DO RISK-AVERSE LOTTERY PLAYERS BECOME RISK-SEEKING LOTTERY WINNERS?

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Abstract

Models of rational lottery play assume that lottery winners are risk-seekers. Models of skewness preference are not an exception to this. Skewness preference is not, in general, sufficient to make risk-averse individuals play, if one does not assume a risk-seeking attitude at windfall gains. In light of the anecdotal evidence on lottery winners remaining risk-averse, we fear that a consistent theory of rational betting may only have a limited domain of application: that of lottery losers.

Key words
expected utility; lotteries; risk aversion; skewness preference

JEL classification
D81;G10

Introduction

From time to time, flashlights focus on the sinister fate of a lottery winner. Michael Allen, a former winner of $5.8 million jackpot, was found in a dreadful motel dead ten years after his win; at the time of his death he was bankrupt [Yankee, December 1998]. Curtis Sharp Jr, winner of a $5 million jackpot in 1982, was broke 17 years later and was training to become a minister of the Baptist Church [Gentleman's Quarterly, April 1999]. In March 1984, Jet Magazine revealed that a former $5 million winner was at that time $200,000 in debt while his wife was drawing on unemployment. The most dramatic case, however, was that of Jimmy Cohoon, winner of a half million dollars jackpot of the Canadian Lotto in 1984, who went broke only 11 weeks later, inspiring folk singer Ron Park "The Ballad of Jimmy Cohoon" [Saturday Night, August 1994]. Given the large number of lottery winners around the globe, these stories seem to focus on exceptional, and for this reason, exciting cases.

Many people are curious to know how it feels to be a big-prize winner. Writer Hunter Davies tells the story of 24 UK Lotto jackpot winners a year after their win in the following words:

> "I wanted to see how ordinary people coped with sudden, massive wealth. I followed ten jackpot wins, which came to 24 individuals, over their first year. Before their win, they'd all had the same thoughts, almost in the same words. 'You won't see me for dust. I'll be on a tropical beach, mate, no problem.' A year later not one is living abroad and only one owns a foreign home...Yes, they all bought new cars, those who could drive, but even then, only a few notches better. They have bought houses for their relations, been very generous to those who were good to them, giving away between 20 and 30 percent of their win... With what's left, they have been

> sensible and solid to the point of stupidity, which means the Royal Bank of Scotland has been the outright winner. They don't want the slightest risk...so they leave their money on deposit at risible interest rates... Camelot has helped, with its winner's advisers, its legal and accountancy experts offering free advice, but it's mainly due to the average person turning out to be incredible sensible...There is a widespread belief that winning a lot of money will make you unhappy. It's a consolatory myth, encouraged by stories in the tabloids, to make up for not winning...One year is, of course, too short a period to judge ultimate reactions. A danger I see ahead is an undramatic and very boring one – boredom itself..."
> [From Hunter Davies' book, *Living on the Lottery*. Excerpts were taken from the author's article "Still driving the old Montego", *New Statesman*, November 1996, pp.20-21]

Most interestingly the point of knowing how people feel when they hit the jackpot and, in particular, whether their attitude towards risk is affected, is central to the study of decision-making under uncertainty. Models of rational betting assume either risk-seeking players, which contradicts people buying insurance policies at the same time, or risk-seeking winners, which implies a risk attitude inflexion [Friedman and Savage (1948)] and should be tested against the evidence on lottery winners.

Recent empirical work on racetracks and lotteries [Golec and Tamarkin (1998), Garrett and Sobel (1999)] reintroduced the old hypothesis under a new heading, i.e. skewness preference. According to both studies, bettors are not risk-seekers, but risk-averse individuals strongly inclined for skewness. However, both studies tell us nothing about the risk attitude of lottery winners. Skewness seekers are globally risk-averse or only locally? Even

worse, these studies may tell us nothing about the risk attitude of lottery players at all. Given a cubic utility (or a cubic polynomial approximation), a positive third order coefficient and a negative second order coefficient only prove that lottery players exhibit risk aversion at some positive range of their wealth; it does not follow from this that their *current wealth* definitely falls into this range.

In a recent theoretical exchange between Prakash *et al.* (1996,1998) and Horowitz (1998) the compatibility of global risk aversion, skewness preference and rational betting was put under scrutiny. Prakash *et al.* (1996, 1998) demonstrated that a cubic utility function exhibiting risk aversion and skewness preference at the level of normal wealth, coupled with a sufficiently skewed lottery, may still explain rational betting. Horowitz (1998), on the other hand, demonstrated that there does not exist a general utility function and a finite payoff skewness that could entice globally risk-averse, skewness seeking individuals to a neutral lottery.

