Πανεπιστήμιο Μακεδονίας Οικονομικών και Κοινωνικών Επιστημών

Τμήμα Οικονομικής Επιστήμης

Διδακτορική Διατριβή

«Τρία Δοκίμια στη Μακροοικονομική Θεωρία και Πολιτική: Εμπειρική και Θεωρητική Διερεύνηση των Χαρακτηριστικών της Οικονομικής Ανάπτυξης»

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Θεσσαλονίκη

Νοέμβριος 2009

ΕΥΧΑΡΙΣΤΙΕΣ

Η παρούσα εργασία αποτελεί την τελική εκδοχή της διδακτορικής μου διατριβής με τίτλο «Τρία Δοκίμια στη Μακροοικονομική Θεωρία και Πολιτική: Εμπειρική και Θεωρητική Διερεύνηση των Χαρακτηριστικών της Οικονομικής Ανάπτυξης» η οποία υποβάλλεται προς έγκριση στο τμήμα Οικονομικής Επιστήμης του Πανεπιστημίου Μακεδονίας. Η συγγραφή και ολοκλήρωση της θα ήταν αδύνατη χωρίς τη βοήθεια και την καθοδήγηση καθηγητών και καταξιωμένων επιστημόνων καθώς επίσης και την κατανόηση και τη συμπαράσταση της οικογένειας μου και αγαπητών φίλων.

Δεν θα μπορούσα να ξεκινήσω αυτές τις ευχαριστίες με κάτι διαφορετικό από την ευγνωμοσύνη μου απέναντι στο ακαδημαϊκό και διοικητικό προσωπικό του τμήματος Οικονομικής Επιστήμης του Πανεπιστημίου Μακεδονίας το οποίο στήριξε ποικιλοτρόπως την προσπάθεια μου από την εποχή των μεταπτυχιακών μου σπουδών. Η πλήρης υλικοτεχνική στήριξη κατά τη διάρκεια του διδακτορικού μου κατέστησε δυνατή την υλοποίηση του σε ένα άριστο περιβάλλον με όλες τις αναγκαίες παροχές που απαιτεί ένα τέτοιο εγχείρημα. Ιδιαίτερες ευχαριστίες θα ήθελα να αποδώσω στην κα. Ελένη Ιωαννίδου η οποία με στήριξε με υπερβάλλοντα ζήλο και με βοήθησε να ανταπεξέλθω σε όλες τις γραφειοκρατικές δυσκολίες.

Η άρτια διδασκαλία των καθηγητών μου στο τμήμα Οικονομικής Επιστήμης κατά τη διάρκεια των μεταπτυχιακών μου σπουδών και η μετέπειτα βοήθεια και καθοδήγηση τους με προετοίμασε άριστα ώστε να μπορέσω να ανταπεξέλθω στις αυξημένες απαιτήσεις του διδακτορικού. Συγκεκριμένα θα ήθελα να ευχαριστήσω τα μέλη της επταμελούς επιτροπής που την έκριναν και τη βελτίωσαν με τα σχόλια και τις παρατηρήσεις τους, καθώς επίσης και τα μέλη της τριμελούς επιτροπής, τους κ. Ιωάννη Καραγιάννη και κ. Στυλιανό Φουντά για τις υποδείξεις τους οι οποίες υπήρξαν καθοριστικές στην ολοκλήρωση του κειμένου. Τους κ. Παναγιώτη Κωνσταντίνου και κ. Κωνσταντίνο Ρουμανιά για το χρόνο που αφιέρωσαν στην ανάγνωση διαφόρων τμημάτων της εργασίας και τις εκτενείς συζητήσεις μας˙ τον κ. Χρήστο Κωνσταντάτο, ο οποίος συνέβαλλε καθοριστικά στην προετοιμασία μου για διδακτορικές σπουδές από την εποχή των μεταπτυχιακών μου σπουδών και παρέμεινε υπομονετικός δάσκαλος σε όλη τη διάρκεια των διδακτορικών σπουδών μου. Τέλος, τον κ. Ευτύχιο Σαρτζετάκη, ο οποίος μέσα από τη συμμετοχή μου σε διάφορα ερευνητικά προγράμματα κατέστησε δυνατή την απρόσκοπτη ενασχόληση με την έρευνα και μου έδωσε τη δυνατότητα να συμμετάσχω σε πολύτιμες ακαδημαϊκές δραστηριότητες.

Η ερευνά μου θα ήταν ελλιπής χωρίς την σημαντική συμβολή των συμμετεχόντων καθηγητών και φοιτητών στο σεμινάριο Μακροοικονομικής Επιστήμης του Οικονομικού Πανεπιστημίου Αθηνών, οι οποίοι μου έδωσαν τη δυνατότητα να παρουσιάσω διάφορα στάδια της δουλειάς μου τόσο στη σειρά σεμιναρίων όσο και στο συνέδριο CRETE και από τους οποίους έλαβα εποικοδομητικά σχόλια και υποδείξεις. Ειδικά θα ήθελα να ευχαριστήσω τους κ. Βαγγέλη Βασιλάτο, κ. Σαράντη Καλυβίτη (ο οποίος «ενέπνευσε» ένα κεφάλαιο της διατριβής με τις εύστοχες υποδείξεις του), την κα. Νατάσα

Μιαούλη, τον κ. Γιώργο Οικονομίδη και τον κ. Αποστόλο Φιλιππόπουλο, καθηγητές του Οικονομικού Πανεπιστημίου Αθηνών. Επίσης, θα ήθελα να ευχαριστήσω τους συμμετέχοντες στο σεμινάριο Οικονομικής Επιστήμης του Πανεπιστημίου Ιωαννίνων για την δυνατότητα να παρουσιάσω την έρευνα μου και τις υποδείξεις τους.

Μολονότι αναφέρθηκα στην τριμελή επιτροπή και στους καθηγητές μου παραπάνω, θα ήθελα να επιμείνω σε έναν από αυτούς. Θα μου ήταν εξαιρετικά δύσκολο να αναφέρω έστω ονομαστικά όλους τους λόγους για τους οποίους η σκέψη μου στρέφεται με ευγνωμοσύνη στον επιβλέποντα καθηγητή της διδακτορικής μου διατριβής κ. Θεόδωρο Παλυβό. Θα ήταν ελάχιστο να πω ότι αυτή η διατριβή δεν θα μπορούσε να υπάρξει χωρίς τη βοήθεια, την υπομονή και την καθοδήγηση του στις ατέρμονες πνευματικές περιπλανήσεις μου, την ηθική στήριξη του στις δύσκολες στιγμές που περνάει ένας ερευνητής και σε όλα τα μικρά και μεγάλα πράγματα που συνθέτουν ένα νέο επιστήμονα. Ίσως θα ήταν λίγο πιο ενδεικτικό να τον ευχαριστήσω για την καθοριστική συμβολή του στη διαμόρφωση της επιστημονικής μου ταυτότητας.

Όσο σημαντική και πολύτιμη ήταν η συμβολή των δασκάλων μου, άλλο τόσο ανεκτίμητη ήταν η συμπαράσταση των δικών μου ανθρώπων. Ένα θερμό ευχαριστώ στον Χαράλαμπο και τη Δέσποινα Παπαστάθη για την αγάπη και τη φροντίδα με την οποία με περιέβαλαν. Την οικογένεια μου για την καθοδήγηση τους, την αγάπη τους και την αμέριστη στήριξη τους όλα αυτά τα χρόνια, που με ενθάρρυνε να επιδιώξω τις επιθυμίες μου. Και στον Κώστα που με βοήθησε να τις πραγματοποιήσω. Χωρίς τη στήριξη του, την αμέριστη υπομονή του και την καθοδήγηση του, θα ήταν αδύνατη η συγγραφή αυτής της διατριβής. Αυτή τη διατριβή δεν τους την αφιερώνω απλώς, τους τη χρωστώ.

Στον Σπύρο, την Ιωάννα, το Μάρκο
Στον Κώστα

Table of Contents

List of Figures

Chapter 1 Introduction

The field of economic growth is one of the most interesting fields in macroeconomic theory. As Lucas (1988, p.5) has put it about the stylized facts of economic growth, "once you start thinking about them, its hard to think about anything else". During the last 30 years researchers have extensively explored this field and came up with significant policy implications for economic growth. It is rather hard to describe in a few sentences the mechanics of economic growth and it is even more difficult to account for the poverty of nations and for differences in growth rates, which has always been the major focus of growth theorists. It is also difficult to precisely describe the factors that enhance economic growth since there are many channels that directly or indirectly affect growth. The plethora of (in most cases innovative) papers on this field indicates the complexity of the problem and the variety of approaches that can be adopted.

The aim of this thesis is to contribute to the current literature and extend the research one step forward into accounting for the mechanics of growth. As I recognize the complexity and the multidimensionality of the topic I attempt to approach it in a two-fold manner. On the one hand I attempt to explain the mechanics of growth as described by the simple but intuitive workhorse model of Ramsey-Cass-Koopmans (henceforth neoclassical growth model), while on the other hand I attempt a more institutional approach by focusing on topics such as corruption and its effect on the provision of public goods.

The thesis can be divided into three distinct parts, each of which focuses on macroeconomic aspects of economic growth from a different perspective. I have chosen to treat each topic separately. Therefore each chapter contains its own introduction, literature review, conclusions and references.

More analytically in Chapter 2 of the thesis, I focus on the behavior of the saving rate in the neoclassical growth model and I attempt to fully characterize its behavior during the transition. The saving rate has always been of the utmost importance as it directly affects an economy and its growth rate. It is highly important for policy makers to be able to identify the saving patterns that occur in an economy and find the appropriate ways to affect them. Despite the fact that this model limits our ability to account for complex causes that may affect the saving rate and the growth rate in turn, however it is rather intuitive into identifying the driving forces behind economic decisions. These driving forces could be handily summarized to the income and the substitution effect that occur as an economy develops. Even in the case where more sophisticated models are constructed which endogenize other factors affecting saving rates, these factors as well are going to be subject to an income and a substitution effect, therefore it is very important to comprehend their interaction.

The outcome of this interaction is the observed pattern in many OECD countries, i.e. an inverted U-shaped pattern of the saving rate after WWII. The initial and in some cases dramatic increase in saving rates was followed by an equally significant decrease which in some cases follows until nowadays. WWII is considered to be a turning point for many economies since the destruction of capital and its very low values at the time are appropriately considered as initial conditions for explaining the transitional behavior of many countries.

Another aspect of the neoclassical model that affects growth and therefore one should study its pattern, is the participation rate of an economy which is the topic of Chapter 3. The majority of the literature focuses either on the long-run evolution of working hours or on the factors explaining unemployment. Much as important these factors may be, they neglect a third aspect which is equally important for an economy, not only in terms of growth but also for other social aspects of it. The aspect neglected in the literature is the evolution of participation rates during the transition of an economy. Participation rates do not always reflect unemployment, as in some cases unemployment is voluntary. Participation rates reflect social dynamics such as the entering of women in the remunerated workforce or the increased schooling rates of children. These dynamics in turn significantly affect the growth rate of an economy either due to the increase in human capital or even due to increases in social capital or simply due to increased participation rates and increased production. Therefore it is highly important to be able to identify these patterns and account for them. This is the novelty of this chapter which can analytically account for the empirically observed patters, such as the U-shape in female participation observed by Goldin (1994). As was the case with the first chapter, still income and substitution effects play a significant role. Moreover, these two chapters highlight the role of the elasticity of factor substitution and the intertemporal elasticity of substitution towards explaining saving and participation rates.

Finally the last chapter adopts a broader scope on the topic of growth, by focusing on institutions and their indirect effect on economic growth. More specifically it focuses on corruption, and how it affects the provision of public goods. In this case I abstract from the neoclassical growth model and I use an overlapping generations model to account for intergenerational interactions. Corruption is a complex issue which can take various forms within a society and can affect it through various channels. My intention lies in highlighting how some forms of corruption (in this case tax evasion on the part of citizens and embezzlement on the part of the politicians) may interact with each other and how they affect some aspects of the economy such as the provision of public goods which in turn directly affect economic growth. As expected, in such cases where agents interact and form beliefs about the behavior of other agents, multiple equilibria may arise. Resolving this multiplicity of equilibria is a highly difficult issue and as Matsuyama (1996, p.18) has put it, "....there is only one way of being perfect, but there are millions of ways of being imperfect". What this chapter aims, is to provide the appropriate framework to think about corruption, the analytical tools to study the emergence of corruption and more importantly the identify the reasons responsible for its persistence. Moreover it aims to highlight that policy makers should not only focus on standard deterrence policies in fighting corruption but should also take other factors into account such as moral

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Chapter 2 The Behavior of the Saving Rate in the Neoclassical Optimal Growth Model

2.1 Introduction

The growth performance of each economy is perhaps a major issue for all policy makers. Attaining a significant rate of growth and more importantly a sustainable or even increasing rate of growth, makes it more likely for countries to escape poverty and join the group of developing or developed countries, or it allows an already industrialized country to become more powerful and grow even faster. In any case, growth is the ultimate goal in macro-level and there are various ways in achieving it. A whole area of economic science is devoted to growth theory and is trying to identify the factors that affect growth as well as to explain differences among countries. Some researchers focus on fundamental issues related to growth such as institutions and governance and examine how these affect the rate of growth in an economy and the economy as a whole. Others take for granted issues such as institutions and try to explain the mechanics of growth.

A huge literature has been developed over the last issue and the relationship between savings and growth is a part of that literature. The causality of this relationship has been extensively debated and controversed, what is certain though is that such a relationship indeed exists. The aim of this chapter is to lay emphasis on the mechanics of growth and more specifically on the behavior of the saving rate. The behavior of the saving rate and more specifically the exploration of its monotonicity properties will partially explain its effects on growth as well as on the speed of convergence to the steady state or the balanced growth path which in turn affects growth.

The saving rate is one of the most significant indicators of the course of an economy. Firstly the saving rate, which to some extend can be translated as capital accumulation, comprises a significant part of the investment in a country, which in turn is the driving force behind economic growth. More significantly, if I consider a closed economy then all savings transform into investment. Additionally, high rate of savings can partially insure a country against the consequences of weak financial systems. The case of Mexico highlights the consequences of the lack of high domestic savings which could have made the country more durable to a sudden move of international capital flows due to unforeseen events or even to self-fulfilling expectations. Saving is finally the major safety valve against retirement as well as the major financing tool for the pension system of a country, an issue that has gained much attendance under the threat of ageing population and cohorts of elderly citizens, especially in European countries.

For the above reasons as well as for a variety of others not mentioned here, each economy seeks to attain the optimal saving rate. While it is evident that a very low saving rate is bad for an economy, it is a fact that a high level of savings may as well be insufficient. The Solow model provides a clear explanation of the reasons why an economy should not over save. Overall the optimal saving choice is an intertemporal choice. Each economy has to identify the optimal saving rate given its initial conditions and make an effort to achieve it. If it manages to find itself on the trajectory that will lead it to its steady state or to its balanced growth path this means that it will have achieved its optimum. Of course all the above are true when we are in an ideal first-best world. In a second-best world, intertemporal choices can be significantly affected by external factors such as market failures, structural breaks and a variety of externalities.

In this chapter, the aim is to conduct a simple and intuitive analysis. Hence I will study the behavior of the saving rate in the neoclassical growth model, namely the Ramsey–Cass-Koopmans optimal growth model (henceforth Ramsey model). By choosing this model I isolate the saving rate from all external factors that may affect it, and there is a significant number of them, and choose to focus on the factors that are inherent in an economy such as the income and the substitution effect as well as the elasticity of factor substitution which gives rise to these effects.

If the patterns of the saving rate are identified then it will be easier to identify the policies that will guide it to the desired direction. It will also go a long way towards explaining, in terms of the elasticity of factor and intertemporal substitution as well as in terms of the income and the substitution effect, why saving rates differ significantly among countries, whatever the effects of such differences may be. Overall, the contribution of this chapter can be summarized as follows. It extends the analysis to any concave production function and derives general conditions, involving among others the elasticities of factor and intertemporal substitution, under which the saving rate manifests non-monotonic behavior during the transition to its steady-state value. Furthermore, it contributes to the existing literature in three more ways. First, it points out cases where, by imposing conditions on the intertemporal elasticity of substitution, the saving rate path becomes monotonic. Second, it extends the work of Kurz (1968) and Barro and Sala-i-Martin (2004), by identifying the class of production functions that render the saving rate constant along the entire transition path and hence make the Ramsey-Cass-Koopmans model isomorphic to that of Solow-Swan. Finally, it does all this not only in the case of exogenous, but also in that of endogenous unbounded growth, as in the convex model of Jones and Manuelli (1990, 2005).

I view the results of this chapter important for the following reasons that have already been clarified above. First of all, the resulting non-monotonic behavior of the saving rate accords well with abundant empirical evidence. Second, the results have important implications regarding the transitional dynamics of the economy and the speed of convergence. Finally, this chapter adds to a growing literature that tries to move away from the Cobb-Douglas restraint.

The outline of the chapter is the following. Part 2 comprises a literature review of the topic. The literature review gives evidence of how the saving rate behaves in real world and presents the patterns this chapter will try to interpret. It describes the various issues that are related to the saving rate, such as the elasticity of factor substitution and the income and substitution effect and it points out the various factors that affect its behavior or get affected by it, such as the speed of convergence. Finally it gives an overview of what has been done in the current literature and highlights what needs to be done.

In part 3 I briefly summarize the Ramsey-Cass-Koopmans model which will form the basis for the analysis as well as the methodology Barro and Sala-i-Martin use to describe the behavior of the saving rate. As my results are related to the general form of the Ramsey model, I provide, in part 4, a brief analysis concerning the generalization of the model. In Part 5 I provide the main results of the paper describing the behavior of the saving rate for the general case of the Ramsey model and for every concave production function. By solving the "inverse optimal" problem, in part 6, I identify the case where the saving rate remains constant along the entire transition path. The last part concludes the chapter.

Three appendices accompany the paper. The first Appendix analyzes the Brunner and Strulik (2002) numerical method that has been used for the numerical parts of the chapter and exhibits some more graphs which make the dynamics of the saving rate more clear under the use of more flexible production functions as well as for the Cobb-Douglas. Appendix B attempts to directly relate the saving rate to the speed of convergence using the Reiss (2000) paper. Finally Appendix 3 gives the proof of Proposition 1 obtained in a following sub-chapter, for the case of exogenous growth.

2.2 Literature Review

The behavior of the saving rate is a complex topic and thus various aspects of it have to be analyzed. In the literature review I will try to cover most of these issues. In brief, I will first cite a number of empirical papers that describe its behavior in the real world. The patterns that become evident from these papers are the ones to be interpreted. The importance of the saving rate becomes evident when one notices its effect on the economy through various channels. The transitional dynamics and the speed of convergence are affected by the saving rate and this has been highlighted by a plethora of papers. The main body of these papers will be the topic of the second part of this literature review.

Recently a number of papers have abstracted from the Cobb-Douglas technology for various reasons. As I share most of their arguments, I have also abstracted from the use of the Cobb-Douglas hypothesis and have favored the use of more flexible functional forms and more specifically of a general concave production function that entails some of the most well-known production functions as special cases. In order to justify the reasons for abstraction I line up a number of papers, which shed some light on the weaknesses of the Cobb-Douglas production function.

Finally as the aim of this chapter is to analyze the behavior of the saving rate in the context of the Ramsey model, I will array a number of papers that have analyzed the saving rate in the same or a similar context. The purpose of this part is to reveal the gaps in the literature as well as to highlight what needs to be done on this topic.

2.2.1 Empirical Evidence

As is the case with economic science, economic models should always try to stay in line with empirical evidence and more importantly to give some intuition concerning the various economic phenomena. This is the case with the saving rate, which according to numerous papers does not seem to behave monotonically in the long run as the Ramsey model with Cobb-Douglas production predicts. Of course this behavior varies from country to country and for different time periods however one could generally say that the saving rate seems to behave according to various different patterns.

One of the most analytical studies concerning the behavior of the saving rate is that of Maddison (1992) who examines several European countries for the time period after World War II. More specifically he provides evidence on gross saving rates in 11 countries, Australia 1870-1988, Canada 1870-1999, France 1820-1913 and 1950-1988, Germany 1870-1913, 1925-1939 and 1950-1988, Korea 1911-1938 and 1953-1988, India 1870-1988, Japan 1885-1988, Netherlands 1921-1939 and 1950-1988, Taiwan 1903-1938 and 1951-198, UK 1870-1988 and USA 1870-1988. The data he uses come from different sources for each country. His choice of sample is based not only on the existence of historical data, which he considers crucial for the validity on the findings concerning the saving rate, but also on the fact that these countries represent 50% of world savings. Graphs 1 and 2 present the saving rate in some of the countries analyzed by Maddison.

As Maddison points out, and becomes evident from the graphs, in most countries the saving rates of the 1950's and 1960's were very high, relatively to the evidence until that time. However, after this "postwar boom", as Maddison characterizes it, most of them reverted to their "normal" levels. One of the aims of this chapter is to account for this inverted U pattern (henceforth called "overshooting") observed in post World War II data. WWII is a crucial time threshold since much capital had been destroyed by the end of the war and therefore this time period can be characterized as an initial condition for the economy with low initial capital and high economic activity. As will become evident in the main part of this chapter, a low initial value for capital stock is a necessary condition for overshooting to occur, an assumption that fully accords with empirical evidence.

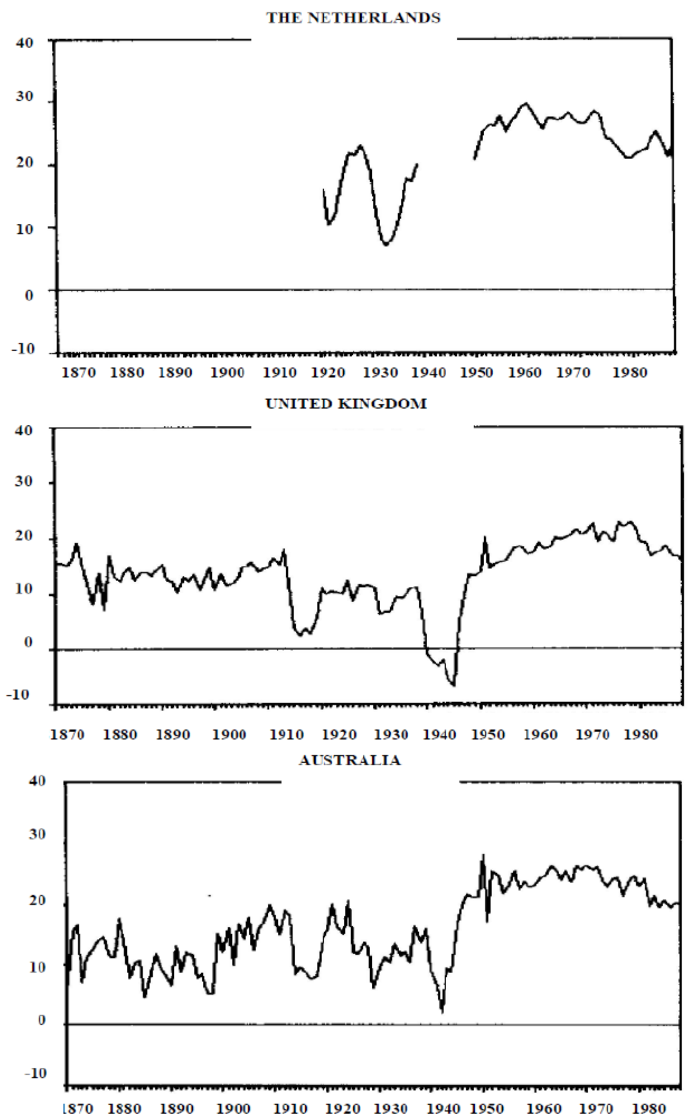

Figure 2.1: Saving Rate in OECD Countries. Source: Maddison (1992)

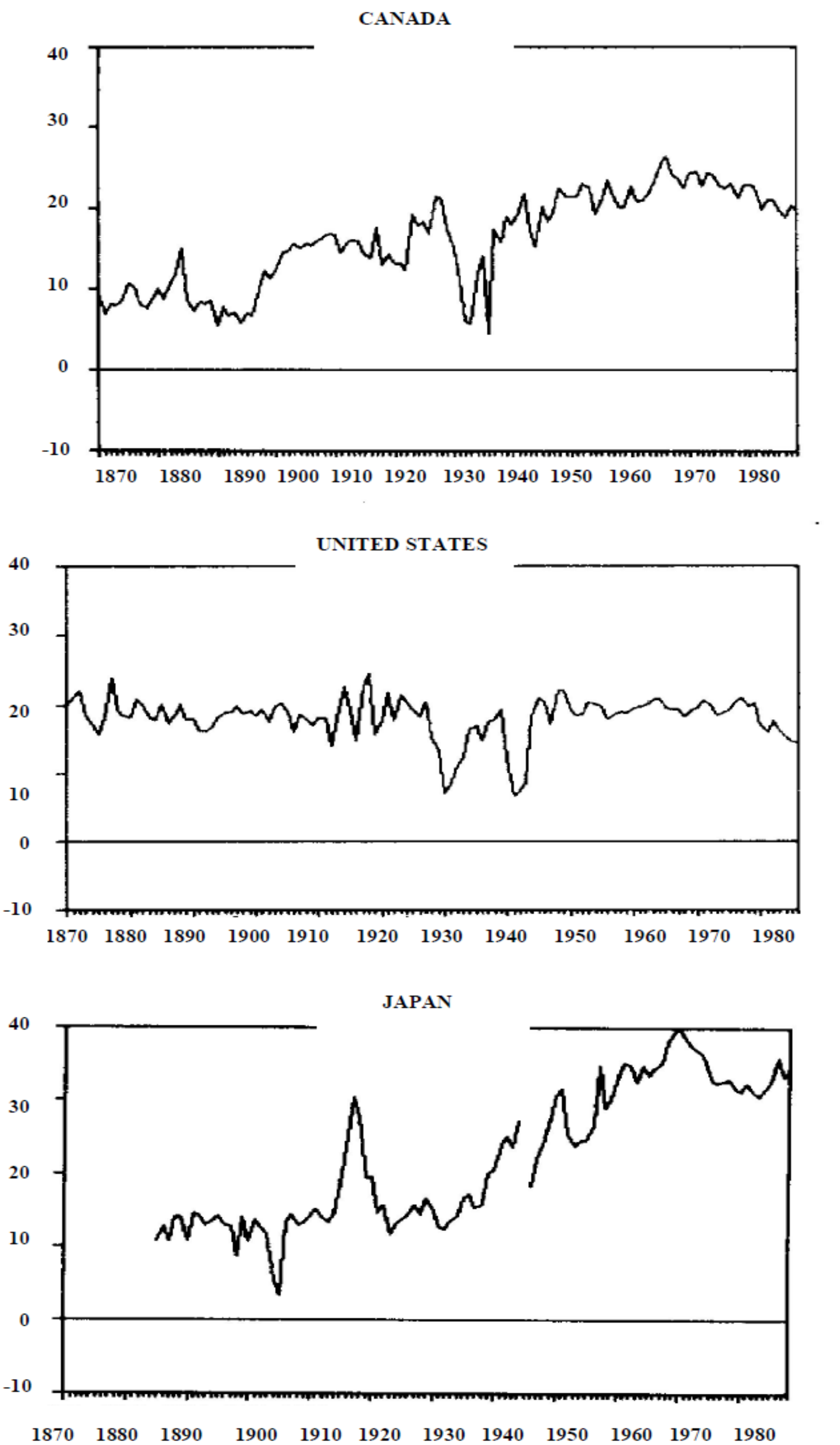

Figure 2.2: Saving Rate in OECD Countries. Source: Maddison (1992)

Maddison gives a variety of factors that affect savings. Among them, I would distinguish the income and the substitution effect. As Maddison points out there seems to be a general positive relationship between the income per head of a country and the acceleration in saving rates. Notably he states that USA which is the country with the least variation in the saving rate in the long-run is also a country which has experienced the smallest postwar growth in per capita income when compared to the other countries of the group. At the same time though, the increase of income has also a negative effect on the rate of savings. This is the so-called substitution effect and it comes from the fact that an increase in income, results in diminishing marginal products and thus diminishing interest rates. Hence consumption is a more attractive choice than investment. The fact that the saving rate is not only related to income but also to investment may be a plausible explanation for the fact that a low income country like India has a higher saving rate than USA. Another possible factor given by Maddison is the leader-follower relationship that exists among countries. A follower country has the incentive and a variety of means to catch up with the leader country. These means have already been used by the leader country and thus the acceleration rates are higher for the follower. Other reasons may be the oil-shocks, the choice of anti-inflationary policies (instead of full-employment targeted policies) etc.

Similar evidence is provided by Tease, Dean, Elmeskov and Hoeller (1991) who examine a similar set of countries for a shorter time period though. Tease et al, using data from the OECD database, examine the saving rate for US, Japan, Germany, France, Italy, UK, Canada as well as for the total of OECD. One of the reasons why they have chosen to study OECD countries is because this group of countries has not been essentially influenced by the oil-crises. Instead they consider that saving in those countries is highly affected by government saving. Overall their findings suggest that saving rate patterns are far from monotonous.

Loyaza, Schmidt-Hebbel and Serven (2000) in the context of a World Bank research project concerning saving in developing countries, report various patterns for several countries as well as regions. The period they cover is also short mainly varying from 1960 onwards, depending on the country they study. They list a number of factors that affect the saving rates in each region. Among them the most important are income, growth, demographics and uncertainty. These factors mainly explain the differences in saving rates among countries and regions that have different characteristics. Apart from the external factors that may affect the saving behavior in a country there are other factors, which are related to the policies followed within each country and these are fiscal policy, pension reforms, financial liberalization and external borrowing and foreign aid

Other evidence is provided by Christiano (1989) and Chari, Kehoe and McGrattan (1996), who report evidence of overshooting for the case of Japan and South Korea correspondingly.

These non-monotonous patterns are the focus of this chapter. What has to be pointed out is that as noted above by the authors of the papers, a variety of reasons may affect the saving rates. This is more than evident when one looks at the graphs, taking into account that they examine many different countries. The aim here is neither to conduct a case study analysis for each of these groups, nor to take historical events into account. What is pursued in this chapter is to interpret the intertemporal saving decision made by individuals in terms of the interaction between the income and the substitution effect. This approach is definitely just a part of the whole story, however even when external factors affect the fundamentals of the economy and consequently the behavior of the individuals in the economy, their final saving decision can be attributed to income and substitution effect. In some sense people always face the same trade-off and follow the same principles in making their decisions. External factors just force them to make new decisions based on new data and different initial conditions. The mechanism though remains the same. For these reasons, I believe that the kind of analysis used here goes a long way towards explaining the patterns of the saving rate observed in real world.

2.2.2 Speed of Convergence and Transitional Dynamics

As was noted in the introduction of this chapter the saving rate affects an economy in many different ways. One of them is related to the effect of the rate of saving on the transitional dynamics of the economy and the speed of convergence towards the steady state. The speed of convergence, usually denoted by the parameter β, is a crucial parameter in terms of policy. If an economy approaches its steady state rapidly, then the economy most of the time is around the vicinity of the steady state and its behavior is going to be determined by the properties of its steady state. If on the other hand the speed of convergence is very low then the transition lasts for a long while and transitional dynamics gain more importance. Consequently, being able to calculate the speed of convergence may reveal some crucial information concerning the economy under examination.

The relationship between the speed of convergence and the saving rate has not been directly analyzed in the literature. What has been analyzed is the relation between the speed of convergence and the value of the elasticity of factor substitution. What is attempted here is an indirect connection between these two variables. In following parts of this chapter it will become evident that the saving rate and the elasticity of factor substitution are interchangeably related to each other. Having established this relation one can talk about the relation between the saving rate and the speed of convergence β.

What will be analyzed here is mainly the literature relating the elasticity of substitution to the speed of convergence. In some papers an immediate connection with the saving rate is attempted. The results however are rather conflicting.

One of the earliest studies was that of Sato (1963). Sato examines fiscal policy in a neoclassical growth model. His results summarize to the following lines:

A change in fiscal policy does not seem to affect the natural rate of growth which depends only on technology parameters as well as on the growth rate of labor force. However the adjustment time seems to be affected by various parameters. Sato gets the following equation:

$$t_k = \frac{\log[1 + \frac{\alpha_0 k}{\alpha_1 (1-k)}]}{p + bn}$$

He obtains the following results:

-The greater the initial savings ratio, the longer the adjustment ratio.

-The higher the new savings ratio, the shorter the adjustment period.

Note that Sato establishes a direct relationship between the savings ratio and the speed of convergence. Additionally in a footnote he also establishes a relationship between the elasticity of factor substitution and the speed of convergence. He claims that the larger σ, the shorter the adjustment period. As an example he mentions the case where $\sigma = \infty$, K and L are no longer different factors and therefore adjustment is instantaneous.

If one considers a variable savings ratio during the adjustment period then an increase in σ decreases the adjustment period (increases speed of convergence).

In a following paper (1964), Sato attempts a comparison between the Harrod-Domar model and the Solow-Swan model. He shows that the adjustment process in the neoclassical system takes place at an extremely slow rate. He also shows that when the system deviates from the natural rate of growth it will always come back to the original equilibrium position. As far as the saving rate is concerned he finds that a sharp rise (fall) in the saving rate will shorten (lengthen) the time length of adjustment.

Ramanathan (1975) examines the role of the elasticity of factor substitution in an one-sector and a two sector growth model with CES production function. The transition period he estimates varies between 22 and 150 years. The main results of the one sector model are given below:

1) If α (the weight for capital) is high (around 0.75) then an increase in σ (the elasticity of factor substitution) increases the adjustment time and an increase in s (saving rate) decreases the adjustment time.

2) If α is low (around 0.25) then an increase in σ does not affect the adjustment time very much, but an increase in s increases the adjustment time.

King & Rebello (1993) test the various neoclassical growth models in order to test how efficient they are in describing real economic life. They end up with the conclusion that when one tries to explain sustained economic growth with transitional dynamics, there are extremely counterfactual implications attributed to the fact that implied marginal products are extraordinarily high in early stages of development.

The point they make concerning the issue of transitional dynamics is that when they vary intertemporal preference parameters, which directly affect the savings rate, then they encounter differing paths for output growth. To be more specific, with less intertemporal substitution in preferences, agents choose smoother consumption profiles, so there is a higher initial level and smaller subsequent growth. In turn higher consumption requires smaller investment. This is a similar result with that of R. Sato (1963) who indicated (in a Solow model) that there was a very lengthy dynamic response to a savings rate shift induced by fiscal policy. This response was quite shortened when K. Sato (1966) introduced depreciation in R. Sato's model, however both models actually indicate that the savings rate must become endogenous.

Barro & Sala-i-Martin (2004) test the behavior of β both for the Solow-Swan and the Ramsey growth model.

For the Solow-Swan growth model, they find the following: After linearization around the steady state they find that β is given by

$$\beta = (1 - \alpha)(\chi + n + \delta)$$

What they claim is that the saving rate does not affect the speed of convergence β. They claim that this result is attributed to the offsetting impact of two opposing forces. On the one hand an increasing saving rate (as k increases) leads to higher investment and therefore to a higher speed of convergence, on the other hand a higher saving rate raises the steady state capital intensity k^* and thereby lowers the average product of capital in the vicinity of the steady state thereby reducing the speed of convergence. In the Cobb-Douglas case these two effects exactly cancel each other hence the saving rate does not seem to affect β. What has to be taken into account in this case is that the elasticity of factor substitution is equal to unity hence the saving rate behaves monotonically, depending on the value of θ, the intertemporal elasticity of substitution.

Note however that this equation is obtained after linearization around the steady state. This implies that β is everywhere constant which is not a plausible result. It is highly likely that along the transition the speed of convergence varies and is affected by the saving rate, as has been pointed out by future research. After all note that $\gamma_k = s\frac{f(k)}{k} - (\chi + n + \delta)$. If s, which in this case is exogenous, increases, then γ_k increases as well. The greater the distance, the faster is the convergence to the steady state. Therefore it seems that during the transition the saving rate affects the speed of convergence. Notice also that the fact that the saving rate differs in various countries (for instance rich economies save more than poor economies) is the main argument behind conditional convergence. Intuitively it seems that the saving rate affects the speed of convergence during the transition, otherwise in the Solow model conditional convergence would not hold.

Barro & Sala-i-Martin extend their analysis to the case of a CES production function. In this case they find that β is given by:

$$\beta = -(\chi + n + \delta)[1 - \alpha(\frac{bsA}{\chi + n + \delta})^{\psi}]$$

If $\psi > 0$ then as $s \uparrow \Rightarrow \beta \uparrow$ (as s increases then β increases)

If $\psi < 0$ then as $s \uparrow \Rightarrow \beta \downarrow$ (as s increases then β decreases)

where $-\infty < \psi < 1$

In this case the saving rate seems to directly affect the speed of convergence β despite the fact that this equation also occurs after linearization around the steady state. Its effect depends on ψ. For $\psi > 0 \Rightarrow \sigma < 1$ and as $s \uparrow \Rightarrow \beta \uparrow$. For $\psi < 0 \Rightarrow \sigma > 1$ and as $s \uparrow \Rightarrow \beta \downarrow$.

For the Ramsey model they find the following: When the saving rate is variable then β is given by (it comes from a log-linearization around the steady state):

$$2\beta = \{\zeta^2 + 4\frac{(1-\alpha)}{\theta}(\rho + \theta\chi + \delta)[\frac{\rho + \theta\chi + \delta}{\alpha} - (\chi + n + \delta)]\}^{1/2} - \zeta$$

where $\zeta = \rho - n - (1-\theta)\chi > 0$.

According to B&S-i-M if the saving rate falls with k then the convergence speed would be higher than otherwise and vice versa. They also showed that a higher value of θ, makes it more likely that the saving rate would rise with θ. Through this mechanism, a higher θ reduces the speed of convergence β. They point out that the effect on the convergence speed does not depend on the level of the saving rate but on the tendency of the saving rate to rise or fall, which is actually the case with overshooting or undershooting.

The point made above, i.e. whether one should measure the speed of convergence based on equations that occur after linearization, was further analyzed by Reiss (2000). According to Reiss a common mistake of studies concerning convergence speed measurement is that the convergence speed measures are usually obtained after linearization around the steady state. However, Reiss claims that the knowledge of the convergence speed is important only if an economy is not already in the vicinity of the steady state. For this reason the paper introduces a method which allows to assess whether the errors introduced by linearization are negligible or not.

His main results can be summarized in the following three propositions:

Proposition 1) Y's speed of convergence is given by:

$$\beta_Y(t) = -\frac{d(Y^* - Y(t))/dt}{Y^* - Y(t)}$$

β_Y is the rate at which the steady state displacement declines relative to its size as time goes by.

Proposition 2) If Y^* is constant he proposes the following measure:

$$\beta_Y(t) = -\frac{dY(t))/dt}{Y^* - Y(t)}$$

Proposition 3) In general any variable's convergence speed off-steady state is not constant over time, it usually changes along a transition path towards its long run equilibrium.

Even though his results do not establish an explicit relationship between elasticity of factor substitution and speed of convergence or the saving rate and the speed of convergence, he paves the way though towards accepting that it is highly likely that along the transition the convergence speed is affected by various parameters, which do not always appear in a convergence speed measure that is obtained after linearization.

In the rest of the paper he makes an effort to quantify the linearization error. Since an economy cannot reach its steady state within a finite period time, he uses half-life time in order to be able to approximately calculate adjustment time. He applies this method in a Solow growth model with a Cobb-Douglas and a CES production function, as well as in a Ramsey growth model. According to his conclusions the key parameter that contributes to the linearization error is the share of capital as well as the economy's initial position.

Rappaport (2000) argues that the speed of convergence is neither constant nor decreasing as predicted by linearization but instead it increases as the economy approaches steady state (note though that this prediction occurs from multiple sector models of capital accumulation). All else being equal, poor economies grow faster than rich ones; but the poor economies' faster growth is less than proportional to their extra distance from their steady state. His results are based on empirical estimates.

Whether the speed of convergence varies along the transition depends also on the choice of the functional form. According to Rappaport in a growth system where there is only a single state variable, a linearization suggests that income grows at a speed proportional to its distance from its steady state level and hence the speed of convergence is constant.

An extensive analysis concerning the relationship between elasticity of substitution and speed of convergence is that of Turnovsky (2002). Turnovsky examines the speed of convergence in an one-sector Ramsey growth model with CES production function and a CIES utility function. His aim is to find the effect of the various elasticities, namely i) the intratemporal elasticity of substitution in production, ii) the intertemporal elasticity of substitution in consumption , iii) the intratemporal elasticity of substitution in consumption. Concerning the elasticity of factor substitution, which is the issue here, Turnovsky proves that if we hold the other two elasticities fixed then an increase (decrease) in σ (elasticity of factor substitution) leads to a decrease (increase) in β (the speed of convergence). The intuition behind this result is that on the one hand an increase in the elasticity of substitution in production raises the long-run change in the capital-labor ratio k, and ceteris paribus this slows down the speed of convergence. At the same time, it tends to have a relatively small effect on the short-run labor supply, which thus has a relatively small effect on the productivity of capital and the short run incentive to invest. To the extend that as σ increases the long-run labor supply is reduced, this tends to reduce the productivity of capital and reduce the speed of convergence.

Turnovsky also reports some evidence concerning the intertemporal elasticity of substitution. Establishing a relationship between the intertemporal elasticity of substitution and the savings rate is also a part of this chapter. Thus, following the same rational as before, if the intertemporal elasticity affects both the saving rate and the speed of convergence, we could assume that there is a direct relation between the latter as well. According to Turnovsky, the speed of convergence is highly sensitive to the intertemporal elasticity of substitution denoted by ε. More specifically a reduction in ε reduces dramatically the speed of convergence and vice versa. This is because, while the long-run response of an increase in capital to the productivity shock is independent of ε, a decrease in ε reduces the short-run response in labor supply, thus reducing the productivity of capital and the incentive to invest thereby slowing down the rate of convergence.

Finally a last paper related to the issue is that of Papageorgiou and Perez-Sebastian (2004), which follows the line of the literature that attempts to correctly estimate the speed of convergence. More specifically what is claimed is that the asymptotic speed of convergence is not a good proxy for characterizing the performance of a model and it can give only an incomplete picture of the transitional adjustment path. What needs to be seen is the behavior of the convergence coefficient along the entire transition path. Additionally they state that far away from the steady state (or the balanced growth path), policy actions (a policy action could as well affect the savings rate), can have a larger (when compared to the effect predicted by Barro&Sala-i-Martin) effect on the speed of convergence, over subsequent periods, because their model (non-scale, R&D based model of economic growth) allows the convergence speed to vary across time.

This paper concludes the analysis over the issue. It is obvious so far that the picture is not clear concerning the direction of the relationship between the speed of convergence as well as between the transitional dynamics and the saving rate. The appropriate measure of the speed of convergence which will clarify the variables affecting it and the exact direction of the relationship between the various variables are issues that need further clarification. However, the existing literature has highlighted this issue and has established that there actually exists some kind of relation between these variables. Thus it has already become clear that characterizing the behavior of the saving rate will go a long way towards characterizing the transitional dynamics of the model as well.

2.2.3 Flexible Functional Forms

Another aim of this chapter is to add to a growing literature that tries to move away from the Cobb-Douglas restraint and towards more flexible and realistic functional forms that enhance our understanding of complex economic phenomena without masking the underlying relations. The modern growth theory has extensively relied on the Cobb-Douglas production function. However, the elasticity of substitution between capital and labor for this specific production function is known to be equal to one. Furthermore, each factor's share of income is constant over time. The linear homogeneity and constant elasticity of substitution have rendered this function quite popular and useful. Additionally, this specification is also consistent with one of Kaldor's (1961) stylized facts. However, the advantages of this functional form are at the same time its disadvantages. A number of economists have traced these disadvantages of the Cobb-Douglas production function and have attempted to explain various economic phenomena by employing different production functions.

Solow (1956), despite the fact that he was among the first to suggest the use of the Cobb-Douglas specification, he put forward various objections to the use of it. For instance he pointed out that Kaldor's stylized facts had held for the data that were available to him at that time, but these data though were quite short. In support of his point he said that the factor shares were not absolutely constant, on the contrary they were relatively constant for the short time period he examined. Five years later, in 1961, he and his coauthors would find the general form of the two-factor constant elasticity of substitution (CES) production function by integrating the observed relationship between output per capita and wage rate.

Revankar (1971), based on the arguments of Arrow et al (1961), Hicks (1948) and Allen (1956) that the elasticity of substitution may not be constant and that it may actually depend upon output and/or factor combinations, has claimed that the assumption of a unitary or a constant elasticity of factor substitution may contain a specification bias. For this purpose he introduced a class of variable elasticity of substitution production functions (VES). The substitution parameter in these functions varies linearly with the capital-labor ratio around the intercept term of unity. CES production function comprises a special case of the VES function. The existence of a VES production function has been estimated and verified empirically for US manufacturing industries, in Revankar (1967).

The empirical examination of the VES production function has also been conducted by Karagiannis, Palivos and Papageorgiou (2005) in the context of an one-sector model. They examine a panel of 28 countries for a 28-year period and they estimate an aggregate production function. The results of their study not only admit the possibility of an aggregate VES production function but also support the possibility of the existence of endogenous unbounded growth, as they calculate an elasticity greater than unity.

Klump and De La Grandville (2000) have attempted to account for growth using a normalized CES production function and a neoclassical growth model in the tradition of Solow (1956). The two theorems they have come up with can be summarized as follows: a) When two countries start from common initial conditions, the one with the higher elasticity of substitution will always experience, other things being equal, a higher income per head, and b) any equilibrium values of capital-labor and income per head are increasing functions of σ. What is made clear through their propositions is that this variance of σ is the one that allows these effects to take place and actually the degree of factor substitution is an engine of growth. A Cobb-Douglas specification with an elasticity of factor substitution equal to unity does not allow for any such interpretations. Similar conclusions are reached by Klump and Preissler (2000) who argue that a higher elasticity of substitution leads to a higher steady state and makes the emergence of sustained growth more probable. This conclusion though has been challenged by Miyagiwa and Papageorgiou (2003) who used an OLG model to explore the growth effects of the elasticity of factor substitution and did not find a monotonic relationship between them. As a matter of fact they even reported cases where a higher elasticity negatively affected growth.

The role of elasticity as an engine of growth is also highlighted by Palivos and Karagiannis (2008) who point out that an elasticity of factor substitution greater that unity is a necessary, not sufficient though, condition, for unbounded endogenous growth. More precisely, an elasticity greater than unity is a necessary condition for the existence of a lower bound on the marginal product of capital which according to Jones and Manuelli (1990) may lead to endogenous unbounded growth.

Klump, Mc Adam and Willam (2007), using a normalized CES function with labor-augmenting technical progress have found that the elasticity of factor substitution is significantly below unity for the US economy. A similar analysis has been conducted by Antras (2004), for US industry for the time period 1948-1998, who has found that when he allows for biased technical change, the Cobb-Douglas specification is not appropriate as he estimates an elasticity less than unity.

Duffy and Papageorgiou (2000) have used panel data techniques to estimate an aggregate CES production function for 82 countries. The inputs they considered were the physical capital stock of each country and the supply of labor. What they found using the whole sample is that the CD production function is rejected in favor of the CES specification and that actually the value of the elasticity is greater than unity. When they splitted the sample into sub-samples they still found that the elasticity takes values other than unity, however for rich countries the elasticity is significantly higher than unity while for poor countries the elasticity is significantly less than unity.

It has become evident from the above literature that the Cobb-Douglas specification restricts the analysis in various ways. For this purpose, in this chapter I adopt a general functional form which allows a variety of results. Additionally it contains the various functional forms (CD, CES, VES) as special cases of the general production function that is going to be used, so as to make the results of each case immediately comparable among them.

2.2.4 Saving rate in the Ramsey model

Despite its paramount importance, little is known about the behavior of the saving rate in perhaps the most popular workhorse model of macroeconomics and growth theory, the Ramsey optimal growth model. Only recently have there been some analytical breakthroughs on this issue. Barro and Sala-i-Martin (2004) examine analytically the case where the production function takes the Cobb-Douglas form and show that the saving rate path to the steady state is monotonic. Smetters (2003) extends the analysis to the constant elasticity of substitution (CES) case and proves that if the elasticity of factor substitution is different from unity, then the saving rate may exhibit non-monotonic behavior. Gomez (2008) characterizes the global dynamics of the saving rate, also in the neoclassical model with CES production function, using qualitative phase diagram techniques. Finally Guha (2008) using a Ramsey model with heterogeneous agents provides the necessary conditions for a monotonous saving rate, thus implying that it does not always behave monotonically.

More analytically, Barro and Sala-i-Martin, examine the case of a Ramsey economy with a CIES utility function and a Cobb-Douglas production function. In appendix 2B they prove the following:

$s^* > \frac{1}{\theta}$ implies $s(t) > \frac{1}{\theta}$ and $\dot{s} > 0$ i.e. the saving rate monotonically increases.

$s^* = \frac{1}{\theta}$ implies $s(t) = \frac{1}{\theta}$ i.e. the saving rate remains constant.

$s^* < \frac{1}{\theta}$ implies $s(t) < \frac{1}{\theta}$ and $\dot{s} < 0$ i.e. the saving rate monotonically decreases.

where s^* denotes the steady state saving rate and θ denotes the inverse of the intertemporal elasticity of substitution. Hence Barro and Sala-i-Martin are the first to correlate the behavior of the saving rate with the interaction between the income and the substitution effect. When $s^* > \frac{1}{\theta}(s^* < \frac{1}{\theta})$ income (substitution) effect prevails. When $s^* = \frac{1}{\theta}$ the two effects offset each other.

The result of Barro & Sala-i-Martin has been overturned by Smetters via the use of a CES production function. He has proved that when the elasticity of factor substitution is less than unity ($\sigma < 1$) then the saving rate might have an inverted U-shaped path, i.e. it might manifest overshooting. Respectively when the elasticity of factor substitution is greater than unity

($\sigma > 1$) then the saving rate might have a U-shaped path, i.e. it might manifest undershooting. Undershooting or overshooting is not always the case. In order to ensure this kind of behavior Smetters imposed certain conditions which are closely related to the initial value of capital stock, $k_{0,}$ as well as to the value of θ, i.e. to the intertemporal elasticity of substitution. His results are verified numerically, using the Mulligan and Sala-i-Martin time elimination method however the magnitude of the undershooting is rather insignificant.

Gomez (2008) has criticized the analysis of Smetters on the grounds that the conditions provided by Smetters, necessary for the existence of overshooting or undershooting, require knowing the value of the saving rate (a jump variable) at the initial time. According to Gomez this is not available unless the transition path of the saving rate is numerically calculated for specific parameter values. Thus the argument of Smetters lacks generality and does not allow for the full characterization of the global dynamics of the saving rate. This is the gap Gomez attempts to fill in by using different analytical tools. More specifically he analyzes the Ramsey model with CES production function, as is the case with Smetters, but he uses qualitative phase diagram techniques. He comes up with the same results as those of Smetters, namely that the saving rate might manifest undershooting or overshooting, but he is able to globally characterize the behavior of the saving rate given the parameters' configuration. His analysis is on (r, z) plane, where r denotes the interest rate and z denotes consumption rate and which exhibit exactly the opposite behavior compared to (k, s).

Finally Guha, utilizing a slightly different version of the Ramsey, identifies certain conditions under which the saving rate becomes monotonic. His analysis and his conditions are also related to the value of the elasticity of factor substitution but they differ from those of Smetters in that they do not depend on the initial value of capital stock. The fact that he uses heterogeneous agents, finally, allows him to examine the behavior of various other variables such as inequality of wealth distribution and the consumption-capital ratio.

2.3 The Ramsey Model

In this part of the chapter I will briefly analyze the Ramsey model as presented in Barro and Sala-i-Martin, since it forms the basis for future analysis as well as for the generalized version of the model that will be presented in the next section. The production function that is used is Cobb-Douglas and it will become evident why the saving rate behaves monotonically in this specific version of the model. In this way I will make the two versions of the model immediately comparable to each other as well as to the results of the analysis in section 5.

2.3.1 The households

The households in the Ramsey economy provide labor in exchange for wages, receive interest income on assets, purchase goods for consumption and save by accumulating additional assets. Each generation contains one or more adults, and adults are the working members of the current generation. Adults in this model are considered to be altruists in the sense that when making plans they take into account the welfare of their descendants either actual or prospective. In order to introduce this behavior in the model it is assumed that the current generation maximizes utility and incorporates a budget constraint over an infinite horizon. In this way there is an immortal extended family despite the fact that individuals have finite lives. Each parent provides transfers to his children and this process take place infinitely, thus the process ends up to an immortal family connected with altruistic bonds.

The size of the family grows at the rate n because of the net influences in fertility and mortality. I make the assumption that n is exogenous and constant. If the number of adults is normalized at time 0 at unity, then the family size that corresponds to the adult population, at time t is:

$$L(t) = e^{nt} \tag{2.1}$$

Each household seeks to maximize overall utility, U, where:

$$U = \int_0^\infty u(c_t)) \exp\{-(\rho - n)t\} dt, \qquad \rho,\ \theta > 0, \tag{2.2}$$

where c_t denotes consumption at time t, ρ is the rate of time preference and n is the population growth rate. This formulation assumes that the household's utility at time 0 is a weighted sum of all future flows of utility. I make the assumption that $u(c_t)$ is increasing and concave (the concavity assumption implies that households tend to smooth consumption over time). Another assumption I make is that $u(c_t)$ satisfies the Inada conditions, i.e. $\lim_{c \to \infty} u'(c) = 0$ and $\lim_{c \to 0} u'(c) = \infty$

Households hold assets in the form of ownership claims on capital or as loans. Negative loans represent debts. Since a closed economy is assumed, no assets can be traded internationally. Households are allowed to lend to and borrow from other households, but in the end of the time period, in equilibrium, the representative household will end up holding zero net loans. The two forms of assets, capital and loans, are considered to be perfect substitutes as stores of value, thus they must pay the same real rate of return, r_t. The household's net assets per person are denoted by α_t and they are measured in real terms, that is in units of consumables.

Households exist in a competitive environment in the sense that they take as given the interest rate r_t as well as the wage rate w_t, paid per unit of labor services. Each adult supplies inelastically one unit of labor per unit of time. In equilibrium, the labor market clears and the household obtains the desired quantity of employment, avoiding in this way "involuntary unemployment". The wage income per adult person equals w_t. Total income per capita received by a household is the sum of labor income and financial or interest income, $r_t\alpha_t$

The flow budget constraint for the household is:

$$\dot{\alpha} = w_t + r_t\alpha_t - c_t - n\alpha_t \tag{2.3}$$

More analytically this equation states that assets per person rise with per capita income, $w_t + r_t\alpha_t$, fall with per capita consumption, c_t, and fall because of expansion of the population in accordance with the term $n\alpha_t$.

Another constraint that has to be taken into consideration is:

$$\lim_{t\to\infty} \alpha_t e^{-\int\limits_0^t (r(v)-n)dv} \geq 0 \tag{2.4}$$

This constraint on the amount of borrowing is imposed by the credit market in order to rule out the case of Ponzi game or chain letter, and its meaning is that in the long run a household's debt per person cannot grow as fast as $r_t - n$, so that the level of debt cannot grow as fast as r_t.

To solve the maximization problem imposed by eqs. (2.2), (2.3), and (2.4), I formulate the Hamiltonian and maximize with respect to c_t and k_t. From the F.O.C's the following equations are obtained:

$$v = u'(c_t)e^{-(\rho-n)t} \tag{2.5}$$

$$\dot{v} = -v(r-n) \tag{2.6}$$

where v is the present-value shadow price of income. It represents the value of an increment of income received at time t in units of utils at time 0.

Equation (2.6) is known as the Euler equation or the Ramsey rule of optimal saving. If I take logarithms of the variable in equation (2.5) and differentiate with respect to time and then substitute for v, I get the basic condition for choosing consumption over time:

$$r = \rho - \frac{U_{cc}c}{U_c}\frac{\dot{c}}{c} \tag{2.7}$$

According to equation (2.7), households choose consumption so as to equate the rate of return, r, to the rate of time preference, ρ, plus the rate of decrease of the marginal utility of consumption. To be more precise, households would prefer a flat consumption profile. In order for households to depart from this flat pattern and sacrifice some consumption today for more consumption tomorrow, i.e. for $\frac{\dot{c}}{c} > 0$, they should be compensated by an interest rate r that is sufficiently above ρ. The amount of compensation amounts to $\frac{U_{cc}c}{U_c}\frac{\dot{c}}{c}$, where $\frac{U_{cc}c}{U_c}$ is the reciprocal of the elasticity of intertemporal substitution.

If the Constant Intertemporal Elasticity of Substitution (CIES) utility function is applied, then equation (2.7) reduces to:

$$\frac{\dot{c}}{c} = \frac{1}{\theta}(r - \rho) \tag{2.8}$$

Another condition that should be introduced is the transversality condition:

$$\lim_{t \to \infty} [v_t \alpha_t] = 0 \tag{2.9}$$

What this condition indicates is that the value of household's assets must approach 0 as time approaches infinity. The intuition is that optimizing agents do not want to have any valuable assets left over at the end of the time period, because their utility would increase if the assets were used to raise consumption.

2.3.2 Firms

Firms produce goods, pay wages for labor input and make rental payments for capital input.

Each firm has access to the production technology:

$$Y = F(K, \hat{L}) \tag{2.10}$$

Y is the flow of output, K is capital input in units of commodities, $\hat{L} = LA_t$ is the effective amount of labor input and A is the level of technology which grows at the constant rate χ. Y exhibits constant returns to scale in K and L and each input exhibits positive and diminishing marginal product. In per capita effective units the production function can be written as:

$$\hat{y} = f(\hat{k}_t) \tag{2.11}$$

where y and k are expressed per unit of effective labor, i.e. $\hat{y} = \frac{Y}{\hat{L}}$ and $\hat{k} = \frac{K}{\hat{L}}$.

The marginal products of the factors are given by:

$$\frac{\partial Y}{\partial K} = f'(\hat{k}_t) \tag{2.12}$$

$$(2.13)$$

$$\frac{\partial Y}{\partial L} = f(\hat{k}_t) - kf'(\hat{k}_t)$$

Firms rent the services of capital from the households that own the capital. Hence the firms' costs for capital are the rental payments which are proportional to K. Suppose R is the rental price for a unit of capital services as well as that capital depreciates at the constant rate δ. The net rate of return to a household that owns a unit of capital is then $R - \delta$. Furthermore households also receive the interest rate r. Equivalently one can write the following equation: $R = r + \delta$.

The representative firm seeks to maximize the following profit function at any point in time:

$$\Pi = F(K, L) - (r + \delta)K - wL \tag{2.14}$$

or in per capita terms:

$$\Pi = L(f(\hat{k}_t) - (r + \delta)\hat{k}_t - w_t e^{-\chi t}) \tag{2.15}$$

Since firms are competitive, they maximize their profits by setting:

$$f'(\hat{k}_t) = r + \delta \tag{2.16}$$

This relationship actually indicates that the firm chooses the ratio of capital to effective labor to equate the marginal product of capital to the rental price. Furthermore in a full market equilibrium, w_t must be such that profit equals 0. Thus w_t has to satisfy the following relationship:

$$w_t = (f(\hat{k}_t) - kf'(\hat{k}_t))e^{-\chi t} \tag{2.17}$$

which implies that factor prices should be equated with marginal products for the market to clear.

2.3.3 Equilibrium

In this part the behavior of the households is combined with the behavior of the firms in order to analyze the structure of competitive market equilibrium. Firstly in equilibrium α equals k, since no other assets are available in this model and since each household finds it optimal to end up with zero net debt. Using equations (3.30) and (3.12) and the flow budget constraint:

$$\dot{\hat{k}} = f(\hat{k}_t) - \hat{c}_t - (n + \delta + \chi)\hat{k}_t \tag{2.18}$$

Correspondingly the dynamic equation for consumption is given by:

$$\frac{\dot{\hat{c}}}{\hat{c}_t} = \frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho - \theta\chi) \tag{2.19}$$

The T.V.C expressed in terms of k is given by:

$$\lim_{t\to\infty} \hat{k}_t e^{-\int\limits_0^t (r(v)-n)dv} \tag{2.20}$$

Equations (2.18), (2.19) and (2.20) form the dynamic system that describes the dynamics of the Ramsey model. As is evident from the phase diagram (Figure 3.1), the system is saddle path stable and the steady state is given by the intersection between the $\dot{\widehat{k}} = 0$ locus and the $\dot{\hat{c}} = 0$ locus and the steady state values of $\hat{c}_t$ and $\hat{k}_t$ are denoted by c^* and k^*. The stable arm is an upward-sloping curve that goes through the origin and the steady state. Starting from a low (high) value of $\hat{k}_t$, $\hat{c}_t$ and $\hat{k}_t$ increase (decrease) over the transition towards their steady state values. Note that in this model, the steady state value of $\hat{k}$ is at the left of $\hat{k}_{gold}$ thus implying that inefficient oversaving cannot occur in this model as in the Solow-Swan case since agents realize that such a behavior is not optimal.

