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UNIVERSITY OF MACEDONIA
ECONOMIC AND SOCIAL SCIENCES

I.P.P.S. in Economic Science

Thesis Title:

The Effects of Social Security on Growth in Debt-constrained Economies

by
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July 2009

A thesis submitted in partial fulfilment of the requirements for the degree of Master in Economics

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Acknowledgements

I am deeply grateful to a number of people for their assistance and support for the fulfilment of this thesis.

First of all, I gratefully acknowledge my supervisor, *Professor Palivos Theodore*, for all the helpful discussions and his advice. The assistance was really invaluable and his apposite remarks helped me to accomplish a meticulous work.

Furthermore, I would like to thank *Professor Konstantinou Panagiotis* for the time devoted to examine my thesis and for useful comments and suggestions.

I also thank my *friends* and *my fellow students* for their moral support and academic experience we share.

Finally, I could not forget my *family* that were always close to me and encouraged me thus overcoming obstacles in my studies and my thesis.

Dedicated with gratitude to

My parents,

Apostolos and Androniki

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Abstract

All too often, asset markets' imperfections raise borrowing constraints that young people face when they contract loans. The distortions to the allocation of resources that debt constraints cause have led to the intervention of the public sector in an attempt to apply an intergenerational transfer policy that will mitigate them. Social security is such a policy which affects incentives to save and educate thus affecting welfare and growth. However, in the presence of a social security policy borrowing constraints arise endogenously as an outcome of rationality constraints. In this thesis, I examine the impact of a social security policy on the long-run growth employing two different pension schemes (unfunded and fully funded) and directly comparing results that arise from exogenous and endogenous borrowing constraints setup. Under parametric analysis some results are vague and for this reason a numerical analysis becomes necessary.

1. Introduction

Since the emergence of New Growth Theory in the 1980's human capital has played a primal role to the development of macroeconomic models, considered as a source of growth. Human capital production requires, among others, investment in human capital (education). However, asset markets' imperfections raise borrowing constraints that young people face when they contract loans. These constraints arise due to the inalienability of human capital and the limited punishments that creditors are able to impose on those who default. In general, credit constraints have been for a long time concern for policy makers and economic analysts as they crucially affect individuals' welfare and growth. More precisely, borrowing constraints are viewed as an obstacle to an efficient allocation of resources throughout the lifetime. The deviation of the complete market allocation and the resulted distortion of the individual saving behavior affect aggregate savings (physical capital accumulation) and the incentives to educate (human capital accumulation) which in turn affect growth.

The distortions that debt constraints cause have led to the intervention of the public sector in an attempt to apply an intergenerational transfer policy that will mitigate them. Social security is such a policy which affects incentives to save and educate thus affecting welfare and growth. However, in the presence of a social security policy borrowing constraints arise endogenously as an outcome of rationality constraints. In other words, creditors will lend up to the point that agents do not have incentive to default. This point is crucially determined by the social security tax rate.

To the best of my knowledge, there is no study directly addressing the effects of a social security program on growth when there are debt constraints in human capital investment. Consequently, in this thesis I focus on an economy with only human capital production and examine the impact of a social security policy on the long-run growth. I employ two different pension schemes; a pay-as-you-go (unfunded) and a fully funded program both financed by a payroll proportional tax. Under parametric analysis some results are vague and for this reason a numerical analysis becomes necessary.

The thesis is organized as follows. In section 2, I make a brief literature review. In section 3, I present the benchmark model used in the rest of my thesis. In the same section I also present the setup of borrowing constraints and the two social security schemes. Section 4 describes the equilibrium of an unconstrained economy and economies with binding borrowing constraints. I then make comparative statics analysis and examine the effects of unfunded and fully funded social security on the equilibrium and long-run growth. In section 5, I calibrate the model and derive more explicit results through numerical analysis. Finally, in the last section I draw some basic conclusions.

2. Literature Review

Romer (1986) and Lucas (1988) were among the first to emphasize the role of human capital investment as the engine of growth. Since then many papers have been written exploring the human capital effects on basic economic variables. However, as mentioned before human capital investment is hindered by asset markets' imperfections in the form of liquidity constraints that young people face. Among the first that introduced exogenous liquidity constraints were Hubbard and Judd (1987). Aiyagari (1994) also built a growth model modified to include uninsured idiosyncratic risk and a "fixed" (exogenously determined) limit up to which agents can borrow. Exogenous debt constraints were then used for analyzing the effects on growth. In the seminal paper of Japelli and Pagano (1994) cross-country regressions showed that liquidity constraints on households raise the saving rate thus inducing physical capital accumulation and increasing growth. However, when there is human capital production the effects of liquidity constraints do not end with the effects on savings. There is also a detrimental impact of debt constraints on human capital production and thus on growth. Among the first to study the above effect were Buiter and Kletzer (1995) who assumed that agents are self-financed for their training costs. Assuming that young individuals make a trade-off between working and studying, De Gregorio (1996) concludes that the inability of the young to borrow against future income reduces the incentives for human capital accumulation thus reducing growth. Christou (1993) develops a neoclassical growth model with borrowing constraints and obtains similar results by simulating the model. Hence, in a model with human capital there are two opposite effects of debt constraints on growth; the subsequent beneficial increase in aggregate savings and the detrimental hindrance of human capital accumulation.

As noted before, a government policy could alleviate the harmful effects of borrowing constraints on welfare and growth. This policy would be ideal if the allocation of resources in a debt-constrained economy could be the same as the allocation obtained with complete markets. Rangel (2003) and Boldrin and Montes (2005) show that a policy with joint provision of public education and social security can generate such a complete market allocation. However, the borrowing constraints are no longer exogenous if we deviate from the laiser-faire economy. The interaction between borrowing constraints and economic policy seems to be more complicate since borrowing constraints arise endogenously as a result of the economic policy. For instance, the level of education or social security tax affect the incentives for the borrowers to default thus making debt constraints bind more or less tightly. Endogenous constraints have been developed in various ways but the most prevalent setup until now is that of Kehoe and Levine (1993). The authors introduce endogeneity in the form of individual rationality constraints assuming that upon default agents are excluded from future asset market trading and their assets are seized.

Following the above setup of endogenous constraints that arise as a result of government policy, Lochner and Monge-Naranjo (2002) simulate a model and study the accumulation of human capital. They conclude that individual default costs vary (i) across the population, (ii) over the life cycle and (iii) by the government policy. Increasing education subsidies should be accompanied by more extensive loan programs. In contrast to exogenous debt constraints models, their model predicts greater human capital investment for any level of observable debt and smaller impact from a wage tax on human capital investment.

Andolfatto and Gervais (2006) also employ Kehoe and Levine’s setup for endogenous constraints and apply the government policy suggested by Boldrin and Montes (2005). The authors conclude that endogenous constraints can generate multiple equilibria. Specifically, when endogenous human capital formation interacts with endogenous debt constraints the constraint set becomes non-convex. This means that for a given level of policy intervention there exist more than one optimal levels of human capital investment. Hence, ex ante identical individuals may exhibit ex post heterogeneity. However, by assuming proportional rather than lump-sum tax I show that this result is not guaranteed.

So far, existing literature has studied the effects of borrowing constraints in a partial equilibrium framework. De la Croix and Michel (2007) extend to a general equilibrium model assuming that endogenous constraints arise as a result of the interest rate and they examine how debt limits on education funding affect growth. The authors show that too low and too high interest rates increase the incentives to default thus harming growth. They also conclude that multiplicity of equilibria arises and that the growth maximum may be attained in equilibrium where debt constraints bind. In other words, countries with relatively weak financial market can exhibit higher growth only if the elasticity of human capital to education is sufficiently high. This last result contradicts to former studies defending that financial deepening leads monotonically to higher long-run growth. The results in this thesis support the latter studies.

As noted above, the state intervention is desirable especially when the attained competitive equilibria are inefficient or they do not achieve the maximum of certain goals related to welfare and growth. However, the causality between government policies and liquidity constraints is bidirectional (reciprocal). Not only can a state policy change the effects of borrowing constraints but also credit constraints affect the efficiency of such policies. A first attempt to examine the interaction between liquidity constraints and social security was made by Hubbard and Judd (1985) who studied the welfare implications of a social security program in a life-cycle economy. Andolfatto and Gervais (2008) investigate whether the welfare implications of a social security policy are sensitive to the specification of the borrowing constraint. For this reason the authors compare welfare effects resulting from models with exogenous and endogenous constraints. They conclude that exogenous constraints are adequately appropriate for modeling such an economy if we focus only on the aggregate impact of social security and not on the heterogeneous effects at the individual level.

Apart from welfare implications, social security policy can affect growth. For instance, a pay-as-you-go (unfunded) pension scheme has been criticized as detrimental to growth since it discourages private saving and physical capital accumulation (Feldstein (1974)). An extra channel that such a policy affects growth is through the decision for education and human capital accumulation (Lambrecht, Michel and Vidal (2005), Docquier and Paddison (2003)). Since social security benefits enjoyed by a generation depend on the income (human capital) of the same or the next generation, individuals are encouraged to invest in human capital production. However, their optimal choice for investment may be hindered by borrowing constraints. A suboptimal level of human capital investment not only reduces welfare but also harms growth. In the next sections I show that the interaction between borrowing constraints and social security results to non-trivial effects on human capital accumulation and long-run growth.

3. The Benchmark Model

3.1 Preliminaries

Our model depicts a small open economy consisting of individuals, firms and the government. Individuals invest time and resources in human capital production in order to accumulate wealth and consume it afterwards. In our model there is no physical capital production neither physical capital as input, since we focus on the role of human capital as the engine of growth.

3.2 Individuals

We employ an overlapping generations framework with identical individuals who live for three periods; youth, middle age and retirement age. The generation born at t period consists of N_t individuals and the population of generations increases at a constant rate n. An agent born at t period draws utility according to the following utility function:

$$u(e_t, c_{1,t+1}, c_{2,t+2}) = \ln(1-e_t) + \ln c_{1,t+1} + \ln c_{2,t+2}$$

The agents draw utility from consumption during the middle and old age (c_1, c_2 respectively) and there is also a disutility term resulted from the time devoted to education e_t during the young age. For simplistic reasons, we drop the utility term regarding the consumption when young since it does not alter our qualitative results.

When young, individuals allocate their time between education and leisure, and borrow with the aid of a financial market. Under the assumption of partial equilibrium and a small open economy the interest rate r with which individuals borrow is exogenously determined. Finally, throughout our analysis we assume that the interest rate is invariant to time and always higher than the population growth rate in order to guarantee dynamic efficiency. When middle-aged, individuals supply labor inelastically and earn the return to human capital according to a production process. Middle-aged individuals have to pay off their loans, pay a social security tax and allocate the rest of their earnings between current consumption and savings for future consumption. Finally, the old retire and consume their savings and the social security benefits. The pay-roll tax is proportional and the social security benefits are distributed in a lump-sum way. Above all, the individuals face the following constraints:

$$s_{1,t} = -q_t \qquad (1)$$
$$(1-\tau)\mathrm{w}_{t+1}h_{t+1} = c_{1,t+1} - (1+r)s_{1,t} + s_{2,t+1} \qquad (2)$$
$$(1+r)s_{2,t+1} + b_{t+2} = c_{2,t+2} \qquad (3)$$

where

s_1,s_2 = savings during the first and second stage of life

q = education expenditure (borrowing) when young

w = labor wage

τ = social security tax rate

b = social security benefits

h = units of efficient labor (private human capital)

Human capital is produced only if both time and resources are devoted to investment in education. Individuals, when young, also can pick up a fraction $v \in [0,1]$ of the existing average level of human capital H_t, without effort, simply by observing what the previous generation does. That is the human capital production is described by the following function:

$$h_{t+1} = v \cdot H_t + \varphi \cdot e_t \cdot q_t \quad , \quad \mathrm{v}, \varphi > 0 \qquad (4)$$

3.3 Firms

There is a single good in the economy produced by firms that employ only efficient labor as an input. The production process is described by the following production function

$$\mathrm{Y} = \mathrm{AH}_t \quad , \quad \mathrm{A}>0$$

where H_t is the total units of efficient labor (aggregate/average human capital)[1]. Profit maximizing implies that the wage per unit of efficient labor is $w_t = A \quad \forall t$.

3.4 The Government

Household liquidity constraints are market imperfections that justify state intervention. In the present model the government intervenes applying a social security scheme under which the state levy pay-roll taxes from middle-aged individuals and allocate them to the old. We also assume that the tax rate is proportional and constant in time, and the social security benefits are distributed in a lump-sum way. At first glance, the introduction of social security does not make sense since people do have access to a financial market in order to save and transfer resources from the second to the third stage of life or reversely. Furthermore, there is no physical capital in order for a social security to affect savings and in turn growth. However, in a model with human capital, a social security tax can modify the incentives to invest in human capital thus affecting growth.[2] We focus on two basic pension schemes; unfunded (pay-as-you-go) and fully funded scheme. By introducing human capital, both schemes can affect capital accumulation thus growth, in contrast to models with only physical capital where a fully funded scheme is neutral.

3.4.1 Unfunded (Pay-As-You-Go) pension system

According to the unfunded program, the taxes levied from a generation (middle-aged) are distributed to the previous generation (old-aged) at the same period so that the government's budget is always balanced every period of time.

$$G_t = T_t \Rightarrow N_{t-2} \cdot b_t = N_{t-1} \cdot \tau \cdot w_t h_t \Rightarrow b_t = (1+n) \cdot \tau \cdot w_t h_t \qquad (5)$$

where

G = total government spending

T = total tax revenue

[1] We normalize the population to 1.

[2] Social security is also analyzed in models with heterogeneous agents when the government aims at distributional effects. (Kaganovich and Zilcha (2008), Docquier and Paddison (2003))

3.4.2 Fully funded pension system

In a fully funded system, the taxes levied from a generation (middle-aged) are distributed to the same generation at the next period so that the government's budget remains balanced:

$$G_{t+1} = (1+r)\cdot T_t \Rightarrow N_{t-2}\cdot b_t = (1+r)\cdot N_{t-2}\cdot \tau \cdot w_{t-1}h_{t-1} \Rightarrow b_t = (1+r)\cdot \tau \cdot w_{t-1}h_{t-1} \quad (6)$$

3.5 Debt constraints setup

Young people often do not have either own resources or parental financial aid for their education funding and they resort to contract a loan which they have to pay off when making wealth. We assume an economy with capital market imperfections which take the form of inability to borrow against future wage income. Capital market imperfections may arise due to two reasons. The first reason is the high costs for enforcing loan repayment due to bankruptcy laws and other legal protections afforded to debtors. Secondly, adverse selection effects can prevent the regular operation of credit markets. We focus on two different kinds of constraints depending on the mechanism that can guarantee the loan repayment; exogenous and endogenous constraints.

Quite interesting is also the case that the middle-aged find themselves debt-constrained. This is the case when social security benefits are extremely high in contrast with the few resources left for consumption when middle-aged. In an attempt to transfer resources back to their middle age and smooth their consumption in a better way, agents may become debt-constrained. However, this scenario is out of our scope and for this reason we assume that the savings in the middle age are always non-negative.

3.5.1 Exogenous constraints

Exogenous debt constraints are considered as an upper limit that the young face when borrowing. This limit is exogenously determined and taken as given in the optimization problem.