The objective of this paper is to demonstrate that cubic utility, risk-averse, skewness-seeking, rational bettors necessarily become risk-seeking winners. This result is useful in several respects. First, it stresses that it is a risk attitude reversal at windfall gains, and not skewness preference alone, that makes risk-averse, cubic utility individuals bet. Second, it proves that skewness preference is a necessary, but insufficient condition for a rational theory of betting. Finally, it entails that evidence on the risk attitude of lottery *winners* is crucial in inferring on the rationality of lottery *players*.

The rest of the paper is structured as follows. In Section 2 we demonstrate that the ultimate assumption producing rational risk-averse gambling is a risk attitude inflexion at the level of windfall gains. In Section 3 we question the relevance of "skewness preference". Concluding remarks are presented in a closing Section.

2. The lottery player and the lottery winner theorem

Assume an individual possesses a cubic utility function given by:

$$U = c_3 w^3 + c_2 w^2 + c_1 w \tag{A1}$$

with:

$$U'(w) = 3c_3 w^2 + 2c_2 w + c_1 > 0 \tag{A2}$$

$$U''(w) = 6c_3 w + 2c_2 < 0 \tag{A3}$$

$$U'''(w) = 6c_3 > 0 \tag{A4}$$

Skewness preference is identified with c_3 being positive because of (A4). Risk aversion over some *positive range* of wealth implies that c_2 be negative because of (A3) and (A4). If the domain of (A1) is nonnegative wealth, i.e. $w \in \Re^+$, inflexion necessarily occurs at $w = -c_2 / 3c_3$ and the individual becomes a risk-seeker for $w \in (-c_2 / 3c_3, +\infty)$. It is clear then that, unless the domain of (A1) is restricted to $w \in [0, -c_2 / 3c_3)$, *skewness preference and risk aversion over some positive range of wealth, together imply a risk attitude inflexion.* No further elaboration is needed on marginal utility in (A2) because the proposed lottery will be neutral.

Our individual is offered a lottery paying $\tilde{z} = \{\vartheta, 0; p, 1-p\}$ and priced at one monetary unit. Net payoff is $\tilde{v} = \tilde{z} - 1$. The lottery is neutral and has the following moments up to the third order:

$$E(\tilde{v}) = p\vartheta - 1 = 0 \tag{A5}$$

$$E(\tilde{v})^2 = \vartheta - 1 \tag{A6}$$

$$E(\tilde{v})^3 = (\vartheta - 2)(\vartheta - 1) \tag{A7}$$

Higher moments in (A6) and (A7) are calculated making use of $p\vartheta = 1$ in (A5).

Prakash *et al.* (1996,1998) demonstrate the following proposition.

Proposition 1: *A rational, risk-averse, skewness seeking individual with cubic utility will accept a neutral lottery provided its payoff is enough skewed.*

Proposition 1 holds if combinations of initial wealth w_0 and prize ϑ exist such that the following conditions are satisfied:

$$U''(w_0) < 0 \tag{1}$$

$$E[U(w_0 + \tilde{v}) \geq U(w_0) \tag{2}$$

$$E(\tilde{v})^3 > 0 \tag{3}$$

Note that the individual's attitude towards risk is examined at the level of initial wealth and that the decision to accept the lottery is taken on the basis of expected utility. Given the utility function in (A1) and the net payoff's moments in (A5)-(A7), condition (2) is equivalent to:

$$\frac{1}{2}E(\tilde{v})^2 U'' + \frac{1}{6}E(\tilde{v})^3 U''' \geq 0 \tag{4}$$

by means of the Taylor's theorem and noting that the term $E(\tilde{v})U'$ vanishes.