2.3.4 Behavior of the Saving Rate

Following the analysis of Barro and Sala-i-Martin I will show how one can study the behavior of the saving rate in the model with Cobb-Douglas. The methodology followed by Barro and Sala-i-Martin concerning the study of the saving rate is rather intuitive and is the same that has been used in Smetters (2003) and the one that will be used in this chapter. The difference among these studies lies actually in the choice of the production function. Barro& Sala-i-Martin and Smetters study the CD and the CES production functions respectively, whose properties are easily identified and known. The main difference in the present study lies in the fact that to study a

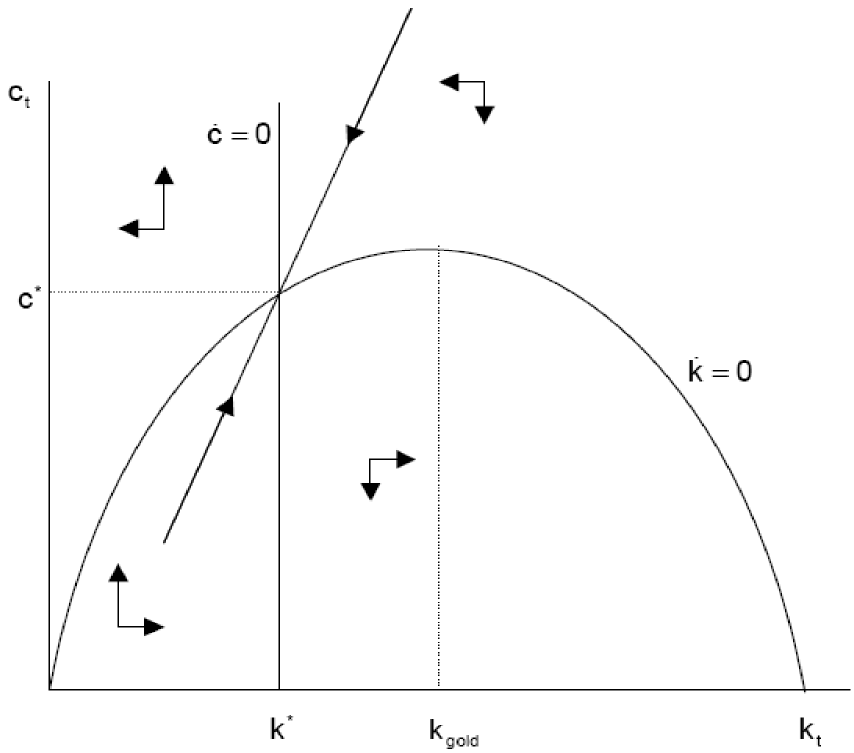

Figure 2.3: Phase diagram in the Ramsey model for the Cobb-Douglas case

general production function, one needs to identify its limiting properties and distinguish among various subcases. All these issues though will be clarified in following chapters.

Beginning with the Barro &Sala-i-Martin case, the saving rate is given by the following equation:

$$\begin{aligned}
\gamma_{z(t)} &= \frac{\dot{z}}{z_t} = \frac{\dot{\widehat{c}}}{\widehat{c}_t} - \frac{f'(\widehat{k}_t)}{f(\widehat{k}_t)}\dot{\widehat{k}} = \frac{\dot{\widehat{c}}}{\widehat{c}_t} - \alpha\frac{\dot{\widehat{k}}}{\hat{k}_t} \qquad (2.21) \\
&= \frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho - \theta\chi) - \alpha(\frac{f(\hat{k}_t)}{\hat{k}_t} - \frac{\hat{c}_t}{\hat{k}_t} - (n + \delta + \chi)) \\
&= \frac{1}{\theta}f'(\hat{k}_t) - \frac{1}{\theta}(\delta + \rho + \theta\chi) - \alpha\frac{f(\hat{k}_t)}{\hat{k}_t} + \alpha\frac{\hat{c}_t}{\hat{k}_t} + \alpha(n + \delta + \chi) \\
&= \frac{1}{\theta}f'(\hat{k}_t) - \frac{1}{\theta}(\delta + \rho + \theta\chi) - f'(\hat{k}) + \alpha\frac{\hat{c}_t}{f(\hat{k}_t)}\frac{f(\hat{k}_t)}{\hat{k}_t} + \alpha(n + \delta + \chi) \\
&= f'(\hat{k}_t)(\frac{1}{\theta} - 1) - \frac{1}{\theta}(\delta + \rho + \theta\chi) + z_t f'(\hat{k}_t) + s^*(\delta + \rho + \theta\chi) \\
&= .f'(\hat{k}_t)(z_t - \frac{\theta - 1}{\theta}) + (\delta + \rho + \theta\chi)(s^* - \frac{1}{\theta})
\end{aligned}$$

In order to obtain eq.(2.21) I have used the following properties of Cobb-Douglas: $\frac{f(\hat{k}_t)}{\hat{k}_t} = \frac{f'(\hat{k}_t)}{\alpha}$ and $s^* = \alpha\frac{\delta+\chi+n}{(\delta+\rho+\theta\chi)}$. Note that s^* denotes the steady state saving rate and can be found by setting $\dot{\widehat{c}} = 0$ and $\dot{\widehat{k}} = 0$ and replacing in $s_t = 1 - \frac{\widehat{c}_t}{f(\hat{k}_t)}$. Also note that from the T.V.C $s^* < \alpha$ since $\rho + \theta\chi < \chi + n$.

From eq. (2.21) I distinguish the following three cases:

A1) If $s^* = \frac{1}{\theta}$ then $z_t = \frac{\theta-1}{\theta}$ as it is consistent with $\gamma_{z(t)} = 0$. If this was not the case and z_t were greater than $\frac{\theta-1}{\theta}$ for some t', then $\gamma_{z(t)} > 0$ for every value of $t > t'$ which is inconsistent with the economy approaching a steady state ($\gamma_{z(t)}$ must always equal 0 in the steady state). Similarly if $z_t < \frac{\theta-1}{\theta}$ for t''then $\gamma_{z(t)} < 0 \; \forall t > t''$which is also inconsistent with the existence of a steady state.

B1) If $s^* > \frac{1}{\theta}$ then $z_t < \frac{\theta-1}{\theta}$

C1)If $s^* < \frac{1}{\theta}$ then $z_t > \frac{\theta-1}{\theta}$

The intuition behind cases (B) and (C) is similar to that of case (A).

Differentiating eq. (2.21) with respect to time I get

$$\dot{\gamma}_{z(t)} = f^{''}(\widehat{k}_t)\dot{\widehat{k}}\left[z_t - \frac{\theta-1}{\theta}\right] + f^{'}(\widehat{k}_t)\gamma_{z(t)}z_t \tag{2.22}$$

As previously I can distinguish the following cases:

A2) If $s^* > \frac{1}{\theta}$ and thus $z_t < \frac{\theta-1}{\theta}$ then $\gamma_{z(t)} < 0$. Following a similar rational with the previous case note that if $\gamma_{z(t)}$ were positive for some t' then $\dot{\gamma}_{z(t)} > 0 \; \forall \; t > t'$ which is inconsistent with the economy approaching a steady state.

Similarly for the other two cases:

B2) If $s^* < \frac{1}{\theta}$ and thus $z_t > \frac{\theta-1}{\theta}$ then $\gamma_{z(t)} > 0$.

C2) If $s^* = \frac{1}{\theta}$ and thus $z_t = \frac{\theta-1}{\theta}$ then $\gamma_{z(t)} = 0$.

The results mentioned in the literature review of this chapter are obtained:

$s^* > \frac{1}{\theta}$ implies $s(t) > \frac{1}{\theta}$ and $\dot{s} > 0$ i.e. the saving rate monotonically increases.

$s^* = \frac{1}{\theta}$ implies $s(t) = \frac{1}{\theta}$ i.e. the saving rate remains constant.

$s^* < \frac{1}{\theta}$ implies $s(t) < \frac{1}{\theta}$ and $\dot{s} < 0$ i.e. the saving rate monotonically decreases.

What is clarified in this chapter is that the Cobb-Douglas production function implies a monotonous saving rate over the entire transition path. As this case is quite limited and does not offer any intuition over the non-monotonous saving rate observed in real world data, in the rest of the chapter I will extend the analysis of Barro and Sala-i-Martin by abstracting from this specific case. For this purpose a CIES utility function is used (which is necessary for the existence of balanced growth) and also a general production function, which does not necessarily satisfy all the properties of a neoclassical production function. In this way not only can I examine richer patterns of the saving rate, but I can also extend the analysis to the case of endogenous unbounded growth.

2.4 A Generalized version of the Ramsey Model

In this section I will set the basis for the subsequent analysis as I will derive the basic equations of the Ramsey model using a general production function. In order to be able to do so, I have to study the limiting values of the production function for each value of the elasticity of factor substitution, i.e. for $\sigma < 1$ and for $\sigma > 1$. The utility function that is used is a CIES utility function, and this is done so because it is necessary for balanced growth to be obtained.

The production function per effective unit of labor $f(\widehat{k})$ is at least twice continuously differentiable, strictly increasing and concave $\forall\, \widehat{k} \in \mathbb{R}_+$, where $\widehat{k}_t = k_t e^{-xt}$ denotes the capital stock per effective unit of labor, k_t is capital per labor unit and x is the rate of labor-augmenting technical progress; furthermore, $f(0) \geq 0$.

Applying standard techniques and the equilibrium conditions $a_t = k_t$, $r_t = f^{'}(\widehat{k}_t) - \delta$ and

$w_t = f(\widehat{k}_t) - f'(\widehat{k}_t)\widehat{k}_t$ leads to the following equations of motion:

$$\frac{\dot{\widehat{c}}}{\widehat{c}_t} = \frac{f'(\widehat{k}_t) - (\delta + \rho + \theta x)}{\theta}, \tag{2.23}$$

$$\dot{\widehat{k}} = f(\widehat{k}_t) - \widehat{c}_t - (x + n + \delta)\widehat{k}_t, \tag{2.24}$$

which are the same as the ones in the Barro&Sala-i-Martin model.

The dynamics of the system (2.23)-(2.24) are well understood. In particular, if $0 \leq \lim_{\widehat{k}(t)\to\infty} f'(\widehat{k}_t) \equiv A < \rho + \delta + \theta x$, then there exists a steady-state equilibrium, which is saddle-path stable. Capital and consumption in effective units, $\widehat{k}_t$ and $\widehat{c}_t$, converge monotonically towards the steady state, so that $\dot{\widehat{k}} > 0$ and $\dot{\widehat{c}_t} > 0\ \forall\ t > 0$. If, on the other hand, $\lim_{\widehat{k}(t)\to\infty} f'(\widehat{k}_t) \equiv A > \rho + \delta + \theta x > 0$ then, in addition to the exogenous part generated by technical progress, there will be unbounded endogenous growth (see Jones and Manuelli 1990, 2005). In the latter case, if I set $x = 0$, then the exogenous part disappears; nevertheless, there will still be perpetual growth, as it can be seen from equation (2.23).

To clarify the issue of endogenous unbounded growth as introduced by Jones and Manuelli I must state that initially theory predicted that in order for unbounded growth to exist, one of the following should hold:

-A constant returns to scale model, with fixed labor and exogenous technological progress, or

- Increasing returns to scale, or

-Exogenous increases in factor supplies (i.e. population growth)

However the above conditions either could not explain cross-country differences or were based on non-convexities of the production function. Instead, the paper by Jones and Manuelli (1990) provided the theoretical background on the endogenous unbounded growth.

They suggested that if we use a concave production function, which violates the Inada, then we can achieve endogenous unbounded growth. As $\hat{k}_t$ approaches infinity, the marginal product of capital according to the Inada approaches 0. If we use a production function that bounds the marginal product of capital from below and if then the marginal product of capital remains positive and consumption grows at a steady rate and then the economy as a whole grows at a steady rate.

Palivos and Karagiannis (2007) extended the analysis by proving that in an economy with concave production function the marginal product of capital remains positive as the economy grows, if the elasticity of substitution between capital and labor becomes greater than unity in the limit. Therefore when the marginal product is positive, endogenous unbounded growth is feasible.

Throughout the paper I denote the limiting values of each variable with an asterisk. If there exists a steady state in terms of effective units, then $\widehat{k}^*$ and $\widehat{c}^*$ take finite values. If, on the other hand, there exists unbounded endogenous growth then $\widehat{k}^*$ and $\widehat{c}^*$ approach infinity.[1] In either case, ratios such as the saving rate, s_t, and the share of income consumed, $z_t = 1 - s_t$, take finite positive values, which are denoted as s^* and z^*, respectively.

Next, recall that the elasticity of substitution between capital and labor is defined as

$$\sigma(\widehat{k}_t) \equiv -\frac{f'(\widehat{k}_t)}{\widehat{k}_t f(\widehat{k}_t)} \frac{f(\widehat{k}_t) - \widehat{k}_t f'(\widehat{k}_t)}{f''(\widehat{k}_t)} > 0.^2 \tag{2.25}$$

When deviating from the Cobb-Douglas (CES) production function, various parameters, such as the share of capital (the elasticity of factor substitution), become endogenous variables. In

1 Hence, expressions such as $f'(\widehat{k}^*)$ should be understood as $\lim_{\widehat{k}\to\widehat{k}^*} f'(\widehat{k}_t)$ if there is exogenous growth only and as $\lim_{\widehat{k}\to\infty} f'(\widehat{k}_t)$ if there is endogenous growth. I abuse somewhat the notation in order to make the two cases of just exogenous and of exogenous as well as endogenous growth immediately comparable.

2 We remind the reader that for a general production function $F(K, L)$ the elasticity of substitution is defined as $\frac{d\ln(K/L)}{d\ln(F_L/F_K)}$. Under constant returns to scale and with two factors of production this can be written as $\frac{F_K F_L}{F F_{KL}}$, which, in terms of the intensive form f, is equal to the one given in (2.25).

addition, the Inada conditions may not be satisfied anymore. In fact, the limiting properties of the production function and the behavior of the capital share are directly linked to the elasticity of factor substitution. As all these play an important role in determining the behavior of the savings rate, the following properties are very useful to the analysis.

Lemma. Denote the share of capital as $\frac{f'(\widehat{k}_t)\widehat{k}_t}{f(\widehat{k}_t)} \equiv \alpha(\widehat{k}_t)$. Then

a) $\lim\limits_{\widehat{k}(t)\to\infty} \dfrac{f(\widehat{k}_t)}{\widehat{k}_t} = \lim\limits_{\widehat{k}(t)\to\infty} f'(\widehat{k}_t)$ b) If $\sigma(\widehat{k}_t) \gtreqless 1$ then $\dfrac{d}{d\widehat{k}_t}\left(\alpha(\widehat{k}_t)\right) \gtreqless 0$

c) If $\lim\limits_{\widehat{k}(t)\to 0} \sigma(\widehat{k}_t) > 1$ then $\lim\limits_{\widehat{k}(t)\to 0} f(\widehat{k}_t) > 0,\quad \lim\limits_{\widehat{k}(t)\to 0} f'(\widehat{k}_t) \leq \infty,\quad \lim\limits_{\widehat{k}(t)\to 0} \alpha(\widehat{k}_t) = 0$

d) If $\lim\limits_{\widehat{k}(t)\to\infty} \sigma(\widehat{k}_t) > 1$ then $\lim\limits_{\widehat{k}(t)\to\infty} f(\widehat{k}_t) = \infty,\quad \lim\limits_{\widehat{k}(t)\to\infty} f'(\widehat{k}_t) > 0,\quad \lim\limits_{\widehat{k}(t)\to\infty} \alpha(\widehat{k}_t) = 1$

e) If $\lim\limits_{\widehat{k}(t)\to 0} \sigma(\widehat{k}_t) < 1$ then $\lim\limits_{\widehat{k}(t)\to 0} f(\widehat{k}_t) = 0,\quad \lim\limits_{\widehat{k}(t)\to 0} f'(\widehat{k}_t) < \infty,\quad \lim\limits_{\widehat{k}(t)\to 0} \alpha(\widehat{k}_t) = 1$

f) If $\lim\limits_{\widehat{k}(t)\to\infty} \sigma(\widehat{k}_t) < 1$ then $\lim\limits_{\widehat{k}(t)\to\infty} f(\widehat{k}_t) < \infty,\quad \lim\limits_{\widehat{k}(t)\to\infty} f'(\widehat{k}_t) = 0,\quad \lim\limits_{\widehat{k}(t)\to\infty} \alpha(\widehat{k}_t) = 0.$

Proof: a) If $\lim_{\widehat{k}(t)\to\infty} f(\widehat{k}_t) = \infty$ then apply L'Hôpital's rule. If $\lim_{\widehat{k}(t)\to\infty} f(\widehat{k}_t) < \infty$ then $\lim_{\widehat{k}(t)\to\infty} f'(\widehat{k}_t) = 0$ and the result follows. b) Differentiate $f'(\widehat{k}_t)\widehat{k}_t/f(\widehat{k}_t)$ with respect to $\widehat{k}_t$ and use (2.25). c-f) See Barelli and Pessôa (2003) and Palivos and Karagiannis (2007).

2.4.1 Note on Barelli and Pess$\hat{o}$a Limiting Properties of the Production Function

The above Lemma, and especially cases c-f, which are crucial for future analysis, has been based upon the research of Barelli and Pess$\hat{o}$a (2003) (henceforth B&P). B&P actually show that a strictly increasing and strictly concave (per capita) production function that satisfies the Inada conditions must be asymptotically Cobb-Douglas. This is an important result since the vast majority of papers in macroeconomics and growth theory nowadays adopt the Inada conditions.

More specifically, B&P show that the limiting value of the elasticity of substitution, as k approaches either zero or infinity, of a production function $f(k)$ that satisfies the Inada conditions must be equal to one. That is,

$$f(0) = 0 \;\; and \;\; f'(0) = \infty \;\Rightarrow\; \sigma(0) = 1. \tag{1}$$

(A similar result is shown for the case where $k \to \infty$).

What I want to show in this sub-chapter is that although (1) is correct, one of the intermediate results in B&P, used to establish (1), is not.

2.4.1.1 Result

The proof of (1) is based on the following two intermediate results, which are also interesting in their own right (see Proposition 1 in B&P):

$$\text{If } \sigma(0) \;<\; 1 \text{ then } \; f(0) = 0 \; \text{ and } f'(0) < \infty \tag{2a}$$

$$\text{If } \sigma(0) \;>\; 1 \text{ then } f(0) > 0 \; \text{ and } \; f'(0) = \infty \tag{2b}$$

Indeed, combining the contrapositive statements of $(2a)$ and $(2b)$ yields (1). Nevertheless consider the following mixed CES-linear production function found in, among others, de la Croix and Michel (2002, p. 122): $f(k) = b + \frac{ak}{1+k}$, a, $b > 0$. For this production function note that $\sigma(0) > 1$, $f(0) = b > 0$ but $f'(0) = a < \infty$. A second counter-example to $(2b)$ is the following: $f(k) = Ak + \alpha - \beta \exp(-\beta k)$, $A > 0$, $\alpha > \beta > 0$, where $\sigma(0) > 1$, $f(0) = \alpha - \beta > 0$ but $f'(0) = A + \beta^2 < \infty$. Finally, another way to present the problem is the following: combining the contrapositive statements of $(2a)$ and $(2b)$ yields, in addition to (1), $f(0) > 0$ and $f'(0) < \infty$ $\Longrightarrow \sigma(0) = 1$, which, as each of the two examples demonstrates, is false. Obviously, these results cast doubt to the validity of (1).

By analyzing carefully the proof of (1) in B&P, it is obvious that what they actually show is (see the last part of their proof of Proposition 1 on p. 363)

$$\text{If } \sigma(0) < 1 \text{ then } f(0) = 0 \text{ and } \lim_{k \to 0} \frac{f(k)}{k} < \infty \qquad (2a')$$

$$\text{If } \sigma(0) > 1 \text{ then } f(0) > 0 \text{ and } \lim_{k \to 0} \frac{f(k)}{k} = \infty \qquad (2b')$$

Then by assuming implicitly that $\lim_{k \to 0} \frac{f(k)}{k} = f'(0)$, they arrive at $(2a)$ and $(2b)$ and hence at (1) (a similar argument is developed for the case where $k \to \infty$). Nevertheless, while $\lim_{k \to \infty} \frac{f(k)}{k} = f'(\infty)$ is always true, $\lim_{k \to 0} \frac{f(k)}{k} = f'(0)$ is not. Indeed, consider the following lemma.

Lemma. If either $f'(0) = \infty$ or $f(0) = 0$, then $\lim_{k \to 0} \frac{f(k)}{k} = f'(0)$.

Proof. If $f'(0) = \infty$ and $f(0) \neq 0$ then the $\lim_{k \to 0} \frac{f(k)}{k} = \infty = f'(0)$ follows immediately. If $f(0) = 0$ then the result follows from the definition of the derivative. □

In both counter-examples provided above $f(0) > 0$ and $f'(0) < \infty$; hence $\lim_{k \to 0} \frac{f(k)}{k} = \infty \neq f'(0) < \infty$ and thus $(2b)$ is incorrect. Fortunately, however, (1) is still valid, since when the two Inada conditions are combined then $\lim_{k \to 0} \frac{f(k)}{k} = f'(0)$; thus the transition from

$$f(0) = 0 \ \textit{and} \ \lim_{k \to 0} \frac{f(k)}{k} = \infty \ \Rightarrow \ \sigma(0) = 1, \qquad (1')$$

which follows from $(2a')$ and $(2b')$, to (1) is legitimate.[3]

2.5 Variable Saving Rate: Under- and Over-shooting

It is well known that in the Ramsey-Cass-Koopmans model along the (asymptotic) balanced growth path, the limiting values of the growth rates of consumption and capital, measured both in effective units, are equal to each other; that is, $\gamma^*_{\widehat{c}(t)} = \gamma^*_{\widehat{k}(t)} = \gamma^*$. Using then (2.23) and (2.24),one can write γ^* and the limiting value of the saving rate $s^* = 1 - \widehat{c}^*/f(\widehat{k}^*)$ as

[3] Note that the other pair that results from the combination of the contrapositive statements of $(2a')$ and $(2b')$, namely $f(0) > 0$ and $\lim_{k \to 0} \frac{f(k)}{k} < \infty$, is incompatible.

$$\gamma^* = \frac{f'(\widehat{k}^*) - (\delta + \rho + \theta x)}{\theta}, \tag{2.26}$$

$$s^* = (\gamma^* + x + n + \delta)\frac{\widehat{k}^*}{f(\widehat{k}^*)}. \tag{2.27}$$

Of course, if there exists only exogenous growth, then $\gamma^* = 0$.

Next let $\gamma_{z(t)}$ denote the growth rate of $z_t = 1 - s_t = \widehat{c}_t / f(\widehat{k}_t)$. The following equation is derived with the use of (2.23), (2.24), (2.26) and (2.27) and is analogous to equation (10) in Smetters (2003) and (2.95) in Barro and Sala-i-Martin (2004):

$$\begin{aligned}
\gamma_{z(t)} &= \frac{\dot{z}}{z_t} = \frac{\dot{\widehat{c}}}{\widehat{c}_t} - \frac{f'(\widehat{k}_t)}{f(\widehat{k}_t)}\dot{\widehat{k}} \qquad (2.28)\\
&= \frac{1}{\theta}(f'(\hat{k}) - \delta - \rho - \theta\chi) - \frac{\hat{k}f'(\widehat{k}_t)}{f(\widehat{k}_t)}(\frac{f(\hat{k})}{\hat{k}} - \frac{\hat{c}}{\hat{k}} - (n + \delta + \chi))\\
&= \frac{1}{\theta}f'(\hat{k}) - \frac{1}{\theta}(\delta + \rho + \theta\chi) - f'(\widehat{k}_t) + \frac{f'(\widehat{k}_t)}{f(\widehat{k}_t)}\hat{c} + \frac{\hat{k}f'(\widehat{k}_t)}{f(\widehat{k}_t)}(n + \delta + \chi)\\
&= \frac{1}{\theta}f'(\hat{k}) - \frac{1}{\theta}(\delta + \rho + \theta\chi) - f'(\hat{k}) + z_t f'(\hat{k}) + \frac{\hat{k}f'(\widehat{k}_t)}{f(\widehat{k}_t)}(n + \delta + \chi) + \gamma^*\frac{\hat{k}f'(\widehat{k}_t)}{f(\widehat{k}_t)} - \gamma^*\frac{\hat{k}f'(\widehat{k}_t)}{f(\widehat{k}_t)}\\
&= f'(\widehat{k}_t)\left[z_t - \frac{\theta - 1}{\theta}\right] - \frac{1}{\theta}(\delta + \rho + \theta\chi) + \frac{\hat{k}f'(\widehat{k}_t)}{f(\widehat{k}_t)}(n + \delta + \chi + \gamma^*) - \gamma^*\frac{\hat{k}f'(\widehat{k}_t)}{f(\widehat{k}_t)}\\
&= f'(\widehat{k}_t)\left[z_t - \frac{\theta - 1}{\theta}\right] + s^*\frac{f(k^*)}{k^*}\frac{f'(\widehat{k}_t)\widehat{k}_t}{f(\widehat{k}_t)} - (\frac{f'(\widehat{k}^*)}{\theta} - \gamma^*) - \gamma^*\frac{\hat{k}f'(\widehat{k}_t)}{f(\widehat{k}_t)}\\
&= f'(\widehat{k}_t)\left[z_t - \frac{\theta - 1}{\theta}\right] - \gamma^*\left[1 - \frac{f'(\widehat{k}_t)\widehat{k}_t}{f(\widehat{k}_t)}\right] + s^*\frac{f(k^*)}{k^*}\frac{f'(\widehat{k}_t)\widehat{k}_t}{f(\widehat{k}_t)} - \frac{f'(\widehat{k}^*)}{\theta}\\
&= f'(\widehat{k}_t)\left[z_t - \frac{\theta - 1}{\theta}\right] + \frac{f(\widehat{k}^*)}{\widehat{k}^*}\left[s^*\frac{f'(\widehat{k}_t)\widehat{k}_t}{f(\widehat{k}_t)} - \frac{1}{\theta}\frac{f'(\widehat{k}^*)\widehat{k}^*}{f(\widehat{k}^*)}\right] + \gamma^*\left[1 - \frac{f'(\widehat{k}_t)\widehat{k}_t}{f(\widehat{k}_t)}\right].
\end{aligned}$$

Equation (2.28) applies to the case where there exists only exogenous growth as well as to the case where there exists both exogenous and endogenous growth. In particular, if there is only exogenous growth then the last term in (2.28) disappears since $\gamma^* = 0$. On the other hand, if

$\lim_{\widehat{k}(t)\to\infty}\sigma(\widehat{k}_t) > 1$ and there is endogenous growth as well, then (2.28) can be simplified further since $f^{'}(\widehat{k}^*) = f(\widehat{k}^*)/\widehat{k}^*$ (Lemma, part a).[4] In either case, $\gamma_z^* = 0$.

Finally, I also use the following expression, which gives the change of $\gamma_{z(t)}$ with respect to time, $\dot{\gamma}_{z(t)} \equiv d(\gamma_{z(t)})/dt$, and is obtained after differentiating (2.28) and using (2.25) and (2.27)

$$\dot{\gamma}_{z(t)} = f^{''}(\widehat{k}_t)\dot{\widehat{k}}\left[z_t - \frac{\theta-1}{\theta}\right] + f^{'}(\widehat{k}_t)\gamma_{z(t)}z_t + (x+n+\delta)\left[1-\sigma(\widehat{k}_t)\right]\frac{f^{''}(\widehat{k}_t)\widehat{k}_t}{f(\widehat{k}_t)}\dot{\widehat{k}} \tag{2.29}$$

I am now ready to establish my first proposition. Throughout the paper, I consider the (realistic) case where a country starts with a capital stock below its steady state value $\widehat{k}^*$.

Proposition 1. a) Let $\widehat{k}_1$ be a value of $\widehat{k}$ such that $\widehat{k}_1 < \widehat{k}^*$. If $\sigma(\widehat{k}_t) > 1\ \forall\ \widehat{k}_t$ and

$$s^*\frac{f'(\widehat{k}_1)\widehat{k}_1}{f(\widehat{k}_1)} > \frac{1}{\theta}\frac{f^{'}(\widehat{k}^*)\widehat{k}^*}{f(\widehat{k}^*)} \tag{1A}$$

then the saving rate is increasing along the transition path from $\widehat{k}_1$ to $\widehat{k}^*$.

b) Moreover, there exists $\widehat{k}_0 < \widehat{k}_1$, such that

$$s^*\frac{f'(\widehat{k}_0)\widehat{k}_0}{f(\widehat{k}_0)} < \frac{1}{\theta}\frac{f'(\widehat{k}^*)\widehat{k}^*}{f(\widehat{k}^*)}. \tag{1B}$$

In addition, the saving rate is decreasing at $\widehat{k}_0$ iff

$$1 > z_0 > 1 - \frac{1}{\theta} - \left[s^*\frac{f'(\widehat{k}_0)\widehat{k}_0}{f(\widehat{k}_0)} - \frac{1}{\theta}\right]\frac{f'(\widehat{k}^*)}{f'(\widehat{k}_0)} - \frac{\gamma^*}{f'(\widehat{k}_0)}\left(1 - \frac{f'(\widehat{k}_0)\widehat{k}_0}{f(\widehat{k}_0)}\right). \tag{1C}$$

Proof: *a*) I prove this proposition for the case where there is both exogenous and endogenous growth ($\widehat{k}^* \to \infty$). The proof for the case where there is only exogenous growth and hence the economy accepts a steady state in effective units is similar and can be found in Appendix

4 Recall that if there is endogenous growth $\widehat{k}^* \to \infty$.

C of this chapter. Recall from the Lemma that if $\sigma(\widehat{k}_t) > 1$ and there is endogenous growth, then $f'(\widehat{k}^*)\widehat{k}^*/f(\widehat{k}^*) = 1$. Also, since $f'(\widehat{k}_1)\widehat{k}_1/f(\widehat{k}_1) < 1$, it follows from condition (1A) that $s^* > 1/\theta$ or $z^* < (\theta - 1)/\theta$. Next suppose that there is a value of $t > t_{k_1}$, say t', such that $z_{t'} > (\theta - 1)/\theta$, where t_{k_1} denotes the value of time that corresponds to $\widehat{k}_1$. Then (2.28) implies that $\gamma_{z(t')} > 0$ (since every term on the RHS is positive) and $\gamma_{z(q)} > 0 \; \forall \; q > t'$; hence, $z^* > (\theta - 1)/\theta$, which is a contradiction. Thus, $z_t < (\theta - 1)/\theta \; \forall \; t > t_{k_1}$. Also, equation (2.29) implies that $\gamma_{z(t)} < 0 \; \forall \; t > t_{k_1}$, for if $\gamma_{z(t)} \geq 0$ for some value of t, say t'', then $\dot{\gamma}_{z(q)} > 0 \; \forall$ $q > t''$, which means $\gamma_z^* > 0$; this is also a contradiction, since along the balanced growth path s and hence z remain constant ($\gamma_z^* = 0$). But then if $\gamma_{z(t)} < 0 \; \forall \; t > t_{k_1}$, this implies that $\dot{s} > 0 \; \forall$ $t > t_{k_1}$, or that s_t is increasing along the transition path from $\widehat{k}_1$ to $\widehat{k}^*$.

b) The proof that there exists $\widehat{k}_0$ such that (1B) holds follows immediately from the Lemma (parts c and d). If (1B) holds, then $z_t \lesseqgtr (\theta - 1)/\theta$ and $\gamma(z)$ in equation (2.29) can take either positive or negative values. The rest of the proof is straightforward; namely, set $\gamma_{z(0)} > 0$ in equation (2.28) and solve for z_0, taking into account the Lemma (parts a and d). ■

According to Proposition 1 the behavior of the saving rate can be non-monotonic. More specifically, if the elasticity of factor substitution exceeds unity and condition (1A) holds, then the saving rate will be increasing before it reaches the balanced growth path and becomes constant. Furthermore, under condition (1C) there exists a value of capital $\widehat{k}_0 < \widehat{k}_1$ such that the saving rate is decreasing over the range from $\widehat{k}_0$ to $\widehat{k}_1$. Hence, looking over the entire transitional path one can see that if $\sigma(\widehat{k}_t) > 1$ and conditions (1A) and (1C) hold, then the saving rate undershoots its long-run value.

As expected, conditions (1A) and (1B) in Proposition 1 encompass the conditions that appear in Barro and Sala-i-Martin (2004) and in Smetters (2003).[5] Barro and Sala-i-Martin (2004) show that if the production function is Cobb-Douglas and $s^* = 1/\theta$ then the income and substitution effect of a change in $\widehat{k}$ offset each other. If $s^* > (<) 1/\theta$ then the income (substitution) effect dominates and the saving rate increases (decreases) during the transition. Naturally, with a general production function what also matters is the share of capital, $f'(\widehat{k}_t)\widehat{k}_t/f(\widehat{k}_t)$, relative to its steady state value, $f'(\widehat{k}^*)\widehat{k}^*/f(\widehat{k}^*)$. That is why these terms appear in conditions (1A) and (1B). Note that if condition (1A) holds at some point $\widehat{k}_1$, then the assumption that $\sigma(\widehat{k}_t) > 1 \; \forall \; \widehat{k}_t$ ensures that it holds everywhere between $\widehat{k}_1$ and $\widehat{k}^*$ (Lemma, part 2). The same holds for condition (1B). Finally, if $\sigma(\widehat{k}_t) = 1$ (Cobb-Douglas) and hence $f'(\widehat{k}_t)\widehat{k}_t/f(\widehat{k}_t) = \alpha$ then either (1A) is true or (1B) is true but not both. If (1A) is true $(s^* > 1/\theta)$ then only part a) of the proposition holds and the saving rate increases during the transition. If, on the other hand, (1B) is true $(s^* < 1/\theta)$ then only part b) holds and the saving rate decreases along the entire transition path (This is the result shown in Barro and Sala-i-Martin (2004)).

I can also illustrate the results of Proposition 1 graphically (Figure 5.1). I do so in the in $(\widehat{k}, z)$-plane because this enables me to deduce easily the behavior of the saving rate (recall that $s_t = 1 - z_t$). I analyze only the case where there is only exogenous growth, i.e. $\gamma^* = 0$. The endogenous growth case is similar. The only difference is that one has to transform the system into one that has a steady state, i.e. I have to transform the system into variables whose growth rate is equal to 0 in the steady state. An example of such variables would be the ones used by Gomez, i.e. (r_t, z_t). However this would not add to the intuition given by these graphs. Setting $\dot{\widehat{k}} = 0$ (equation 2.24) I obtain

[5] For example, if the production function is Cobb-Douglas then $f'(\widehat{k}_t)\widehat{k}_t/f(\widehat{k}_t) = f'(\widehat{k}^*)\widehat{k}^*/f(\widehat{k}^*) = \alpha$ and (1A) simplifies to $s^* < \frac{1}{\theta}$, as in Barro and Sala-i-Martin. Also, if the production function is CES then condition (1A) simplifies to $s^*[f'(\widehat{k}_1)/f'(\widehat{k}^*)]^{1-\sigma} > \frac{1}{\theta}$ as in Smetters (2003), Proposition 1B.

$$\dot{\widehat{k}} = 0 \text{ locus:} \qquad \widehat{z}_t = 1 - (x + n + \delta)\frac{\widehat{k}_t}{f(\widehat{k}_t)}, \tag{2.30}$$

which is downward-sloping and convex under the assumption that $\sigma(\widehat{k}_t) > 1 \; \forall \; \widehat{k}_t$ and decreases from *unity* to $\Omega = 1 - (\chi + n + \delta)\frac{1}{A}$, as k goes to infinity, where $A = \lim_{\widehat{k}(t) \to \infty} \frac{\widehat{k}_t}{f(\widehat{k}_t)}$. Note that $\lim_{\widehat{k}(t) \to \infty} \widehat{z}_t = \Omega$ can take either negative or positive values (to make Figure 3.1, though, less cluttered, it is drawn as a straight line, since this does not affect my conclusions). Moreover, by setting $\gamma_{z(t)} = 0$ (equation 2.28) I get

$$\dot{z} = 0 \text{ locus:} \qquad \widehat{z}_t = 1 - \frac{1}{\theta} - \frac{f(\widehat{k}^*)}{\widehat{k}^*}\frac{1}{f'(\widehat{k}_t)}\left[s^* \frac{f'(\widehat{k}_t)\widehat{k}_t}{f(\widehat{k}_t)} - \frac{1}{\theta}\frac{f'(\widehat{k}^*)\widehat{k}^*}{f(\widehat{k}^*)}\right]. \tag{2.31}$$

The $\dot{\widehat{z}} = 0$ locus is more complex, so I must briefly analyze its behavior asymptotically and specify the conditions that lead to undershooting in the savings rate. The limits characterizing the asymptotical behavior of the locus are the following:

$$\begin{aligned}
a) \lim_{\widehat{k}(t) \to 0} \widehat{z}_t &= 1 - \frac{1}{\theta} - \frac{f(\widehat{k}^*)}{\widehat{k}^*}\frac{1}{B}\left[-\frac{1}{\theta}\frac{f'(\widehat{k}^*)\widehat{k}^*}{f(\widehat{k}^*)}\right] \text{ if } \lim_{\widehat{k}(t) \to 0} f'(\widehat{k}_t) = B > 0 \\
& \quad or \\
\lim_{\widehat{k}(t) \to 0} \widehat{z}_t &= 1 - \frac{1}{\theta} \text{ if } \lim_{\widehat{k}(t) \to 0} f'(\widehat{k}_t) = \infty \\
b) \lim_{\widehat{k}(t) \to \infty} \widehat{z}_t &= 1 - \frac{1}{\theta} - \frac{f(\widehat{k}^*)}{\widehat{k}^*}\frac{1}{C}\left[s^* - \frac{1}{\theta}\frac{f'(\widehat{k}^*)\widehat{k}^*}{f(\widehat{k}^*)}\right] = X
\end{aligned}$$

The above limits can take values either negative or positive value depending on the value of θ and the limiting values of the share of capital and of the marginal product of capital. Also note that $\widehat{z}'_t$ can take positive values as $\widehat{k}_t$ approaches zero and can take negative values as $\widehat{k}_t$ approaches infinity. The exact shape of the curve is based on the properties that appear in the Lemma.

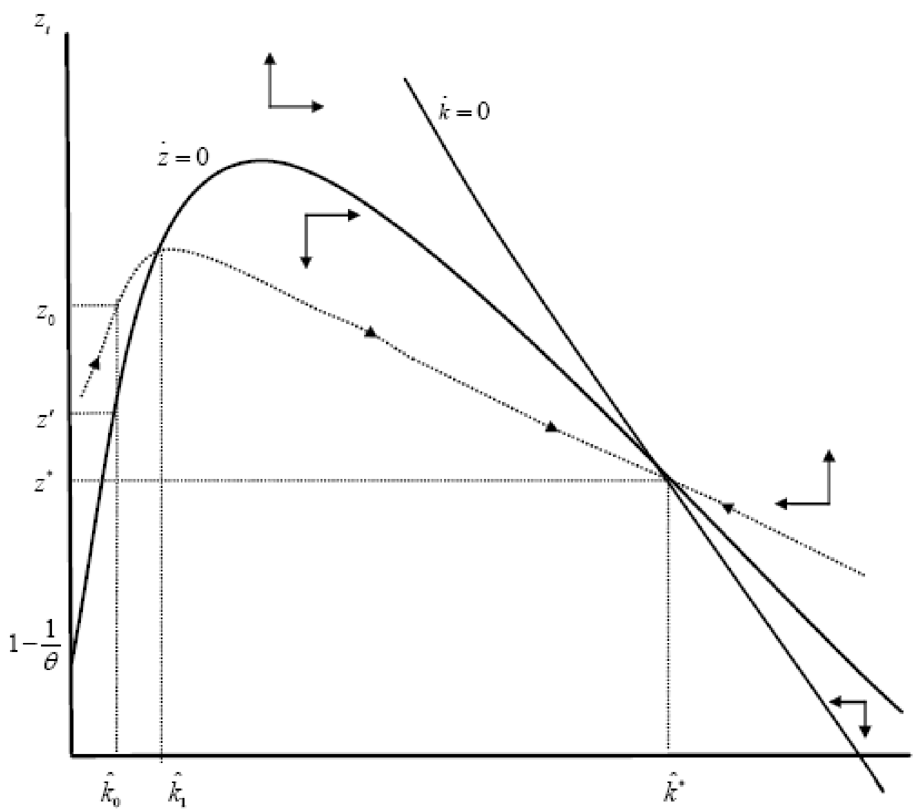

Figure 2.4: Overshooting of the consumption ratio - undershooting of the saving rate

I analyze the case where $\theta > 1$, $\Omega < 0$, $X < 0$ and $\lim_{\widehat{k}(t)\to 0} \widehat{z}_t = 1 - \frac{1}{\theta}$, as well as that the maximum of the $\widehat{\dot{z}} = 0$ locus occurs at a value of $\widehat{k}_t < \widehat{k}^*$ (at the left of the steady state). All these restrictions can hold at the same time thereby the existence of undershooting is likely. Given the configuration of the two loci, the arrows point west (east) above (below) the $\widehat{\dot{k}} = 0$ locus, and north (south) above (below) the $\widehat{\dot{z}} = 0$ locus.

The steady-state equilibrium $(\widehat{k}^*, z^*)$ exhibits saddle-path stability. As can been seen in Figure 5.1, for values of the initial capital stock that are greater than $\widehat{k}_1$, the consumption-output ratio z_t is monotonically decreasing and, hence, the saving rate is monotonically increasing. However, if the initial capital stock is sufficiently low, then z_t is first increasing and then decreasing; hence, z_t (s_t) exhibits over-shooting (undershooting). Notice from Figure 5.1 that the non-monotonicity of the $\dot{z} = 0$ locus is necessary for z to overshoot its steady-state value. Indeed, conditions 1A and 1B concern the behavior of this locus. More specifically, condition 1A is sufficient for the $\dot{z} = 0$ locus to be eventually decreasing, whereas condition 1B is necessary for the same locus to be initially increasing. The figure provides also an intuitive explanation of condition 1C. Consider a point $(\widehat{k}_0, z_0)$ on the stable arm. Holding the capital stock the same $(\widehat{k}_0)$, if I extend this point to the $\dot{z} = 0$ locus I reach another value z'. Condition 1C requires that $z_0 > z'$; that is, it requires that the consumption ratio be higher than the one that corresponds to a constant value (given by the $\dot{z} = 0$ equation). Up to point $\widehat{k}_1$, where $z_{0i} > z_i'$, as k rises so does z, whereas from point $\widehat{k}_1$ onwards, where $z_{0i} < z_i'$, as k rises z falls (z_{0i} denotes values of z on the stable arm and z_i' values on the $\dot{z} = 0$ locus).

The following example shows that undershooting can well arise when there exist plausible functional forms and reasonable parameter values.

Example 1. Consider the production function $f(\widehat{k}_t) = A(\widehat{k}_t)^\alpha - \gamma$, where $A > 0 > \gamma$ and

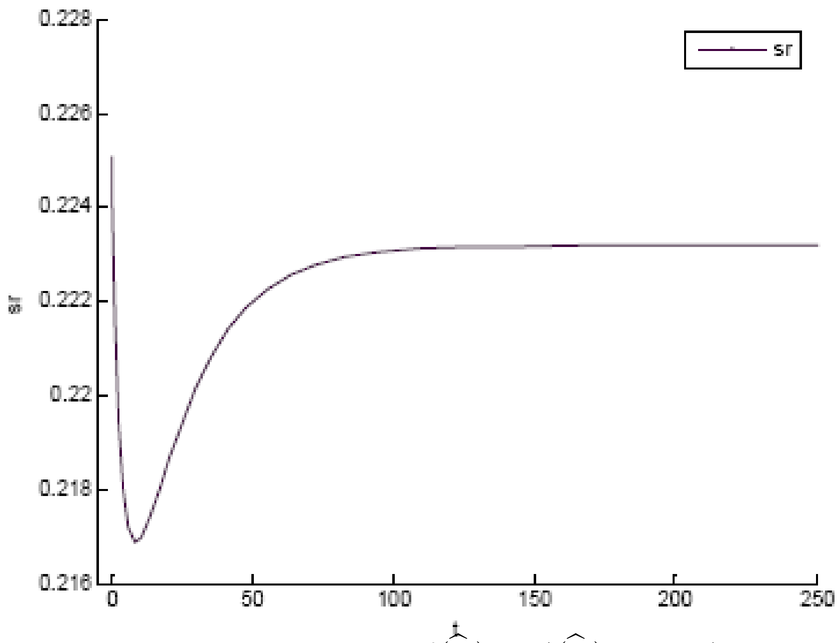

Figure 2.5: Undershooting of the saving rate. $f(\widehat{k_t}) = A(\widehat{k_t})^{\alpha} - \gamma$, $A = 1$, $\alpha = 0.695$, $\gamma = -61$, $n = 0.01$, $\delta = 0.0645$, $\rho = 0.035$, $\theta = 2.4$, $x = 0.01$

$1 > \alpha > 0$. The elasticity of substitution for this production function is $1 - \alpha\gamma/[(1-\alpha)f(\widehat{k_t})]$, which is greater than unity for every finite value of k.[6] To compute the transitional dynamics of the model, I use the backward integration method proposed by Brunner and Strulik (2002), which transforms an unstable boundary value problem into a stable initial value problem by a time reversal of the dynamic system. Figure 5.2 presents the transitional path; clearly, the saving rate undershoots its long-run value.

Corollary 1. If $\sigma(\widehat{k_t}) > 1 \ \forall \ \widehat{k_t}$ and θ is sufficiently high then the saving rate increases monotonically along the entire transition path.

Proof: If θ is sufficiently high then $z_t \leq (\theta - 1)/\theta, \forall \ t$ and not just for $t > t_{k_1}$ as in Proposition 1. Consider next equation (2.29). Since the first and the third terms on the RHS are both

[6] Note, however, that $\lim_{k\to\infty} f'(\widehat{k}) = 0$ and hence this production function cannot yield endogenous unbounded growth.

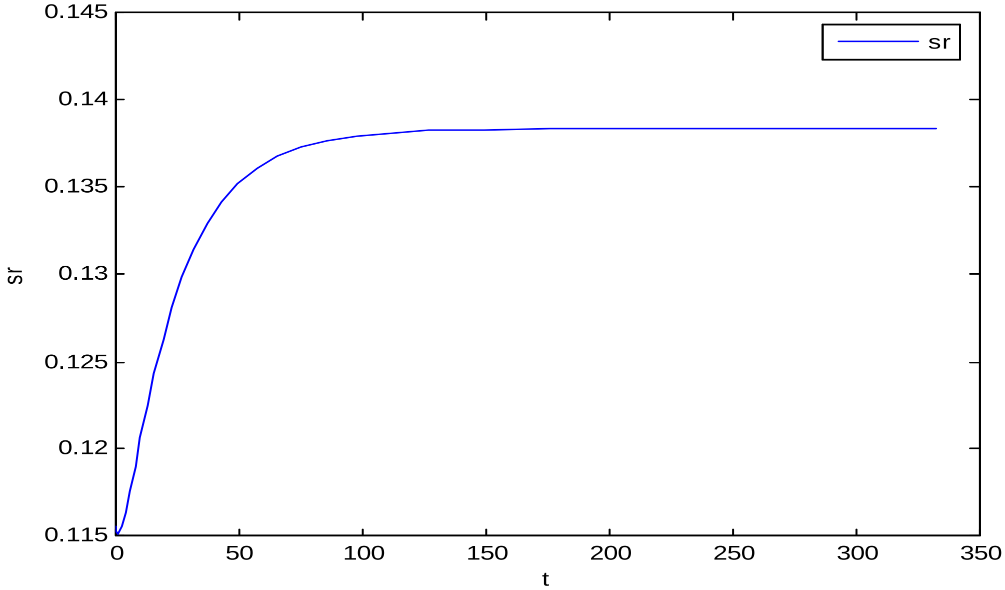

Figure 2.6: Increasing saving rate. $f(\widehat{k}_t) = A(\widehat{k}_t)^{\alpha} - \gamma$, $A = 1$, $\alpha = 0.695$, $\gamma = -61$, $n = 0.01$, $\delta = 0.0645$, $\rho = 0.035$, $\theta = 5$, $x = 0.01$

positive, it follows that $\gamma_{z(t)} < 0$, for if $\gamma_{z(t)} > 0$ then $\dot{\gamma}_{z(t)} > 0$ as well, which means that γ_z^* will never become zero. But then if $\gamma_{z(t)} < 0$, it follows that $\dot{z}_t < 0$ and $\dot{s}_t > 0 \; \forall \; t$.

■

The intuition behind Corollary 1 is that a sufficiently high θ weakens the substitution effect and makes the income effect dominate; hence, the saving rate path is increasing. Figure 5.3 presents the same production function with the same parameter values used in Figure 5.2 with the only difference being the value of θ, which is now greater than in the previous figure. It is evident that this undershooting effect has vanished.

Proposition 2. a) Let $\widehat{k}_1 < \widehat{k}^*$. If $\sigma(\widehat{k}_t) < 1 \; \forall \; \widehat{k}_t$ and

$$s^* \frac{f'(\widehat{k}_1)\widehat{k}_1}{f(\widehat{k}_1)} < \frac{1}{\theta} \frac{f'(\widehat{k}^*)\widehat{k}^*}{f(\widehat{k}^*)} \tag{2A}$$

then the saving rate is decreasing along the transition path from $\widehat{k}_1$ to $\widehat{k}^*$.

b) If there exists $\widehat{k}_0 < \widehat{k}_1$, such that

$$s^* \frac{f'(\widehat{k}_0)\widehat{k}_0}{f(\widehat{k}_0)} > \frac{1}{\theta}\frac{f'(\widehat{k}^*)\widehat{k}^*}{f(\widehat{k}^*)}. \tag{2B}$$

then the saving rate is increasing at $\widehat{k}_0$ iff

$$0 < z_0 < 1 - \frac{1}{\theta} - \left[s^* \frac{f'(\widehat{k}_0)\widehat{k}_0}{f(\widehat{k}_0)} - \frac{1}{\theta}\frac{f'(\widehat{k}^*)\widehat{k}^*}{f(\widehat{k}^*)} \right] \frac{f(\widehat{k}^*)}{f'(\widehat{k}_0)\widehat{k}^*}. \tag{2C}$$

Proof: *a*) First note that since $\sigma(\widehat{k}_t) < 1$, $\gamma^* = 0$. Furthermore, the Lemma and assumption (2A) imply that $s^* < 1/\theta$ and $z^* > (\theta - 1)/\theta$. Suppose there is a value of $t > t_{k_1}$, say t', such that $z_{t'} < (\theta - 1)/\theta$. Then (2.28) implies that $\gamma_{z(t')} < 0$ (since every term on the RHS is negative) and $\gamma_{z(q)} < 0 \; \forall \; q > t'$; hence, $z^* < (\theta - 1)/\theta$, which is a contradiction. Thus, $z_t > (\theta - 1)/\theta$ $\forall \; t > t_{k_1}$. Also, equation (2.29) implies that $\gamma_{z(t)} > 0 \; \forall \; t > t_{k_1}$, for if $\gamma_{z(t)} \leq 0$ for some value of t, say t'', then $\dot{\gamma}_{z(q)} < 0 \; \forall \; q > t''$, which means $\gamma_z^* < 0$; this is inconsistent with the economy approaching a steady state where s and hence z are constant ($\gamma_z^* = 0$). But then if $\gamma_{z(t)} > 0 \; \forall$ $t > t_{k_1}$, I have $\dot{s} < 0 \; \forall \; t > t_{k_1}$, or s_t is decreasing along the transition path from $\widehat{k}_1$ to $\widehat{k}^*$.

b) The rest of the proof is straightforward.[7] Condition (2C) follows once again from equation (2.28); namely, set $\gamma_{z(0)} < 0$ and solve for z_0, taking into account the fact that $\gamma^* = 0$.

■

Note that according to Proposition 2, the path of the saving rate, taken as a whole, first increases and then decreases. Thus, under the specified conditions, if $\sigma(\widehat{k}_t) < 1$, then the saving rate overshoots its long-run value. Furthermore, the intuition behind conditions (2A) and (2B) is

[7] Notice that, in contrast to Proposition 1, the existence of $\widehat{k}_0$ such that condition (B) in Proposition 2 holds is not guaranteed any more. The same is true in Smetters (2003). We note that there is a small typo in Smetters (2003) towards the end of the proof of Proposition 1 case (A) on page 705; namely, the paper appears to claim that if $f(\widehat{k}) = \left[\alpha \widehat{k}^{1-1/\sigma_{KL}} + (1-\alpha)\right]^{1/(1-1/\sigma_{KL})}$ and $\sigma_{KL} < 1$ then $\lim_{\widehat{k}\to 0} f'(\widehat{k}) = \infty$. This should read, instead, $\lim_{\widehat{k}\to 0} f'(\widehat{k}) = \alpha^{1/(1-1/\sigma_{KL})} < \infty$.

similar to that given after Proposition 1. Finally as it can be easily shown, this proposition nests the cases of Cobb-Douglas and CES.

Proposition 2 is illustrated in Figure 5.4. The two loci $\dot{\widehat{k}} = 0$ and $\dot{z} = 0$ are given by equations (2.30) and (2.31), respectively. I plot the case where $\theta > 1$ and the minimum of the locus occurs at a value of $\widehat{k}_t < \widehat{k}^*$ (the exact shape of each curve is again easily derived after using the Lemma). The steady-state equilibrium is the point $(\widehat{k}^*, z^*)$, which is saddle-path stable. For values of the initial capital stock that are greater than $\widehat{k}_1$, the consumption ratio z_t is monotonically increasing and, hence, the saving rate is monotonically decreasing. However, if the initial capital stock is sufficiently low, then z_t is first decreasing and then increasing; hence, z_t exhibits under-shooting. The saving rate behaves exactly the opposite and, therefore, it exhibits overshooting. Just as in Proposition 1, the conditions that appear in Proposition 2 concern the behavior of the $\dot{z} = 0$ locus. Notice that for undershooting of z to occur this locus must be non-monotonic. Condition 2A is sufficient for it to be eventually increasing, whereas condition 2B is necessary for it to be initially decreasing. Finally, condition 2C follows the same way as condition 1C in Proposition 1 and Figure 3.1. Consider first a point $(\widehat{k}_0, z_0)$ on the stable arm. Holding the capital stock the same $(\widehat{k}_0)$, if I extend this point to the $\dot{z} = 0$ locus I reach another value z'. Condition 2C requires that $z_0 < z'$; that is, it requires that the consumption ratio be lower than the one that corresponds to a constant value (given by the $\dot{z} = 0$ equation). Thus, up to point $\widehat{k}_1$, as k rises, z falls.

Example 2. Consider the production function $f(\widehat{k}_t) = 1 - e^{-b\widehat{k}_t}$, $b > 0$. The elasticity of substitution for this function is $(1/b\widehat{k}_t) - (e^{-b\widehat{k}_t}/f(\widehat{k}_t))$, which is less than unity for every value of k. Using the same numerical method as in Example 1 I can compute the transitional dynamics of the economy. The path of the saving rate, which is depicted in Figure 3.4, shows that in this case s overshoots its long-run value.

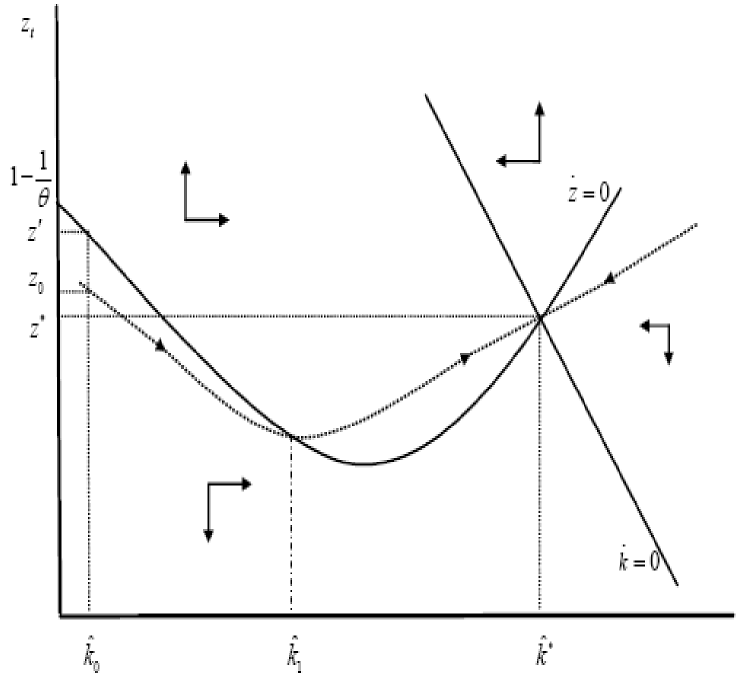

Figure 2.7: Undershooting of the consumption ratio - Overshooting of the saving rate

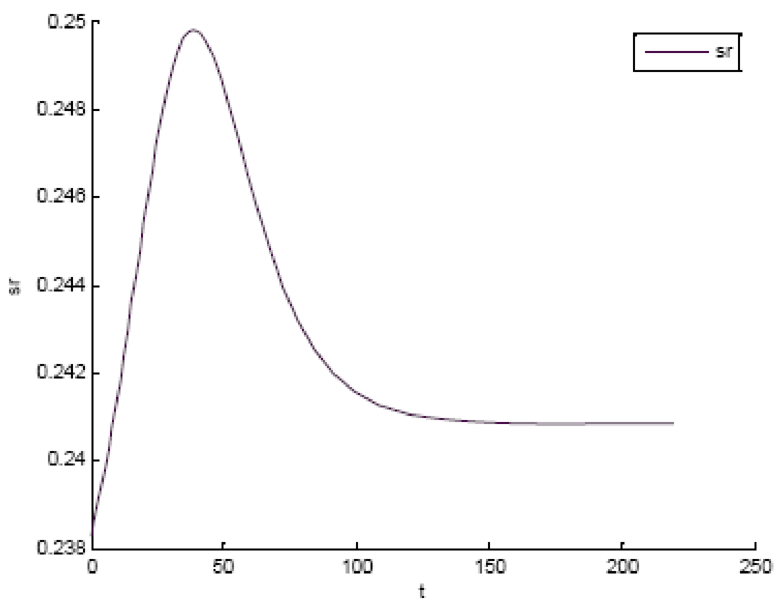

Figure 2.8: Overshooting of the saving rate. $f(\widehat{k}_t) = 1 - e^{-b\widehat{k}_t}$, $b = 0.64$, $n = 0.01$, $\delta = 0.0645$, $\rho = 0.028$, $\theta = 7.3$, $x = 0.01$

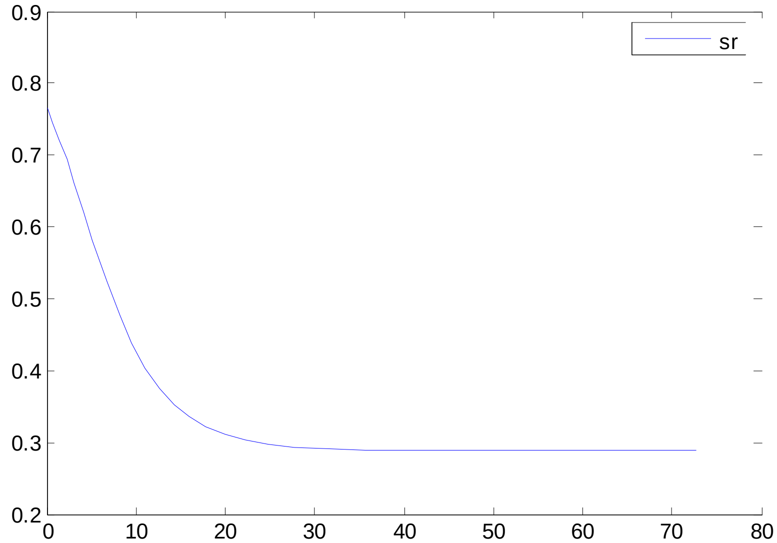

Figure 2.9: Decreasing saving rate. $f(\widehat{k}_t) = 1 - e^{-b\widehat{k}_t}$, $b = 0.64$, $n = 0.01$, $\delta = 0.0645$, $\rho = 0.028$, $\theta = 0.95$, $x = 0.01$

Notice in Proposition 2 that given condition (2B) and the fact that $z_t > 0$, a necessary condition for (2C) to hold is $\theta > 1$. In other words, if $\theta \leq 1$, then condition (2C) in Proposition 2 cannot hold and, thus, the saving rate path will be monotonic. More formally, consider the following corollary.

Corollary 2. If $\sigma(\widehat{k}_t) \leq 1 \; \forall \; \widehat{k}_t$ and $\theta \leq 1$ then the saving rate declines monotonically along the entire transition path.[8]

Proof: First note that since $\sigma(\widehat{k}_t) \leq 1$, $\gamma^* = 0$. Furthermore, if $\theta \leq 1$ then $s_t < 1 \leq 1/\theta$ and $z_t > 0 \geq (\theta - 1)/\theta \; \forall \; t$ and not just for $t > t_{k_1}$ as in Proposition 2. Consider next equation (2.29). Since the first and the third terms on the right hand side are both negative, it follows that $\gamma_{z(t)} > 0$, for if $\gamma_{z(t)} < 0$ then $\dot{\gamma}_{z(t)} < 0$ as well, which means that γ_z^* will never become zero. But then if $\gamma_{z(t)} > 0$, it follows that $\dot{z}_t > 0$ and $\dot{s}_t < 0 \; \forall \; t$, that is, the saving rate declines monotonically over time along the entire transition path. $\blacksquare$

8 The case where $\sigma = 1$ is shown also in Barro and Sala-i-Martin (2003).

Example 3. Consider the case where the production function is Cobb-Douglas $f(\widehat{k}_t) = k^\alpha$, $\alpha < 1$, and the share of capital equals the inverse of the intertemporal elasticity of substitution, that is $\alpha = \theta$. It is known that in this case the model can be solved analytically and the saving rate is (see Smith 2006, equation 20)

$$s_t = 1 - \frac{\rho + (1-\alpha)\delta - \alpha n}{\alpha}\left[(\widehat{k}^*)^{1-\alpha} + \left[(\widehat{k}_0)^{1-\alpha} - (\widehat{k}^*)^{1-\alpha}\right] e^{-(1-\alpha)\frac{\delta+\rho+\theta x}{\alpha}t}\right].$$

If $\widehat{k}_0 < \widehat{k}^*$, then $ds_t/dt < 0$, that is, the saving rate decreases monotonically over the transition to its steady-state value; the substitution effect dominates the income effect along the entire transition path.