We follow De Gregorio (1996) for the setup of an exogenous debt constraint but in a slightly different way. Since young do not work, their borrowing limit is considered as a fraction of the current income (average level of human capital) that previous generation hold:

$$q_t \leq \theta \cdot w_t H_t \qquad (7)$$

where $\theta \in [0,1]$.

3.5.2 Endogenous constraints

Exogenous borrowing constraints though easy to handle, they do not seem realistic and cannot capture every aspect of a borrowing behavior. In the real world, there is no a mechanism that can guarantee the repayment of a loan. For this reason, creditors will lend up to the point that the act of default is costly in terms of welfare. The default penalties can affect the cost of defaulting thus preventing it. The most prevalent penalties for a defaulter are her permanent or temporary exclusion from the financial market or/and the garnishment of fraction of her income. We follow Kehoe, Levine (1993, 2000) in the setup of the endogenous constraints, assuming that defaulters are permanently excluded from the financial market. That is, private creditors are able to garnishee 100% of any future savings planned by individuals thus making the act of default irrational. However, a crucial assumption is that creditors cannot garnishee social security benefits. In this setup, there are two individual rationality constraints (IRC); constraint (8) dictates that an individual will not default if the welfare drawn by consumption allocation upon default (right-hand side) is lower than the welfare enjoyed by consumption allocation without default (left-hand side). Constraint (9) implies that the savings when middle-aged must be positive, that is there is only possibility to transfer resources from the middle age to the old and not reversely.

$$\ln c_{1,t+1} + \ln c_{2,t+2} \geq \ln[(1-\tau)w_{t+1}h_{t+1}] + \ln b_{t+2} \qquad (8)$$

$$s_{2,t+1} \geq 0 \Rightarrow c_{2,t+2} - b_{t+2} \geq 0 \qquad (9)$$

4. Equilibrium and growth effects

Firstly, we are going to define the competitive equilibrium that is the general solution to the problems that individuals, firms and the government face.

Definition 1

A competitive equilibrium for this economy is a sequence of prices $\{w_t\}_{t=0}^{\infty}$ *, a sequence of allocations* $\{e_t, q_t, c_{1t}, c_{2t}\}_{t=0}^{\infty}$ *and a sequence of human capital stock* $\{H_t\}_{t=0}^{\infty}, H_0 > 0$ *, such that given the population growth n, the constant and internationally determined interest rate r, the constant tax rate* τ *, and the social security benefits b, individuals' utility is maximized, firms' profits are maximized and the government's budget is satisfied.* ■

Secondly, since individuals are identical, in equilibrium the average human capital stock equals to the individual human capital stock. That is, we have:

$$h_t = H_t \quad (10)$$

At this point we can proceed to the solution of each case that can arise from the combination of the debt constraints and the social security programs.

4.1 No (or not binding) debt constraints

Profits' maximization problem requires that:

$$w_t = A \qquad \forall t \qquad (11)$$

Substituting the human capital equation (4) and (11) into constraint (2) and combining the constraints (1), (2) and (3) we take the lifetime budget constraint that an individual faces:

$$\frac{c_{t+1}}{1+r} + \frac{c_{t+2}}{(1+r)^2} = \frac{(1-\tau)\mathrm{A}(v\mathrm{H_t} + \phi e_t q_t)}{1+r} - q_t + \frac{b_{t+2}}{(1+r)^2} \qquad \text{(B.C.)}$$

To mention that we consider the budget constraint as binding throughout the thesis. When there are no borrowing constraints or they simply do not bind, individuals find optimal allocations of $\{e_t, q_t, c_{1t}, c_{2t}\}_{t=0}^{\infty}$ so as to maximize their utility subject to the budget constraint.

$$L_t = \ln(1-e_t) + \ln c_{1,t+1} + \ln c_{2,t+2} + \lambda_t \left(\frac{(1-\tau)\mathrm{A}(v\mathrm{H_t} + \phi e_t q_t)}{1+r} - q_t + \frac{b_{t+2}}{(1+r)^2} - \frac{c_{t+1}}{1+r} - \frac{c_{t+2}}{(1+r)^2} \right)$$

The first order conditions related to the above maximization problem are:

$$e_t: \quad -\frac{1}{1-e_t} + \lambda_t \frac{(1-\tau)\mathrm{A}\phi\mathrm{q_t}}{1+r} = 0 \qquad (12\mathrm{a})$$

$$q_t: \quad \lambda_t \left(\frac{(1-\tau)\mathrm{A}\phi\mathrm{e_t}}{1+r} - 1\right) = 0 \qquad (12b)$$

$$c_{1,t+1}: \quad \frac{1}{c_{1,t+1}} - \frac{\lambda_t}{1+r} = 0 \qquad (12c)$$

$$c_{2,t+2}: \quad \frac{1}{c_{2,t+2}} - \frac{\lambda_t}{(1+r)^2} = 0 \qquad (12d)$$

Directly from (12b) we obtain the optimal time spend on education:

$$e_t = \frac{1+r}{(1-\tau)\cdot \mathrm{A} \cdot \phi} = e^* \qquad \forall t \quad (13\mathrm{a})$$

Combining (BC), (12c), (12d) and (13a) we take:

$$c_{1,t+1} = \frac{1}{2}\left[(1-\tau)\mathrm{A}v\mathrm{H_t} + \frac{b_{t+2}}{1+r}\right] \qquad (13b)$$

Substituting (13b) in (12c) and then (12c) in (12a) we take:

$$q_t = \frac{(1-\tau)\mathrm{A}v\mathrm{H_t} + \frac{b_{t+2}}{1+r}}{2[(1-\tau)\mathrm{A}\phi - (1+\mathrm{r})]} \qquad (13c)$$

At this point we should discriminate two different solutions according to the two different social security programs.

4.1.1 Unfunded pension system

Making use of eq.(4), (5) and (10) in eq.(13c) we derive the optimal education expenditure:

$$q_{t+1} = B(\tau)\cdot q_t - \mathrm{A}v(1\text{-}\tau)\left[\frac{1\text{-}\tau}{\tau(1+\mathrm{n})} + \frac{v}{1+\mathrm{r}}\right]\cdot H_t \qquad (14\mathrm{a})$$

where

$$B(\tau) = \frac{2(1-\tau)[(1\text{-}\tau)\mathrm{A}\phi - (1+r)]}{\tau(1+\mathrm{n})} - v$$

Having determined the optimal time and expenditure devoted to education, we can derive the optimal consumption paths from the equations below:

$$c_{1,t+1} = [(1-\tau)\mathrm{A}\phi - (1+\mathrm{r})]\cdot \mathrm{q_t} \qquad (14b)$$
$$c_{2,t+2} = (1+r)c_{1,t+1} \qquad (14c)$$

Note that the optimal time spent on education (eq.13a) is a stationary allocation chosen by every generation. The positive relation between the tax rate and the time devoted to education is obvious; an increase in the tax rate depresses after-tax income thus inducing agents to invest more in human capital production increasing time devoted to education. Obviously, the disutility resulting from an increase in schooling time is less than the extra utility derived from the increase in lifetime income resulting from this increase in e.

On the contrary, the optimal level of expenditure (eq.14a) change over time and more importantly it carries recursive dynamics.

<u>*Effects on growth*</u>

At this point we are able to examine the effects of the social security tax on the long-run growth of the economy. Let us first define the stationary or balanced growth path (BGP) which will characterize the long-run growth rate of our economy.

Definition 2

Consider a path $\{e_t, q_t, c_{1t}, c_{2t}\}_{t=0}^{\infty}$ *that satisfies equations (13a,14a,b,c). It is a balanced growth path (BGP) if the growth rates of* q_t, c_{1t}, c_{2t} *equal to the growth rate of the human capital stock* H_t *and the growth rate of schooling time is zero. In other words, the variables* $\frac{q_t}{H_t}, \frac{c_{1t}}{H_t}, \frac{c_{2t}}{H_t}, e_t$ *are stationary on a BGP.* ■

Note that in this case the growth rate of schooling time is zero in every period because it is a stationary allocation. (eq.(13a))

Economy's growth rate in period t is given by the following expression:

$$\gamma_{t+1} = \frac{Y_{t+1}}{Y_t} = \frac{H_{t+1}}{H_t} = v + \phi e_t \frac{q_t}{H_t} \quad (15a)$$

In order to derive the balanced growth rate in a more tractable way we follow De la Croix and Michel (2007) and we define the debt-to-income ratio z_t[3]. This ratio shows the degree of development of the financial markets.[4] It can also be considered as a transformed capital-to-labor ratio since education expenditure q is the physical capital (resources) that agents invest and H is the efficient labor. We define it as:

$$z_t = \frac{q_t}{AH_t} \quad (16)$$

Hence, the growth rate in eq.(15a) is transformed as:

$$\gamma_{t+1} = \frac{H_{t+1}}{H_t} = v + A\phi e_t z_t \quad (15b)$$

Now dividing equation (14a) with A H_t and substituting equations (15b) and (16) in (14a) we take:

[3] We define the ratio in terms of current income because agents do not have income when young.
[4] Azariadis, Kaas (2004).

$$z_{t+1} = \frac{B(\tau)\cdot z_t - C(\tau)}{v + \frac{1+\mathrm{r}}{1-\tau}\cdot z_t} \qquad (17a)$$

where $\mathrm{C}(\tau) = v(1-\tau)\left[\frac{1-\tau}{\tau(1+\mathrm{n})} + \frac{v}{1+\mathrm{r}}\right]$

The above first-order difference equation characterizes the dynamic behavior of q_t/AH_t in time, which in turn dictates the dynamic behavior of the growth rate γ_{t+1} according to eq.(15b). The dynamic behavior of debt-to-income ratio and growth results from the intergenerational links that the unfunded pension system generates; that is, social security benefits depend on next generation's income.

First, note that in order for z_{t+1} to be always positive (q and H take only positive values) z_t must be confined to values higher than $\frac{C(\tau)}{B(\tau)}$ if $B(\tau) > 0$ or lower than $\frac{C(\tau)}{B(\tau)}$ if $B(\tau) < 0$. However, the latter restriction cannot hold true since it implies that z_t is negative. Hence, we restrict to the former constraint and we also assume from now on that $B(\tau)$ is always positive. This assumption confines the parameters to take certain values.

In order to derive the balanced growth path we have to estimate the steady state of $z_{t+1}=f(z_t)$ setting $z_{t+1}=z_t=z$. We obtain two steady-state values for z:

$$z^*_{1,2} = \frac{-(1-\tau)\left[(1-\tau)(1+\mathrm{r}-\mathrm{A}\phi(1-\tau)) + v\tau(1+\mathrm{n})\right] \pm \sqrt{D}}{(1+n)(1+r)\tau} \qquad (17b)$$

where $D = \left[2A\phi(1-\tau)-(1+r)\right]\cdot\left[(1+r)(1-\tau)+(1+n)v\tau\right] - (A\phi)^2(1-\tau)^3$

Lemma 1.*(Existence and uniqueness of steady state)*

If $\left[2A\phi(1-\tau)-(1+r)\right]\cdot\left[(1+r)(1-\tau)+(1+n)v\tau\right]-(A\phi)^2(1-\tau)^3=0$ *for* $\tau\in[0,1]$ *then an equilibrium z* exists and is unique.*

Proof. It is straightforward from eq.(17b). ■

If the discriminant D is positive, two discrete steady states will exist. However, both steady states are either positive or negative depending on the sign of the bracket

$$[\,(1-\tau)(1+r-A\phi(1-\tau))+v\tau(1+n)\,]$$

Lemma 2. *Assume that*

$$(1-\tau)(1+r-A\phi(1-\tau))+v\tau(1+n)\ <0\ ;$$

then the steady state values for z are positive.

Proof. See Appendix A. ■

Notice that the condition in Lemma 2 also guarantees that B(τ) will always be positive.

Multiplicity of equilibria in unconstrained economies is not common in literature. However, here it is the nature of the unfunded pension system (the intergenerational dynamics) that generates the multiplicity.

We are going to approach the form that the difference equation (17a) has in the space of z. In appendix A it is proved that $z_{t+1}=f(z_t)$ is an increasing and concave hyperbole having asymptotes other than the axes.

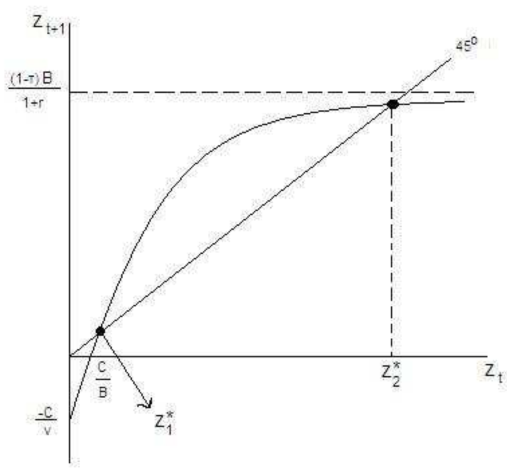

Fig.1 The two steady states of the unconstrained economy

We can easily verify from figure 1 that z_1^* is unstable and z_2^* asymptotically stable equilibrium. For z_1^* there is *local determinacy*, that is any solution other than z_1^* generates a trajectory that monotonically diverges from the steady state. In other words, there is a unique initial condition z(0)= z_1^* that leads to the equilibrium z_1^*. On the other hand, z_2^* is *locally indeterminate*, that is a continuum of initial conditions z(0) around z_2^* that generate trajectories converging to the equilibrium z_2^*. Hence, we conclude that *global indeterminacy* for z(t) and thus for the BGP exists. This result arises simply from the multiplicity of steady-state equilibria regardless of the stability properties of these equilibria. However, as z is a jump (non-predetermined) variable there are no transitional dynamics. The variable z jumps immediately in a unique way to take its long-run value and stay there until the system is disturbed in some way. Put it differently, given the initial human capital stock H(0), the choice of investment in education q immediately adjusts to keep the debt-to-income ratio constant over time ensuring a balanced growth path.

Now we are ready to examine the effects of a change in tax on the balanced growth rate. We have two balanced growth rates depending on the level of debt-to-income ratio z that the economy will balance. If the economy is balanced with a low value for debt-to-income ratio z_1^*, then it will experience low long-run growth. If the economy is balanced with a high debt-to-income ratio z_2^*, then it will experience high long-run growth. Combining eq.(13a), (15b) and (17b) the balanced-growth rate is:

$$\gamma_{1,2}^{BGP} = v + \phi \cdot e^* \cdot z_{1,2}^* \quad (18)$$

However, the parameterized computation of the effects on the growth rate requires difficult algebra that cannot give us explicit results. Hence, we are going to examine this case through simulation in the next section.

4.1.2 Fully funded pension system

Making use of eq.(4), (6) and (10) in eq.(13c) we derive the optimal education expenditure:

$$q_t = \frac{AvH_t}{\eta(\tau)} \qquad (19)$$

where $\eta(\tau) = 2A\phi(1-\tau) - \frac{2-\tau}{1-\tau}(1+r)$.

