A. Introduction

Hotelling's paper (1929) was the first to approach horizontal differentiation. Assuming a linear market line, defined in the interval $[0, 1]$, each consumer is located in a different spot of the market, buying one unit of the product or nothing at all. The paper assumes that the price each consumer pays consists of each individual transportation cost, which is linear in distance per unit of length, plus the price each firm charges. Two firms, also restricted to be located in $[0, 1]$, compete in prices. Despite the fact that the products the two firms produce is homogeneous, if the firms locate ate different spots consumers view their products as differentiated, differentiation occurring from the different locations. For convenience, production costs are assumed to be zero. According to Hotelling's results, when firms' locations are given, a small cut in a firm's price does not attract the entire market share of the rival, but only a fraction of it. Moreover, when firms choose their prices after choosing their location, a tendency for product homogeneity occurs, *i.e.*, both firms locate near the center of the line. This result was termed *Principle of Minimum Differentiation*.

D'Aspremont *et.al*. (1979) pointed out that equilibrium in the model as presented in Hotelling (1929) cannot exist when firms are located sufficiently close to each other. To restore the existence of equilibrium, it modified a basic parameter of the model, the linearity of transportation cost function. By assuming a quadratic transport cost function it is shown that firms, not only avoid locating at the center of the market, but instead they locate at the two extremes of the line, a result named *Principle of Maximum Differentiation*. A plethora of papers have followed investigating changes in most basic components of the original Hotelling's paper.

When firms are allowed to choose for their financial structure, they actually choose the amounts of equity and debt needed, in order to operate. Jensen and Meckling (1976), shows that when a firm borrows money its behavior changes, due to the fact that, once debt has been contracted, the manager-owner maximizes equity instead of total firm value. Under limited liability in case of bankruptcy debt-holders become residual claimants. It follows that the manager-owner puts no weight in the amount of losses in case of bankruptcy. This being known by potential lenders it increases the cost of debt, thus, reducing total firm value in case of borrowing.

Brander and Lewis (1986) showed that despite its negative direct effect, leverage may have beneficial strategic effects in an oligopolistic market where rival interaction is important. Using general demand functions, it is shown that acting on behalf of equity holders induces the manager to follow a more aggressive policy in terms of quantities. The intuition behind this result is the fact that the manager of a leveraged firm maximizes expected profits defined only on the range of non-bankruptcy outcomes. In any other case, *i.e.*, when negative demand shocks occur and the firm goes bankrupt, all the operating profits are used for repaying the debt holders. Showalter (1995) examined the aforementioned strategic effects in a context of price competition and showed that there is a significant difference between demand uncertainty and cost uncertainty.

The only work on the long run effects of leverage on product specification is Constantatos, Perrakis and Lefoll (2006), where it is shown that in a vertically differentiated market the role of leverage depends crucially on whether it is undertaken before of after qualities have been chosen. Although the horizontal differentiation literature is enormous covering a great variety of related topics, little work has been done concerning the interaction between horizontal differentiation and financial structure. While Showalter's work examines the impact of leverage on the prices of differentiated products, no work has been done on the impact of leverage on firm's location.

In this work we try to fill this lacuna in the literature and shed light on location competition under demand uncertainty and leverage. We use a quadratic transportation cost model and assume uncertainty over the exact realization of the right end of the consumer distribution, where firm 2 is located. We show that when leverage is not decided before locations, a) some leverage is always beneficial for the firm that faces uncertainty in its turf (firm 2), and b) leverage moves both firms to the left, resulting in firm 2 being located closer to the center and firm 1 further away from it.

In all the cases examined in this work, leverage increases equilibrium prices, as in Showalter (1995). The substitutability between leverage and location in relaxing price competition allows the leveraged firm to locate closer to the center. That the non leveraged firm 1 moves further away from the center is a bit surprising. More thorough investigation shows that this is due to two reasons. First, since the presence of leverage is shown to move firm 2 towards the center, firm 2 moves away from the center in order to increase its distance (differentiation) from its rival (first strategic effect). Second, by moving to the left, firm 1 induces firm 2 to take up more leverage, which results to higher prices for both firms (second strategic effect). The latter effect is absent when either location and leverage are decided simultaneously, or the optimal leverage is a corner solution, unaffected by rival decisions. In both these cases we find that the leftward movement of both firms is tempered, relative to our basic case (game 2).

Defining product differentiation as the distance between firms' locations, we find that when the optimal leverage is an interior solution, product differentiation is in its highest degree, comparing to the rest cases. However, when the optimal leverage is a corner solution, or when location and leverage are decided simultaneously, product differentiation degree is higher than that when financial structure stage is not included in the model. In addition to that, leverage increases firm 2's equilibrium price more than firm 1's.

In section B we present a two-part literature review: the first part examines the relevant papers related to location issues in horizontal product differentiation, while the second reviews the literature related on the impact of financial structure on oligopolies. In section C we solve a benchmark game (game 1), our basic model (game 2) with its special case of corner solution, and game 3 where location and leverage are decided at the same time. Section D concludes.

B. Literature Review

Introduction

A market is an oligopoly when only a few firms provide their products. The high market concentration implied by oligopoly is considered *per* se as a prime suspect for high prices and the presence of above normal (economic) profits. Besides market concentration, a very important feature of market structure is product homogeneity, as expressed by the degree of substitutability between rival products. Perfect substitutability between products occurs when consumers consider products to be identical in their characteristics or attributes. On the other hand, imperfect substitutes are those products that are similar, but not identical. In the former case we view the product as homogeneous, while in the latter as differentiated.

As a determinant of market power, product differentiation rivals concentration in importance. Markets with relatively low concentration can exhibit positive values of the Lerner index if products are imperfect substitutes (for instance, monopolistic competition markets), while others with very high concentration may have prices close to the competitive ones, due to products being relatively homogeneous. In the extreme case of perfectly homogeneous products, when competition takes place in prices, the Lerner index is zero and the oligopoly problem may cease to exist, even if only two firms operate in the market (*Bertrand* paradox). Given the importance of product substitutability, it is no surprise that an important part of the Industrial Organization literature is devoted to the study of product differentiation, usually following one of the following two approaches: the goods approach and the characteristics approach, which gives rise to the well-known address models.

The goods approach, or monopolistic competition model, assumes that in an industry each firm produces a unique product, but though consumers have a taste of variety, *i.e.* their utility increases when consuming several goods, they consider the products to be close substitutes. In this way a producer cannot be a monopolist, since demand is more elastic, but he certainly prices above the marginal cost. In other words, market power exists, but it is restricted.

This theory examines whether the number of brands, and thus the degree of variety, is the optimal or not. Assuming free entry, the positive profits in the short run will attract new firms in the long run. In equilibrium, the number of brands depends on the elasticity of substitution and thus the degree of substitutability that consumers set, and on the fixed costs of entry, as well. Actually, increased scale economies and low degree of differentiation imply low degree of variety.

In the goods approach, the introduction of a new product affects all the existing products symmetrically. This implies that the introduction of a new Sushi bar in a neighborhood will affect both international cuisine and fast food restaurants alike. Thus, while the goods approach is well suited to capture questions related to the number of varieties available in the market, it remains silent when questions of localized competition are raised. This shortcoming is corrected by the "characteristics approach," where consumers are assumed to be interested in consuming not the goods *per se,* but some of their characteristics. For instance, when consumers are assumed to only be interested in a single characteristic, goods can be placed on a straight line according to the "amounts" of that characteristic they contain. Thus, the position of the good, or its "address" on the line, indicates the characteristic in question it contains. Goods that are in neighboring positions on the line are closer substitutes between them than compared to any good that is far away. Consequently, the introduction of a new product has more pronounced consequences for its neighbor products than for products in further distance. Assumptions over consumer's preferences distinguish this approach between vertical and horizontal differentiation.

Vertical differentiation assumes that all consumers rank a product in the same way, implying that in the case of duopoly, the one product is better than the other. Since the usual characteristic of the good is quality, all consumers prefer the product with the better quality when prices are equal. When we examine the case of search goods, *i.e.* when consumers are able to observe quality before buying, the main question is whether the market offers the right quality or not. In case that quality is revealed only after consuming, the products of the firms are experience goods and techniques such as signaling of the firms are also investigated, in order to see firm's behavior. In the extreme case of credence goods, quality is never observed.

The second case of address models occurs when consumers, though identical in everything, have heterogeneous taste preferences for a particular product. As a consequence, products that are imperfect substitutes can coexist in the same market when their prices are equal or slightly different. In this case products are horizontally differentiated and the main research of this case focuses on the degree of differentiation that firms will decide for their products. This degree is conveniently expressed by the relative product location on the line.