2.6 Constant Saving Rate

As mentioned above when the production function is Cobb-Douglas and $s^* = 1/\theta$, then the saving rate is constant over the entire transition (See Kurz 1968 and Barro and Sala-i-Martin 2004).[9] In other words, this is a case where the income and substitution effect cancel each other. The following proposition generalizes this result. It identifies the general class of production functions that yield a constant saving rate and hence make the Ramsey-Cass-Koopmans and the Solow-Swan models isomorphic.

Proposition 3. The saving rate is constant over the entire transition if and only if the inverse of the production function $\widehat{k}(f)$ belongs to the following class

$$\begin{aligned} \widehat{k}(f) &= f^b\left(A - \frac{\theta s^* - 1}{\delta + \rho + \theta x}\frac{f^{-b+1}}{-b+1}\right), \qquad b \neq 1 \\ &= f\left(A - \frac{\theta s^* - 1}{\delta + \rho + \theta x}\ln f\right), \qquad b = 1 \end{aligned} \tag{2.32}$$

where $b \equiv \theta(x+n+\delta)/(\delta+\rho+\theta x)$ and A is a constant of integration.

9 Kurz (1968) reaches this conclusion by analyzing the inverse optimal problem for the Solow-Swan model; that is, given a consumption path he determines a class of objective functionals that would optimally imply such a path.

Proof. ($\Rightarrow$): If the saving rate is constant, $s_t = s^* \, \forall \, t$, then $\gamma_{z(t)} = 0 \, \forall \, t$. Setting equation (2.28) equal to zero and using equations (2.26) and (2.27) yields

$$s^* = \frac{1}{\theta} + (x+n+\delta)\frac{k_t}{f(k_t)} - \frac{\delta+\rho+\theta x}{\theta}\frac{1}{f'(k_t)} \tag{2.33}$$

or solving for $f'(k_t)$

$$\begin{aligned}
f'(k_t) &= \frac{1}{\theta s^*} f'(k_t) + (x+n+\delta)\frac{k_t f'(k_t)}{f(k_t)}\frac{1}{s^*} - \frac{\delta+\rho+\theta x}{\theta s^*} \\
\frac{1}{f'(k_t)} &= \frac{(1 - \frac{1}{\theta s^*} - (x+n+\delta)\frac{k_t}{f(k_t)}\frac{1}{s^*})}{\delta+\rho+\theta x}\theta s^* \\
\frac{dk}{df} &= \frac{-\theta s^* + 1 + (x+n+\delta)\frac{k_t}{f(k_t)}\theta}{\delta+\rho+\theta x}
\end{aligned}$$

or

$$\frac{dk}{df} - \frac{\theta(x+n+\delta)}{\delta+\rho+\theta x}\frac{k}{f} = \frac{1-\theta s^*}{\delta+\rho+\theta x},$$

which is a first-order differential equation. The solution of this equation according gives eq. (2.32)

.

($\Leftarrow$): If the production function is (2.32) then (2.33) holds. Upon using (2.26), (2.27) and (2.33), (2.28) becomes

$$\gamma_{z(t)} = f'(\widehat{k}_t)\left[s^* - s_t\right].$$

Suppose $s^* > s_t$ for some $t = t'$ then $\gamma_{z(t)} > 0$ and $\gamma_{s(t)} < 0$ for all $t > t'$, a result that is inconsistent with s approaching its steady state value s^*. Similarly, $s^* < s_t$ for any $t = t'$ can be ruled out because it implies that $\gamma_{z(t)} < 0$ and $\gamma_{s(t)} > 0$ for all $t > t'$, a result that is again inconsistent with s approaching its steady state value. Thus, $s_t = s^* \, \forall \, t$. ■

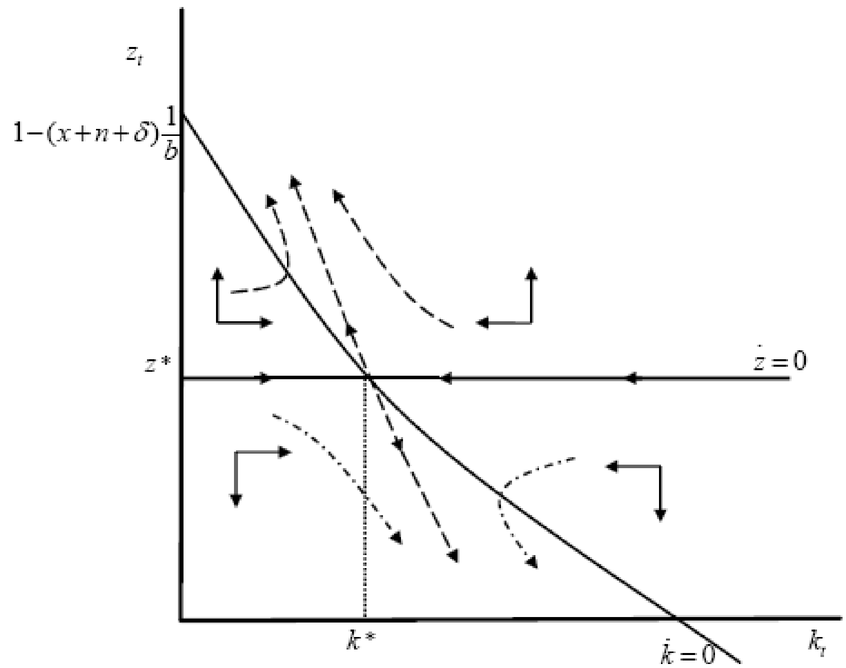

Figure 2.10: Constant saving rate

Of course, in general, more restrictions on the parameter values might be needed to ensure that the inverse of (2.32) exists and that it is a proper production. I can illustrate the dynamics of this economy as well. In Figure 4.1 I consider the case where $\gamma^* = 0$. The locus $\dot{\widehat{k}} = 0$ is the same as before (equation 2.30). Also the $\dot{z} = 0$ locus is given by equation (2.31), which after using equations (2.26) and (2.27), it becomes

$$\widehat{z}_t = 1 - \frac{1}{\theta} - (x + n + \delta)\frac{\widehat{k}_t}{f(\widehat{k}_t)} + \frac{1}{\theta}\frac{\delta + \rho + \theta x}{f'(\widehat{k})} \tag{2.34}$$

Moreover, it follows from equation (2.33) that the term

$$(x + n + \delta)\frac{k_t}{f(k_t)} - \frac{\delta + \rho + \theta x}{\theta}\frac{1}{f'(k_t)}$$

is independent of time and, hence, the $\dot{z} = 0$ locus becomes parallel to the horizontal axis.

Given that z_t is independent of time I can write equation (2.24) as

$$\dot{\widehat{k}}_t = (1 - z^*) f(k) - (x + n + \delta) k,$$

which is the fundamental Solow equation. As shown, however, in Proposition 4 below, there is still an essential difference between the Ramsey model with a (2.32) production function and the Solow model; namely, the saving rate in the Ramsey model is related to the underlying parameters and cannot be chosen arbitrarily.

Example 4. Note that if $s^* = 1/\theta$ then (2.32) yields $f(\widehat{k}) = A^{-1/b}(\widehat{k})^{1/b}$. Of course, one needs to assume that $A > 0$ for positive values of output and $b \geq 1$ for concavity. If $b > 1$ then I obtain the Cobb-Douglas production function; this is the result found in Kurz (1968) and Barro and Sala-i-Martin (2004). If $b = 1$, then one has the so-called "Ak" production function.

Example 5. Consider next the case where $b = 2$. Then (2.32) becomes

$$Af^2 + \frac{\theta s^* - 1}{\delta + \rho + \theta x} f - \widehat{k} = 0,$$

which yields

$$f(\widehat{k}) = \frac{-\dfrac{\theta s^* - 1}{\delta + \rho + \theta x} + \left[\left(\dfrac{\theta s^* - 1}{\delta + \rho + \theta x} \right)^2 + 4A\widehat{k} \right]^{1/2}}{2A} \tag{2.35}$$

(the other root is rejected because it yields negative values for output). I assume that $A > 0$ so that output takes positive values for all possible parameter values. Without further restrictions, the function (2.35) is a proper production function whose properties depend on the values of its parameters. For example, if $\theta s^* < 1$, then $\lim_{\widehat{k} \to 0} f(\widehat{k}) > 0$, $\lim_{\widehat{k} \to \infty} f(\widehat{k}) = \infty$, $f'(\widehat{k}) > 0 > f''(\widehat{k})$, $\lim_{\widehat{k} \to 0} f'(\widehat{k}) > 0$, $\lim_{\widehat{k} \to \infty} f'(\widehat{k}) = 0 = \lim_{\widehat{k} \to 0} \widehat{k} f'(\widehat{k}) / f(\widehat{k})$,

$\lim_{\widehat{k}\to\infty} \widehat{k}f'(\widehat{k})/f(\widehat{k}) = 1/2$, $\lim_{\widehat{k}\to 0} \sigma(\widehat{k}) = \infty$ and $\lim_{\widehat{k}\to\infty} \sigma(\widehat{k}) = 1$.

Just as in the case of Section 2, there is a close connection between the elasticity of substitution, the share of capital and the saving rate. Consider the following proposition.

Proposition 4. The saving rate is constant if and only if

$$\sigma(\widehat{k}_t)\alpha(\widehat{k}_t) = 1/b \quad \forall\, \widehat{k}_t. \tag{2.36}$$

where it may be recalled that $b \equiv \theta(x+n+\delta)/(\delta+\rho+\theta x)$.

Proof. Notice from (2.33) that the saving rate is constant if and only if

$$(x+n+\delta)\frac{k_t}{f(k_t)} - \frac{\delta+\rho+\theta x}{\theta}\frac{1}{f'(k_t)}$$

is independent of time. Upon differentiation one can see that this would be the case if and only if (2.36) holds. ■

Proposition 4 determines when the income and intertemporal substitution effect offset each other. Using equation (2.29), I see that in the case of exogenous growth, i.e., $\gamma^* = 0$, $s^* = (1/\sigma^*\theta)$ (once again in the case of Cobb-Douglas $\sigma = 1$ where I obtain the familiar result that if $s^* = 1/\theta$ then s_t is constant). In the endogenous growth case, on the other hand, where $\lim_{\widehat{k}\to\infty} \alpha(\widehat{k}_t) = 1$, I have $\sigma^* = 1/b$ and $s(t) = s^* = (\gamma^* + x + n + \delta)/(\theta\gamma^* + \delta + \rho + \theta x)$.

2.7 Conclusions

The significant role played by the saving rate in each economy has rendered its study a crucial issue with major policy implications. An extended strand of the literature has dealt with various issues related to the saving rate. As the effects of the saving rate are multiple and the factors that affect it are numerous, one has to specify the area of study. This chapter aimed at studying the fundamentals concerning the behavior of the saving rate. In other words it ignores the external factors that may be related to international conditions, to policy-oriented effects on the saving rate and numerous other factors it chooses to study how the various elasticities (intertemporal elasticity and elasticity of factor substitution) as well as how the income and the substitution effect, affect the saving rate.

More specifically it characterizes analytically the behavior of the saving rate along the transition path for the Ramsey-Cass-Koopmans model for any concave production function and for both cases of endogenous and exogenous growth. It has shown that for an elasticity of factor substitution greater (less) than unity, the saving rate path may exhibit undershooting (overshooting). Nevertheless, by imposing conditions on the elasticity of intertemporal substitution, one can ensure monotonicity. Similar results were obtained in Smetters, however the results of this chapter are viewed as important since they generalize the Smetters results to any concave production function and fully characterize the behavior of the saving rate. Finally it has identified the general class of production functions that render the saving rate constant over the entire transition path, as in the Solow-Swan model. The crucial difference, however, is that in the Solow model the saving rate is exogenously determined while in this model the saving rate is endogenously determined.

As mentioned in the introduction, there are various reasons that render the results of this research significant. Abstraction from the Cobb-Douglas production function, a better understanding of the different saving patterns that are observed in the data and characterization of the transitional dynamic of the saving rate are some of them.

The interpretation of the results and the use of the occurring information should vary for each economy. Additionally, specific country parameters and policy issues as well as current events that may as well affect the saving rate should also be taken into account before deciding on saving policy.

Further research on this topic should include endogenization of the labor supply, which will make the model more realistic and can additionally explain labor force patterns also observed in society. This extension will be the main topic of the second chapter of this thesis.

Appendix A Brunner and Strulik Numerical Method and Numerical Examples

The Brunner and Strulik (2002) numerical method is an alternative approach to the study of transitional dynamics via the means of numerical analysis. Important as the steady state dynamics might be, they are only a part of the whole story and tell nothing about the transition. As linearization is not a solution to this problem, more sophisticated methods are required to study non-linear systems.

The method implemented by Brunner and Strulik is that of backward integration. Backward integration is based on the idea of time-elimination as well as on the idea that the approximation of the infinite time horizon is endogenously determined. More analytically, the idea of time elimination was first introduced by Mulligan and Sala-i-Martin (1993), who have transformed an unstable boundary value problem into a stable initial value problem. In order to do so, they have considered the steady state as the initial value of the problem and they have solved the problem backwards. Additionally they have eliminated time and for that reason their method is called time elimination method. A similar approach has been adopted by Brunner and Strulik who do not eliminate time though, instead they reverse it and solve a backward looking system.

Concerning the second issue of endogeneity, it is based on the fact, as the authors mention, that the time horizon depends on the initial deviation of the backward-looking system from its steady state. This time horizon can be easily derived by ODE solution algorithm. This also explains why the method is named backward integration and not backward shooting. The term shooting implies that one may miss the target and have to try a different path each time until he hits the right one. On the contrary the term integration implies that one step is taken each time and there is no chance that the target is missed. The mechanism through which backward-integration method works, lies

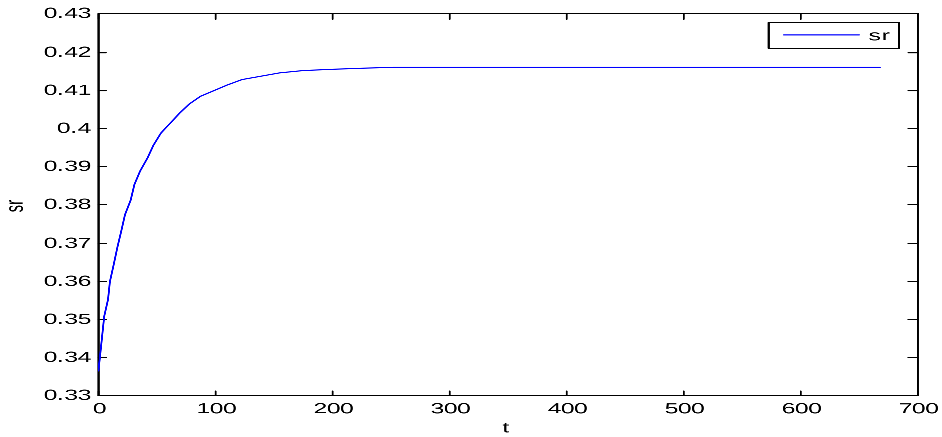

Figure A.1: Increasing saving rate. $f(\widehat{k_t}) = A(\widehat{k_t})^{\alpha} - \gamma$, $A = 1$, $\alpha = 0.695$, $\gamma = -5$, $n = 0.01$, $\delta = 0.0645$, $\rho = 0.035$, $\theta = 3$, $x = 0.01$

in exploiting the numerical stability of an unstable but attractive manifold in a saddlepoint stable system.

This latter feature is one of the main advantages of the Brunner and Strulik method when compared to shooting methods. Another advantage is that it does not need the specification of a terminal time and it is accurate. Finally it is quite simple and easy to implement and does not need the re-introduction of time which is the case with the Mulligan and Sala-i-Martin method.

These are the reasons why this numerical method was employed in the current chapter. A variety of production functions were used for the analysis. The ones chosen for the main body of the chapter were the most representative ones, in the sense that they exhibited the greatest magnitude and approached realistic saving rate values. However a number of other graphs can be produced when using different production functions or different parameter values. The numerical analysis makes the results more evident and highlights some details of the topic.

Following I will present some graphs not presented in the main text:

$\sigma > 1$

As was analyzed in Proposition 1, when $\sigma > 1$ the saving rate might manifest undershooting.

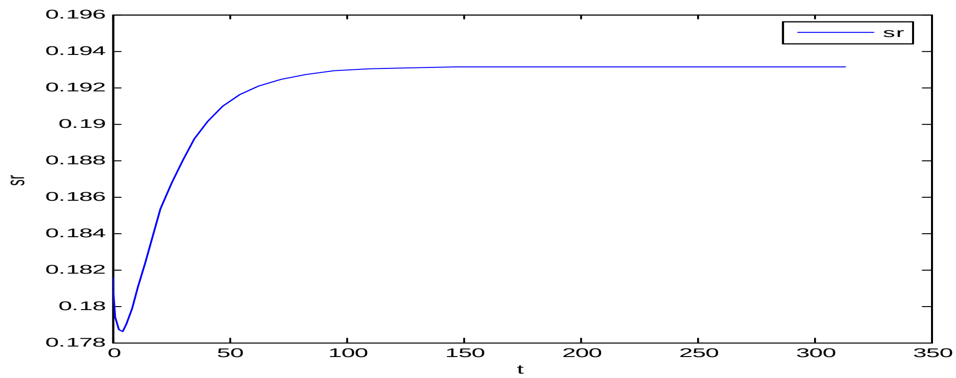

Figure A.2: Undershooting of the saving rate. $f(\widehat{k}_t) = A(\widehat{k}_t)^{\alpha} - \gamma$, $A = 1$, $\alpha = 0.695$, $\gamma = -70$, $n = 0.01$, $\delta = 0.0645$, $\rho = 0.035$, $\theta = 3$, $x = 0.01$

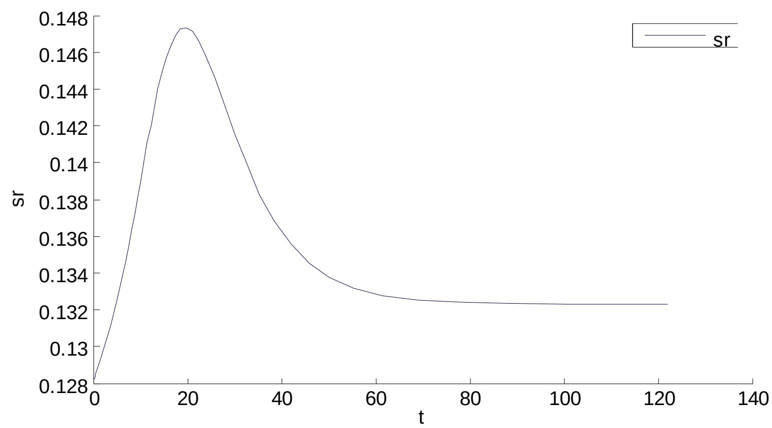

Figure A.3: Overshooting of the saving rate: $(\widehat{k}_t) = 1 - e^{-b\widehat{k}_t}$, $b = 1.8$, $n = 0.01$, $\delta = 0.07$, $\rho = 0.04$, $\theta = 12$, $x = 0$

If this is the case and the saving rate does not behave monotonically, slight changes in parameter values may alter the transitional dynamics of the saving rate.

For instance, if I lower the value of parameter γ in the numerical example of Proposition 1 and for a low value of θ the saving rate may as well not manifest undershooting.

Another case is the one where undershooting does actually exist however the transition period, the magnitude of the undershooting and the estimated rate are significantly different.

$\sigma < 1$

When the elasticity of substitution $\sigma < 1$ the saving rate might manifest overshooting as has

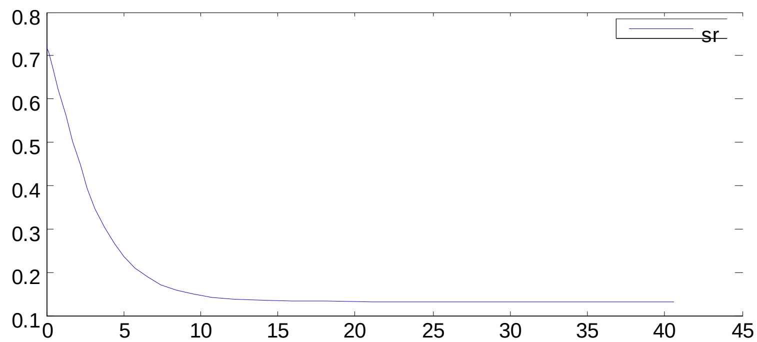

Figure A.4: Decreasing Saving Rate. $f(\widehat{k_t}) = 1 - e^{-b\widehat{k}_t}$, $b = 1.8$, $n = 0.01$, $\delta = 0.07$, $\rho = 0.04$, $\theta = 0.98$, $x = 0$.

been proved in proposition 2. As was the case with $\sigma > 1$, a change in parameter values may alter the dynamic behavior of the saving rate.

In the overshooting figure, note that after having changed some parameter values, when compared to the overshooting figure of proposition 2, the figure presents a different picture. More specifically, the transition period has significantly increased, the magnitude of the overshooting has slightly decreased and the predicted value of the saving rate is much lower (approximately 10%). However there still exists overshooting which is not always the case.

One can observe the large differences that occur when different functional forms are chosen. The most significant effect is noticed on the estimated value of the saving rate as well as on the transition period which varies significantly with parameter changes. These results are rather expected, when taking into account the vast literature that has dealt with the issue and the effects of the various elasticities on the saving rate as well on the speed of convergence. The numerical simulations are an illustrative way to comprehend the effect of the various parameters, analyzed in this chapter, on the saving rate and consequently on the economy as a whole.

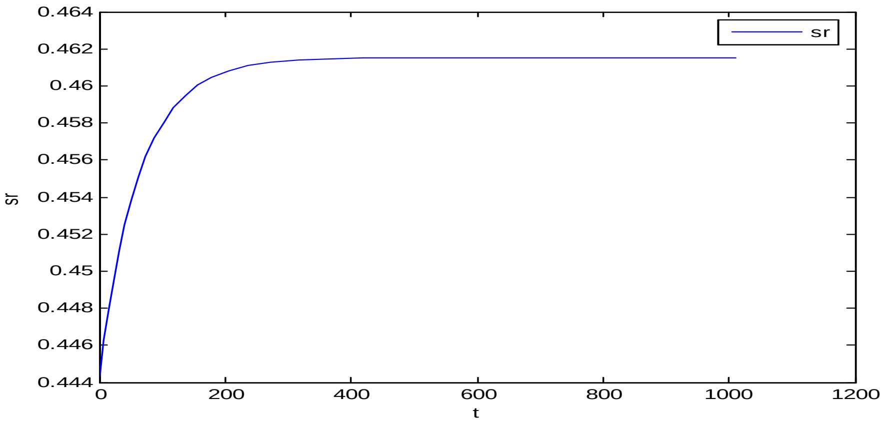

Figure A.5: Cobb-Douglas production function, increasing saving rate: $n = 0.01$, $\delta = 0.05$, $\rho = 0.02$, $\theta = 3$, $x = 0.02$, $\alpha = 0.75$

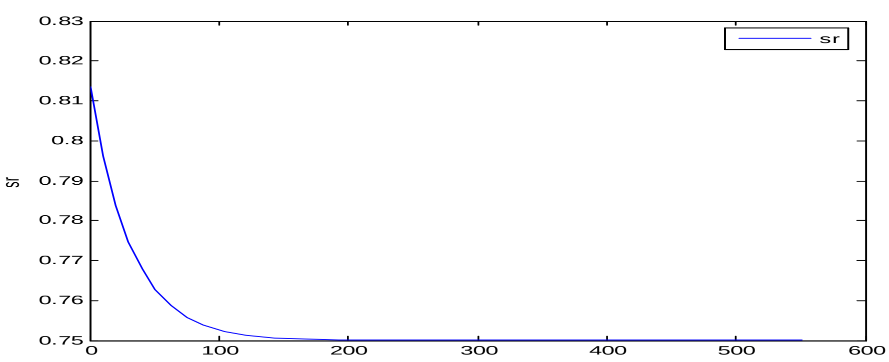

Figure A.6: Cobb-Douglas production function, decreasing saving rate: $n = 0.01$, $\delta = 0.05$, $\rho = 0.02$, $\theta = 0.5$, $x = 0.02$, $\alpha = 0.75$

Appendix B The Effect of Saving Rate on the Speed of Convergence

Following the analysis of Reiss (2002) I will use his equations in order to directly introduce the saving rate as a variable into the equation of convergence. Using the Reiss equation makes the analysis of the transition feasible, as his suggested equation is not the result of linearization, but instead refers to the whole transition path.

$$\begin{aligned}
\beta_Y &= \frac{\dot{Y}}{Y^* - Y(t)} = \frac{f'(k_t)\dot{k}}{f(k^*) - f(k_t)} \\
&= \frac{f'(k_t)[f(k_t - c - (\chi + n + \delta)k_t)]}{f(k^*) - f(k_t)} \\
&= \frac{f'(k_t) - \frac{cf'(k_t)}{f(k_t)} - (\chi + n + \delta)\frac{k_t f'(k_t)}{f(k_t)}}{\frac{f(k^*)}{f(k_t)} - 1} \\
&= \frac{f'(k_t)[1 - \frac{c}{f(k_t)}] - (\chi + n + \delta)\frac{k_t f'(k_t)}{f(k_t)}}{\frac{f(k^*)}{f(k_t)} - 1} \\
&= \frac{f'(k_t)s_t - (\chi + n + \delta)\frac{k_t f'(k_t)}{f(k_t)}}{\frac{f(k^*)}{f(k_t)} - 1}
\end{aligned}$$

The above equation indicates a positive correlation between the speed of convergence and the saving rate. This results accord well with the predictions of Sato (1963, 1964) as well as with some cases of B&S-i-M.

More specifically:

When $s \uparrow \Rightarrow \beta \uparrow$

When $s \downarrow \Rightarrow \beta \downarrow$

It is important though to point out that this result cannot be taken at face value, as there are other factors on this equation that affect the speed of convergence, such as the share of capital, and which are closely related to the saving rate. However they give an idea for the direction of their relationship.

Alternatively if I want to test the predictions of Barro & Sala–i-Martin who state that it is not the level of the savings rate that affects the speed of convergence but instead its direction of its change, I can take the following equation which introduces the growth rate of the saving rate:

$$
\begin{aligned}
\beta_Y &= \frac{\dot{Y}}{Y^* - Y(t)} = \frac{f'(k_t)\dot{k}}{f(k^*) - f(k_t)} \\
&= \frac{f'(k_t)[f(k_t - c - (\chi + n + \delta)k_t)]}{f(k^*) - f(k_t)} \\
&= \frac{f'(k_t) - \frac{cf'(k_t)}{f(k_t)} - (\chi + n + \delta)\frac{k_t f'(k_t)}{f(k_t)}}{\frac{f(k^*)}{f(k_t)} - 1} \\
&= \frac{f'(k_t)[1 - \frac{c}{f(k_t)}] - (\chi + n + \delta)\frac{k_t f'(k_t)}{f(k_t)}}{\frac{f(k^*)}{f(k_t)} - 1} \\
&= \frac{f'(k_t)[1 - \frac{c}{f(k_t)}] - (\chi + n + \delta)\frac{k_t f'(k_t)}{f(k_t)} - f'(k^*) + f'(k^*)}{\frac{f(k^*)}{f(k_t)} - 1} \\
&= \frac{f'(k_t) - f'(k^*) - \gamma_z}{\frac{f(k^*)}{f(k_t)} - 1} \\
&= \frac{f'(k_t) - f'(k^*) + \gamma_s}{\frac{f(k^*)}{f(k_t)} - 1}
\end{aligned}
$$

According to this equation when γ_s is positive (negative), i.e. when s increases (decreases) then the speed of convergence increases (decreases) as well. This results at first sight contradicts the results of B&S-i-M however note that here the relationship between θ and the saving rate is not clear, instead it depends on other factors as well.

The above results are far from an analytical study over the relationship between the interest rate and the speed of convergence. After all this is not the object of this study. They are actually an attempt to directly correlate these two variables in order to get a rough idea for the direction of the effect.

Appendix C Proof of Proposition 1 for the Case of Exogenous Growth

If I assume that there is only exogenous growth then $\gamma^*_{\widehat{c}(t)} = \gamma^*_{\widehat{k}(t)} = \gamma^* = 0$. The the limiting value of the saving rate $s^* = 1 - \widehat{c}^*/f(\widehat{k}^*)$ is given by:

$$s^* = (x + n + \delta)\,\frac{\widehat{k}^*}{f(\widehat{k}^*)}. \tag{C.1}$$

Let again $\gamma_{z(t)}$ denote the growth rate of $z_t = 1 - s_t = \widehat{c}_t/f(\widehat{k}_t)$. The following equation is derived with the use of (2.23), (2.24), (2.26) and (C.1) with the crucial difference that now $\gamma^* = 0$:

$$\begin{aligned}
\gamma_{z(t)} &= \frac{\dot{z}}{z_t} = \frac{\dot{\widehat{c}}}{\widehat{c}_t} - \frac{f'(\widehat{k}_t)}{f(\widehat{k}_t)}\dot{\widehat{k}} \\
&= \frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho - \theta\chi) - \frac{\hat{k}f'(\widehat{k}_t)}{f(\widehat{k}_t)}(\frac{f(\hat{k}_t)}{\hat{k}_t} - \frac{\hat{c}_t}{\hat{k}_t} - (n + \delta + \chi)) \\
&= \frac{1}{\theta}f'(\hat{k}_t) - \frac{1}{\theta}(\delta + \rho + \theta\chi) - f'(\widehat{k}_t) + \frac{f'(\widehat{k}_t)}{f(\widehat{k}_t)}\hat{c} + \frac{\hat{k}f'(\widehat{k}_t)}{f(\widehat{k}_t)}(n + \delta + \chi) \\
&= \frac{1}{\theta}f'(\hat{k}_t) - \frac{1}{\theta}(\delta + \rho + \theta\chi) - f'(\hat{k}_t) + z_t f'(\hat{k}_t) + \frac{\hat{k}f'(\widehat{k}_t)}{f(\widehat{k}_t)}(n + \delta + \chi) \\
&= f'(\widehat{k}_t)\left[z_t - \frac{\theta - 1}{\theta}\right] - \frac{1}{\theta}(\delta + \rho + \theta\chi) + \frac{\hat{k}f'(\widehat{k}_t)}{f(\widehat{k}_t)}(n + \delta + \chi) \\
&= f'(\widehat{k}_t)\left[z_t - \frac{\theta - 1}{\theta}\right] - \frac{f'(\widehat{k}^*)}{\theta} + s^*\frac{f(k^*)}{k^*}\frac{f'(\widehat{k}_t)\widehat{k}_t}{f(\widehat{k}_t)} \\
&= f'(\widehat{k}_t)\left[z_t - \frac{\theta - 1}{\theta}\right] + \frac{f(\widehat{k}^*)}{\widehat{k}^*}\left[s^*\frac{f'(\widehat{k}_t)\widehat{k}_t}{f(\widehat{k}_t)} - \frac{1}{\theta}\frac{f'(\widehat{k}^*)\widehat{k}^*}{f(\widehat{k}^*)}\right].
\end{aligned} \tag{C.2a}$$

Equation (C.2a) applies to the case where there exists only exogenous growth. As mentioned in proposition one, here again, $\gamma^*_z = 0$.

Finally, I also use the following expression, which gives the change of $\gamma_{z(t)}$ with respect to time, $\dot{\gamma}_{z(t)} \equiv d(\gamma_{z(t)})/dt$, and is obtained after differentiating (C.2a) and using (2.25) and (C.1)

$$\dot{\gamma}_{z(t)} = f^{''}(\widehat{k}_t)\dot{\widehat{k}}\left[z_t - \frac{\theta-1}{\theta}\right] + f^{'}(\widehat{k}_t)\gamma_{z(t)}z_t + (x+n+\delta)\left[1-\sigma(\widehat{k}_t)\right]\frac{f^{''}(\widehat{k}_t)\widehat{k}_t}{f(\widehat{k}_t)}\dot{\widehat{k}} \tag{C.3}$$

Proposition 1. a) Let $\widehat{k}_1$ be a value of $\widehat{k}$ such that $\widehat{k}_1 < \widehat{k}^*$. If $\sigma(\widehat{k}_t) > 1 \,\forall\, \widehat{k}_t$ and

$$s^* \frac{f'(\widehat{k}_1)\widehat{k}_1}{f(\widehat{k}_1)} > \frac{1}{\theta}\frac{f^{'}(\widehat{k}^*)\widehat{k}^*}{f(\widehat{k}^*)} \tag{1A}$$

then the saving rate is increasing along the transition path from $\widehat{k}_1$ to $\widehat{k}^*$.

b) Moreover, there exists $\widehat{k}_0 < \widehat{k}_1$, such that

$$s^* \frac{f'(\widehat{k}_0)\widehat{k}_0}{f(\widehat{k}_0)} < \frac{1}{\theta}\frac{f'(\widehat{k}^*)\widehat{k}^*}{f(\widehat{k}^*)}. \tag{1B}$$

In addition, the saving rate is decreasing at $\widehat{k}_0$ iff

$$1 > z_0 > 1 - \frac{1}{\theta} - \left[s^* \frac{f'(\widehat{k}_0)\widehat{k}_0}{f(\widehat{k}_0)} - \frac{1}{\theta}\right]\frac{f'(\widehat{k}^*)}{f'(\widehat{k}_0)} \tag{1C}$$

Proof: *a*) Recall from the Lemma that if $\sigma(\widehat{k}_t) > 1$ then $f^{'}(\widehat{k}^*)\widehat{k}^*/f(\widehat{k}^*) = 1$. Also, since $f^{'}(\widehat{k}_1)\widehat{k}_1/f(\widehat{k}_1) < 1$, it follows from condition (1A) that $s^* > 1/\theta$ or $z^* < (\theta-1)/\theta$. Next suppose that there is a value of $t > t_{k_1}$, say t', such that $z_{t'} > (\theta-1)/\theta$, where t_{k_1} denotes the value of time that corresponds to $\widehat{k}_1$. Then (C.2a) implies that $\gamma_{z(t')} > 0$ (since every term on the RHS is positive) and $\gamma_{z(q)} > 0 \,\forall\, q > t'$; hence, $z^* > (\theta-1)/\theta$, which is a contradiction. Thus, $z_t < (\theta-1)/\theta \,\forall\, t > t_{k_1}$. Also, equation (C.3) implies that $\gamma_{z(t)} < 0 \,\forall\, t > t_{k_1}$, for if $\gamma_{z(t)} \geq 0$ for some value of t, say t'', then $\dot{\gamma}_{z(q)} > 0 \,\forall\, q > t''$, which means $\gamma_z^* > 0$; this is also a contradiction, since along the balanced growth path s and hence z remain constant ($\gamma_z^* = 0$). But then if $\gamma_{z(t)} < 0$ $\forall\, t > t_{k_1}$, I have that $\dot{s} > 0 \,\forall\, t > t_{k_1}$, or that s_t is increasing along the transition path from $\widehat{k}_1$ to

$\widehat{k}^*$.

b) The proof that there exists $\widehat{k}_0$ such that (1B) holds follows immediately from the Lemma (parts c and d). If (1B) holds, then $z_t \lesseqgtr (\theta - 1)/\theta$ and $\gamma(z)$ in equation (C.3) can take either positive or negative values. The rest of the proof is straightforward; namely, set $\gamma_{z(0)} > 0$ in equation (C.2a) and solve for z_0, taking into account the Lemma (parts a and d).

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Chapter 3 Accounting for Long-Run Patterns in Labor Force Participation Rates

3.1 Introduction

Trends in working hours and labor supply have not always been central in economic theory. Not until economists realized that working hours were approximately one third of total working time did they start paying attention to leisure. All this available time that can be devoted to various activities, most of which entail consumption of goods or result in forgone earnings, has attracted economists attention as well as their effort to subsume leisure in their models and investigate its role in more detail.

Initially the vast majority of the literature examining labor supply, both theoretically and empirically, usually referred to working hours an agent is willing to supply. In due course, as female labor supply increased, as well as the average duration of schooling, it became evident that there was a necessity for an approach of labor supply conceived as labor force participation rates.

The labor supply research can be subdivided in two main categories. The first comprises the empirical research that studies empirically the labor supply patterns both in terms of hours worked and of participation rates. In this strand of literature that will be extensively analyzed below, the main outcome lies in the fact that working hours have decreased intertemporally, both for men and women. The second outcome is derived from the study of various groups participation rates and most notably for that of women or of young adults, where researchers find evidence of a U-shaped path in female participation and of a decreasing pattern for the young. Of course many other aspects of this topic such as male or elder participation rates have been studied as well and have also come up with interesting results.

The second category focuses its research on theoretical models that study labor supply. In this context labor supply is usually conceived as per worker working hours and the horizon is finite as is the case with life cycle models. This literature has come up with significant results concerning the role of labor supply in economic decisions. The hypothesis of endogenous labor supply has been enriched with various plausible and realistic assumptions such as the production process and/or the structure and the realization of leisure, however in most cases the analysis is usually limited to the life cycle model.

Throughout the historical process the issue of working hours had always had a dominant role. As economies moved to different market formulations, the main argument at stake was a decrease in working hours. This has always been the main object of labor unions and other coalitions. Not only did they bargain about wages but also about the horaire. The attention attached to this topic makes one wonder why it is important to study labor hours and labor supply patterns. Starting with working hours, the first and most important reason, as has already been stated in the introduction is that working hours compared to the rest of the hours available to an agent are much less. Thereby taking this fact into account, especially when it comes to models that attempt to interpret economic phenomena, is of the utmost importance. Doing this is even more significant when leisure involves consumption of other goods as Becker (1965) has pointed out. He also pointed out that foregone earnings are another significant reason why leisure should be counted for. An important point made in his analysis, which applies especially for developed economies, is that the maximization of utility does not necessarily coincide with maximization of worker productivity. In other words workers may prefer not to maximize their earnings and their productivity and may actually prefer to work less in order to gain more pleasure from leisure. For this reason, leisure as a pleasant activity has directly entered the utility function and is actually positively related to it. Another important reason to account for non-working hours is because this time may be devoted to other activities such as educational activities which create human capital and make workers more productive. There is a huge literature studying the effect of human capital on various aspect of economic life which has initially departed from the observation that non-working time can be directed towards various productive or non-productive activities.

The case of human capital or more precisely of schooling applies not only to the study of leisure but also to the study of participation rates. As an economy develops, it has been observed that the level of schooling increases and that young adults enter the labor market in later ages. The returns from schooling have proved to be higher thereby agents must balance their benefits from entering the labor market immediately or at a later age. This is more evident in the case of women, where women in the distant past did not have the option to participate in the educational process. Today the participation of women in schooling has rapidly increased and is even exceeding that of men both in percentage rates as well as in duration. This later entering of young adults in labor force has tremendous impact on the social process as a whole, on the production process as human capital becomes an increasingly important factor of production, as well as on the economic life of agents as it is one additional economic decision to be made which interacts with all the others.

Especially for the case of women, they traditionally abstracted not only from schooling, but also from the formal production process and did not participate in the formal labor market. It has not been until relatively recently (just a few decades) that women have started participating in the working force officially. This participation has been significant by many aspects. The first and very important is that the structure of the production process has changed. Women, which initially actively participated in home production, without getting directly reimbursed for their participation, have gradually entered the large scale production. As a matter of fact there is a causality between their participation in the labor force and the transformation of the production structure. This change had major effects in the structure and the economic behavior of the household as well. Especially in economies driven by consumption, the newly obtained ability of a large part of the population to consume on its own money and not necessarily as a part of the household, has tremendous economic implications. This is evident from the fact that in real world terms a whole industry addressing to women has been set up during the latter half of 20th century. Last but not least all these newcomers in the labor market, which enter it at various ages, have material effects on social security issues, one of the most important economic decisions of a government. This makes clear the importance of trying to comprehend and describe participation patterns. The better informed a government is on this topic, the better policies it can pursue.

I tried to focus on the benefits of policy makers and economists from studying working hours and labor force participation rates. If one takes into account social effects as well, then this literature gains even more importance, however this analysis is far beyond the scope of this paper. And while the analysis concerning working hours has been extensively undertaken, the same does not hold concerning labor force participation rates, at least not analytically. A large number of empirical studies has attempted to explain the participation rates of various age and sex groups, however very few studies have actually induced the notion of the labor force participation rate in their model and in their utility functions. This is the gap that this paper aspires to fill in by introducing the notion of labor force participation rate in the utility function. In doing so I abstract from one of the main assumptions used in the majority of the literature. This assumption is that all workers are "employed" and that their choice lies in working hours. I actually assume exactly the opposite: all agents who work, do not have a choice on working hours and their decision lies in whether to work or not. In doing so I implicitly make the utility of the household dependent on the number of workers within the household. I believe that the assumption of choosing working hours is indeed realistic, since most people work in blocks of hours and not even people who run their own firms can actually choose to work less than a minimum amount of hours for fear of not withstanding the competition. The second assumption is also realistic as many adult members of the household do not participate in the working force either on their own choice or because the abide by social norms.

My analysis does not aspire to account for all these social factors that affect participation rates. It actually aims at indicating that one should more carefully examine this topic and explain why participation rates may not be monotonous intertemporally, as pointed out by empirical evidence. For this purpose I will use a standard Ramsey model with endogenous labor supply and flexible assumptions about the value of intertemporal elasticity of substitution and the elasticity of factor substitution. My main finding is that participation rates are not always monotonous and as a matter of fact I can obtain a U-shaped pattern that is empirically observed for women. Moreover I can, under different parametrization, account for the decreasing rates in young adults participation. The main intuition behind my non-monotonicity result lies in the income and substitution effects. This intuition is rather simple and standard and more sophisticated models can be constructed that take explicitly into account the various factors such as schooling, norms or structure of production. However these specific factors should be endogenous in the model and thus subject to income and substitution effects themselves. Overall the model's contribution lies in the fact that it examines participation rates theoretically, a topic widely neglected in the current literature, and moreover it goes a long way towards explaining the observed patterns.

The outline of the paper goes as follows: the first part sets up an extensive literature review concerning the various aspects related to labor supply. As my model relies on the standard

Ramsey model and aspires to obtain significantly different results in the same context, I appose the analysis of Barro and Sala-i-Martin (2004) on this topic who account for the decline in working hours. When setting up my model in the following part, I obtain complex patterns under the presence of more flexible assumptions about elasticities. Illustrative dynamics fill in the gap whenever analytical approach is too complicated. The stability analysis of the model takes place in Appendix A of the chapter whereas in Appendix B I examine a special case of indeterminacy under flexible normality assumptions.

3.2 Literature Review

The purpose of this literature review is to highlight the various aspects related to labor force participation rates and their trends. On the one hand I want to summarize the empirical evidence that I will attempt to explain, as well as to indicate through the existing empirical analysis, the necessity for a theoretical model that accounts for the observed trends. These trends as I will state below are far form monotonous and the need for a theoretical model that accounts for this non-monotonicity becomes evident when observing these graphs.

On the other hand I want to strengthen the various assumptions used in my model and indicate their plausibility. More analytically I want to highlight the literature supporting the argument that people usually work in blocks of hours. For completeness I will also refer to the literature that focuses on working hours instead of participation rates and point out how these models differ from mine. The reasons why one should abstract from the Cobb-Douglas hypothesis have already been extensively analyzed in the first chapter of this thesis and for this reason will be omitted from the analysis, however I will provide empirical evidence that supports the hypothesis that utility function is not necessarily separable.

3.2.1 Empirical Evidence

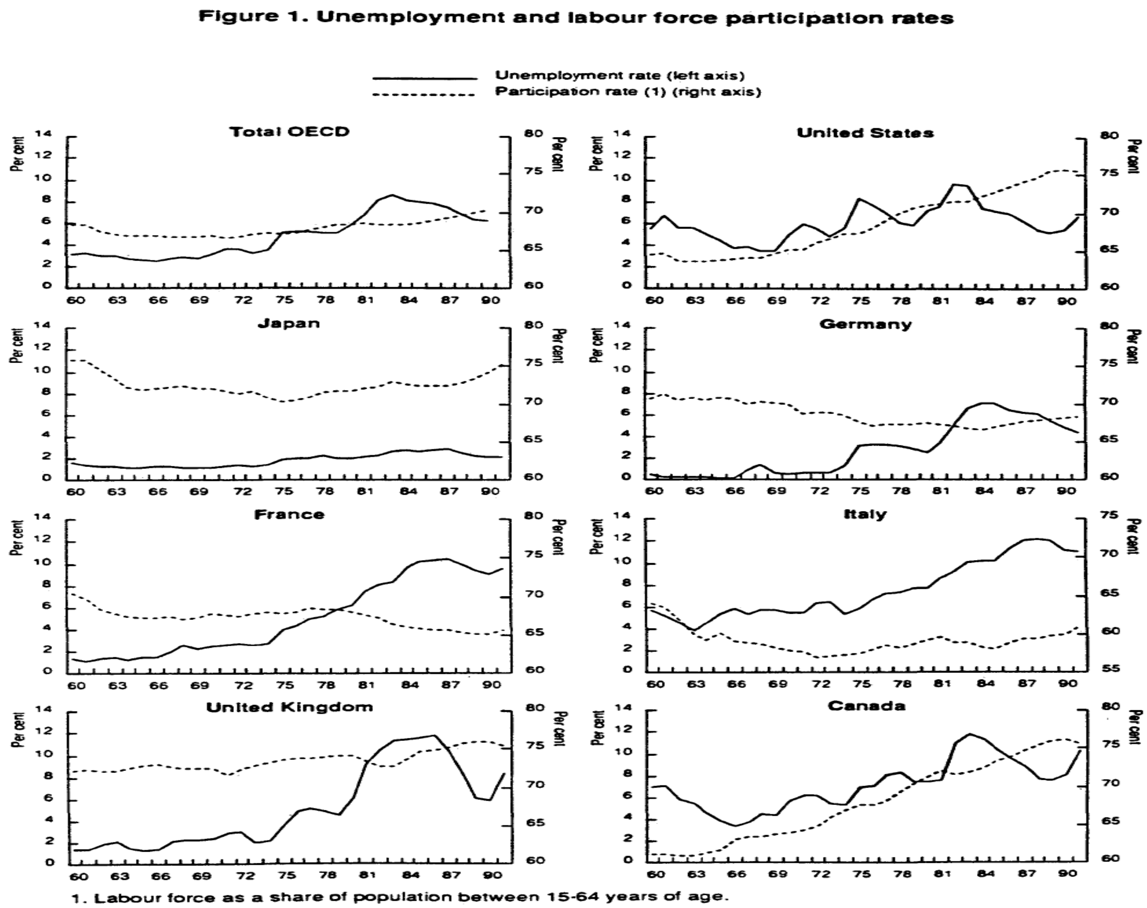

Figure 1: Participation Rates in OECD Countries, Source: Elmeskov and Pichelman (1993)

If one focuses on studying labor force participation rates he finds out that these patterns are far from monotonous not only for women, as one would expect, but in some cases for other categories of the population such as young adults. One of the most representative papers in terms of empirical evidence is the paper of Elmeskov and Pichelman (1993) as it indicates graphically the non-monotonous behavior of labor force participation rates in OECD countries. The aim of the paper is to analyze unemployment and labor force participation in a cross-section of countries and interpret the observed trends and cycles.

The overall observation for OECD countries is that with some exceptions, labor force participation rates do not behave monotonically. For the Total OECD (Figure 1) participation rates have slightly fell during the 1960's but during 1970-1990's they steadily rose. Many factors may have contributed to this U-shaped pattern. According to Elmeskov and Pichelman, the initial decrease may be attributed to increased schooling attendance, i.e. decrease in young adults' participation rates, to income effects leading to increased demand for leisure as well as to the public pension schemes. The subsequent rise in participation rates is attributed entirely to the increase in female labor force participation. The actual pattern of participation rates and the intensity of its variations differs among countries. For instance the US participation rate (Figure 1) which was rather stable during 1960's, experienced an increase that continued until 1990's. The same goes for Australia (Figure 2). In Greece and Finland (Figure 3) the initial decrease was followed by an increase in participation rates therefore forming a U-shaped pattern. In Netherlands and Turkey the initial decrease continued until 1990's.

Concerning the impact of unemployment on participation rates, it is observed that in countries that experience large increases in unemployment they also experience smaller increases in participation rates and moreover in countries were the unemployment rate is small, the participation rate is large (i.e. Japan and Sweden) and the opposite (i.e. Spain and Ireland). Therefore in the long-run there appears to be a negative relationship between unemployment and labor force participation.

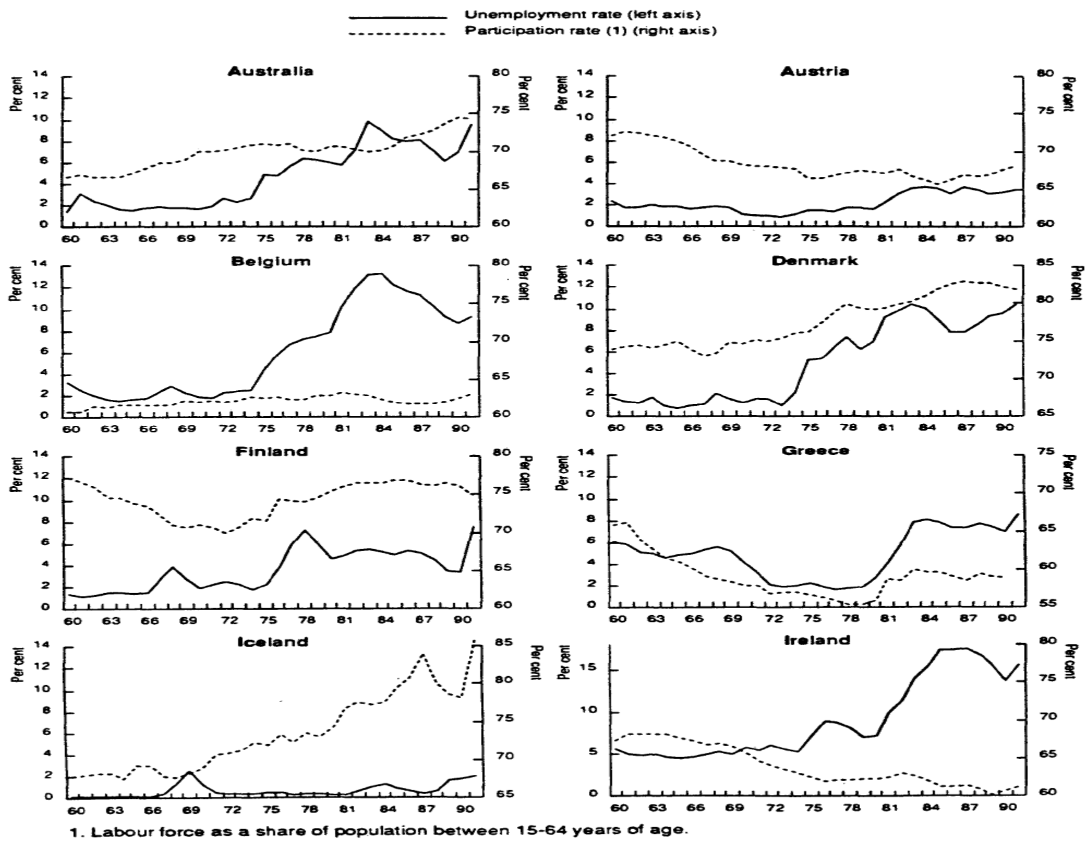

Figure 2: Participation Rates in OECD Countries. Source: Elmeskov and Pichelman (1993)

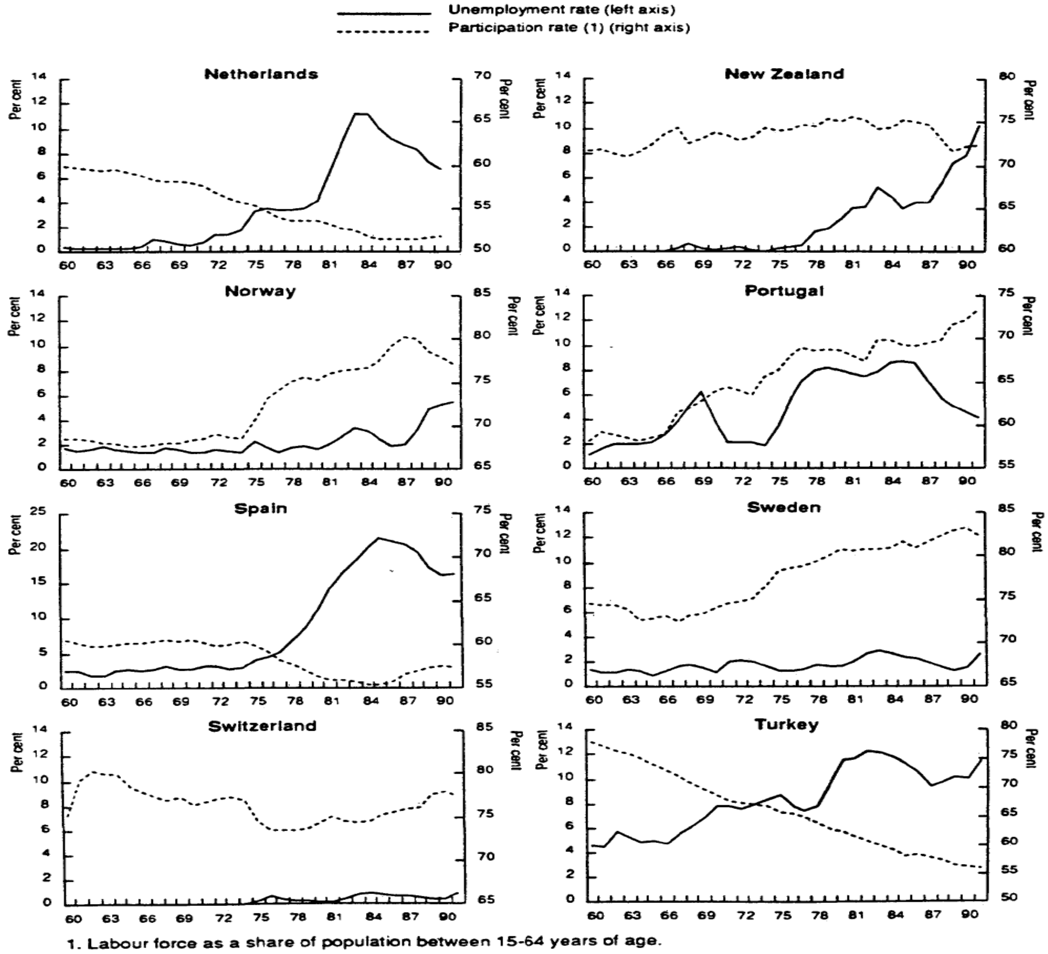

Figure 3: Participation Rates in OECD Countries. Source: Elmeskov and Pichelman (1993)

With some exceptions, as in the case of Elmeskov and Pichelman who study aggregate participation rates, the vast majority of the literature that studies participation rates focuses on specific population groups and most notably to adult women as they have experienced dramatic variations in their participation rates. It is true that the participation rates of male adults do not dramatically change intertemporally. In this case, if one wants to interpret the overall rates then he should probably make the assumption that the female participation rates are sufficient to affect the overall trend of the total population, which is an empirically plausible assumption. One of the most interesting papers in this literature that focuses on participation rates of women is that of

Goldin (1994). The aim of this study is to highlight the existence of a U-shaped pattern in female participation rates, manifested as the economy develops. The driving forces behind this pattern are income and substitution effects.

More analytically as incomes rise in most societies, then the income effect takes place and hence female labor participation rate falls. This can be attributed to various reasons. Among them is technical progress that makes women "redundant" since female jobs are replaced by machines used by men, therefore the income effect may delay the official participation of women in labor force. Another significant reason for non-participation of women in labor force is the so-called "stigma". In early societies, female participation in official labor force and specifically in industry (since services were not widespread at the time) was "stigmatizing" the males of the household as incapable of taking care of their family. Therefore only women coming from very poor households participated in labor force. As income started rising it was easier for these households to let women out of workforce. Therefore initially the income effect dominated and female participation decreased.

However in due course new jobs were developed, the so-called white collar jobs (indicating clerical jobs in contrast to blue-collar jobs indicating heavier jobs), in which even women coming from middle and upper class participated, therefore they were not associated with stigma. Moreover as wages rose, the returns to education became significantly higher and therefore more women started getting education and entering the workforce. This effect is the so-called substitution effect and is responsible for the increasing part of the U.

This kind of evidence is clear for the US but is also observed in many other countries in which women only recently have started getting education and are thus expected to enter the increasing area of the U, like Latin America which is currently considered to be at the bottom of the U.

According to Goldin the bottom of the U for the US was reached in 1920. If it is true that income and substitution effects are telling a significant part of the story then it comes as no surprise that most countries follow similar patterns in different time periods though due to different economic conditions.

An interesting case which contradicts the standard literature reporting constant working hours along the balanced growth path is the study of Gali (2005). He uses evidence from G-7 countries where both participation rates and working hours per worker display non-stationarity features. His analysis is conducted within an RBC context where fluctuations may occur. His series though are rather long, especially for some countries, thereby they are indicative for long-run trends.

His findings concerning participation rates are interesting because they contradict the standard literature concerning working hours and they verify the findings of this study for some countries such as the US and Canada concerning participation rates. More analytically Gali argues that not only the patterns are far from monotonous but in some cases they might even be U-shaped. For instance in the US and Canada (Graphs 4 and 5) it is evident that hours per capita (which are the hours worked per person aged 16 to 65), in the late part of the sample display an upward trend which can be attributed to increases in employment rates combined with a relatively flat pattern for hours per worker. This result accords with the assumption and the results of the model of this chapter as it is assumed that hours per worker are constant (workers work in blocks of hours), while participation rates within the household vary. As far as the decreasing part of the figure is concerned, it can be attributed to the decrease in working hours that took place in earlier years. However the U is not the only pattern that occurs. In some cases decreasing or increasing participation rates may occur which is another result that this chapter aspires to account for, i.e. for the fact that participation rates are far from constant along the transition.

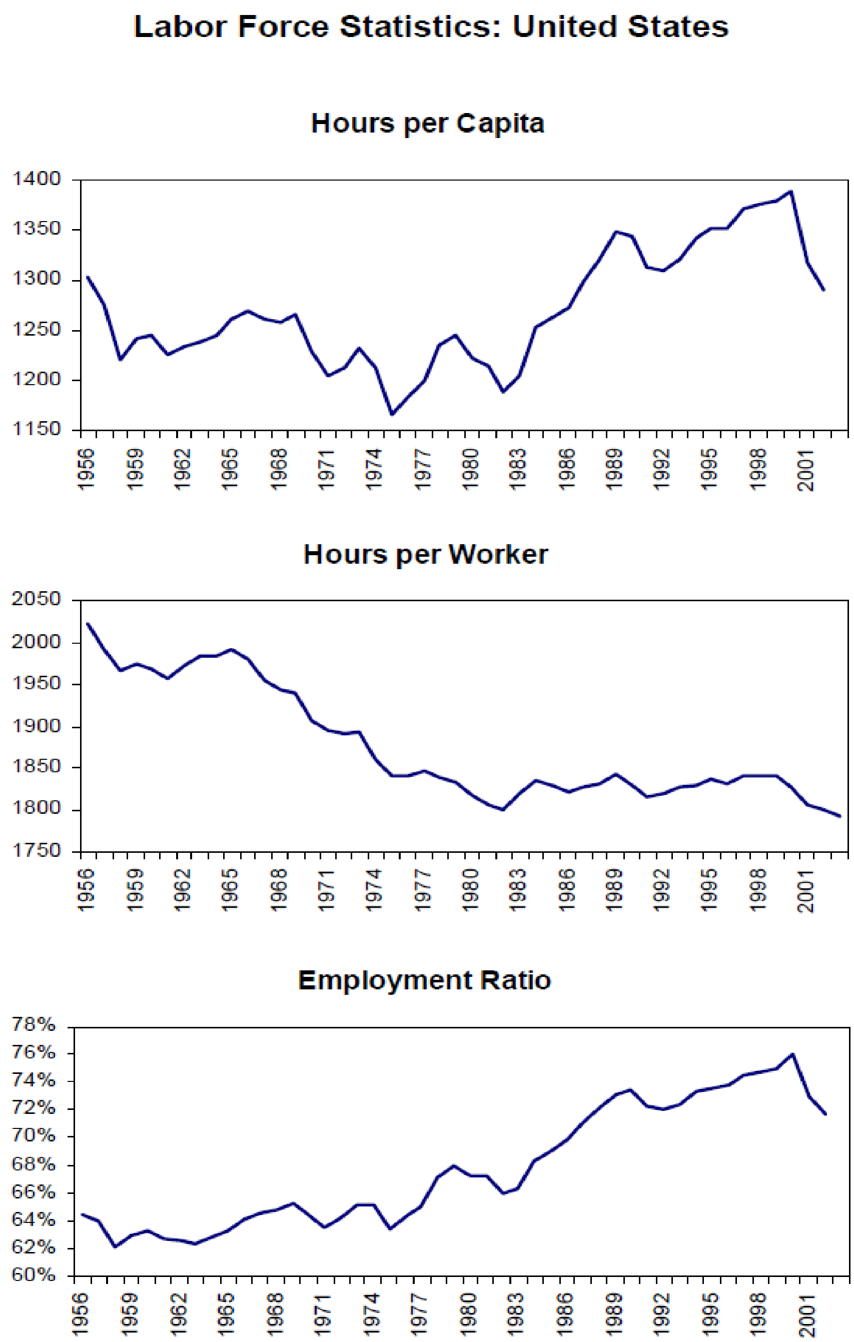

Figure 4: Participation Rates in the US. Source: Gali (2005)

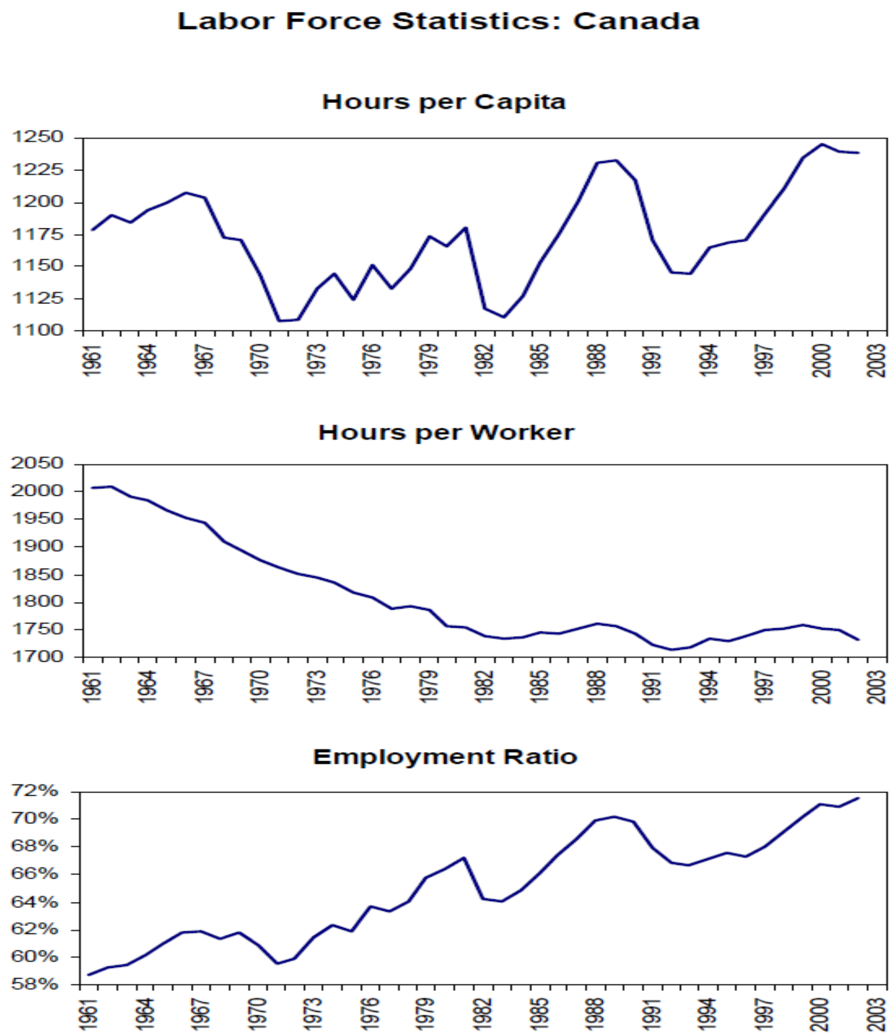

Figure 5: Participation Rates in Canada. Source: Gali (2005)

One of the most extensive surveys is that of Mosisa and Hopple (2006). In this paper there is an extensive analysis of the trends in labor force participation rates in the United States. The data of the paper extend from 1950 in some cases and mainly come from Current Population Survey, a monthly sample survey of 60.000 households. The labor force participation rate is defined as the proportion of the working age population whether working or actively looking for work. More specifically in US the working age population consists of the civilian, non-institutional population aged 16-65 who reside in the 50 states. The groups exclude inmates of institutions (i.e. mental or penal institutions and homes for the aged) and people active on active duty in Armed Forces.