<u>*Effects on growth*</u>

Using eq.(13a) and (19) we derive the growth rate:

$$\gamma_{t+1} = \frac{H_{t+1}}{H_t} = v + \phi\frac{e_t q_t}{H_t} = v\left(1 + \frac{1+r}{(1-\tau)\cdot\eta(\tau)}\right) = \gamma^{BGP} \qquad (20)$$

Note that, in case of a fully funded social security, there is no recursive behavior of the optimal education expenditure and the growth rate is stationary. That is economies with fully funded social security exhibit balanced growth forever. According to Definition 2, the above result is expected since e is stationary (eq.(13a)) and q and H always grow at the same rate (eq.(19)). The effect of a change in the tax rate on the growth rate is given by the following differential:

$$\frac{\partial\gamma^{BGP}}{\partial\tau} = \frac{1+r}{(1-\tau)^2\eta(\tau)} - \frac{1+r}{(1-\tau)\eta(\tau)^2}\frac{\partial\eta(\tau)}{\partial\tau}$$

The determinant term is the first differential of $\eta(\tau)$:

$$\frac{\partial\eta(\tau)}{\partial\tau} = -2A\phi + \frac{1+r}{1-\tau} - (1+r)\frac{2-\tau}{(1-\tau)^2} = -\left(2A\phi + \frac{1+r}{(1-\tau)^2}\right) < 0$$

Hence, $\frac{\partial\gamma^{BGP}}{\partial\tau} > 0 \quad \forall\tau\in[0,1]$. The growth-maximizing tax rate is the upper bound.

Conclusion 1.

Increases in the tax rate of a fully funded social security benefit growth of an unconstrained economy. ■

The increasing growth rate is somewhat expected since it results from the increasing in τ time devoted to education eq.(13a) and education expenditure eq.(19). This can be explained as follows; an increase in the tax rate enhances social security benefits but reduces after-tax income. Hence, agents are induced to increase time devoted to education in order to increase after-tax income. Similarly they increase education expenditure in order to create more human capital thus attaining higher social security benefits. The increase in expenditure q reduces resources for consumption, but also enhances human capital production thus increasing future income. Here, increasing q leads to a positive net lifetime income.

4.2 Exogenous debt constraints

When debt constraints bind individuals have to maximize their utility subject to the budget constraint and the exogenous debt constraint. Combining the utility function, the (BC), eq.(7) and eq.(11) we take the following Lagrange equation:

$$L_t = \ln(1-e_t) + \ln c_{1,t+1} + \ln c_{2,t+2} + \lambda_t \left(\frac{(1-\tau)\mathrm{A}(v\mathrm{H}_t + \phi e_t q_t)}{1+r} - q_t + \frac{b_{t+2}}{(1+r)^2} - \frac{c_{t+1}}{1+r} - \frac{c_{t+2}}{(1+r)^2} \right) + \mu_t(\theta A H_t - q_t)$$

The FOC's related to the above Lagrange are:

$$e_t: \quad -\frac{1}{1-e_t} + \lambda_t \frac{(1-\tau)\mathrm{A}\phi \mathrm{q}_t}{1+r} = 0 \qquad (21a)$$

$$q_{t:} \quad \lambda_t \left(\frac{(1-\tau)\mathrm{A}\phi \mathrm{e}_t}{1+r} - 1 \right) - \mu_t = 0 \qquad (21b)$$

$$c_{1,t+1}: \quad \frac{1}{c_{1,t+1}} - \frac{\lambda_t}{1+r} = 0 \qquad (21c)$$

$$c_{2,t+2}: \quad \frac{1}{c_{2,t+2}} - \frac{\lambda_t}{(1+r)^2} = 0 \qquad (21d)$$

Since the rationality constraint binds we directly obtain the optimal education expenditure:

$$q_t = \theta A H_t \quad \text{(22a)}$$

Equation (22a) represents the dynamic path of education expenditure over time and, given an initial condition for H, investment in education q is always determined by the average human capital stock of the same period.

From (21c) and (21d) we take:

$$c_{2,t+2} = (1+r)c_{1,t+1} \quad \text{(22b)}$$

With simple substitutions of (22b) in (B.C.) we take an expression for $c_{1,t+1.}$ Then setting this expression in (21c) we take a similar expression for λ. Finally, setting expression of λ and eq.(22a) in (21a) we derive an expression for e_t:

$$e_t = K(\tau) - \frac{b_{t+2}}{3(1+r)(1-\tau)\phi\theta A^2 H_t} \qquad (22c)$$

where $K(\tau) = \frac{2\phi\theta A - v}{3\phi\theta A} + \frac{1+r}{3\phi A(1-\tau)}$.

At this point we can distinguish the solutions of the two social security programs.

4.2.1 Unfunded pension system

Implementing eq.(5) and then eq.(4) and (10) in (22c) we obtain a first-order difference equation that characterizes the optimal path of the time devoted to education.

$$e_{t+1} = \frac{3(1+r)(1-\tau)(K(\tau) - e_t)}{\tau(1+n)(v + \phi\theta A e_t)} - \frac{v}{\phi\theta A} \qquad (23)$$

Equation (23) shows the optimal path that schooling time follows, which depends negatively on its previous level. We can consider the recursive equation as either forward or backward-looking. In fact, it is a forward-looking equation since by assumption agents maximize their utility taking their future social security benefits into account, which depend on their offspring's income. However, the causality between e_{t+1} and e_t can be explained in the same way no matter if the agents look in the past or in the future since there is no uncertainty in our model. The negative relation between e_{t+1} and e_t can be explained as follows; if the schooling time of generation t+1 increases, both their future income and as a result the social security benefits of generation t will increase. Taking this implication into account, agents of generation t have incentive to cut their time devoted to education since they enjoy increased social security benefits, thus increased lifetime income. In other words, the increased income induces agents to cut schooling time thus increasing their lifetime utility.

If we substitute (22a) and (23) in (4) we take the equilibrium path of human capital. Making similar substitutions, we take the optimal consumption paths.

At this point we should examine the properties of the difference equation (23) in order to guarantee an equilibrium (steady-state) solution. First, some restrictions on the schooling time apply, which are given below:

$$\begin{aligned} &0 \le e_{t+1} \le 1 \\ &0 \le e_t \le 1 \quad \text{when} \quad \tau = 0 \end{aligned}$$

The first restriction yields that e_t must lie in the range $(\underline{e}, \bar{e})$ which is explicitly given in appendix B. In other words, out of this range e is restricted to take the corner values (0 and 1). The second restriction yields a lower bound for the exogenous borrowing limit θ. That is

$$\theta > \underline{\theta} = \frac{v}{2\phi \mathrm{A} + 1 + \mathrm{r}} \quad (24)$$

Dropping time indexes in equation (23) we obtain the steady-state level of time devoted to education, which is:

$$\hat{e} = \frac{-A\phi\theta\left[(3(1-\tau)(1+\mathrm{r})+2v\tau(1+\mathrm{n})\right]+\sqrt{D}}{2\tau(1+n)(A\phi\theta)^2} \quad (25)$$

where $D = 9(1+r)(1-\tau)^2 + 4\tau(1+\mathrm{n})\left[\theta(1+\mathrm{r}) + 2(1\text{-}\tau)(\mathrm{A}\phi\theta + v)\right]$

Lemma 3.

Under an unfunded pension rule and with binding exogenous constraints, a steady state for time devoted to education exists and is unique.

Proof. In equation (25) the discriminant D is strictly positive for any $\tau \in [0,1]$. ■

We have disregarded the second root of e since it is negative and will never be reached. In addition, it is computed that $\hat{e}$ is a decreasing function of θ. This can be explained as follows; a decline in θ depresses education expenditure and agents increase time for education in order to preserve their human capital (income) at the initial level.

For stability of the steady state we should check $f'(\hat{e})$. In order to be lower than one in absolute value, computations yield that the following inequality must hold:

$$2v(1-\tau) + \theta(1+\mathrm{r}+2\mathrm{A}\phi(1\text{-}\tau)) < 0$$

Obviously this inequality does not hold for $\tau \in \left[0,1\right]$ thus the steady state is unstable for any values of the parameters.

$$|\ f'(\hat{e})\ | > 1$$

However, since e is a control (non-predetermined) variable the economy always jumps on the steady state value $\hat{e}$. Hence, the equilibrium is globally determinate since there is a unique value e(0)= $\hat{e}$ that leads the economy to equilibrium. Any other value of e generates trajectories that diverge from $\hat{e}$.

Effects on growth

Substituting equation (22a) in (15a), the growth rate each period equals

$$\gamma_{t+1} = \frac{H_{t+1}}{H_t} = v + \phi \frac{e_t q_t}{H_t} = v + \phi \frac{\mathrm{e}_t \theta A H_t}{H_t} = v + \phi\theta \mathrm{A} \mathrm{e}_t$$

According to Definition 2, our economy is on a balanced growth path if e takes its steady-state value. Note that the BGP is globally determinate since the equilibrium of e is globally determinate.

$$\gamma^{BGP} = v + \phi\theta \mathrm{A}\hat{\mathrm{e}} \quad (26)$$

The balanced growth rate will move to the same direction as the steady state of e corresponding to changes in the tax rate.

$$\frac{\partial \gamma^{BGP}}{\partial \tau} = \phi\theta \mathrm{A} \frac{\partial \hat{e}}{\partial \tau}$$

The differential $\frac{\partial \hat{e}}{\partial \tau}$ is a second-degree polynomial of θ. Hence the exogenous borrowing limit θ is a crucial parameter that determines how time devoted to education (thus growth rate) reacts to changes in the tax rate. The critical values for θ are:

$$\theta^*_{1,2} = \frac{9(1+r)^2 - 4v(1+\mathrm{n})(2\mathrm{A}\phi + 1 + \mathrm{r}) \pm \sqrt{(1+\mathrm{r})^2\,[(1+\mathrm{r})\,(9 + 9\,\mathrm{r} - 8\,(1+\mathrm{n})\,v) - 16\,\mathrm{A}\phi\, v(1+\mathrm{n})]}}{2\,(1+\mathrm{n})\,(1+\mathrm{r}+2\,\mathrm{A}\phi)^2} \quad (27)$$

Table 1 shows how θ affects the differential $\frac{\partial \hat{e}}{\partial \tau}$.

		θ_1^*		θ_2^*	
$\frac{\partial \hat{e}}{\partial \tau}$	$-$	0	$+$	0	$-$

Table 1: Critical values of the debt-to-income ratio

Conclusion 2.

If the debt-to-income ratio θ lies in $[0,\theta_1^*)\cup(\theta_2^*,1]$, *where* $\theta_{1,2}^*$ *are given by eq.(27), increases in the tax rate of an unfunded pension scheme harm long-run growth. On the other hand, for borrowing limits that lie in* (θ_1^*,θ_2^*) *increases in the tax rate benefit growth.* ■

Conclusion 2 confirms theory stating that the efficiency (here not only quantitative but also qualitative results) of a social security tax depends crucially on the tightness of the borrowing constraints. The ambiguity of the sign of $\frac{\partial \hat{e}}{\partial \tau}$ results from two opposite forces explained below. When θ is high ($>\theta_2^*$) education expenditure q of generation t is also high thus offspring's (generation t+1) income exceeds generation's t income. Assuming unfunded social security, this means that an increase in the tax rate increases lifetime income[5]. Hence individuals have incentive to lower time devoted to education. For lower $\theta\in(\theta_1^*,\theta_2^*)$, the opposite holds; an increase of tax rate reduces lifetime income thus inducing agents to increase time for education. Note that increases in time for education bear a cost in term of reduced utility and a benefit in terms of increased income. In the cases discussed above θ is sufficiently high (equally e is sufficiently low) to ensure that the benefit prevails the cost of an increase in e[6]. However, for very low θ ($<\theta_1^*$), time devoted to education is very high. This means that for subsequent increases in education time the utility cost prevails over the income benefits due to the concavity of the utility with respect to schooling time. Hence, for low θ an increase in the tax rate will finally make agents cut time devoted to education.

Although results crucially depend on θ, at least the model can ensure that for a given θ the growth rate will behave in a monotonic way with respect to the tax rate. This means that there is no interior growth-maximizing tax rate. For $\theta\in[0,\theta_1^*)\cup(\theta_2^*,1]$ the growth-maximizing tax rate is zero, while for $\theta\in(\theta_1^*,\theta_2^*)$ it equals to 1. The above results hold only if the exogenous constraint binds for any τ.

[5] $\tau A h_{t+1} < (1+n)\tau A h_{t+2}$

[6] We discussed before that e is a decreasing function of θ. In addition, the utility is a decreasing and concave function of e.

We will see in the simulation section that this result may not be guaranteed since the constraint is not always binding.

4.2.2 Fully funded pension system

Implementing eq.(6) and then eq.(4) and (10) in (22c) we obtain the optimal time devoted to education which is stationary:

$$e_t = \frac{2\phi\theta \mathrm{A}(1-\tau) + (1+\mathrm{r})\theta - v}{[3-2\tau]\phi\theta \mathrm{A}} \qquad \forall t \qquad (28a)$$

It is easily derived that e increases with higher θ:

$$\frac{\partial e_t}{\partial \theta} = \frac{v}{(3-2\tau)\phi \mathrm{A}\theta^2} > 0 \quad (28b)$$

<u>*Effects on growth*</u>

Substituting equation (22a) and (28a) in (15a), the growth rate each period equals

$$\gamma_{t+1} = \frac{H_{t+1}}{H_t} = v + \phi\frac{\mathrm{e_t q_t}}{H_t} = v + \frac{2\phi\theta \mathrm{A}(1-\tau) + (1+\mathrm{r})\theta - v}{3-2\tau} = \gamma^{BGP} \qquad \forall t \quad (29a)$$

which is stationary. Hence, again the economy under fully funded social security is always on the balanced growth path. It is trivially concluded that tighter borrowing constraints (lower θ) harm growth:

$$\frac{\partial \gamma^{BGP}}{\partial \theta} = \frac{2\phi \mathrm{A}(1-\tau) + (1+\mathrm{r})}{3-2\tau} > 0$$

The above result is expected since according to eq.(22a) and (28b), q and e move to the same direction as θ. Hence, if θ decreases, time and resources for education decrease thus causing growth rate to fall. This result confirms De Gregorio (1996).

The effect of a change in the tax rate on growth is given by the differential:

$$\frac{\partial \gamma^{BGP}}{\partial \tau} = 2\frac{(1+r-A\phi)\theta - v}{(3-2\tau)^2} \quad (29b)$$

Lemma 4. *(Sufficient condition)*

If $A\phi > 1+r$, then increasing social security contributions harm growth.

Proof. It is straightforward from the differential (29b). ■

We already know from the unconstrained case that a condition that guarantees e_t<1 is $(1-\tau)A\phi > 1+r$.[7] Hence, if we want to be consistent with the parameter restrictions throughout all the cases we study, the sufficient condition in Lemma 4 should hold.

An alternative way would be to set θ as the crucial parameter for determining the sign of $\frac{\partial \gamma}{\partial \tau}$. More specifically, three formal cases arise, given in Conclusion 3.

Conclusion 3.

Under a fully funded pension rule, the response of the growth rate to changes in the tax rate is described in the next table.

<table>
<tr><td colspan="2">1+r < Aφ</td><td>$\forall\theta$</td><td>$\frac{\partial \gamma}{\partial \tau} < 0$</td></tr>
<tr><td rowspan="2">1+r > Aφ</td><td>ν > 1+r-Aφ</td><td>$\forall\theta$</td><td>$\frac{\partial \gamma}{\partial \tau} < 0$</td></tr>
<tr><td>ν < 1+r-Aφ</td><td>$\theta > \frac{v}{1+r-A\varphi}$</td><td>$\frac{\partial \gamma}{\partial \tau} > 0$</td></tr>
</table>

■

However, the last two cases may not be realistic since, according to real data, the marginal factor productivities (A, φ) are much greater than 1 while the interest rate ranges in low levels. Hence, there is strong evidence that social security tax harms growth.

