be deterministic and not stochastic. Furthermore positive Lyapunov exponent values indicate the existence of noisy chaos in all three series examined.

Several, but not many, papers have been published the last few years concerning the behavior of Athens Stock Exchange Market. According to the findings of Koutmos et al. (1993) paper the first and second moments of the distribution of returns are dependent on time and hence the white noise model does not hold. Barkoulas.and Travlos (1998) in their paper claim that they do not find strong evidence in support of a chaotic structure in the Athens Stock Exchange. In their work Curtis, Hanias and Thalassinos (2007), support that the data set is not simply stochastic implying that there is a deterministic component of dynamics that should be further examined. Finally Saraidaris and Margaris (2008) find a weak but clear evidence of chaos in the Athens Stock Exchange returns series.

Based on our results we find a weak but clear evidence of high-dimensional chaotic structure in ASE high-frequency returns series. Something like that could indicate that although Greek stock market is officially considered mature market in reality it has characteristics of an emerging market.

Conclusions

EMH has been for many years the cornerstone of economic science. According to it the observed price reflects all available information. It also states that all information is public and rational traders process the information through their purchase or selling procedure. Stock prices instantly absorb all new information. From a mathematical point of view EMH supports that financial time series are independent and identically distributed (i.i.d.), and hence behave in a random way, known as white noise. As a consequence, no prediction of future prices is possible from current prices since all information is already included in current prices. Price movements do not follow any patterns or trends. Investors are assumed to be rational and homogenous and they only use the fundamentals of stocks. That means that all economic risks and observed volatility are due to causes which are external to the information system (exogenous processes). So in the absence of exogenous shocks the economy and financial markets are stable.

This paper investigates whether the behavior of the Athens Stock Exchange returns series is governed by noisy chaotic dynamics. Descriptive statistics reveal that for all three series examined, namely the FTSE 20-FTSE40-FTSE80, the Efficient Market Hypothesis does not hold and high values of skewness, kurtosis and the Jarque-Bera coefficient imply that the market is highly inefficient.

To test for chaos the diagnostic tool of correlation dimension was employed. Although in all three cases the correlation dimension does not converge to a single value the fact that its rate of change is slower than the one of the embedding dimension, meaning that slope is less than 1, implies the presence of underlying determinism.

By applying Lyapunov Exponent tests we further examine for the presence of chaos. Again in all three cases the Larger Lyapunov Exponent is found to be positive and close to zero. This is a strong indicator that the series are generated by a noisy chaotic system.

The findings of the study can have some interesting implications. The existence of chaos in Athens Stock Exchange could be exploitable and of great use for market agents. The presence of a high-dimensional chaotic structure implies that short-term predictability may be possible. Finally it seems that Greek Stock market shares a lot of the characteristics of an emerging market rather than a developed one.