The overall trends, according to Mosisa and Hopple, could be summarized as follows: After rising fairly steadily for more than five decades, the labor force participation rate in the US has peaked a 67,1% in the later 1990's and has started decreasing ever since. The overall behavior can be decomposed in various demographic groups, each of which manifests a rather different behavior originating from a number of social factors. Before decomposing the overall change a significant factor that has to be mentioned is the appearance of the "baby-boom" generation, - those born between 1946-1964 - which has moved across various age-cohorts thus influencing the observed trend in labor force participation rate all these years.

More analytically, the age-groups that exhibited the greatest variation, or actually the greatest decline in participation rate are the groups of teenagers, 16-19, and the group of young adults, 20-24. The main parameter affecting the participation rates of this group is participation in schooling. As the economy develops not only are there established more firm laws concerning the mandatory education of children and adults, but also the returns from schooling are more and more increasing, especially in the presence of immigration, which enforces the competition in the search of a job (as both categories, teens and immigrants are both unskilled workers). The decline in teenager labor force participation may as well be attributed to other social factors, such as rising family incomes or personal choice. Despite the fact that these two age cohorts manifest the same behavior, it is a fact that changes are sharper for teenagers than for young adults, as the participation in schooling is greater for the first group. Additionally there is not much differentiation between men and women in this group.

What one notices is that one of the most striking labor market developments of the post WWII period is the dramatic rise in the participation rate of women, which has been the driving force behind the rise observed in the overall participation rate during the latter half of the 20th century.

This rise however has stopped and after 2000 there has been observed a slight decline. This overall trend is a fact for all women, mothers or not and there is some variation in the participation rates of women depending on their level of education, with more educated women participating more actively in the labor force. However during the decline more educated women leaving the labor force were more than less educated women.

Finally concerning men, one notices that there has been a steady decline in the men participation rates, dropping by an average of 0.8% since 1990. After that date the decrease was even larger. Of course a variety of reasons could be invoked to explain this trend. Among them the most significant is Social Security benefits, the more developed they are the more easily one leaves the labor force. However in men participation, aged 55 and over there has been observed an increase in participation rates and this may be attributed to an improvement in their health status or an increasing cost in health benefits, which makes working a more advantageous choice.

Another extensive analysis is that of Kapsos (2007). The paper aims at estimating the economic activity rates as well as the size of the economically active population for various age cohorts, for the two sexes and for various regions around the world for the time period between 1980-2005. To conduct his analysis he uses the Key Indicators of the Labor market (KILM) database. As the gathering of this kind of data is a rather complicated issue he initially sets some criteria which define the non-comparability among various sources of labor force participation data, such as survey types, standard age groupings, definitions of economically active population, institutional differences among countries that affect labor force participation, etc.

Overall Kapsos uses the following definition for working force: Working age population is considered to be the population aged 15-65. Of course in some countries, as in Greece for instance, this may be illegal, unless the adolescent has fulfilled the obligatory education while in other countries, most notably developing ones, even younger children actively participate in the working force. What is evident form this definition is that the issue of labour force and its definition as well as its measurement is a rather complicated issue that must be carefully handled.

The results of this paper are many and significant, as they approach a variety of issues. Firstly he notes that countries with low per capita income (or GDP growth rate) tend to exhibit high participation rates, and actually higher than average. This fact takes places especially among young people, the elderly (independently of their sex) and women. This can be attributed to the fact that for most poor people the only asset they can live on is their labor. As an economy grows, an income effect takes place and people start working less hours, especially the groups mentioned above. Young people go to school, hence schooling is positively affected by growth (or the inverse causality) and women can exit the labor market during maternity, while the elder live in a country which may provide them social security and can leave their jobs when old.

-In 2005 there has been a more than 35% increase in global labor force compared to 1980 mainly attributed to population growth.

-The 1990 increase in global activity rates compared to 2008 has stopped in 2008. The initial rise was due to an increase in female participation rates while the fall was due to a decrease in youth participation rates.

-The female participation rates have increased up to 4% from 1980 to 2005 and women comprised 40% of total labor force.

-The male participation rate has fallen by 1.2% for the same period. The youth participation rates have rapidly declined by 9% approximately for the same period.

The above observations are drawn from global data. Naturally there are large variations among regions that make the income effect more evident. Especially vulnerable to regions are the data related to women and youth. The poorer a country the more insignificant are the variations in female and youth participation rates.

The above mentioned literature can be summarized in various observations among which the most important is that participation rates are far from being characterized as monotonous. They may manifest a U-shaped path, they may be increasing or decreasing, in all cases though they vary along the transition. This is the fact this chapter aims to account for. In looking for the sources of this non-monotonicity I will follow the analysis of Goldin, i.e. I will attempt to explore the role played by the income and the substitution effect. As has become evident may factors affect participation rates which cannot all be incorporated in a simple growth model, however some of this factors, such as schooling, are also endogenous and therefore subject to income and substitution effects themselves. Therefore a simple growth model may not account for all the factors affecting labor market, it can though provide a clear intuition about the driving forces behind it.

3.2.2 Labor-Leisure Choice

The purpose of this part is to analyze the models that endogenize labor-leisure choice. This kind of analysis is dominant in the literature and can be divided in two sub-categories. The first one comprises models that examine the long-run behavior of labor supply, while the second comprises models that examine labor supply in life-cycle models. In both cases however the focus is on leisure instead of participation rates. Much as important this analysis may be it fails to account for the patterns observed in the literature. In this chapter I will focus on the literature that examines long-run labor supply and leisure choices, as the issue of life cycle decisions is beyond the scope of this analysis.

3.2.2.1 Long-Run Labor Supply

This literature could hardly be characterized as extensive. Significant efforts have been made but the assumptions are very restrictive and actually aim at interpreting the decrease in working hours. However as has already been pointed out at the paper of Gali, this may not always be the case. One of the first papers in this literature is that of Davis (1969). Davis does not make a clear distinction between labor hours and labor force participation rate. But as he directly relates labor supply with per-capita income, and he assumes that increases in labor supply coincide with increases in population growth, it could be assumed that he refers to participation rates. Davis is trying to introduce labor supply in the Solow model. What he assumes is that labor supply grows at the same rate with population, i.e. everyone in the economy works. Then he renders the labor growth rate positively dependent on per capita income, i.e. when income grows then labor supply and population grow as well. What he proves is that with the introduction of this specification he obtains a modified golden rule instead of the one in the standard Solow model and more specifically the optimal savings ratio is less than the Golden Rule case. However he does not obtain any results related to the dynamic behavior of labor supply dynamics, which is not surprising since the labor force and the population are identical. What he suggests is that it would be more appropriate to distinguish labor force from population.

This extension has been undertaken by Samuelson (1980). Samuelson distinguishes between labor supply and population and as he relates the labor supply with the wage it is natural to assume that he refers to labor supply in terms of working hours instead of participation rates. As was the case with Davis, Samuelson obtains results that are interesting in terms of the Solow model, i.e. he obtains a modified golden rule. As Samuelson states, the result of varying participation rates is a simple theory of optimal steady-state savings and employment, where interest rates affect participation rates and consumption. However it is obvious that the paper examines the behavior of the participation rate only in terms of these variables and not in the long-run. In concluding the paper, Samuelson points out the need for a theory that will examine economies not in steady state, where the response of the participation rate to variations in the wage rate will be examined.

Hahn (1990) examines working hours and as a matter of fact he considers the intertemporal fall in hours to be a stylized fact. He attempts to give an explanation for the fact that historically, working hours seem to have decreased. He tries to explain this fact by introducing leisure in a Ramsey model. In his model he introduces exogenous technical progress. All members of the household work, as is usually the assumption made in these models, and their choice lies in how many hours they choose to work. His main finding is that the growth rate of labor supply varies in the steady state and under certain assumptions it may even become negative.

However the model of Hahn is not in accordance with the notion of steady state. What he does is that he mathematically solves for the steady state of his model. In all these models, time variables such as leisure and working time must be zero in the steady state otherwise they will eventually become either negative or greater than unity. What Hahn finds is an optimal path for the utility maximization, which however does not lead to a steady state. Had Hahn followed the same course of analysis as Barro and Sala-i-Martin did, he would have obtained the same results i.e. that labor supply rate is negative during the transition, when the production is Cobb-Douglas.

Eriksson (1996) has followed a similar analysis to that of Hahn. He examines various versions of standard growth models with endogenous labor supply, and aims at finding how the steady state growth rate is affected by preference parameters. In all the models he examines there is technical progress, either exogenous or endogenous. The models he examines are a) a standard Ramsey model with exogenous technological progress, b) a similar model with endogenous technological progress and finally c) a learning-by-doing model. What is interesting in terms of this chapter, is the Ramsey model analysis. What is characteristic in this literature and has already be mentioned in this chapter is Eriksson's mentioning that the theory of economic growth has not laid emphasis on the long-run behavior of labor supply. Actually the vast majority of the literature has dealt with the issue of fluctuations in employment instead of long-run movements. However, as Hahn (1991) points out, the working hours fall historically and thus this should be theoretically examined and explained. For this purpose the author introduces leisure in the standard neoclassical growth model as well as labor augmenting exogenous technological progress.

His main findings are that all the variables including labor supply grow in the steady state. Additionally if σ, the marginal utility of consumption, is greater than unity, which is supported by some empirical papers for some countries, then the labor supply rate in the steady state is negative (an the opposite holds). Also preferences affect the growth rate of the economy contrary to the standard model, but not the rate of time preference as in the endogenous growth model. What actually becomes evident in this model is the interaction between technological progress and labor supply. These results are viewed as important because they partially justify the fact that sudden variations in the growth rate of production or consumption could as well be attributed to preference parameter shifts. Additionally his results extend the analysis of the topic beyond business cycles which are dominant in this literature (Rebello, 1991).

The problem with his approach though is that it does not examine the various variables in per capita terms or in effective units. As expected, variables grow in the steady state but this is not a significant result since they grow because they are not in per capita terms. The same is true for leisure along with the fact that exogenous technological progress is the reason why he obtains a positive growth rate in labor supply in the steady state. After all this is the reason why he obtains different results than that of Barro and Sala-i-Martin, who use a CD production function but no technical progress and get a zero labor supply rate in the steady state. Finally he takes as a stylized fact that working hours decline. There is actually such evidence for some countries, however in his model he could also prove that working hours increase (when $\theta<1$) a case for which he does not provide any intuition especially when taking into account that $\theta < 1$ which is not an implausible assumption.

Solow in his book, "Growth Theory: An exposition" analyzes the topic of labor supply and leisure in the context of the Lucas model and comments on the analysis of Hahn. He does not do so in order to find evidence concerning the behavior of labor supply in the long run; he is actually interested in the different results that occur when he inserts leisure in the model. A first important point is that all time variables in the model such as working time and leisure have zero growth rates in the steady state because otherwise they will eventually exceed unity or become negative. Specifically he says that they could be negative but he finds that strange, especially the fact that people would spend all their time studying. An equally strange issue concerning the case with negative growth rates, would be that if working time becomes zero then there would be no production and eventually the economy would collapse. For this reason he examines the case where u and l are constant in the steady state thus their growth rate is equal to zero. This case, i.e. a negative growth rate for l, is found, as stated above in Hahn. As Solow mentions, this may be some path that maximizes the utility integral but this path does not lead to steady state at all. In this path u and l may have all kinds of different behaviors. One could even say that despite the fact that this is a mathematically feasible solution, it does not have an economic meaning.

Galor (1996) attempts to explain variations in female labor supply when compared to that of men. For this purpose he introduces in his model "a couple" comprised by a man and a woman who are endowed with the same amount of mental ability. However men are also endowed with physical strength. As Galor assumes, as the economy develops, mental ability is becoming more productive and for this reason, women leave home production and child breeding in order to enter the labor market. This according to Galor is what explains the increasing labor supply of women. This model though is a fertility model that does not explain the U-shaped path of female labor supply mentioned in Goldin (1994) and Durand (1975). In order to account for this effect he suggests two variations of his model. The first is a combination of his model for decreased fertility at high-income levels with a model for increasing fertility at low-income levels. Such a combination would be consistent with the U-shaped pattern. However a critique would be that fertility choice is endogenous hence it cannot actually explain variations of labor or if they are interdependent. The second model would be a model where a second technology for female production exists which is not fully rival to raising children. Such an assumption would be consistent with the point Goldin made, namely that during early industrialization, married women were engaged in work done at home, however the progressive separation of home and work and the creation of new (white-collar) jobs made it feasible for them to enter the paid workforce.

Ortigueira (2000) studies labor-leisure choices in a dynamic context. The aim of the paper is to analyze an endogenous growth model with leisure where human capital accumulation acts as the engine propelling economic activity. Agents in the model derive utility from consumption and leisure where leisure is defined as the amount of time devoted to those activities augmented by the level of education. The topic of the paper in not directly related to this chapter however it contains some important elements.

More analytically he establishes a positive relationship between the long-run growth rate and schooling. Additionally he makes clear the income and substitution effect concerning leisure. He states that the income and the substitution effect play a significant role in the dynamic behavior of leisure. On the one hand an increase in physical capital raises the wage rate paid in the output sector. Then there exist incentives to spend more time working in this sector. As the wage rate is the price of the leisure time, the representative consumer finds it more expensive to stay out of work. On the other hand the increase in physical capital reduces its relative price making less attractive its production (r decreases) and thus leading to a decrease in working time (substitution effect). Overall in his setting the weight of leisure in utility does not affect the long-run rate of growth. However, his results on transitional dynamics show that this parameter plays a major role in the determination of the long-run consequences derived from temporary endowment shocks.

Ngai and Pissarides (2006) study the pattern of hours as they are distributed among the market and the home, as well as in the three sectors of the economy, namely agriculture, manufacturing and services. They set up a model which provides for all these sectors and kinds of production and after solving it they calibrate it to see if they can reproduce the data observed in empirical research. As far as calibration is concerned, they can rationalize the observed falling or U-shaped pattern for aggregate hours, the complete marketization of agriculture and manufacturing. They also find that even though the economy is in a balanced growth path, or in a steady state where the aggregate hours are constant, since the growth rate of time variables is zero, there is a shift of hours from agriculture to services without violating balanced aggregate growth. According to their explanations, this hump-shaped path, is found for hours of work spent on home-produced services.

In terms of modeling, one of their main contributions, is that while other models can yield a fall in market hours during economic growth, due to a rise in returns to education or a rise in demand for leisure, their model achieves to explain the turning point in market hours. In solving their model and interpreting their results they make the following basic assumptions: a) Market goods are poor substitutes to each other but home-produced goods have close substitutes in the market, b) Agriculture and industry have higher rates of Total Factor Productivity (TFP) growth than do services, but within each sector market production has higher TFP growth than home production. This model follows an intuitive approach towards this issue, however it does not give any evidence concerning participation rates.

If one abstracts from continuous time models then different results may occur. de Hek (1998) introduces labor-leisure choice into a discrete time model and he finds that the dynamics that may occur depend on the interaction between consumption and leisure. If they are substitutes then multiple steady states may occur in the model, one low-level (poverty trap) and one high level, while when they are complements, monotone behavior towards the steady state may be disturbed as the optimal path may turn out to be cyclical.

3.2.3 Indivisibility of Labor

One of the main assumptions of this paper is the indivisibility of labor. More specifically I make the assumption that if an agent supplies the labor unit he possesses then he supplies the whole of it. This is a rather plausible assumption especially if one considers the fact that only a very small percentage of workers has an actual choice concerning its working hours. In most workplaces there is a specific amount of hours one is obliged to work. In some cases there is the possibility of working beyond the minimum horaire and get overpaid, however in most jobs this is not the case and it is not definitely what the literature of leisure has in mind when referring to labor-leisure choices. This plausible assumption is widely used in a strand of the literature but the most characteristic papers in this literature are the appears of Hansen (1985) and Rogerson (1988).

Hansen (1985) does not actually aim in studying the dynamic behavior of labor supply. His aim is to find a plausible explanation for variations in working hours, taking for granted that labor is indivisible. For this purpose he uses a growth model with technology shocks for the post-war US economy. A main assumption of this model is that these technology shocks force workers to enter and exit the labor market instead of increasing or decreasing working hours which is the usual assumption. His main finding is that the economy experiences large fluctuations in aggregate working hours. This result can be attributed to fluctuations in the total number of workers employed and not in variation in hours worked by each worker. Additionally the elasticity of intertemporal substitution does not coincide with these fluctuations as it is not the same as the individual elasticity of factor substitution. This is after all the main difference with respect to other models with divisible labor. This assumption is crucial in order to obtain the fluctuations observed in the US economy, an issue in which the standard RBC literature has failed.

Rogerson (1988) examines a general equilibrium model in which not all agents are employed at every instant. As he clearly states, the non-convexities he introduces in the model have an important effect on the nature of the equilibrium. More specifically there are major implications for the aggregate response of the economy to shocks, and the economy will display much larger fluctuations in hours of work in response to a given shock in technology. In terms of modeling he assumes that labor is indivisible, i.e. each person supplies either one unit of labor per period, or no labor at all. Whether a person will work next period or not depends on a lottery. In this way, Rogerson introduces unemployment in the model. What is important in this model is that it can account for fluctuations in total labor supply. As he states, few other papers have attempted to study fluctuations, for example Altonji and Ashenfelter (1980) and Kydland and Prescott (1982). However their estimates of labor supply using micro data are much smaller that required reconciling aggregate fluctuations with equilibrium theory. In this paper, non-convexities are of substantial interest for this problem. While in the above-mentioned papers movements in aggregate hours are small relative to movements in real wages, Hansen (1985) who was the first to introduce indivisible labor delivered too much movement in aggregate hours. To calculate the aggregate hours he "multiplies" the population by the block of hours worked by a household and thus he calculates the total number of hours worked. According to Hansen the fact that all agents are simultaneously indifferent between working and not working is what causes the large response in employment relative to productivity. Another way to reproduce similar results in the presence of indivisibilities is to introduce heterogeneous agents. Here there are no lotteries, and their decision to work or not is based on the wage rate and to the degree of heterogeneity. As a whole the paper gives evidence that supports the large fluctuations observed in working hours, but does not extensively analyze the intertemporal variations in participation rates.

This kind of heterogeneity is analyzed by Sorger (2000) and Turnovsky and Penalosa (2006). They both come up with the result that the labor dynamics can be rather complex, however they do not conduct their analysis within the indivisibility context, but they instead examine labor-leisure choices. The issue of heterogeneity with indivisible labor and its effect on dynamic labor supply has not been studied in the literature.

3.2.4 Abstraction from Separable Utility Function

Separable utility functions have been extensively used in the literature, mainly due to their analytical plausibility and the intuitive and clear results they usually produce. In this chapter non-separability, as well as non-unitary elasticity of factor substitution, are crucial assumptions for obtaining non-monotonous dynamics. As I have already analyzed in the first chapter, abstraction from the Cobb-Douglas hypothesis is a plausible assumption. The same goes for the intertemporal elasticity of substitution, since there seems to be much evidence in favor of the non-separable utility functions. These studies either study directly the degree of substitutability or calculate it in a general context. The findings are various and in some cases conflicting. There is not always an agreement on whether the values of the intertemporal elasticity are greater or lower than unity, however what is important is that they support the non-separable utility hypothesis.

Hall (1988) studies the degree of substitutability in a model where consumption is affected by expectations on real interest rates. In all his estimates, in which he uses different databases for postwar US data, the obtained values for intertemporal elasticity are below unity, they do not exceed 0.2 and may well be zero.

Attanasio and Weber (1993) estimate intertemporal elasticity of substitution for aggregate US data and find not only that it is lower than unity but also that it is lower than the one for average cohort data.

Laitner and Silverman (2005) estimated the substitutability between leisure and consumption and find that the drop in consumption observed when agents retire, is not consistent with the separable preferences assumption. Jacobs (2005) cannot as well reject the assumption of non-separability when estimating the various parameters in the utility function using investment data. Furthermore, a strand of the business cycles literature often uses the estimates of Hansen and Singleton (1983), who find a range of point estimates between 0 and 2.

3.3 The Ramsey Model with Endogenous Labor Supply-The Standard Model

In this part of the chapter I will extensively analyze the model presented by Barro and Sala-i-Martin (2004) for the case of endogenous labor supply. As I have already analyzed in an earlier part of the chapter, this kind of analysis is dominant in the endogenous labor supply literature and actually aims at interpreting the decline in working hours observed intertemporally. However if one takes into account the number of hours per worker, including the increase in hours caused by new-coming working force such as women, or the decrease in hours caused by younger people who spend more time on schooling, then it may become evident that the pattern of working hours is not really monotonous, let alone decreasing. Such an argument is also supported in Ngai&Pissarides (2006) who mention that an increase in the labor force participation rates of women has increased overall hours supplied in the United States. This is not always the case of course since in Europe the hours per employee continued to decrease despite the increase in female labor force participation.

I firmly believe though that what should be first interpreted is the behavior of the labor force participation rate intertemporally. The change in working hours is an issue that is directly related to the change in labor hours. In this chapter I intend to generalize this version of the Barro and Sala-i-Martin (henceforth B&S-i-M) to the case of a general production function and a general utility functional form. The analysis bears little resemblance to that of Barro and Sala-i-Martin especially when one departs form the C-D and the separable utility case. Starting from the standard case, I will set the basis for further analysis of the model.

3.3.1 Households

The households in the Ramsey economy provide labor in exchange for wages, receive interest income on assets, purchase goods for consumption and save by accumulating additional assets. Each generation contains one or more adults, and adults are the working members of the current generation. Adults in this model are considered to be altruists in the sense that when making plans they take into account the welfare of their descendants either actual or prospective. In order to introduce this behavior in the model it is assumed that the current generation maximizes utility and incorporates a budget constraint over an infinite horizon. In this way there is an immortal extended family despite the fact that individuals have finite lives. Each parent provides transfers to his children and this process take place infinitely, thus the process ends up to an immortal family connected with altruistic bonds.

The size of the family grows at the rate n because of the net influences in fertility and mortality. We make the assumption that n is exogenous and constant. If the number of adults is normalized at time 0 at unity, then the family size that corresponds to the adult population, at time t is:

$$N(t) = N_o e^{nt} \text{ where } N_0 = 1 \tag{3.1}$$

The difference with the Ramsey model with exogenous labor supply is that now population N is distinguished from labor input $L(t)$ which varies endogenously for given $N(t)$. If one defines $l(t)$ as the typical person's intensity of work effort at time t, as B&S-i-M do then:

$$L(t) = l(t)N(t) \tag{3.2}$$

A significant point made by B&S-i-M is that $l(t)$ may denote either time spent working or work effort or labor force participation rates. The first and the latter definition imply an upper bound while the second does not actually have a bound and more importantly cannot be readily measured. For this reason the authors choose one of the two definitions that impose a bound on $l(t)$.

Each household seeks to maximize overall utility, U, where:

$$U = \int_0^\infty u(c_t, l_t) \exp\{-(\rho - n)t\} dt, \qquad \rho,\ \theta > 0, \tag{3.3}$$

where l_t has been defined above, c_t denotes consumption at time t, ρ is the rate of time preference and n is the population growth rate. This formulation assumes that the household's utility at time 0 is a weighted sum of all future flows of utility. We make the assumption that

$u(c_t, l_t)$ is concave as we usually assume. Additionally $U_l < 0, U_{ll} < 0$.

Households hold assets in the form of ownership claims on capital or as loans. Negative loans represent debts. Since a closed economy is assumed, no assets can be traded internationally. Households are allowed to lend to and borrow from other households, but in the end of the time period, in equilibrium, the representative household will end up holding zero net loans. The two forms of assets, capital and loans, are considered to be perfect substitutes as stores of value, thus they must pay the same real rate of return, r_t. The household's net assets per person are denoted by α_t and they are measured in real terms, that is in units of consumables.

Households exist in a competitive environment in the sense that they take as given the interest rate r_t as well as the wage rate w_t, paid per unit of labor services. The wage income per household equals w_t. Total income per capita received by a household is the sum of labor income and financial or interest income, $r_t\alpha_t$

The flow budget constraint for the household is:

$$\dot{\alpha} = w_t l_t + r_t\alpha_t - c_t - n\alpha_t \tag{3.4}$$

More analytically this equation mentions that assets per person rise with per capita income, $w_t l_t + r_t\alpha_t$, fall with per capita consumption, c_t, and fall because of expansion of the population in accordance with the term $n\alpha_t$.

Another constraint that has to be taken into consideration is:

$$\lim_{t\to\infty} \alpha_t e^{-\int_0^t (r(v)-n)dv} \geq 0 \tag{3.5}$$

This constraint on the amount of borrowing is imposed by the credit market in order to rule out the case of Ponzi game or chain letter, and its meaning is that in the long run a household's debt per person cannot grow as fast as $r_t - n$, so that the level of debt cannot grow as fast as r_t.

3.3.2 Firms

Firms produce goods, pay wages for labor input and make rental payments for capital input. Each firm has access to the production technology:

$$Y = F(K, L) \tag{3.6}$$

where Y is the flow of output, K is capital input in units of commodities, $L = l_t N_t A_t$ is the effective amount of labor input and A is the level of technology which grows at the constant rate χ. Note that in this model, all agents in the economy work but they not supply all of their labor. Hence labor supply is given by the number of hours worked, $l_t N_t$. Y exhibits constant returns to scale in K and L and each input exhibits positive and diminishing marginal product. In per capita effective units the production function can be written as:

$$\hat{y} = f(\hat{k}_t) \tag{3.7}$$

where y and k are expressed per unit of effective labor, i.e. $\hat{y} = \frac{Y}{\hat{L}}$ and $\hat{k} = \frac{K}{\hat{L}}$.

The marginal products of the factors are given by:

$$\frac{\partial Y}{\partial K} = f'(\hat{k}_t) \tag{3.8}$$

$$\frac{\partial Y}{\partial L} = f(\hat{k}_t) - kf'(\hat{k}_t)$$

Firms rent the services of capital from the households that own the capital. Hence the firms' costs for capital are the rental payments which are proportional to K. Suppose R is the rental price for a unit of capital services as well as that capital depreciates at the constant rate. The net rate of return to a household that owns a unit of capital is then $R - \delta$. Furthermore households also receive the interest rate r. Equivalently one can write the following equation: $R = r + \delta$.

The representative firm seeks to maximize the following profit function at any point in time:

$$\Pi = F(K, L) - (r + \delta)K - wL \tag{3.9}$$

or in per capita terms:

$$\Pi = L(f(\hat{k}_t) - (r + \delta)\hat{k}_t - w_t e^{-\chi t}) \tag{3.10}$$

Since firms are competitive, they maximize their profits by setting:

$$f'(\hat{k}_t) = r + \delta \tag{3.11}$$

This relationship actually indicates that the firm chooses the ratio of capital to effective labor to equate the marginal product of capital to the rental price. Furthermore in a full market equilibrium, w_t must be such that profit equals 0. Thus w_t has to satisfy the following relationship:

$$w_t = (f(\hat{k}_t) - kf'(\hat{k}_t))e^{-\chi t} \tag{3.12}$$

which implies that factor prices should be equated with marginal products for the market to clear.

3.3.3 Equilibrium

To solve the maximization problem imposed by eq. (3.3), (3.4), and (3.5), I formulate the Hamiltonian and maximize with respect to c_t, l_t and k_t. From the F.O.C's the following equations are obtained:

$$v = u'(c_t)e^{-(\rho-n)t} \tag{3.13}$$

$$u_l e^{-(\rho-n)t} = -vw_t \tag{3.14}$$

This new first order condition reflects the relationship between consumption and leisure at a point in time.

$$\dot{v} = -v(r-n) \tag{3.15}$$

where v is the present-value shadow price of income. It represents the value of an increment of income received at time t in units of utils at time 0.

Equation (3.15) is known as the Euler equation or the Ramsey rule of optimal saving. If I take logarithms of the variable in equation (3.14) and differentiate with respect to time, and then substitute for v then I get the basic condition for choosing consumption over time:

$$r = \rho - \frac{u_{cc}c}{u_c}\frac{\dot{c}}{c} - \frac{u_{cl}l}{u_c}\frac{\dot{l}}{l} \tag{3.16}$$

According to equation (3.16), households choose consumption so as to equate the rate of return, r, to the rate of time preference, ρ, plus the rate of decrease of the marginal utility of consumption, plus the rate of change of marginal utility of labor supply. If $u_{cl} > 0$. i.e. if consumption and labor supply are complements then a higher value of $\frac{\dot{l}}{l}$ subtracts from the rate of time preference, which means that households prefer to consume a lot in the future when l will be high. If $u_{cl} < 0$. i.e. if consumption and labor supply are substitutes, then this effect is reversed.

B& S-i-M use the following utility function:

$$u(c, l) = \log(c) + \omega(l)$$

where $\omega(l) = -\zeta l^{1+\sigma}, \zeta > 0$ and $\sigma \geq 0$.

This is a separable utility function, in other words $u_{cl} = 0$ hereby eq.(3.16) reduces to

$$r = \rho - \frac{u_{cc} c}{u_c} \frac{\dot{c}}{c} \tag{3.17}$$

which is the standard Euler equation.

Up to this point they have been working with per capita variables c and k. The next step is to introduce the variable per unit of effective labor to include the effect from variable labor supply l, that is,

$$\hat{k} = \frac{K}{lNe^{xt}} \tag{3.18}$$

$$\hat{c} = \frac{C}{lNe^{xt}} \tag{3.19}$$

From (3.19), (3.18), the market clearing condition $r = f'(k) - \delta$ and the steady state condition $\alpha = k$ they obtain:

$$\gamma_{\hat{c}} = (f'(\hat{k}_t) - \delta - \rho - \chi) - \gamma_l \tag{3.20}$$

$$\gamma_{\hat{k}} = \frac{f(\hat{k}_t)}{\hat{k}_t} - \frac{\hat{c}_t}{\hat{k}_t} - (n + \delta + \chi) - \gamma_l \tag{3.21}$$

It is obvious that these dynamic equations differ from the ones in the exogenous labor supply model only due to the growth rate of labor supply. As it is known, in the steady state all the time variables have zero growth rate, hence the steady state values for $\hat{k}_t$ and $\hat{c}_t$ are the same with the standard model. What needs to be done is to find the dynamic equation for the growth rate of l_t and its steady state value. To do this they define

$$\omega(l_t) = -\zeta l^{1+\sigma} \qquad \zeta > 0, \sigma > 0$$

Dividing eq.(3.13) by eq. (3.14) gives the following equation:

$$\zeta(1+\sigma)l_t^{1+\sigma} = (1-\alpha)A\frac{\hat{k}_t^{\alpha}}{\hat{c}_t} \tag{3.22}$$

and using a Cobb-Douglas production function and the above utility function for $\omega(l_t)$ they obtain:

$$\gamma_l = \frac{\alpha}{1+\sigma}\gamma_{\hat{k}} - \frac{1}{1+\sigma}\gamma_{\hat{c}} \tag{3.23}$$

which give the growth rate of labor supply.

Using the dynamic equations for $\hat{k}_t, \hat{c}_t$ and l_t they formulate the dynamic system:

$$\gamma_{\hat{k}} = A\hat{k}_t^{\alpha-1} - \frac{1}{\alpha+\sigma}\left[\sigma\frac{\hat{c}_t}{\hat{k}_t} + (1+\sigma)(\chi+\delta) + \rho + \sigma n\right]$$

$$\gamma_{\hat{c}} = \alpha A\hat{k}_t^{\alpha-1} + \frac{1}{\alpha+\sigma}\left[\alpha\frac{\hat{c}_t}{\hat{k}_t} - (1+\sigma)(\chi+\delta) - (1+\alpha+\sigma)\rho + \alpha n\right]$$

The steady state values of $\hat{k}_t, \hat{c}_t$ are the same as in the standard model with exogenous labor supply, while the l_t steady state value is given by :

$$l^* = \left[\left(\frac{1-\alpha}{\zeta(1+\sigma)}\right)\left(\frac{\rho+\chi+\delta}{\rho+\chi+\delta-\alpha(\chi+\delta+n)}\right)\right]^{\frac{1}{1+\sigma}}$$

The system is saddle-path stable and the phase diagram is the same as in the standard Ramsey model.

The next important step, especially with respect to this chapter is how they figure the intertemporal behavior of labor supply. The dynamic equation for γ_l is given by:

$$
\begin{aligned}
\left(\frac{1}{1+\frac{\alpha}{1+\sigma}-\frac{1}{1+\sigma}}\right)\gamma_l &= -\frac{\alpha}{1+\sigma}(\chi+\delta+n)-\frac{\alpha}{1+\sigma}\frac{\hat{c}_t}{\hat{k}_t}+\frac{1}{1+\sigma}(\rho+\chi+\delta) \Rightarrow \\
(\frac{\sigma+\alpha}{1+\sigma})\gamma_l &= -\frac{\alpha}{1+\sigma}(\chi+\delta+n)-\frac{\alpha}{1+\sigma}X+\frac{1}{1+\sigma}(\rho+\chi+\delta) \Rightarrow \\
\gamma_l &= -\frac{\alpha}{\alpha+\sigma}(\chi+\delta+n)-\frac{\alpha}{\alpha+\sigma}X+\frac{1}{\alpha+\sigma}(\rho+\chi+\delta) \\
\gamma_l &= -\frac{\alpha}{\alpha+\sigma}(X-X^*)
\end{aligned}
$$

where $X = \frac{\hat{c}_t}{\hat{k}_t}$ and $X^* = \frac{\hat{c}^*}{\hat{k}^*} = \frac{\rho+\chi+\delta}{\alpha} - (\chi+\delta+n)$. B&S-i-M have proved that X falls monotonically and hence $X > X^* \Rightarrow \gamma_l \forall \hat{k}_t$. This result implies that l_t falls monotonically during the transition, which means that poor people work more hours than rich people.

Their analysis includes not only the decreasing pattern of labor supply, which they consider as plausible due to the empirical observation that work effort declines during early stages of development, but also the behavior of the saving rate. Using eq. (3.22) they find that when $\theta = 1$, l_t moves at the same direction with the saving rate which also decreases monotonically during the transition. As they clearly state they consider this as a problem and their suggested solution is abstraction from the assumption that $\theta = 1$. Although I partially agree with their suggestion, since their is much evidence against $\theta = 1$, I have already argued in the first chapter of this thesis that the saving rate may not behave monotonically. This may hold here as well especially when l_t does not behave monotonically as well. And more importantly this result can be achieved even if $\theta = 1$, so long as we abstract from the Cobb-Douglas hypothesis.

Furthermore they state that if $\theta > 1$, then $U_{cl} > 0$ thus a decreasing l_t would imply an increasing saving rate. What they do not take into account though is that when one abstracts from the hypothesis that $\theta = 1$ then various patterns may arise and the behavior of l_t is no longer easily characterized

This kind of analysis will be the main topic of this chapter. My aim is to thoroughly analyze the behavior of labor supply for every possible value of θ and σ. To achieve this I must abstract from both cases of Cobb-Douglas and of the separable utility function. Arguments in favor of these alternative functional forms have already been analyzed in the literature review of this chapter.

3.4 The Ramsey Model with Endogenous Labor Force Participation-The General Case

3.4.1 Description

At this point, after having analyzed the behavior of labor supply intertemporally, in the context of a Ramsey model with Cobb-Douglas production and CES utility, I am going to use more general functional forms in an attempt to fully characterize the behavior of l.

Few points must be made at this point which are essential for the analysis that follows. The first point is how is l defined. In the context of this chapter I have already clearly defined that my purpose is not to interpret variations in per *worker* working hours observed in the data. This approach has been extensively analyzed in the literature, the analysis conducted is both empirical and theoretical and it is usually related to life-cycle models. What I am actually interested in, is the behavior of labor supply intertemporally or what I will henceforth call labor force participation rate (LFPR). Participation rates can either be expressed as employment rates (note though that this model cannot account for unemployment) or as per *capita* working hours. In the empirical literature both kinds of measurement have been used. In terms of this model if I assume that involuntary unemployment does not exist then both measures are equivalent. To be able to distinguish this interpretation from the interpretation given in other related papers, one has to seek the differences in various aspects of modelling.

A first aspect and perhaps the most important is to clarify how is participation rate introduced in this and other models, especially when long-run analysis is involved. A typical example is the Barro and Sala-i-Martin model. On the one hand, Barro and Sala-i-Martin claim that it is a matter of choice how one interprets l. For instance they claim that it can be the fraction of time spent working, or variations in work effort or expansions and contractions of labor-force participation. The difference among all the above interpretations lies in measurement. If l denotes time spent working, then it can be readily measured and it would have a natural upper bound of 100 percent. If it allows for variations in work effort then it cannot be measured as it is a subjective measure. Finally if it denotes labor force participation rates then it can as well be measured and have an upper bound in terms of the household and for this reason Barro and Sala-i-Martin do not distinguish between these two interpretations.

Is that correct? If we assume a zero population growth rate, $n = 0$, and normalize initial population to unity, then the two measures are directly comparable as they both have the same upper bound and can be readily measured. In this case the utility function takes the form $u(c_t, l_t)$ where c_t denotes per capita consumption and l_t denotes labor force participation rate of the household. As it is usually the case we assume one representative household in the economy with N members. The "problem" may occur though when the population growth rate is positive, $n > 0$. In this case utility may have the form $u(c_t, l_t)$, same as above, where the utility of each person in the household depends on its own consumption and on the relative number of members of the household working. To make this idea more clear, this formulation would imply that the utility is the same in a household were one member out of two works and in a household where

2 out of 4 members work. One could claim though that the appropriate formulation would be to have a utility function of the form $u(c_t, l_t N_t)$ where the utility of each member of the household depends on the average consumption as well as on the total number of members of the household working. Which of the two formulations is preferred is open to discussion. At the main body of the chapter I choose to analyze the case where there is zero population growth rate, were such an issue does not arise.

A second topic that needs further clarification concerning this approach is that of indivisible labor. Despite the fact that I use the notion of indivisibility of labor in my model, this does not imply that indivisibilities are introduced. The cause of this result lies in the fact that "unemployment " is introduced not across households but within households. This means that contrary to the Hansen model all households in my model have the same utility with the representative one. In Hansen not all households are identical since some households are employed and other unemployed.

A third topic is whether it is a plausible assumption to make that utility is decreasing in participation rates. In a context where working hours vary it is easier to accept that people derive joy from leisure and disutility from labor and thus choose how may hours of work maximize their utility. However when participation rate is introduced and agents work in blocks of hours it is harder to justify, at least in sociological terms, that agents derive disutility from their participation in workforce. However in the same sense agents in the standard model do not derive any joy from their jobs I will make the same assumptions leaving workaholism and job satisfaction related issues to a future analysis.

To further clarify the argument of this chapter, note that in all the above cases, contrary to the literature that examines variations in working hours, members of the household have one unit of labor and they either supply it all or they do not supply labor at all. In terms of utility function this could be represented by $u(c_t, tl_t)$ where c_t denotes per capita consumption, and tl_t denotes the time each member of the household works times the members of the household that actually work. The dominant part of the literature chooses to set l_t equal to unity, thereby implying that all household members actually go to work, and maximize with respect to t, i.e. they choose how much to work. What I choose to do is to set t equal to unity, thereby assuming that each person that works does supply all its available hours and maximizes with respect to l_t. This is not an unrealistic hypothesis as in real world few people have actually a choice about their working hours. Most people work in blocks of hours and what they usually choose is whether to work or not. Even when they have to make decisions concerning their working hours they still decide in terms of blocks, i.e. they choose whether to work part time or full time. Especially within a household which comprises heterogeneous agents, men, women, children or older people, the choice of which members of the household are going to work is often met in real world.

Of course in the context of a Ramsey model, where all agents are homogeneous such a distinction is not clear. However it is still possible to assume that the household has reasons for not wishing everyone to work. For the same reasons why one person may choose not to work all the time he has available, since his utility depends on leisure as well, if we examine household as a unity where agents with identical preferences derive utility collectively through the household, then the household may choose not all its members to work if this does not maximize its utility. It goes without saying that since in Ramsey all members are homogeneous it makes no difference which members are going to work and which will stay out of work. Such decisions are important in real world cases but these assumptions are beyond the scope and the potential of the model. It is easy though to come up with plausible interpretations. For instance when an economy develops, various age or sex groups enter or abandon working force, young people and female labor force among them. If I hypothetically assume that all members choose to work and supply all their labor, this would be equivalent in terms of utility to the case where a person chooses zero leisure and supplies all its labor. We know from standard textbooks that this is not utility maximizing in models with endogenous labor supply, unless certain assumptions are made. In a similar manner though I assume that utility maximizing households will not fully participate in workforce and it is not hard to imagine why families some decades ago were represented in the workforce only be their male counterparts.

Another restriction of the Ramsey model is that it does not distinguish between men and women, young or old. Therefore even though it can predict a U-shaped pattern which is empirically observed and attributed to female participation, such a clear distinction is not feasible in terms of modelling. However since the model attributes these patterns to the "competition" between income and substitution effects, which in turn depend on the marginal products of labor and capital and on parameter values, it would not be mistaken to conclude that in some cases the models fits with female participation, i.e. when the U-pattern occurs and the marginal product of labor is higher than is usually for men. Moreover since various patterns, monotonous or non may occur, therefore accounting for all cases of empirical evidence, the distinction between sexes is unimportant.

Finally one last topic I want to make clear is that involuntary unemployment is not examined in the model at least not in the sense that agents are not able to find jobs. The labor force participation rate is defined as the proportion of the working age population whether working or actively looking for work and each member of the household has the option to work if he wishes to. Of course unemployment is a very important topic however this is not what I want to focus on and a different context would be much more appropriate for such a study.

As this literature is rather limited, numerous issues must be further examined and analyzed. Additionally much more detailed analysis is required to fully characterize participation patterns. In this chapter I will make a first attempt to characterize the dynamic behavior of labor supply as driven by forces such as income and substitution effect. In doing so I will employ flexible utility and production assumptions in the context of the standard Ramsey model. I hope that what is forfeited due to the simplicity of the model is gained in intuition.

3.4.2 Households

The model used here is very similar to the one analyzed above. Therefore I will not extensively analyze the various aspects of the model. I will just set the basic equations and highlight the differences introduced via the use of general functional forms.

Agents maximize the following utility function:

$$\max \int_{o}^{\infty} e^{-\rho t} u(c_t, l_t) \tag{3.24}$$

where c_t denotes per capita consumption and l_t denotes labor force participation rate within the household. There is one representative household with N members and the growth rate of population, n equals zero, i.e. there is no population growth. $l_t N_t$ is the number of the members of the household that participate in the labor force.

A first important point to be made is related to the utility function. More analytically I will use the following functional form as proposed by King, Plosser and Rebello (1987) (henceforth KPR).

$$\begin{aligned} u(c_t, l_t) &= \frac{c_t^{1-\theta}}{1-\theta} v(l_t), \text{ when utility is multiplicatively separable} \qquad (3.25) \\ & \quad and \\ u(c_t, l_t) &= \log(c_t) + v(l_t), \text{ when utility is additively separable and } \theta = 1 \end{aligned}$$

The reason for using this utility function is analyzed in the KPR paper. Quoting them, the feasible steady state is compatible with an optimal competitive equilibrium, by imposing two restrictions on preferences: (i) the intertemporal elasticity of substitution in consumption must be invariant to the scale of consumption, and (ii) the income and substitution effects associated with sustained growth in labor productivity must not alter labor supply (in steady state).

The first restriction must hold because the marginal product of capital, which equals one plus the real interest rate in equilibrium, must be constant in the steady state. Consumption is growing at a constant rate and the ratio of discounted marginal utilities must equal one plus the interest rate, hence the intertemporal elasticity of substitution must be constant and independent of the level of consumption. The second restriction is necessary since in the steady state the hours worked (KPR define l as hours worked) are constant in the steady state. In case where there exists a growing marginal productivity of labor -induced by labor augmenting technical change- income and substitution effects of productivity growth must have exactly offsetting effects on labor supply (in steady state).

The properties of the above utility function are the following:

$$\begin{aligned} u_c &= c_t^{-\theta} v(l_t) > 0, \quad u_{cc} = -\theta c_t^{-\theta-1} v(l_t) < 0 \\ u_l &= \frac{c_t^{1-\theta}}{1-\theta} v'(l_t) < 0, \quad u_{ll} = \frac{c_t^{1-\theta}}{1-\theta} v''(l_t) < 0 \\ u_{cl} &= u_{lc} = c_t^{-\theta} v'(l_t) \lesseqgtr 0 \text{ when } \quad \theta \lesseqgtr 1 \end{aligned}$$

while $\theta > 1 \Rightarrow v'(l_t) > 0 \Rightarrow v''(l_t) > 0 \Rightarrow u_{cl} > 0$.

The period-by-period budget constraint for each household is

$$\frac{da_t}{dt} = w_t l_t + r_t a_t - c_t \tag{3.26}$$

where a denotes assets per household member and r and w are the competitive interest rate and wage, respectively.

The no-Ponzi game condition is given by:

$$\lim_{t\to\infty} \alpha_t e^{-\int_0^t r(v)dv} \geq 0 \qquad (3.27)$$

3.4.3 Firms

Firms produce goods, pay wages for labor input and make rental payments for capital input. Each firm has access to the production technology:

$$Y = F(K, \hat{L}) \qquad (3.28)$$

where Y is the flow of output, K is capital input in units of commodities, L_t is the amount of labor input. For expositional convenience I will assume that there is no technical progress, thus $\chi = 0$, where χ denotes rate of growth of technical progress and $A_0 = 1$, where A_0 denotes the initial level of technology. Y exhibits constant returns to scale in K and L and each input exhibits positive and diminishing marginal product. In per worker units the production function can be written as: $\hat{y} = f(\hat{k}_t)$ where y and k are expressed per unit of effective labor, i.e. $\hat{y} = \frac{Y}{\hat{L}} = \frac{Y}{lN}$ and $\hat{k} = \frac{K}{\hat{L}} = \frac{K}{lN}$, where l denotes the fraction of people of the household who actually participate in the working force. Note now that per capita variables are different from variables per worker, since not all people of the economy work. More analytically the per capita variables are given by: $y = \frac{Y}{N}$, $c = \frac{C}{N}$ and $k = \frac{K}{N}$.

The marginal products of the factors are given by:

$$\frac{\partial Y}{\partial K} = F'(K, lN) = F'(k, l) = f'(\hat{k}_t)$$

$$\frac{\partial Y}{\partial L} = f(\hat{k}_t) - kf'(\hat{k}_t)$$

Firms rent the services of capital from the households that own the capital. Hence the firms' costs for capital are the rental payments which are proportional to K. Suppose R is the rental price for a unit of capital services as well as that capital depreciates at the constant rate . The net rate of return to a household that owns a unit of capital is then $R - \delta$. Furthermore households also receive the interest rate r. Equivalently one can write the following equation: $R = r + \delta$.

The representative firm seeks to maximize the following profit function at any point in time:

$$\Pi = F(K, L) - (r + \delta)K - wL \tag{3.29}$$

or in per capita terms:

$$\Pi = L(f(\hat{k}_t) - (r + \delta)\hat{k}_t - w_t)$$

Since firms are competitive, they maximize their profits by setting:

$$f'(\hat{k}_t) = r + \delta \tag{3.30}$$

This relationship actually indicates that the firm chooses the ratio of capital to effective labor to equate the marginal product of capital to the rental price. Furthermore in a full market equilibrium, w_t must be such that profit equals 0. Thus w_t has to satisfy the following relationship:

$$w_t = f(\hat{k}_t) - kf'(\hat{k}_t)$$

which implies that factor prices should be equated with marginal products for the market to clear.

3.4.4 Equilibrium

3.4.4.1 Case A: $\theta \neq 1$

After maximizing eq. (3.24) with respect to eq. (3.26) and eq. (3.27) I obtain the following F.O.C's:

$$u_c e^{-\rho t} = \lambda \tag{3.31}$$

$$u_l e^{-\rho t} = -\lambda w_t \tag{3.32}$$

$$\dot{\lambda} = -r\lambda \tag{3.33}$$

where λ is the present-value shadow price of income. It represents the value of an increment of income received at time t in units of utils at time 0. Note that (3.32) reflects the relationship between consumption and leisure at a point in time, as was the case above.

Dividing eq.(3.31) by eq. (3.32) gives the following equation:

$$\frac{U_c}{U_l} = -\frac{1}{w} \Rightarrow \frac{U_c}{U_l} = -\frac{1}{f(\hat{k}) - \hat{k}f'(\hat{k})} \Rightarrow \frac{U_c}{-U_l} = \frac{1}{f(\hat{k}) - \hat{k}f'(\hat{k})}$$

From the above equation I can obtain a relationship between $\hat{c}$ and l which will prove useful for future analysis:

Setting $\theta \neq 1$ I obtain

$$\begin{aligned} \frac{U_c}{U_l} &= -\frac{1}{w} \Rightarrow \frac{c^{-\theta}v(l)}{\frac{c^{1-\theta}}{1-\theta}v'(l)} = -\frac{1}{f(\hat{k}) - \hat{k}f'(\hat{k})} \Rightarrow \\ c &= -(1-\theta)(f(\hat{k}) - \hat{k}f'(\hat{k}))\frac{v(l)}{v'(l)} \Rightarrow \hat{c} = -(1-\theta)(f(\hat{k}) - \hat{k}f'(\hat{k}))\frac{v(l)}{v'(l)l} \end{aligned} \tag{3.34}$$

which comes from $\hat{c} = \frac{c}{l}$

Setting $\theta = 1$ I obtain

$$\begin{aligned} \frac{U_c}{U_l} &= -\frac{1}{w} \Rightarrow \frac{\frac{1}{c}}{v'(l)} = -\frac{1}{f(\hat{k}) - \hat{k}f'(\hat{k})} \Rightarrow \\ c &= -(f(\hat{k}) - \hat{k}f'(\hat{k}))\frac{1}{v'(l)} \Rightarrow \hat{c} = -(f(\hat{k}) - \hat{k}f'(\hat{k}))\frac{1}{v'(l)l} \end{aligned} \tag{3.35}$$

If I take logarithms and differentiate with respect to time the terms of the equation (3.34) I obtain:

$$\begin{aligned} \left[\frac{U_{cc}c}{U_c} - \frac{U_{lc}c}{U_l}\right]\frac{\dot{c}}{c} + \left[\frac{U_{cl}l}{U_c} - \frac{U_{ll}l}{U_l}\right]\frac{\dot{l}}{l} &= \frac{\hat{k}f''(\hat{k})\dot{\widehat{k}}}{f(\hat{k}) - \hat{k}f'(\hat{k})} = \\ \left[\frac{U_{cc}c}{U_c} - \frac{U_{lc}c}{U_l}\right]\frac{\dot{c}}{c} + \left[\frac{U_{cl}l}{U_c} - \frac{U_{ll}l}{U_l}\right]\frac{\dot{l}}{l} &= -\frac{\hat{k}f'(\hat{k})}{f(\hat{k})}\frac{1}{\sigma}\frac{\dot{\widehat{k}}}{\widehat{k}} \end{aligned}$$

where $\sigma(\widehat{k}_t) \equiv -\frac{f'(\widehat{k}_t)}{\widehat{k}_t f(\widehat{k}_t)}\frac{f(\widehat{k}_t) - \widehat{k}_t f'(\widehat{k}_t)}{f''(\widehat{k}_t)} > 0$. Replacing in the above equation for $\frac{U_{cc}c}{U_c} - \frac{U_{lc}c}{U_l} = -1$, I obtain the following equation:

$$\left[\frac{U_{cl}l}{U_c} - \frac{U_{ll}l}{U_l}\right]\frac{\dot{l}}{l} = -\frac{\hat{k}f'(\hat{k})}{f(\hat{k})}\frac{1}{\sigma}\frac{\dot{\widehat{k}}}{\widehat{k}} + \frac{\dot{c}}{c}$$

Using the fact that $\frac{\dot{c}}{c} = \frac{\dot{\hat{c}}}{\hat{c}} + \frac{\dot{l}}{l}$ (which comes from differentiating $\hat{c} = \frac{c}{l}$ with respect to time) I obtain the following dynamic equation for the evolution of l :

$$\left[\frac{U_{cl}l}{U_c} - \frac{U_{ll}l}{U_l} - 1\right]\frac{\dot{l}}{l} = -\frac{1}{\sigma}\frac{\hat{k}f'(\hat{k})}{f(\hat{k})}\frac{\dot{\widehat{k}}}{\widehat{k}} + \frac{\dot{\hat{c}}}{\hat{c}} \tag{3.36}$$

In order to find an explicit formula for $\frac{\dot{l}}{l}$ I must find the equations for $\frac{\dot{\hat{c}}}{\hat{c}}$ and $\frac{\dot{\widehat{k}}}{\widehat{k}}$. From (3.26), the market clearing condition $r = f'(\hat{k}) - \delta$, from $\hat{k} = \frac{k}{l}$ and the steady state condition $\alpha = k$ I obtain:

$$\frac{\dot{\widehat{k}}}{\widehat{k}} = \frac{f(\hat{k}_t)}{\hat{k}_t} - \frac{\hat{c}_t}{\hat{k}_t} - \delta - \frac{\dot{l}}{l} \tag{3.37}$$

The dynamic equation for $\frac{\dot{\hat{c}}}{\hat{c}}$ is a little bit more complex to obtain. If I take logarithms of the variable in equation (3.31) and differentiate with respect to time, and then substitute for $\dot{\lambda}$ from (3.33) then I get the basic condition for choosing consumption over time:

$$\begin{aligned}
r &= \rho - \frac{u_{cc}c}{u_c}\frac{\dot{c}}{c} - \frac{u_{cl}l}{u_c}\frac{\dot{l}}{l} \Rightarrow r = \rho + \theta\frac{\dot{c}}{c} - \frac{u_{cl}l}{u_c}\frac{\dot{l}}{l} \Rightarrow \\
\frac{\dot{c}}{c} &= \frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho) + \frac{1}{\theta}\frac{u_{cl}l}{u_c}\frac{\dot{l}}{l} \Rightarrow \\
\frac{\dot{\hat{c}}}{\hat{c}} &= \frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho) + \left[\frac{1}{\theta}\frac{u_{cl}l}{u_c} - 1\right]\frac{\dot{l}}{l}
\end{aligned} \tag{3.38}$$

Replacing equations (3.38) and (3.37) in eq. (3.36) I obtain the following equation:

$$\left[\left(1 - \frac{1}{\theta}\right)\frac{U_{cl}l}{U_c} - \frac{U_{ll}l}{U_l} - \frac{1}{\sigma}a(\hat{k})\right]\frac{\dot{l}}{l} = -\frac{1}{\sigma}a(\hat{k})\left[\frac{f(\hat{k}_t)}{\hat{k}_t} - \frac{\hat{c}_t}{\hat{k}_t} - \delta\right] + \frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho) \tag{3.39}$$

Using (4.5) I can obtain the full system of dynamic equations characterizing the system:

$$\frac{\dot{\widehat{k}}}{\widehat{k}} = \left[\frac{f(\hat{k}_t)}{\hat{k}_t} - \frac{\hat{c}_t}{\hat{k}_t} - \delta\right]\left[1 + \frac{1}{A}\frac{1}{\sigma}a(\hat{k})\right] - \frac{1}{A}\frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho) \tag{3.40}$$

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho)\left(1 + \left(\frac{1}{\theta}\frac{u_{cl}l}{u_c} - 1\right)\frac{1}{A}\right) - \left[\frac{1}{\theta}\frac{u_{cl}l}{u_c} - 1\right]\left[\frac{1}{A}\frac{1}{\sigma}a(\hat{k})\left(\frac{f(\hat{k}_t)}{\hat{k}_t} - \frac{\hat{c}_t}{\hat{k}_t} - \delta\right)\right] \tag{3.41}$$

where $A = \left(1 - \frac{1}{\theta}\right)\frac{U_{cl}l}{U_c} - \frac{U_{ll}l}{U_l} - \frac{1}{\sigma}a(\hat{k}) \gtrless 0$ and $a(\hat{k}) = \frac{\hat{k}f'(\hat{k})}{f(\hat{k})} > 0$ denoting the share of capital.

In order to obtain an equation that relates $\hat{c}$ and l, I must divide $\frac{v_c}{u_l}$ from the FOC's to obtain:

$$\hat{c} = -(1-\theta)(f(\hat{k}) - \hat{k}f'(\hat{k}))\frac{v(l)}{v'(l)l} \tag{3.42}$$

3.4.4.2 Case A: $\theta = 1$

When the utility function is logarithmic the dynamic system is given by the following equations:

$$\left[-\frac{U_{ll}l}{U_l} - \frac{1}{\sigma}a(\hat{k})\right]\frac{\dot{l}}{l} = -\frac{1}{\sigma}a(\hat{k})\left[\frac{f(\hat{k}_t)}{\hat{k}_t} - \frac{\hat{c}_t}{\hat{k}_t} - \delta\right] + (f'(\hat{k}_t) - \delta - \rho) \tag{3.43}$$

$$\frac{\dot{\widehat{k}}}{\widehat{k}} = \left[\frac{f(\hat{k}_t)}{\hat{k}_t} - \frac{\hat{c}_t}{\hat{k}_t} - \delta\right]\left[1 + \frac{1}{A}\frac{1}{\sigma}a(\hat{k})\right] - \frac{1}{A}(f'(\hat{k}_t) - \delta - \rho) \tag{3.44}$$

and

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho)\left(1 - \frac{1}{A}\right) + \frac{1}{A}\frac{1}{\sigma}a(\hat{k})\left(\frac{f(\hat{k}_t)}{\hat{k}_t} - \frac{\hat{c}_t}{\hat{k}_t} - \delta\right) \tag{3.45}$$

where $A = -\frac{U_{ll}l}{U_l} - \frac{1}{\sigma}a(\hat{k}) < 0$ and

$$\hat{c} = -(f(\hat{k}) - \hat{k}f'(\hat{k}))\frac{1}{v'(l)l} \tag{3.46}$$

3.4.5 Stability

Stability of a dynamic system with a concave production function and endogenous labor supply can always exist under the assumption of constant returns to scale. This result has been partially proved by Hintermaier (2003) who has proved that indeterminacy cannot occur unless one assumes increasing returns to scale and actually a large degree of returns from plausible labor shares. Of course many other cases may arise, instability for instance, so for this reason we must make specific assumptions that depend on the formulation of the model. The important is that stability can exist and thus the system may have a unique steady state equilibrium. A more extensive analysis on the stability of the system is provided in Appendix D.

Concerning indeterminacy the relevant literature assumes that consumption is a normal good. In Appendix E I attempt to approach the issue of indeterminacy by making more flexible assumptions concerning the normality of consumption. In brief, what I find is that the standard result in the indeterminacy literature, i.e. that the labor supply and demand curves cross with the wrong slopes and the latter is steeper than the former, is reversed when non-normal consumption is assumed. This result cannot be obtained under the assumption of leisure as an inferior good neither under the standard assumptions. Despite the implausibility of such an assumption, there are instances in the empirical and theoretical literature that report such evidence, which will be extensively analyzed in Appendix E.