[7] See eq.(13a)

4.3 Endogenous debt constraints

In an economy with endogenous debt constraints, individuals maximize their utility subject to the lifetime budget constraint (B.C.) and the two rationality constraints (8), (9). Thus combining the equations above with eq.(11) we obtain:

$$L_t = \ln(1-e_t) + \ln c_{1,t+1} + \ln c_{2,t+2} + \lambda_t \left(\frac{(1-\tau)\mathrm{A}\mathrm{h}_{t+1}}{1+r} - q_t + \frac{b_{t+2}}{(1+r)^2} - \frac{c_{t+1}}{1+r} - \frac{c_{t+2}}{(1+r)^2} \right) + \mu_{1t}[\ln c_{1,t+1} + \ln c_{2,t+2} - \ln((1-\tau)\mathrm{A}\mathrm{h}_{t+1}) - \ln b_{t+2}] + \mu_{2t}(\ln c_{2,t+2} - b_{t+2})$$

where $h_{t+1} = v \cdot H_t + \varphi \cdot e_t \cdot q_t$

The FOC's related to the Lagrange equation are:

$$e_t: \quad -\frac{1}{1-e_t} + \lambda_t \frac{(1-\tau)\mathrm{A}\phi \mathrm{q}_t}{1+r} - \mu_{1t}\frac{\phi q_t}{h_{t+1}} = 0 \qquad (30\mathrm{a})$$

$$q_t: \quad \lambda_t \left(\frac{(1-\tau)\mathrm{A}\phi \mathrm{e}_t}{1+r} - 1\right) - \mu_{1t}\frac{\phi e_t}{h_{t+1}} = 0 \qquad (30b)$$

$$c_{1,t+1}: \quad \frac{1}{c_{1,t+1}} - \frac{\lambda_t}{1+r} + \frac{\mu_{1t}}{c_{1,t+1}} = 0 \qquad (30c)$$

$$c_{2,t+2}: \quad \frac{1}{c_{2,t+2}} - \frac{\lambda_t}{(1+r)^2} + \frac{\mu_{1t}}{c_{2,t+2}} + \frac{\mu_{2t}}{c_{2,t+2}} = 0 \qquad (30d)$$

Intuitively, the case that both rationality constraints (8), (9) bind results to zero optimal education expenditure. If the constraint (9), which implies non-negative savings, binds, it means that agents find it optimal either to consume under autarky or to borrow when middle-aged due to excessive resources in old age. However, in both cases they result living under autarky thus making no savings.

$$s_{2,t+1} = 0 \Rightarrow c_{2,t+2} = b_{t+2} \qquad (31\mathrm{a})$$

This means that creditors cannot impose any penalty in case of default in the second period, since the only penalty they could use is the garnishment of future savings. Hence, individuals always find it optimal to default and creditors will never lend them.

We can also verify this result using algebra and the result is the same no matter which social security program applies. Setting eq.(31a) in eq.(30d) we solve for λ in terms of b_{t+2}, μ_1, μ_2. Then substituting it in (30c) we take an expression for $c_{1,t+1}$ and then substituting it with (31a) in the (B.C.) we take:

$$\frac{b_{t+2}}{(1+r)^2}\frac{1+\mu_{1t}}{1+\mu_{1t}+\mu_{2t}}=\frac{(1-\tau)Ah_{t+1}}{1+r}-q_t \quad (31b)$$

Also substituting $c_{1,t+1}$ and $c_{2,t+2}$ in the binding constraint (8) with the expressions we found above, we have:

$$\frac{b_{t+2}}{1+r}\cdot\frac{1+\mu_{1t}}{1+\mu_{1t}+\mu_{2t}}=(1-\tau)Ah_{t+1} \quad (31c)$$

It is straightforward from eq.(31b) and (31c) that q_t=0. In this trivial case, the economy will contract (poverty trap) since there will be no formation of new human capital, depending only on the human capital of the previous generations. The net growth rate will be v-1 < 0 or equally the gross growth rate:

$$\gamma_{t+1}=v+\phi e_t q_t=v+\phi e_t\cdot 0=v<1$$

We are now going to examine the most intricate case in which only constraint (8) binds. Again we are going to distinguish two solutions, one for each social security program.

4.3.1 Unfunded pension system

We have μ_2=0 and s_t >0. Also from (30c) and (30d) we take

$$c_{2,t+2}=(1+r)c_{1,t+1} \quad (32a)$$

Substituting (31a), (30c) and (5) in (8) we take

$$\frac{(1+r)^3(1+\mu_{1t})^2}{\lambda^2}=A^2\tau(1-\tau)(1+\mathrm{n})\mathrm{h}_{t+1}h_{t+2} \quad (32b)$$

Similarly setting (31a), (30c) and (5) in the (B.C.)

$$2\frac{1+\mu_{1t}}{\lambda}=\frac{(1-\tau)\mathrm{Ah}_{\mathrm{t+1}}}{1+r}-q_t+\frac{(1+n)\tau\mathrm{Ah}_{\mathrm{t+2}}}{(1+r)^2} \qquad (32c)$$

In order to make the system of equations more tractable we define a new variable as:

$$x_t=\frac{e_t q_t}{H_t} \qquad (33\mathrm{a})$$

Thus $$h_{t+1}=(v+\phi\mathrm{x}_{\mathrm{t}})H_t \Rightarrow \frac{h_{t+1}}{H_t}=(v+\phi\mathrm{x}_{\mathrm{t}}) \qquad (33b)$$

and $$q_t=\frac{x_t H_t}{e_t}=\frac{x_t h_{t+1}}{e_t}\frac{H_t}{h_{t+1}}=\frac{x_{\mathrm{t}} h_{t+1}}{e_t(v+\phi\mathrm{x}_{\mathrm{t}})} \qquad (33c)$$

Substituting eq.(33a), (33b), (33c) in (30a), (30b), (32b) and (32c) we obtain the following system of equations:

$$-\frac{1}{1-e_t}+\lambda_t\frac{(1-\tau)\mathrm{A}\phi x_{\mathrm{t}} h_{t+1}}{(1+r)e_t(v+\phi\mathrm{x}_{\mathrm{t}})}-\mu_{1t}\frac{\phi x_t}{\mathrm{e}_{\mathrm{t}}(v+\phi x_t)}=0 \qquad (34\mathrm{a})$$

$$\lambda_t(\frac{(1-\tau)\mathrm{A}\phi\mathrm{e}_{\mathrm{t}}}{1+r}-1)-\mu_{1t}\frac{\phi e_t}{h_{t+1}}=0 \qquad (34b)$$

$$\frac{(1+r)^3(1+\mu_{1t})^2}{\lambda^2}=A^2\tau(1-\tau)(1+\mathrm{n})\mathrm{h}^2_{\mathrm{t+1}}(v+\phi\mathrm{x}_{\mathrm{t+1}}) \qquad (34c)$$

$$2\frac{1+\mu_{1t}}{\lambda}=\frac{(1-\tau)\mathrm{Ah}_{\mathrm{t+1}}}{1+r}-\frac{x_t h_{t+1}}{e_t(v+\phi x_t)}+\frac{(1+n)\tau\mathrm{Ah}_{\mathrm{t+1}}(v+\phi\mathrm{x}_{\mathrm{t+1}})}{(1+r)^2} \qquad (34d)$$

Solving (34a) and (34b) for λ and μ and substituting them in (34c) and (34d) we derive two equations in terms of e_t, x_{t+1}, x_t. Solving them for e_t and equating, they yield a first-order difference equation. Then substituting $x_{t+1}=x_t=x$, we derive an expression in terms of the steady state x:

$$\frac{4\mathrm{A}\phi(1+\mathrm{r})[\mathrm{v}\tau(1-\mathrm{v})-\phi x(1-\tau)]+3(1+\mathrm{r})^2\phi x+2\mathrm{v}(1+\mathrm{r})(1+\mathrm{r}-2\mathrm{A}\phi)-4\mathrm{A}\tau\phi^2(1+n)(2\mathrm{v}-\phi x^2)+\sqrt{D}}{64(1+\mathrm{r})\phi^2(\mathrm{v}+\mathrm{x}\,\phi)^2}-\mathrm{A}^2(1+\mathrm{n})(1-\tau)\tau(\mathrm{v}+\mathrm{x}\,\phi)=0 \qquad (35)$$

where $\mathrm{D}=(1+\mathrm{r})^4(2\,\mathrm{v}+3\,\mathrm{x}\phi)^2+8\,\mathrm{A}\,(1+\mathrm{r})^2\mathrm{x}\phi^2(\mathrm{v}+\mathrm{x}\phi)[(1+\mathrm{n})\tau)\tau+\phi\mathrm{x})-(1+\mathrm{r})(1-\tau)]$

At this point we should note that equation (35) as a polynomial yields multiple solutions for x (thus for q). The multiplicity of equilibria emerges due to the intergenerational dynamics resulted from the nature of the unfunded pension system as mentioned in section 4.1.1. It is known that multiplicity of equilibria may also result from non-convexities of the constraint set of the optimization problem. This source of multiplicity is called *endogenous heterogeneity*[8]. This means that certain values of the tax rate are consistent with more than one optimal choice of q, thus ex ante identical agents can exhibit ex post heterogeneity. For instance, Andolfatto and Gervais (2006) show that some levels of policy intervention can generate one low and one high optimal level of human capital investment. These two equilibria can equally be selected by agents since they yield the same level of utility.

According to Andolfatto and Gervais (2006), non-convexities would arise if for any level of tax rate there was a non-negative level of optimal investment, say $\hat{q}(\tau)$, that is consistent with the act of default. The restriction that the optimal investment cannot lie in the neighborhood of $\hat{q}(\tau)$ cuts a path through the set of feasible human capital investment allocations, leaving the constraint set non-convex.

When q=0 and e>0 there is always a level of tax rate $\tilde{\tau}$ that results to perfect consumption smoothing for a defaulter. That is:

$$c_1^D(\tilde{\tau}) = c_2^D(\tilde{\tau}) \Rightarrow \tilde{\tau} = \frac{1}{(1+n)v+1}$$

Obviously, at $\tau = \tilde{\tau}$ and q=0 the act of non-default and default yield the same level of utility, thus non-default is expected. If q increases, net income (thus utility) of a defaulter is always greater than that of a non-defaulter since the non-defaulter has to pay off the loan. Hence, at $\tau = \tilde{\tau}$ for any q>0 there is incentive to default. For $\tau > \tilde{\tau}$ perfect consumption smoothing can no longer be attained for a defaulter since more resources are transferred to the retirement age. Hence, for $\tau > \tilde{\tau}$ and q=0 the act of default yields lower utility than that of non-default. As q increases a defaulter gains in terms of income and future social security benefits, since investment enhances not only her own income but also average human capital thus their offspring's income. This is the crucial difference between a model with lump-sum social security tax (like in Andolfatto, Gervais (2006)) and a labor income tax assumed in our model. With lump-sum tax an increase in investment q enhances second-period income thus the

[8] Andolfatto, Gervais (2006) provide long discussion on this topic.

second-period consumption and there will be a q>0 such that the defaulter achieves perfect consumption smoothing. However, with labor income tax an increase in q of generation t enhances second-period income (and second-period consumption) but also enhances offspring's (generation t+1) income thus increasing generation's t social security benefits (and their third-period consumption)[9]. The fact that both second and third-period consumptions increase cannot ensure perfect consumption smoothing for a q>0. In this case, a positive $\hat{q}(\tau)$ will be desirable for a defaulter only if second-period consumption grows at a greater rate than third-period consumption such that at $\hat{q}(\tau)$ the two consumption allocations become equal. That is $\frac{\partial c_1^D}{\partial q} > \frac{\partial c_2^D}{\partial q}$. However, for very high q, c_1^D exceeds c_2^D and the act of default becomes again costly. Hence, there is a neighborhood of finite $\hat{q}(\tau)$ that features default and these $\hat{q}(\tau)$ will never be reached. On the other hand, if $\frac{\partial c_1^D}{\partial q} \leq \frac{\partial c_2^D}{\partial q}$, for any q>0 c_2^D always exceeds c_1^D and as q increases the consumption smoothing even more deteriorates.

Now considering non-default, as the investment q increases, a non-defaulter gains in terms of future income but less than a defaulter since she pays off the debt.

To sum up, for $\tau > \tilde{\tau}$ and q>0 the act of default may bear a cost in terms of consumption smoothing and the act of non-default bears a cost in terms of loan repayment. There will be a $\hat{q}(\tau)$ >0 that features default only if the utility upon default increases/declines at a higher/lower rate than the utility upon non-default. That is:

$$\frac{\partial u^D}{\partial q_t} > \frac{\partial u^{ND}}{\partial q_t}$$

The explanation and the results when $\tau < \tilde{\tau}$ are similar to the above discussion.

9 See equation (5).

Conclusion 4.

In an economy with unfunded social security and endogenous debt constraints endogenous heterogeneity (multiple equilibria) will not emerge iff

$$\frac{\partial c_1^D}{\partial q} \leq \frac{\partial c_2^D}{\partial q} \quad and \quad \frac{\partial u^D}{\partial q_t} \leq \frac{\partial u^{ND}}{\partial q_t}. \qquad \blacksquare$$

<u>*Effects on growth*</u>

The growth rate in terms of x is:

$$\gamma_{t+1} = v + \phi \frac{\mathrm{e}_{\mathrm{t}} q_t}{H_t} = v + \phi \cdot x_{\mathrm{t}}$$

Thus, from Definition 2, the balanced growth rate is derived when the variable x_t takes its steady state value:

$$\gamma^{BGP} = v + \phi \frac{\mathrm{e}_{\mathrm{t}} q_t}{H_t} = v + \phi \cdot x \qquad (36)$$

However, the expression with x that we derive above is a high-degree polynomial that, when solved explicitly, yields complicate expressions making it hard to do qualitative analysis. We will examine the effects of the tax rate on the equilibrium through numerical analysis in the next section. Indeed, we are going to show that the results are ambiguous due to the emergence of multiple equilibria. However, these equilibria may arise as a result of the polynomial form of eq.(35) and not only due to endogenous heterogeneity.