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Appenix A

Correlogram of FTSE20

Date: 02/02/12 Time: 20:19
Sample: 3 11639
Included observations: 11637

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.016	-0.016	2.9457	0.086
		2	-0.004	-0.004	3.0920	0.213
		3	0.021	0.021	8.2246	0.042
		4	0.020	0.021	13.049	0.011
		5	0.009	0.010	14.052	0.015
		6	0.005	0.005	14.364	0.026
		7	0.015	0.014	16.834	0.018
		8	0.014	0.014	19.072	0.014
		9	0.011	0.011	20.611	0.014
		10	0.002	0.002	20.677	0.023
		11	0.019	0.018	24.786	0.010
		12	-0.009	-0.009	25.659	0.012
		13	0.013	0.012	27.525	0.011
		14	0.007	0.005	28.025	0.014
		15	0.031	0.030	38.907	0.001
		16	0.016	0.016	41.772	0.000
		17	0.011	0.010	43.070	0.000
		18	0.003	0.001	43.160	0.001
		19	0.007	0.005	43.685	0.001
		20	-0.010	-0.012	44.781	0.001
		21	-0.011	-0.013	46.256	0.001
		22	0.006	0.004	46.708	0.002
		23	0.011	0.011	48.230	0.002
		24	-0.014	-0.014	50.426	0.001
		25	0.004	0.003	50.600	0.002
		26	-0.003	-0.005	50.734	0.003
		27	-0.004	-0.004	50.946	0.004
		28	0.005	0.005	51.269	0.005
		29	0.012	0.012	52.837	0.004
		30	-0.023	-0.024	58.920	0.001
		31	0.002	0.000	58.958	0.002
		32	0.007	0.005	59.508	0.002
		33	-0.008	-0.007	60.196	0.003
		34	0.015	0.015	62.710	0.002
		35	0.002	0.004	62.770	0.003
		36	0.003	0.003	62.873	0.004
		37	-0.018	-0.018	66.729	0.002
		38	-0.004	-0.005	66.918	0.003
		39	0.006	0.006	67.348	0.003
		40	0.023	0.023	73.353	0.001
		41	0.010	0.012	74.415	0.001
		42	0.006	0.006	74.821	0.001
		43	0.017	0.016	78.007	0.001
		44	-0.002	-0.003	78.045	0.001
		45	0.003	0.002	78.128	0.002
		46	-0.012	-0.013	79.909	0.001
		47	0.008	0.006	80.577	0.002
		48	0.007	0.006	81.177	0.002
		49	0.003	0.003	81.313	0.003
		50	0.003	0.002	81.434	0.003
		51	0.004	0.002	81.646	0.004
		52	0.006	0.007	82.084	0.005
		53	0.000	0.001	82.085	0.006
		54	-0.004	-0.006	82.272	0.008
		55	-0.014	-0.015	84.420	0.007
		56	0.005	0.002	84.772	0.008
		57	0.010	0.008	86.020	0.008
		58	-0.011	-0.012	87.453	0.007
		59	-0.029	-0.030	97.537	0.001
		60	0.032	0.031	109.70	0.000
		61	0.019	0.021	113.95	0.000
		62	0.004	0.007	114.14	0.000
		63	0.012	0.011	115.77	0.000
		64	-0.004	-0.004	115.93	0.000
		65	0.003	0.001	116.03	0.000
		66	-0.011	-0.011	117.36	0.000
		67	-0.004	-0.006	117.55	0.000
		68	-0.010	-0.012	118.67	0.000
		69	-0.000	-0.000	118.67	0.000
		70	-0.011	-0.010	120.17	0.000
		71	-0.007	-0.007	120.68	0.000
		72	-0.014	-0.014	123.05	0.000
		73	0.003	0.004	123.13	0.000
		74	0.003	0.004	123.25	0.000
		75	-0.005	-0.005	123.53	0.000
		76	0.003	0.001	123.64	0.000
		77	0.024	0.024	130.11	0.000
		78	-0.013	-0.012	132.06	0.000
		79	0.011	0.009	133.41	0.000
		80	0.004	0.004	133.61	0.000

Correlogram of FTSE20

	81	0.006	0.008	134.00	0.000
	82	0.003	0.004	134.13	0.000
	83	0.016	0.015	137.02	0.000
	84	0.002	0.002	137.05	0.000
	85	0.015	0.015	139.75	0.000
	86	-0.008	-0.008	140.59	0.000
	87	-0.001	0.001	140.60	0.000
	88	0.001	-0.002	140.61	0.000
	89	0.006	0.005	141.04	0.000
	90	0.009	0.008	141.97	0.000
	91	-0.007	-0.007	142.51	0.000
	92	-0.009	-0.015	143.50	0.000
	93	0.022	0.022	149.19	0.000
	94	0.000	-0.001	149.19	0.000
	95	0.009	0.009	150.11	0.000
	96	0.019	0.015	154.53	0.000
	97	-0.011	-0.010	155.93	0.000
	98	-0.012	-0.012	157.52	0.000
	99	-0.010	-0.011	158.61	0.000
	100	0.052	0.050	190.69	0.000