Hotelling's model

Hotelling's paper (1929) was the first to approach horizontal differentiation and study the degree of differentiation in a duopoly market. It assumed that consumers are no longer located at a point, but along a market line of length *l*, with uniform distribution. On the same line two firms are located, firm 1 and firm 2, producing at zero cost a product with identical characteristics. Producers charge mill prices and each consumer buys one unit of the product or nothing at all. In order to get this unit, a consumer has to pay a transportation cost, *t*, per unit of length traveled. Since the location and thus the FOB price are different for each consumer, imperfect substitutability occurs between the products of the two firms, although having simplified assumptions over demand.[1]

Assuming transportation cost to be linear in distance, Hotelling (1929) first examined a one-stage model, in which firms' locations, L_1 and L_2, are fixed and producers choose at the same time their prices, P_1 and P_2. Each firms' demand is determined by the consumer X, who is indifferent between the two products. Following a simplifying assumption quite standard in the literature, we assume that the market is defined in the interval $[0,\ 1]$, though

[1] A FOB price consists of the mill price, which is identical to every consumer, plus the individual transportation cost. Mill pricing is defined as the pricing resulted by the production cost.

the same conclusions are acquired for every interval; X's location is defined by:

$$P_1 + t(X - L_1) = P_2 + t(L_2 - X) \quad . \tag{1}$$

Solving (1) for X we get firms' respective demands:

$$D_1(P_1, P_2) \equiv X = \frac{P_2 - P_1}{2t} + \frac{L_2 + L_1}{2} \tag{2}$$

$$D_2(P_1, P_2) = 1 - \frac{P_2 - P_1}{2t} - \frac{(L_2 + L_1)}{2} \tag{3}$$

Each firm's profits are defined as

$$\Pi_i(P_i, P_j) = P_i(D_i(P_i, P_j)), \qquad i, j = 1,2$$

Maximizing each profit function with respect to the firm's price, we get the first order conditions:

$$\frac{P_2}{2t} + \frac{(L_2 + L_1)}{2} - \frac{P_1}{t} = 0 \tag{4a}$$

$$\frac{P_1}{2t} + \frac{(L_2 + L_1)}{2} - \frac{P_2}{t} = 0 \quad , \tag{4b}$$

while the second order conditions are satisfied.

Assuming that firms define their prices instantaneously, the above conditions, which also represent the reaction functions, must be satisfied at the same time. Solving the system of (4a) and (4b), we get the equilibrium prices:

$$P_1^e = \frac{t(2 + L_2 + L_1)}{3} \tag{5a}$$

$$P_2^e = \frac{t(4 - L_2 - L_1)}{3} \tag{5b}$$

Consequently, the equilibrium profits will be:

$$\Pi_1 = \frac{t}{2}\left(\frac{2 + L_1 + L2}{3}\right)^2 \tag{6a}$$

$$\Pi_2 = \frac{t}{2}\left(\frac{4 - L_1 - L2}{3}\right)^2 \tag{6b}$$

In case firms are located in the same position, *t* equals zero and so do prices and profits, demonstrating Bertrand's Paradox for price competing firms. In every other case according to Hotelling, market power can actually exist in a model endowed with the above characteristics.

Although the convexity of the profit functions in their respective locations excludes location optimality, as noticed by Hotelling, he examined the location tendency of each firm. Supposing the location of firm 1 is fixed at point L_1, but firm 2 is able to choose its location, he found that firm 2 would actually prefer a location as near that of the first firm as possible, since $\frac{d\Pi_2^e}{dL_2} < 0$. With a similar reasoning we find that firm 2 would like to locate as close to firm 1 as possible. Given that firms are not allowed to cross each other's location, firms will prefer being located in the center of the market line, an outcome named as *The Principle of Minimum Differentiation.*

Due to the simplified structure, the assumptions over Hotelling's model form crucial determinants for the minimum differentiation result. In what followed, critics of this model searched the role of demand function and elasticity, transportation cost, pricing strategy, consumer's distribution and number of firms, challenging the result of minimum differentiation. Indeed, in several cases when even only one parameter of the model is changed, not only minimum differentiation does not occur, but also firms choose to be located as far as possible, an outcome called *Maximum Differentiation*. However, in many other cases scientists concluded that minimum differentiation or intermediate degrees of differentiation can actually be an equilibrium outcome.

Due to the great volume of the horizontal differentiation literature, we restrict our review to papers that assume a) market is a straight line, b) firms take their decisions simultaneously (Stackelberg leadership is ignored), and c) that consider only pure strategies.

The role of transportation cost

Transportation cost does not only affect location decisions, but most importantly, it has a crucial impact on the existence of equilibrium itself. D'Aspremont, Gabszewicz and Thisse (1979) questioned firms' tendency to agglomerate at the center of the line. Examining the price stage with fixed locations and using identical assumptions, they searched conditions under which X can be set not only between the two firms, but in the backyard demand of each of them, as well[2]. Having three possible ranges of locations of X is equivalent to having three possible domains for the prices. In case that X belongs to firm 2's backyard demand, rearranging X's definition condition we get:

$$P_2^e - P_1^e = t(L_2 - L_1) \qquad (1)'$$

Since L_2 is to the right of L_1, P_1 must be less than P_2. When P_1 equals zero, firm 1 gets zero profits and has an incentive to slightly increase its price, so that P_1 be positive and yet less than P_2, thus yielding positive profits. This reasoning shows that the price of firm 1 as given by (1)', is not the equilibrium one. When P_1 is already positive but still substantially less than P_2, firm 1 gets the entire market, and in response, firm 2 has an incentive to cut its price, which implies that the price of firm 2 in (1)' is not the equilibrium one. When P_1 is once again smaller, but rather close to P_2, firm 1 gets only a fraction of the market, while through a small cut in its price it can capture the entire demand. Hence, the pair (P_1, P_2), as obtained in (1)', is once again not equilibrium.

Due to symmetry, this analysis applies equally when X is situated to the left of both firms. As a result, a necessary and sufficient condition for price equilibrium to exist is that $|P_1^e - P_2^e| < t(L_2 - L_1)$. If and only if this condition is satisfied, the equilibrium prices are the ones presented in Hotelling (1929) and any move near the center increases the profits of each firm. The case where $P_1^e < P_2^e - t(L_2 - L_1)$, firm 1 gets the entire market and the opposite holds true for $P_1^e > P_2^e - t(L_2 - L_1)$: in both cases equilibrium cannot exist.[3]

According to D'Aspremont *et.al*, the technical explanation for the absence of equilibrium is related to the discontinuity of profits functions that occurs when

[2] By backyard demand or consumption we mean the consumers located to the right of firm 2 and to the left of firm 1, respectively.

[3] A similar analysis using a two-stage game reaches the same conclusion.

the indifferent consumer is not located between the two firms. In order to restore equilibrium, a quadratic transport cost function is introduced, while keeping all other assumptions as in Hotelling's original work.

Using a quadratic transportation cost function (QTC), the indifferent consumer is now defined by:

$$P_1 + t(X - L_1)^2 = P_2 + t(L_2 - X)^2 , \tag{7}$$

and equilibrium prices become:

$$P_1^e = \frac{t(2 + L_2 + L_1)}{3}(L_2 - L_1) \tag{8a}$$

$$P_2^e = \frac{t(4 - L_2 - L_1)}{3}(L_2 - L_1) \ . \tag{8b}$$

Since quadratic cost ensures continuous profit functions for all locations, no further modification of the original assumptions is needed in order to ensure equilibrium existence.

Surprisingly, the QTC yields $\frac{d\Pi_2^e}{dL_2} > 0$ and $\frac{d\Pi_1^e}{dL_1} < 0$, implying the *Principle of Minimum Differentiation* under consideration.[4] Using the derivative of firm 1 as an example, we can write:

$$\frac{d\Pi_1^e}{dL_1} = \underset{(+)}{\frac{\partial \Pi_1^e}{\partial L_1}} + \underset{(+)}{\frac{d\Pi_1}{dP_2}} \cdot \underset{(-)}{\frac{dP_2^e}{dL_1}} < 0 \tag{9}$$

Though relocation toward the center increases firm 1's demand, as shown in the first term of (9), the overall outcome is negative, implying that firm 1 will move as far as possible from the market center. What forces a firm far away from the center is the effect of the opponent's price in the firm's profits. Specifically, the first term of (9), which represents the direct effect of firm 1's location on its profits, is positive. This implies that a movement to the center increases firm 1's profits, due to an increase in its respective demand. However, relocation to the center has an indirect impact, as well; it triggers price competition, *i.e.,* it decreases firm 2's price, which in turn decreases firm 1's profits, as shown in the second term of (9). Moreover, the indirect or strategic effect is dominant, implying that any movement towards the market segment would create severe competition in prices. It can be shown (see

[4] The analysis here follows Tirole (p.281).