4.3.2 Fully funded pension system

We have μ_2=0 and s_t >0. Also from (30c) and (30d) we take

$$c_{2,t+2} = (1+r)c_{1,t+1} \qquad (37a)$$

Substituting (31a), (30c) and (6) in (8) we take

$$\frac{(1+r)^2(1+\mu_{1t})^2}{\lambda^2} = A^2\tau(1-\tau)\mathrm{h}_{\mathrm{t+1}}^2 \qquad (37b)$$

Similarly setting (31a), (30c) and (6) in the (B.C.)

$$2\frac{1+\mu_{1t}}{\lambda}=\frac{\mathrm{Ah}_{t+1}}{1+r}-q_t \qquad (37c)$$

Equations (30a), (30b), (37b) and (37c) are a system that contains four variables (λ_t, μ_{1t}, e_t, q_t). Solving the system we derive explicit expressions for e_t and q_t. The optimal allocation of time devoted to education is stationary:

$$e_t=\frac{A\phi(1-2\sqrt{\tau(1-\tau)})-(1+r)}{A\phi(\tau-\sqrt{\tau(1-\tau)})} \qquad \forall t \quad (38a)$$

On the other hand, the optimal education expenditures are a function of the average human capital:

$$q_t=\frac{\left(1-2\sqrt{\tau(1-\tau)}\right)\left(\tau-\sqrt{\tau(1-\tau)}\right)AvH_t}{(1+r)\left(1+\tau-3\sqrt{\tau(1-\tau)}\right)-A\phi-4A\phi\left((1-\tau)\tau-\sqrt{\tau(1-\tau)}\right)} \quad (38b)$$

As before, we should note that there is possibility of multiple equilibria. If this is the case, they will result from non-convexities of the constraint set. The explanation is similar to that in the previous section. When q=0 and e>0 there is always a level of tax rate $\tilde{\tau}$ that results to perfect consumption smoothing for a defaulter. That is:

$$c_1^D(\tilde{\tau})=c_2^D(\tilde{\tau})\Rightarrow\tilde{\tau}=\frac{1}{2+r}$$

Obviously, at $\tau=\tilde{\tau}$ and q=0 the act of non-default and default yield the same level of utility, thus non-default is expected. If q increases net income (thus utility) of a defaulter is always greater than that of a non-defaulter since the non-defaulter has to pay off the loan. Hence, at $\tau=\tilde{\tau}$ for any q>0 there is incentive to default. For $\tau>\tilde{\tau}$ perfect consumption smoothing can no longer be attained for a defaulter since more resources are transferred to the retirement age. Hence, for $\tau>\tilde{\tau}$ and q=0 the act of default yields lower utility than that of non-default. As q increases a defaulter gains in terms of income and future social security benefits. As mentioned before, with lump-sum tax an increase in investment q enhances second-period income thus the second-period consumption and there will be a q>0 such that the defaulter achieves perfect consumption smoothing. However, with labor income tax an increase in q enhances second-period income thus increasing both the second-period and third-period consumption. The third-period consumption increases because income directly affects social security benefits under a fully funded system[10]. The fact that both second and third-period consumptions increase cannot ensure perfect consumption smoothing for a q>0. In this case, a positive $\hat{q}(\tau)$ will be desirable for a defaulter only if second-period consumption grows at a greater rate than third-period consumption such that at $\hat{q}(\tau)$ the two consumption allocations become equal. That is:

$$\frac{\partial c_1^D}{\partial q} > \frac{\partial c_2^D}{\partial q} \Rightarrow (1-\tau)A\phi e_t > (1+r)\tau A\phi e_t$$
$$\Rightarrow \tau < \frac{1}{2+r}$$

which does not hold since by assumption $\tau > \tilde{\tau} = \frac{1}{2+r}$. Hence, for any q>0 c_2^D always exceeds c_1^D and as q increases the consumption smoothing even more deteriorates.

On the other hand, as the investment q increases, a non-defaulter gains in terms of future income but less than a defaulter since she pays off the debt.

To sum up, for $\tau > \tilde{\tau}$ and q>0 the act of default bears a cost in terms of consumption smoothing and the act of non-default bears a cost in terms of loan repayment. There will be a $\hat{q}(\tau)$ >0 that features default only if the utility upon default increases/declines at a higher/lower rate than the utility upon non-default. That is:

$$\frac{\partial u^D}{\partial q_t} > \frac{\partial u^{ND}}{\partial q_t}$$
$$\Leftrightarrow \frac{2\phi e_t}{h_{t+1}} > \frac{A\phi e_t - (1+r)}{c_1^{ND}}$$

The explanation and the results when $\tau < \tilde{\tau}$ are similar to the above discussion.

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[10] See equation (6).

Conclusion 5.

In an economy with fully funded social security and endogenous debt constraints endogenous heterogeneity (multiple equilibria) will not emerge iff

$$\frac{\partial u^D}{\partial q_t} \leq \frac{\partial u^{ND}}{\partial q_t} \qquad \blacksquare$$

<u>Effects on growth</u>

The growth rate is stationary, that is the fully funded social security ensures balanced growth in every period (Definition 2):

$$\gamma_{t+1} = v + \phi \frac{\mathrm{e}_t q_t}{H_t} = v + \frac{v\left(A\phi(1-2\sqrt{\tau(1-\tau)}) - (1+r)\right)\left(1-2\sqrt{\tau(1-\tau)}\right)}{(1+r)\left(1+\tau-3\sqrt{\tau(1-\tau)}\right) - A\phi - 4A\phi\left((1-\tau)\tau - \sqrt{\tau(1-\tau)}\right)} = \gamma^{BGP} \qquad \forall t \qquad (39)$$

We are going to see in the next section which is the relation between the tax rate and the growth rate since this expression does not generate explicit results.

5. Simulation

Simulation is done in order to derive more explicit results for cases that theory cannot yield. Hence, we do not focus so much on quantitative results but rather on qualitative results, since we concern for the general pattern that each case generates. In section 5.1 we calibrate the model according to data from U.S.A. in order to approach each case as much as possible.

5.1 Calibrating the model

The model's parameters are those that describe final product's technology (A), human capital production (v,φ), the population growth rate (n), the real interest rate (r), the debt-to-income ratio (θ) and the tax rate (τ).

Individuals live for three periods, so we consider that each period lasts 20 years of adult life. Following Hubbard and Judd (1987) the population grows at a rate of 1% per annum or $(1.01^{20}-1)\cdot 100\% = 22\%$ per period. We also assume an annual interest rate equal to 4% or $1.04^{20}-1=1.19$ per period.

The unsecured debt to disposable income ratio over the 1995-1999 period for USA was 8.4% according to Livshits et al. (2007). Since disposable income over the same period was 72.7% of GDP, the debt-to-income ratio is 6.1%.[11]

Finally, according to the data of National Longitudinal Survey of Youth (NLSY), adults devote 3.1 hours per weekday on average[12], so we assign a steady-state value for time devoted to education equal to 0.13.

The other variables (v, φ, A) are computed such that, with an unfunded social security tax τ=0.09, young face binding exogenous constraints in the steady state. Table 3 contains the values of the calibrated model.

Parameter values	**Target data**
A = 3	e = 0.13
v = 0.82	θ = 0.061
φ = 2.4	r = 1.19
	n = 0.22

[11] Andolfatto, Gervais (2008)

[12] Data include individuals, ages 15 to 49, who were enrolled full time at a university or college. Data include non-holiday weekdays and are averages for 2003-07.

5.2 Unfunded social security and growth

In this section we are ready to directly compare unconstrained and constrained cases and find whether a maximum growth rate can be attained.

5.2.1 Unconstrained economy

According to section 4.1.1, two steady state values for z arise. Hence, there are two balanced growth paths and we can explore their characteristics. Substituting the benchmark parameters in eq.(18) we take the following figures that depict the balanced growth rate as function of the social security tax.[13]

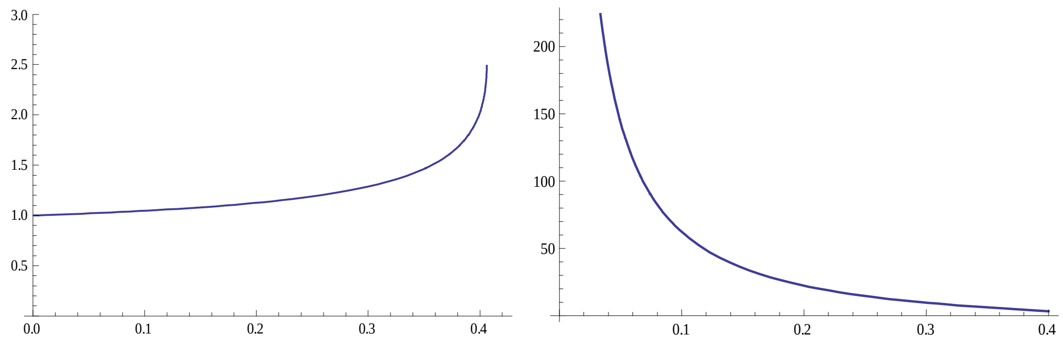


Fig.2 The unconstrained balanced growth as a function of the social security tax

(Left: low steady state z_1^*, right: high steady state z_2^*)

The two steady states generate opposite results regarding the response of the balanced growth rate to changes in the social security tax rate. The growth patterns depicted above are the same as the respective patterns of debt-to-income ratios. The left figure features the low steady state z_1^* (negative root of eq.17b) and confirms studies concluding that increasing tax induces people to invest more in human capital thus enhancing long-run growth. On the other hand, the right figure stands for the high steady state z_2^* (positive root of eq.17b) and shows that the growth rate will decline if the tax rate increases. Two opposite forces, which determine the net lifetime

[13] We must ensure the existence and non-negativity of the steady state z. According to the assumptions of Lemmas 1 and 2, steady-state values for z exist and are positive for tax rates lower than a $\tau_{\max}$ (here 0.4), so we confine to this range of tax rates.

income, drive optimal education expenditure (thus z) when tax rate increases; the decline in after-tax income encourages investment in education and the increased social security benefits induce agents to cut education expenditure. When education expenditure q is low the former effect dominates the latter and when q is high the opposite holds.

More interestingly, the low equilibrium debt-to-income ratio yields low long-run growth while the high equilibrium ratio drives economy to a high-growth path.[14] This means that there will be "convergence clubs" of economies[15]; those which have a very low initial level of debt-to-income ratio will experience low long-run growth while those initially with high debt-to-income ratios will balance on a high-growth path.

5.2.2 Exogenous debt constraints

Firstly, we must ensure that the optimal choice of time devoted to education will be non-negative. The lower bound of debt-to-income ratio θ (eq.24) that guarantees the above assumption is:

$$\theta > \underline{\theta} = 0.06$$

In addition, the steady-state values for the time spent on education must lie between [0,1]. Applying the calibrated parameters to eq.(25), the steady-state value for e is proved to be in the above range for any $\theta > \underline{\theta}$ and for any $\tau < \tau_{\max} (= 0.4)$.

According to the theoretical part, when exogenous debt constraints bind, there are two critical values for θ that determine the relation between tax rate and growth (table 1). We recall it below.

		θ_1^*		θ_2^*	
$\frac{\partial \hat{e}}{\partial \tau}$	$-$	0	$+$	0	$-$

Table 1: Critical values of the debt-to-income ratio

[14] We should remark that the characterizations "low" and "high" used in this section may be misleading. They are used only for discriminating the two equilibria when referring to them. In fact, the first equilibrium features from null to high growth and the second equilibrium features extremely high growth.

[15] The term is borrowed from Azariadis, Kaas (2004).

The above rule with respect to the debt-to-income ratio applies only when the exogenous constraint binds. However, not for all tax rates the constraint binds. For this reason, in order to determine the optimal growth curve as a function of the tax rate we should compare the constrained with the unconstrained level of education expenditure q. When there is an exogenous constraint the optimal allocation will be the one with the lowest value of q. That is:

$$q_t = \min\{q^*, \hat{q}\} \quad (40a)$$

where $\hat{q} = \theta A H_t$.[16]

We can transform the above decision rule in terms of the auxiliary variable z defined before as the debt-to-income ratio $z_t = \frac{q_t}{AH_t}$. Dividing eq.(40a) with AH_t we take:

$$z_t = \min\{z^*, \hat{z}\} \quad (40b)$$

where $\hat{z} = \theta$.

Hence, the balanced growth rate under exogenous debt constraints will be given according to eq.(18) if $z^*_{1,2} < \hat{z}$ and according to eq.(26) if $z^*_{1,2} > \hat{z}$. However, since $\hat{z}$ is a function of θ the above comparisons give different results for different values of θ. In addition, we should note that θ is a stationary debt-to-income ratio thus it is also the value of the initial debt-to-income ratio of period 0, z_0. Four cases arise with respect to θ and they are depicted in figure 3. Figure 3 shows the debt-to-income ratio as a function of the social security tax rate.

- If $\theta < \theta_1$ (fig.3(a)), initial $z_0 < z_1^*$ and the economy will balance at the low steady state z_1^*. Comparing the constrained with the unconstrained z, we observe that the exogenous constraint binds ($z_1^* > \hat{z}$) for any $\tau \in [0, \tau_{\max}]$. Thus $\hat{z}$ prevails as the equilibrium debt-to-income ratio.

[16] For consistency with the notation we denote unconstrained allocations of q and z with an asterisk (*) and constrained allocations with a hat (^).

- If $\theta \in (\theta_1, \theta_2)$ (fig.3(b)), $z_1^* < z_0 < z_2^*$ for $\tau < \tau^*$ and $z_0 < z_1^*$ for $\tau > \tau^*$. As a result, for $\tau < \tau^*$ the economy will balance at the high steady state z_2^* since it is stable and for $\tau > \tau^*$ the economy will balance at the low steady state z_1^*. Hence, we take the equilibrium z will be given according to the following rules:

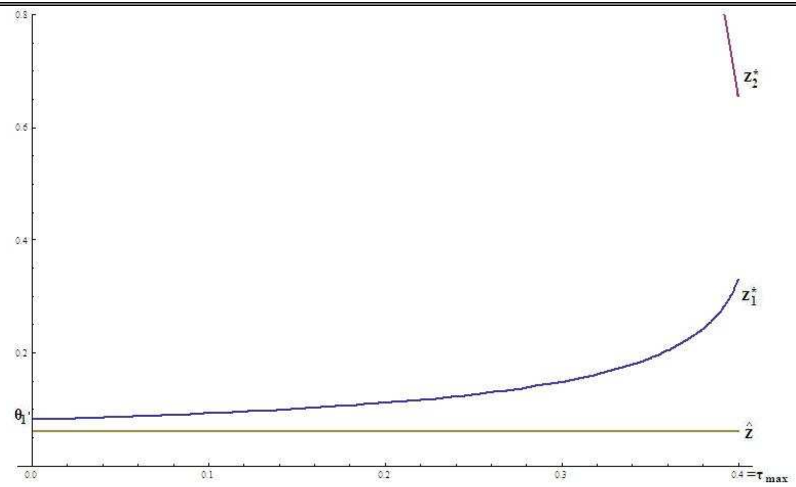
$$z_t = \min\{z_2^*, \hat{z}\} \text{ for } \tau < \tau^*$$

$$z_t = \min\{z_1^*, \hat{z}\} \text{ for } \tau > \tau^*$$

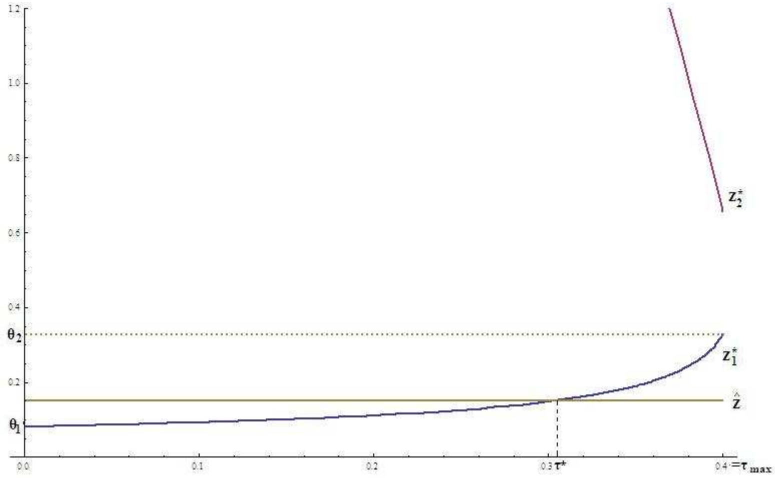
  It is obvious that $\hat{z}$ prevails in both rules according to figure 3(b).