It follows from (4) and from the definitions of the net payoff's higher moments in (A6) and (A7) that conditions (1), (2) and (3) are equivalent to:

$$w_0 + \frac{c_2}{3c_3} < 0 \tag{5}$$

$$w_0 + \frac{\vartheta}{3} - \frac{2}{3} + \frac{c_2}{3c_3} \geq 0 \tag{6}$$

$$\vartheta > 2 \tag{7}$$

A formal proof that (ϑ, w_0) combinations satisfying conditions (5)-(7) exist is offered in Prakash *et al.* (1996,1998). The result is, however, straightforward by inspection of Figure 1. The shadowed area shows all combinations of initial wealth and prize that would make a risk-averse, skewness seeking individual accept a neutral lottery. If $-c_2/3c_3$ were negative, no such combination would exist. Note also that all three lines intersect at $(2, -c_2/3c_3)$ reminding us of the fact that *a locally risk neutral individual is just indifferent for a neutral symmetric lottery.*

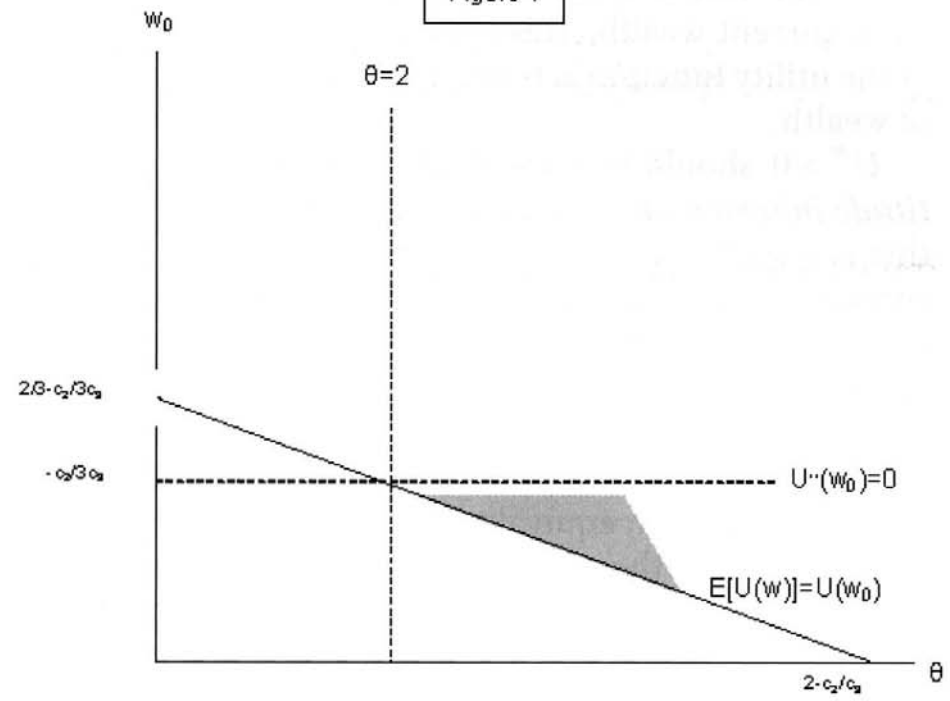
However, characterizing our individual as risk-averse is somehow misleading. In fact, she is risk-averse only in the neighborhood of her initial wealth, i.e. locally, and cannot be risk-averse in the neighborhood of her windfall gain. Cubic or not, a utility function that is strictly concave over its entire domain cannot violate Jensen's inequality, i.e. $E[U(\tilde{w})] < U[E(\tilde{w})]$. If the converse holds for some specified wealth and promised prize, it must be because, at some level of wealth (in fact at $w_0 + \vartheta - 1$), the utility function becomes strictly convex. A formal statement of the argument is given in the following proposition.

Proposition 2: *A rational, risk-averse, skewness seeking individual with cubic utility will become risk-seeker after having won the prize of a neutral lottery with enough skewed payoff.*

Proposition 2 requires that there do not exist combinations of initial wealth and prize such that the individual rationally accept the lottery while remaining risk-averse, or at least becoming risk neutral, at his terminal wealth. Consequently, proof of proposition 2 requires that there do not exist combinations of initial wealth and prize satisfying conditions (5)-(7) and :

$$U''(w_0 + \vartheta - 1) \leq 0 \qquad (8)$$

Figure 1



It is easily demonstrated that condition (8) is equivalent to:

$$w_0 + \vartheta - 1 + \frac{c_2}{3c_3} \leq 0 \qquad (9)$$

As depicted in Figure 2, conditions (5)-(7) and (9) are verified in pairs for some combinations of initial wealth and prize but cannot hold simultaneously. All (θ, w_0) combinations that would attract our individual imply a windfall gain lying in the risk-seeking range. As a result, *if risk-averse rational players are attracted to lotteries because of their love for skewness, lottery winners must necessarily be risk-seekers.*

3. Skewness preference or risk attitude reversal

Skewness preference alone cannot turn a risk-averse individual into a gambler, if it is not coupled with a risk at-