Tirole p.281) that when firms cannot locate outside the market interval, a corner solution is obtained, setting each firm at the respective edge of the market.

Lambertini (1994) presented a Hotelling model with quadratic transport cost and firms unrestricted to locate anywhere in the real axis line and showed that the optimal locations are $-\frac{1}{4}$ and $\frac{5}{4}$, while the consumer's line remains at [0, 1].

D'Aspremont *et.al* was not the only work that related the linearity of transport cost with the failure of minimum differentiation. Economides (1986) examined a family of transportation cost functions, defined as $f(t) = t^a$, where $1 \leq a \leq 2$. After solving for the equilibrium prices and assuming that equilibrium always exist for the symmetric case, it analyzes the effect of location on firms' profits of firm 1. According to its findings, when $a \in \left[1, \frac{5}{3}\right]$ an interior solution is obtained, with locations within the interval given by $L_1^e = \frac{5}{4} - \frac{3a}{4}$ and $L_2^e = 1 - \left(\frac{5}{4} - \frac{3a}{4}\right)$. When $a \in (\frac{5}{3}, 2]$, and requiring that firms be located inside the market interval, a corner solution is given by $L_1^e = 0$, $L_2^e = 1$.

Interestingly, the presence of interior solution sheds a different light on the results of D'Aspremont *et.al.* (see Equation (9)). Specifically, while for $a = 1$, discontinuity in the profit functions does occur, this does not seem to be the reason for the equilibrium non-existence. For $a < 1$, the profit functions become again continuous and yet there is no price equilibrium. This brings forward the fact that it is the *degree* of convexity in the transportation cost rather than the convexity itself that entails the quasi-concavity in the profit functions, needed to ensure continuous reaction functions and equilibrium existence. However, Economides explored only the symmetric location case.

Gabszewicz and Thisse (1986) explored demand's characteristics in the presence of a linear-quadratic transport cost function. Examining price competition in the case of symmetric and fixed locations, it finds that price equilibrium does not exist when firms are too close to each other. Since price cycles may occur, optimal locations cannot exist. According to its interpretation, it is the lack of concavity in the demand function that prevents equilibrium existence.[5]

Anderson (1988) reexamined the transport cost function, used in Gabszewicz and Thisse (1986) and concluded that only if the linear part of this function is zero can equilibrium exists. When this condition is not met, reaction functions do not intersect due to their discontinuity, imposed by the failure of the quasi-concavity in profits. It further noticed that even when firms are located very close, price equilibrium might exist, while this is impossible when they are slightly further apart.

The strategic effects of relocating

Using either a linear or a quadratic transportation cost function and changing one of the rest determinants, the strategic effect of relocating towards center changes quantitatively, *i.e.,* it becomes more or less intense, or qualitatively, *i.e.*, its sign becomes positive or negative.

An interesting aspect of horizontal differentiation arises from the possibility of price discrimination. Assuming that producers can discriminate among consumers, it is examined whether setting the same price and letting each consumer pay its own transportation cost or setting different prices among consumers, affects firm's location decisions. Price discrimination in the linear model was first examined by Hoover (1937). Under a set of assumptions that eliminate the possibility of reselling and considering the elasticity of demand to be the same and constant for all buyers, price discrimination applies against the remote consumers. On the other hand, freight absorption is suggested when considering the more distant consumers being more elastic, or when assuming linear demand.

[5] In this paper piecewise, linear as well as continuous demand functions are analyzed. As it turns out, concavity in demand is only a sufficient condition for the existence of price equilibrium.

Using a game-theoretic approach, Thisse and Vives (1988) explored a one-stage game where firms choose prices and pricing policy. The pricing policy options are whether to price uniformly and let consumers pay the respective transport cost, or to discriminate in prices. Assuming fixed but not identical locations, uniform pricing never entails equilibrium, unlike discrimination. Using a two-stage game where firms first decide whether to commit or not to a price policy and then set their prices, it concludes that discrimination is the dominant strategy, though a mutual commitment to uniform pricing would lead to increased profits. Firms are restricted in a Prisoner's Dilemma game, in which mill pricing, representing the Pareto optimal of this case, is not equilibrium.[6]

Assuming inelastic individual demand implies that reservation prices are not only identical among consumers, but also high enough to rule out the non-purchase option in equilibrium. Smithies (1941) introduced a linear demand function at every location and examined a one-stage game, where prices and locations are chosen simultaneously. Using economic intuition rather strict mathematic analysis, that paper supported the relocation tendency of firms towards the center. Due to inelastic demand in Hotelling's model, backyard consumption is unaffected by the location of the nearest firm, hence backyard consumers cannot attract firms outwards. In Smithies' model, any increase in transportation cost may diminish the now elastic backyard demand, thus weakening the tendency of locating towards the center.

Developing the intuition coming from elasticity, Bockem (1994) uses a two-stage game with quadratic transportation cost,[7] focusing on the fact that each consumer is willing to pay for the product just as much as his own reservation price. Allowing high reservation prices to exist implies that some consumers may not buy at all, introducing this way an elastic demand. As it is expected, firms neither follow minimum differentiation, in order to avoid price competition, nor confirm D'Aspremont *et.al*, since a hole in the market center will occur.

[6] In the second game firms are located at the two extremes of the market, since *t* is assumed linear in distance. In the first model it is considered as a convex function.

[7] Following her terminology, *t* is normalized to 1, since what Bockem defines as "transportation cost" is the difference between the ideal product and the one offered by a producer. Hence, this cost consists of the consumer-firm distance.

Since the reservation price must always be less than the total cost from buying one unit, *i.e.* the mill price plus the transport cost, Hinloopen and Marrewijk (1999) showed that when the reservation price is the same for each consumer and low enough, any relocation towards center generates a trade-off between the transport cost and the price of a firm. Therefore, firms act as monopolists and a continuum of equilibrium locations exist, including maximum differentiation. If the reservation price has an intermediate value, firms act as oligopolies, supplying the whole market, while choosing an intermediate differentiation level.

Existence of equilibrium may also arise in the presence of non-uniform distributions. Shilony (1981) resets the one-stage game by using a general form of consumers' distribution. Equilibrium always exists for symmetric locations, implying equality in prices. Moreover, a fixed location advantage, i.e., when one firm is located nearer the center, increases its price comparing to its rival, reflecting this way the substitutability between mill pricing and transport cost. Finally, when firms are located close, there is always a density function postulating equilibrium, explaining this way Hotelling's intuition for minimum differentiation.

Neven (1986) examines a range of concave distributions and proves that a unique Nash equilibrium in prices exists. Continuing to the location sub-game, it shows that when center concentration is not too high, D'Aspremont *et.al* are confirmed. However, moving away from the center is less profitable than that in the uniform case, due to a greater loss of demand, provided demand concentration. Consequently, when demand is highly concentrated near the center, firms choose to locate inside the market line.

The above models cannot account for observed asymmetries in demand concentration. Using log-concave general distributions, Anderson, Goeree and Ramer (1997) include asymmetry in demand concentration. In this way it ensures price equilibrium by using a weaker assumption than concavity. According to this research, when distribution is too tight firms have a strong incentive to move towards center, decreasing their prices. When both asymmetry and concavity are in low levels, the location equilibrium existence is unique, while in symmetric distributions with high concavity, asymmetric equilibria take place.

Introduction of Uncertainty

Rhee, De Palma, Fornell and Thisse (1989) were the first to introduce uncertainty in horizontal differentiation, supporting with their results the Principle of Minimum Differentiation. Consumers' utility is stochastic, depending on both an observable and an unobservable product attribute. The latter introduces a stochastic element in the utility function, for which uncertainty is assumed. It is shown that when consumers have increased heterogeneity for the uncertain variable, *i.e.,* when their tastes are sufficiently different, minimum differentiation is the only result. Consequently, the level of differentiation increases when there is low degree of uncertainty, since the two attributes play a similar role.

Meagher and Zauner (2004) introduce uncertainty in the total demand location. Assuming the market center, M, deviates symmetrically by an amount less than $|0.5|$, both firms face uncertainty in a location-then-price game. According to the results, when uncertainty is revealed before the price decisions are taken, firms prefer high differentiation, while equilibrium in every stage is unique.[8] Moreover, the optimal locations are not corner solutions, since the firms are allowed to move outside the market line. It is also worth mentioning that increases in uncertainty lead to increased differentiation.