3.5 Transitional Dynamics

The analysis that has taken place so far concerning the transitional behavior of labor supply either focuses on logarithmic utility and CD utility (Barro and Sala-i-Martin), or is based solely on the behavior of the variable around the steady state (i.e. Turnovsky and Penalosa). As has already been mentioned in the literature review it is widely supported that labor supply behaves monotonically, a conviction though which is not being supported by real world data especially when it comes to labor force participation rates. Furthermore this monotonicity result is not surprising if one takes into account the assumptions that lead to it.

It is evident though, from the dynamic equation of labor supply, that there is no reason why it should behave monotonically, especially when we abstract from standard utility and production assumptions. One can see for example, from eq. (4.5) that not only the RHS can behave non-monotonously, but the same goes for the LHS as well since A varies intertemporally when $v(l)$ is non-homogeneous. Unfortunately the functional forms are rather complicated for explicit results to be obtained, at least for some cases, however I will attempt to analytically characterize l_t up to some point and then numerically display the possibility of non-monotonicity.

To analytically characterize the model I must make some simplifying assumptions which do not alter the basic results though. A more detailed analysis would simply complicate things without adding to the results.

The first simplification to be made is to transform the 3x3 system of dynamic equations into a 2x2 system. To do so I must assume a homogeneous, in l, utility function. Such an assumption implies that $\frac{U_{cl}l}{U_c}$ and $\frac{U_{ll}l}{U_l}$ are constants and thus A is constant and $A \gtrless 0$. Had I assumed that utility is not homogeneous, then A could behave non-monotonously and could even change signs. Furthermore the transformation of the 3x3 system into a 2x2 system would be very messy and would require specific functional forms.

A second assumption to be made is that the economy approaches a steady state and does not experience unbounded growth. This assumption applies especially for the case where $\sigma > 1$ where unbounded growth may occur. More specifically, if $\lim_{\widehat{k}(t)\to\infty} \sigma(\widehat{k}_t) > 1$ then $\lim_{\widehat{k}(t)\to\infty} f^{'}(\widehat{k}_t) \equiv A > 0$; hence, if $A > \rho + \delta + \theta$, then there will be unbounded growth (for details see Palivos and Karagiannis 2007). This hypothesis is not very restrictive and my results are robust in both cases of endogenous and exogenous growth.

Keeping these assumptions I will make the necessary transformations and then make more assumptions about the values of θ and σ.

3.5.1 The 2x2 System: An Analytical Approach

Using equations (3.39), (3.40) and (**??**) for the case of $\theta \neq 1$ I obtain the following system in terms of $\hat{k}$ and l.

$$A\frac{\dot{l}}{l} = -\frac{1}{\sigma}a(\hat{k})\left[\frac{f(\hat{k}_t)}{\hat{k}_t} + \left[(1-\theta)\left(\frac{f(\hat{k})}{\hat{k}_t} - f'(\hat{k})\right)\gamma\right] - \delta\right] + \frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho) \tag{3.47}$$

$$\frac{\dot{\widehat{k}}}{\widehat{k}} = \left[\frac{f(\hat{k}_t)}{\hat{k}_t} - \left[(1-\theta)\left(\frac{f(\hat{k})}{\hat{k}_t} - f'(\hat{k})\right)\gamma\right] - \delta\right]\left[1 + \frac{1}{A}\frac{1}{\sigma}a(\hat{k})\right] - \frac{1}{A}\frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho) \tag{3.48}$$

where $A = \left(1 - \frac{1}{\theta}\right)\frac{U_{cl}l}{U_c} - \frac{U_{ll}l}{U_l} - \frac{1}{\sigma}a(\hat{k})$ is now constant and $\gamma = \frac{v(l)}{v'(l)l}$, which can be interpreted as the elasticity with respect to l, is also constant and its sign depends on θ.

Correspondingly, for $\theta = 1$ and using equations (3.43), (3.44) and (3.46) I obtain the following system:

$$A\frac{\dot{l}}{l} = -\frac{1}{\sigma}a(\hat{k})\left[\frac{f(\hat{k}_t)}{\hat{k}_t} + \left[\left(\frac{f(\hat{k})}{\hat{k}_t} - f'(\hat{k})\right)\tilde{\gamma}\right] - \delta\right] + (f'(\hat{k}_t) - \delta - \rho) \tag{3.49}$$

$$\frac{\dot{\widehat{k}}}{\widehat{k}} = \left[\frac{f(\hat{k}_t)}{\hat{k}_t} - \left[\left(\frac{f(\hat{k})}{\hat{k}_t} - f'(\hat{k})\right)\tilde{\gamma}\right] - \delta\right]\left[1 + \frac{1}{A}\frac{1}{\sigma}a(\hat{k})\right] - \frac{1}{A}(f'(\hat{k}_t) - \delta - \rho) \tag{3.50}$$

where $A = -\frac{U_{ll}l}{U_l} - \frac{1}{\sigma}a(\hat{k}) < 0$ and constant and $\tilde{\gamma} = \frac{1}{v'(l)l} < 0$, which is also constant for the case of the homogeneous utility function.

Using the above equations I will try to study the dynamic behavior of labor supply. To do so I will analytically examine all possible values of σ and θ with general functional forms. Whenever I cannot obtain analytical results I revert to specific functions and use numerical analysis.

3.5.1.1 The Cobb-Douglas

Case 1: $\sigma = 1, \theta = 1$

This is the standard case analyzed in most papers, which actually predicts a decreasing labor supply as $\hat{k}_t$ increases. For this reason it is the ideal benchmark case to test whether my model actually works and nests this special case. The theory predicts that when I have separable utility and Cobb-Douglas production, labor supply decreases monotonically.

Under $\sigma = 1, \theta = 1$ the dynamic equations take the following form:

$$A\frac{\dot{l}}{l} = -a\left[\frac{f(\hat{k}_t)}{\hat{k}_t} + \left[\left(\frac{f(\hat{k})}{\hat{k}_t} - f'(\hat{k})\right)\tilde{\gamma}\right] - \delta\right] + (f'(\hat{k}_t) - \delta - \rho) \tag{3.51}$$

$$\frac{\dot{\widehat{k}}}{\widehat{k}} = \left[\frac{f(\hat{k}_t)}{\hat{k}_t} - \left[\left(\frac{f(\hat{k})}{\hat{k}_t} - f'(\hat{k})\right)\tilde{\gamma}\right] - \delta\right]\left[1 + \frac{1}{A}a\right] - \frac{1}{A}(f'(\hat{k}_t) - \delta - \rho) \tag{3.52}$$

where $A = -\frac{U_{ll}l}{U_l} - a < 0$ and constant and $\tilde{\gamma} = \frac{1}{v'(l)l} < 0$. Note that when $\sigma = 1 \Rightarrow a(\hat{k})$ =constant= a.

I will try to transform eq.(3.51) in a way that allows me to study its sign. Introducing some notation, $X = \frac{\hat{c}}{\hat{k}}$ and $X^* = \frac{\hat{c}^*}{\hat{k}^*} = \frac{f(\hat{k}^*)}{\hat{k}^*} - \delta = \frac{f'(\hat{k}^*)}{a} - \delta$ I can rewrite eq.(3.51) as follows:

$$\begin{aligned} A\frac{\dot{l}}{l} &= aX + a\delta - \delta - \rho \Rightarrow \\ \frac{\dot{l}}{l} &= \frac{1}{A}a(X - X^*) \Rightarrow \end{aligned}$$

Barro and Sala-i-Martin have proved that X decreases monotonically and thus $X > X^*$, $A < 0 \Rightarrow \frac{\dot{l}}{l} < 0 \,\forall\, \hat{k}_t$, thus obtaining the same results as in the standard literature. Of course when one abstracts from these specific values for θ and σ it is not as clear to obtain the behavior of l.

For each specific case I will provide numerical results. To obtain these results I have used the Brunner and Strulik (2002) numerical method, which has been extensively analyzed in the first chapter of this thesis. In realizing the simulations I have used widely accepted values for the various parameters. The results I obtain are rather close to real world data, as presented in the literature review even though they cannot explicitly account for factors such as female participation rates or unemployment. However the reproduction of observed patterns and of plausible rates is feasible. In some cases I even use non-homogeneous utility functions since numerical analysis eliminates difficulties rising form their use. Finally note that in thee following graphs l_t always denotes participation rates.

Numerical analysis for the case where $\sigma = 1, \theta = 1$ returns the following results:

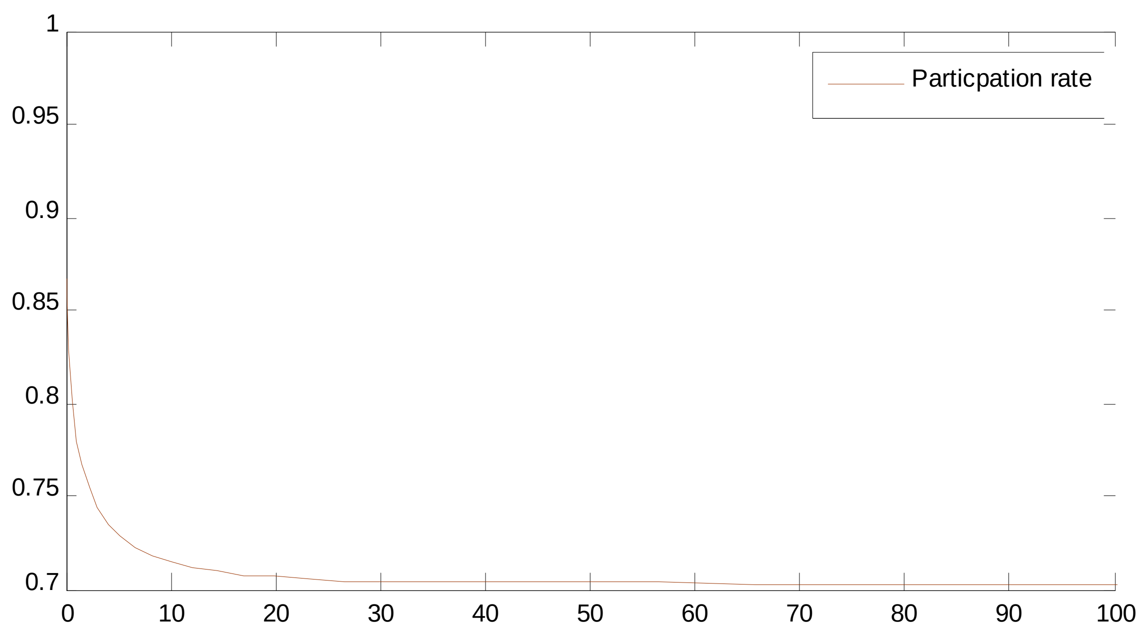

Figure 6: Participation Rate with Cobb Douglas Production and Homogeneous and Separable Utility: $U(c,l) = \ln c - (l)^h : \delta = 0.041, \rho = 0.04, a = 0.25, h = 5$

In this numerical example I have used a Cobb-Douglas production function as well as a homogeneous utility function with plausible parameter values for the rest of the variables. If we observe the graphs provided in the literature review we observe that participation rates are within

0.85 and 0.70 for low unemployment rates and between 0.70-0.55 for high unemployment rates As I cannot account for unemployment using the Ramsey model, I consider that a plausible range of rates in the context of my model is between the observed rates, i.e. between 0.55-0.85. As expected for this particular case the participation rate is monotonically decreasing towards the steady state and the rates I obtain are well within empirical estimates. This monotonicity result is robust to other parameter values.

Case 2: $\sigma = 1, \theta \neq 1$

The dynamic equations for this case take the following form:

$$A\frac{\dot{l}}{l} = -a(\hat{k})\left[\frac{f(\hat{k}_t)}{\hat{k}_t} + \left[(1-\theta)\left(\frac{f(\hat{k})}{\hat{k}_t} - f'(\hat{k})\right)\gamma\right] - \delta\right] + \frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho) \tag{3.53}$$

$$\frac{\dot{\hat{k}}}{\hat{k}} = \left[\frac{f(\hat{k}_t)}{\hat{k}_t} - \left[(1-\theta)\left(\frac{f(\hat{k})}{\hat{k}_t} - f'(\hat{k})\right)\gamma\right] - \delta\right]\left[1 + \frac{1}{A}a(\hat{k})\right] - \frac{1}{A}\frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho) \tag{3.54}$$

where $A = \left(1 - \frac{1}{\theta}\right)\frac{U_{cl}l}{U_c} - \frac{U_{ll}l}{U_l} - a(\hat{k})$ and $\gamma = \frac{v(l)}{v'(l)l} < 0$. Since $\theta \neq 1$ we cannot be certain about the sign of A which can either be negative or positive. What we can be certain about is that since $\sigma = 1$ and $a(\hat{k})$ is constant then A is a constant and is either negative or positive.

In this case the dynamic behavior of $\dot{l}$ depends on the RHS of eq. (E.13). Following the procedure above I will try to transform the dynamic equation for l.

$$A\frac{\dot{l}}{l} = a(X - X^*) + (\frac{1}{\theta} - 1)(f'(\hat{k}) - \delta - \rho) \tag{3.55}$$

Again using the fact that X behaves monotonically we can distinguish the following two cases:

Case 2.A: $\theta < 1$

In this case the RHS of eq. (E.15) is everywhere positive and hence the sign of $\dot{l}$ depends on the sign of A. In any case though the saving rate behaves monotonically.

Case 2.B: $\theta > 1$

This case is more complicated and one must study the derivative of the RHS of eq. (E.13) to find out whether it behaves monotonically or not. More specifically by transforming eq. (E.13) I obtain:

$$
\begin{aligned}
A\frac{\dot{l}}{l} &= -\left[f'(\hat{k}_t) + \left[(1-\theta)\left(f'(\hat{k}_t) - f'(\hat{k}_t)a(\hat{k})\right)\gamma\right] - \delta a(\hat{k})\right] + \frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho) \Rightarrow \\
A\frac{\dot{l}}{l} &= -f'(\hat{k}_t)\left[1 + (1-\theta)(1-a)\gamma - \frac{1}{\theta}\right] + \delta a - \frac{1}{\theta}(\delta + \rho)
\end{aligned}
$$

If we take derivatives of the RHS of the above equation we obtain $-f''(\hat{k}_t)\left[1 + (1-\theta)(1-a)\gamma - \frac{1}{\theta}\right]$. It is obvious that the the sign of this derivative depends on the sign of $1 + (1-\theta)(1-a)\gamma - \frac{1}{\theta} \lesseqgtr 0$ when $\theta \lesseqgtr 1$. It is evident that since the RHS behaves monotonically the same goes for $\dot{l}$ and its sign depends on the sign of A.

One last point to be made concerning A and its significant role in determining the dynamic behavior of labor supply, is that its sign completely parameter dependent, a point that will become evident in the following parts of the chapter where I will use specific functional forms.

The following propositions conclude the dynamic behavior of labor supply for the case of the Cobb-Douglas.

Proposition 1. a) When the production function is Cobb-Douglas, i.e. $\sigma = 1$ and $\theta = 1$ labor supply decreases monotonically as the economy approaches a steady state.

b) When the production function is Cobb-Douglas, i.e. $\sigma = 1$ and $\theta \gtreqless 1$ labor supply behaves monotonically, either increasing or decreasing, depending on parameter values, as the economy approaches a steady state

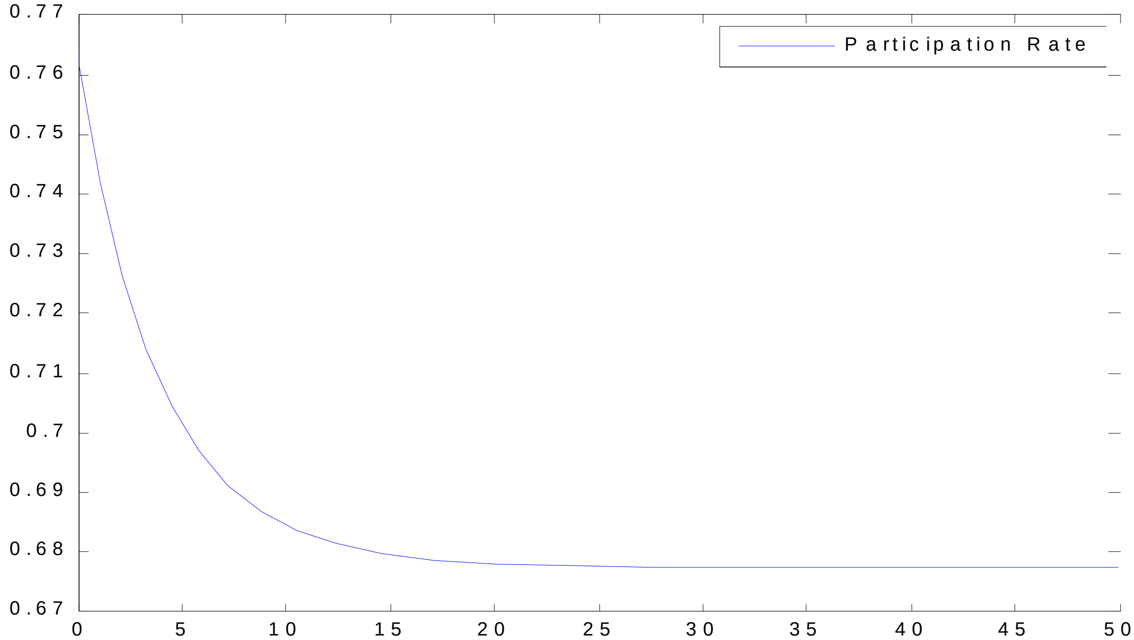

Figure 7: Participation Rate with CD Production and Non-Homogeneous, Non-Separable Utility: $u(c,l) = \frac{c_t^{1-\theta}}{1-\theta}(1-l_t)^{\theta}$ $\delta = 0.06, \rho = 0.03, a = 0.25, \theta = 0.3, \sigma = 1$

The numerical analysis returns, as expected, a monotonous participation rate, a result robust to parameter variations:

3.5.1.2 Constant and Variable Elasticity of Substitution Production Functions

When CES and VES production functions are studied it is harder to obtain analytically results because non-monotonicity may arise form both sides of eq. (E.8). I will again study each case separately.

Case 3: $\sigma \neq 1, \theta = 1$

The dynamic equation of $\dot{l}$ for this case is given by:

$$A\frac{\dot{l}}{l} = -\frac{1}{\sigma}a(\hat{k})\left[\frac{f(\hat{k}_t)}{\hat{k}_t} + \left[\left(\frac{f(\hat{k})}{\hat{k}_t} - f'(\hat{k})\right)\tilde{\gamma}\right] - \delta\right] + (f'(\hat{k}_t) - \delta - \rho)$$

where $A = -\frac{U_{ll}l}{U_l} - \frac{1}{\sigma}a(\hat{k}) < 0$ and constant and $\tilde{\gamma} = \frac{1}{v'(l)l} < 0$.

It is evident that in this case, for $\sigma \gtrless 1$ it is hard to analytically prove that the RHS of the above equation behaves non-monotonically however it is equally hard to preclude this likelihood. Following there is a numerical example that illustrates the possibility of a non-monotonous labor supply intertemporally. The exact shape of $\dot{l}$ also depends on A as well which in this case is always negative.

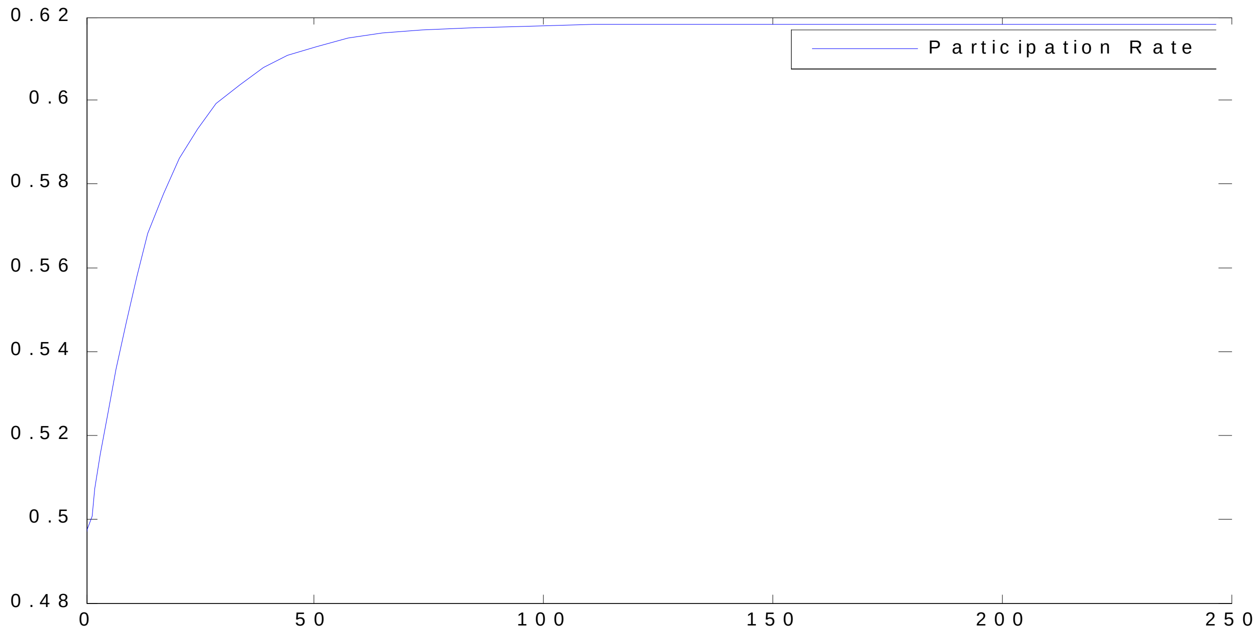

Figure 8: Participation Rate with CES Production and Homogeneous and Separable Utility $U(c,l) = \ln c - ($
$\delta = 0.041, \rho = 0.04, a = 0.75, \theta = 1, h = 2, \sigma = 0.7$

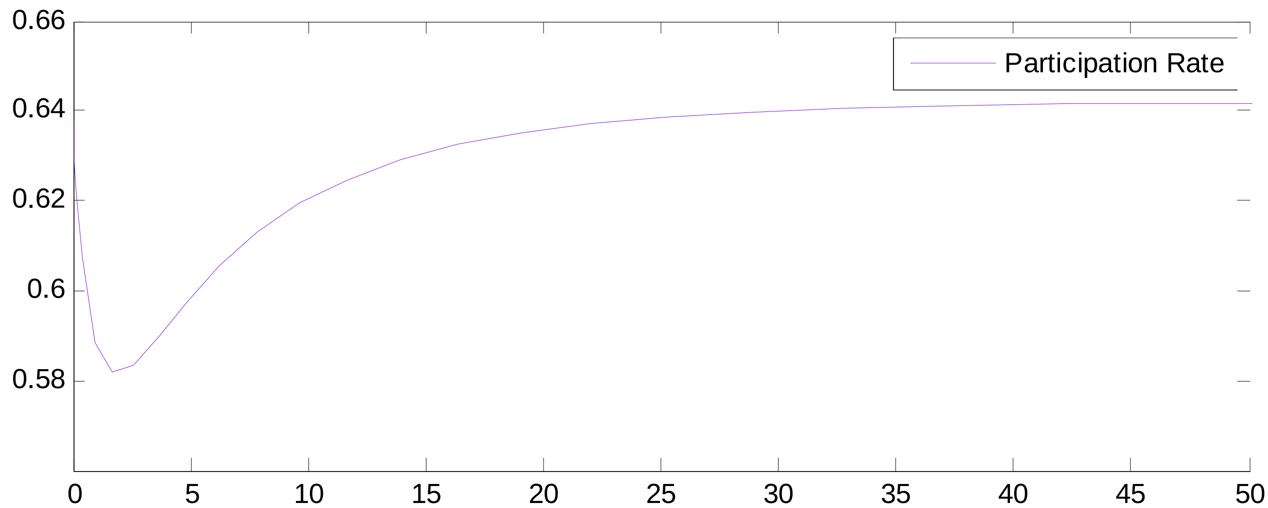

Figure 9: Participation Rate with CES Production and Homogeneous and Separable Utility $U(c,l) = \ln c -$
$\delta = 0.041, \rho = 0.04, a = 0.75, \theta = 1, \sigma = 0.59, h = 2$

It is evident that when abstracting from unitary elasticity of factor substitution, various patterns of participation rates may arise. Mainly driven by the values of the factor elasticity of substitution but also from the intertemporal elasticity of substitution as well, participation rates may have a U-shaped path, as Goldin has described, or even be monotonous, as is observed in some cases.

Case 4: $\sigma \neq 1, \theta \neq 1$

This is the last and most complicated case as the dynamic behavior of $\dot{l}$ is now given by:

$$A\frac{\dot{l}}{l} = -\frac{1}{\sigma}a(\hat{k})\left[\frac{f(\hat{k}_t)}{\hat{k}_t} + \left[(1-\theta)\left(\frac{f(\hat{k})}{\hat{k}_t} - f'(\hat{k})\right)\gamma\right] - \delta\right] + \frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho)$$

or if I express the above equation in terms of X :

$$A\frac{\dot{l}}{l} = -\frac{1}{\sigma}a(\hat{k})\left[X - X^*\right] + \frac{1}{\theta}\left(f'(\hat{k}_t) - f'(\hat{k}^*)\right) - \frac{1}{\sigma}a(\hat{k})\left[\frac{f(\hat{k}_t)}{\hat{k}_t} - \frac{f(\hat{k}^*)}{\hat{k}^*}\right]$$

where $A = \left(1 - \frac{1}{\theta}\right)\frac{U_{cl}l}{U_c} - \frac{U_{ll}l}{U_l} - \frac{1}{\sigma}a(\hat{k}) \gtrless 0$. One could probably some to some conclusion concerning the RHS of the latter equation, i.e. it is evident that when $\sigma > 1 \Rightarrow f'(\hat{k}_t) - f'(\hat{k}^*) >$

$\frac{f(\hat{k}_t)}{\hat{k}_t} - \frac{f(\hat{k}^*)}{\hat{k}^*}$ and thus for the RHS to be monotonously positive the condition $\frac{1}{\theta} > \frac{1}{\sigma}a(\hat{k}_t)$ must hold. But even if this is the case non-monotonicity may again occur since in this case $a(\hat{k})$ varies as $\hat{k}$ grows towards the steady state and thus A may vary as well. Overall there may be various sources of non-monotonicity. Even if one can identify various cases for which non-monotonicity may arise, the assumptions required may be so restrictive and parameter sensitive that may reduce the intuitive value of the result. What is at stake is to indicate the likelihood for such a pattern to arise. For this purpose I will resort to a numerical example that illustrates the complex dynamics of the model. For this specific case I will use a non-homogeneous utility function, as this option is feasible when using numerical analysis, to indicate that non-monotonicity results can as well be obtained when non-homogeneous utility functions are used.

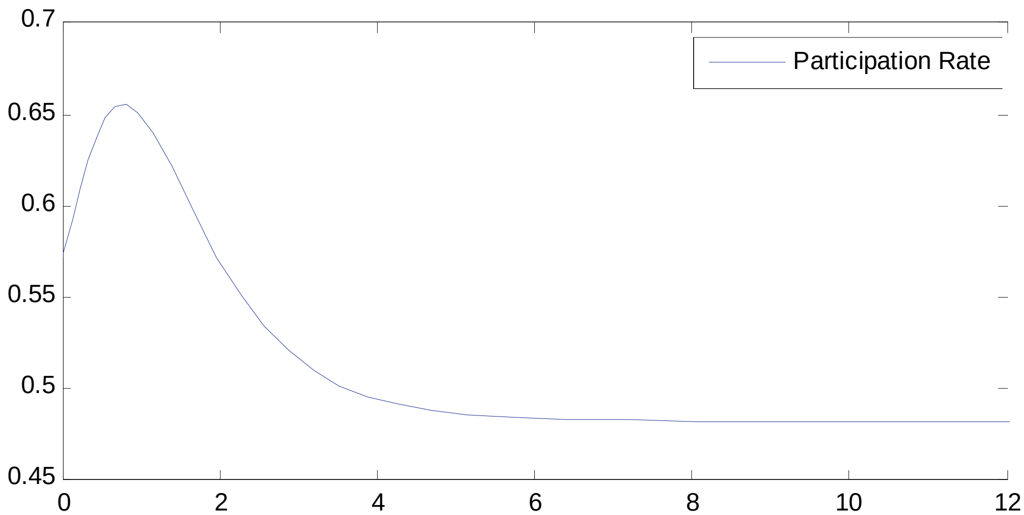

Figure 10: Participation Rate with CES Production and Non-Homogeneous, Non-Separable Utility: $u(c,l) = \frac{c_t^{1-\theta}}{1-\theta}(1-l_t)^{\theta}$ $\delta = 0.041, \rho = 0.04, a = 0.25, \theta = 0.5, \sigma = 0.41$

What we can obtain in this case, apart from monotonous patterns that may arise, is that an overshooting in the behavior of the participation rate.

3.5.2 Intuition and Results

As I have already pointed out from the beginning of the chapter the driving forces behind the participation rates dynamics are the income and substitution effects. Given that I use flexible utility and production functions these two effects do not offset each other. Which of the two effects prevails depends on the relative relationship between the elasticity of factor substitution and the intertemporal elasticity of substitution. If the reference point is labor supply then the income effect decreases labor force participation rate while the substitution effect increases participation rate. When income rises members of the household find it less attractive to work since the household income is greater now. On the other hand a substitution effect implies that as the economy proceeds, wages increase at an increasing rate. Hence agents have an incentive to enter the working force as the cost of voluntary unemployment has increased.

As far as numerical results are concerned the following must be noted: First the participation rates observed via the numerical analysis are rather realistic. Of course there are large differences within countries which are partially explained by the unemployment rates however in most numerical results rates above 50% and 60% are obtained which is the case for most countries with limited unemployment. Second, if we observe the Elmeskov graphs it is evident that all kinds of patterns may arise. In some cases these patterns are monotonous while in others are not (note that fluctuations around these trends are not taken into account). What is important with this modelling is that when numerically analyzed and under plausible assumptions about parameter values and certain assumptions about the values of the intertemporal elasticity of substitution and the elasticity of factor substitution, these patterns can be reproduced. Whenever the theory predicts monotonicity, this monotonicity is robust in the numerical analysis. While on the other hand when overshooting or undershooting occurs, the magnitude of it is close to the magnitude obtained for empirical estimates. What there graphs fail to account for is the time period during which this U pattern occurs, since they usually reach a steady state within 15-20 years, however they do better in the case of monotonous patterns.

As far as parameters are concerned there has been an effort to make realistic choices and most of the parameter values used in this chapter come from Barro and Sala-i-Martin textbook (2004). In some cases the elasticities vary either above or below unity (both cases are met in empirical literature), however they never take unrealistically large values. What is interesting, especially in cases where U-shaped or inverted U-shaped patterns occur is that even small variations in the elasticity of factor substitution are enough to yield such a pattern. Therefore this could be met in societies that experience only small differentiations in elasticity of factor substitution, holding all other parameter values constant.

Concerning the analytical results obtained using this approach, I believe that the model goes a long way towards explaining the observed patterns especially when one takes into account that not all the population is employed, i.e. $L \neq N$. In the standard models in the literature examining working hours intertemporally, their main finding is that working hours per worker decrease in the long run. This is not the case though when working hours per person are taken into account. As became evident from the graphs of Gali, even when working hours per worker are decreasing which is usually the case, working hours per person may vary in a non-monotonous manner. This of course is due to the fact that participation rates are changing and when years after 1950 are examined they include the baby-boom generation and the dramatic changes in female participation rates. Much as plausible these explanations may be, none of the standard models examining working hours can account for that. Their standard assumption is that working force is identified with population or is a percentage of it, and population growth rate is either constant or increasing at a constant rate. Hence the least one would expect from such a formulation is that working hours per worker are behaving in an identical way with working hours per person. The main advantage of this model is that it can actually reproduce this variability in participation rates. Of course one could say that it does so at the cost of assuming constant number of working hours per worker but even in this case this assumption is more plausible. Furthermore hours per worker (which are constant) are different than hours per capita which depend directly on the participation rate.

3.5.3 Real World Evidence

Income and substitution effects are rather primitive economic motives. They are always present in theoretical models as well as in real world decisions even though we are not always able to conceptualize them. In some cases they are directly transubstantiated into economic decisions and in other cases they find their way to the economy through other forces, such as increased schooling, that in turn lead to economic decisions. Ortigueira in a model with human capital and Goldin suggest ways through which income and substitution effects become the locomotive for economic activities that in turn affect participation rates. Goldin further extends the analysis and tries to find what caused this switch between the income and substitution effect that is responsible for the observed U-shaped path in female participation rates. In this model I have based my analysis on these two effects only. Departing from them there are other more evident reasons behind participation rates patterns. Note though that factors affecting participation rates such as schooling or fertility are endogenous in the model and thus subject to income and substitution effects themselves.

Below I cite some real world evidence which go further towards explaining the observed patterns. These are not directly related to the analysis of this specific model however they indicate the direction where this research should head to. I will refer to the factors that increase and decrease participation rates as traced in the economic literature.

Among the factors that increase participation rates the most important is the increased participation of women. Many social changes as well as changes in the production process have occurred before this increase took place, however in most countries female participation rates dramatically increased and thus aggregate working hours increased as well. Towards this direction the role of publicly financed day-care institutions taking over selective parts of the traditional family roles of women, was very significant.

The composition of the population is also a very important factor affecting observed participation rates. A typical example is baby-boom generation which is evident in participation data. Another issue is that of immigration where increases in immigration rates are directly translated into increases in participation rates. If we abstract from such factors and focus on broader economic conditions we observe that the rate increases during economic expansions, since more jobs are available (pro-cyclical influence).

On the other hand if we examine factors that reduce participation rates then schooling is among the dominant ones. As the economy has developed and the role of human capital and the returns to it have become evident, an increasing number of young adults prolong their stay in educational sector and delay their participation in the labor market.

Other reasons are coverage and generosity of tax financed public pension schemes or unemployment benefits which reduce the incentives to participate or stay long in the labor market as well as economic downturns, since there are fewer job opportunities (cyclical influence).

3.6 Conclusions

The purpose of this paper was to introduce a completely different approach towards labor market and differentiate not only from life-cycle and RBC models, but also from long-run models that focus on the trends related to labor-leisure choices. Significant as this approach may be, and there is no doubt about that, leisure is only one side of the coin. When aggregate labor supply is at stake both sides must be taken into account, and the second side is that of participation rates. This has already become evident by the strand of empirical research that predicts not only decreasing but also fluctuating rates in working hours. It has been implied and concluded that fluctuations in aggregate working hours should not only be attributed to leisure choices and substitution effects, but also to employment decisions. This is also a very likely explanation for the differences observed between per capita and per worker working hours.

In this chapter I have chosen to explore the other side of the coin and specifically the patterns of the labor force participation rates. In doing so I neglected, or to be more precise I have adopted a very specific, highly plausible though assumption concerning working hours, i.e. that of indivisible labor which is strongly represented in the literature. Certainly this is not always the case, however I made this simplification for expositional purposes. A model where both employment and working hours decisions are both at stake has been partially developed in the Appendix, however further research toward this direction is necessary for the analysis to be completed.

My initial motivation was thus a theoretical model that would explain these non-monotonous patterns observed in real world data and the attainment of my model was that is actually managed to reproduce this data in numerical examples whenever analytical results were too complicated to be intuitive. The U-shaped path predicted especially for the dynamic behavior of labor supply in women was reproduced as well, along with other results concerning monotonicity such as the decreasing participation of young adults. Towards this direction, extremely crucial is the role played by abstraction of intertemporal and factor elasticity from unity, assumptions that are empirically plausible for economies like the US. The framework chosen is purposely very simple, i.e. the Ramsey model, in order to gain more intuition and find the economic forces behind the various patters. The main driving forces were found to be the interaction between the income and substitution effect in the presence of leisure. If one wants to take a closer look over the issue should definitely resort to various social parameters, some of which can and have to be induced in economic models, such as schooling and female emancipation. These factors though do not contradict the intuition behind my results as social evolution is directly linked or even motivated by primordial economic instincts and income and substitution effects could definitely be characterized as such.

This argument does not imply that such an analysis is not needed as it would explicitly trace various factors affecting participation rates and would act in a complementary manner to this research. Thus, this could be a suggestion for further research, as well as an exploration of both working hours and participation rates interacting simultaneously. The analysis must be dynamic and for the whole transition path, not around the steady state as is usually the case when participation rates are examined. When one abstracts form the standard cases analyzed in the literature, he should also conduct analytical research on the dynamic properties of the model. Motivated by the research on dynamic properties of my model I explored the issue of indeterminacy. Further research is required though to fully characterize the dynamics of labor force participation rates and reproduce the observed patterns.

Appendix D Stability Analysis

In order to analyze the stability of the system I will first provide the general form of the Jacobian obtained by the linearization around the steady which can be adjusted for every sub-case. Then for expositional purposes I will use the case of Cobb-Douglas production function which is more convenient. To conduct the analysis I will transform the dynamic system of three equations into a 2x2 system and more specifically into a system of the variables $\hat{c}$ and $\hat{k}$. The reason for choosing these variables instead of $\hat{k}$ and l is because l does not explicitly appear in any of the dynamic equations whenever utility is homogeneous, thus linearization would not be feasible. The assumptions made in the main body of the chapter, i.e. homogeneity and exogenous growth are retained in this appendix as well.

As we saw in the main body of the chapter the dynamic equations of the system, for $\sigma \neq 1$ and $\theta \neq 1$ are given by:

$$\frac{\dot{\widehat{k}}}{\widehat{k}} = \left[\frac{f(\hat{k}_t)}{\hat{k}_t} - \left[(1-\theta)\left(\frac{f(\hat{k})}{\hat{k}_t} - f'(\hat{k})\right)\gamma\right] - \delta\right]\left[1 + \frac{1}{A}\frac{1}{\sigma}a(\hat{k})\right] - \frac{1}{A}\frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho)$$

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{1}{\theta}(f'(\hat{k}_t) - \delta - \rho)\left(1 + \left(\frac{1}{\theta}\frac{u_{cl}l}{u_c} - 1\right)\frac{1}{A}\right) - \left[\frac{1}{\theta}\frac{u_{cl}l}{u_c} - 1\right]\left[\frac{1}{A}\frac{1}{\sigma}a(\hat{k})\left(\frac{f(\hat{k}_t)}{\hat{k}_t} - \frac{\hat{c}_t}{\hat{k}_t} - \delta\right)\right]$$

By linearization around the steady state and using the steady state condition I obtain:

$$\dot{\hat{k}} = \left[(f'(\hat{k}^*) - \delta)(1 + \frac{a}{A}\frac{1}{\sigma}) - \frac{1}{A}\frac{1}{\theta}\hat{k}^* f''(\hat{k}^*)\right]\left(\hat{k}_t - \hat{k}^*\right) - (1 + \frac{a(\hat{k})}{A}\frac{1}{\sigma})(\hat{c}_t - \hat{c}^*)$$

$$\dot{\hat{c}} = \hat{c}^* \left[f''(\hat{k}^*)\frac{1}{\theta}(1+\frac{\gamma}{A}) - \frac{\gamma}{A}a(\hat{k})\frac{1}{\sigma}\left(\frac{f'(\hat{k}^*)\hat{k}^* - f(\hat{k}^*)}{\left(\hat{k}^*\right)^2} + \frac{\hat{c}^*}{\left(\hat{k}^*\right)^2} \right) \right] \left(\hat{k}_t - \hat{k}^*\right) + \left(\hat{c}^* \frac{\gamma}{A}\frac{1}{\sigma}a(\hat{k})\frac{1}{\hat{k}^*} \right)(\hat{c}_t - \hat{c}^*)$$

and the Jacobian of the system is given by:

$$J = \begin{bmatrix} (f'(\hat{k}^*) - \delta)(1+\frac{a}{A}\frac{1}{\sigma}) - \frac{1}{A}\frac{1}{\theta}\hat{k}^* f''(\hat{k}^*) & -(1+\frac{a(\hat{k})}{A}\frac{1}{\sigma}) \\ \hat{c}^* \left[f''(\hat{k}^*)\frac{1}{\theta}(1+\frac{\gamma}{A}) - \frac{\gamma}{A}a(\hat{k})\frac{1}{\sigma}\left(\frac{f'(\hat{k}^*)\hat{k}^* - f(\hat{k}^*)}{\left(\hat{k}^*\right)^2} + \frac{\hat{c}^*}{\left(\hat{k}^*\right)^2} \right) \right] & \left(\hat{c}^* \frac{\gamma}{A}\frac{1}{\sigma}a(\hat{k})\frac{1}{\hat{k}^*} \right) \end{bmatrix}$$

To proceed with the analysis I will simplify the above matrix to the Cobb-Douglas case which can provide more intuitive results concerning the stability properties of the model. The matrix obtained in this case is:

$$J = \begin{bmatrix} (f'(\hat{k}^*) - \delta)(1+\frac{a}{A}) - \frac{1}{A}\frac{1}{\theta}\hat{k}^* f''(\hat{k}^*) & -(1+\frac{a}{A}) \\ f''(\hat{k}^*)\frac{1}{\theta}(1+\frac{\gamma}{A}) - \frac{\gamma}{A}a\left(\frac{f'(\hat{k}^*)\hat{k}^* - f(\hat{k}^*)}{\left(\hat{k}^*\right)^2} + \frac{\hat{c}^*}{\left(\hat{k}^*\right)^2} \right) & \left(\hat{c}^* \frac{\gamma}{A}a\frac{1}{\hat{k}^*} \right) \end{bmatrix}$$

Calculating the determinant of the Jacobian and using the steady state conditions $\frac{\hat{c}^*}{\hat{k}^*} = \frac{f(\hat{k}^*)}{\hat{k}^*} - \delta$ as well as $\frac{f(\hat{k})}{\hat{k}_t} = \frac{f'(\hat{k})}{a}$ which holds for the Cobb-Douglas, I obtain:

$$\begin{aligned} Det\,|J| &= \frac{\hat{c}^*}{\theta}\frac{f''(\hat{k}^*)}{A}\left(\frac{u_{cl}l}{u_c} - \frac{u_{ll}l}{u_l} - 1 \right) \text{ and} \\ Det\,|J| &< 0 \text{ for saddle path stability} \end{aligned}$$

It is evident that when $\theta = 1 \Rightarrow Det\,|J| = \hat{c}^* \frac{f''(\hat{k}^*)}{A}\left(-\frac{u_{ll}l}{u_l} - 1 \right) < 0 \forall \hat{k}_t$ which implies that saddle path stability always exists and no further assumptions must be made.

When $\theta \neq 1$ however more assumptions need to be made to ensure a negative determinant sign.

More specifically when $\theta < 1$ the term in parentheses in negative and thus A must also be negative for saddle path stability to exist i.e. $\left(1 - \frac{1}{\theta}\right)\frac{U_{cl}l}{U_c} < \frac{U_{ll}l}{U_l} + a.$

When $\theta > 1$ the term in parentheses is either negative or positive. If it is negative then A

must also be negative for saddle path stability to exist i.e. $\left(1-\frac{1}{\theta}\right)\frac{U_{cl}l}{U_c} < \frac{U_{ll}l}{U_l}+a$. If the term in parentheses is positive, i.e.$\frac{u_{cl}l}{u_c} > \frac{u_{ll}l}{u_l}+1$ then A must also be positive for saddle path stability to hold i.e. $\left(1-\frac{1}{\theta}\right)\frac{U_{cl}l}{U_c} > \frac{U_{ll}l}{U_l}+a$.

The analysis for more complex cases is identical and the difference lies in the assumptions one has to make as well as on the fact that in some cases one has to study a 3x3 system. What is important in the analysis of this simple case is that saddle path stability can be ensured under plausible assumptions.

Appendix E On Indeterminacy Under Flexible Normality Assumptions

It is a standard result in the indeterminacy literature that under certain assumptions, usually that of increasing returns, for indeterminacy to occur the slope of the labor demand curve must be steeper that the slope of the constant labor supply curve, when the utility is separable in consumption and leisure. Alternatively for the non-separable case, it must be steeper than the slope of the Frisch labor supply curve. Ayiagari (1994) summarizing the various cases for which indeterminacy may rise, points out that a necessary condition is the labor demand curve to be steeper than the labor supply curve. However he refers to separable utility functional forms which do not allow for the possibility that consumption is an inferior good. The same result applies and in all other cases that study indeterminacy taking for granted normality of consumption, among others Benhabib and Farmer (1994), Bennett and Farmer (2000), Meng and Yip (2008).

I investigate a case that applies to non-separable utility, where this result is reversed i.e. the slope of the Frisch labor supply must be steeper than the slope of the labor demand curve, as well as that the constant labor supply and labor demand curves both cross with the wrong slopes. For this case to arise consumption must be an inferior good while labor supply must be a normal good. The assumption that consumption is an inferior good in not empirically plausible and is usually not met in this strand of literature, however it has occurred in empirical estimates. Mankiw, Rotemberg and Summers (1985) use aggregate US data to estimate consumption and labor decisions and investigate whether they satisfy the standard model of a representative optimizing individual. Their findings suggest that the actual data do no fit well the model, the utility functional form that occurs from the data is non-separable and it is usually convex. But even when utility is concave then either consumption or leisure is an inferior good. They attribute this result to the fact that over the business cycle, consumption and leisure move in the opposite direction.

Apart from the interpretation that consumption is countercyclical with respect to inferior goods, there is as well the literature that attributes non-normality of consumption to the other elements of the utility function. Gould (1981) states the case where consumption of one good (leisure would be the case in our model) may interfere in the enjoyment of the other good and may reduce the marginal utility of the former good. He also refers to time constraints, as defined by Becker (1965) where time appears explicitly as a constraint on household production and consumption activities, as a plausible reason why consumption of goods that are not inferior, may decrease as output increases. Another explanation, which would be plausible for certain labor supply forms would be that of Garrat (1997), where the presence of an indivisible good (labor could be an indivisible good if I assume that workers work in blocks of hours) may cause a divisible good to be inferior and possibly Giffen.

I also find that the constant labor supply curve has negative slope, a result met in Farmer&Guo (1995), in which case the normal curves are simply reversed. In the Farmer and Guo case, an upward shift in labor supply will increase labor, output and future capital thus ratifying the optimist expectations. This result has attached much criticism since according to Ayiagari, the authors are considered to have actually induced their result by inducing an upward sloping labor demand curve along with an estimation procedure that reconciles the model with roughly zero productivity/labor correlation.

Our approach is not aimed to empirically test the model under the non-normality assumption. A breakthrough in this literature would be to decrease the degree of returns to plausible levels, an aim that cannot be achieved via the use of such a standard model as Hintermaier (2003) has proved. Our aim is to reverse the standard argument concerning the labor supply and demand curves by making more flexible assumptions concerning normality of the arguments in the utility function. Specifically I focus on relaxing the assumption of consumption normality, an issue that has not been analyzed, contrary to the case where normality of leisure is relaxed.

E.1 The Model

The model follows closely the analysis of Bennett and Farmer. I adopt the same production technology and I just allow for a generalization of the utility function, following the one given in King, Plosser and Rebello (1987) to allow for negative values of the labor supply elasticity and non-normality of consumption.

E.1.1 Production

I adopt the model with externalities in production technology at the aggregate level, while individuals and firms behave competitively. Each firm produces a homogeneous commodity using the following technology:

$$Y = K^a L^b A \tag{E.1}$$

where $a + b = 1$, thus implying a constant returns to scale technology and $A > 0$. A denotes the aggregate productivity shock which is taken as given by individuals. In aggregate terms though, A is determined by the aggregate activity of firms and is thus given by:

$$A = \bar{K}^{\alpha} L^{\beta} \tag{E.2}$$

Taking this into account, the social production function is given by:

$$Y = K^{\alpha} L^{\beta} \tag{E.3}$$

where $1 > \alpha > a$, $\beta > b$ and $\alpha + \beta > 1$.

Factors' shares of national income are given by:

$$b = \frac{wL}{Y} \quad \text{(E.4)}$$

$$a = \frac{rK}{Y} \quad \text{(E.5)}$$

where w and r denote respectively the wage rate and the rental rate. It is evident that factor shares in national income, a and b differ form the social marginal products α and β due to the presence of externalities. The degree of externalities is given by m where $\alpha = am$ and $\beta = bm$.

E.1.2 The Consumer's Problem

Contrary to the functional form Bennett and Farmer use, I use the following functional form as proposed by King, Plosser and Rebello.

$$u(C,L) = \frac{C^{1-\sigma}v(L)}{1-\sigma}, \text{ when utility is multiplicatively separable}$$

$$and \quad \text{(E.6)}$$

$$u(C,L) = \log(C) + v(L), \text{ when utility is additively separable and } \sigma = 1 \quad \text{(E.7)}$$

As the paper extends the analysis for the non-separable utility functional form, I will stick with the multiplicatively separable form and more analytically I will use the following utility function:

$$u(C,L) = \frac{C^{1-\sigma}e^{-(1-\sigma)\frac{L^{1+\gamma}}{1+\gamma}} - 1}{1-\sigma}, \text{ i.e. } V(L) = e^{-(1-\sigma)\frac{L^{1+\gamma}}{1+\gamma}}$$

The reasons for using this utility function can be derived from the King, Plosser and Rebello paper. Quoting them, the feasible steady state is compatible with an optimal competitive equilibrium by imposing two restrictions on preferences: (i) the intertemporal elasticity of substitution in consumption must be invariant to the scale of consumption, and (ii) the income and substitution effects associated with sustained growth in labor productivity must not alter labor supply (in steady state).

In this paper I have one more reason for using this functional form and the reason is that this formulation allows for negative values of γ, an assumption crucial for our results. The plausibility of negative values for γ will be discussed in a later part of the paper.

From U_{LL} it is obvious that when $\sigma < 1 \Rightarrow \gamma > 0$ as well as that when $\gamma < 0 \Rightarrow \sigma > 1$. Further restrictions are necessary to ensure that U_{LL} takes negative values, more analytically:

$$\begin{aligned}
\text{if } \gamma &> 0 \;\&\; \sigma > 1 \;\Rightarrow U_{LL} < 0 \\
\text{if } \gamma &> 0 \;\&\; \sigma < 1 \;\Rightarrow U_{LL}, < 0 \text{ when } L^{1+\gamma} < \frac{\gamma}{1-\sigma} \\
\text{if } \gamma &< 0 \;\Rightarrow \sigma > 1 \;\Rightarrow U_{LL} < 0 \text{ when } L^{1+\gamma} > \frac{\gamma}{1-\sigma}
\end{aligned}$$

Also note that as far as general functional forms are concerned, $\sigma < 1 \Rightarrow v'(L) < 0 \Rightarrow v''(L) < 0$, while $\sigma > 1 \Rightarrow v'(L) > 0 \Rightarrow v''(L) > 0$. Bennett and Farmer have examined only the case where $v'(L) < 0 \Rightarrow v''(L) < 0$.

Finally for concavity to hold, the following conditions are necessary:

$$U_{CC} < 0 \quad \text{and} \quad U_{CC}U_{LL} - U_{CL}^2 > 0$$

For the last condition to hold:

$$\begin{aligned}
\text{if } \gamma &> 0 \;\&\; \sigma > 1 \;\Rightarrow L^{1+\gamma} > \frac{\sigma\gamma}{1-\sigma} \\
\text{if } \gamma &> 0 \;\&\; \sigma < 1 \;\Rightarrow L^{1+\gamma} < \frac{\sigma\gamma}{1-\sigma} \\
\text{if } \gamma &< 0 \;\Rightarrow \sigma > 1 \;\Rightarrow\; L^{1+\gamma} > \frac{\sigma\gamma}{1-\sigma}
\end{aligned}$$

or if I use general functional forms:

$$-\frac{\sigma}{1-\sigma}v(L)v''(L) > (v'(L))^2$$

The maximization problem is given by:

$$\max \int_0^\infty u(C, L)e^{-\rho t}dt$$

$$\begin{aligned} s.t \qquad \dot{K} &= (r-\delta)K + wL - C \\ K(0) &= K_0 \end{aligned}$$

and the no-Ponzi constraint. $0 < \delta < 1$ is the depreciation rate and $\rho > 0$ is the discount rate.

The FOC's of this problem are:

$$\frac{\partial H}{\partial C} = 0 \Rightarrow C^{-\sigma}v(L) = \Lambda \tag{E.8}$$

where Λ is the co-state variable.

$$\frac{\partial H}{\partial L} = 0 \Rightarrow -\frac{C^{1-\sigma}}{1-\sigma}v'(L) = \Lambda w \tag{E.9}$$

From equations (E.8), (E.9) and (E.4) I obtain:

$$b\frac{Y}{L} = w = -\frac{1}{1-\sigma}C\frac{v'(L)}{v(L)} \tag{E.10}$$

which gives an expression for the real wage in terms of marginal disutility of labor supply and the marginal utility of consumption.

Finally the dynamic equation for Λ is given by:

$$\dot{\Lambda} = \Lambda\left(\rho + \delta - a\frac{Y}{K}\right) \tag{E.11}$$

Using equation (E.11) and the dynamic equation for the evolution of capital which is given by $\frac{\dot{K}}{K} = \frac{Y}{K} - \delta - \frac{C}{K}$, as well as the fact that lowercase letters denote the logarithms of the respective uppercase letters, I obtain a system of dynamic equations:

$$\dot{\lambda} = \rho + \delta - ae^{y-k} \tag{E.12}$$

$$\dot{k} = e^{y-k} - \delta - e^{c-k} \tag{E.13}$$

Our aim is to formulate a dynamic system in terms of the variables k and λ and then study its stability properties around the steady state.

E.1.3 Steady State Analysis

To further analyze the system and be able to find the steady state of the system I must introduce some notation:

$$h(L) = -\frac{1}{1-\sigma}\frac{v'(L)}{v(L)} > 0$$

and

$$h'(L) = -\frac{1}{1-\sigma}\frac{v''(L)v(L) - v'(L)^2}{v(L)^2} \gtrless 0$$

It is now evident that there are some differences to the analysis of the model when compared to the Bennett and Farmer model, especially when it comes to the behavior of $h'(L)$. More analytically about $h'(L)$:

$$\text{when } \sigma < 1 \Rightarrow h'(L) > 0$$

$$\text{when } \sigma > 1 \Rightarrow h'(L) \gtrless 0 \text{ when } v(L)v''(L) \gtrless (v'(L))^2$$

What is important concerning the implications of this utility specification is to test whether there actually exists a steady state in this model. $v'(0)$ is bounded, $v(0)$ is bounded and positive and the same goes for $h(0)$. Following the notation introduced above I denote the logarithms of steady state uppercase variables with the lowercase variables l^*, k^*, y^*, c^*. It is easy to obtain steady state variables for $y^* - k^*$ and $c^* - k^*$ by setting $\dot{\lambda} = 0$ and $\dot{k} = 0$ from equations (E.12) and (E.13).

$$y^* - k^* = \ln(\frac{\rho + \delta}{a}) \tag{E.14}$$

$$c^* - k^* = \ln(\frac{\rho + \delta(1-a)}{a}) \tag{E.15}$$

$$y^* - c^* = \ln(\frac{\rho + \delta}{\rho + \delta(1-a)}) \text{ which can be derived from the first two equations}$$

By taking logarithms of equation (E.10) at the steady state I obtain:

$$l^* + \ln h(L^*) = \ln(b) + y^* - c^* \tag{E.16}$$

I notice that as $L^* \rightarrow 0 \Rightarrow f(L^*) \rightarrow -\infty$ while as $L^* \rightarrow 0 \Rightarrow f(L^*) \rightarrow \infty$, where $f(L^*) = \log(L^*) + \ln h(L^*)$. The limiting values remain the same and this happens due to the fact that $h(L^*)$ is finite. However $f(L^*)$ may be non-monotonous when $\sigma > 1$, since now $h(L^*)$ may manifest different behavior than that of $\log(L^*)$. This implies that for $\sigma > 1$ more restrictions are required in order to ensure only one positive value for L^* for which eq. (E.16) holds. After having established a value for L^* I can compute l^* and by using equations (E.14) and (E.15) solve for y^*, k^* and c^*.

E.1.4 Dynamic Equilibria

To define the equilibrium, I need a time path for λ and k which are determined by the system:

$$\dot{\lambda} = \rho + \delta - ae^{y-k} \tag{E.17}$$

$$\dot{k} = e^{y-k} - \delta - e^{c-k} \tag{E.18}$$

and a set of time paths for the variables c, l and y that satisfy the conditions:

$$y = \alpha k + \beta l \tag{E.19}$$

$$\log(b) + y - l = c + \log h(L) \tag{E.20}$$

$$-\sigma c + \log v(L) = \lambda \tag{E.21}$$

which come from taking logarithms of equations (E.3), (E.10) and eq.(E.8).

I want to linearize the above equations around the steady state

$$\begin{aligned} y &= y^* + \alpha(k - k^*) + \beta(l - l^*) \Rightarrow \qquad (E.22) \\ \tilde{y} &= \alpha\tilde{k} + \beta\tilde{l} \end{aligned}$$

where tilde's denote deviations from the steady state.

Correspondingly:

$$\begin{aligned} l &= l^* - (c - c^*) + (y - y^*) + \frac{h'(L^*)}{h(L*)} L^* \frac{\partial L^*}{\partial l}(l - l^*) \Rightarrow \qquad (E.23) \\ \tilde{l}(1+\gamma) &= \tilde{y} - \tilde{c} \end{aligned}$$

where $\gamma = \frac{L^* h'(L^*)}{h(L^*)}$. Note that when

$$\sigma < 1 \Rightarrow h'(L) > 0 \Rightarrow \gamma > 0$$

$$\sigma > 1 \Rightarrow h'(L) \gtrless 0 \Rightarrow \gamma \gtrless 0$$

Finally

$$\begin{aligned} \lambda &= \lambda^* - \sigma(c - c^*) + \frac{v'(L^*)}{v(L*)} L^* \frac{\partial L^*}{\partial l}(l - l^*) \Rightarrow \\ \tilde{\lambda} &= -\sigma\tilde{c} + \psi\tilde{l} \end{aligned} \tag{E.24}$$

where $\psi = \frac{L^* v'(L^*)}{v(L^*)}$ and $\psi \gtrless 0$ when $\sigma \gtrless 1$.

Concerning the interpretation of γ and ψ, γ is the slope of the labor supply curve, while ψ denotes the share of wages relative to consumption.

E.1.5 Local Dynamics

In order to analyze the dynamics of the system around the steady state I must express it only in terms of k and λ. For this to be done I must find analytic expressions for $\tilde{y} - \tilde{k}$ and $\tilde{c} - \tilde{k}$, in terms of k and λ and replace them in equations (E.12) and (E.13). I first rewrite the system of (E.12) and (E.13) in matrix form:

$$\begin{bmatrix} 1 & 0 \\ 0 & \sigma \end{bmatrix} \begin{bmatrix} \tilde{y} - \tilde{k} \\ \tilde{c} - \tilde{k} \end{bmatrix} + \begin{bmatrix} -\beta \\ -\psi \end{bmatrix} \tilde{l} + \begin{bmatrix} 0 & 1-\alpha \\ 1 & \sigma \end{bmatrix} \begin{bmatrix} \tilde{\lambda} \\ \tilde{k} \end{bmatrix} = 0 \tag{E.25}$$

I also rewrite eq.(E.23) in terms of the same linear combination of variables:

$$\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} \tilde{y} - \tilde{k} \\ \tilde{c} - \tilde{k} \end{bmatrix} + (1+\gamma)\tilde{l} = 0 \tag{E.26}$$

Divide the second row of (E.25) by σ :

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{y}-\tilde{k} \\ \tilde{c}-\tilde{k} \end{bmatrix} + \begin{bmatrix} -\beta \\ -\frac{\psi}{\sigma} \end{bmatrix} \tilde{l} + \begin{bmatrix} 0 & 1-\alpha \\ \frac{1}{\sigma} & 1 \end{bmatrix} \begin{bmatrix} \tilde{\lambda} \\ \tilde{k} \end{bmatrix} = 0 \tag{E.27}$$

Divide eq. (E.26) by (1+γ):

$$\begin{bmatrix} -\frac{1}{1+\gamma} & \frac{1}{1+\gamma} \end{bmatrix} \begin{bmatrix} \tilde{y}-\tilde{k} \\ \tilde{c}-\tilde{k} \end{bmatrix} + \tilde{l} = 0 \tag{E.28}$$

I solve eq. (E.28) for $\tilde{l}$ and substitute it in eq. (E.28).