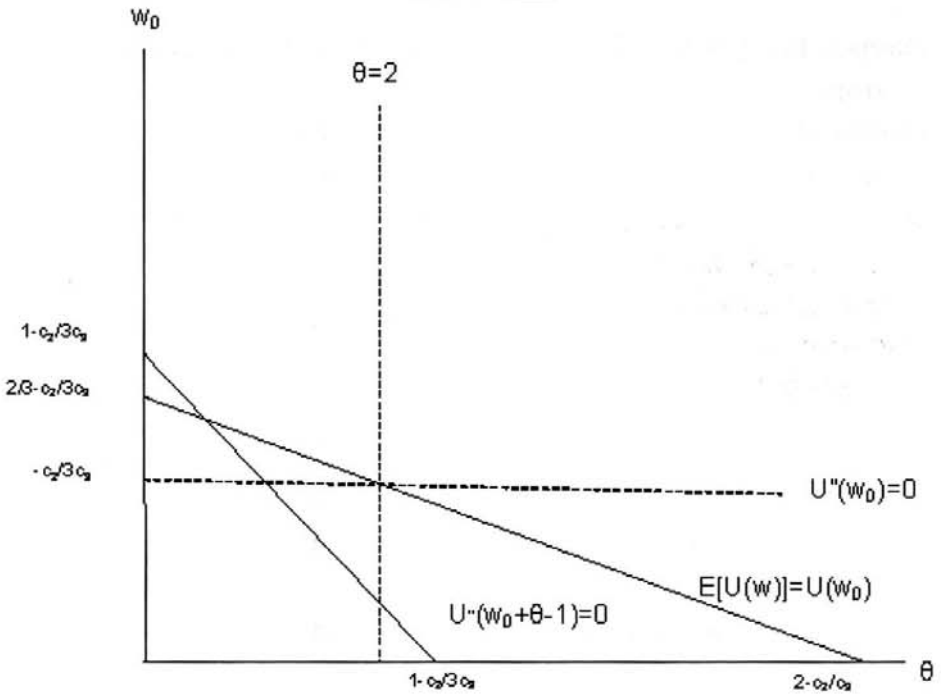
titude reversal at some level of wealth, however remote from current wealth. This is easily verified if the domain of the utility function is restricted to the risk-averse range of wealth.

$U''' > 0$ should be viewed as a *condition for a risk attitude inflexion and not as skewness preference.* Note that this is a matter of interpretation of a positive third derivative of the utility function. A highly positive third derivative means that the individual is a skewness seeker or, equivalently, that she is close to her inflexion point and an attitude reversal is eminent. In particular, a rising c_3 in a cubic utility function as in (A1) entails a rising skewness preference or, equivalently, a shrinking range of risk aversion. For all these reasons, skewness preference is, in our opinion, a misleading notion concealing that we ultimately need a risk attitude inflexion to explain rational gambling.

Proposition 2 has also a more direct effect on our understanding of how individuals really behave. It predicts that lottery winners should become risk-seekers and regress quickly to their initial, if not a lower, level of wealth. In other words, it predicts that from time to time gamblers become "shooting stars", quickly projecting themselves to high levels of wealth and then vanishing because of their adopting a catastrophic risk-seeking attitude. Michael Allen, Curtis Sharp Jr. and Jimmy Cohoon, the broke millionaires, conform to this prediction whereas the normal life of the others does not.

We do not definitely know whether lottery winners become indeed reckless risk-seekers or, on the contrary, go on living the way they used to do. Unfortunately, evidence on the behavior of lottery winners is only sparse and anecdotal and it will continue to be so; lottery winners prize anonymity most of all. However, most headlines and front pages seem to suggest that lottery winners seek expert advise on how to invest their win, that they care providing for their relatives, family and, especially,

Figure 2



their heirs, that they reluctantly quit their jobs and past occupations, and that they fear about their security. If all this evidence is taken to mean that real flesh and blood lottery winners remain risk-averse, then our theory of rational decision-making under uncertainty will only have a limited domain of application: that of lottery losers.

Concluding remarks

Skewness preference may be an explanation of rational, risk-averse individuals participating in lotteries provided it does not conceal that what we ultimately assume is risk-seeking lottery winners. In light of the anecdotal evidence on lottery winners remaining risk-averse, we fear that rational theories of lottery play may have a limited domain of application: that of lottery losers. Further research on the attitude of lottery winners is needed in order to infer on rational betting.

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