In order to study the degree of differentiation in an uncertain environment, Casado-Izaga (1999) assumes consumers are located in $|\theta,\ \theta + 1|$, with θ~U$|0,\ 1|$. Firms do not know the value of θ, but only its distribution. In the first stage firms choose locations, then θ is revealed and finally they choose their prices. Optimal locations of firm 1 and 2 are $\frac{2}{9}$ and $\frac{16}{9}$ respectively. Comparing these results to those in Lambertini's (1977), it reveals that uncertainty over consumers' distribution increases product differentiation.

[8] Transportation cost is once gain assumed to be of quadratic form

Balvers and Szerb (1996) use a game with fixed prices, in order to study the location choice when firms are uncertain over the differentiation degree of their products. Equality in prices guarantees uniqueness of equilibrium. Introducing a small probability of firms producing differentiated products, firms realize the actual level of differentiation after choosing their locations, while every decrease of substitutability between the products is considered to be a positive shock to a firm's demand. Since locating towards the center increases the risk of producing a less differentiated product, the principle of minimum differentiation does not hold.

On the other hand, Harrington (1992) studies a fixed-location game, instead of fixed prices, introducing uncertainty over the degree of differentiation. In this case, firms face a probability of producing similar but not identical products, while at the same time infinite horizon is assumed. In this way, a cut in price of a firm, imposing decrease in the rival's price as well, implies a trade off between current and future profits. Specifically, equality in prices in a present round of the game, even when these prices are above the positive marginal cost, is considered to be an admission of the firm that it produces a product identical to its rival. This fact in turn would intensify price competition in the future. On the contrary, not competing the rival's present price would preserve future uncertainty. In this way, firms would avoid price competition.

Matsumura and Matsushima (2003) studied uncertainty in the production cost of each firm. In this model, firms first choose locations, then they decide for their cost of production and finally they choose their prices. Assuming insignificant level of uncertainty, firms locate at the edges of the market. On the contrary, minimum differentiation is achieved when the degree of uncertainty is substantial. Intermediate values of uncertainty lead to multiple equilibria, including both minimum and maximum differentiation.

Financial Structure

Assuming that a firm needs to finance its operation, its ownership structure is determined by whether its financing is done through debt, equity markets or both. The resulting "financial structure' of the firm, *i.e.*, the amounts of equity and leverage that finance its operation, may have significant implications on its behavior, as well as on the behavior of the rival firm. While many works study the relation between financial structure and pricing, only a few examine the relation between financial structure and product quality. Our main subject, *i.e.*, the relation between structure and location has been rather neglected. The basic difference between debt and equity holders is that the former borrow and pay back a fixed amount of money. On the other hand, in case of bankruptcy, though the debt holders are the first to be considered for repayment, and since there is no sufficient revenue to cover the entire debt, equity holders become residual claimants.

Jensen and Meckling (1976) (J&M hereafter) studied the factors that determine firms' decisions over their ownership structure. It assumes that even when managers are the sole owners of their firms, they maximize their utility rather than maximizing their profits. Manager's utility depends on both the market value of their wage contract, defined as wealth, and expenditures, such as on-the-job consumption, defined as non-pecuniary benefits. The indifference curves of a manager are convex, since the marginal rate of substitution between non-pecuniary benefits and wealth diminishes when the level of benefits increases. Utility is maximized under the restriction of the current market value of the firm. It further assumes that any on the job consumption decreases the pecuniary returns a manager receives from the enterprise.

However, if a manager is the enterprise's only residual claimant, he bears the full cost of any on the job consumption. Specifically, when the firm is all equity financed, the manager maximizes his utility by choosing optimal combinations of total value of the firm and cost of non-pecuniary benefits. Non-pecuniary benefits decrease the value of the firm, but increase the manager's utility by consuming such benefits. As long as the manager is the only owner, his wealth is reduced by the full cost of his consumption.

J&M further investigates the case where the manager decides to sell a fraction $(1-\alpha)$ of the equity to outside investors, while retaining a fraction α for himself. In this case, consuming non-pecuniary benefits decreases his wealth only by α. Since the cost of consuming decreases, the manager increases the level of consumption and that represents the *agency costs* or residual loss. Since perfect information is assumed throughout the paper, any potential share-holders will take into account the agency costs when examining the value of the firm. Specifically, they will be willing to pay only a fraction $(1-\alpha)$ of the expected firm value, given the change in the manager's behavior, implied by the change in ownership. In other words, the shareholders will anticipate the change in the manager's behavior, and discount it by $(1-\alpha)$. Taking the real cost of his slackness and non-pecuniary benefits, the manager wishes to take less. He needs however to find credible ways to commit ex ante in taking less of these benefits. Consequently, it is in the agent's interest to limit the agency costs of equity.

J&M shows that debt implies excessive risk-taking. Specifically, the manager in this case chooses riskier projects with lower expected value, increasing this way the probability of default. The reason behind this result is the asset substitution effect that occurs, since a risky project with negative net present value increases the equity value more than a safer project with positive net present value. The fact that equity holders can extract value from debt holders by increasing investment risk, after the debt level is chosen, represents the agency cost in the debt financing case. Since rationality still holds, debt holders anticipate shareholders' behavior by valuing less the firm before committing to any loan.

Brander and Lewis (1986) (B&L hereafter) were the first to examine the interaction between a firm's financial structure and its decisions in an oligopoly market in the presence of uncertainty. Furthermore, uncertainty, z, is uniformly distributed along the interval $\left[\underline{z}, \bar{z}\right]$ and may occur in demand or production cost of each firm. Though a positive value of z may either increase or reduce profits, both cases are examined in B&L We present only the first case, since the outcomes of the second case are just the opposite.

The model assumes that two Cournot firms play a two-stage game, choosing debt at the first stage and output levels at the second. Uncertainty, which is common for both firms, is revealed only after the output levels are decided. [9]

Let V be the total value of the firm, *i.e.,* the sum of the equity value, E, and the debt value, W. The output chosen by the manager maximizes either E, or W, depending on whether the manager acts in the interest of shareholders or debt holders, respectively. In the first case, firm i's Equity value is defined as the expected operating profits minus debt, in the range of the *good states of the world*, *i.e.*, the expectation taken over the states in which the firm is non bankrupt. Since we consider the case where $\frac{\partial R}{\partial z} > 0$, there must be a value of z, call it $\hat{z}$, such that $R_i\left(q_i, q_j \hat{z}_i\right) - D_i = 0$. Hence, equity value is $\int_{\hat{z}}^{\bar{z}} (R - D)\, dz$.

It follows that a small increase in debt increases firm i's output, since $\frac{\partial q_i^*}{\partial D_i} > 0$.

Therefore the leveraged firm will produce, ceteris paribus, more output than an all-equity financed firm. This in turn implies that debt shifts the reaction function of the leveraged firm to the right, thus increasing that firm's output and decreasing its rival's output in equilibrium. In other words, debt induces an aggressive behavior, since taking into account only the good states of the world is equivalent to making a riskier investment. [10]

Using second stage's results and continuing to the first stage, firm i's maximized value is:

$\max V_i\left(q_i(D), q_j(D), D\right)$,

---

[9] Firms are considered to be symmetric throughout the article.

[10] On the other hand, when the manager represents the debt holders, it maximizes W, *i.e.*, the operating profits of the firm as defined in the interval $\left[\underline{z}, \hat{z}\right]$ and the output that results is less than before.

where $D = (D_i, D_j), i, j = 1,2.$

Analyzing the maximization problem, we get:

$$\frac{dV_i}{dD_i} = 0 \Rightarrow \frac{\partial V_i}{\partial q_i} \cdot \frac{dq_i}{dD_i} + \frac{\partial V_i}{\partial q_j} \cdot \frac{dq_j}{dD_i} = 0 \Rightarrow$$

$$\underset{(+)}{\frac{dq_i}{dD_i}} \cdot \left(\underset{(0)}{\frac{\partial E_i}{\partial q_i}} + \underset{(-)}{\frac{\partial W_i}{\partial q_i}} \right) + \underset{(-)}{\frac{dq_j}{dD_i}} \left(\underset{(-)}{\frac{\partial E_i}{\partial q_j}} + \underset{(-)}{\frac{\partial W_i}{\partial q_j}} \right) = 0 \qquad (10)$$

The first term of the above equation represents the direct effect of debt on firm's value and the second the indirect or strategic effect.