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{y}-\tilde{k} \\ \tilde{c}-\tilde{k} \end{bmatrix} + \begin{bmatrix} -\beta\frac{1}{1+\gamma} & \beta\frac{1}{1+\gamma} \\ -\frac{\psi}{\sigma}\frac{1}{1+\gamma} & \frac{\psi}{\sigma}\frac{1}{1+\gamma} \end{bmatrix} \begin{bmatrix} \tilde{y}-\tilde{k} \\ \tilde{c}-\tilde{k} \end{bmatrix} + \begin{bmatrix} 0 & 1-\alpha \\ \frac{1}{\sigma} & 1 \end{bmatrix} \begin{bmatrix} \tilde{\lambda} \\ \tilde{k} \end{bmatrix} = 0 \Rightarrow \tag{E.29}$$

$$\begin{bmatrix} 1-\beta\frac{1}{1+\gamma} & \beta\frac{1}{1+\gamma} \\ -\frac{\psi}{\sigma}\frac{1}{1+\gamma} & 1+\frac{\psi}{\sigma}\frac{1}{1+\gamma} \end{bmatrix} \begin{bmatrix} \tilde{y}-\tilde{k} \\ \tilde{c}-\tilde{k} \end{bmatrix} + \begin{bmatrix} 0 & 1-\alpha \\ \frac{1}{\sigma} & 1 \end{bmatrix} \begin{bmatrix} \tilde{\lambda} \\ \tilde{k} \end{bmatrix} = 0 \tag{E.30}$$

I set $A = \begin{bmatrix} 1-\beta\frac{1}{1+\gamma} & \beta\frac{1}{1+\gamma} \\ -\frac{\psi}{\sigma}\frac{1}{1+\gamma} & 1+\frac{\psi}{\sigma}\frac{1}{1+\gamma} \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1-\alpha \\ \frac{1}{\sigma} & 1 \end{bmatrix}$ and thus the above equation can be written as:

$$A \begin{bmatrix} \tilde{y}-\tilde{k} \\ \tilde{c}-\tilde{k} \end{bmatrix} + B \begin{bmatrix} \tilde{\lambda} \\ \tilde{k} \end{bmatrix} = 0$$

and solving for $\begin{bmatrix} \tilde{y}-\tilde{k} \\ \tilde{c}-\tilde{k} \end{bmatrix}$ I obtain:

$$\begin{bmatrix} \tilde{y}-\tilde{k} \\ \tilde{c}-\tilde{k} \end{bmatrix} = \Phi \begin{bmatrix} \tilde{\lambda} \\ \tilde{k} \end{bmatrix}$$

where

$$\Phi = -A^{-1}B = \begin{bmatrix} \phi_1 & \phi_2 \\ \phi_3 & \phi_4 \end{bmatrix}, A^{-1} = \frac{1}{Det\,|A|} \begin{bmatrix} 1+\frac{\psi}{\sigma}\frac{1}{1+\gamma} & -\beta\frac{1}{1+\gamma} \\ \frac{\psi}{\sigma}\frac{1}{1+\gamma} & 1-\beta\frac{1}{1+\gamma} \end{bmatrix}$$

and

$$Det\,|A| = \frac{\sigma(1+\gamma)+\psi-\beta\sigma}{\sigma(1+\gamma)}$$

I also set $\eta = -\sigma(1+\gamma)-\psi+\beta\sigma$ and thus $Det\,|A|$ can be written as: $Det\,|A| = -\frac{\eta}{\sigma(1+\gamma)}$.

Using all the above I can easily derive the elements of matrix Φ.

$$\Phi = \begin{bmatrix} -\frac{\beta}{\eta} & \frac{(1-\alpha)\sigma(1+\gamma)+(1-\alpha)\psi-\beta\sigma}{\eta} \\ \frac{(1+\gamma)-\beta}{\eta} & \frac{(1-\alpha)\psi+\sigma(1+\gamma)-\beta\sigma}{\eta} \end{bmatrix}$$

Using the elements of Φ I can now approximate:

$$\begin{bmatrix} \tilde{y}-\tilde{k} \\ \tilde{c}-\tilde{k} \end{bmatrix} = \Phi \begin{bmatrix} \tilde{\lambda} \\ \tilde{k} \end{bmatrix}$$

or :

$$\tilde{y}-\tilde{k} = \phi_1\tilde{\lambda}+\phi_2\tilde{k} \tag{E.31}$$

$$\tilde{c}-\tilde{k} = \phi_3\tilde{\lambda}+\phi_4\tilde{k} \tag{E.32}$$

I substitute eq. (E.31) and (E.32) into the system of (E.12) and (E.13) to obtain:

$$\dot{\lambda} = \rho+\delta-ae^{\phi_1\tilde{\lambda}+\phi_2\tilde{k}+0_0} \tag{E.33}$$

$$\dot{k} = e^{\phi_1\tilde{\lambda}+\phi_2\tilde{k}+0_0}-\delta-e^{\phi_3\tilde{\lambda}+\phi_4\tilde{k}+0_1} \tag{E.34}$$

where $0_o = y^*-k^*$ and $0_1 = c^*-k^*$ are constants that do not influence the dynamics.

I linearize the system of (E.33) and (E.34) around the steady state to obtain:

$$\begin{bmatrix} \dot{\lambda} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} -a\phi_1 e^{0_0} & -a\phi_2 e^{0_0} \\ \phi_1 e^{0_0}-\phi_3 e^{0_1} & \phi_2 e^{0_0}-\phi_4 e^{0_1} \end{bmatrix} \begin{bmatrix} \lambda-\lambda^* \\ k-k^* \end{bmatrix}$$

Provided that a steady state actually exists, I can replace for $0_o = y^*-k^*$ and $0_1 = c^*-k^*$ with their steady state values which are given by: $0_o = y^* - k^* = ln(\frac{\rho+\delta}{a})$ and $0_1 = c^* - k^* = \ln(\frac{\rho+\delta}{a} - \delta)$. Using these steady state values I obtain:

$$\begin{bmatrix} \dot{\lambda} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} -a\phi_1 \frac{\rho+\delta}{a} & -a\phi_2 \frac{\rho+\delta}{a} \\ \phi_1 \frac{\rho+\delta}{a} - \phi_3(\frac{\rho+\delta}{a} - \delta) & \phi_2 \frac{\rho+\delta}{a} - \phi_4(\frac{\rho+\delta}{a} - \delta). \end{bmatrix} \begin{bmatrix} \lambda - \lambda^* \\ k - k^* \end{bmatrix}$$

The Jacobian of the system, based on which I will determine the necessary conditions for indeterminacy to hold, is given by: $J = \begin{bmatrix} -a\phi_1 \frac{\rho+\delta}{a} & -a\phi_2 \frac{\rho+\delta}{a} \\ \phi_1 \frac{\rho+\delta}{a} - \phi_3(\frac{\rho+\delta}{a} - \delta). & \phi_2 \frac{\rho+\delta}{a} - \phi_4(\frac{\rho+\delta}{a} - \delta). \end{bmatrix}$. Calculating for the sign of the determinant of matrix J I obtain the following formulation, $Det\,|J| = a(\phi_1\phi_4 - \phi_2\phi_3)(\frac{\rho+\delta}{a})\frac{\rho+\delta(1-a)}{a}$. It is obvious that since $\rho > 0, \delta > 0, a > 0$, the sign of $Det\,|J|$ depends on the term $(\phi_1\phi_4 - \phi_2\phi_3)$ which equals $Det\,|\Phi|$.

So the following holds $sign\,(Det\,|J|) = sign\,(Det\,|\Phi|)$ and $Det\,|\Phi| = \frac{Det|B|}{Det|A|}$. It is easy to find that $Det\,|B| = -\frac{1}{\sigma}(1-\alpha) < 0$, which means that $sign\,(Det\,|\Phi|) = -sign\,(Det\,|A|)$. To sum up:

$$sign\,(Det\,|J|) = sign\,(Det\,|\Phi|) = -sign\,(Det\,|A|)$$

For indeterminacy to occur, the number of predetermined variables must be less than the number of negative roots. As I have one initial value for k_0, I must have two negative eigenvalues and thus $Det\,|J| > 0 \Rightarrow Det\,|A| < 0$. I saw earlier that

$$Det\,|A| = -\frac{\eta}{\sigma(1+\gamma)} \tag{E.35}$$

$$where\;\eta = -\sigma(1+\gamma) - \psi + \beta\sigma \tag{E.36}$$

E.1.6 Labor Supply and Demand Functions

Before studying the $sign\,(Det\,|A|)$ I must obtain the slopes of the constant labor supply, the Frisch labor supply and the labor demand function, which will prove useful for the analysis that follows.

To analyze labor supply functions I must distinguish between constant labor supply and Frisch labor supply. The constant labor supply is defined as, labor supply as a function of the real wage holding constant the consumption, while the Frisch labor supply is defined as, labor supply as a function of the real wage holding constant the marginal utility of consumption. To obtain the constant labor supply I replace for $v(L)$ and take logarithms of eq. (E.10).

$$\ln w = \text{constant} + \gamma l \tag{E.37}$$

where γ is the slope of the labor supply curve The constant labor supply curve is the same as the one in Bennett and Farmer and Benhabib and Farmer.

To get the Frisch labor supply I must take logarithms of eq.(E.8) and totally differentiate:

$$dc = -\frac{1}{\sigma}d\lambda + \frac{1}{\sigma}\frac{1}{L}\psi dL$$

Doing the same for eq.(E.10) I get:

$$dc = \frac{1}{w}dw - \gamma\frac{1}{L}dL$$

Equating the two equations, integrating and solving for $\ln w$:

$$\ln w = \text{constant} - \frac{1}{\sigma}\lambda + (\frac{1}{\sigma}\psi + \gamma)l$$

which gives the Frisch labor supply. An important note concerning the Frisch labor supply which is equal to $(\frac{1}{\sigma}\psi + \gamma)$, is that if concavity holds, the Frisch labor supply must always be positively sloped as Hintermaier has proved for a general utility functional form which satisfies standard assumptions such as concavity, continuity and differentiability. This point will prove to be very useful for our future analysis as it indicates that the Frisch labor supply will always be positively curved as the theory predicts.

Finally the labor demand curve is derived from maximizing the social production function and is given by:

$$\ln w = \text{constant} + \alpha k + (\beta - 1)l$$

Hintermaier has also proved that for indeterminacy to hold, $\beta - 1 > 0$, otherwise the trace of the J matrix, which must be negative for indeterminacy to hold, will always be positive. This means that for indeterminacy to occur, the slope of the labor demand curve, i.e. $\beta - 1$, must be positive, contrary to the standard theory that predicts a negative slope.

E.2 Indeterminacy

For indeterminacy to occur two assumptions must be satisfied:

i) $Det\,|A| < 0$

ii) $Trace\,|J| < 0$

E.2.1 Determinant

From this point onwards I must make more assumptions for the study of $sign\,(Det\,|A|)$.

Case A. The first case I will study is the case where $\gamma > 0$. Since $Det\,|A| = -\frac{\eta}{\sigma(1+\gamma)}$ and $(1+\gamma) > 0$ the sign of $Det\,|A|$ depends on the sign of $-\eta$.

$$\begin{aligned} Det\,|A| &< 0 \Rightarrow -\eta < 0 \Rightarrow \eta > 0 \Rightarrow \\ -\sigma(1+\gamma) - \psi + \beta\sigma &> 0 \Rightarrow \beta - 1 > \gamma + \frac{1}{\sigma}\psi. \end{aligned}$$

The economic meaning of the above inequality is that the labor demand and the Frisch labor supply curve should cross with the wrong slopes, and since both terms of the inequality are positive, this means that the labor demand curve should have positive slope and be flatter than the

Frisch labor supply curve. Note that the constant labor supply curve has the right slope, since $\gamma > 0$.

It is evident that this inequality nests the special case of Bennett and Farmer. If I use their specification of utility I obtain:

$$\beta - 1 > \gamma + \frac{\sigma - 1}{\sigma}\psi$$

Case B. I will assume that $\gamma < 0$ ($\Rightarrow \sigma > 1$) and $\gamma > -1$. This case is similar as above, since the sign of $Det\,|A|$ still depends on the sign of $-\eta$. The only difference now lies in the fact that $\gamma < 0$ which implies that the constant labor supply curve is negatively sloped and thus the labor demand curve and the constant labor supply curve cross with the wrong slopes, a result that has been empirically obtained by Guo and Farmer and will be further analyzed below.

Case C. The last case and the one emphasized in this paper is that $\gamma < 0$ ($\Rightarrow \sigma > 1$) and $\gamma < -1$. In this case the sign of $Det\,|A|$ does not depend only on the sign of $-\eta$ since now $1 + \gamma < 0$. This implies that :

$$-\eta > 0 \Rightarrow \eta < 0 \Rightarrow \beta - 1 < \gamma + \frac{1}{\sigma}\psi \tag{E.38}$$

$$and$$

$$\beta - 1 > \gamma \tag{E.39}$$

E.2.2 Non-Normality in Consumption

As far as case C is concerned, the importance of our parametrization lies in the fact that I may still have increasing returns, however the slope of the labor demand curve does not have to be steeper than the Frisch labor supply curve, which is positively sloped. Furthermore, the constant labor supply curve is negatively sloped since $\gamma < -1$ which means that both curves have the "wrong slope". This result occurs due to the fact that in the case where $\gamma < 0$, I note that leisure is a normal good, i.e. $U_l U_{cc} - U_c U_{cl} \geqslant 0$, however consumption is not a normal good since $U_l U_{cl} - U_c U_{ll} \geqslant 0$.

Additionally note in case B, where the standard indeterminacy condition is obtained consumption is still a non-normal good since $\gamma < 0$. So overall the non-normality assumption can reproduce the standard in the literature result and reverse it as well.

E.2.3 Trace

The second condition that must be satisfied for indeterminacy to occur is that the trace of $Det\,|J| < 0$,

$$Tr\,|J| = \frac{\rho}{\eta}\beta - \frac{\rho\alpha\sigma(1+\gamma)}{a\eta} + \frac{\delta}{\eta}\beta(1-\sigma) + \frac{\delta}{\eta}(1-\alpha)\psi + \frac{\delta}{\eta}\sigma(1+\gamma)\left(1-\frac{\alpha}{a}\right) < 0$$

which can hold under certain assumptions, that vary depending on the value of γ. Note however that when $\gamma < -1$, the trace is always negative.

I can derive the following proposition:

Proposition 1. Under the assumption of increasing returns and a negative slope of the constant labor supply curve, that implies $\sigma > 1$ and $U_{cl} > 0$, indeterminacy can occur when the slope of the Frisch labor supply curve is steeper than the slope of the labor demand curve, i.e. $\beta - 1 < \gamma + \frac{1}{\sigma}\psi$ and the $Tr\,|J| < 0$.

E.2.4 Applying the Hintermaier Theorem

It is important to make sure that this case does not violate the impossibility theorem of Hintermaier. For this purpose I use his general functional form in which I implement my utility functional form to find out if I can obtain the same results.

First note that with my utility function:

$$\delta_{cc} = -\sigma, \quad \delta_{cL} = -(1-\sigma)L^{1+\gamma}$$
$$\delta_{Lc} = 1-\sigma, \quad \delta_{LL} = \gamma - L^{1+\gamma}(1-\sigma)$$

Using his definitions for the determinant and the trace I observe that:

$$\begin{aligned} Det\,|J| &= -(\rho+\delta(1-a))\frac{\rho+\delta}{a\tau}\left[-\beta(1+\delta_{cc}-\delta_{Lc})+(1-\alpha)(1-\delta_{cL}+\ \delta_{LL})\right] \\ where\,\tau &= -\delta_{cc}\delta_{LL}-\delta_{cc}+\beta\delta_{cc}+\delta_{cL}\delta_{Lc} \end{aligned}$$

If I apply the assumption I made for case C above, i.e. $\gamma < 0$ ($\Rightarrow \sigma > 1$ and $U_{cl} > 0$) and $\gamma < -1$ I notice that the sign of $Det\,|J|$ depends on the sign of τ which must be positive. If I substitute with the appropriate form I obtain condition (E.38), $\beta - 1 < \gamma + \frac{1}{\sigma}\psi$.

Following the same procedure for $Tr\,|J|$ I find that $Tr\,|J| = \frac{-\beta(\rho+\delta)}{\tau} - \frac{\alpha}{a\tau}(-\sigma(1+\gamma))(\rho+\delta)) < 0$ since $\gamma < -1$ and $\tau > 0$.

Thereby indeterminacy holds without violating the Hintermaier Impossibility Theorem.

E.3 Empirical Evidence

As Ayiagari was quick to point out, constructing empirically plausible business-cycles models with animal spirits needs more work. In this strand of literature, empirically plausible models would imply increasing returns whose degree approximates 1.09, as Basu and Fernald (1997) have estimated for the American economy, as well as other estimations for the values of σ, γ and other parameters. For instance, a non-separable utility function is a plausible assumption and there seems to be econometric evidence against a logarithmic utility function in consumption.

Pelloni and Waldman (1997), in an endogenous growth model, find evidence that when $\sigma<1$, indeterminacy occurs around any given balanced growth path, at the cost of unrealistically high degree of increasing returns. Additionally Bennett and Farmer mention that in their model indeterminacy can occur for values of $\sigma \gtrless 1$ however they claim that indeterminacy is more likely for low values of σ. In our model, $\sigma > 1$ is a prerequisite for this specific case to arise.

When labor supply is fixed, evidence is also in favor of a value of σ greater than unity. A strand of the business cycles literature often uses the estimates of Hansen and Singleton, who find a range of point estimates between 0 and 2.

Our model fails to decrease the degree of returns to empirically plausible levels, as is the case with all the other one-sector models, since Hintermaier has proved that if $\beta = bm > 1$ which implies that for a plausible b, i.e. for $b = 0.7$ then m must be at least 1.47 for increasing returns to exist. The degree of increasing returns around 1.47 though, is still consistent with the Caballero and Lyons (1992) as well as with the estimations of Baxter and King (1991) who suggest an elasticity of social output with respect to labor of around 1.5.

However I manage to reproduce the Farmer and Guo results who estimate an upward sloping labor-demand curve and a downward sloping labor-supply curve. Note that in the Farmer and Guo paper the authors use a logarithmic utility function which renders the constant labor supply and the Frisch labor supply functions equivalent. So my results differ from theirs in the sense that while the constant labor supply crosses with the wrong curve, the same does not hold for the Frisch labor supply which crosses with the right slope.

How plausible is the assumption for $\gamma < 0$? Pencavel (1986) summarizes estimates of male labor supply elasticity where the estimates based on the static model yield negative labor supply elasticities. Mankiw, Rotemberg and Summers also calculate a long-run elasticity of leisure with respect to wage of 0.26, thus implying a backward-bending long-run labor supply curve.

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Chapter 4 Moral Considerations in a Model with Political Corruption and Tax Evasion in Publicly Provided Goods

4.1 Introduction

The chapter aims to incorporate three strands of literature in a simple model of growth. The first is that of political corruption, the second is tax compliance and the third is coordination failures. More analytically I want to focus on the interaction between corruption and tax evasion in an economy with publicly provided goods. For simplicity and in order to make the context more specific I will focus on the case where the publicly provided good is education. My intention though is to formulate a model that can account for any publicly provided good that can be accumulated such as infrastructure, health, education etc.

A second reason for choosing education is to fill in a gap in the theoretical literature concerning corruption and education. There are several empirical studies (Mauro, 1997, Tanzi and Davoodi, 1997) that predict a negative relationship between corruption and public expenditure devoted to education. Strangely there is no analytical model that accounts for the direction of this relationship.

Corruption takes many forms and as Andvig (1990) has claimed, "corruption may corrupt". Two forms of corruption are introduced in the model: embezzlement and tax evasion while the purpose of this chapter is to analyze the interrelationship between these two forms. Two groups of agents are introduced in the model. The first group comprises politicians while the second comprises citizens. They both have the opportunity to behave in a corrupt[10] way, using different means though. My intention is to find out how the behavior of one group affects the behavior

[10] Note that the terms "corrupt behavior" or "corruption" will be henceforth used to describe both embezzlement on the part of the politicians and tax evasion on the part of the citizens.

of the other, or more specifically how expectations on the part of the group concerning the behavior of the other affect the group's behavior. Scholz and Lubell (1998 a,b) report evidence that tax compliance behavior is significantly negatively associated with the level of dishonesty in the government. The combination of these two behaviors results in coordination failures and multiple equilibria. Many papers have been written on coordination failures and it seems that this theory goes a long way towards explaining differences in (sometimes similar with respect to fundamentals) countries. The occurring equilibria in this model are "self-fulfilling" i.e. driven by agents expectations.

More importantly I try to find ways to eliminate multiple equilibria by developing the means or the policy tools that will help the agents coordinate to an equilibrium, hopefully to a"good" one. In this quest of policy tools I first resort to widely used policies such as deterrence policies. They prove to be unsuccessful though, since their only effect may be an increase in the relative costs associated with corruption. This surprising result is another contribution of this chapter which contradicts the standards literature of tax evasion and deterrence policies. More interestingly I find that under certain assumptions, which are related to the provision of the public good, an increase in fines may even increase tax evasion. This failure indicates that an economy needs something more than that to coordinate in a unique equilibrium. This can be realized by a real world observation where the imposition of fines is widely spread, with totally different results in different countries. It would be mistaken to claim that Scandinavian countries are so less corrupt due to the heavy penalty rates imposed on tax evasion or corruption. In any case they are definitely not stricter than the penalties in China, which may even involve death penalties for "grand theft", still though it experiences more severe corruption.

These results and these observations indicate that the solution of this problem lies somewhere else. A first explanation that comes to mind is that differences in corruption lie in differences in culture. And perhaps such a guess would be correct, at least for a majority of countries. As it is very difficult to precisely define the meaning of the word "culture" and incorporate it in a model, I have chosen to isolate some elements of it, in a way that would be easy to introduce in a simple model and more importantly to give directions for policy.

The element used is that of moral considerations and specifically the notion of "bad conscience". Moral considerations have been widely used in the literature as a means to improve the results of the standard models which predicted lower than reality compliance rates. Various kinds of moral considerations have been used in the tax evasion literature such as "social stigma" (Allingham and Sandmo, 1972), "bad conscience" (Sandmo, 2005), "individual morality" (Gordon, 1989) and "fairness" (Bodrignon, 1993). The main difference of this paper with respect to the majority of the literature is that moral considerations may actually result in a coordination failure, while in the context of this model, "bad conscience" eliminates the bad equilibrium by rendering it non-optimal for the agents. The purpose is to establish a positive result that indicates the direction towards which government policies should be oriented.

Overall the central idea is that deterrence policies alone, are not sufficient as measures against corruption. In some cases they may alleviate the problem by significantly increasing the "cost" of corrupt actions, however there is an upper bound on penalty and auditing rates, associated with costs of auditing or political cost. This paper signifies that policy makers and especially governments should turn their attention to "measures" or "policies" that cultivate the morality of agents. Moreover, the source of the coordination failures indicates that one of the most significant problems is lack of trust, an issue recently highlighted in the economic literature (Aghion et al, 2008). Only measures that improve the morals of a country and enhance trust between citizens and politicians, combined with other standard policies can go a long way towards fighting corruption.

The chapter goes as follows: In the second part, which comprises the literature review, I analyze the main aspects of each of the three topics analyzed in the paper, namely corruption, tax evasion and coordination failures. Since the relevant literature is rather extensive I try to focus on the interrelation between these strands of literature and issues that are relevant to this chapter. The analysis includes both the empirical evidence and the theoretical aspects of these topics. Part 3 analytically examines the baseline model that will form the basis for more elaborate ones. The baseline model provides simple results and clear intuition while subsequent models build upon the baseline case in order to make the results directly comparable between different models. Subsequent models, examine sequential games (Stackelberg) and in part 4 I introduce deterrence policies such as fines on evaded tax and on evaded income. Finally in part 5 of the chapter, I introduce moral considerations in the model as a means to resolve the occurring multiplicity of equilibria in the baseline model.

4.2 Literature Review

This literature review will analyze three main topics. The first and perhaps the most extensive is the issue of corruption. Corruption can take many forms and can affect the economy in various different ways. Even though in the model I will assume that political corruption is expressed through embezzlement of the collected tax revenue by the politicians, a rather simple but convenient for the model way, in this review I will refer to various forms of corruption. More importantly I will extensively refer to the effect of corruption on growth. Its disputable effect on growth is not the focus of the model, even though it is briefly studied, what is important though is that this effect takes place though various channels. One of this channels is public spending in general and spending on education more specifically, which is the focus of the model. Furthermore the interrelationship between tax evasion or shadow economy (as a proxy for tax evasion) and corruption will be examined among others. It is interesting to see if the model accords with some of the predictions of the empirical studies concerning the effects of corruption on growth.

The second topic is that of tax evasion. This has been a widely analyzed topic especially in the field of public economics. When one thinks of taxation, which possesses a central role in public finance, it comes as natural to think about tax evasion. One challenge of this literature was to define the optimal tax under the presence of tax evasion. Another challenge was to find the appropriate policies that diminish tax evasion. A last but not least interesting question in this context, was to find why the first analytical models failed in accounting for the observed compliance rates. These kinds of considerations contributed to the development of behavioral public finance, a strand of literature that despite the criticism it has received, has accounted for many empirical observations that abstract from the axiom of the rational agent.

The last topic to be reviewed in this chapter is that of coordination failures. Coordination failures have been extensively analyzed in various topics and in some cases provide a satisfactory explanation for the differentiated evolution of economies which share many common characteristics. In different contexts different methods have been used to resolve these failures while the choice of methods depends on whether the occurring multiple equilibria are self-fulfilling or path dependent.

4.2.1 Corruption

The role of the state has a prominent position in most definitions of corruption, which define corruption as a particular and rather "perverted" state-society relationship. There is a plethora of definitions, but the definitions mentioned here will be confined to the economic context. The most popular and widely used definition of corruption is the one used by the World Bank. Corruption is defined as the use of public authority for private gain, or, in other words, use of official position, rank or status by an office bearer for his own personal benefit. Economically, it is often modeled as a principal-agent problem: A principal delegates some decision power to an agent. The agent is aware of the principal's rules of preference in exercising the power. The principal's problem is that the agent may serve his own interest instead of the principal's. As Bardhan (1997) mentions, it is a common misconception that a corrupt transaction and an illicit transaction imply the same thing. It should be made clear that neither all illegal transactions are corrupt, nor are all instances of corruption illegal (for instance post-retirement jobs in private firms to bureaucrats of agencies meant to regulate them).

Alternative definitions of corruption are the following:

-"Intentional noncompliance with arm's length relationships aimed at deriving some advantage from this behavior for oneself or for related individuals." (Tanzi, 2005)

- "The sale of governmental property, by government officials, for personal gain." (Shleilfer & Vishny, 1993)

- "Where political opportunities are scarce, corruption occurs as people use wealth to buy power and where economic opportunities are few, corruption occurs when political power is used to pursue wealth" (Huntington, 1968)

4.2.1.1 Corruption Classifications

There are several classifications of corruption showing the variety of different dimensions of this phenomenon. The most important are the following:

-Briber or Bribee initiated depending on whether the bribe is offered or demanded (Tanzi, 2000).

-Self-enforcing or spontaneous (Tanzi, 2000).

-Monetary or non-monetary depending on whether an official requires monetary compensation for his service or he expects a service in return.

-Grand or Petty, depending on the amount of bribe paid or assets stolen.

-Political or bureaucratic, depending on whether corruption takes place in a political level or in a bureaucratic level (Ackerrman, 1997).

-Another classification of corruption concerning its incidence is that of well-organized and chaotic corruption (Mauro, 1997). It is also mentioned as predictable or arbitrary. Bribery is predictable when the briber knows exactly what to pay and for what. He can also be sure that his aim will be achieved and that he will not have to bribe again. The opposite case, where there is confusion and the outcome of the bribery is uncertain, constitutes arbitrary corruption.

-Rare-Widespread-Systemic: When corruption is rare, it is relatively easy to detect, isolate, punish and prevent it from becoming widespread. When corruption is widespread it is more difficult to prevent it and cure it. Finally, when corruption is systemic the institution's rules and peoples' behavior and attitudes adapt to the corrupt way of functioning, and corruption becomes a way of life. It is very difficult to overcome it and has many consequences on several domains, which are going to be discussed in further detail in the next part of this paper.

These are not the only classifications concerning corruption they are the most important though. Few others could be found in Tanzi (1998) .

4.2.1.2 Forms of Corruption

One of the studies that have characterized some main forms of manifestations of corruption according to a number of main characteristics is that of Amudnsen (1999). The main forms are:

-Bribery: Bribery is the payment (in money or kind) that is given or taken in a corrupt act. We can distinguish between bribery with theft, where the official charges a lower price than the one defined by the government and hides the money, and bribery without theft. In this case, the official charges the anticipated fee, plus the amount of the bribe.

-Embezzlement: It refers to theft of resources by people who are put to administer them. Embezzlement is not considered to be a corrupt action from a legal point of view. It is rather viewed as theft, since corruption legally occurs when there is a transaction between two individuals, one state agent and one civilian. In the case of embezzlement, there is no participation of civilians. Nevertheless, it is included in the broader definitions.

-Straddling: It is the process by which some powerful individuals use their power in order to enter or expand their private business interests.

-Fraud: Fraud is, when a public agent, who is responsible of carrying out tasks assigned by his principal, manipulates the flow of information, to his private profit. It is an economic crime that involves some kind of trickery or deceit.

-Extortion: Extortion occurs when money or other recourses are extracted by the use of coercion, violence or the threats of using force (i.e. blackmailing).

-Favoritism or Cronyism: It is a mechanism implying highly biased distribution of state resources, due to the natural human proclivity to favour family, friends or anyone else close and trusted. In the most corrupt countries, favoritism is not just a political mechanism; it is rather a common practice, especially during the period of elections.

-Nepotism: It is a special case of favoritism, where the office holder prefers appointing his friends and family in various positions and thus securing his position this way more than meritocratically.

-Influence peddling.

-Appropriation of public assets for private use.

In the above list, some activities such as fraud and embezzlement can be undertaken by an official alone and without the participation of a second party. Others, such as bribery or extortion involve two parties, the giver and the taker in the corrupt act. Such corrupt acts are: government contracts, government benefits, time saving and regulatory avoidance, influencing outcomes of legal and regulatory processes. All the above activities are usually referred as rent-seeking activities, in the sense that a bureaucrat owns an amount of economic rent. Some researchers, such as Rose-Ackerman and Wei (1999) seem to identify corruption with bribery, not a widespread view though.

4.2.1.3 Corruption Indicators

A proper corruption indicator should have several basic qualities in order to be used in empirical analysis. Namely a good corruption indicator should be:

a) Trustworthy.

b) Measuring the proper variable.

c) Accurate.

d) Precise.

In Johnston (2000), there are few other desirable qualities for corruption indicators.

A first classification of corruption indicators is that of objective and subjective indicators.

Objective Indicators: Objective quantifications based on objectively veritable information, such as the number of corruption charges in a given year, or the amount of money embezzled, or “the number of internet search engine hits of corruption charges in a year” (Tanzi, 1998). As a result, they are trustworthy and relatively precise. Nevertheless, they are not valid because corruption acts vary from country to country and what is considered to be a corrupt act in one country may be legal in another.

Subjective Indicators: “The difficulty in coming up with a valid objective quantification of corruption may explain why research has seemed to focus on “subjective measures.” (Ades, Alberto and Di Tella, 1996). These indicators are mainly based on questionnaires, where individuals are asked to assess corruption. Therefore, they are usually valid and trustworthy, but not accurate. An overview of a number of subjective governance measures is provided in Kaufmann, Kraay & Zoido-Lobaton (1999) .

Subjective indicators may be further classified to perception-based indicators and experience-based indicators.

Perception based indicators:

a) Standard and Poor’s DRI unit and Political Risk Services.

b) Data published by the Economist Intelligence Unit. It is a set of measures on risk and business attractiveness for more than 180 countries.

c) International Country Risk Guide (ICRG) Index. Produced annually since 1982, it is a private international investment risk service.

d) Global Competitiveness Report.

e) Business International (BI) Index. It is an extensive survey about a large number of political risk factors, including corruption data for 52 countries.

There are several problems concerning these indicators. Firstly, it is highly likely that perception indices are endogenous. For instance, corruption perception is influenced by the attention paid by the media to corrupt acts. It is also possible, that a scandal may lead to an increased perception of corruption. Secondly, since many indices are based on questionnaires, they are by definition untrustworthy since most agents, especially the corrupt ones, have a reduced incentive or fear to answer honestly to questionnaires. Finally, in a very corrupt country, where bribery and other forms of corruption are a common practice, individuals may be so exposed to corruption, that they consider it to be natural and, as a result, they have a wrong perception of corruption.

Experience based indicators: This kind of indicators is based on subjective experiences with corruption. Their validity and precision depend on the questions asked, while their trustworthiness and accuracy depend on the implementation of the survey. However they seem to offer the greatest potential for comparability. Furthermore, experience does not lag true corruption level the way perception does, and it is unbiased by the "headline effect".

Aggregate Corruption Indicators: All the indicators mentioned above, are directly based on polls and surveys. But the most recent and important class is that of indicators that are constructed by aggregating several primary measures.

The most popular among them, is Corruption Perception Index (CPI), which is annually published since 1995 by Transparency International (TI). In the CPI evaluated countries are ranked from 0 (the most corrupt) to 10 (the least corrupt), representing the degree to which corruption is perceived to exist among public officials and politicians. CPI uses information from various different sources and is usually referred as the "poll of polls". Its validity and trustworthiness are rather good, since the aggregate indicators are chosen to be valid and trustworthy.

Another aggregate indicator is the one provided by Kaufmann, Kraay & Zoido-Lobaton (1999), where they use twelve different corruption indicators to create a composite indicator for 155 countries. The crucial assumption they make is that each component indicator is a linear transformation of an underlying graft quantity. The composite indicator is an estimation of this quantity. Like the CPI, this indicator is trustworthy and valid, but it is imprecise and has a large standard deviation.

4.2.1.4 Causes of Corruption

Before proceeding with the various causes of corruption it is necessary to present the equation of corruption as provided by Klitgaard (1988).

C=R+D-A

In the above equation, C stands for corruption, R for economic rent and A for accountability. In brief, the equation implies that the more opportunities for economic rent (R) exist in a country, the larger the corruption will be. Similarly, the greater the discretionary powers (D) granted to administrators, the greater the corruption will be. Finally, the more administrators are held accountable (A) for their actions, the less the corruption will be.

An analytical study, concerning the causes of corruption, is provided by Tanzi (1998). In this paper only the most important causes will be presented.

-Regulations and authorizations: A large number of regulations and authorizations, not only results in heavy bureaucracy, but also gives a kind of monopoly power to the officials who inspect or authorize the various activities. Therefore the larger the number of regulations and authorizations in a country, the more extensive corruption becomes in that country.

-Taxation: Under certain circumstances such as complicated laws, excessive discretionary power of tax administrators, frequent contact between tax administrators and tax payers, corruption is far too possible to become a major problem in tax and customs administrations (De Soto, 1989).

-Provision of goods and services at below-market prices: A number of vital goods such as education, electricity, national security etc. is provided by the government at below-market prices. Despite the necessity of such a measure, limited supply and excess demand result in the creation of “black-markets” which are a form of corruption.

-Financing of parties: When public money is not available for the activities of political parties, financing may be generated by illegal and corrupt activities or by individuals who will expect a reward to their financing, if the party is finally elected.

-Quality of the bureaucracy: The existence of politically motivated hiring, patronage, nepotism and favoritism result in bad quality of bureaucracy, which in turn, results in the perpetuation of the problem. In fact, the system feeds itself.

-Level of public sector wages: The level of wages paid to civil servants is negatively correlated with the level of corruption. In fact, the lower the wage, the higher the corruption. This has also been empirically proved. (Haque and Sahay, 1996).

-Inspection ability and penalty systems: The lower the inspection ability in a country and the more lenient the penalty system is, the more likely the expansion of corruption is.

-Transparency of rules, laws and processes: The lack of transparency in rules, laws and processes creates a fertile ground for corruption.

-Examples by the leadership: A corrupt leadership is certainly a bad example for the bureaucrats. As a result, the higher the political corruption is, the higher bureaucratic corruption is.

-"Political modernization defined as the transition from an autocratic to a more democratic government is usually accompanied by increases in corruption" (Huntington, 1968). According to this view, the problem is attributed to underdeveloped institutions and thus, to a weak state machine. Development of the legal institutions has to begin first, before reforms take place. This explains the extensive corruption usually observed in transition economies.

Various other arguments and empirical tests concerning the cause of corruption can be found in Ades and Di Tella (1996).

4.2.1.5 Corruption and Growth

The issue of corruption is not a new one. Only the last three decades there has been a growing concern over the issue of corruption and its impact on the economy. What seemed to interest most researchers was the effect of corruption on economic growth. It would make sense to assume that one of the reasons for this increasing concern was the rapid and rather impressing economic development of countries that were considered to be highly corrupt.

As far as the effects of corruption on growth are concerned one can distinguish three approaches. According to the first, corruption is beneficial for growth; however, there is a limited bibliography supporting this view and it comes, mainly, from political science. The main drawback of this approach is that there is no empirical proof that such a relationship exists. The second approach indicates that corruption has a negative or even devastating effect on growth through several channels, such as direct investment. There have been many theoretical models trying to prove this effect and even more empirical models, all of them showing a negative correlation between corruption and growth. The last and less represented approach is the one that supports that corruption has absolutely no effect on growth.

Benefits from Corruption: Over thirty years ago, the political scientist Samuel Huntington (1968) stated that "...in terms of economic growth, the only thing worse than a society with a rigid, over-centralized, dishonest bureaucracy is one with a rigid, over-centralized and honest bureaucracy". What this statement implies, is that excessive regulations, authorizations and everything else implied by the word bureaucracy would remain excessive without bribery. However, with the possibility of bribery, the likelihood of a reduced "real red tape" is higher, since officials are likely to bypass some rules and regulations. Thus, the positive effect of corruption suggested by Huntington is that bribery (which is a form of corruption) results in de-regulation.

His views were as well extended to the domain of politics. As he argued "….corruption provides immediate, specific and concrete benefit to groups which might, otherwise, be thoroughly alienated by society. Corruption may, though, be functional to the maintenance of a political system in the same way that reform is". His view, though representative, is not unique. Several other political scientists, as well as economists, have supported the idea that corruption is growth enhancing. Among them, Nathaniel Leff (1964) has stated that "…if the government has erred in its decision, the course made by corruption may well be the better one" and "Corruption may introduce an element of competition into what is otherwise a comfortably monopolistic industry….and payment of the highest bribes (becomes) one of the principal criteria for allocation…. Hence a tendency towards efficiency is introduced into the system". He claims that corruption is efficiency enhancing since it may allow economically beneficial actions that erroneous policy or rigid bureaucracy would otherwise prevent.

Similar notions can be found in Leys (1965) who claims that "It is natural but wrong to assume that the results of corruption are always both and important…Were bureaucracy is both elaborate and inefficient, the provision of strong personal incentives to bureaucrats to cut red tape may be the only way of speeding the establishment of the new firm".

The above ideas are not new. They are simply an extension of the second-best idea expressed by economists. To be more specific, this idea implies that in the second-best world, a possible way of improving welfare, when there are pre-existing, policy- induced distortions, is to create more distortions (i.e. black market), even when some resources have to be spent in these activities. Of course, the ideal solution is the first-best solution, more specifically the elimination of the initial distortions.

Even in the case where the pre-existing distortions are absent, corruption could be seen as part of a Coasean bargaining process whereby bribery does not affect the allocation efficiency in government procurement contracts. The size of bribes by different economic agents could reflect their different opportunity costs. This means that better firms are more capable or more willing to buy lower effect red tape. Consequently, a license or a contract awarded on the basis of a bribe could achieve a Pareto-optimal allocation. The fact that the producer's surplus ends up to the bureaucrat instead of the principal does not seem to affect the allocation efficiency.

Sheilfer & Vishny (1994) have studied the possibility implied by the Coase Theorem and have shown that bribery can facilitate an efficient allocation of resources. Beck & Maher (1986) and Lien (1986), have modelled a situation in which the exact cost levels are unknown to the briber and, thus, he has to take strategic considerations in making his offer of a bribe. They came up with the result that the firm that is able to offer the biggest bribe and consequently win the contract, is the lower-cost firm.

Another model aimed at proving the efficiency-enhancing role of corruption is that of Lui (1985). Using a queuing model, Lui counter argues the propositions of Mydral (1989), namely that bribery is inefficient because bureaucrats may cause delays for attracting more bribes. The author derives the Nash equilibrium strategies that minimize the average value of the time costs of the queue. Thus, he concludes that the server, who would like to maximize the bribe revenue, is able to choose the speed of the service (provided that bribery is allowed). It is natural to think that the consequence of this choice is that the waiting time is cut down. Consequently, firms that are able and willing to pay the bribes are more likely to save time, since they can avoid delays caused by bureaucracy. This is the reason why an alternative name for the "efficient grease" hypothesis is "speed money". Under this perspective, corruption works like a piece rate for bureaucrats, acts as a lubricant that smooths operations, and, hence, raises the efficiency of the economy. The "speed money" hypothesis seems to be beneficial only in countries with burdensome bureaucratic regulations, while the piece-rate mechanism would operate regardless of the level of red tape.

Khan (1997) argues that corruption may have different efficiency effects, depending on the distribution of power within the agent-client networks: strong agents can maximize their rents over the long horizons. Thus, the most rent-maximizing strategy for them would be to create a system of rights which would lead to the most efficient productive environment, one in which long-run profits and rents are the highest. The opposite happens when weak agents maximize over the short-run period and tend to grab every rent available, whether it hurts or helps production efficiency.

Tullock, Becker and Stigler (1980) argued that corruption allows the government to maintain a lower tax burden, since bribery adds to the low wages of state officials. Lower taxes in turn, are likely to result in a higher growth rate.

Werner (1989), having studied corruption in Israel, argues that corruption has positive effects in foreign direct investment by "bypassing cumbersome, genuinely hampering, governmental economic regulations".

Graziano (1980) claimed that corruption might be useful political glue since it allows politicians to get funds that can be used to hold a country together. This outcome may be a necessary condition for growth.

Mendez and Sepulveda (2000) examined the impact of corruption on growth by a dynamic general equilibrium model. They found that corruption has two contradicting effects. On the one hand, it increases economic growth by giving the possibility to overcome the burdensome regulations, but on the other hand, it damages investment. The total impact of corruption on economic growth depends on the relative size of these effects.

It is claimed that under certain circumstances, a bribe is simply a transfer from a client to an official and hence, does not have a social cost. However, in practice, the necessary conditions for this situation to rise are rare. According to Schleilfer & Vishny, what makes the outcome of the bribery sub-optimal, is the fact that it must be kept secret. What is claimed is that if bribes were made legal, the problem of sub-optimality could have been avoided.

Another benefit from corruption is that it is considered to be a mechanism, which equalizes supply and demand.

If we support the view that economic growth increases through political stabilization, then few other arguments could be presented. As Bequart-Leclercq (1989) states "….Corruption functions like grease in the gears, it has an important redistributive effect, it is a functional substitute for direct participation in power, it constitutes the cement between elites and parties and it affects the effectiveness with which power is exercised." Corruption plays a beneficial role in authoritative regimes since it "….guarantees certain zones of freedom and of free movement in the face of the totalitarian tendencies inherent in states and in political parties… Political corruption has another important function, to redistribute public resources by parallel means accessible to groups that would otherwise be excluded."

Another supporter of the view that corruption enhances political development and economic growth is Nye (1967). He supports that economic development could be benefited through the following channels that are affected by corruption:

-Capital formation

-Cutting red tape

-Entrepreneurship and incentives

Nye also argues that corruption plays a positive role in political development through the integration of the elites and the non-elites, as well as by improving the governmental capacity.

Negative Effects of Corruption: Many researchers have heavily criticized the view that corruption increases growth. Before proceeding to the literature supporting that corruption is highly harmful for growth, it is necessary to present the main counter arguments to the ones mentioned above.

The first of them is provided by Bardhan, who supports that in the second best case made above, it is usually taken for granted that some of the pre-existing distortions are alleviated from corruption. However, these distortions are not exogenous and the fact that it is ignored that these distortions and corruption are caused or at least preserved by the same cause, constitutes a mistaken approach.

Mydral has questioned the argument of speed money by arguing that officials may be motivated to delay more in order to attract more money, while Kaufmann and Wei (1999) have empirically proved that the time wasted increases with the level of corruption. Andvig (1989) has criticized Lui's model by denoting that queuing models as allocation mechanisms are so many-sided and complex that can give more than one results concerning the average waiting time.

The problem with looking upon bribes as side payments in a Coasean bargaining process is that, firstly, due to asymmetric information, the two parts might disagree over the size of the bribe and, secondly, there is a long distance between the payment of the bribe and the realization of the promise. Additionally, as Boycko, Sheilfer and Vishny (1996) and Farrel (1996) notify, the result of a corrupt transaction is quite uncertain and arbitrary, since corrupt contracts cannot be enforced by courts and the insecurity of property rights along with the problems of asymmetric information is likely to prevent corruption from playing the role envisaged by the Coase Theorem.

The argument that corruption acts as a deregulator is countered by Kaufmann and Wei by pointing out that this view is true only in a very narrow sense and only when the bad regulations and official harassment are taken as exogenous. What the original argument ignores is that the officials have discretionary power over the creation, proliferation and interpretation of these regulations.

Viewing bribes as a mechanism that equalizes supply and demand, ignores the fact that many public goods ought not to be allocated to the highest bidder, because they are addressed to the less privileged as well. Thus, corruption not only deprives some people of the necessary, but also makes the implementation of government policies extremely difficult.

Apart from the above counter arguments, there is an extensive literature emphasizing the negative effects of corruption on growth. Since however there are many other factors that affect growth it is not easy to identify the direct effect between these two variables. This becomes evident if we graph the corellation between CPI and Growth Rate for a number of countries, where we cannot predict a clear relationship between the two or we even predict a positve relationship. However if we categorize the countries into developed (Figure 4.1) and developing (Figure 4.2) then a negative corellation between corruption and growth occurs.

The majority of the researchers purports that the effect of corruption on growth occurs through various channels, which, in turn, affect growth. The focus of this chapter will be the effect of corruption on public expenditure. Special attention will be given to this part as well as to the effect of corruption on tax revenue and on shadow economy which is a proxy of tax evasion. Apart from these important channels, a few others will as well be stated in order to have a complete picture for the effect of corruption on growth.

Corruption and Public Expenditure

The impact of corruption on public investment is explored in a number of studies. High-level

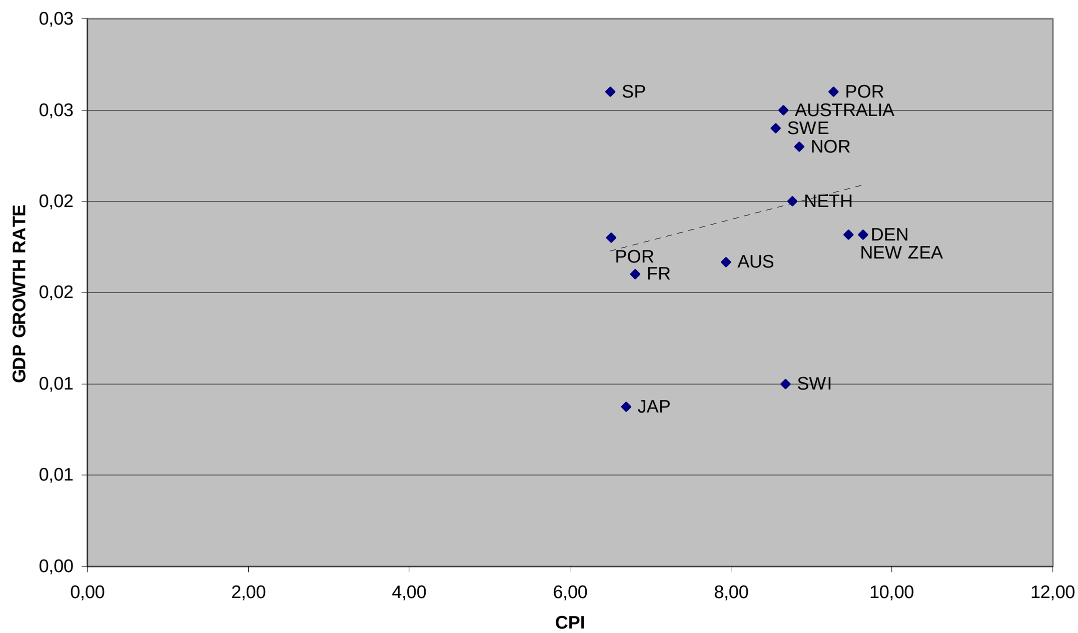

Figure 4.1: Correlation Between Growth and CPI Index in Developed Countries. Sources: CPI, Wolrdbank.

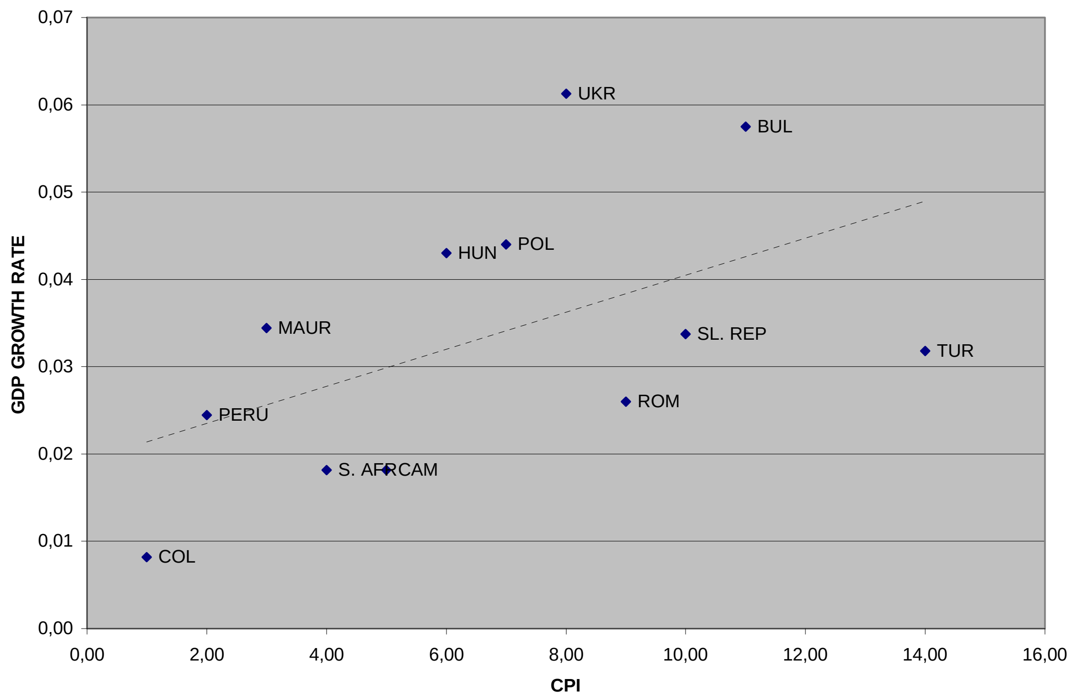

Figure 4.2: Correlation Between Growth and CPI Index in Developing Countries. Sources: CPI, Wolrdbank.

corruption results in distorting the entire-decision making process related to public investment projects. It is a usual phenomenon to favour large and capital intensive projects at the expense of smaller social infrastructure problems since those civil servants who allocate sources seek for higher illegal payments.

Tanzi and Davoodi (1997) using cross-country data have found that corruption increases public investment but reduces its productivity. Since politicians can easily manipulate public investment, political corruption can highly affect public investment. They have also presented evidence supporting the idea that high corruption is associated with poor quality of infrastructure and with low operation and maintenance expenditure. This occurs for two reasons. Initially, bribery enables service providers to ignore established standards and secondly, new projects imply larger budgets than those of maintenance projects, which in turn imply higher bribes. Thus while corruption is likely to increase public investments by favouring large projects which are going to bring a lot of money into the pockets of the agents, it is also likely to cause a deterioration in the quality of infrastructure and, thus, reduce the country's growth prospects.

Another channel through which corruption is suspected to negatively affect growth is that of the composition of public spending and more specifically it is believed that corruption reduces social spending such as education or health. This has been empirically proved by Mauro (1998) and confirmed by Gupta, Davoodi and Rosa (1998). It has also been claimed that military contracts are budgeted at the expense of social projects, as well as that urban areas are favoured at the expense of rural areas. Angelopoulos et al (2007) show that corruption and rent-seeking directly hurt growth by limiting the resources available to finance public infrastructure and other socially useful services.

An extensive study was conducted by Mauro (1998), on the composition of government expenditure. Mauro based his analysis on the fact that not all kinds of public spending are equally profitable in terms of rent-seeking. As Kreueger (1974) was early to point out, the lower the degree of competition the higher the rents that can be extracted. This is especially true when it comes to high technology goods, such as military equipment, which are produced by a limited, worldwide, number of firms. It is evident that text books and teachers' salaries do not offer any substantial perspective for rent-seeking and for this reason it is highly likely that government funds, under the presence of corruption, are directed to other, more "profitable activities". This is the reasonable argument that Mauro wanted empirically test, along with the composition of public spending.

To conduct his study he takes into account the issue of likely causality between composition of public spending and corruption. To make sure that he studies the direction from corruption to the composition of public expenditure he employs some instrumental variables such as ethno-linguistic fractionalisation, and the black market premium.

His findings strongly support his hypothesis. There seems to be a significant relationship between corruption and growth. He finds that if a country improves its "grade" on corruption from 4 to 5 $\frac{1}{2}$ out of 6 (recall that he uses the BI index where the higher the score, the less corrupt is the country with a maximum of 6), then on average, education spending would increase by about 0,6% of GDP. What is significant concerning his results is that they are robust to a variety of other hypothesis, as well as that this result holds not only for developing and very corrupt countries but also for developed countries.

What one should pay attention to though, is that this effect is partially enhanced by the fact that when a country grows and becomes richer, then government expenditure tends to rise, an effect known as Wagner effect. Still when this effect is accounted for, the results remain robust to this

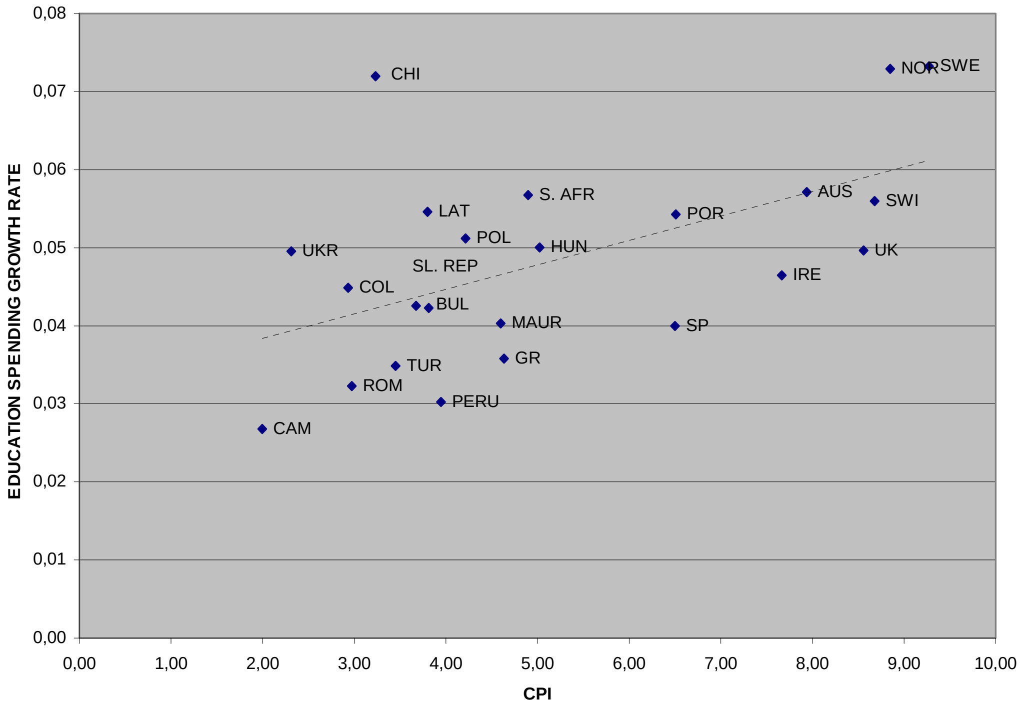

Figure 4.3: Correlation Between Spending on Education and CPI Index. Sources: CPI, Wolrdbank.

specification.

In Figure (4.3), which graphs the corellation between spending on education and corruption[11], it is evident that there seems to be a negative relationship between spending on education rate and tax evasion. The same strong relationship seems to hold if we split the countries into developed and developing countries. For instance in Figure (4.4) I graph some Former Soviet Democracies which spent much on education (significantly more that other developing coutnries) and at the same time they suffer from extensive corruption. Still in this case there seems to be a negative correlation between corruption and spending on education.

Corruption and Taxes

[11] Data on corruption come from CPI index while spending on education is based on World Bank data for a variety of coutnries.

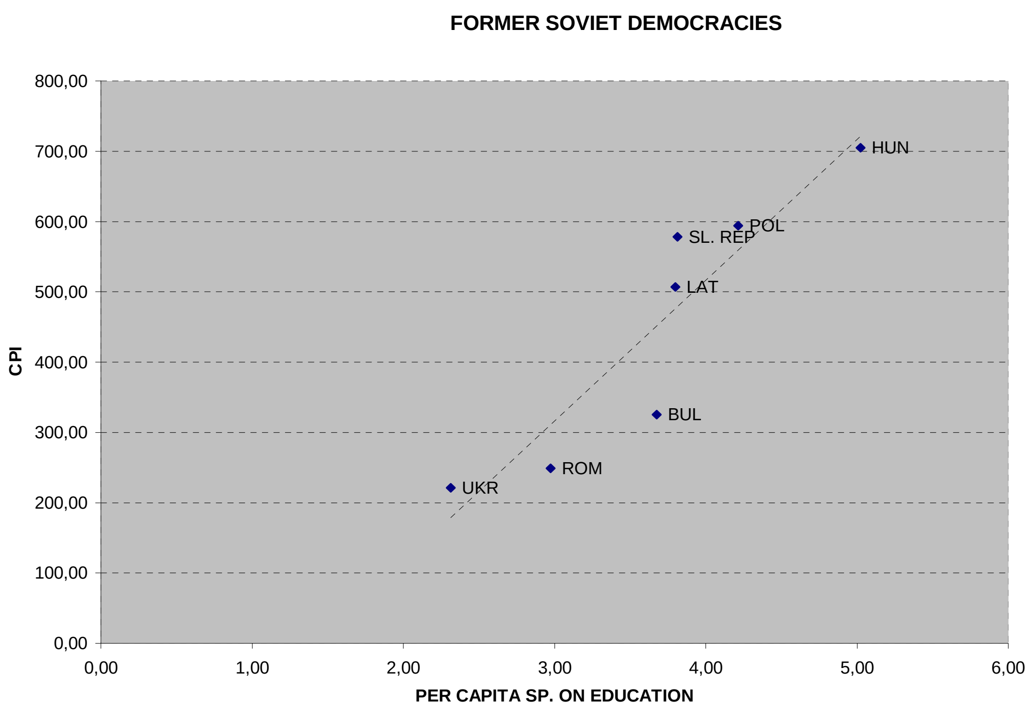

Figure 4.4: Correlation Between Spending on Education and CPI Index in Former Soviet Democracies. Sources: CPI, Wolrdbank.

That corruption has an impact on tax revenue and the taxed structure generally is not a new notion. The main idea is that the more corrupt a country is, the less revenue from taxes it gathers. In other words, an amount of money collected by tax administrators remains to their hands. The literature is rather extensive and what is remarkable is that many theoretical models use this notion (i.e. the willingness of a tax administrator to be bribed in order to lower the tax burden of the bribee) in order to prove the impact of corruption. This whole situation is highly likely to alter the optimal level of public spending, reduce its productivity and lead to higher fiscal deficits. Thus, growth is negatively affected in a twofold manner when tax evasion occurs with the tolerance of the tax administrators.

An indirect impact of tax evasion is that this tax loss is compounded because the bribe often serves to circumvent the reporting of normal business transactions that would otherwise have produced tax consequences such as construction permits or sales taxes etc. In a simple correlation between tax revenue as a percentage of GDP and CPI index for various countries (Figure 4.5) we observe that there seems to be a negative relationship between the two variables.

Moreover, corruption alters the tax structure. As Barreto and Alm (2003) have pointed out, the socially optimal tax structure of a corrupt economy, as opposed to that of a less corrupt economy, involves a greater reliance upon consumption taxes and a smaller use of income taxes. Obviously, this kind of tax structure hurts the less privileged groups and leads to income inequality whose impact on growth has been mentioned above.

Corruption resembles tax in various ways. First of all it is a mark-up over the price, just like taxation. The difference, however, is that taxation is collected by the government and it comprises a part of the budget, while bribery is collected by the agent and never reaches the government. This is the reason why in a monarchy or in a regime where there is monopolistic corruption (in

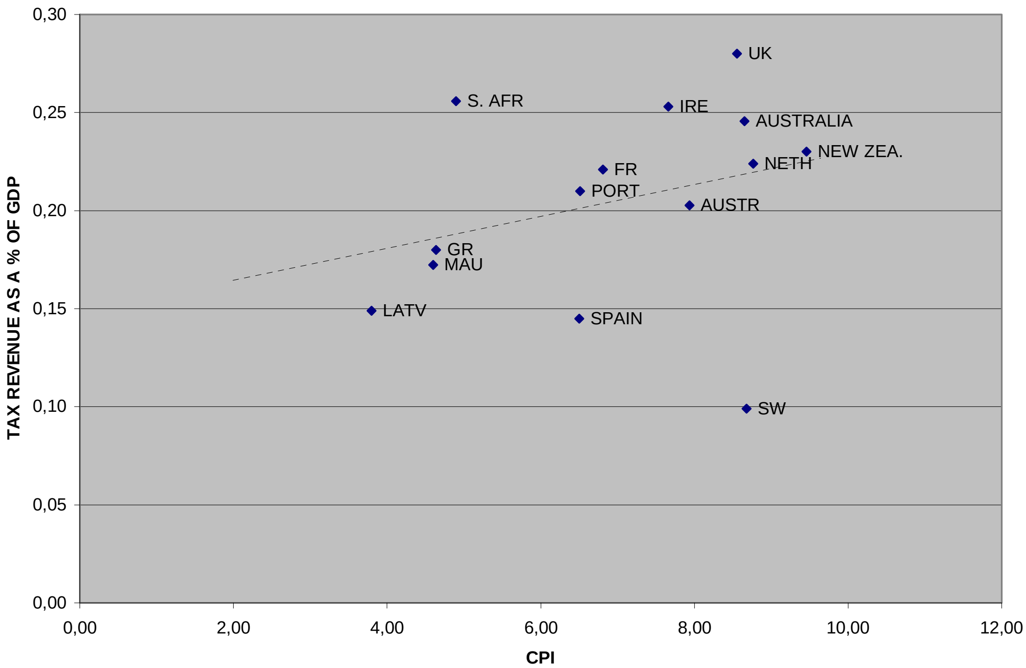

Figure 4.5: Correlation Between Tax Revenue as a % of GDP and CPI Index. Sources: CPI, Wolrdbank.

other words the corrupt system is organized by the government itself) corruption and tax almost imply the same thing since revenues from both end up to the same person (the monarch) or to the government. Furthermore, both corruption and taxation deter FDI and private investment.