- If $\theta \in (\theta_2, \theta_3)$ (fig.3(c)), in the initial period $z_1^* < z_0 < z_2^*$ for any $\tau \in [0, \tau_{\max}]$ the economy will balance at the high steady state z_2^* since it is stable. Comparing the constrained with the unconstrained z, we observe that the exogenous constraint binds ($z_2^* > \hat{z}$) for any $\tau \in [0, \tau_{\max}]$. Thus $\hat{z}$ prevails as the equilibrium debt-to-income ratio.

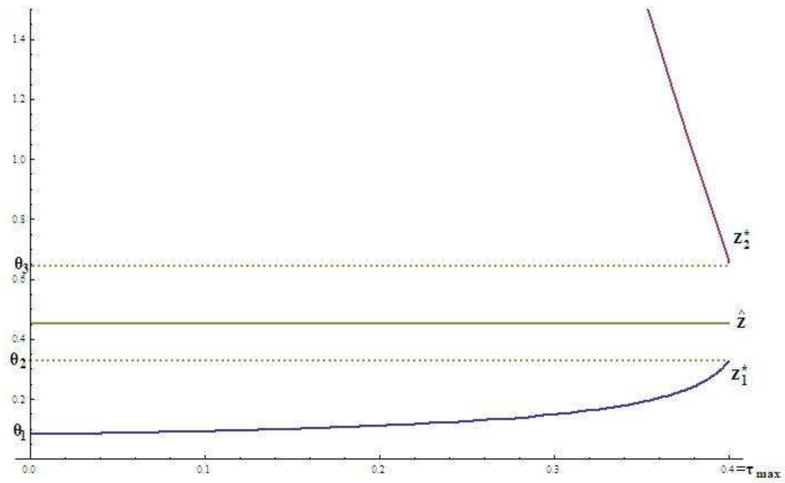
- If $\theta > \theta_3$ (fig.3(d)), initial $z_0 > z_2^*$ and the economy will balance at the high steady state z_2^*. Comparing the constrained with the unconstrained z, we observe that the exogenous constraint binds ($z_2^* > \hat{z}$) for $\tau < \hat{\tau}$ and does not bind ($z_2^* < \hat{z}$) for $\tau > \hat{\tau}$. Hence, the equilibrium debt-to-income ratio will be $\hat{z}$ for $\tau < \hat{\tau}$ and z_2^* for $\tau > \hat{\tau}$.



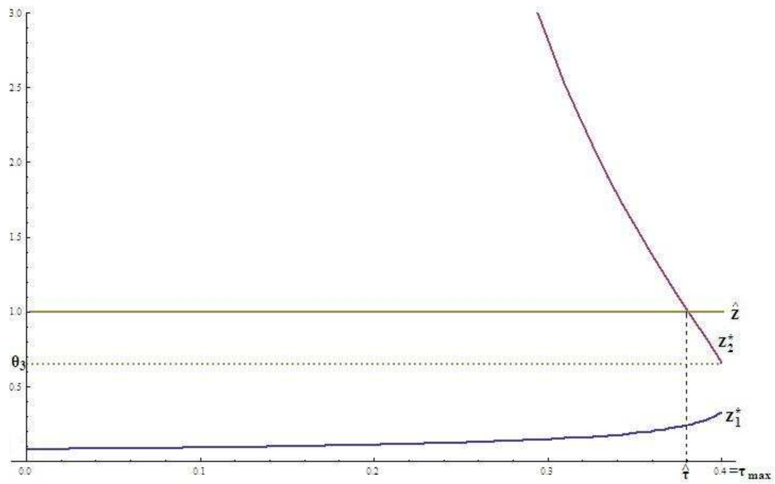
(a) $\theta < \theta_1$



(b) $\theta \in (\theta_1, \theta_2)$



(c) $\theta \in (\theta_2, \theta_3)$



(d) $\theta > \theta_3$

Fig.3 The equilibrium debt-to-income ratio as a function of the social security tax under exogenous debt constraints

Recapitulating from all the cases above, we conclude that the equilibrium debt-to-income ratio equals:

$$\hat{z} \quad \begin{cases} \text{for } \theta < \theta_3 \text{ and } \forall\,\tau \\ \text{for } \theta > \theta_3 \text{ and } \tau < \hat{\tau} \end{cases}$$

$$z_2^* \qquad \text{for } \theta > \theta_3 \text{ and } \tau > \hat{\tau}$$

Taking the above results and table 1 into account, we can derive the pattern of the balanced growth rate. We discriminate two cases according to the table 1:

- For $\theta < \theta_1^*$ and $\theta > \theta_2^*$, the equilibrium growth rate will always be a decreasing function of the tax rate. The growth-maximizing tax rate is zero.
- For $\theta \in (\theta_1^*, \theta_2^*)$ the equilibrium growth rate will be increasing for $\tau < \hat{\tau}$ and decreasing function of the tax rate for $\tau > \hat{\tau}$. That is growth will have an inverted-U shape and there is an interior maximum growth at $\tau = \hat{\tau}$.

According to the calibrated parameters the first case arises and maximum growth will be attained at θ=1 and $\tau = 0$. At this point maximum growth equals 6.07.

Under binding exogenous constraints and for a wide range of values for the parameters, it is computed that

$$\frac{\partial \gamma^{BGP}}{\partial \theta} > 0$$

This means that economies with undeveloped financial markets (low θ) experience low long-run growth while financial deepening (high θ) relates to higher growth.

These results support those of De Gregorio (1996) who shows that increasing borrowing constraints harm growth. However, we should make two remarks; first, De Gregorio’s results come from a general equilibrium framework and secondly that we include both time and resources as inputs in the human production process. The latter means that an increase in θ encourages agents to borrow more thus boosting human capital formation and growth. De Gregorio assumes only time as an input and concludes that time for education increases in response to increases in θ. In contrast, in our model the response of optimal time devoted to education to increases in θ is ambiguous.

5.2.3 Endogenous debt constraints

Applying the benchmark parameters to eq.(35) we derive three steady-state values for the auxiliary variable x. Debt-to-income ratio is trivially derived from eq.(33a) as:

$$z_t = \frac{q_t}{AH_t} = \frac{x_t}{Ae_t}$$

Hence, there are three steady states for z and three discrete balanced growth paths as functions of the tax rate which are depicted in figure 4. Figure 4(a) features economies with very low initial financial development (low initial debt-to-income ratio) thus low equilibrium ratio. In other words, creditors cannot sufficiently ensure the repayment of loans and they provide suboptimal level of credit. Figures 4(c) features economies starting with very high debt-to-income ratio thus converging to a high equilibrium level and experiencing high long-run growth. This means that credit markets are well-developed to provide greater amount of credit. Economies with intermediate initial debt-to-income ratios may converge to any of the three cases depending on the stability of the equilibria.[17] However, in general we conclude that economies initially with missing financial markets experience low long-run growth forever whereas economies with well-developed financial markets will balance at high growth. In other words, there is a strong positive relation between financial deepening and long-run growth. The result that under endogenous debt constraints there are "convergence clubs" of economies was initially drawn by Azariadis and Kaas (2004).

Furthermore, social security harms growth for economies that balance at high growth (equilibria $\hat{z}_2$ and $\hat{z}_3$). This result is expected from the nature of the rationality constraints; as tax rate increases so do social security benefits thus inducing agents to default easier since consumption smoothing improves. The fact that agents default easier for higher tax rates makes constraints bind more tightly and the debt-to-income ratio falls. In contrast, in economies balanced at the low steady state, debt-to-income ratio (and growth) has a hump-shaped pattern in response to the tax rate. This pattern seems to be distorted from what rationality constraints dictate due to implications of the unfunded pension system. In fact, it can be viewed as U-shaped pattern starting from tax rate about 0.4 and ending to 1 thus confirming

[17] The difference equation of z was implicitly derived as it is a complicate expression. Thus we cannot examine the stability of equilibria.

rationality constraints theory. We explain in detail this approximately U pattern in the section of the fully funded scheme.

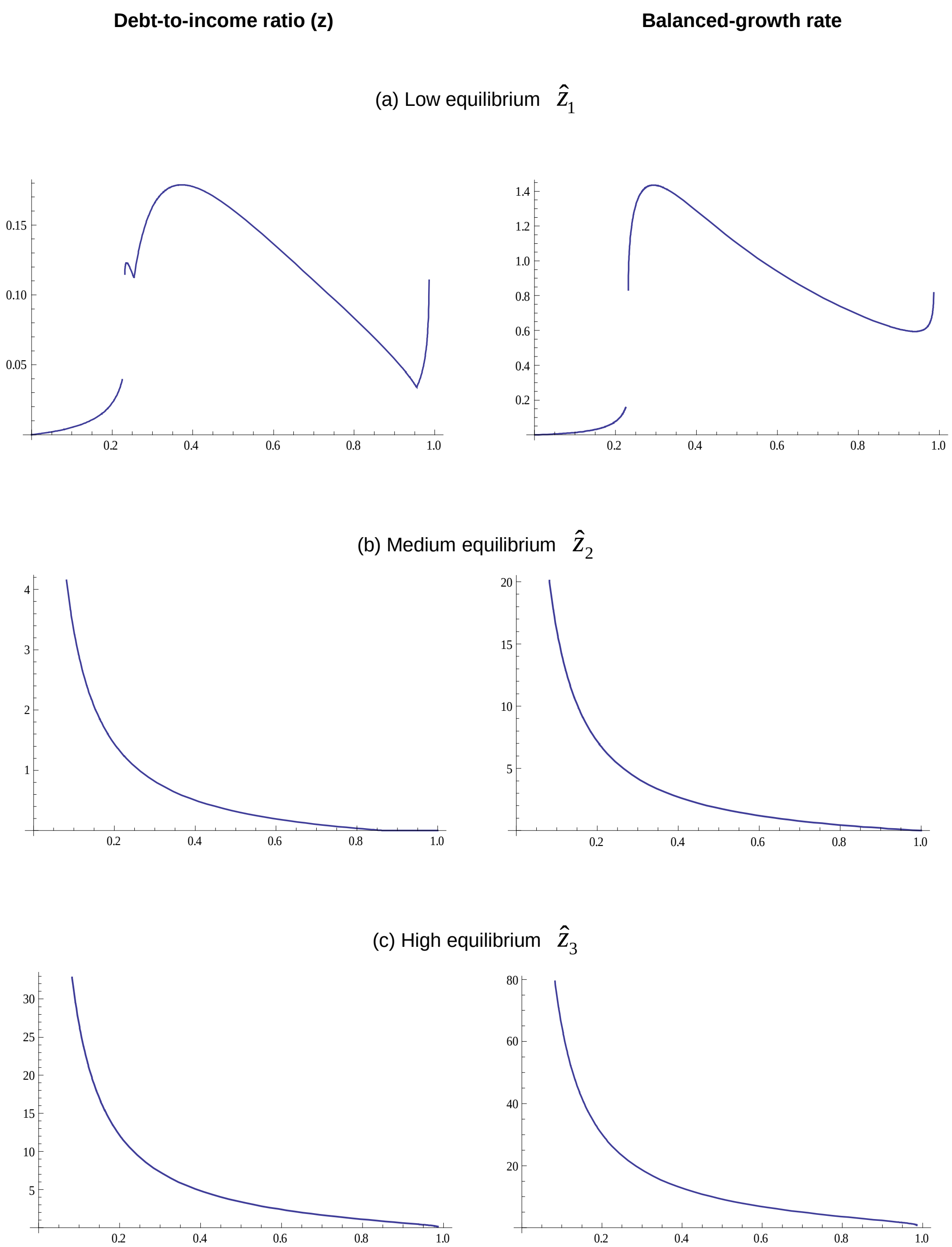


Fig.4 Debt-to-income rations and balanced-growth rates
under binding endogenous debt constraints

However, as noted before constraints do not bind for any level of the tax rate. Thus when there are constraints the optimal allocation will be the one with the lowest value for education expenditure q (or equally z). We also do this task in order to derive the growth-maximizing regime.

Applying the benchmark parameters to the optimal allocations for z and e we obtain three discrete steady-state values for z and e, $\hat{z}_{1,2,3}$ and $\hat{e}_{1,2,3}$. Six cases arise depending on the initial value of z since there are two unconstrained and three endogenously constrained steady states for z. Optimal constrained z will be given by the following rule:

$$z_t = \min\{z_i^*, \hat{z}_j\} \qquad i = 1,2 \quad j = 1,2,3$$

Table 2 recapitulates the results from comparing unconstrained and constrained z and determining the optimal level of debt-to-income ratio (or equally education expenditure). Examining for different initial values for z we conclude that one of the cases above will never be reached so we exclude it. Cases 1, 3 and 5 will certainly arise. Unfortunately, we cannot ensure what happens with cases 2 and 4 since the difference equation of z is not tractable and does not allow to examine the stability of the constrained equilibria $\hat{z}_{1,2,3}$.

Case 1 $(z_1^*, \hat{z}_1)$	$\tau \in [0, \tau_1)$	$z_1^* > \hat{z}_1$	$z_t = \hat{z}_1$
	$\tau \in [\tau_1, \tau_2)$	$z_1^* < \hat{z}_1$	$z_t = z_1^*$
	$\tau \in [\tau_2, \tau_{\max}]$	$z_1^* > \hat{z}_1$	$z_t = \hat{z}_1$
Case 2 $(z_1^*, \hat{z}_2)$	$\tau \in [0, \tau_{\max}]$	$z_1^* < \hat{z}_2$	$z_t = z_1^*$
Case 3 $(z_2^*, \hat{z}_1)$	$\tau \in [0, \tau_{\max}]$	$z_2^* > \hat{z}_1$	$z_t = \hat{z}_1$
Case 4 $(z_2^*, \hat{z}_2)$	$\tau \in [0, \tau_{\max}]$	$z_2^* > \hat{z}_2$	$z_t = \hat{z}_2$
Case 5 $(z_2^*, \hat{z}_3)$	$\tau \in [0, \tau_3)$	$z_2^* > \hat{z}_3$	$z_t = \hat{z}_3$
	$\tau \in [\tau_3, \tau_{\max}]$	$z_2^* < \hat{z}_3$	$z_t = z_2^*$

Table 2: Optimal z under endogenous debt constraints

The five cases of the growth rate as a function of the tax rate are depicted in figure 5. Instead of γ which is the notation for growth, we purposely denote the curves with their respective z in order to make the match between table 2 and figure 5 more obvious.