Through the second stage's results, it is already stated that an increase in firm's debt increases its output and decreases the rival's output. Moreover, in the second stage the firm chooses the output that maximizes its equity value, so the first term of the first parenthesis is zero, due to the envelope theorem. On the other hand, when the manager acts on behalf of the debt holders, any increase in debt decreases the debt value of the firm. Consequently, the direct effect has a negative sign. On the other hand, any decrease in the rival's output, caused by the increased debt of the firm, increases operating profits and hence both E and W, which explains the positive sign of the strategic effect.

Though taking over debt triggers two opposite forces, B&L shows that the strategic effect dominates and an interior solution is obtained. This is demonstrated by the fact that when D equals zero, *i.e.*, when the firm is all-equity financed, the second term of the first parenthesis becomes zero, which implies that debt does not cause any direct effect on firm's value. Since the indirect effect is always positive, any positive amount of debt will increase firm's value. Hence, firms may use debt strategically, in order to increase their market share.

Showalter (1995) applied the B&L idea in a price competition game, investigating the effects of uncertainty on firms' decision to take over debt. Firms again choose debt at the first stage, but now at the second stage the choice variable is price. As before, demand contains a random shock, the exact value of which is revealed after prices have been set. It is shown that debt shifts upwards both firms' reaction functions, which in turn results in higher equilibrium prices.

Haan and Toolsema (2004) examined the case of Bertrand competition, in which firms first choose their debt levels and then compete in prices, using linear demand functions. Demand uncertainty is revealed after the final decisions are taken. In its linear demand functions, it uses an additional term, z , representing a common exogenous shock in each firm's demand function. According to its results, leveraged firms have higher prices, but lower quantities and profits, comparing to all-equity financed firms. Moreover, increases in uncertainty imply less debt in equilibrium.

The model

We begin our analysis with the Game 1, a two-stage duopoly model, which mainly serves as the benchmark case. Consumers are uniformly distributed along the market interval $\lfloor 0, \alpha \rfloor$. The right end of the consumer interval is random, fluctuating uniformly between (1-σ , 1+σ). $E(\alpha) = 1$ and density equals 1. Each consumer buys one unit of the product or none at all. In addition to the price, consumers pay a transportation cost of *t* per unit of length traveled. We assume transportation cost to be quadratic in distance traveled.

In the first stage firms choose their locations simultaneously. While all consumers are located on the $\lfloor 0, \alpha \rfloor$ interval, firms are allowed to locate outside the consumer interval. Without loss of generality we further assume that firm 2 is always to the right side of firm 1, that is $L_2 \leq L_1$. Consequently, firm 2 faces uncertainty directly in its turf, since the right edge of consumers' distribution, and thus its market size, are not *a priori* known. For simplicity we set both firms' marginal and fixed cost equal to zero.

In the second stage each firm decides its price, taking the locations as given. The two firms choose prices simultaneously before the value of α is revealed. After all decisions are made, the value of α is revealed and sales take place.

In the second and the third games the assumptions over demand and supply remain the same, but one more stage is introduced, during which the firm directly facing uncertainty, *i.e.* firm 2, decides about its financial structure.[11] This structure is determined by the amount of debt, D, the firm borrows from bond- holders. We assume that once debt has been contracted, the manager of the firm acts on behalf of the equity-holders.

The distinction between Games 2 and 3 comes from the number of the stages. In game 2, firms simultaneously choose their locations during the first stage. In the second stage firm 2 decides for the level of D; in the third stage firms simultaneously choose their prices. In Game 3, debt and location are chosen both at the first stage, while the price stage follows.

GAME 1

The consumer indifferent between the two products is located at X, defined by equation (7).

Solving (1) we get firm 1's respective demand:

$$D_1(P_1, P_2) \equiv X = \frac{P_2 - P_1}{2t(L_2 - L_1)} + \frac{L_2 + L_1}{2} \qquad (2)'$$

And consequently firm 2's respective demand:

$$D_2(P_1, P_2) = \alpha - \frac{P_2 - P_1}{2t(L_2 - L_1)} - \frac{(L_2 + L_1)}{2} \qquad (3)'$$

We solve the model using backward induction. In stage 1 firm 1 maximizes its profits with respect to its price, P_1.

$$\max_{P_1} \Pi_1(P_1, P_2) = P_1\left(\frac{P_2 - P_1}{2t(L_2 - L_1)} + \frac{L_2 + L_1}{2}\right) \qquad (4)$$

From the FOC we derive the best response of firm 1 to any P_2

$$P_1(P_2) = \frac{P_2}{2} + \frac{t(L_2 - L_1)(L_2 + L_1)}{2} \qquad (5)$$

While the SOC is satisfied.

The maximization problem for the second firm is quite different, since firm 2 needs to choose its price before the realization of the consumers distribution

[11] The financial structure of the other firm is irrelevant.

is known. In order to find its reaction function, we maximize its profits according to the expected value of α :

The expected profits function will be:

$$\max_{P_2} E\left(\Pi_2\left(P_1,P_2\right)\right) = \int_{1-\sigma}^{1+\sigma} P_2\left(\alpha - \frac{P_2 - P_1}{2t\left(L_2 - L_1\right)} - \frac{(L_2 + L_1)}{2}\right) f(\alpha)\cdot d\alpha \qquad (6)$$

Taking into account that $E(\alpha) = 1$, the reaction function of firm 2 is:

$$P_2\left(P_1\right) = \frac{P_1}{2} + \frac{t\left(L_2 - L_1\right)\left(2 - L_2 - L_1\right)}{2} \qquad (7)'$$

Substituting (7) into (5) we get the equilibrium prices:

$$P_1^e = \frac{t\left(L_2 - L_1\right)\left(2 + L_2 + L_1\right)}{3} \qquad (8a)$$

$$P_2^e = \frac{t\left(L_2 - L_1\right)\left(4 - L_2 - L_1\right)}{3}, \qquad (8b)$$

which are identical to (8a) and (8b).

Proceeding to the second stage, firm 2 maximizes its profits, taking firm 1's location as given:

$$\max_{L_2} E\left(\Pi_2\left(P_1^e\left(L_1,L_2\right),P_2^e\left(L_1,L_2\right)\right)\right) = \int_{1-\sigma}^{1+\sigma} P_2^e\left(\alpha - \frac{P_2^e - P_1^e}{2t\left(L_2 - L_1\right)} - \frac{(L_2 + L_1}{2}\right) f(\alpha)\cdot d\alpha$$

The reaction function of firm 2 to any L_1 is:

$$L_2 = \frac{4 + L_1}{3} \qquad (9)'$$

Similarly for firm 1:

$$\max_{L_1} \Pi_1\left(P_1^e\left(L_1,L_2\right),P_2^e\left(L_1,L_2\right)\right) = P_1^e\left(\frac{P_2^e - P_1^e}{2t\left(L_2 - L_1\right)} + \frac{L_1 + L_2}{2}\right)$$

The reaction function of firm 1 is:

$$L_1 = \frac{L_2 - 2}{3} \qquad (10)'$$

Equating (9) and (10) we get the equilibrium locations:

$$L_1^e = -\frac{1}{4} \qquad (11a)$$

$$L_2^e = \frac{5}{4} \qquad (11b)$$

It is useful to note at this point that in the absence of uncertainty over the consumers' distribution, the optimal prices and locations would have been the same. Specifically, the results of every stage are the same with those obtained in Lambertini (1977).

Once located at the optimal spots, firms sell their products at:

$$P_1^* = P_2^* = \frac{3t}{2} \tag{12}$$

Substituting (11a), (11b) and (12) into (4) and (6), we calculate the expected values of firm 1's and 2, respectively:

$$E(\Pi_1) = \frac{3t}{4}, \; E(\Pi_2) = \frac{3\sigma \cdot t}{2} \tag{13}$$

We note that the expected profits of firm 2 are an *increasing* function of σ. In other words, firm 2's profits increase with the variance of its part of the market. When σ is sufficiently large, $(\sigma > 0.5)$, $E(\Pi_2) > E(\Pi_1)$

FINANCIAL STRUCTURE SELECTION

To examine the second and third games we first solve the price stage, which is common in both.

We assume that the manager of firm 2 protects the share- holders' interest. We further assume limited liability, *i.e.*, in demand states where net revenues are insufficient to cover the promised debt payment, debt holders become residual claimants. Consequently, in those states shareholders earn nothing, which explains their indifference among all possible bankrupt outcomes. On the other hand, their interest concentrates on outcomes where they do earn a positive amount of money, *i.e.*, cases with operating profits greater than debt.