Despite their similarities, many researchers have argued that corruption is more taxing than tax and this is caused by the fact that corruption is illegal while taxes are legal. The illegality of corruption involves many sources wasted, since bureaucrats are trying to secure their position and avoid detection and punishment. It also causes uncertainty and embeds arbitrariness since it is not transparent, and carries much poorer enforcement of an agreement between a briber and a bribee. The degree of uncertainty depends on how corruption is organized. If it is arbitrarily organized but extensive there is much uncertainty, but if we talk about monopolistic corruption the degree of uncertainty is quite low.

Another reason that makes corruption more taxing than tax is that since the budget suffers from tax evasion (which occurs through corruption) more taxes could be imposed. Finally, since taxation cannot serve the interests of powerful politicians, corruption could play this role. To be more specific, supposing the existence of a monopoly is very profitable for a country and the country wants to maintain the monopoly by imposing high taxes. These taxes though would discourage FDI in other sectors. Then the role of taxation could be played by corruption, which, in the form of excessive bribery, would prevent FDI and contribute to the existence of the monopoly.

In all the above cases, corruption would reduce growth more than taxes would have done.

Corruption and Underground Economy

Numerous empirical and theoretical studies have shown that there is a positive relationship between corruption and the underground economy. When referring to underground economy, two kinds of activities are implied. The first involves illegal activities such as drug trade, while the second refers to those who are legal but not officially recorded such as tax evasion. Corruption gives rise to both of these kinds and contributes directly to the rise of the underground economy. The more corrupt a country is, the more likely it is to face the problem of underground economy (black-markets etc). The existence of underground economy results in the undermining of economic growth.

De Soto (1989) mentions that the size of underground economies is expected to be larger in corrupt countries and that illegal establishments have to pay a larger fraction of their net income in bribes than the legal ones.

In an extensive empirical study of the size of underground economy in 69 countries, he concludes that corruption, rather than tax rates, is the main determinant of the size of the underground economy.

The problem with the existence of underground economy is that when a large portion of an economy goes underground, official macroeconomic data (i.e. inflation rate or volume of imports-exports) will mostly cover the formal sector, become unreliable to assess economic performance or to provide a basis for policy making and analysis. In the absence of reliable data, transparent policies and macroeconomic management, it is scarce for economic growth, efficiency and modernization to occur. In case these macroeconomic policies are instructed and implemented by corrupt politicians, the situation deteriorates.

Corruption and Investment

A lot of researchers have studied the impact of corruption on investment, which is an important determinant of growth. There are many ways in which corruption can affect investment. The most important are:

-total investment

-the size and the composition of foreign direct investment

-the size of public investment, which has already been analyzed and

-the quality of the investment

In several papers Mauro (1993, 1995, 1997, 1998) has found a negative correlation between corruption and investment. Having collected data from Business International for a sample of 68 countries, he found a statistically significant negative estimator for corruption and thus a negative impact of corruption on the rate of investment to GDP, even in countries where bureaucratic regulations are very cumbersome. Similar results have been found by Fishman and Svensson (2000).

One study that questions the results of Mauro is that of Wedeman (1997) who denotes that correlation between corruption and investment may be strong for countries that are perceived to have low levels of corruption but it loses power for countries with high level of corruption.

Corruption seems to have a devastating impact on foreign direct investment (FDI) according to Wei (1997).

As far as whether FDI enhances economic growth, there is a strong literature supporting that view (Borensztein and Lee, 1998) and pointing out that especially in a poor country, FDI provides the so much needed capital resources, technology, know-how and managerial and marketing skills that improve the country's competitiveness. Furthermore it creates thousands of new jobs and gives the opportunity to the country to link with the capital markets and the financial and business services of other countries.

Corruption and International Trade

One of the most important studies of the impact of corruption on international trade is that of Lambsdorff (1999), who makes an extensive inquiry into the effects of corruption on the geographic composition of international trade. He has come to the conclusion that countries that are perceived to be less corrupted have a competitive advantage when compared to countries that are perceived to be more corrupt, due to the fact that there are differences between the exporter's willingness to offer bribes.

Wei, in his study finds a two-way relationship between corruption and the openness of the economy. On the one hand, low degree of openness results in high levels of corruption and, on the other hand, high levels of corruption result in low degree of openness. Similar results can be found in Gatti (2000), where it has been shown that greater variation of import duties increases the opportunities for customs officials to earn corrupt income. Thus, more corrupt economies will show a tendency towards developing greater diversity in tariffs.

Corruption and Public Sector Regulations

An intriguing question is whether corruption is caused by extensive regulations or the regulations are caused by corruption. According to Mydral (1989), Rose-Ackerman (1994) and Tanzi (1998), the extensive regulations are a deliberate strategy of civil servants in order to attract more bribes. Any corrupt, rent-seeking bureaucrat has the motive to maximize his revenue from bribes. Thus, the existence of numerous, complicated regulations and their implementation is highly convenient for the bureaucrat. Suffice it to say that when the bureaucrat has extensive discretionary power, the situation is even worse. To sum up, if rent-seeking officials endogenously choose regulations, red tape and their bureaucratic enforcement, the result will be their dramatic increase, to the official's best interest. A representative example comes from Winters (1996), who mentions that the stronger resistance to tax reforms in Indonesia came from the tax officials themselves, since the simplification of the tax system would no longer serve their interests.

Even if all these regulations are not deliberately created and enforced, it is possible that a number of new regulations are created in order to fight the problem of corruption, which of course makes the problem worse.

According to the views of the industrial organization on corruption, the reason why corruption reduces growth is because the numerous regulations provide the agents with the ability to impose independent bribes on private agents seeking complementary permits from these agencies. If the entry of these agencies into regulation is free, the bribe burden on private agents will go to infinity.

Corruption and Income Distribution

In a corrupt country, the privileged and the well connected are definitely benefited and enjoy economic rents, which, in turn, represent abnormal or monopoly profits. Thus, the wealth tends to be concentrated in the hands of a privileged minority. It is obvious that the income distribution becomes uneven and, as a result, the burden of corruption falls on the less privileged or to the poor who cannot afford to pay bribes.

Income inequality is very harmful both for economic growth and political stabilization, since it might result in stronger incentives for the groups at the bottom of the distribution to engage in illegal or violent activities for material benefits or as a reaction to inequality. This instability creates uncertainty over the protection of property rights and, hence, reduces investment and productivity, which, consequently, reduce growth. It also affects the consumption pattern, since a number of luxurious goods, which are beyond the reach of the poor, are either produced or imported. Similar notions could be found in Alesina and Perroti, in Murphy as well as in North (1990).

Corruption and Enterprise Growth

“The growth of enterprises as a building lock of growth is an old and respected topic in economics, dating back to Adam Smith’s notion of scale economies....In general the lessons are that factors that impede the growth of SME's and stifle the entry of new firms, which tend to be small and important in a dynamic economy, will also tend to slow down the growth rate of the economy" (Tanzi and Hamid, 2000). Despite the fact that large enterprises are considered to be very important sources of jobs, innovation and growth, the attention of many researchers has turned to small and medium enterprises. Schumpeter has sited several arguments in favour of the enhancing for the economy role of small and medium enterprises (SME's). However, empirical studies show the differential impact of corruption on these enterprises. In general, anti-competitive practices and corruption are considered to be one of the most difficult obstacles for the SME's to overcome. Especially when the agents are not interested in establishing a lasting relationship with the firms, the amount of bribery becomes a very heavy burden for the new firms and some of them may even be driven out of competition. Furthermore, these studies suggest that what is so harmful for the firms, is the difficulty they face when trying to access essential business services.

The reason why large firms are less affected by corruption is that they can more easily protect themselves since whole departments are devoted to dealing with bureaucrats and they already have a share in the market. Additionally, they might have some gains since bribing could provide them with monopolistic power or lower taxation. On the contrary, the SME's are obliged to pay a lot of money without enjoying any of these privileges.

Buscaglia and Ratliff (1997) have shown that corruption can reduce the rates of return on capital for small firms, more than those of large enterprises. According to their estimations (concerning firms in Argentina) public sector corruption reduces the expected rate of return on invested capital by 1 to 1.5% for large firms 2 to 2.5% for medium sized firms and 3 to 3.6% for small firms.

Corruption and the Allocation of Talent

A relatively new notion, supported by various studies is that there may exist a connection between rent seeking and the allocation of talent (Murphy, Schleifer and Vishny, 1993). Furthermore, rent seeking and corruption are believed to have a negative impact on growth, if they motivate talented individuals to devote to rent seeking and unproductive activities. Despite the difficulty of collecting the necessary data, several outcomes have occurred, suggesting the existence of such a correlation. For instance, Murphy, Schleifer and Vishny suggest that corruption and rent seeking may influence growth through the allocation of talent and the proof for their argument is that in a corrupt country the number of lawyers is greater than the number of engineers and when regressing they find that this inequality or in other words, this higher allocation of talent to law has a negative impact on growth. Angelopoulos et al (2007) and Angelopoulos and Economides (2008) show that corruption indirectly affects growth since the fact that they can extract money from state coffers diverts agents away from productive work adn pushes them into rent-seeking activities.

Krueger (1974) has suggested that had it not been for economic rent, corruption and especially bribery should not impose a social cost since it is nothing more than transfer payments. Nevertheless, since bureaucrats have the power to pursue rent seeking through corruption, individuals have an incentive to compete over the privilege of becoming bureaucrats. This is usually mentioned as investment to political capital. Thus, they consume economic recourses that could, otherwise, be used for production or investment in human capital.

Neutral View: There seems to be a neutral view supporting that corruption is neither beneficial nor harmful to growth. However, the literature supporting this notion is rather limited. Barreto suggests that in a neoclassical growth model, where corruption is endogenous, the level of corruption and its impact on growth, income and consumption is visible. When red tape is added to the model, it is proved that corruption has no impact on growth, but what finally happens is a redistribution of income.

On empirical grounds, a large survey in 58 countries has been conducted as a part of the World Development Report 1997. Almost 3000 firms were asked a number of questions related to the reliability, predictability and quality of government services. Using that data, Brunneti, Kisunko and Weder (1997) constructed an indicator measuring the "credibility of rules", one component of which is perceived corruption. In a regression of growth and investment rates on corruption, they found that even though the impact of corruption on investment is negative and significant, its impact on growth is insignificant.

4.2.2 Tax evasion

One of the most significant and rather old problems, that comes naturally when one talks about tax is that of tax evasion. It is a very compound issue since it can be linked to many areas of economics and can raise various issues. To define tax evasion, the term denotes the effort to avoid tax by illegal means. It should not be confused with tax avoidance which involves the effort to avoid taxes via the use of legal means, such as donations, or loopholes in the tax law. From a legal standpoint tax evasion is illegal while tax avoidance is not. In terms of economic results though the two activities are usually hard to distinguish (Feldman and Kay, 1981, Cowell, 1990). The same holds from a moral standpoint where tax avoidance is not always ethical.

According to Franzoni (1998) tax compliance entails the following: 1) true reporting on tax base, 2) correct computation of the liability, 3) timely filing of the return and 4) timely payment of the amount due. The tax evasion literature usually focuses on the first activity which consists the main problem. This is not only due to the fact that false reporting reduces government revenue but also due to the fact that even when auditing takes place it is sometimes hard or even impossible to verify the actual income.

One of the most difficult issues is how to treat tax evasion. One must take into account two things when deciding to deal with tax evasion. The first is that taxation per se introduces some distortions in the economy. Therefore the optimal tax rate must be chosen to minimize these distortions. whereas under the presence of tax evasion, finding the optimal tax rate is an even more difficult task. Moreover if taxes are introduced as a policy (i.e. redistribution, efficiency), tax evasion renders this policy ineffective. The tax revenues are reduced and the same goes for public expenditure that heavily relies on this revenue. What the government should do in the face of these difficulties it to find the appropriate policies that minimize tax evasion, such as deterrence policies, and implement them in an efficient way. The choice of this kind of policies is not an easy task, since they are not all equally effective, and the whole attempt becomes even more difficult if one takes into account the costs incurred by such policies.

This is only one side of the coin. Another important issue is the view that the size of tax evasion is calculated by subtracting the realized tax revenue from the estimated tax revenue (estimated on the current tax base), which is not correct. Had policy makers gone after all tax evaders, not only the cost would have been unbearable, but the tax base would significantly shrink. Therefore, it is implied that tax evasion is to some extend desirable. Deterrence policies should be designed in such a way to be effective in reducing evasion but not shrinking the tax base. This task is far from easy especially when it must be combined with the target of choosing the optimal tax rate.

All the above difficulties are the main topic of the tax evasion literature which at a later point accounted for the provision of public goods as well. As Musgrave (1959) has put it, public finance consists of two parts, a governments revenue-raising and an expenditure aspect. The analysis raised to address these problems always based on a standard axiom. That of the rational agents.

However, it became evident in due course that agents did not always behave rationally and there are imposed difficulties in defining the optimal policy. This problem was addressed by behavioral public finance, which made an effort to account for "paradox" behaviors.

All the above issues will be addressed in this part starting with empirical evidence on tax evasion.

4.2.2.1 Empirical Evidence

As is the case with every illegal activity, the size of tax evasion is hard to estimate. According to Andreoni et al (1998), one of the most widely used measures is the tax gap, i.e. the difference between the income taxes households actually owe, which is based upon estimations, and what they actually report and pay. In the US the most reliable information comes from the Taxpayer Compliance Measurement Programme (TCMP) of the Internal Revenue Service (IRS) which audits on a regular basis a sample of 45000-55000 tax returns. For the year 1988 in the US, the TCMP has estimated that only a 53% has paid its taxes correctly. Of course non compliance does not apply for all these cases, since a 7% has overpaid its taxes while a part of the remaining 40% has underpaid due to errors that result from the complicated procedure involved. According to Fanzoni (1998) the federal income tax gap of the US had been estimated for 1998 at 17%.

To improve compliance rates and deter tax evasion the IRS realizes audits based on past data. Depending on the extent of tax evasion the household may be subject to civil or criminal penalties. Other penalties such as imprisonment may be imposed as well but in terms of modeling they can always be translated into a penalty (this happens in courts occasionally).

Audits are not very extensive in the US (they approach 20% of underpaying tax payers, or according to McCaffery and Slemrod, 2004, the average audit rate for individual tax returns in the US is less that 1%) which makes surprising the fact that tax evasion is not more extensive and also makes evident the need for accounting for tax morale in the models, since they originally predicted higher evasion rates. The evidence in other developed countries is similar to that of the US however in some developing countries the rates of tax evasion are much higher. For instance in the Western developed countries the rates of tax evasion are estimated around 5%-25% of potential tax revenue (Feige, 1989, Pyle, 1989, Thomas, 1992) while for developing countries higher rates may appear (Tanzi and Shome, 1994).

Similar to real world evidence, empirical studies on tax evasion are rather limited and should be cautiously handled because data sources are scarce. Additionally due to the form of the data, empirical studies do not actually answer to all the questions posed by theoretical models. The first and most important drawback concerning tax evasion data is that it is an activity people try to conceal. This holds even in case where people answer anonymous questionnaires thus rendering the results non-reliable or at least less trusted.

The basic sources of tax evasion data are four: 1) Audit data, 2) Survey data, 3) Tax amnesty data and 4) Laboratory generated data. Of them the most reliable are considered to be those of the first category that are derived from audits of the local tax authorities. Each year a percentage of households are being audited and some of them have been used by researchers to study tax evasion. This kind of data have been used to study tax evasion in the US, Netherlands and Jamaica (Elffers, Weigel and Hessing, 1987, for Netherlands and Alm, Bahl and Murray, 1993, for Jamaica).

Surveys are also a good source for tax evasion data since they often include socioeconomic data thus provide a more thorough picture of the problem. However as I mentioned earlier these data are not always accurate since respondents do not always tell the truth about their tax attitude. The same goes with tax amnesty data with the additional problem that the self-reporting tax evaders are not a representative sample of the overall population. Finally laboratory experimental data are faced with the critique that applies to all experiments i.e. that the setting is usually not realistic and the participants do not take some important factors such as morals into consideration. However they sometimes give a picture about the primary motives of tax evasion.

The main factors accounting for tax compliance, according to empirical studies are the following:

-Income and tax rates: The effect of tax rates on compliance has not yet been clearly explained by existing theoretical models since an income and a substitution effect take place. Clotfelter (1983) in an empirical analysis finds that the after-tax income and marginal tax rates positively affect tax evasion. The problem with his analysis is that there is a strong positive relationship between tax rates and income. Feinsteinn (1991) has tried to circumvent this problem by using pooled data in order to separate the effects of these two variables. He contradicts the result of Clotfelter and agrees with the intuition of Yitzhaki by finding that there is significant negative relationship between the marginal tax rate and evasion.

-Demographic and social factors: compliance rates seem to differ depending on the marital status of the household, across occupations, the age of the taxpayer and the sex. For this kind of analysis mostly census and experimental data have been used since they provide all the necessary variables. Klepper and Nagin (1989a) support that older people are more compliant.

-Penalties and audit probabilities: Witte and Woodbury (1985), analyzing data from the TCMP for 1969, focused on the effect of enforcement parameters on tax evasion. They found that an increase in auditing probability negatively affected underreporting, while increases in income and in the "opportunities" for tax evasion, positively affected underreporting. Their results were criticized by Dubin and Wilde (1988) who stressed that audit rates may be endogenous in the system. Therefore in case where underreporting is reduced then the IRS devotes less effort to auditing since the net return from it has been reduced.

-Prior audits: In experimental studies researchers have found that if a household gets audited in one year this is likely to affect its future behavior. However this result has not been verified by actual audit data (Long and Schwartz, 1987).

-Objective versus subjective enforcement measures: Another factor that affects tax compliance rates is the perception of people concerning the probability of being audited (Scholz and Pinney, 1993). Tax payers usually overestimate this probability and this probably explains the high compliance rates mentioned earlier.

-The influence of tax practitioners: tax practitioners seem to affect tax compliance in many ways. For instance they reduce payer uncertainty about their legal tax obligations, (Scotchmer, 1989), they reduce the time and anxiety costs related to tax return preparations and tax audits (Reinganum and Wilde, 1991), etc. Overall, even though they reduce the likelihood of decreased tax compliance due to error, on the other hand they may promote non-compliance by reducing the physical and monetary costs.

One last determinant which I want to stress out is sociological and ethical factors. In empirical studies such factors are hard to measure. Among the studies that have taken place is that of Erard and Feinstein (1994) who have used guilt and shame to account for compliance rates. They argue that when a tax payer is involved in underreporting and does not get caught may anticipate guilt, while if he does not escape being caught he may anticipate shame. They find that such variables improve the models' fit to real data. Their results have been criticized on the basis of the introduction of such terms in the utility functions are well as for the fact that sentiments such as guilt and shame are not directly observable. Spicer and Becker (1980) in a lab experiment find that taxpayers tend to evade more taxes if they believe that their tax burden is not fair. Unfairness of the tax system makes people morally legalize tax evasion. Their result though has been called into question by Webley et al (1991) who argued that the perceived tax burden does not affect the decision to tax evade. Fortin, Lacroix and Villeval (2006) in a model with tax evasion and social interactions, have conducted a lab experiment and have found, among other results, that fairness negatively affects tax evasion.

4.2.2.2 Tax Compliance Models-No Public Goods

The literature on tax evasion departed from the seminal contribution of Allingham and Sandmo (1972), who wanted to find out whether higher tax rates increase or decrease tax compliance. In a simple model they introduced a system of audits and penalties where the penalty in their model is imposed on the income evaded. What they find is that an increase in tax rates may either increase or decrease tax evasion depending on how fast absolute risk aversion declines. This result though is directly related to the form of penalty in the model. When fines are imposed on evaded income two effects take place, an income effect and a substitution effect. When tax rate increases, then the agent becomes more poor and therefore less willing to risk to evade. This is the income effect and results in a decrease in tax evasion. The substitution effect occurs because the penalty rate is held fixed when the regular tax rate increases, so the difference between the penalty rate and the regular tax rate goes down, and this increases the incentive to underreport income. Which of the two effects prevails defines the overall effect of an increase in tax rate on tax evasion.

Concerning deterrence policies, they have conducted comparative statics analysis which indicates that increases in the probability of detection negatively affect tax evasion and the same goes with an increase in penalty rates. Thus they suggest that these two measures should substitute each other. This suggestion is further enhanced if one accounts for costs associated with auditing.

As Kolm (1973) has put it, the optimum deterrence policy is to "hang them (the tax evaders) with zero probability". Note that when they endogenize the probability of detection they obtain similar results. The effect of variations in income on tax evasion is not clear and depends on the relationship between risk aversion and income.

Allingham and Sandmo also developed a dynamic model to study the "stigma associated with tax evasion. They find that it is "short-sighted" to evade taxes and thus consistent individuals will evade less. In case the individual gets caught, he will start declaring everything. Overall the fear of "stigma" does not totally deter tax evasion it just makes more "costly" the decision to tax evade. The Allingham and Sandmo model was rather simple and therefore neglected various aspects that were later covered by other researchers. The most important criticism it has received though is that of Kolm (1973)[12] which commented on the fact that they did not assume a government budget constraint, or in other words the tax revenue does not return back to the economy. The literature generated to fill in this gap will be the focus of the next subchapter.

Yitzhaki (1974) building upon the A-S model proved that when the penalty is imposed on the tax evaded, tax evasion is reduced with an increase in the tax rate and therefore only the substitution effect takes place. His model made the A-S predictions concerning the effect of variations in tax on tax evasion more clear, however it contradicted the common sense results, i.e. that an increase in tax rate would increase tax evasion.

When labor supply is included in the model, as in Pencavel (1979) and Sandmo (1981) the effects of increases in tax on tax evasion are far more complicated. On the one hand an increase in tax may decrease labor supply, due to reduction in the effective wage rate but on the other hand it may increase labor supply since individuals want to insure themselves against the losses

12 Quoting him: "But this is hardly public economics; in fact it is very private".

due to audits. Other extensions (Engel and Hines, 1994) are related to repeated games, since tax reporting is usually an annually repeated procedure and produces equally complicated results.

Some extensions are emphasizing the role of audits. For instance they correlate the probability of being audited to the income reported. These models can also account for a tax agency's optimal strategy. Issues of commitment can also be linked to these models, i.e. whether the tax authority will commit to the auditing strategy or not. The optimal strategy of the tax agency depends on its decision to commit or not. When the tax agency can commit then it is optimal for the tax agency to impose a cut-off rule, which consists of a threshold level of income below which a percentage of the reports q will be audited. When the tax authority cannot commit then a sequential game is played between the tax payers and the tax authority and many equilibria may arise depending on the ability of tax payers to forecast the ability to be audited.

Much as extensive the research may be, there are many problems related to the existing models. For instance the models that introduce a cut-off rule predict a much less percentage of tax evasion than the existing one. Additionally such rules are simplifications of the rules actually imposed and followed. All these models are based on the crucial assumption that if an audit takes place the tax authority will be able to trace the real income evaded which is not always the case. Other simplifying assumptions are adopted as well as that being audited does not involve any costs on the part of the tax payer.

An important topic related to tax evasion is the so-called principal agent problem. Most models consider that the tax authority and the rest of the government have the same targets. This is not always the case and in some cases or countries the objectives differ significantly. Melumad and Mokherjee (1989) examine whether a government can indeed control the behavior of the tax authority. They find that incentives such as rewards proportional to the fines collected may make the control of the tax authority by the government more feasible.

4.2.2.3 Tax Compliance Models-Provision of Public Goods

A main drawback of the Allingham and Sandmo as well as the Yitzhaki model was that they did not account for the use of the tax revenues. Kolm (1973) was the first to point this out and made an attempt to introduce government to the model. He finds that when public goods are introduced in the model the optimal tax rates, fines and auditing rate are different than in the model with no public goods. Gottlieb (1985), incorporates in the A-S context public goods which are financed by revenue from taxation and penalties. He made two different assumptions. Under the first assumption, which he called the competitive case because agents were isolated and did not take into account the tax evasion of the other agents, he found that the equilibrium was Pareto inferior. In the interdependent case, where each agent could take into account tax evasion of the other players he found that a paradox occurred similar to the Prisoner's dilemma. Even though the full compliance outcome is Pareto superior, a government led by utilitarian welfare standards cannot lead to this equilibrium by imposing the optimal fine rate.

Cowell and Gordon (1988) attempted to adjust the Yitzhaki model of tax evasion to the real world evidence as well as to address the concern of Kolm, relating the issue of tax evasion to public economics. Overall what they achieved in their paper was to provide theoretical evidence that under certain assumptions, the Yitzhaki results, i.e. a decrease in tax evasion when the tax rate increases, is reversed. These assumptions were related to the provision of a public good, financed by the tax revenues.

In a large economy, with Ziff public goods (Ziff stands for zero income effect of z, where z in their model accounts for possible effects of the size of the economy), they find that if individuals display absolute risk aversion then an increase in the tax rate positively affects tax evasion if public goods are under-provided, and negatively affects the tax evasion if public goods are over-provided. Thus this is the basic mechanism that alters Yitzhaki's prediction, i.e. the insertion of public goods. This result survives the introduction of heterogeneous agents.

Another approach was to deduct the element of public good in their model, in a way that introduces rivalness. The fundamental characteristics of a public good are non-rivalness and non-excludability. If rivalness is introduced then each agent when tax-evading can no longer free ride, instead he must take into account his tax evasion as well. To formulate such as model they drop the assumption of a large economy. The good is no longer public good, as it does not satisfy the two necessary conditions, however it is publicly provided and is non-excludable but rival. After this transformation, each player, contrary to the original Cournot competition where he maximized expecting a zero response from the others, now he conjectures that others will respond to his own tax evasion.

Since the number of people is now limited everyone is very thoughtful concerning tax evasion because he expects that if he starts tax evading, other people will do the same. Thus tax evasion is reduced. However as the size of the economy increases it is easier to ignore the effects of the other people evading and thus an increase in tax evasion is expected. What is different in the small economy is that now the under or over abundance of public goods cannot solely determine the effect on tax evasion of an increase in the tax rate. The same can be said about a large economy with some conformity.

What seems paradoxical, according to the common sense and the authors' opinion, is that an increase in tax rate decreases tax evasion when goods are under-provided (it is just a case that cannot be precluded contrary to the no conformity or large economy model). However when public goods are under-provided, and thus agents are not satisfied by the way the revenues from taxation are used, one would expect agents to increase tax evasion, not decrease it. The authors attribute this weakness of the model to the fact that they do not account for interactions between the government and taxpayers.

4.2.2.4 Tax Evasion-Moral and Social Dynamics

A significant issue directly related to tax compliance is moral and social dynamics. These factors had initially been neglected in the literature however they were eventually employed as an interpretative factor. Guilt, shame, stigma and fairness are some of the issues that will be analyzed in this context. Allingham and Sandmo (1988) were the first to introduce the notion of stigma in tax evasion literature, finding that it leads to a more restrictive condition for tax evasion to be optimal. Spicer and Lundstedt (1976) and Smith (1992b) find that if tax payers are not satisfied by the government they tend to evade more taxes. Pommerehne, Hart and Frey (1994) analyze the relationship between the provision of a public good, tax compliance and fairness considerations. They find that the greater the deviation between the individual's choice of public good and the actual level, the stronger incentives they have to evade taxes.

An interesting model in the literature is that of Bordignon (1993). Bordignon attempts to give another dimension to the literature on tax evasion, more specifically that of fairness, and how does fairness influence the choice of agents concerning tax evasion. In his analysis he adopts the model of Cowell and Gordon (1988) who had introduced the notion of public goods and he extends it to include fairness considerations. Fairness interacts with the provision of public goods, and it is their level that mainly determines whether an agent will evade taxes or not. Of course apart from the level of public goods it is also the behavior of the other agents that also influences one's decision to evade. This interaction among agents and this reciprocity in behavior is introduced through the notion of fairness. For fairness constraint to be introduced in the model, the Kantian notion has been used which can be summarized as following: individuals consider it fair to pay as much as they would wish the other individuals to pay, or in our case the politicians (Laffont, 1975). Furthermore, following Sugden (1984) he adds reciprocity in tax behavior.

His main findings can be summarized as following: First of all multiple equilibria may occur as a result of the interaction among agents. Secondly, a result that is consistent with many empirical findings is that not all people evade taxes even in the case were it would be to their interest to do so. If the amount of public goods is below a certain level, individuals will not tax evade, while tax evasion may be positive and increasing after some threshold level of public goods provision. What is significant about his results is that they accord to many empirical and experimental studies that highlight the importance of ethics and fairness in issues of tax evasion (Sandmo 1981). In his primary analysis, he highlighted the need for considering such issues as a solution to the problem that all people will evade if they are given the chance, a prediction far from reality.

Spicer and Lundstedt, (1976) and Wallschutzky, (1984) also examined issues of equity and how they affect the trade off between tax evasion and public expenditure. They find that there is a positive interaction between tax evasion and the perceived inequity. A similar positive relationship between tax evasion and perceived tax evasion by other taxpayers is found in Spicer and Hero (1985) and Porcano (1988).

Sandmo (2005) suggested that people may have"bad conscience" when they evade taxes. He introduced "bad conscience" in the model by adding a term in the utility function that is increasing with the level of evasion and reduces utility. He finds that when such considerations exist, a positive expected gain may not be sufficient for an agent to decide to tax evade. More interestingly he found that when under the presence of "bad conscience" the effect of the penalty tax is diminished, since the extrinsic incentive to be compliant reduces the intrinsic incentive to be honest. Even though this kind of models have been criticized on the grounds of expected utility maximization, the results are robust to the use of the rank-dependent expected utility (RDEU, Eide, 2002).

4.2.3 Coordination Failures

One of the major challenges in economics for many decades has been to account for differences between developed and developing countries. The understanding of this crucial topic would be the key to the development of poor countries. Many attempts have been made ever since to address this issue, none of them though can be characterized as fully satisfactory. Neoclassical economics had always focused on fundamentals, i.e. resources, technology and preferences. If one assumes homogeneous preferences for different countries then differences in the capital across countries are due to the fact that some countries started accumulating at a later point in time and therefore all countries will converge at some point in time. Moreover an extensive provision of capital can cover for the differences and speed up the convergence. However such practices when actually implemented were far from successful. The next plausible explanation was government failure. Governments prevented market from functioning smoothly and attaining optimal outcomes. But this explanation as well was not sufficient and could not account for differences among countries especially for government failures exogenous in the model.

In the neoclassical theory institutions, distribution and history did not matter since the equilibrium of the economy could not be affected by such factors; fundamentals would lead to optimal Pareto outcomes. When neoclassical theory failed to answer this question in a satisfactory way, it became evident that these factors should be given more attention. A new literature developed which pointed out that economic outcomes depend among others on: institutions, distribution of wealth, history and ecology. Each of the four factors has been extensively analyzed in the literature with very interesting implications for economic theory. For the purpose of this chapter though I will mainly focus on "ecology".

The term "ecology" was used to emphasize that according to the newly developed theory, economic results are influenced by various endogenous factors apart from the fundamentals. According to the neoclassical theory the dynamics of an economy are rather mechanical and given the initial conditions it is easy to anticipate the evolution of this economy and the attained equilibrium. This is not the case with modern development economics, where multiple equilibria may occur due to various factors. As Darwin has put it, ecosystems may have multiple equilibria, despite the fact that they may share some common characteristics. Which equilibrium will be determined may be affected by various factors or even by lack. In terms of policy this was "bad" and "good" news at the same time because what was needed to move from one equilibrium to the other was just a "push" to the right direction. The good news was that this "push" could be less costly than transfers of capital (which were the policy suggested by neoclassical theory), however if it was not to the correct direction the economy could find itself to a worse equilibrium. As Hoff (2001) has put it, "individuals' failure to coordinate complementary changes in their actions leads to a state of affairs that is worse for everyone than some alternative state of affairs that is also an equilibrium". If agents cannot optimally allocate the resources by themselves, or in other words if the "invisible hand" does not work, a low equilibrium may indeed occur.

The notion of coordination failure and multiple equilibria is not a new one. Actually, its first emergence dates back to 1943, in Rosenstein-Rodan, who argued that at early stages of development, an equilibrium with low development was feasible, and this was not due to technology or resources but due to lack of coordination. What the government should do was to intervene in the industrialization process. Few others (Nurkse, 1953; Leibenstein, 1957) also accepted the possibility of "underdevelopment equilibria" and "vicious circles of poverty" (contrary to the "virtuous circle" which refers to the "good" equilibrium) but at that time their impact on neoclassical economists was rather limited.

It was the formal modelling of externalities, technological progress and scale economies that made the theory more robust. Gradually the term "coordination failure" took on a different meaning. As Cooper and John (1988) had put it, coordination failure meant the existence of multiple equilibria, that can occur in games with strategic complementarity (as they put it "strategic complementarity can arise when the optimal strategy of an agent depends positively upon the strategies of the other players"). Moreover if the game is symmetric, the equilibria can even be Pareto-ranked.

These equilibria may either be history dependent or self-fulfilling. In the case of history dependent equilibria not only past events will define current behavior but also initial conditions have a major role in the final outcome. An example of history dependence is the distribution of wealth (Bardhan 2000; Hoff 1994, 1996 are some authors that have analyzed this topic and found that high inequality may result in stagnation). In the case of "self-fulfilling equilibria" expectations define the final outcome.

Independently of the source of multiplicity of equilibria it is easy to realize what action should be taken so that the economy will move from the "bad equilibrium" to the good "equilibrium". Current literature suggested various ways to escape "poverty traps". Tirole (1996), in model of group reputation, suggested that an anticorruption programme could potentially drive the economy to the "good" equilibrium, which should last though for a while and be strictly followed. Stiglitz (1974) suggested that affirmative action programmes can eliminate equilibria, i.e. a change in legal status that would force an equilibrium. In a similar spirit is the suggestion of

Cooter (2000) who finds that enactment of a social norm into law can eliminate self-fulfilling equilibria. Interestingly, especially in the context of this model, Cooter (1997) argues that when a state is governed by laws that reflect social norm, it is hardly corruptible (a rule of law state), whereas it is easily corruptible when the law is enforced and does not mirror social norms (the rule of state law). Commitment is also another way to eliminate equilibria. Commitments can be credible in many different ways. Storing capacity, stockpiling resources or deep interventions are some intuitive ways that can eliminate equilibria.

According to Howitt (2001) the models with multiple equilibria have two main drawbacks: The first drawback is that these models do not account for the coordination process, they just focus on the existence of the equilibria. Even when action is taken towards moving to the good equilibrium, it is not clear how this is achieved and what cost it entails.

The second drawback is that when coordination failure is present it is not certain that there are going to be two equilibria, a "good" and a "bad" one. Actually there may exist many other equilibria in between to which agents may as well coordinate. This last drawback has been emphasized by Matsuyama (1995) who claims that resolution of coordination failures by the government is not feasible since economies just move from one equilibrium to another (which is just one of the many possible equilibria, and most likely not the "best"). As he characteristically put it, "There is only one way of being perfect, but there are millions of ways of being imperfect".

His critique does not aim to diminish the importance of the models with multiple equilibria. Instead his critique is against government policies aimed to resolve this multiplicity. He states that a government policy, even if it achieves anything, it may "push" the economy to a worse equilibrium. However he totally argues with the intervention of the government on the first hand, since he considers that market actions could result in coordination as well without the intervention of the government.

Part of this critique has been confronted by Hoff who suggested that the government usually has powers that private sector does not have, and these powers are important in addressing the public good and the externality problems.

4.3 The Baseline Model

4.3.1 Introduction

The purpose of this chapter is to develop the baseline model that will form the basis for more elaborate ones. Starting from a standard model that generates multiple equilibria and coordination failures, I will try to identify how to resolve the failure and make agents coordinate to choose a unique equilibrium. In games with multiple equilibria there are various ways to obtain a unique equilibrium. Of the most widely used are the following two: a) a change in the rules of the games (i.e. sequential games) and b) the introduction of policy that will act as a selection equilibrium device. The first suggestion, which is applied in this chapter and indeed leads to a unique equilibrium is of reduced importance in terms of policy since changes in the rules of the game are not always feasible in real world situations. Therefore I will focus on policy reforms. As a selection equilibrium device I will assume that the "government" (which does not explicitly appear in the model) will resort to two kinds of reforms. The first is a rather widely used measure, namely the imposition of fines and the development of an auditing mechanism while the second is the undertaking of actions that cultivate the morals of the economy (i.e. anti-corruption campaigns). These considerations impose a kind of a "conscience tax". This direct effect on the utility follows the tradition of Becker (1968) and Sandmo (2004).

While the first reform does not eliminate the failure, the same is not true for the second. Despite the fact that the imposition of fines is a measure that may increase the cost of corruption, however it does not resolve the multiplicity of equilibria. Instead it makes them adjust their behavior to the new data and behave accordingly. In other words low corruption and high corruption are still two options belonging to the agents' reaction functions despite the fact that they are most costly. In this sense this kind of reform can bee viewed as a "shallow" intervention, following the terminology of Hoff (2001), which cannot effectively resolve the problem, and may even, as we will see below, make the situation worse. This danger is also stressed by Matsuyama (1996) who calls for great caution when such reforms are undertaken.

On the contrary, the "cultivation" of social morals in the economy, once it is achieved, it has better results in fighting corruption. The agents, whose utility is directly affected if they get caught behaving in a corrupt manner, manage to coordinate in a unique, and luckily "good" equilibrium. The source of this result is that extensive corruption, the former "bad" equilibrium, does no longer belong to their reaction function and therefore is no longer a best response. In this sense it can be viewed as "deep" intervention, since once morals exist in the society it is a kind of "irreversible" action. Much as simplistic this may be, at least in terms of modelling, it is a challenging task to find the ways to cultivate moral behavior. It is a procedure that must be persistent, as Tirole (1996) pointed out in a different context, and universally applied in order to become a norm. "Shallow" reforms may be a part of or accompany or enhance this effort.

4.3.2 The Model

I consider an overlapping generations model. Agents are divided in two distinct categories, namely private citizens[13] and politicians. The choice of occupation is exogenous in the model. This makes the model more tractable by restricting changes in the size of the group of politicians

[13] From this point onwards I will use the terms private citizen and citizen interchangeably to signify a citizen who does not hold any official or public position. Even though the term citizen is usually associated with a native or naturalized member of a state or other political community in general, the term private citizen highlights the difference between this group and politicians (Webster Dictionary).

and of citizens. Each generation of politicians and private citizens consists of a unit mass cohort of homogeneous agents[14].

The aggregate production function of the economy is

$$Y_t = BH_t = B\int_0^1 h_t^i g_t(h_t^i)dh_t^i \qquad A > 0$$

where H_t is the aggregate stock of human capital at time t and $g_t(h_t^i)$ is the density function which characterizes the distribution of human capital at time t and for which $\int g_t(h_t^i)dh_t^i = 1$. B which is equal to the marginal product, i.e. the wage, is normalized to unity for analytical convenience. Note however that since all agents are homogeneous we can omit the i superscript and thus use $Y_t = H_t = H$. Also note that according to the mass-unit assumption the average human capital is denoted with h_t. Agents live for one period and they are endowed with one unit of labor which they entirely devote to production. Accumulation of human capital is given by:

$$h_{t+1} = AE_t + \upsilon H_t \text{ and} \tag{4.1}$$

$$E_t = \mu_t \varphi z_t \tau H_t$$

where E_t is government spending on education and A is a technology parameter. Spending must be non negative, therefore.

$$\mu_t \varphi z_t \tau H_t \geq 0 \tag{4.2}$$

Private citizens are taxed at the exogenous rate τ on their declared income which is given

14 This formulation allows to focus on the interactions between the two groups instead of interactions within each group or "peer" effects. In future extensions one can use heterogeneous agents within each group, so as to have interactions not only between groups, but also within groups.

by zh_t. Thereby z denotes the *declared income* of the citizens. Politicians, contrary to citizens, do not participate in the production process. Their source of income is the tax revenue of the economy $z_t \tau H_t$. The tax revenue is allocated in the three following activities. A part $(1-\varphi)$ of the tax revenue goes to politicians as wages[15]. The rest of the tax revenue, $\varphi z_t \tau H_t$ is planned to go on education. However a part $(1-\mu)$ out of $\varphi z_t \tau H_t$ is embezzled by the politicians. Thereby μ denotes the portion of tax revenue that goes to education *without having been embezzled.* vH_t denotes the minimum level of human capital obtained by agents without exerting any effort or to put it differently the rate of human capital transferred freely to the next generation. For instance, nowadays the use of telephone or the TV does not require any specific skills or any effort on the part of the young. This human capital accumulation process shares common characteristics with several papers in the literature; see, among others, De Gregorio and Kim (2000) and Ceroni (2001). $z, \tau, \mu < 1$ since they denote rates.

Private citizens and politicians maximize the following utility function which is defined over consumption at time t+1 and an altruistic motive of the parent concerning their child's human capital stock.

$$u^i = c^i_{t+1} h^{\beta}_{t+2}$$

where c^i_{t+1}, i=c,p, denotes citizens (c), or politicians (p) consumption and h^{β}_{t+2} denotes offsprings' human capital[16] Therefore the current generation is related to future generations by manifesting an altruistic motive whose strength is denoted by β. Similar altruistic motives can be

15 φ is in the model is exogenously determined. However it is a first form of allocation of the tax revenue, which could later become endogenous or expand to other categories of the budget. This part φ could also be considered as the part of GDP that goes to the public sector for wages, or for consumption of the public sector. The wages in the public sector as a percentage of GDP lies between 5%-25%. For instance in Greece, wages for the public sector are approximately 10%.

16 Alternatively i could have used c^i_{t+2} i.e offsprings' consumption, with no qualitative difference in terms of results.

found in Ceroni (2001)

Private Citizen's Problem

The private citizen's maximization problem solved at time $t+1$ is given by

$$\max_{c^c_{t+1},z} c^c_{t+1} h^{\beta}_{t+2}$$

$$\begin{aligned} s.t \quad c^c_{t+1} &= (1 - z_{t+1}\tau)h_{t+1} \qquad (4.3) \\ h_{t+1} &= AE_t + vH_t \\ c^c_{t+1} &\geq 0,\ 1 \geq z_{t+1} \geq 0, \end{aligned}$$

Equation (4.3) gives the budget constraint of the citizen whose consumption equals his disposable income after having evaded taxes to a rate z. Agents born in period t+1 choose $\{c^c_{t+1}, z\}$to maximize their lifetime utility subject to their budget constraint, to the learning technology and the non-negativity constraints.

Concavity always holds when $\beta < 1$, while in the case where $\beta > 1$ the following condition must be satisfied:

$$v > \frac{(\beta - 1)(1 - z_{t+1}\tau)A\mu\varphi}{2} - Az_{t+1}\tau\mu_{t+1}\varphi \qquad (4.4)$$

Maximization yields the following first order conditions:

$$c^c_{t+1} : \ (A\mu_{t+1}\varphi z_{t+1}\tau h_{t+1} + vh_{t+1})^{\beta} = \lambda \qquad (4.5)$$

$$z_{t+1}: \; c^c_{t+1}\beta(A\mu_{t+1\varphi}z_{t+1}\tau h_{t+1}+vh_{t+1})^{\beta-1}A\mu_{t+1}\varphi\tau h_{t+1}=\lambda\tau h_{t+1} \tag{4.6}$$

From (4.5), (4.6) and (4.3) I obtain

$$z_{t+1}=\frac{\beta A\mu_{t+1}\varphi-v}{A\mu_{t+1}\varphi\tau(1+\beta)} \tag{4.7}$$

where (4.7) is the reaction function of the private citizen.

Politician's Problem

The politician's maximization problem solved at time $t+1$ is given by

$$\max_{c^p_{t+1},\mu} c^p_{t+1}h^{\beta}_{t+2}$$

$$\begin{aligned} s.t \;\; c^p_{t+1} &= (1-\mu_{t+1}\varphi)z_{t+1}\tau h_{t+1} && (4.8)\\ h_{t+1} &= AE_t+vH_t \\ c^p_{t+1} &\geq 0, \; 1\geq \mu_{t+1}\geq 0, \end{aligned}$$

Equation (4.8) gives the budget constraint of the politician whose consumption, c^p_{t+1}, is equal to the amount of money he embezzles plus his wage income. Agents born in period t+1 choose $\{c^p_{t+1},\mu\}$ to maximize their lifetime utility subject to their budget constraint, to the learning technology and to the non-negativity constraints.

Similarly for concavity, when $\beta<1$ concavity always holds, while when $\beta>1$ the following condition must hold:

$$v > \frac{(\beta - 1)(1 - z_{t+1}\tau)A\mu_{t+1}\varphi}{2} - Az_{t+1}\tau\mu_{t+1}\varphi$$

Maximization yields the following first order conditions:

$$c^p_{t+1} : \ (A\mu_{t+1}\varphi z_{t+1}\tau h_{t+1} + vh_{t+1})^{\beta} = \lambda \tag{4.9}$$

$$\mu_{t+1} : c^p_{t+1}\beta(A\mu_{t+1}\varphi z_{t+1}\tau h_{t+1} + vh_{t+1})^{\beta-1}Az_{t+1}\tau h_{t+1} = \lambda z_{t+1}\tau h_{t+1} \tag{4.10}$$

From (4.9), (4.10) and (4.8) solving for z I obtain

$$\mu_{t+1} = \frac{\beta Az_{t+1}\tau - v}{A\varphi z_{t+1}\tau(1+\beta)} \tag{4.11}$$

where (4.11) is the reaction function of the private citizen.

4.3.3 Simultaneous Game - Nash Equilibria

We can view the above situation as a simultaneous game between private citizens and politicians. Each agent when maximizing with respect to his choice variables has to trade-off between his current consumption, which is reduced by taxation, and the human capital stock of his offspring which increases with taxation. Additionally each player chooses the optimal level of tax evasion and embezzlement taking the choice of the other player into account. Combining equations (4.7) and (4.11) which are the reaction functions of the citizens and the politicians respectively, with non-negativity constraints I obtain:

$$z_{t+1} = \begin{array}{cc} 0 & \text{if } \beta A\mu_{t+1}\varphi < v \\ \frac{\beta A\mu_{t+1}\varphi - v}{A\mu_{t+1}\varphi\tau(1+\beta)} & \text{if } \beta A\mu_{t+1}\varphi > v \end{array}$$

and

$$\mu_{t+1} = \begin{matrix} 0 & \text{if } \beta A z_{t+1}\tau < v \\ \frac{\beta A z_{t+1}\tau - v}{A z_{t+1}\tau\varphi(1+\beta)} & \text{if } \beta A z_{t+1}\tau > v \end{matrix}$$

Furthermore note that

$$\frac{\partial z_{t+1}}{\partial \mu_{t+1}} = \frac{v}{A\mu_{t+1}^2\varphi\tau(1+\beta)} : \; > 0 \quad : z_{t+1} \text{ Strategic Complement with } \mu_{t+1}$$

$$\frac{\partial \mu_{t+1}}{\partial z_{t+1}} = \frac{v}{A z_{t+1}^2\tau\varphi(1+\beta)} : \; > 0 \quad : \mu_{t+1} \text{ Strategic Complement with } z_{t+1}$$

which indicates that decisions on z_{t+1} and μ_{t+1} are strategic complements. Games of strategic complementarity, according to the literature (Cooper and John, 1988, Vives, 2005) are those in which the best response of any player, is increasing in the actions of the rival. Therefore since z_{t+1} and μ_{t+1} are strategic complements we infer that the amount evaded on the part of the citizens positively depends on the amount embezzled on the part of the politicians and the same holds for the politician as well. A reduction in the amount embezzled by the politician, increases the marginal return of the citizen who chooses to increase the amount declared in the tax authorities.

The parameter responsible for the occurrence of strategic complementarity in the model is v. If I set $v = 0$ I obtain solutions for the politicians and the citizens that do not depend on the strategy of the other player. The intuition behind this result is that the appearance of v makes the marginal utility of the citizen ($z_{t+1}(\mu_{t+1})$) dependent on the marginal utility of the politician ($\mu_{t+1}(z_{t+1})$) by bounding the returns of education at zero spending. Therefore even at zero spending, agents have an incentive to coordinate to the bad equilibrium, i.e. $z = \mu = 0$, whereas if $v = 0$ there would exist a unique, symmetric equilibrium $z = \mu = \frac{\beta}{1+\beta}$ imposed by optimization. Each player would realize that the other player would not have an incentive to deviate from this equilibrium.

4.3.3.1 Equilibrium

A Nash equilibrium in this economy consists of sequences $\{c^i_{t+1}\}^{\infty}_{t=0}, \{z_{t+1}\}^{\infty}_{t=0}, \{\mu_{t+1}\}^{\infty}_{t=0}, \{Y_{t+2}\}^{\infty}_{t=0}$, $\{H_{t+2}\}^{\infty}_{t=0}, \{h_{t+2}\}^{\infty}_{t=0}$, i=c,p, such that given h_0 in every period t=0,1,...

1. Private citizens choose z_{t+1} to maximize their utility function, taking μ_{t+1} as given.
2. Politicians choose μ_{t+1} to maximize their utility function, taking z_{t+1} as given.
3. The labor market clears.
4. $h_{t+1} = H_{t+1,}$ since all agents are identical in equilibrium.

The steady state values of (4.7) and (4.11) are:

$$z_1^* = 0$$

$$z_2^* = \frac{A\beta - \sqrt{A(A\beta^2 - 4v\beta - 4v)}}{2A\tau(1+\beta)}$$

$$z_3^* = \frac{A\beta + \sqrt{A(A\beta^2 - 4v\beta - 4v)}}{2A\tau(1+\beta)}$$

$$\mu_1^* = 0$$

$$\mu_2^* = \frac{A\beta + \sqrt{A(A\beta^2 - 4v\beta - 4v)}}{2A\varphi(1+\beta)}$$

$$\mu_3^* = \frac{A\beta + \sqrt{A(A\beta^2 - 4v\beta - 4v)}}{2A\varphi(1+\beta)}$$

As Cooper and John (1988) had pointed out, under the presence of strategic complementarity, multiple equilibria may occur. The occurring equilibria, are not fundamentals driven, instead they are self-fulfilling and they critically depend on the expectations of one group concerning the behavior of the other group. Coordination failures with multiple, self-fulfilling or path-dependent equilibria abound in the literature, e.g. bank-runs, poverty traps, R-D investment, etc.

To graphically analyze the solutions:

$$z: \begin{array}{ll} \text{if } \mu \to 1 \text{ then} & z \to \frac{\beta A\varphi - v}{A\varphi\tau(1+\beta)} > 0 \\ \text{if } \mu \to 0 \text{ then} & z \to \frac{-v}{0} \to -\infty \\ & \text{since I assumed earlier that } v > 0 \end{array}$$

$$\mu: \begin{array}{ll} \text{if } z \to 1 \text{ then} & \mu \to \frac{\beta A\tau - v}{A\varphi\tau(1+\beta)} > 0 \\ \text{if } z \to 0 \text{ then} & \mu \to \frac{-v}{0} \to -\infty \\ & \text{since I assumed earlier that } v > 0 \end{array}$$

Figures (4.6) and (4.7) illustrate the pure strategy equilibria in the case where the two reaction curves are increasing and concave. Concerning the stability of equilibria the two corner Nash equilibria are stable with respect to the best-reply dynamics, while the other equilibrium is unstable. Mixed strategy equilibria are not considered in this context. The two corner equilibria are the following: In the "bad" equilibrium citizens fully evade taxes and politicians embezzle the total part of the tax revenue that was supposed to become spending on education. In the second corner equilibrium compliance is much higher (not equal to unity though, unless for specific parameter values). In this equilibrium the incidence of political corruption and of tax evasion is significantly reduced. Finally the third equilibrium represents an intermediate corruption-evasion regime which is unstable. In this equilibrium the reaction functions cross with the wrong slopes, therefore each agent when anticipating how the other agents react, will never choose this equilibrium. Since this equilibrium is unstable each agent will either choose to behave rather honestly or will choose to fully evade-embezzle resources depending on his beliefs concerning the action of the other group.

Relating these three equilibria with the growth rate of the economy as given by eq. (4.12)

$$\frac{Y_{t+1}}{Y_t} = \frac{H_{t+1}}{H_t} = Az\mu\varphi\tau + v \tag{4.12}$$

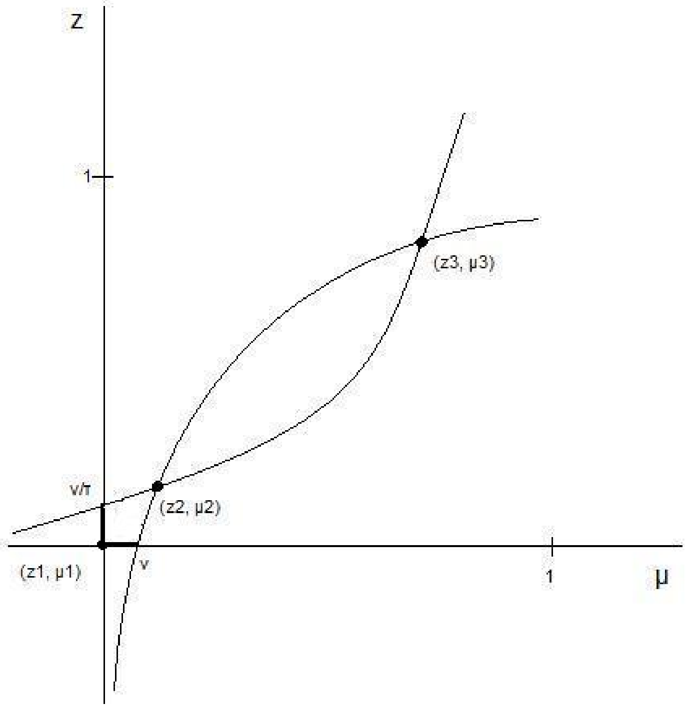

Figure 4.6: Multiple Equilibria in the Baseline Model

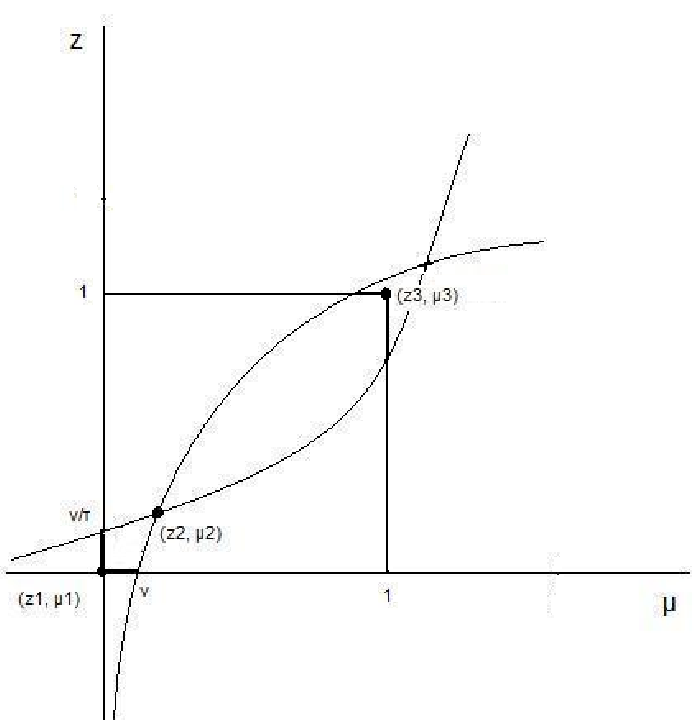

Figure 4.7: Multiple Equilibria in the Baseline Model-Full Compliance

it is evident that the three equilibria correspond to three development regimes. The "bad" equilibrium corresponds to a low development-high corruption regime, the intermediate equilibrium to an intermediate development-corruption regime and the "good" equilibrium to a high development-low corruption regime. The evolution of the economy critically depends on its ability to find itself on the fast development regime. As mentioned above the evolution of this economy is not path-dependent and the occurring equilibria are self-fulfilling equilibria. Which of the two stable equilibria will prevail depends on the strategic interactions between the two agents concerning their choice of z and μ. The focus of the following chapters, will be to find the mechanism that can eliminate multiple equilibria and drive the economy to one unique equilibrium [17].

A comparative static result related to the growth rate of the economy is that the allocation of budget directly affects growth. It is easily identified that an increase (decrease) in spending on education, positively (negatively) affects the growth rate of the economy. This result accords with empirical evidence by Barro (1991), who argues that an increase in resources devoted to non-productive activity (in this case $1-\varphi$) is associated with lower per capita growth, while Devarajan et al. (1996) find that some categories of spending on education (net of administration, management, etc.) are positively associated with the growth rate.

4.3.3.2 Welfare

Another important issue is to examine the welfare properties of the model. The approach of Cooper and John (1988) cannot be applied in this model since the equilibria are not symmetric. Therefore I will resort to the analysis of Vives (2005a, 2005b) on supermodular games. Quoting

[17] The importance of initial conditions under the presence of corruption has extensively been analyzed in the literature (see for examble Blackburn et al, 2005, 2006). Even though it requires just a simple modification to examine this topic and make the evolution of the economy path-dependent, I have chosen not to analyze this case in the baseline model in order to focus on social interactions. A simple exposition of this case could be the focus of future research.

Vives, a game $(A_i, \pi_i; i \in N)$ is smooth supermodular if each A_i (which denotes the strategy set) is a compact cube in Euclidean space, and $\pi_i(\alpha_i, \alpha_{-i})$ (where denotes π_i the payoff of player $i \in N$, N denotes the set of players and $\alpha_i \in A_i$ denotes the strategy profile) is twice continuously differentiable with:

(i) $\frac{\partial^2 \pi_i}{\partial \alpha_{ih} \partial \alpha_{ik}} \geq 0$ for all $k \neq h$ and

(ii) $\frac{\partial^2 \pi_i}{\partial \alpha_{ih} \partial \alpha_{jk}} \geq 0$ for all $j \neq i$ and h and k

where α_{ih} denotes the h_{th} component of the strategy α_i of player i.

The first condition denotes strategic complementarity in a players' own strategies, which always holds in this model since each player has only one strategy. The second condition is strategic complementarity in the strategies of rivals.

When a game is smooth supermodular, it has the following properties among others:

1) Existence and order structure.

2) Pareto ranked equilibria

3) Stability

Note that these properties hold for non-symmetric equilibria as well.

What is of interest here is the second property, i.e. Pareto dominance. According to Vives, if the payoff to a player is increasing in the strategies of the other players, then the largest equilibrium point is the Pareto best equilibrium and the smallest is the Pareto worst. Therefore if the game is smooth supermodular then the low corruption equilibrium is Pareto best equilibrium contrary to the high corruption equilibrium which is Pareto worst equilibrium.

Following the definition of Vives and after having proved in an earlier part that the strategic complementarity holds between the strategies of the two players, then the game is smooth supemodular and the equilibria are Pareto ranked with the Pareto superior equilibrium corresponding to the low corruption equilibrium.

4.3.3.3 Numerical Analysis

Following I illustrate numerically the solutions as well as some comparative statics. Concerning the numerical values that are used they are primarily chosen so as to apply to all following models and therefore make the results directly comparable among each other. For most values it is not easy to resort to policy parameters, except perhaps the tax rate which usually ranges from 10% to 40% of total income of agents. Moreover it is know that wages in public sector are approximately a rate of 5%-20%[18] of total GDP while education is approximately 5%-8%, however in the context of this model, since there are no other allocations of budget it is hard to identify what the value of φ should be. Similar problems hold for v as well, which is hard to measure, however it is evident that it must be quite low. Finally A is technology parameter indicating productivity of spending on education. Since there is no clear evidence in the literature for the value of A I will use a minimum value.

Assigning parameter values to the model,

$$A = 1.1, \beta = \frac{1}{2}, v = \frac{1}{30},\ \varphi = 0.4\ and\ \tau = \frac{1}{3}$$ [19]

I obtain the following equilibria:

$$z : \begin{array}{l} z_1^* = 0 \\ z_2^* = 0.238 \\ z_3^* = 0.761 \end{array} \text{ and } \mu : \begin{array}{l} \mu_1^* = 0 \\ \mu_2^* = 0.199 \\ \mu_3^* = 0.634 \end{array}$$

From the numerical results and Figure (4.8) I observe that the good equilibrium makes predictions that are lower than real evidence. This is rather expected though since in the model

[18] In Euro area it is approximately 10% on average. More analytically, according to Eurostat and ECB, Germany and Ireland report the lowest ratios of 8-9% of GDP while France, Finland and Portugal spend about 14-15% of GDP. The Scandinavian countries report a significantly higher ratio, approaching 20%. UK and the US report public wage spending ratios similar to that of the euro area average.

[19] The chosen parameter values satisfly all constraints imposed in the model.

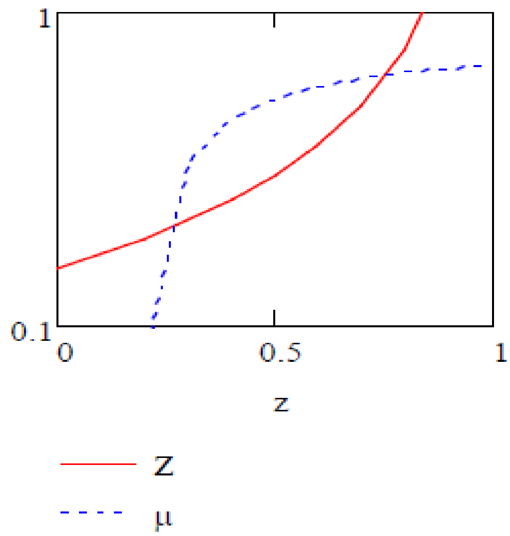

Figure 4.8: Multiple Equilibria in the Baseline Model.

there are no policies deterring agents from being corrupt, and therefore they make their decision by maximizing their payoffs only taking into account the behavior of the other group which behaves in a similar manner.