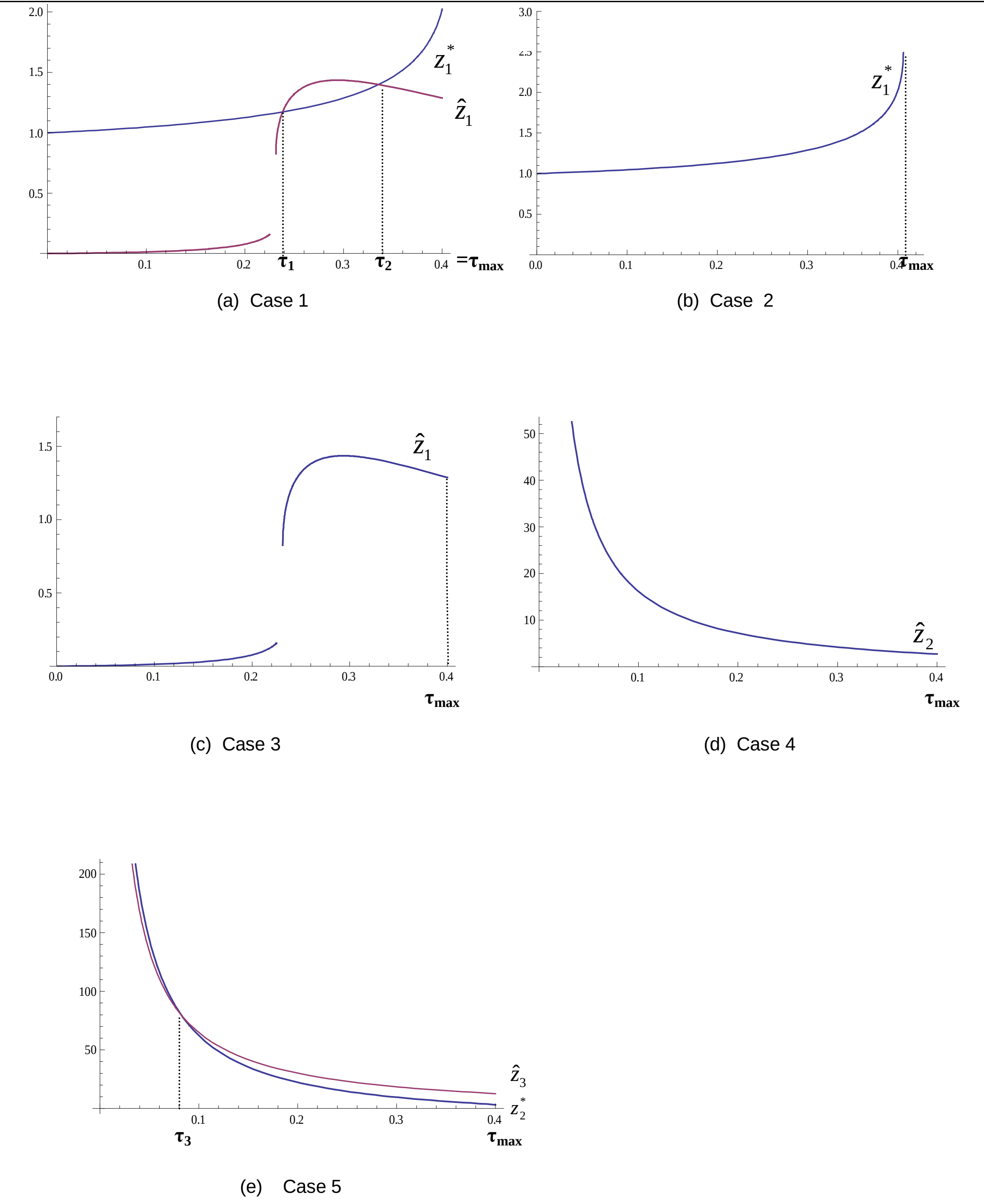


Fig.5 The balanced-growth rate under endogenous debt constraints

According to figure 5(a), **case 1** results to an inverted-U growth curve consisting of the unconstrained curve for tax rates $\tau \in [\tau_1, \tau_2)$ and the constrained curve for tax rates $\tau \in [0, \tau_1)$ and $\tau \in [\tau_2, \tau_{\max}]$. The growth rate is increasing for tax rates lower than τ_2 and decreasing for higher tax rates. Hence, the there is an interior growth-maximizing tax rate equal to τ_2 (kink point).

Case 2, depicted in figure 5(b), generates the unconstrained low level of debt-to-income ratio z_1^*, which means that endogenous constraints do not bind for any $\tau<\tau_{max}$. In other words, the unconstrained q is always lower than the constrained q which means that agents will always choose to borrow the optimal unconstrained level. Hence, the balanced growth rate is monotonically increasing in the social security tax. The growth-maximizing tax rate is the upper limit τ_{max}.

Case 3 in figure 5(c) yields a pattern in which the constrained low-level allocation $\hat{z}_1$ prevails over the unconstrained high-level allocation z_2^* for any $\tau< \tau_{max}$. The long-run growth rate follows an inverted-U route according to the constrained growth curve and there is an interior growth-maximizing tax rate.

Case 4 generates the constrained intermediate steady state $\hat{z}_2$ for which increasing social security contributions harm growth. Hence, long-run growth is a monotonically decreasing function of the social security tax (figure 5(d)) and the growth-maximizing tax rate is zero.

Case 5, described in figure 5(e), results to a crooked curve. Up to $\tau= \tau_3$ the curve is identical to the constrained growth curve corresponding to the high debt-to-income ratio $\hat{z}_3$. For higher tax rates the curve becomes identical to the unconstrained growth curve corresponding to the high debt-to-income ratio z_2^*. Hence, long-run growth is a decreasing function of the tax rate and the growth-maximizing tax rate equals to zero.

Table 3 shows the minimum and maximum growth rates generated by each case using the calibrated parameters.

Case 1	0 (τ=0)	1.37 (τ=0.33)
Case 2	1 (τ=0)	2.5 (τ=0.4)
Case 3	0 (τ=0)	1.45 (τ=0.28)
Case 4	2.69 (τ=0.4)	∞ (τ=0)
Case 5	3.21 (τ=0.4)	∞ (τ=0)

Table 3

According to figure 5 we observe that for economies balanced at high debt-to-income ratio (equally growth), i.e. cases 4 and 5, social security harms growth. On the other hand, economies balanced at the low level of debt-to-income ratio (thus growth) benefit from a social security tax.

Comparing exogenous with endogenous constraints model under the specific parameterization we observe that *exogenous constraints* can generate a long-run growth rate 6.07 as the best scenario when $\theta=1$ and $\tau = 0$. However, this case that θ and τ take corner values is a very special and rather unrealistic case. On the other hand, under *endogenous constraints* a wide and feasible range of the tax rate can guarantee high long-run growth rates for economies with initial well-developed financial markets (cases 4 and 5).

In general, we observe that *exogenous constraints* can generate a decreasing or inverted-U growth curve depending on the exogenous debt-to-income ratio θ. Similarly, *endogenous constraints* yield an increasing, decreasing or inverted-U growth curve depending on the initial value of debt-to-income ratio z. If case 2 does not emerge, obviously both constraints setups suggest that too high social security contributions harm growth.

More interesting results are derived from the comparison between economies with perfect financial markets and debt-constrained economies. As discussed before the growth-maximizing tax rate for unconstrained economies is zero or τ_{max} depending on the "convergence club" that an economy is placed. For low-growth economies a τ_{max} tax rate is appropriate while for high-growth economies social security harms growth. On the other hand, in debt-constrained economies a zero or an interior level of tax rate turns out to be the most appropriate considering growth.

In addition, improvements in financial markets (higher θ) result to higher long-run growth when there are exogenous debt constraints. Similarly, with endogenous constraints, a financial reform that would lead to perfect credit markets would enhance growth (comparison between figures 2 and 4). This result confirms previous studies that find a monotonically increasing relation between financial development and long-run growth.[18] In contrast, De la Croix and Michel (2007) conclude to a hump-shaped pattern.

[18] Aghion, Howitt, Mayer-Foulkes (2005)

5.3 Fully funded social security and growth

5.3.1 Unconstrained economy

Applying the benchmark parameters to eq.(20) we take the growth rate as a function of the social security tax, which is depicted in figure 6.

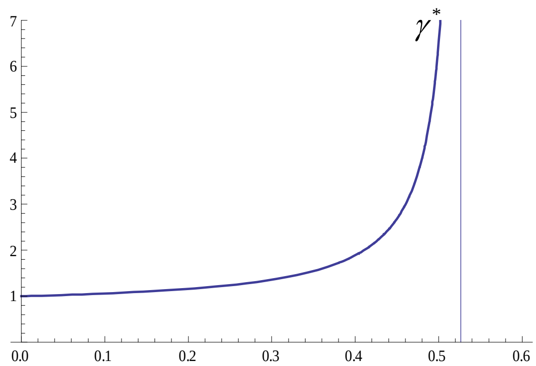


Fig.6 The unconstrained balanced-growth rate as a function of the social security tax

We observe that, with fully funded social security, increasing contributions (tax rate) benefit long-run growth.[19] This result was expected from the theoretical analysis (section 4.1.2).

5.3.2 Exogenous debt constraints

Conclusion 3 in section 4.2.2 gives a quite clear view of how long-run growth reacts to changes in the tax rate. Under this parameterization the sufficient condition of Lemma 4 is satisfied, hence the constrained growth rate will certainly be a decreasing function of the social security tax.

However, as noted before, when there are constraints they do not necessarily bind. As a consequence, we are not ex ante certain for the behavior of the equilibrium growth rate. More precisely, for some tax rates the constraint may not bind and the growth rate will be an increasing function of the tax rate. For other tax rates the constraint will bind and the growth rate will have negative slope. Hence, we have to

[19] An unconstrained balanced-growth rate exists for tax rate up to a τ_{max} (here 0.52). Hence, in our analysis we are going to confine to this range of tax rates.

integrate the unconstrained and the constrained growth curves in a diagram and determine the optimal route of the growth rate as the tax changes.

The optimal allocation will be the one with the lowest value for education expenditure q (or equally debt-to-income ratio z):

$$z_t = \min\{z^*, \hat{z}\}$$

From eq.(19) the unconstrained expenditure is:

$$q_t^* = \frac{AvH_t}{\eta(\tau)} \Rightarrow z_t^* = \frac{v}{\eta(\tau)}$$

From eq.(22a) the constrained expenditure is:

$$\hat{q}_t = \theta AH_t \Rightarrow \hat{z}_t = \theta$$

Obviously, θ is a crucial parameter since it affects the qualitative results regarding the route of the growth rate. Put it differently, θ determines the intersection point of the constrained and unconstrained curve. There are two cases:

- If $\theta < \hat{\theta}$ (here 0.08), $\hat{z} < z^*$ for any tax rate and the exogenous constraint will always bind. Hence, the growth rate has a decreasing behavior as tax rate increases (figure 7(a)) and the maximum growth rate is attained for tax rate equal to zero.
- If $\theta > \hat{\theta}$, for low tax rates the constraint will not bind and for high rates the constraint becomes binding. Thus the balanced growth curve is initially increasing and then decreasing in tax (figure 7(b)). In this case there is a kink point which yields the maximum growth rate.

The growth-maximizing tax rate will be either zero or interior $\tau \in [0, \tau_{\max}]$ depending on θ. Furthermore, financial deepening (higher θ) results to higher long run growth according to figure 7. This result is expected from the theoretical part since time and resources devoted to education increase with θ[20]. However, as θ increases above 0.08 the tightness of borrowing constraints does not affect long-run growth for some tax rates since the optimal growth curve is identical to the unconstrained curve

[20] See section 4.2.2.

which is irrelevant to θ. It is easily concluded that maximum growth in a constrained economy will always be lower than that of an unconstrained economy (compare figures 6 and 7).

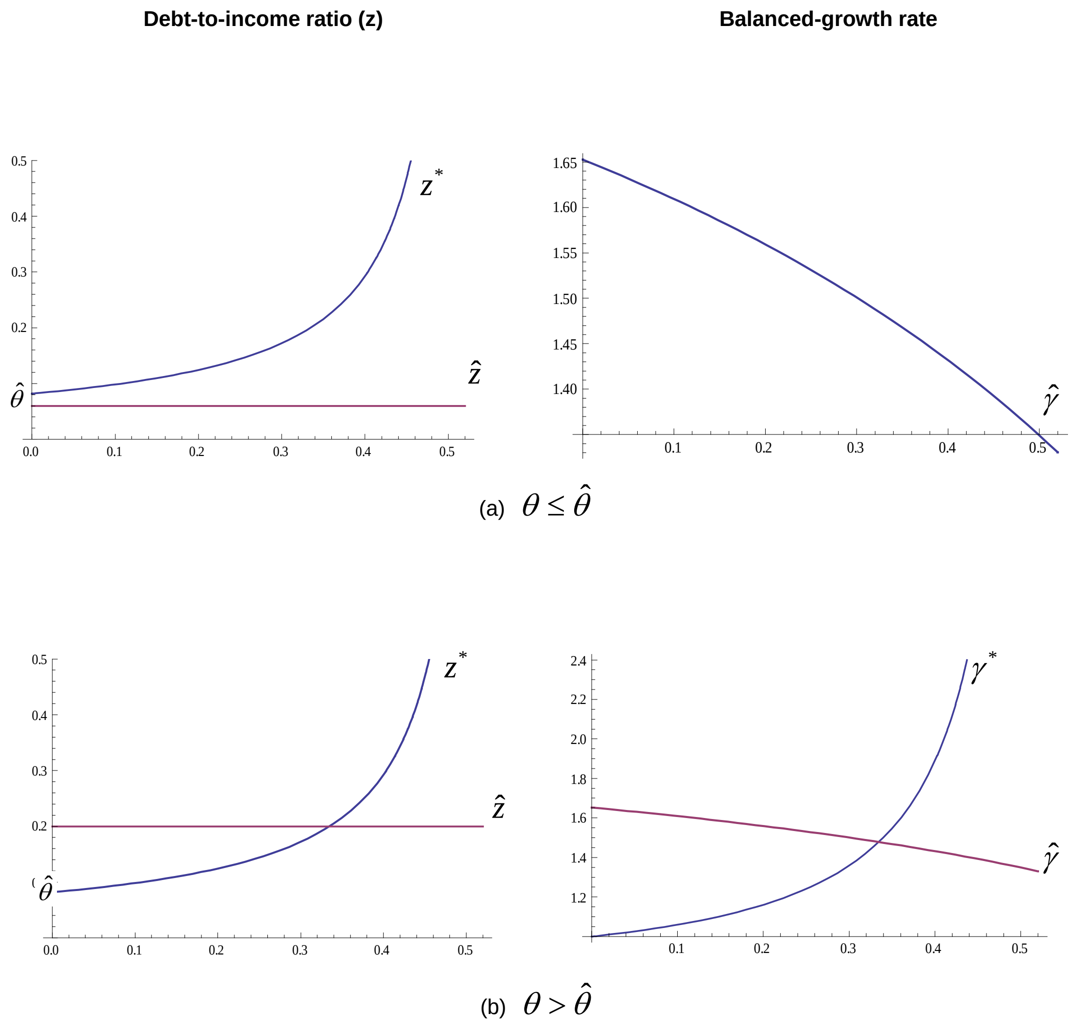
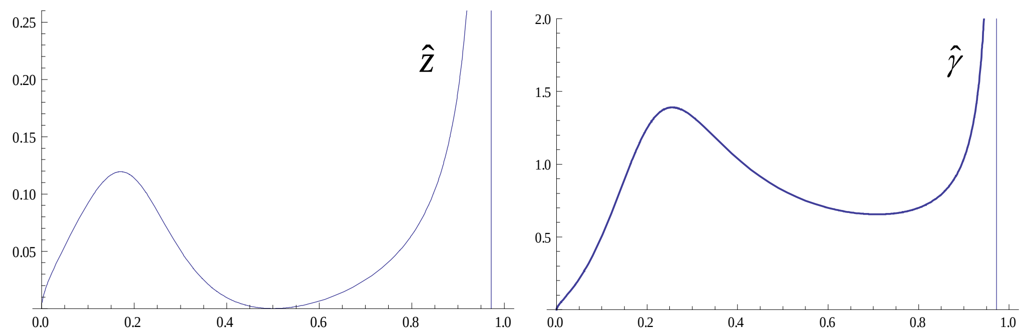


Fig.7 Debt-to-income ratio and balanced-growth rate as functions of the social security tax under exogenous constraints

According to the calibrated model, maximum growth rate can potentially reach 5.1. This is the case when θ=1 and tax rate equals to the interior kink point 0.49 (figure 7(b)). On the other hand, growth cannot be lower than 0.4 which is the case when θ=0 and τ=0.52.