The demand of each firm and the price reaction function of firm 1 are the same as before, given by (2), (3) and (5). To find the reaction function of firm 2, we maximize its Equity Value function with respect to P_2:

$$\max_{P_2} E(P_1, P_2, \alpha) = \int_{\alpha_Z}^{\bar{\alpha}} (\Pi_2(P_1, P_2\alpha) - D) \cdot f(\alpha) \cdot d\alpha \tag{14}$$

Where $\bar{\alpha} = 1 + \sigma$ and α_z is defined by:

$$\Pi_2(P_1, P_2, \alpha) = D \Rightarrow \alpha_Z = \frac{D}{P_2} + \frac{P_2 - P_1}{2t(L_2 - L_1)} + \frac{L_2 + L_1}{2} \qquad (15)$$

Solving (12) we get the reaction function of the firm:

$$P_2(P_1) = \frac{P_1}{2} + \frac{t(L_2 - L_1)\cdot\left(2\hat{\alpha}_Z - L_2 - L_1\right)}{2}, \qquad (16)$$

where $\hat{\alpha}_Z = E(\alpha \mid \alpha \geq \alpha_Z)$. The equilibrium prices will now become:

$$P_1^e = \frac{t(L_2 - L_1)\cdot\left(\hat{\alpha} + \alpha_Z + \sigma + L_2 + L_1\right)}{3} \qquad (17a)$$

$$P_2^e = \frac{t(L_2 - L_1)\cdot\left(2\hat{\alpha} + 2\alpha_Z + 2\sigma - L_2 - L_1\right)}{3} \qquad (17b)$$

Or equivalently:

$$P_1^e = \frac{t(L_2 - L_1)\cdot\left(2\hat{\alpha}_Z + L_2 + L_1\right)}{3} \qquad (17a')$$

$$P_2^e = \frac{t(L_2 - L_1)\cdot\left(4\hat{\alpha}_Z - L_2 - L_1\right)}{3} \qquad (17b')$$

Comparing the last pair of prices with (8a) and (8b), notice that they only differ with respect to the $\hat{\alpha}_z$ term, which is obviously equal to 1 in the fist game,

since $\alpha_Z = 1 - \sigma \Rightarrow \hat{\alpha}_Z = \frac{\bar{\alpha} + 1 - \sigma}{2} \Rightarrow \hat{\alpha}_Z = 1$, where $\hat{\alpha}_Z = E(\alpha / \alpha \geq \alpha_Z) = \frac{\bar{\alpha} + \alpha z}{2}$.

It is obvious that these prices are greater than those in Game 1, $\forall\, \alpha_Z > 1 - \sigma$.

GAME 2

Since rationality is not questioned, the manager of firm 2 knows that the debt- holders do know the actual debt needed for the investment. At the same time the debt- holders know that the manager realizes the way they evaluate an investment. As a result, finding the debt holders' willingness to lend is equivalent to finding the optimal debt level for the firm, along the range of all possible outcomes. It is also assumed that all the agents, including the debt holders, are risk-neutral.

From (13) we find that $\frac{d\alpha_Z}{dD} > 0$, which implies that a change in D changes α_z at the same direction; taking over a greater amount of debt implies an increase at the point where debt equals the operating profits, *i.e.,* α_z. Consequently, the price-maximized profits must be the same, whether using D, as the choice variable, or α_z. We follow the second case, to avoid complex calculations.

Using third stage results, the manager's problem in the second stage becomes:

$$\max_{\alpha_Z} \Pi_2\left(P_1^e, P_2^e, \alpha\right) = \int_{1-\sigma}^{1+\sigma} P_2^e \left(\alpha - \frac{P_2^e - P_1^e}{2t\left(L_2 - L_1\right)} - \frac{(L_2 + L_1)}{2}\right) f(\alpha) \cdot d\alpha \qquad (18)$$

and the optimal α_z is:

$$\alpha_Z^{\ e} = 2\hat{\alpha} - \sigma + \frac{-L_1}{4} + \frac{-L_2}{4} \qquad (19)$$

Next we evaluate the effect of σ on the optimal prices and α_z:

$$\frac{dP_1^e}{d\sigma} = \frac{dP_1^e}{d\alpha_Z}\frac{d\alpha_Z^{\ e}}{d\sigma} + \frac{\partial P_1^e}{\partial \sigma} \qquad (20)$$

$$\frac{dP_2^{\ e}}{d\sigma} = \frac{dP_1^e}{d\alpha_Z}\frac{d\alpha_Z^{\ e}}{d\sigma} + \frac{\partial P_1^e}{\partial \sigma}, \qquad (21)$$

where:

$$\frac{\partial P_1^e}{\partial \sigma} = \frac{dP_1^e}{\alpha_Z} = \frac{t}{3}\left(L_2 - L_1\right) \qquad (22)$$

$$\frac{\partial P_2^e}{\partial \sigma} = \frac{dP_2^e}{d\alpha_Z} = \frac{2t}{3}\left(L_2 - L_1\right) \qquad (23)$$

$$\frac{d\alpha_Z^{\ e}}{d\sigma} = -1 \qquad (24)$$

Substituting (31), (32) and (33) into (29) and (30), we find that

$$\frac{dP_1^e}{d\sigma} = 0 \qquad (20)'$$

$$\frac{dP_2^e}{d\sigma} = 0 \qquad (21)'$$

As it is shown, a change in σ implies an opposite change in the optimal α_z. What is striking is that when σ increases, not only optimal prices, but also L_2 - L_1 increases. Since for every possible value of α, firm 2's demand is less than before due to σ's increase, the optimal α_z decreases in the second stage. However, according to (20)' and (21)', a small change in σ does not affect equilibrium prices. Since α_z can be inside or outside the interval $\left[\underline{\alpha}, \bar{\alpha}\right]$, we examine the two cases separately:

Interior Solution: $\alpha_Z^e \in \left[\underline{\alpha}, \bar{\alpha}\right]$

Substituting the optimal α_z into firm 2's Value and maximizing the latter with respect to L_2:

$$\max_{L_2} \Pi_2\left(P_1^e\left(L_1, L_2, \alpha_Z^*\right), P_2^e\left(L_1, L_2, \alpha_Z^*\right)\right) =$$

$$\int_{1-\sigma}^{1+\sigma} P_2^e\left(\alpha - \frac{P_2^e - P_1^e}{2t\left(L_2 - L_1\right)} - \frac{(L_2 + L_1)}{2}\right) f(\alpha) \cdot d\alpha \qquad (25)$$

The reaction function is:

$$L_2 = \frac{1}{3}\left(4\hat{\alpha} + L_1\right) \qquad (26)$$

Similarly, maximizing firm 1's profit function with respect to its location:

$$\max_{L_1} \Pi_1\left(P_1^e\left(L_1, L_2, \alpha_Z^*\right), P_2^e\left(L_1, L_2, \alpha_Z^*\right)\right) = P_1^e\left(\frac{P_2^e - P_1^e}{2t\left(L_2 - L_1\right)} + \frac{L_1 + L_2}{2}\right) \qquad (27)$$

This yields firm 1's reaction function:

$$L_1 = \frac{1}{3}\left(-4\hat{\alpha} + L_2\right) . \qquad (28)$$

Solving the reaction functions system of (26) and (28), we find the equilibrium locations:

$$L_1^* = -\hat{\alpha} \ , \ L_2^* = \hat{\alpha} , \qquad (29)$$

as well as the equilibrium value of α_z:

$$\alpha_Z{}^* = 2\hat{\alpha} - \sigma \qquad (30)$$

Since we have assumed an interior solution, $\alpha_z^* \leq \bar{\alpha}$ which implies that $\sigma > 0.5$ is a sufficient condition for having mixed financial structure. Assuming $\hat{\alpha} = 1$, since α_Z^* must be positive σ must be less than 2, so $\sigma \in (0.5,2)$.

The equilibrium prices will be:

$$P_1^* = 2t \ , \ P_2^* = 4t \qquad (31)$$

We first notice that the prices of the two firms are no longer equal, which is explained by the fact that P_2 is more sensitive to a change of α, than is P_1, *i.e.*,

$$\frac{dP_1^e}{d\hat{\alpha}_Z} = \frac{2t}{3}(L_2 - L_1) < \frac{dP_2^e}{d\hat{\alpha}_Z} = \frac{4t}{3}(L_2 - L_1) \qquad (32)$$

While changes in the upper end of consumers' distribution affect directly firm 2's revenues, through affecting its demand, they only affect indirectly firm 1, through its reaction function. This means that P_2 increases whenever α increases, while P_1 increases as a reaction to any increase of P_2.

Furthermore, since $\alpha_Z > 1 - \sigma \Rightarrow \hat{\alpha}_Z > 1$ we find that leverage increases both prices, comparing to those in Game 1, where $\hat{\alpha}_Z = 1$, due to the absence of leverage.