Correlogram of FTSE40

Date: 02/02/12 Time: 20:23
Sample: 3 11639
Included observations: 11637

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.036	0.036	15.259	0.000
		2	-0.014	-0.015	17.548	0.000
		3	0.011	0.012	18.988	0.000
		4	0.021	0.020	24.093	0.000
		5	0.003	0.002	24.233	0.000
		6	0.009	0.010	25.279	0.000
		7	0.021	0.020	30.419	0.000
		8	0.013	0.011	32.344	0.000
		9	0.005	0.005	32.645	0.000
		10	0.008	0.007	33.397	0.000
		11	0.009	0.008	34.373	0.000
		12	-0.006	-0.007	34.792	0.001
		13	0.006	0.006	35.167	0.001
		14	0.013	0.012	37.203	0.001
		15	0.020	0.019	41.952	0.000
		16	0.026	0.025	50.104	0.000
		17	0.015	0.012	52.604	0.000
		18	-0.008	-0.009	53.330	0.000
		19	0.027	0.027	61.748	0.000
		20	0.023	0.019	67.765	0.000
		21	0.002	0.000	67.818	0.000
		22	-0.012	-0.014	69.579	0.000
		23	0.010	0.008	70.777	0.000
		24	-0.006	-0.010	71.269	0.000
		25	-0.004	-0.004	71.439	0.000
		26	-0.010	-0.011	72.543	0.000
		27	-0.004	-0.005	72.733	0.000
		28	-0.004	-0.004	72.921	0.000
		29	0.001	0.001	72.930	0.000
		30	-0.007	-0.009	73.545	0.000
		31	-0.003	-0.003	73.671	0.000
		32	0.006	0.006	74.071	0.000
		33	0.009	0.008	74.962	0.000
		34	0.013	0.012	76.986	0.000
		35	0.004	0.002	77.171	0.000
		36	0.016	0.014	79.980	0.000
		37	-0.018	-0.019	83.669	0.000
		38	-0.021	-0.019	88.672	0.000
		39	0.023	0.022	94.724	0.000
		40	0.049	0.046	122.29	0.000
		41	0.025	0.025	129.52	0.000
		42	-0.003	-0.003	129.65	0.000
		43	0.004	0.003	129.85	0.000
		44	0.002	0.000	129.89	0.000
		45	-0.005	-0.004	130.14	0.000
		46	-0.007	-0.007	130.73	0.000
		47	0.004	0.002	130.88	0.000
		48	0.007	0.005	131.52	0.000
		49	-0.010	-0.013	132.78	0.000
		50	0.011	0.010	134.11	0.000
		51	-0.002	-0.004	134.15	0.000
		52	-0.002	-0.002	134.20	0.000
		53	0.002	0.002	134.24	0.000
		54	0.003	0.001	134.33	0.000
		55	-0.012	-0.016	135.88	0.000
		56	-0.010	-0.013	137.11	0.000
		57	0.012	0.012	138.94	0.000
		58	-0.021	-0.022	144.01	0.000
		59	-0.013	-0.014	146.05	0.000
		60	0.081	0.079	221.90	0.000
		61	0.008	0.003	222.68	0.000
		62	-0.005	-0.000	222.96	0.000
		63	0.006	0.006	223.37	0.000
		64	0.002	-0.002	223.40	0.000
		65	0.004	0.006	223.58	0.000
		66	-0.007	-0.006	224.18	0.000
		67	-0.009	-0.012	225.18	0.000
		68	-0.008	-0.009	225.90	0.000
		69	-0.006	-0.006	226.36	0.000
		70	-0.002	-0.001	226.39	0.000
		71	-0.013	-0.012	228.23	0.000
		72	-0.030	-0.027	239.11	0.000
		73	-0.017	-0.016	242.32	0.000
		74	-0.007	-0.007	242.88	0.000
		75	-0.001	-0.003	242.90	0.000
		76	-0.003	-0.006	242.97	0.000
		77	0.005	0.007	243.32	0.000
		78	-0.023	-0.017	249.33	0.000
		79	0.011	0.009	250.64	0.000
		80	0.047	0.041	277.05	0.000

Correlogram of FTSE40

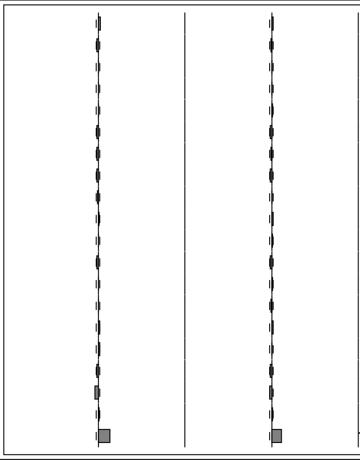
81	0.030	0.027	287.30	0.000
82	0.004	0.007	287.48	0.000
83	0.003	0.002	287.58	0.000
84	0.008	0.007	288.35	0.000
85	-0.003	-0.001	288.44	0.000
86	0.000	0.003	288.44	0.000
87	-0.008	-0.006	289.14	0.000
88	-0.013	-0.012	291.26	0.000
89	0.008	0.010	292.01	0.000
90	0.007	0.007	292.53	0.000
91	0.001	0.003	292.54	0.000
92	0.010	0.012	293.64	0.000
93	0.006	0.003	294.11	0.000
94	0.000	-0.003	294.11	0.000
95	0.017	0.016	297.36	0.000
96	0.013	0.004	299.24	0.000
97	-0.014	-0.016	301.61	0.000
98	-0.015	-0.011	304.22	0.000
99	-0.002	-0.008	304.28	0.000
100	0.065	0.054	353.86	0.000