5.3.3 Endogenous debt constraints

The pattern that debt-to-income (and growth) follows is consistent with what rationality constraints dictate. In order to understand it better in figure 8(a) we present the debt-to-income ratio as a function of the tax rate for the whole range of τ. We observe that the pattern of debt-to-income ratio in response to tax approaches a U shape. For low tax rates (≈0.2) consumption smoothing is bad under autarky (second-period consumption exceeds third-period consumption) thus inducing agents not to default. Similarly for very high tax rates consumption smoothing is bad under autarky (third-period consumption exceeds second-period consumption) thus again inducing agents not to default. Hence, for low and high levels of the tax rate an effective financial market can exist providing high levels of credit. However, for intermediate levels of the tax rate consumption smoothing is relatively good under autarky and agents have strong incentive to default. The latter make creditors restrict the supply of loans. For the above reasons, low and high tax rates feature high debt-to-income ratios while intermediate tax rates feature very low debt-to-income ratios thus generating a U-shaped pattern. Long-run growth in figure 8(b) follows a similar pattern to the debt-to-income ratio.



(a) (b)

Fig.8 (a) Debt-to-income ratio and (b) balanced growth rate
as functions of social security tax

From eq.(38b)

$$z_t = \frac{q_t}{AH_t} = \frac{\left(1-2\sqrt{\tau(1-\tau)}\right)\left(\tau-\sqrt{\tau(1-\tau)}\right)\cdot v}{(1+r)\left(1+\tau-3\sqrt{\tau(1-\tau)}\right)-A\phi-4A\phi\left((1-\tau)\tau-\sqrt{\tau(1-\tau)}\right)}$$

Applying the benchmark parameters, $z_t^* > \hat{z}_t \quad \forall \tau$ (similar to fig.9(a)). Hence, the endogenous constraint binds for every τ and the growth rate is given by the constrained growth curve eq.(39) which is depicted in figure 8(b). The growth-maximizing tax rate is 0.24 where growth maximum equals to 1.4. However, the benchmark parameters restrict us to a special case and for this reason we are going to consider any case may arise. The unconstrained and constrained z may have four different positions presented in figure 9.

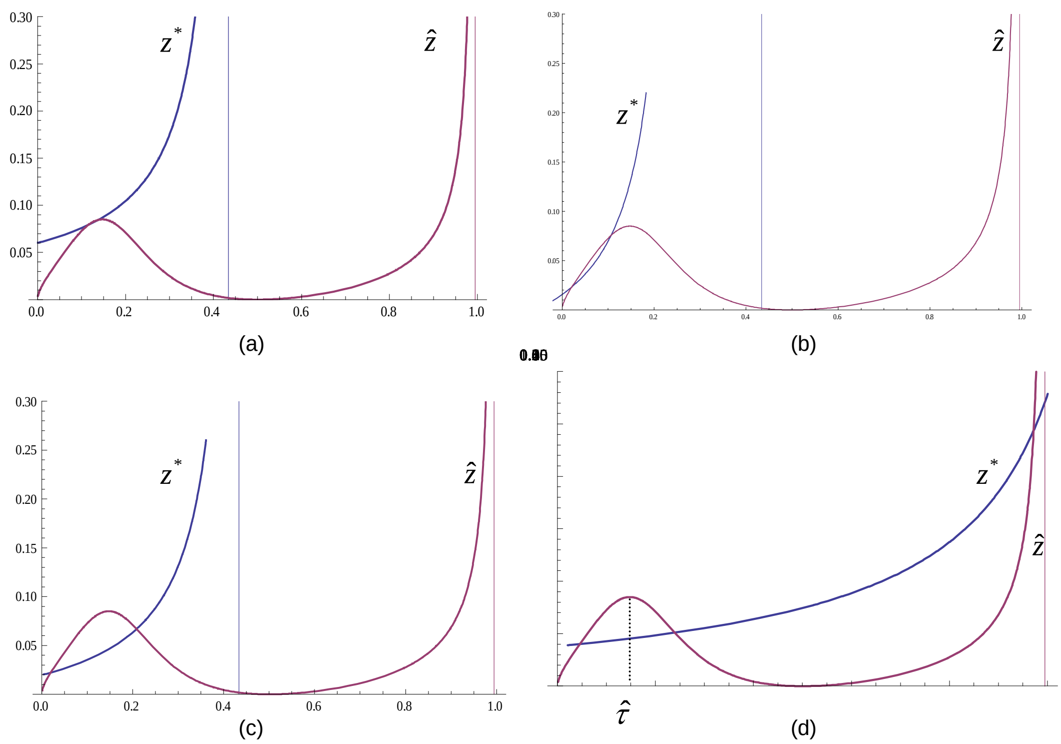


Fig.9 Debt-to-income ratios as functions
of the social security tax under endogenous constraints

According to the above cases, growth may have a monotonically increasing pattern, an inverted-U pattern or the shape of the constrained growth curve (fig.8(b)) depending on the position of τ_{max} (i.e. the ending point of z* curve). This means that maximum growth can be attained at either $\hat{\tau}$ (low peak) or τ_{max}. Furthermore, a financial reform that would lead to perfect financial markets would enhance growth (comparison between figures 6 and 8(b)).

The **comparison between the unconstrained and the constrained cases** under fully funded system is now straightforward. Increasing social security contributions benefit growth when young individuals are unconstrained while they harm growth when there are binding exogenous debt constraints. Finally, with binding endogenous constraints, very high and low tax rates benefit growth since credit markets operate effectively but intermediate levels of tax rate harm growth (according to the U shape).

Furthermore, the growth-maximizing regime for an unconstrained economy is a tax rate equal to the τ_{max}. In an economy with exogenous debt constraints, maximum growth is attained at zero or an interior $\tau \in [0, \tau_{max}]$ tax rate depending on the financial development. Higher degree of financial deepening requires greater social security contributions. For endogenously constrained economies the growth-maximizing tax rate is either $\hat{\tau}$ (low peak) or τ_{max} but not zero. In other words, under endogenous constraints the introduction of social security always benefits growth while this is not the case when there are exogenous constraints.

According to the calibrated model, the growth-maximizing regime for an unconstrained economy is a tax rate equal to 0.52, for an exogenously constrained economy the tax rate equals to 0.49 and for endogenously constrained economy τ=0.24. That is, targeting high long-run growth, exogenous constraints setup requires a higher degree of government intervention. However, more active intervention is sometimes unattractive regarding welfare implications (Hubbard, Judd (1987)). In addition, it is computed that with exogenous constraints long-run growth ranges from 0.4 to 5.1, while with endogenous constraints growth ranges from 0 to 1.4. Hence, endogenous constraints, though more realistic and appropriate to model constraints when there is a government policy, generate more conservative results than exogenous constraints regarding economy's growth potential. Table 4 recapitulates the minimum and maximum values of the growth rate under a fully funded pension system using the benchmark parameters.

	Minimum growth	Maximum growth
No constraints	1 (τ=0)	∞ (τ=0.52)
Exogenous debt constraints	0.4 (τ=0.52, θ=0)	5.1 (τ=0.49, θ=1)
Endogenous debt constraints	0 (τ=0)	1.4 (τ=0.24)

Table 4

5.4 Comparison between social security programs

An additional task is to directly compare the two social security schemes and examine their relative efficiency on the long-run growth.

Let us take first the **unconstrained economy**. As noted before, the unfunded (PAYG) program yields two equilibria, one characterized by low debt-to-income ratio and low growth and the other characterized by high debt-to-income ratio and high growth. On the other hand, the fully funded program results to a unique equilibrium featuring low growth. This means that *the intergenerational dynamics of the unfunded pension system will generate two convergence groups of economies depending on their initial debt-to-income capital while the absence of intergenerational links in a fully funded system will lead to the long-run convergence of all economies.* Furthermore, an unfunded social security tax always benefit growth while unfunded social security may benefit or harm growth depending on the initial situation of an economy; economies with low initial debt benefit from unfunded social security while economies initially with high debt-to-income ratios are harmed. Thus for economies with large amount of credit (high z), the choice of growth regime is crucially depended on the social security scheme and should be carefully chosen.

If there are **exogenous debt constraints**, both social security programs expect either a decreasing or an inverted U-shaped response of the equilibrium growth rate to changes in the tax. Specifically, economies with highly developed financial markets will have an interior growth-maximizing tax rate while financially under-developed (highly constrained) economies will experience maximum growth at zero tax.

Finally, **endogenous constraints** combined with unfunded public pensions generate a monotonically increasing, decreasing or inverted U-shaped growth rate depending on the initial financial development of an economy. For highly developed economies a zero tax is appropriate while for under-developed economies an interior tax is the best regime. On the other hand, the fully funded pension system generates an increasing, inverted-U or an approximately U (fig.8(b)) growth pattern. This means that social security always benefits growth imposing either an interior or τ_{max} tax rate no matter the degree of the initial financial development. Obviously, similar to unconstrained economies, the choice of social security program should be more careful for highly financially developed economies since the two programs yield different growth regimes.

6. Conclusion

In this thesis we examine the effects of social security on long-run growth in debt-constrained economies.

Assuming *unfunded social security,* multiple equilibria (convergence groups of economies) emerge due to the intergenerational dynamics arisen from the nature of the unfunded system. An unconstrained economy with unfunded social security has two equilibria, one featuring low growth and one very high growth. Economies balanced at the low growth benefit from the introduction of social security while for economies with high equilibrium growth social security harms growth. When there are binding exogenous debt constraints the effects crucially depend on financial deepening (debt-to-income ratio); high financial development requires high social security contributions while economies with under-developed financial markets may get harmed from social security. Endogenous constraints result to multiple equilibria thus not giving a clear view of tax effects on growth. However, it is certain that, targeting maximum growth with endogenous constraints, initial financial deepening relates negatively to the social security burden.

Assuming *fully funded pension system* we derive more explicit results. Specifically, in an unconstrained economy increasing social security contributions always benefit growth, while with binding exogenous constraints social security contributions benefit growth up to a point depending on the degree of financial deepening (debt-to-income ratio). If debt constraints become endogenous, fully funded social security always benefits growth.

Finally, employing any constraint setup and social security scheme, financial deepening benefits long-run growth.

The scope of this thesis is confined to partial equilibrium effects assuming a small open economy with constant interest rate. Letting interest rate to be endogenously determined by the equilibrium conditions in the assets market, implications of social security on growth become more complicate. De la Croix and Michel (2007) study the effects of debt constraints on growth in a general equilibrium framework using the interest rate as the key variable. Similarly, analyzing the general equilibrium effects of a social security policy may yield different results from those drawn in this thesis thus giving rise to future research.

Appendix A

❖ *Proof of Lemma 2.*

Firstly, it is straightforward that if the bracket

$$(1-\tau)(1+r-A\phi(1-\tau))+v\tau(1+n)$$

is negative, the positive root of z

$$z_2^* = \frac{-(1-\tau)\left[(1-\tau)(1+r-A\phi(1-\tau))+v\tau(1+n)\right]+\sqrt{D}}{(1+n)(1+r)\tau}$$

is undoubtedly positive.

The negative root of z will also be positive if

$$-(1-\tau)\left[(1-\tau)(1+r-A\phi(1-\tau))+v\tau(1+n)\right]>\sqrt{D}\Rightarrow^{21}$$
$$\Rightarrow(1-\tau)^2\left[(1-\tau)(1+r-A\phi(1-\tau))+v\tau(1+n)\right]^2>D$$

Making some algebraic operations we conclude to the following inequality:

$$(1+n)(1+r)^2(1-\tau)^2 v\tau\left[(1+r)(1-\tau)+v\tau(1+n)\right]>0$$

which always holds true. Hence, the negative root of z has positive value. ■

❖ *Stability analysis of the difference equation (17a).*

One way of checking for stability would be computing the differential $f'(z^*)$. However, it yields an expression with many parameters without an explicit result. For this reason, we do stability analysis approaching the graph of eq.(17a).

The function $z_{t+1}=f(z_t)$ is **continuous in $\mathbf{R}^+$** since the first differential

$$f'(z_t)=\frac{B(\tau)}{v+\frac{1+r}{1-\tau}z_t}-\frac{B(\tau)z_t-C(\tau)}{\left(v+\frac{1+r}{1-\tau}z_t\right)^2}\cdot\frac{1+r}{1-\tau}$$

is well-defined for any $z\in R^+$.

[21] Since both sides of the inequality are positive (Lemma's assumption) we can square them.

The function $z_{t+1}=f(z_t)$ is **monotonically increasing in $\mathbf{R}^+$** iff $f'(z_t)>0 \quad \forall z\in R^+$. Making some algebraic operations in the above inequality we conclude to the following expression:

$$\mathrm{v}B(\tau)+\frac{1+\mathrm{r}}{1-\tau}C(\tau)>0$$

which always holds true since B(τ), C(τ) > 0.

The graph intersects the vertical axis at the point:

$$f(0)=-\frac{C(\tau)}{\mathrm{v}}<0$$

The graph is an **hyperbole** with the following asymptotes:

$$\lim_{z\to-\frac{1-\tau}{1+\mathrm{r}}\mathrm{v}} f(z_t)=-\infty \qquad \lim_{z\to+\infty} f(z_t)=\frac{1-\tau}{1+\mathrm{r}}B(\tau)>0$$

The function $z_{t+1}=f(z_t)$ is **concave in $\mathbf{R}^+$** iff $f''(z_t)<0 \quad \forall z\in R^+$. Making some algebraic operations in the above inequality we conclude to the following expression:

$$2\frac{1+r}{1-\tau}\left(B(\tau)\mathrm{v}+\frac{1+\mathrm{r}}{1-\tau}C(\tau)\right)>0$$

which always holds true since B(τ), C(τ) > 0. ■

Appendix B

$$e_{t+1}\geq 0\Rightarrow e_t\leq\frac{3(1+r)(1-\tau)\mathrm{K}(\tau)-\dfrac{(1+\mathrm{n})\mathrm{v}^2\tau}{\phi\theta A}}{3(1+\mathrm{r})(1-\tau)+\mathrm{v}\tau(1+\mathrm{n})}=\bar{e}$$

$$e_{t+1}\leq 1\Rightarrow e_t\geq\frac{3(1+r)(1-\tau)K(\tau)-\left(1+\dfrac{\mathrm{v}}{\phi\theta\mathrm{A}}\right)\tau\mathrm{v}(1+\mathrm{n})}{3(1+r)(1-\tau)+(\phi\theta A+\mathrm{v})(1+n)\tau}=\underline{e}$$

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