Due to strategic complementarity there are some comparative statics results enhanced by multiplier effects. A multiplier effect denotes the direct effect of variation in one variable in the strategy of an agent who takes as given the strategy of the other agent, which in turn is amplified by the variation in this variable. For instance as v increases both z and μ decrease which is expected, since when individuals can learn a lot from their environment without effort or resources, then they have a reduced incentive to invest on human capital and are inclined towards increasing consumption. Similarly an increase in β makes the altruistic motive of the parent stronger and the parent (either politician or civilian) reduces his corrupt behavior (tax evasion or embezzlement) in favor of his offspring's human capital. An increase in the productivity of spending on education, A, positively affects corruption and tax evasion since the more productive the spending, the more motivated are agents not to deprive funds that are supposed to go to spending on education, while with lower A is it is more "profitable" to conceal (evade or embezzle) this money.

Concerning politician's wage I observe that a decrease in φ increases μ, thus reduces politician's incentive to embezzle money. Note though that it does not reduce the incentive of the citizen to tax evade not even through the increase in μ[20]. The same holds for a change in τ which affects the behavior of the citizens (an increase in τ increases the incentive to tax evade) but not the behavior of the politicians. In terms of modelling this is due to the fact that these terms are multiplicatively related to z and μ and it indicates that agents actually care about the product $z\tau$ and $\mu\varphi$.

To numerically illustrate some comparative statics, an increase in their wage (i.e. a decrease in φ), would significantly reduce embezzlement rates. Keeping all the other values unchanged and decreasing φ to $\varphi = 0.3$ I obtain:

$$z: \begin{array}{l} z_1^* = 0 \\ z_2^* = 0.238 \\ z_3^* = 0.761 \end{array} \text{ and } \mu: \begin{array}{l} \mu_1^* = 0 \\ \mu_2^* = 0.265 \\ \mu_3^* = 0.845 \end{array}$$

I observe (Figure 4.10) that the "good" equilibrium has significantly improved by this change in the allocation of budget in favor of the politicians' wage. The results are in accordance with Haque and Sahay (1996) who find evidence on a negative relationship between corruption and wages.

Similarly for τ if I assume a decrease in tax to $\tau = \frac{1}{3.5}$ then:

$$z: \begin{array}{l} z_1^* = 0 \\ z_2^* = 0.278 \\ z_3^* = 0.887 \end{array} \text{ and } \mu: \begin{array}{l} \mu_1^* = 0 \\ \mu_2^* = 0.199 \\ \mu_3^* = 0.634 \end{array}$$

[20] In later parts i will extensively analyze the effect of a change in the allocation of budget. Not though that this allocation is exogenous in this model. It would be interesting to endogenise the allocation of budget as a policy tool and expand it to other categories as well.

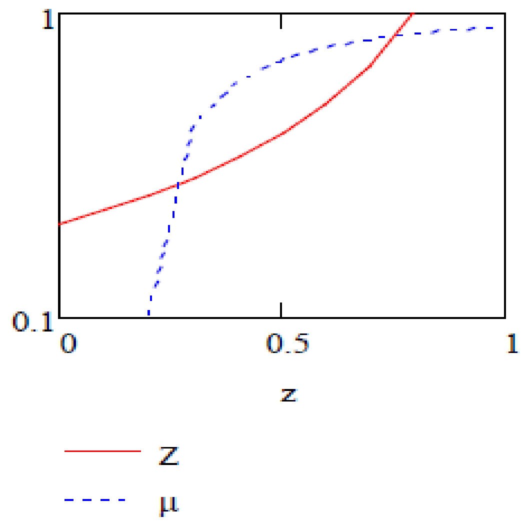

Figure 4.9: Multiple Equilibria in the Baseline Model, φ=0.3.

agents have a decreased incentive to tax evade. The effect of changes in tax rates on tax evasion is not very clear in the literature. This effect will be extensively analyzed in a following part of this chapter where fines will be introduced.

More comparative statics results can also be numerically tested. For instance starting from the first equilibrium and slightly lowering v gives the following results:

$$A = 1.1, \beta = \frac{1}{2}, v = \frac{1}{50}, \ \varphi = 0.4 \ and \ \tau = \frac{1}{3}$$

$$z: \begin{array}{r} z_1^* = 0 \\ z_2^* = 0.124 \\ z_3^* = 0.875 \end{array} \text{ and } \mu: \begin{array}{r} \mu_1^* = 0 \\ \mu_2^* = 0.103 \\ \mu_3^* = 0.729 \end{array}$$

What is interesting in this numerical experiment is not only that the decrease in v improved the "good" equilibrium, which was rather expected since now agents learn less without effort and therefore must spend more on education to get "equally" educated, but also the direction of change in the "intermediate" equilibrium. I observe that the "intermediate" equilibrium has decreased, which is a development though, since this equilibrium acts as a threshold above which the agents will coordinate to the good equilibrium. This result did not take place when φ and τ varied. On the contrary in the above examples the "intermediate" equilibrium moved to the same direction with the "good" equilibrium. This result can again be attributed to the relationship of the variables ($z\tau$ and $\mu\varphi$) and to the fact that this effect is not canalized to the whole economy. The effect of v is canalized and it affects the overall level of corruption. This holds for all the other variables as well.

For instance an increase in the parents altruistic motive yields:

$$A = 1.1, \beta = \frac{1.2}{2}, v = \frac{1}{30}, \ \varphi = \frac{1}{2} \ and \ \tau = \frac{1}{3}$$

$$z: \begin{array}{c} z_1^* = 0 \\ z_2^* = 0.180 \\ z_3^* = 0.944 \end{array} \text{ and } \mu: \begin{array}{c} \mu_1^* = 0.0 \\ \mu_2^* = 0.150 \\ \mu_3^* = 0.787 \end{array}$$

i.e. an improvement in the "good" equilibrium and a lowering of the intermediate equilibrium.

Overall what can be observed is that changes in parameters of the model affect the overall corruption rates of the agents as well as the likelihood to find themselves in one of the two equilibria.

4.3.4 Sequential Game

Despite the improvement in the good equilibrium, multiplicity still persists. To eliminate multiplicity of equilibria we can choose two directions: Either to change the rules of the game, or to induce policy in the model. The first option is not very intuitive since we know from the outset that a unique equilibrium will occur without having introduced any policy in the model. However it will be presented in the context of this chapter as a means to test for the internal consistency of the model as well as to give some intuition for the cases where sequential moves are indeed feasible as an option[21]. The agents will play a sequential game, where in succeeding chapters I will assume that each group may act as a leader.

4.3.4.1 Commitment on the part of the Private Citizens

In the first game citizens are the leaders. How will they react? In a sequential game, they will maximize their own utility, choosing the rate of tax evasion, and predicting politicians' optimal response to their strategy.

The model and the notation is exactly the same as in the baseline model. The difference lies in that to find the SPNE I will use the backward induction method, i.e. to solve for the reaction function of the follower first which in this case is the politician. This is the same with eq. (4.11) i.e.

$$\mu = \frac{\beta A z_{t+1}\tau - v}{A z_{t+1}\tau\varphi(1+\beta)} \tag{4.13}$$

The private citizen's maximization problem solved at time $t+1$ is given by

$$\max_{c^c_{t+1}, z} \ln c^c_{t+1} + \beta \ln h_{t+2}$$

$$\begin{aligned} s.t \quad c^c_{t+1} &= (1 - z_{t+1}\tau)h_{t+1} \\ h_{t+1} &= A z_t \tau \frac{\beta A z_{t+1}\tau - v}{A z_{t+1}\tau\varphi(1+\beta)}\varphi H_t + vH_t \\ c_{t+1} &\geq 0, \; 1 \geq z_t \geq 0, \end{aligned} \tag{4.14}$$

[21] An interesting extension would be to introduce the probability that a politician does not get re-elected if he is corrupt. In this game there is some kind of sequntial move, since agents play first (through voting) knowing that the politician will have a reduced incentive to embezzle money if he fears that he will not be re-elected.

Maximization and further manipulation yields the following equation:

$$z_{t+1} = -\frac{v - A\beta}{A\tau(1+\beta)} \tag{4.15}$$

Equilibrium

A Nash equilibrium in this economy consists of sequences $\{c_{t+1}\}_{t=0}^{\infty}, \{z_{t+1}\}_{t=0}^{\infty}, \{\mu_{t+1}\}_{t=0}^{\infty}, \{Y_{t+2}\}_{t=0}^{\infty}$, $\{H_{t+2}\}_{t=0}^{\infty}$, $\{h_{t+2}\}_{t=0}^{\infty}$, such that given h_0 in every period t=0,1,...

1. Private citizens choose z_{t+1} to maximize their utility function, knowing that the rate of embezzlement on the part of the politicians is related to their decision on the rate of evasion, i.e. $\mu_{t+1} = f(z_{t+1})$

2. Politician choose μ_{t+1} to maximize their utility function, taking z_{t+1} as given.

3. The labor market clears.

4. $h_{t+1} = H_{t+1,}$ since all agents are identical.

Therefore the steady state value is given by:

$$z^* = -\tfrac{v-A\beta}{A\tau(1+\beta)} \quad \text{for } v < A\beta$$

and

$$\mu^* = \tfrac{A\beta^2 - v(1+2\beta)}{(1+\beta)\varphi(A\beta - v)} \quad \text{for } A\beta^2 \gtrless v(1+2\beta) \text{ and } A\beta \gtrless v$$

Therefore there exists one unique equilibrium which maximizes citizens payoffs.

Numerical Exposition

To numerically illustrate the results I will use the same parameter values as in the baseline model:

$$A = 1.1, \beta = \frac{1}{2}, v = \frac{1}{30}, \ \varphi = \frac{1}{2} \ and \ \tau = \frac{1}{3}$$

to obtain the following equilibria:

$$z : z_1^* = 0.939 \text{ and } \mu : \mu_1^* = 0.672$$

In the unique equilibrium, (z_1^*, μ_1^*) obtained in the sequential game, it is evident that the overall corruption has decreased, however the decrease in evasion rates is far more significant than the decrease in embezzlement rates. This situation is reversed if politicians play first.

4.3.4.2 Commitment on the part of the Politicians

In this part the politician is the leader of the game. The reaction function of the citizen is given by (4.7)

$$z_{t+1} = \frac{\beta A \mu_{t+1} \varphi - v}{A \mu_{t+1} \varphi \tau (1 + \beta)}$$

The politicians maximization problem at time $t + 1$ is:

$$\max_{c_{t+1}^p, \mu} \ \ln c_{t+1}^p + \beta \ln h_{t+2}$$

$$
\begin{aligned}
s.t \;\; c^p_{t+1} &= (1-\mu\varphi)\frac{\beta A\mu_{t+1}\varphi - v}{A\mu_{t+1}\varphi\tau(1+\beta)}\tau h_{t+1} \\
h_{t+1} &= A\frac{\beta A\mu_{t+1}\varphi - v}{A\mu_{t+1}\varphi\tau(1+\beta)}\tau\mu_t\varphi h_t + v h_t \\
c_{t+1} &\geq 0,\; 1 \geq \mu_t \geq 0,
\end{aligned}
$$

Maximization and further manipulation yields the following equation:

$$\mu_{t+1} = \mu^* \tag{4.16}$$

I refer to the solution as μ^* since its mathematical form is too complicated to gain intuition over it.

Equilibrium

A Nash equilibrium in this economy consists of sequences $\{c_{t+1}\}_{t=0}^{\infty}, \{z_{t+1}\}_{t=0}^{\infty}, \{\mu_{t+1}\}_{t=0}^{\infty}, \{Y_{t+2}\}_{t=0}^{\infty}$, $\{H_{t+2}\}_{t=0}^{\infty}$, $\{h_{t+2}\}_{t=0}^{\infty}$, such that given h_0 in every period t=0,1,...

1. Politicians choose μ_{t+1} to maximize their utility function, knowing that the rate of tax evasion on the part of the private citizens is related to their decision on the rate of embezzlement, i.e. $z_{t+1} = f(\mu_{t+1})$

2. Private citizens choose z_{t+1} to maximize their utility function, taking μ_{t+1} as given.

3. The labor market clears.

4. $h_{t+1} = H_{t+1,}$ since all agents are identical.

Therefore the steady state values are given by z^* and μ^*[22].

[22] The formulas obtained are too complicated therefore they are represented in a simple form.

Numerical Exposition

Using the same parameter values as in the baseline model:

$$A = 1.1, \beta = \frac{1}{2}, v = \frac{1}{30},\ \varphi = \frac{1}{2}\ and\ \tau = \frac{1}{3}$$

I obtain the following equilibria:

$$z : z^{*} = 0.845 \text{ and } \mu : \mu^{*} = 0.981$$

The results illustrate that there exists a unique equilibrium in which politicians are significantly less corrupt and citizens are less corrupt as well (but not as much as in the game where they acted as leaders).

Overall the intuition we gain out of these result is that both groups of agents have an incentive to behave rather honestly, since increases in z and μ increase their welfare. When each group plays first "realizes" this fact, and behaves more honestly than in the sequential game in order to increase payoffs. However there are always incentives to "cheat" and for this purpose "zero corruption" is not always an equilibrium for agents, not even in the sequential game.

Another interesting result is that strategic substitutability cannot be precluded in these two models. For instance when the politician announces that he will go for a low μ agents may find it profitable to go for a high z. On the contrary if the politician announces that he will go for high corruption (or to be more formal that he will not invest much in education) agents, who take his strategy as given, may not find it profitable to tax evade extensively, because this would dramatically decrease the stock of human capital. Much as interesting this case may be it will not be analyzed in the context of this chapter.

When thinking in terms of policy, many options come in mind. Policy could perhaps take the form of fines, that reduce the incentive of agents to behave in a corrupt way. Alternatively it could intend to increase the social costs of being involved in corrupt activities, through anti-corruption campaigns or exemplary punishments. Some ethical or moral considerations may even not occur as a result of policy but instead be closely related to the cultural characteristics of a society.

In the following part I will attempt to introduce some policy in the simultaneous game in the form of deterrence policies as a means to correct coordination failures. Deterrence policies have attracted much attention in the tax literature and for this purpose, apart from studying the effect of such policies in corruption I will also attempt a comparison between them.

4.4 Enforcement Policy-The Introduction of Fines

When considering the issue of corruption and especially that of tax evasion, the imposition of fines comes naturally as a measure to enforce compliance. This idea is also enhanced by the fact that in most countries imposition of fines is a common practice as a punishment, taking into account though that certain countries, such as China, can even sentence officials to death penalty for extensive embezzlement. Other countries, such as the majority of OECD countries, for large amounts of tax evasion or embezzlement have the alternative of imprisonment. Therefore the form and the extent of fines differs dramatically from country to country and for each specific case. What is important in terms of economic modelling, is that all forms of punishment, except for death penalty, can be translated into fines. For instance imprisonment can be easily translated into foregone consumption. So long as moral issues are not considered, such as social stigma for prisoners or embezzlers, there is a fine rate appropriate for each kind of punishment (Andreoni, 1998).

Based on this argument, I will check whether deterrence policies can eliminate multiplicity of equilibria and coordinate the actions of the two groups of agents by imposing a fine on each group. A fine will make a risk averse agent more unwilling to behave in a corrupt way, compared to the baseline model where the possibility of being caught does not exist. Even though fines as a form of punishment will be used for both groups, their interpretation varies for each group. As far as citizens are concerned, a fine may stand as it is or for imprisonment, or for money that one gives to be bailed out of jail. On the part of the politicians a fine per se is not very meaningful. What sounds more realistic is that if a corrupt politician is discovered to embezzle he may be jailed or more likely he loses his office and the wage associated with it. Moreover he may be induced to return the amount he embezzled by locking up his property. As we will see below all these assumptions can be embodied in simple forms of fines.

An important point concerning the imposition of fines, is the one exposed in the papers of Allingham and Sandmo (1972) and Yitzhaki (1974) on the effect of an increase in tax on tax evasion. This controversy much as interesting it may be it will not be examined in this chapter due to the multiplicative relationship between taxes and tax evasion which implies that the substitution effect always prevails and therefore an increase in the tax rate increases tax evasion. However, at least as far as citizens are concerned, I will impose both kinds of fines, i.e. fines in evaded tax and fines on evaded income, to test whether better results are obtained using one of the two methods. On the part of the politician the fine will be the same in both cases, since politicians do not "evade" money, instead they "embezzle"money and therefore they pay a fine imposed on the amount of money they embezzle.

What will be examined in this part which has not been extensively analyzed in the literature is the effect of fines and auditing probabilities on tax evasion and embezzlement under two different assumptions: 1) provision of public goods and 2) no provision of public goods. Most papers in the literature focus on the effect of taxes on tax evasion under these two assumptions and not on the effect of fines and auditing probabilities. These cases yield interesting results since the effect of fines and auditing probabilities on corruption is totally different for each of the two cases. The only paper in the literature that deals with this topic is that of Gottlieb (1985) which examines two cases: The competitive case where each agent is isolated and a Pareto inferior equilibrium occurs and the interdependent case where the decision of one agent affects the others. The baseline model with fines and provision of public goods are similar to the interdependent case since increases in these variables will contribute to the perpetuation of tax evasion. In the model with no public goods, the results comply with the standard literature on tax evasion i.e. an increase in fines and auditing probabilities (under the assumption of zero cost associated with auditing probabilities) decreases tax evasion.

4.4.1 Fines Financing the Provision of Public Good

In the first part of the analysis I will assume that public goods are provided in the economy which are financed by tax revenue and collected fines. Interestingly I find that when there is provision of public goods, the effect of an increase in fines on tax evasion is not clear. On the one hand agents have an incentive to reduce tax evasion due to increased fines, while on the other hand they have an incentive to increase tax evasion since these fines supplement the provision of public goods. The same holds with auditing probabilities p and q.This is an interesting result not extensively analyzed in the relevant literature that should be carefully accounted for by the government in order to find the optimum penalty rate and auditing probability.

4.4.1.1 Fines on Evaded Tax

Agents

As was the case with the other models I will build on the notation of the baseline model. Starting from the citizens, they maximize the same utility function subject to a different budget constraint. The difference with the previous models is that now agents are confronted with a possibility of being caught to evade, $0 < p < 1$. For the time being this possibility is completely exogenous and does not increase with the level of tax evasion[23] If the citizen gets caught to evade, he must pay a penalty π on the amount of evaded taxes and $\pi > \tau$. Therefore if he gets caught evading he will pay $\pi\tau(1-z)h_{t+1}$ where $\tau(1-z)h_{t+1}$ is the amount of evaded taxes[24].

Correspondingly, politicians are also confronted with a probability to be caught embezzling money, $0 < q < 1$, which is also exogenous and different from the probability of citizens to be caught. In this case the politician will have to pay a penalty rate $\theta > 1$ imposed on the embezzled income $(1-\mu)\varphi z\tau h_t$, i.e. an amount $q\theta(1-\mu)\varphi z\tau h_t$. All these fines directly affect the consumption of each group and enter their utility function indirectly.

The accumulation of human capital is now affected by the probability that some agents will be caught, since all the income from fines is directed and invested back to the economy. Therefore human capital accumulation is now given by:

$$h_{t+1} = AE_t + vH_t \quad \text{where}$$

$$E_t = z_t\mu_t\varphi\tau H_t + p\tau\pi(1-z_t)H_t + q\theta(1-\mu_t)\varphi z_t\tau H_t \text{ [25]}$$

For non-negative spending the following condition must be imposed:

$$z_t\mu_t\varphi\tau H_t + p\tau\pi(1-z_t)H_t + q\theta(1-\mu_t)\varphi z_t\tau H_t \geq 0$$

The citizen's maximization problem solved at time t + 1 is given by

23 It would be an interesting extention to make the probability of being caught to evade endogenous.
24 I implicitly assume that in the event of an audit, the real income of the citizen, h_{t+1}, will be revealed.

$$\max_{c^c_{t+1,}z} c^c_{t+1} h^{\beta}_{t+2}$$

$$s.t \quad c^c_{t+1} = (1 - z_{t+1}\tau)h_{t+1} - p\tau\pi(1-z)h_{t+1} \tag{4.17}$$

$$c_{t+1} \geq 0, \; 1 \geq z_{t+1} \geq 0,$$

Note that in this model for positive consumption to hold I must impose the following constrains:

$$1 - z_{t+1}\tau > \tau p\pi(1 - z_{t+1})$$

Maximization and further manipulation yields the following equation:

$$z_{t+1} = \frac{-1 + p\pi\tau + \frac{p\pi(1-p\pi)\tau}{\beta(\varphi\mu_{t+1} - \rho\pi + \varphi(1-\mu_{t+1})q\theta)} + \frac{(1-\rho\pi)v}{A\beta(\varphi\mu_{t+1} - \rho\pi + \varphi(1-\mu_{t+1})q\theta)}}{-\tau(1-p\pi) - \frac{\varphi\mu_{t+1}(1-p\pi)\tau + p\pi(1-p\pi)\tau - \varphi(1-\mu_{t+1})(1-p\pi)\tau q\theta}{\beta(\varphi\mu_{t+1} - p\pi) + \varphi(1-\mu_{t+1})q\theta}} \tag{4.18}$$

Politician

The politician's maximization problem solved at time t + 1 is given by

$$\max_{c^p_{t+1,}z} c^p_{t+1} h^{\beta}_{t+2}$$

$$s.t \quad c^p_{t+1} = (1 - \mu_{t+1}\varphi)z\tau h_{t+1} - q\theta(1 - \mu_{t+1})\varphi z_{t+1}\tau h_{t+1} \tag{4.19}$$

$$c_{t+1} \geq 0, \; 1 \geq \mu_{t+1} \geq 0$$

For positive consumption the following constraint is imposed:

$$(1-\mu_{t+1}\varphi)z\tau h_{t+1} - q\theta(1-\mu_{t+1})\varphi z_{t+1}\tau h_{t+1} \geq 0$$

Maximization and further manipulation yields the following equation:

$$\mu_{t+1} = \frac{Ap\pi\tau(1-z_{t+1}) + v - A\beta\tau z_{t+1} + A\varphi q\theta z_{t+1}\tau(1+b)}{A(1+\beta)\varphi\tau(q\theta-1)z_{t+1}} \tag{4.20}$$

Equilibrium

A Nash equilibrium in this economy consists of sequences $\{c_{t+1}\}_{t=0}^{\infty}, \{z_{t+1}\}_{t=0}^{\infty}, \{\mu_{t+1}\}_{t=0}^{\infty}, \{Y_{t+2}\}_{t=0}^{\infty}$, $\{H_{t+2}\}_{t=0}^{\infty}$, $\{h_{t+2}\}_{t=0}^{\infty}$, such that given h_0 in every period t=0,1,...

1 Private citizens choose z_{t+1} to maximize their utility function, taking μ_{t+1} as given.

2. Politicians choose μ_{t+1} to maximize their utility function, taking z_{t+1} as given.

3. The labor market clears.

4. $h_{t+1} = H_{t+1,}$ since all agents are identical.

$$z_{t+1:} \quad \begin{array}{c} z_1^* = 0 \\ 0 < z_2^* < z_3^* \\ z_2^* < z_3^* \leq 1 \end{array}$$

$$\mu_{t+1:} \quad \begin{array}{c} \mu_1^* = 0 \\ 0 < \mu_2^* < \mu_3^* \\ \mu_2^* < \mu_3^* \leq 1 \end{array}$$

Since equilibrium values are too complicated to be exposed I will provide some numerical examples that will clarify the results. Before proceeding to the numerical results though I will analyze the case were fines are imposed on evaded income and then I will attempt a direct numerical comparison between the two cases.

4.4.1.2 Fines on Evaded Income

The notation is the same with the model were fines are imposed on evaded taxes. What is different is the budget constraint of the citizen and the human capital accumulation. More analytically:

Agent

$$h_{t+1} = AE_t + vH_t \quad \text{where}$$

$$E_t = z_t\mu_t\varphi\tau H_t + p\pi(1-z_t)H_t + q\theta(1-\mu_t)\varphi z_t\tau H_t$$

For non-negative spending the following condition must be imposed:

$$z_t\mu_t\varphi\tau H_t + p\pi(1-z_t)H_t + q\theta(1-\mu_t)\varphi z_t\tau H_t \geq 0$$

The citizen's maximization problem solved at time t + 1 is given by

$$\max_{c^c_{t+1},z} c^c_{t+1}h^{\beta}_{t+2}$$

$$s.t \quad c^c_{t+1} = (1 - z_{t+1}\tau)h_{t+1} - p\pi(1-z)h_{t+1} \tag{4.21}$$

$$c_{t+1} \geq 0,\ 1 \geq z_{t+1} \geq 0,$$

Note that in this model for positive consumption to hold I must impose the following constraints:

$$1 - z_{t+1}\tau > p\pi(1 - z_{t+1})$$

The term $p\pi(1-z)h_{t+1}$ denotes the fine (if he gets caught) imposed on evaded income.

Maximization and further manipulation yields the following equation:

$$z_{t+1} = \frac{-1 + p\pi\tau + \frac{Ap\pi(\tau - p\pi) + (\tau - \rho\pi)v}{\beta(A\varphi\mu_{t+1}\tau - A\rho\pi + A\varphi(1-\mu_{t+1})q\theta\tau)}}{-\tau + p\pi - \frac{-A\varphi\mu_{t+1}(\tau - p\pi)\tau + Ap\pi(\tau - p\pi)\tau - A\varphi(1-\mu_{t+1})(\tau - p\pi)\theta}{\beta(A\varphi\mu_{t+1_\tau} - Ap\pi) + A\varphi(1-\mu_{t+1})q\theta\tau}} \tag{4.22}$$

Politician

The politician's maximization problem solved at time t + 1 is given by

$$\max_{c^p_{t+1}, z} c^p_{t+1} h^\beta_{t+2}$$

$$s.t \quad c^p_{t+1} = (1 - \mu_{t+1}\varphi)z\tau h_{t+1} - q\theta(1-\mu_{t+1})\varphi z_{t+1}\tau h_{t+1} \tag{4.23}$$

$$c_{t+1} \geq 0, \; 1 \geq \mu_{t+1} \geq 0$$

For positive consumption the following constraints are imposed:

$$(1 - \mu_{t+1}\varphi)z\tau h_{t+1} - q\theta(1-\mu_{t+1})\varphi z_{t+1}\tau h_{t+1} \geq 0$$

Maximization and further manipulation yields the following equation:

$$\mu_{t+1} = \frac{Ap\pi(1 - z_{t+1}) + v - A\beta\tau z_{t+1} + A\varphi q\theta z_{t+1}\tau(1+b)}{A(1+\beta)\varphi\tau(q\theta - 1)z_{t+1}} \tag{4.24}$$

Equilibrium

A Nash equilibrium in this economy consists of sequences $\{c_{t+1}\}_{t=0}^{\infty}, \{z_{t+1}\}_{t=0}^{\infty}, \{\mu_{t+1}\}_{t=0}^{\infty}, \{Y_{t+2}\}_{t=0}^{\infty}$, $\{H_{t+2}\}_{t=0}^{\infty}$, $\{h_{t+2}\}_{t=0}^{\infty}$, such that given h_0 in every period t=0,1,...

1 Private citizens choose z_{t+1} to maximize their utility function, taking μ_{t+1} as given.

2. Politicians choose μ_{t+1} to maximize their utility function, taking z_{t+1} as given.

3. The labor market clears.

4. $h_{t+1} = H_{t+1,}$ since all agents are identical.

$$z_{t+1}: \quad \begin{array}{c} z_1^* = 0 \\ 0 < z_2^* < z_3^* \\ z_2^* < z_3^* \leq 1 \end{array}$$

$$\mu_{t+1}: \quad \begin{array}{c} \mu_1^* = 0 \\ 0 < \mu_2^* < \mu_3^* \\ \mu_2^* < \mu_3^* \leq 1 \end{array}$$

4.4.1.3 Numerical Analysis

Since analytical solutions are rather complicated I will resort to numerical solutions to illustrate the main implications of the model. In each of the following illustrations the first table of results corresponds to the model with fines imposed on evade tax while the second on the model with fines imposed on evaded income.

Using the baseline model parameter values and imposing very small auditing probabilities (smaller than those empirically estimated for the US) as well as a minimum fine rate:

$$A = 1.1, \beta = \frac{1}{2}, v = \frac{1}{30},\ \varphi = 0.4,\ \tau = \frac{1}{3},\ q = \frac{1}{200},\ p = \frac{1}{200},\ \pi = 1.1 \text{ and } \theta = 1.1$$

I obtain the following equilibria:

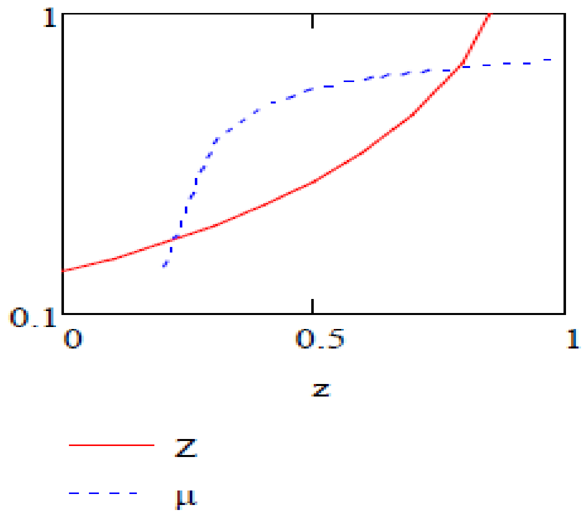

Figure 4.10: Multiple Equilibira to the Model with Detterence Policies.

$$z: \begin{array}{r} z_1^* = 0 \\ z_2^* = 0.140 \\ z_3^* = 0.863 \end{array} \text{ and } \mu: \begin{array}{r} \mu_1^* = 0 \\ \mu_2^* = 0.124 \\ \mu_3^* = 0.725 \end{array}$$

$$z: \begin{array}{r} z_1^* = 0 \\ z_2^* = 0.319 \\ z_3^* = 0.692 \end{array} \text{ and } \mu: \begin{array}{r} \mu_1^* = 0 \\ \mu_2^* = 0.296 \\ \mu_3^* = 0.599 \end{array}$$

The first and most significant result is that the coordination failure still exists (Figure 4.10). The imposition of fines does not function as a signal to agents to coordinate in one equilibrium.

Another important result is that in this model, where the assumption that revenues from fines are invested on the public good is made, the effect of fines on tax evasion and corruption is not always as clear as in Allingham and Sandmo. Directly comparing the results in the baseline model and these results we observe that compliance rates have reduced both for the politicians and the citizens due to the imposition of fines. To highlight this argument note the following example. By decreasing q to $q = 1/300$ and p to $p = 1/300$ I obtain the following equilibria:

$$z: \begin{array}{l} z_1^* = 0 \\ z_2^* = 0.135 \\ z_3^* = 0.867 \end{array} \text{ and } \mu: \begin{array}{l} \mu_1^* = 0 \\ \mu_2^* = 0.117 \\ \mu_3^* = 0.726 \end{array}$$

$$z: \begin{array}{l} z_1^* = 0 \\ z_2^* = 0.288 \\ z_3^* = 0.718 \end{array} \text{ and } \mu: \begin{array}{l} \mu_1^* = 0 \\ \mu_2^* = 0.261 \\ \mu_3^* = 0.613 \end{array}$$

which indicate that a decrease in audit probability, actually increased compliance rates.

As stated earlier agents when "cheating", benefit in terms of consumption however their utility decreases due to reduced human capital stock for their children. Apart from this trade-off, in this model if they get caught, they enjoy an increase in the human capital stock (since fines go to the public good) and this indirectly increases their utility. Therefore the standard result obtained in the majority of tax evasion models concerning the effect of penalty rates and auditing probabilities is changed.

This is a significant result since it contradicts the standard literature on tax evasion and deterrence policies. In all relevant papers (Allingham and Sandmo (1972), Kolm (1973), Yitzhaki (1974), Gottlieb (1985), Cowell and Gordon (1988)) the effect of an increase in fine on tax evasion is not questioned or argued and it is always considered to be negative. However in this model, it is evident that the introduction of a public good and the assumption of a small economy is sufficient alter this results and indicate the possibility of a positive effect of fines on tax evasion.

In terms of policy it is very important to know the direction of this effect in order to be able to define the optimal rates that diminish tax evasion. Additionally contrary to the results that claim that deterrence policies make the decision to evade more costly, in this model the decision to evade is less costly, and instead of resolving the failure it actually aggravated the problem.

When comparing the two kinds of fines, it is evident that the model were fines are imposed in evaded tax gives better results, rendering this policy more effective.

Concerning the comparative statics with respect to the other parameters, A, v, β etc. the results are the same as in the baseline model.

4.4.2 Fines do not Finance Provision of Public Good

This model produces different results with respect to fines and auditing probabilities. In this case the benefit from being caught is lost and agents are only faced with the dilemma to increase their consumption at the expense of their children's human capital. I find that increases in the penalty rates and auditing probabilities decrease tax evasion.

4.4.2.1 Fines on Evaded Tax

The notation is exactly the same as in the previous models. Human capital accumulation is now given by:

$$h_{t+1} = AE_t + vH_t \quad \text{where}$$

$$E_t = z_t \mu_t \varphi \tau H_t$$

For non-negative spending the following condition must be imposed:

$$z_t \mu_t \varphi \tau H_t \geq 0$$

The citizen's maximization problem solved at time t + 1 is given by

$$\max_{c^c_{t+1,}z} c^c_{t+1} h^{\beta}_{t+2}$$

$$\begin{aligned} s.t \quad c^c_{t+1} &= (1 - z_{t+1}\tau)h_{t+1} - p\tau\pi(1 - z_{t+1})h_{t+1} \\ c_{t+1} &\geq 0,\ 1 \geq z_{t+1} \geq 0, \end{aligned} \tag{4.25}$$

Note that in this model for positive consumption to hold I must impose the following constraints:

$$1 - z_{t+1}\tau > \tau p\pi(1 - z_{t+1})$$

Maximization and further manipulation yields the following equation:

$$z_{t+1} = \frac{A\beta\mu_{\ t+1}\varphi(p\pi\tau - 1) + v\,(1 - p\pi)}{(p\pi - 1)\,A\mu_{t+1}\varphi\tau(1 + \beta)}$$

Politician

The politician's maximization problem solved at time t + 1 is given by

$$\max_{c^p_{t+1,}z} c^p_{t+1} h^{\beta}_{t+2}$$

$$s.t \quad c^p_{t+1} = (1 - \mu_{t+1}\varphi)z\tau h_{t+1} - q\theta(1 - \mu_{t+1})\varphi z_{t+1}\tau h_{t+1} \tag{4.26}$$

$$c_{t+1} \geq 0,\ 1 \geq \mu_{t+1} \geq 0 \tag{4.27}$$

For positive consumption the following constraints are imposed:

$$(1-\mu_{t+1}\varphi)z\tau h_{t+1} - q\theta(1-\mu_{t+1})\varphi z_{t+1}\tau h_{t+1} \geq 0$$

Maximization and further manipulation yields the following equation:

$$\mu = \frac{v(1-q\theta) + A\beta\tau z_{t+1}(\varphi q\theta - 1)}{(q\theta-1)\,Az_{t+1}\tau\varphi(1+\beta)}$$

Equilibrium

A Nash equilibrium in this economy consists of sequences $\{c_{t+1}\}_{t=0}^{\infty}, \{z_{t+1}\}_{t=0}^{\infty}, \{\mu_{t+1}\}_{t=0}^{\infty}, \{Y_{t+2}\}_{t=0}^{\infty}$, $\{H_{t+2}\}_{t=0}^{\infty}$, $\{h_{t+2}\}_{t=0}^{\infty}$, such that given h_0 in every period t=0,1,...

1 Private citizens choose z_{t+1} to maximize their utility function, taking μ_{t+1} as given.

2. Politicians choose μ_{t+1} to maximize their utility function, taking z_{t+1} as given.

3. The labor market clears.

4. $h_{t+1} = H_{t+1,}$ since all agents are identical.

$$z_{t+1:} \quad \begin{array}{c} z_1^* = 0 \\ 0 < z_2^* < z_3^* \\ z_2^* < z_3^* \leq 1 \end{array}$$

$$\mu_{t+1:} \quad \begin{array}{c} \mu_1^* = 0 \\ 0 < \mu_2^* < \mu_3^* \\ \mu_2^* < \mu_3^* \leq 1 \end{array}$$

4.4.2.2 Fines on Evaded Income

Agent

$$h_{t+1} = AE_t + \upsilon H_t \quad \text{where}$$

$$E_t = z_t \mu_t \varphi \tau H_t$$

For non-negative spending the following condition must be imposed:

$$z_t \mu_t \varphi \tau H_t \geq 0$$

The citizen's maximization problem solved at time t + 1 is given by

$$\max_{c^c_{t+1,} z} c^c_{t+1} h^{\beta}_{t+2} \tag{4.28}$$

$$s.t \quad c^c_{t+1} = (1 - z_{t+1}\tau)h_{t+1} - p\pi(1-z)h_{t+1} \tag{4.29}$$

$$\tag{4.30}$$

$$c_{t+1} \geq 0, \ 1 \geq z_{t+1} \geq 0,$$

Note that in this model for positive consumption to hold I must impose the following constraints:

$$1 - z_{t+1}\tau > p\pi(1 - z_{t+1})$$

The term $p\pi(1-z)h_{t+1}$ denotes the fine (if he gets caught) imposed on evaded income.

Maximization and further manipulation yields the following equation:

$$z_{t+1} = \frac{A\beta\mu_{t+1}\varphi\tau(1-p\pi\tau)+v\,(p\pi-\tau)}{(\tau-p\pi)\,A\mu_{t+1}\varphi\tau(1+\beta)} \tag{4.31}$$

The politicians problem is exactly the same as in the above model.

4.4.2.3 Equilibrium

A Nash equilibrium in this economy consists of sequences $\{c_{t+1}\}_{t=0}^{\infty}, \{z_{t+1}\}_{t=0}^{\infty}, \{\mu_{t+1}\}_{t=0}^{\infty}, \{Y_{t+2}\}_{t=0}^{\infty}$, $\{H_{t+2}\}_{t=0}^{\infty}$, $\{h_{t+2}\}_{t=0}^{\infty}$, such that given h_0 in every period t=0,1,...

1 Private citizens choose z_{t+1} to maximize their utility function, taking μ_{t+1} as given.

2. Politicians choose μ_{t+1} to maximize their utility function, taking z_{t+1} as given.

3. The labor market clears.

4. $h_{t+1} = H_{t+1,}$ since all agents are identical.

$$z_{t+1}: \quad \begin{array}{c} z_1^* = 0 \\ 0 < z_2^* < z_3^* \\ z_2^* < z_3^* \leq 1 \end{array}$$

$$\mu_{t+1}: \quad \begin{array}{c} \mu_1^* = 0 \\ 0 < \mu_2^* < \mu_3^* \\ \mu_2^* < \mu_3^* \leq 1 \end{array}$$

4.4.2.4 Numerical Analysis

Using the same values with the baseline model with fines

$$A = 1.1, \beta = \frac{1}{2}, v = \frac{1}{30},\ \varphi = 0.4,\ \tau = \frac{1}{3},\ q = \frac{1}{200},\ p = \frac{1}{200},\ \pi = 1.1 \text{ and } \theta = 1.1$$

I obtain (the first table reports results for the model with fines on evaded tax and the second for the model with fines on evaded income):

$$z: \begin{array}{l} z_1^* = 0 \\ z_2^* = 0.234 \\ z_3^* = 0.766 \end{array} \text{ and } \mu: \begin{array}{l} \mu_1^* = 0 \\ \mu_2^* = 0.197 \\ \mu_3^* = 0.638 \end{array}$$

$$z: \begin{array}{l} z_1^* = 0 \\ z_2^* = 0.236 \\ z_3^* = 0.774 \end{array} \text{ and } \mu: \begin{array}{l} \mu_1^* = 0 \\ \mu_2^* = 0.195 \\ \mu_3^* = 0.640 \end{array}$$

At first sight we observe that when this the model where fines are imposed on evaded income produced better results than the model where fines are imposed on evaded tax. Recall that when fines returned to the economy the reverse was true. Also note that an increase in auditing rates, to $q = \frac{1}{100}, p = \frac{1}{100}$ which are the approximate auditing rates for the US yields:

$$z: \begin{array}{l} z_1^* = 0 \\ z_2^* = 0.234 \\ z_3^* = 0.771 \end{array} \text{ and } \mu: \begin{array}{l} \mu_1^* = 0 \\ \mu_2^* = 0.196 \\ \mu_3^* = 0.642 \end{array}$$

$$z: \begin{array}{l} z_1^* = 0 \\ z_2^* = 0.234 \\ z_3^* = 0.788 \end{array} \text{ and } \mu: \begin{array}{l} \mu_1^* = 0 \\ \mu_2^* = 0.192 \\ \mu_3^* = 0.646 \end{array}$$

i.e. an increase in auditing probabilities has decreased tax evasion and embezzlement. The same relationship exists between penalty rates and tax evasion (embezzlement). However changes in the values are very limited especially if we take into account that we doubled the auditing probability. Moreover the numerical results illustrate that the occurring equilibria are worse than in the previous models and this is due to this loss of money that come from fines. Overall, the implications of this model indicate towards different actions, instead of fines. Fines not only do not eliminate multiple equilibria, but the occurring equilibria are in every case worse[26].

Measures that affect utility directly, such as moral considerations can more successfully eliminate multiplicity of equilibria or at least reduce the likelihood that they occur. Such considerations are not always the result of policy, they are rather dependent on cultural

26 The introduction of costs related to auditing probability would even worsen the equilibria.

characteristics, however in some cases it is possible for a government to pursue policies that affect morals. Following I set up a model in which I introduce such considerations.

4.5 Tax Evasion and Moral Considerations

So far it has become evident that deterrence policies are not effective in eliminating the coordination failure. In some cases and under certain parameter configurations they have improved the situation however the case where the economy may find itself in the low equilibrium has not been precluded. Different kinds of policy seem to be needed to resolve this situation.

In this part I will deviate from the standard literature on tax evasion, corruption and policies aimed to combat these "diseases" and turn to moral considerations. A vast literature has been developed to account for this issue. Moral considerations can take various forms such as fairness considerations (Alesina and Angeletos, 2005), social stigma (Allingham and Sandmo, 1988), trust (Aghion et al.(2008), reputation (Tirole,1996), etc.

The focus of this part will be moral considerations and more specifically "bad conscience". Sandmo (2004) introduced "bad conscience" in the tax evasion context and he found that it may lead to a more restrictive condition for tax evasion to be optimal or even make tax evasion non-optimal. The introduction of "bad conscience" was via the insertion of a disutility term in the utility function that evolves with tax evasion. The budget constraint and the equation for the accumulation of human capital remain unaffected which also leaves the interior equilibria unaffected. This negative value related to tax evasion could as well be interpreted as a "conscience tax". The "bad conscience" component may affect the utility in various ways, as Moffitt (1983) suggested (for welfare stigma) For instance it may be a *flat* component that arises from the choice of strategy itself or may also be a *variable* component that varies with the size of the strategic choice.

In this model I will introduce "bad conscience" considerations in a simple way, building on the modelling of Sandmo, i.e. by adding a disutility term in the utility function as a variable component, according to which an increase in tax evasion negatively affects "conscience".

4.5.1 Bad Conscience

To make the analysis more tractable and enable direct comparisons between the two cases I will build again on the baseline model.

The private citizen's maximization problem solved at time $t+1$ is given by

$$\max_{c_{t+1},z} c_{t+1}^{c} h_{t+2}^{\beta} + \gamma\Omega^{C}$$

where

$$h_{t+1} = AE_t + \upsilon H_t \text{ and} \tag{4.32}$$

$$\tag{4.33}$$

$$E_t = z_t \mu_t \varphi \tau H_t \tag{4.34}$$

and

$$z_t \mu_t \tau \geq 0 \tag{4.35}$$

The utility function is the same as in the baseline model with the difference being the addition of the term $\gamma\Omega^{C}$. This term is very important for the model as it represents "bad conscience" considerations. As it will become evident in the following lines, $\Omega^{C} < 0$ represents "bad conscience" as perceived by citizens. $\gamma > 0$ and can be interpreted as the degree of sensitiveness to moral considerations. In a more sophisticated version γ could vary depending on the behavior of other members of the group, however even in this simple form it can account for differences in the behavior of the group as a whole as observed in different countries. For instance γ in Greece would probably take lower values than in Scandinavian countries (since tax evasion and political corruption are more widespread phenomena). Ω^C is given by[27]:

$$\Omega^C = u^{TE} - u^{NTE} = c_{t+1}^{cTE} h_{t+2}^{\beta TE} - c_{t+1}^{cNTE} h_{t+2}^{\beta NTE}$$

Ω is perceived as the difference in utility between the case where the citizen chooses to evade taxes and the utility in the case where the citizen would have chosen not to evade taxes. Note that he does not directly care about the fact that the politician may embezzle money. Disutility comes from considering his own action. The greater is the difference in utilities caused by his evasion the "worse" he feels about his corrupt behavior. c_{t+1}^{cTE} and h_{t+2}^{TE} are consumption and human capital when he evades while c_{t+1}^{cNTE} and $\ln h_{t+2}^{NTE}$ are the corresponding variables had the citizen not evaded.

$$c_{t+1}^{cTE} = (1 - z_{t+1}\tau) h_{t+1}^{TE} \tag{4.36}$$

$$\tag{4.37}$$

$$h_{t+1}^{TE} = A E_t^{TE} + \upsilon h_t^{TE}$$

Equation (4.36) gives the budget constraint of the private citizen after having evaded taxes to a rate z and is the same as in the baseline model contrary to the case of not tax evading:

27 Note that in this model the terms h_{t+1}^{TE} and h_{t+1}^{E} and h_{t+1} will be interchangeably used. The only terms that differ are h_{t+1}^{NTE} and h_{t+1}^{NE}. Interpretation of these terms follows.

$$c_{t+1}^{cNTE} = (1-\tau)h_{t+1}^{NTE} \tag{4.38}$$

(4.39)

$$h_{t+1}^{NTE} = AE_t^{NTE} + vh_t^{NTE}$$

As noted earlier, when the citizen has moral considerations he regards his behavior only. It is his strategy that makes him feel compunction.

Using constraints (4.36) and (4.38) I obtain the following equation for Ω^C :

$$\Omega^C = (1 - z_{t+1}\tau)(Az_{t+1}\mu_{t+1}\varphi\tau + v)^{\beta}h_{t+1}^{(1+\beta)TE} - (1-\tau)(A\mu_{t+1}\varphi\tau + v)^{\beta}h_{t+1}^{(1+\beta)NTE} < 0$$

$\Omega^{''C} < 0$ for concavity to hold

Solving for concavity of Ω I obtain the same concavity condition as in the baseline model.

An important point that will be later clarified is that Ω^C<0, i.e. $u^{TE} < u^{NTE}$. The argument that increased tax evasion and corruption reduces agents utility was proved in the baseline model case, by showing that the three equilibria were Pareto ranked and that the equilibrium with the lowest rates of corruption and tax evasion was the "good" one. Recall that this was true for the case were tax evasion and political corruption were strategic complements. In the following lines I will prove that the equilibria in both models are analogous thereby the same assumptions can apply for both cases.

An important restriction concerning Ω^C is related to how it is affected by z and specifically for the notion of "bad conscience" to be meaningful $\frac{\partial \Omega^C}{\partial z} > 0$. An increase in z (i.e. a reduction in tax evasion) increases Ω^C. Since $\Omega^C < 0$ an increase in z will reduce the "bad conscience" caused by tax evasion. Had $\frac{\partial \Omega^C}{\partial z} < 0$, this would be equal to feel pleased by corrupt actions and everyone would choose to be as much corrupt as he can. Even though this may be the case for some groups, such as the Mafia, I will stick with the more realistic and widely accepted assumption of $\frac{\partial \Omega^C}{\partial z} > 0$.

Contacting comparative static analysis:

$$\frac{\partial \Omega^C}{\partial z} > 0 \Rightarrow v < \beta A \mu_{t+1} \varphi - A z_{t+1} \tau \mu_{t+1} \varphi (1 + \beta) \tag{4.40}$$

Eq. (4.40) is an important restriction that will be later used.

Maximization and further manipulation yields:

$$z_{t+1} = \frac{\beta A \mu_{t+1} \varphi - v}{A \mu_{t+1} \varphi \tau (1 + \beta)} \tag{4.41}$$

The reaction function of the citizen is exactly the same as in the baseline model, a rather expected result since I only altered the utility and not the budget constraint of the citizen. Even though his reaction function remains the same, not all previous strategies are now part of this reaction function and this is due to the introduction of "bad conscience".

Politician's Problem

$$\max_{c^p_{t+1}, z} c^p_{t+1} h^\beta_{t+2} + \delta \Omega^p$$

In the case of the politician I have added the term $\delta \Omega^p$. Again I assume that $\Omega^p < 0$ which represents "bad conscience" as perceived by politicians. $\delta > 0$ is interpreted as the degree of sensitiveness of politician to moral considerations. Additionally for the case of politicians "bad conscience" considerations could also be derived from the fear of going public. Ω^p is given by:

$$\Omega^p = u^E - u^{NE} = c^{pE}_{t+1} h^{\beta E}_{t+2} - c^{pNE}_{t+1} h^{\beta NE}_{t+2}$$

Ω is perceived as the difference in utility between the case where the politician chooses to embezzle part of the tax revenue and the utility in the case where he doesn't. c_{t+1}^{pE} and h_{t+2}^{E} are consumption and human capital when he embezzles money while c_{t+1}^{pNE} and $\ln h_{t+2}^{NE}$ are the corresponding variables when he doesn't.

$$c_{t+1}^{pE} = (1-\mu\varphi)z_{t+1}\tau h_{t+1}^{E} \tag{4.42}$$

$$\tag{4.43}$$

$$h_{t+1}^{E} = AE_t^E + \upsilon h_t^E$$

Equation (4.42) gives the budget constraint of politician when he embezzles tax revenues. If he is not corrupt:

$$c_{t+1}^{pNE} = (1-\varphi)z_{t+1}\tau h_{t+1}^{E} \tag{4.44}$$

$$\tag{4.45}$$

$$h_{t+1}^{NE} = AE_t^{NE} + \upsilon h_t^{NE}$$

Using constraints (4.36) and (4.38) I obtain the following equation for Ω^C :

$$\Omega^P = (1-\mu\varphi)z_{t+1}\tau h_{t+1}^{(1+\beta)E}A(z_t\mu_t\varphi\tau+\upsilon)^{\beta} - (1-\varphi)z_{t+1}\tau h_{t+1}^{(1+\beta)E}A(z_t\varphi\tau+\upsilon)^{\beta} < 0$$

In a similar manner to the citizen problem I assume that $\frac{\partial\Omega^P}{\partial\mu} > 0$. An increase in μ (i.e. a reduction in political corruption) increases Ω^P. Since $\Omega^P < 0$ an increase in μ will reduce the "bad conscience" associated with embezzlement.

Contacting comparative static analysis:

$$\frac{\partial\Omega^P}{\partial\mu} > 0 \Rightarrow v < \beta A z_{t+1}\tau - A z_{t+1}\mu_{t+1}\varphi\tau(1+\beta) \tag{4.46}$$

Maximization and further manipulation yields:

$$\mu_{t+1} = \frac{\beta A z_{t+1}\tau - v}{A z_{t+1}\tau\varphi(1+\beta)} \tag{4.47}$$

Again notice that the reaction function remains the same.

4.5.1.1 Nash Equilibria

Combining equations (4.46) and (4.47) which constitute the reaction functions of the citizens and the politicians respectively, with non-negativity constraints I obtain:

$$z_{t+1} = \begin{array}{cc} 0 & \text{if } \beta A\mu_{t+1}\varphi > v \\ \frac{\beta A\mu_{t+1}\varphi - v}{A\mu_{t+1}\varphi\tau(1+\beta)} & \text{if } \beta A\mu_{t+1}\varphi > v \end{array}$$

and

$$\mu_{t+1} = \begin{array}{cc} 0 & \text{if } \beta A w + \beta A z_{t+1}\tau + A w > v \\ \frac{\beta A z_{t+1}\tau - v}{A z_{t+1}\tau\varphi(1+\beta)} & \text{if } \beta A z_{t+1}\tau > v \end{array}$$

I want to check whether the corner solution is still an equilibrium. Using constraints (4.46) and (4.35) with equality I obtain:

$$v < \beta A z_{t+1}\tau - A z_{t+1}\mu_{t+1}\varphi\tau(1+\beta) \Rightarrow v < \beta A z\tau$$

which contradicts the above inequality.

Respectively using eq. (4.40) and (4.35) I obtain:

$$v < A\mu_{t+1}\varphi - A z_{t+1}\tau\mu_{t+1}\varphi(1+\beta) \Rightarrow v < \beta A\mu\varphi$$

Therefore since inequalities (4.46) and (4.40) should hold under the presence of "bad conscience" the solution (z, μ)=$(0, 0)$ is no longer a feasible solution of the model.

4.5.1.2 Equilibrium

A Nash equilibrium in this economy consists of sequences $\{c_{t+1}\}_{t=0}^{\infty}, \{z_{t+1}\}_{t=0}^{\infty}, \{\mu_{t+1}\}_{t=0}^{\infty}, \{Y_{t+2}\}_{t=0}^{\infty}$, $\{H_{t+2}\}_{t=0}^{\infty}$, $\{h_{t+2}\}_{t=0}^{\infty}$, such that given h_0 in every period t=0,1,...

1 Private citizens choose z_{t+1} to maximize their utility function, taking μ_{t+1} as given.

2. Politicians choose μ_{t+1} to maximize their utility function, taking z_{t+1} as given.

3. The labor market clears.

4. $h_{t+1} = H_{t+1,}$ since all agents are identical.

The steady state values are :

$$z_1^* = \frac{A\beta - \sqrt{A(A\beta^2 - 4v\beta - 4v)}}{2A\tau(1+\beta)}$$

$$z_2^* = \frac{A\beta + \sqrt{A(A\beta^2 - 4v\beta - 4v)}}{2A\tau(1+\beta)}$$

$$\mu_1^* = \frac{A\beta + \sqrt{A(A\beta^2 - 4v\beta - 4v)}}{2A\varphi(1+\beta)}$$

$$\mu_2^* = \frac{A\beta + \sqrt{A(A\beta^2 - 4v\beta - 4v)}}{2A\varphi(1+\beta)}$$

Figure (4.11) illustrates the pure strategy equilibria in the case where the two reaction curves are increasing and concave. Contrary to the previous simultaneous games models the low equilibrium is no longer a part on agents' reaction function and the economy is left with one unstable equilibrium and a stable equilibrium which is the "good" one.

Simulations yield the same results since the reaction functions are the same, however agents' utility differs in each case. Following the same analysis as in model 1 it is easily proved that the equilibria are Pareto ranked.

4.6 Concluding Remarks

Corruption in its various forms is one of the most important problems a society must face

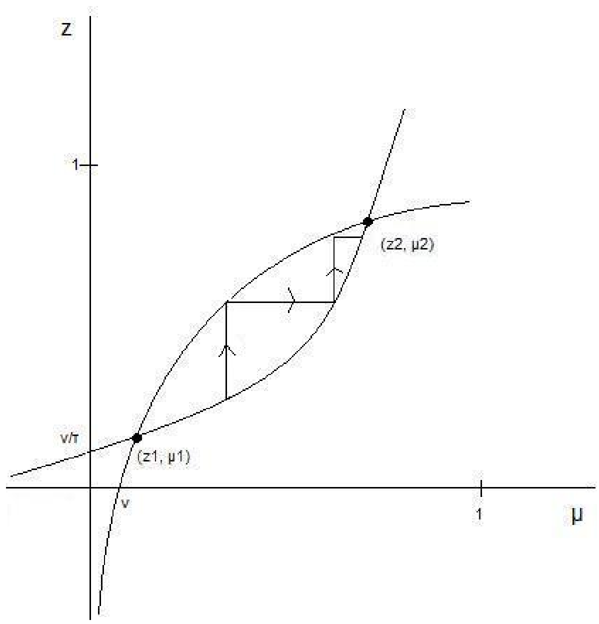

Figure 4.11: Bad Conscience

mostly due to its various detrimental effects on all aspect of the society, one of which is that of economic growth. Treatment of such diseases must be harsh and persistent since corruption may easily corrupt, a point which the current paper tries to emphasize. In a society which is corrupt, the final outcome highly depends on expectations of agents concerning the behavior of the others. Therefore action must be taken and this kind of action should be at the right direction.

In the model I have used some standard policies, such as fines, to deter corruption however the results are not very encouraging, a fact that is often observed in corrupt societies. Moreover in some cases and under certain assumptions related to the provision of public goods, increased fines may even enhance tax evasion. This is not meant to be a critique against measures such as fines and auditing probabilities, after all it would be a mistake not to acknowledge that in some countries such measures seem to be effective. This result acts more as the starting point to think why in some societies deterrence measures are effective while in others are not. Intuition leads to cultural issues, and since corruption and tax evasion are at the center of the analysis, a kind of tax morale or "bad conscience" seems to be an appropriate candidate explanation.

Indeed the introduction of moral considerations in the model yields better results and the elimination of the "bad" equilibrium. The introduction of moral considerations aims not to give specific policy measures. After all such an effort would be rather "controversial" especially if we take into account the critique on the models that generate coordination failures. Neither does it aim to "claim" having achieved the "best" equilibrium of all the others, at least in real world terms. The result of this model aims to highlight the direction towards which any kind policy should be oriented. The raising of society's " morale" should be a necessary condition for the effectiveness of all other policy measures. Morals in the economy can either be pursued by following specific policies, such as anti-corruption campaigns, still though this is not enough. As Tirole (1996) for anti-corruption measures to be effective, it is not enough to implement them but also to be persistent and continuous. Otherwise corruption will retreat. Moreover all policies should be implemented in a manner that increases tax morale, i.e. in a fair, absolute and holistic way with no exceptions In some cases, even this simple formula is enough to raise the morals of the economy and lead to better outcomes.

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Chapter 5 Conclusions

The aim of my thesis was to highlight aspects of a crucial issue, namely that of economic growth. Recognizing the complex nature of this field I attempt to approach it from a different perspective in each of the three chapters comprising this thesis. Thus the contribution of my research varies depending on the approach adopted in each chapter. Chapters 2 and 3 focused on the mechanics of growth as described by the, standard in the literature, neoclassical growth model. Chapter 4 adopted a more institutional approach and attempted to describe the dynamics of a society instead of the dynamics of the aggregate economy. The latter was the subject of Chapters 2 and 3.

Chapter 2 focused on the dynamics of the saving rate in the neoclassical growth model in the Ramsey context and fully accounted for its behavior. To generalize my results and make them applicable for a wide range of models I used a general production function, satisfying standard but not restrictive properties (such as concavity). Imposing conditions on the elasticity of factor substitution and on the elasticity of intertemporal substitution I managed to ensure monotonicity in the behavior of the saving rate, which otherwise could behave non-monotonically and exhibit U-shaped (or inverted U-shaped) patterns. Therefore this research not only accounts for empirically observed patters, but also adopts plausible assumptions such as the abstraction from the Cobb-Douglas production function. Finally it identified a class of production functions that can endogenously derive a constant saving rate over the entire transition. Extensions of this research involves empirically calibrating the model using data from leading economies and see if it can reproduce observed patters[28].

[28] I owe this interesting extention of my model to prof. Jim Malley.

Chapter 3 analytically described the participation rate patterns in the standard neoclassical growth model. The contribution of this research lies in the fact that it abstracted from the standard literature that focuses either on working hours intertemporally or on unemployment, and highlighted this significant aspect of the economy which significantly affects its course. Characteristic examples of its importance are the participation rates of women, which seem that have exhibited a U-shaped pattern intertemporally as well as increases in the schooling rate of young agents. The model introduced in this chapter is transformed in order to be able to account for participation rates and it indeed manages to reproduce the observed patterns by imposing some restrictions on the elasticity of factor substitution and the intertemporal elasticity of substitution. Despite the fact that it cannot account for unemployment, it attains to reproduce realistic participation rates. Future extensions include the construction of a model that can at the same time account for changes in hours and for changes in participation rates. The introduction of involuntary unemployment would as well make the model more realistic.

The last chapter of this thesis focused on more institutional issues and more specifically on corruption. Via the use of an overlapping generations model I pointed out that the various forms of corruption, adopted and applied by different kinds of agents, interact with each other and this may have a detrimental effect on the economy. In this model I highlighted the effect of corruption on the provision of public goods (i.e. public education) where it became evident that if agents expect that other agents will behave in a corrupt way, then coordination failures may occur and therefore the economy may find itself in a bad equilibrium. When attempting to resolve this failure it became clear that deterrence policies alone are not always effective. More importantly it was proved that contrary to the standard literature, the imposition of fines may have exactly the opposite effect under certain assumptions. What was proved in the last part of this chapter is that if a policy maker manages to raise the moral considerations of the society associated with the corrupt activities then a unique, and rather improved, equilibrium may occur. Therefore all kinds of policy should always have an impact on the morals of the society and not be exhausted in pure severity.

Future extensions can take many directions. An interesting approach is to examine whether more sophisticated deterrence policies, such as endogenous auditing rules or cut-off rules (Reinganum and Wilde, 1985) can act as a selection equilibrium device and therefore leave some room for policy. Even more interestingly the model could be transferred to a dynamic context where such policies may have a long lasting effect and issues such as reputation may arise (Tirole, 1996). In terms of the public good, it would be interesting to introduce private education (Glomm and Ravikumar, 1992) and examine how this would affect the decision of agents with respect to public education. As an alternative to the use of fines, other approaches could be used that mostly belong to the area of political economy. Introducing political accountability in the model through voting and reappointment rules (Besley and Smart, 2007) could perhaps eliminate multiplicity of equilibria or at least improve the occurring equilibria.

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