Correlogram of FTSE80

Date: 02/02/12 Time: 20:24
Sample: 3 11639
Included observations: 11637

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.112	0.112	145.61	0.000
		2	-0.017	-0.030	148.87	0.000
		3	0.013	0.018	150.82	0.000
		4	0.019	0.015	155.04	0.000
		5	0.005	0.002	155.34	0.000
		6	0.019	0.019	159.60	0.000
		7	0.009	0.005	160.61	0.000
		8	0.020	0.019	165.17	0.000
		9	0.004	-0.001	165.33	0.000
		10	0.001	0.001	165.34	0.000
		11	0.015	0.014	168.02	0.000
		12	-0.005	-0.010	168.33	0.000
		13	0.009	0.011	169.24	0.000
		14	0.011	0.007	170.67	0.000
		15	0.015	0.013	173.23	0.000
		16	0.012	0.009	174.94	0.000
		17	0.013	0.010	176.89	0.000
		18	-0.026	-0.029	184.52	0.000
		19	0.036	0.042	199.56	0.000
		20	0.046	0.035	224.24	0.000
		21	0.009	0.001	225.27	0.000
		22	-0.015	-0.015	227.79	0.000
		23	0.006	0.007	228.24	0.000
		24	-0.021	-0.025	233.51	0.000
		25	-0.006	-0.002	233.99	0.000
		26	-0.012	-0.014	235.77	0.000
		27	-0.007	-0.006	236.33	0.000
		28	-0.004	-0.004	236.54	0.000
		29	0.005	0.006	236.79	0.000
		30	-0.018	-0.020	240.61	0.000
		31	-0.007	-0.002	241.13	0.000
		32	-0.005	-0.004	241.39	0.000
		33	-0.004	-0.002	241.55	0.000
		34	0.013	0.014	243.63	0.000
		35	0.017	0.014	246.92	0.000
		36	0.016	0.011	249.74	0.000
		37	-0.017	-0.018	253.08	0.000
		38	-0.034	-0.029	266.60	0.000
		39	0.028	0.033	275.87	0.000
		40	0.097	0.087	385.19	0.000
		41	0.015	-0.001	387.72	0.000
		42	-0.009	-0.008	388.76	0.000
		43	0.006	0.007	389.15	0.000
		44	0.007	0.005	389.75	0.000
		45	-0.003	-0.003	389.88	0.000
		46	-0.008	-0.009	390.58	0.000
		47	0.012	0.014	392.34	0.000
		48	0.005	-0.003	392.62	0.000
		49	-0.007	-0.007	393.22	0.000
		50	-0.001	-0.001	393.25	0.000
		51	-0.006	-0.009	393.68	0.000
		52	-0.004	-0.001	393.90	0.000
		53	-0.005	-0.005	394.19	0.000
		54	0.009	0.006	395.06	0.000
		55	-0.015	-0.023	397.64	0.000
		56	-0.021	-0.021	402.83	0.000
		57	0.001	0.007	402.84	0.000
		58	-0.027	-0.023	411.11	0.000
		59	-0.004	-0.004	411.28	0.000
		60	0.111	0.106	555.17	0.000
		61	0.039	0.017	573.12	0.000
		62	-0.011	-0.011	574.67	0.000
		63	0.000	0.000	574.67	0.000
		64	0.002	0.003	574.74	0.000
		65	-0.001	0.002	574.75	0.000
		66	-0.011	-0.009	576.05	0.000
		67	-0.010	-0.009	577.27	0.000
		68	-0.016	-0.019	580.37	0.000
		69	-0.011	-0.008	581.89	0.000
		70	-0.018	-0.013	585.49	0.000
		71	-0.013	-0.010	587.39	0.000
		72	-0.024	-0.016	594.00	0.000
		73	-0.003	0.003	594.09	0.000
		74	-0.023	-0.028	600.30	0.000
		75	-0.002	0.001	600.33	0.000
		76	-0.004	-0.009	600.50	0.000
		77	-0.003	0.002	600.57	0.000
		78	-0.034	-0.021	614.41	0.000
		79	0.028	0.026	623.41	0.000
		80	0.074	0.052	687.61	0.000

Correlogram of FTSE80



81	0.028	0.017	697.01	0.000
82	-0.009	-0.007	697.95	0.000
83	0.007	0.005	698.54	0.000
84	0.001	0.002	698.56	0.000
85	0.006	0.013	699.05	0.000
86	-0.010	-0.010	700.32	0.000
87	-0.008	-0.006	701.16	0.000
88	-0.014	-0.015	703.54	0.000
89	-0.004	0.001	703.68	0.000
90	0.013	0.017	705.77	0.000
91	0.006	0.008	706.22	0.000
92	-0.015	-0.015	708.99	0.000
93	0.006	0.011	709.38	0.000
94	0.007	-0.001	710.00	0.000
95	0.021	0.020	715.00	0.000
96	0.018	0.007	718.95	0.000
97	-0.012	-0.011	720.61	0.000
98	-0.036	-0.024	736.01	0.000
99	0.015	0.014	738.71	0.000
100	0.146	0.119	988.10	0.000