Using the optimal prices and locations, expected firm values become:

$$E(\Pi_1) = t \, , \ E(\Pi_2) = 4\sigma \cdot t . \qquad (33)$$

Since $\sigma > 0.5$, it follows that $E(\Pi_2) > E(\Pi_1)$.

The presence of leverage has further implications, since it affects locations through prices. Comparing Equation (29) to the corresponding ones in Game 1, we see that while firm 2 moves toward the centre of the line, firm 1 chooses a location further away from the centre. Since α_z increases prices, the range of locations that firm 2 may choose, without triggering price competition, is shifted to the left. According to (19), the closer the location of firm 2 to the centre, the greater the value of α_z. On the other hand, having firm 1 moving to the right would decrease α_z, according to (19), so price competition would be severe. This explains the incentive of firm 1 to be located even further.

Corner Solution: $\alpha_Z \to \bar{\alpha}$

When the fluctuation of α is rather narrow, *i.e.*, $\sigma \in (0,0.5)$, α_z must equal $\bar{\alpha}$, otherwise firm 2 goes bankrupt in every possible outcome and has no reason to operate. Once again the price stage remains the same, but now, in order to find optimal locations, we set $\alpha = \bar{\alpha}$ into firms' values.

Maximizing (20) with respect to L_2 and (22) with respect to L_1, where $\bar{\alpha} = \hat{\alpha} + \sigma$, we derive the firms' reaction functions of locations:

$$L_2 = \frac{1}{3}\left(8 - L_1 + 2\sigma - 2\sqrt{4 - 4L_1 + L_1^2 + 2\sigma - L_1\sigma + 7\sigma^2}\right) \qquad (34a)$$

$$L_1 = \frac{1}{3}\left(-2 - 2\sigma + L_2\right) \qquad (34b)$$

Equating (34a) with (34b) we get the optimal locations:

$$L_1^* = \frac{1}{16}\left(2 - 7\sigma - 3\sqrt{4 + 4\sigma + 9\sigma^2}\right) \qquad (35a)$$

$$L_2^* = \frac{1}{16}\left(38 + 11\sigma - 9\sqrt{4 + 4\sigma + 9\sigma^2}\right) \qquad (35b)$$

To compare these results to those of the previous cases, we set $\sigma \in (0,0.5)$ and obtain that $L_1 \in (-0.25,-0.63)$ and that $L_2 \in (1.25,1.1)$, since optimal locations are monotonic in σ. It is clear that, for every value of σ, both firms move to the right on the market line. Since equilibrium prices are higher than those in Game 1, the range of locations that firm 2 may choose without triggering price competition is, once again, shifted to the left. However, in this case the optimal value of α_z does no longer depend on locations. In that sense, this range of firm 2's possible locations is shifted as necessary as it takes to benefit from the increased prices, but no more. This explains the fact that the optimal location of firm 2 is to the left of its location in Game 1 but still to the right of its location when α_Z has an interior solution. On the other hand, firm 1, while it moves further away from the center, compared to its location in Game 1, it does not go that far away, as it does when optimal σ accepts an interior solution. The variable that activates this reaction is uncertainty, as expressed by the value of σ. Comparing (34b) to (10)', we note that the value of L_1, as defined by (34b), is always less by $\frac{2}{3}\sigma$, its value in (10)'.

To illustrate these results, we examine how location affects the respective firm's profits:

$$\frac{d\Pi_1}{dL_1} = \underset{\substack{(+)\\(a)}}{\frac{\partial \Pi_1}{\partial L_1}} + \underset{\substack{(+)\\(b)}}{\frac{d\Pi_1}{dP_2}} \cdot \underset{\substack{(-)\\(c)}}{\frac{dP_2^e}{dL_1}} + \underset{\substack{(+)\\(d)}}{\frac{d\Pi_1}{d\alpha_z}} \cdot \underset{\substack{(-)\\(e)}}{\frac{d\alpha_z^e}{dL_1}} \tag{36}$$

In Game 1 the third term of equation (36), *i.e.*, the product of terms (*d*) and (*e*), equals zero due to all-equity financing, and firm 1 locates at $-\frac{1}{4}$. In the interior solution case of Game 2, this third term exists and is negative due to the fact that an increase in L_1 decreases α_z, as shown in (19). Since both strategic effects of location, the one through affecting rival price (the second term in (36)) and the one affecting rival financial structure (the third term in (36)) are negative, the direct effect (first term in (36)), offsets the *total indirect effect*, presented by the last two terms, in a location further than $-\frac{1}{4}$. The intuition behind this result is presented by (22). When firm 1 locates towards centre, α_z decreases, which in turn implies a decrease in firm 1's equilibrium price. Since prices do not depend only on firms' locations but on debt as well, price competition becomes more severe, *ceteris paribus*.

Returning to the corner solution case, leverage once again implies higher prices, comparing to those in Game 1. However, firm 1's location does no longer affect the level of debt of firm 2, which means that once again the third term is zero. In this way, firm 1 has no incentive to locate that far apart from the centre, as it does in the interior solution case. Despite, however, its higher price relative to that in Game 1, firm 1 does not locate at a spot to the right of $-\frac{1}{4}$ since in every case we examine, $\frac{dL_1}{dL_2} = \frac{1}{3} > 0$. Since firm 1 knows that firm 2 will locate closer to the centre, comparing to Game 1, firm 1 locates further than its location in Game 1.

Using the same argumentation for firm 2 we get:

$$\frac{d\Pi_2}{dL_2} = \underset{\substack{(+)\\(a)'}}{\frac{\partial \Pi_2}{\partial L_2}} + \underset{\substack{(+)\\(b)'}}{\frac{d\Pi_2}{dP_1}} \cdot \underset{\substack{(-)\\(c)'}}{\frac{dP_1^e}{dL_2}} + \underset{\substack{(+)\\(d)'}}{\frac{d\Pi_2}{d\alpha_z}} \cdot \underset{\substack{(-)\\(e)'}}{\frac{d\alpha_z^e}{dL_2}} \qquad (36)'$$

In Game 1 the third term presented by (*d*)' and (*e*)' equals zero for the same reason as before and it is shown that firm 2 will locate at $\frac{5}{4}$. In the interior solution case this term equals zero, due to the envelope theorem. However, as shown in (19), locating towards the centre increases leverage, and thus prices. Consequently, the substitutability between leverage and location allows firm 2 to locate to the right of its previous location in Game 1, since now its price will be greater.

In the corner solution case, the third term equals zero, since firm 2's location cannot affect leverage. This is why firm 2 does not locate to the right of its location in the interior case. Its price is still greater than that in Game 1, which explains the fact that firm 2 moves towards the centre.

GAME 3

Consider now a two-stage game, in which locations and α_z are both decided at the first stage and prices at the second. The price stage remains the same and the equilibrium α_z is given by (19). Assuming that $\hat{\alpha} = 1$, the optimal L2 is found by:

$$\max_{L_2} \Pi_2\left(P_1^e\left(L_1, L_2, \alpha_Z\right), P_2^e\left(L_1, L_2, \alpha_Z\right)\right)$$

According to the FOC, the reaction function of firm 2, to any L^1 is:

$$L_2 = \frac{1}{3}\left(7 + \alpha_Z - L_1 + \sigma - \sqrt{19 + 7\alpha_Z^{\ 2} + 4L_1^2 - 2\alpha_Z(5 + L_1 - 7\sigma) - 10\sigma + 7\sigma^2 - 2L_1(7 + \sigma)}\right)$$

(37)

Similarly, the optimal L_1 is found by:

$$\max_{L_1} \pi_1\left(P_1^e\left(L_1, L_2 \alpha_Z\right), P_2^e\left(L_1, L_2, \alpha_Z\right)\right),$$

which yields:

$$L_1 = \frac{1}{3}\left(-1 - \alpha_Z - \sigma + L_2\right) \quad (38)$$

Solving the system of (34), (35) and (19) we find that:

$$L_1^* = -\frac{4}{7}, \; L_2^* = \frac{8}{7}, \; \alpha_z^* = \frac{13}{7} - \sigma .$$

Obviously, both firms are located to the right of their locations in interior solution case, $(-\hat{\alpha}, \hat{\alpha})$, while the optimal α_z is less than before. To compare these results with those of the previous games, we consider (38) and (38)'.
In this game the third term is still zero, since locations and debt are chosen simultaneously, while prices are increased comparing to those in Game 1. Firm 1 once again has no reason to locate as far as in interior solution case, but it also does not locate at a spot to the right of $-\frac{1}{4}$, since it knows that firm 2 will approach centre.

Using the same argumentation for firm 2, its location does not affect leverage, so it locates to the right of its location in interior solution. Since prices are greater than those in Game 1, firm 2 has an incentive to locate to the left of its location in Game 1. However, prices are not as high as those in corner solution, which explains the fact that firm 2 will not locate to the left of its previous location in corner solution.

D. Conclusions

We first examine Game 1, a Hotelling's model (1929) with quadratic transportation cost .We assume that the market line is defined in the interval $[0, \alpha]$, with the right end being random and fluctuating uniformly between $(1-\sigma, 1+\sigma)$. Allowing firms to locate inside or outside the market line and assuming that firm 1 is always to the left of firm 2, we find that the optimal locations for the two firms are $-\frac{1}{4}$ and $\frac{5}{4}$ respectively. This result is the same as obtained by Lambertini (1977), when a D'Aspremont *et.al*. model is followed and firms are allowed to locate inside or outside the market interval. We can conclude that even when uncertainty is included, firms' location remain the same.

In Game 2 firms choose their locations simultaneously at stage 1, while at stage 2 the firm that faces uncertainty directly in its turf, firm 2, decides for its financial structure, *i.e*., the level of debt that optimizes its profits. We define as α_z the point at which firm 2's debt equals its operating profits. We show that increased debt corresponds to increased value of α_z. Due to this fact, we use the second variable as the choice variable in order to study how leverage affects firms' decisions. In the final stage firms choose simultaneously for their prices. Equilibrium prices are greater than those in Game 1, due to the existence of leverage.

After showing that a positive amount of debt will always be profitable for firm 2, we examine two cases. In the first one uncertainty is sufficiently low and an interior solution for the optimal α_z is found. Firm's optimal locations are -1 and 1 respectively. It is found that when firms' locations increase, the equilibrium value of α_z decreases. This implies that firm 1 has an incentive to locate far away from the center, in order to soften price competition through the increased value of α_z. On the other hand, firm 2 moves towards center. In this way prices increase and firm 2 locates in a spot to the left of its previous location without triggering price competition.

When σ is sufficiently high, a corner solution is obtained. Testing several values of σ when $\sigma \in (0,0.5)$ we find that $L_1 \in (-0.25,-0.63)$ and that $L_2 \in (1.25,1.1)$. In this case, firms' locations do not affect α_z. Consequently, firm 1 moves to the right of its previous location in interior solution. However, it does not locate to the right of its previous location in Game 1. The reason behind this result is the fact that firm 2 has an incentive to locate to the left of 1.25. If firm 1 located even closer to the center, price competition would be severe.

Both the above results follow B&L conclusion that leverage implies a more aggressive behavior of the manager. However, in our case this behavior corresponds only with the firm that decides for its financial structure, since it is the one that faces uncertainty directly. Moreover, the increase in product differentiation is achieved due to its rival softer reaction.

At the first stage of Game 3 firm 2 decides for its financial structure, while both firms decide for their locations simultaneously. In the second stage firms choose for their prices simultaneously. Since locations and leverage are decided at once, α_z is no longer a function of locations. The optimal locations are $-\frac{4}{7}$ and $\frac{8}{7}$ respectively. Obviously, firm 1 has no motive to locate as far as in the interior solution case, but it also does not locate at a spot to the right of $-\frac{1}{4}$, since it knows that firm 2 will locate towards center. Firm 2 locates to the right of its location in interior solution. Since prices are greater than those in Game 1, firm 2 has an incentive to locate to the left of its location in Game 1. However, prices are not as high as those in corner solution, which explains the fact that firm 2 will not locate to the left of its location in corner solution.

According to the results of all cases, leverage increases equilibrium prices, following Showalter's (1995) results. The substitutability between leverage and prices motivates the firm that decides for its financial structure to locate towards center. On the other hand, weakening price competition is once again a strong motive for the other firm to locate far away from the center. Comparing to the results in Haan and Toolsema (2004), the firm that faces uncertainty directly will end up with a greater price than its rival, instead of having equal prices, since uncertainty in our case is not symmetric for both firms.

Though it is clear that leverage affects firms' decisions through prices, further investigation should be done. Essentially results could be obtained if the financial structure stage preceded the location one, in a game where the firm that deals directly with uncertainty chooses the level of debt and then both firms decide simultaneously for their locations and then their prices.

Moreover, uncertainty could be examined under the content of uncertain differentiation for one of the two firms. Though literature has already studied cases in which firms face a probability of not producing identical products, this probability was rather related to a second unrevealed attribute of the product, rather than firms' locations. Specifically, it would be interesting to examine how leverage affects firms, when firms' actual locations can deviate from the locations chosen.

REFERENCES

Anderson, S. (1988). "Equilibrium Existence in the Linear Model of Spatial Competition", *Economica*, 55, p. 479 - 491.

_____ Goeree, J. K. and Ramer R. (1997), "Location, location, location", *Journal of Economic Theory* 77, p. 102–127.

Balvers, R. and Szerb, L., (1996), "Location in the Hotelling duopoly model with demand uncertainty", *European Economic Review*, 40, p. 1453–1461.

Böckem, S. (1994), “A Generalized Model of Horizontal Product Differentiation”, *The Journal of Industrial Economics*, 42(3). p. 287-298.

Brander, J. A. and Lewis, T. R., “Oligopoly and financial structure: The limited liability effect,” *American Economic Review*, 76, p. 956-970.

Casado-Izaga, F-J, “Location decisions: The role of uncertainty about consumer tastes”, *Journal of Economics*, 71(1), p. 31 – 46.

D’Aspremont, C., Gasbszewicz, J. J. and Thisse, J - F. (1979) “On Hotelling’s Stability in Competition,” *Econometrica*, 17, p. 1145-1151.

Economides, N. (1986), "Nash Equilibrium in Duopoly with Products Defined by Two Characteristics," *RAND Journal of Economics*, 17(3), p. 431-439.

Gabszewicz, J. J. and Thisse, J.-F. (1986), "On the Nature of Competition with Differentiated Products," *Economic Journal*, 96 (381), p. 160-72.

Haan, M. A. and Toolsema, L. A., “The strategic use of debt reconsidered”, University of Groningen, *Research Institute SOM* (Systems, Organisations and Management), N° 03E37.

Harrington, J. E., (1992), “Uncertainty over Product Differentiation in a Price-Setting Duopoly: A Non-Robustness Result,“ *Economics Letters*, 39, p. 283-288.

Hinloopen, J. and van Marrewijk, C. (1999), "On the limits and possibilities of the principle of minimum differentiation," *International Journal of Industrial Organization*, 17(5), p. 735-750.

Hoover, E. M. (1937),”Spatial price discrimination", Review of Economic Studies, 4, p. 182-191.

Hotelling, H. (1929), “Stability in Competition**”,** *Economic Journal*, 39, p. 41-57.

Jensen, M. C., and Meckling, W. C. (1976), "Theory of the Firm: Managerial Behavior, Agency Costs and Capital Structure," *Journal of Financial Economics*, 3, p. 305-360.

Lambertini, L. (1994), "Equilibrium locations in the unconstrained Hotelling game", *Economic Notes* 24, p. 438-446.

Lancaster, K. (1990), "The Economics of Product Variety: A Survey", *Marketing Science,* Vol. 9 (3), p. 189-206.

Matsushima, N., Matsumura, T., (2003), "Cost uncertainty and spatial agglomeration".

Meagher, K. J., Zauner, K. G. (2004), "Product differentiation and location decisions under demand uncertainty*", Journal of Economic Theory,* 117(2), p. 201-216

Neven, D. J. (1986), "On Hotelling's Competition with Non-Uniform Customer Distributions", *Economics Letters*, 21, p. 121-126.

Rhee, B-D., de Palma A., Fornell, C. and Thisse, J.-F (1992), "Restoring the principle of minimum differentiation in product positioning", *Journal of Economics and Management Strategy* 1, p. 475–506.

Shilony, Y. (1981), "Hotelling's Competition with General Customer Distributions," *Economics Letters*, 8, p. 39-45.

Showalter, D. M. (1995), "Oligopoly and Financial Structure: Comment", *American Economic Review*, 85, p. 647-653.

Smithies, A. (1941), "Optimum location in spatial competition*", Journal of Political Economy*, 49, p. 423-439.

Thisse, J-F and Vives, X. (1988.), "On the Strategic Choice of Spatial Price Policy", *American Economic Review*, 78, p. 122-137.

Tirole, J., "The Theory of Industrial Organization", MIT press, 1988, Cambridge, MA.